

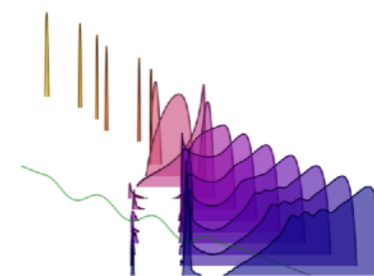
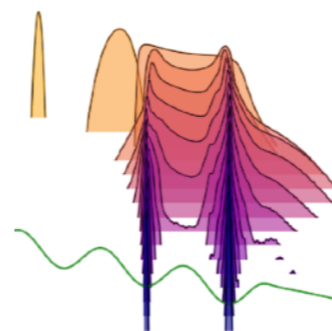
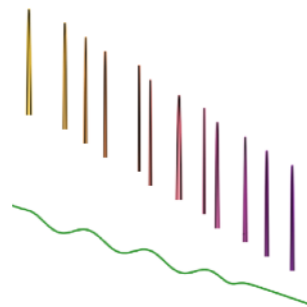
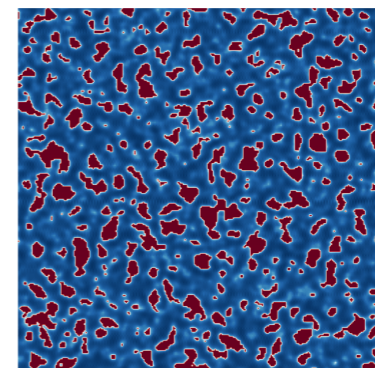
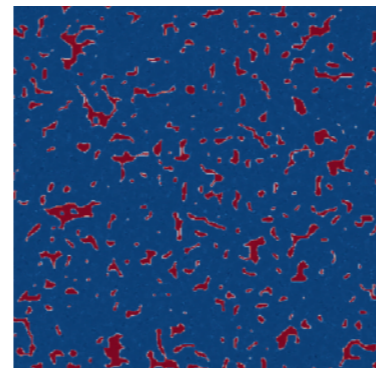
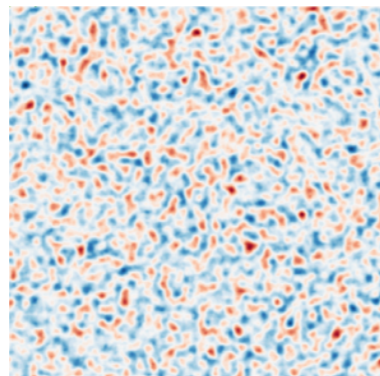
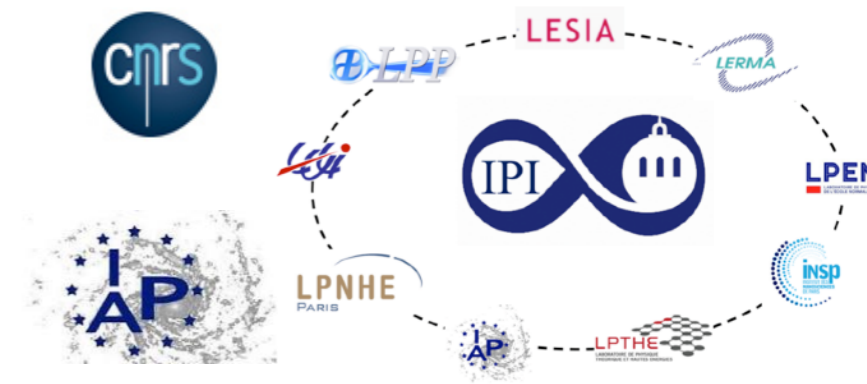
The Inflationary Butterfly Effect



Non-perturbative dynamics from small-scale features

Angelo Caravano (IPI fellow @ IAP, Paris)

collaborators: S.Renaux-Petel, K.Inomata [arXiv:2403.12811](https://arxiv.org/abs/2403.12811)



Roadmap



0) Context and motivation

1) Lattice simulations inflation

AC, E. Komatsu, K. D. Lozanov, J. Weller

arXiv
2102.06378
2110.10695
2204.12874
2209.13616

AC

AC, D. Jamieson, E. Komatsu [in preparation]

2) The Inflationary Butterfly Effect

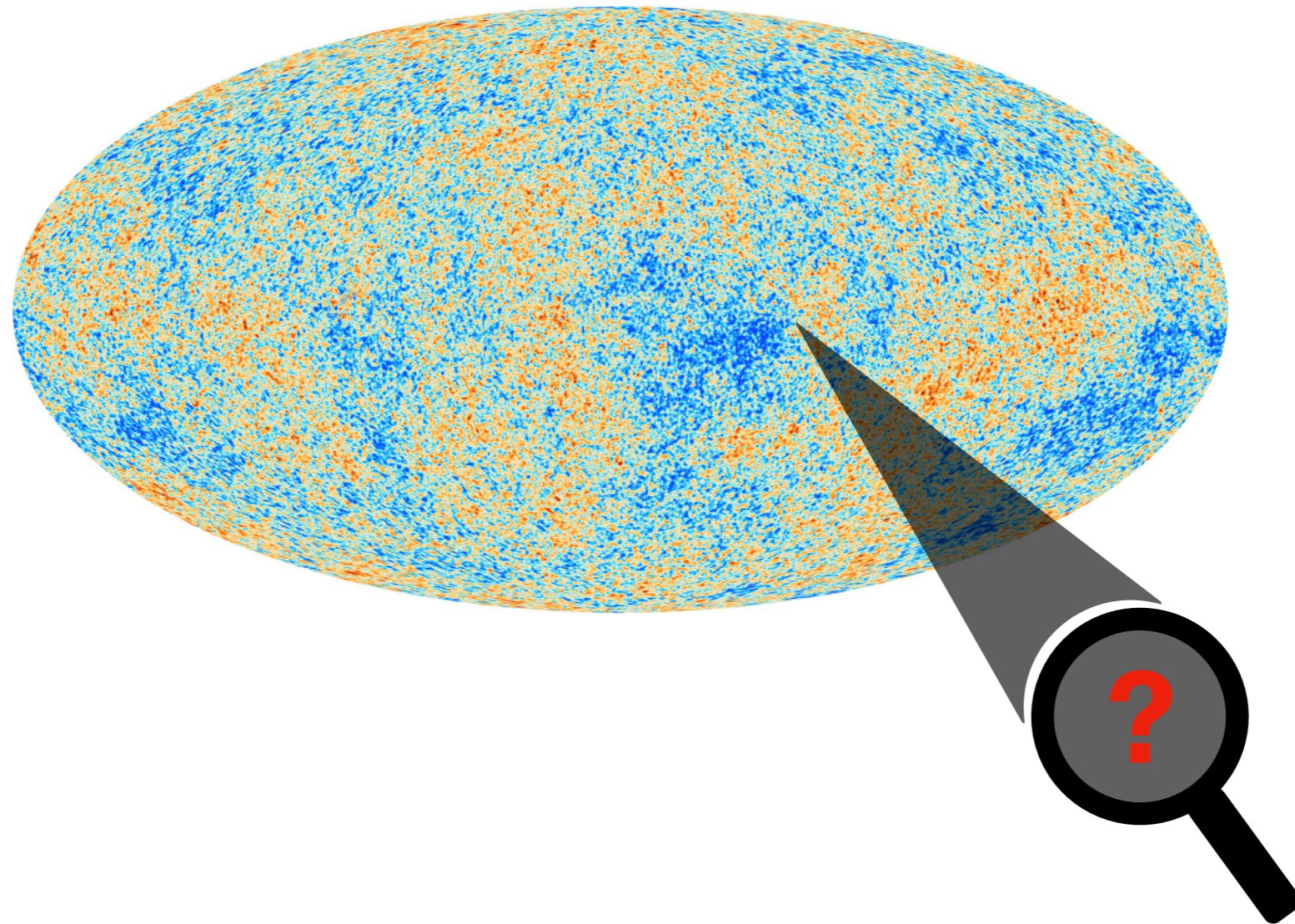
AC, S. Renaux-Petel, K. Inomata

2403.12811

The early Universe at small scales



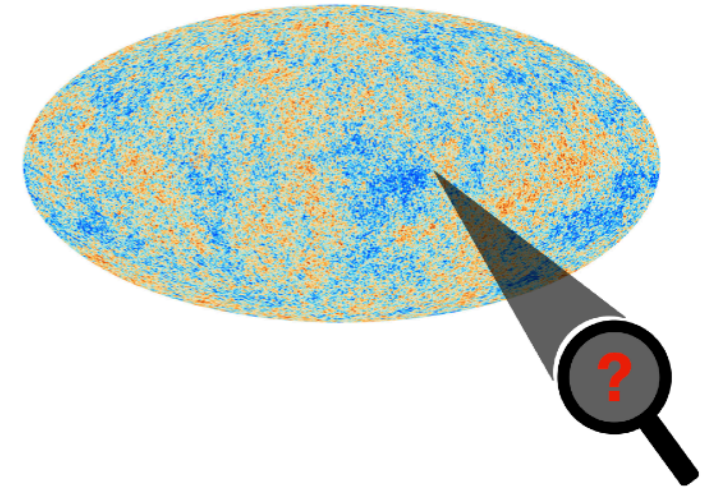
What is the physics of inflation at scales $\lambda \ll \lambda_{CMB}$?



Inflation generates fluctuations at scales $\sim e^{40}$ smaller than CMB scales

Inflation at small scales

What is the physics of inflation at scales $\lambda \ll \lambda_{CMB}$?



Observable thanks to PBH and GWs!

For sizeable effect, however:

$$\mathcal{P}_\zeta \sim 10^{-2} - 10^{-4} \longrightarrow \zeta \sim 10^{-1} - 10^{-2}$$

Possible nonlinear physics?

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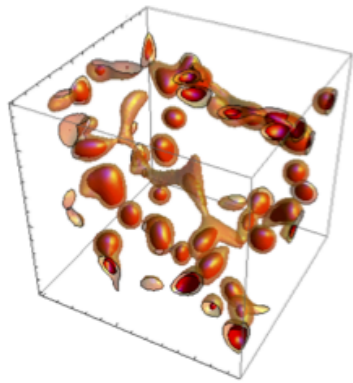
2) The Inflationary Butterfly Effect

AC, S. Renaux-Petel, K. Inomata

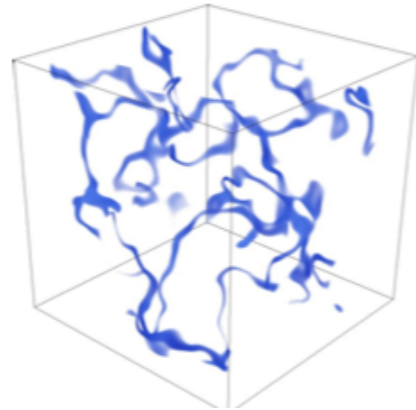
2403.12811

Lattice simulations

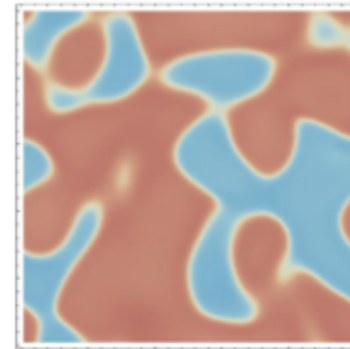
- Numerical tool to study **non-perturbative** cosmological phenomena.
- Examples: **reheating** phase after inflation, cosmological **phase transitions**.



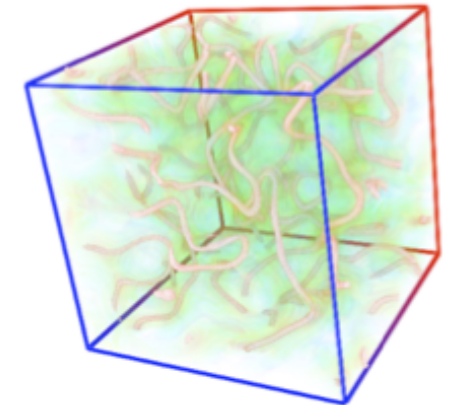
[M. A. Amin, R. Easter, H. Finkel, arXiv:1009.2505]



[A. V. Frolov, arXiv:1004.3559]



[M. A. Amin, J. Fan, K. D. Lozanov, M. Reece, arXiv:1802.00444]



[J. Dufaux, D.G. Figueroa, J. Garcia-Bellido, arXiv:1006.0217]

My goal:

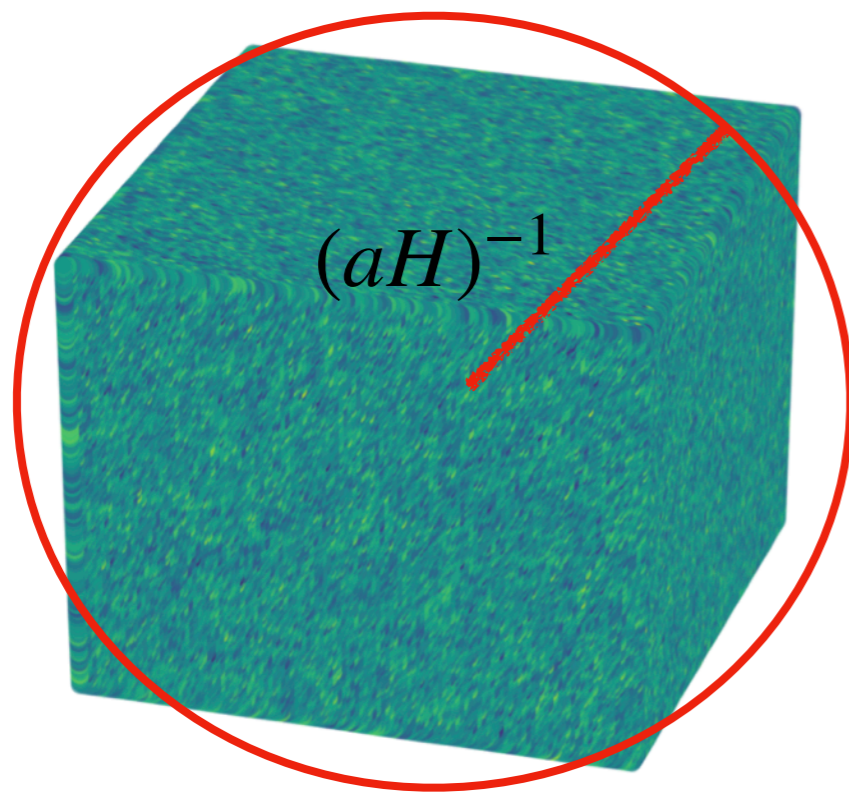
Develop lattice techniques for inflation

AC, E. Komatsu, K. D. Lozanov, J. Weller

AC

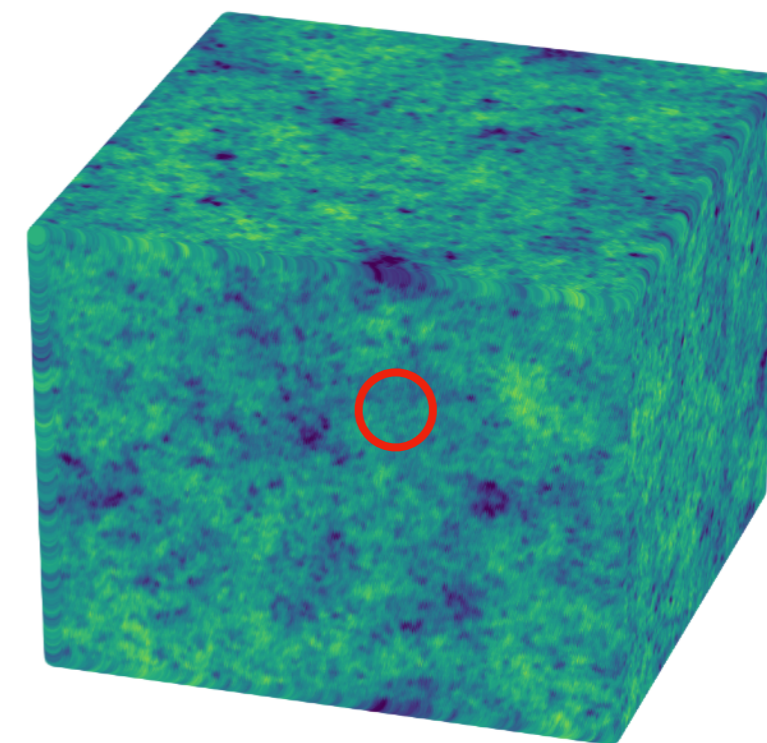
arXiv
2102.06378
2110.10695
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Lattice simulations of Inflation



“sub-horizon” box

Nonlinear
evolution



“super-horizon” box
(frozen)

- Key point: non-perturbative $\phi(\vec{x}, t) \neq \bar{\phi}(t) + \delta\phi(\vec{x}, t)$
- Assumptions: 1) Neglect gravitational interaction fixed metric $ds^2 = a(\tau)(-d\tau^2 + d\vec{x}^2)$
2) Semi-classical approach (neglect quantum tunneling, interference, etc...)

Roadmap



0) Introduction and motivation

1) Lattice simulations inflation

2) The Inflationary Butterfly Effect

AC, S. Renaux-Petel, K. Inomata

2403.12811

Inflationary Butterfly Effect



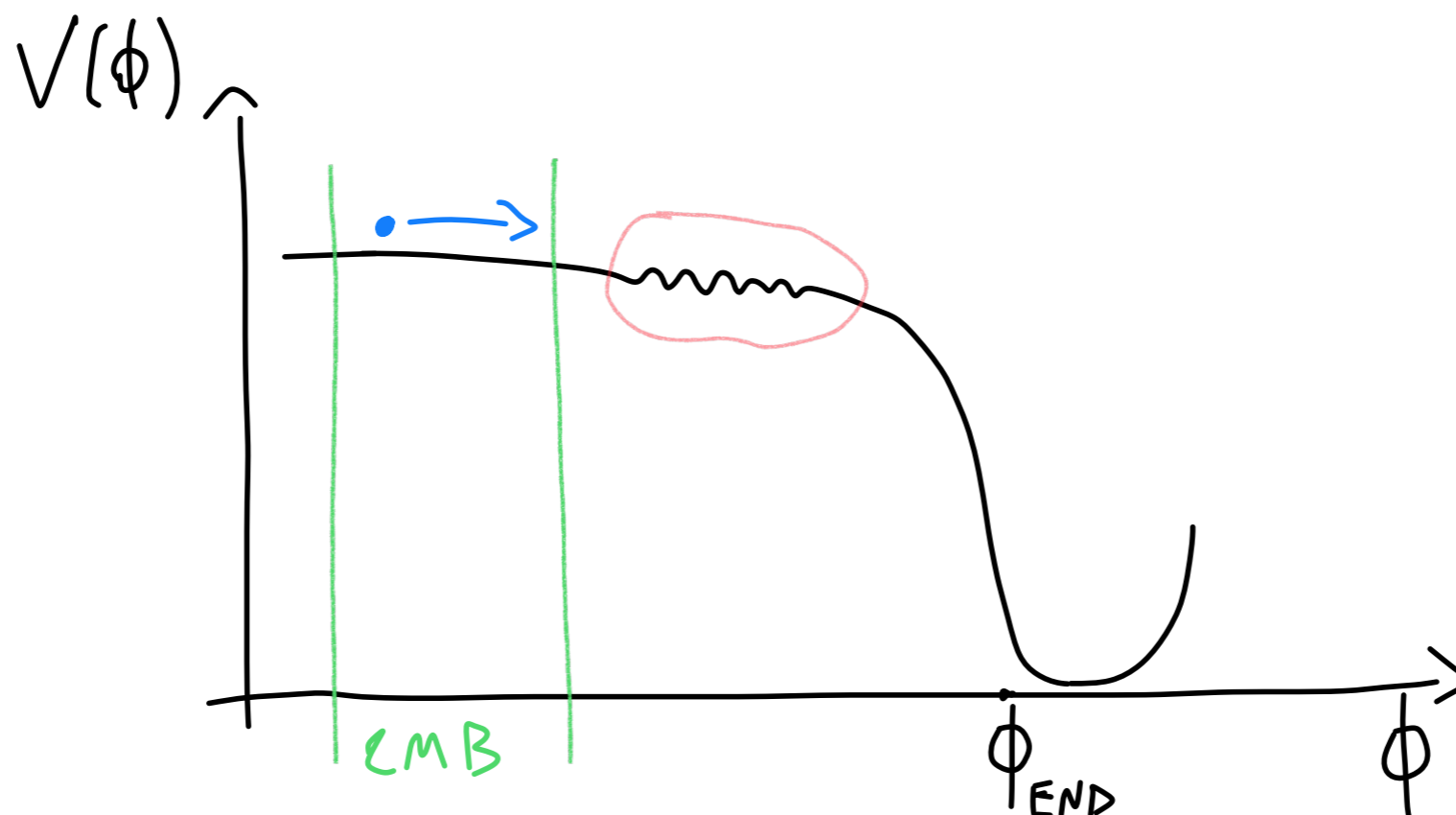
Consider a **small-scale modification** of the inflaton potential

$$V(\phi) = V_{\text{sr}}(\phi) + \Lambda^4 W(\phi) \left[\cos \left(\frac{\phi - \phi_0}{f} \right) - 1 \right]$$

Slow-roll potential

Localised oscillation

$$W(\phi) = \frac{1}{4} \left(1 + \tanh \left(\frac{\phi - \phi_0}{f} \right) \right) \left(1 + \tanh \left(\frac{\phi_0 - \phi + \Delta\phi}{f} \right) \right)$$



Oscillatory potential

Consider a **small-scale modification** of the inflaton potential

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oscillation



parametric
resonance

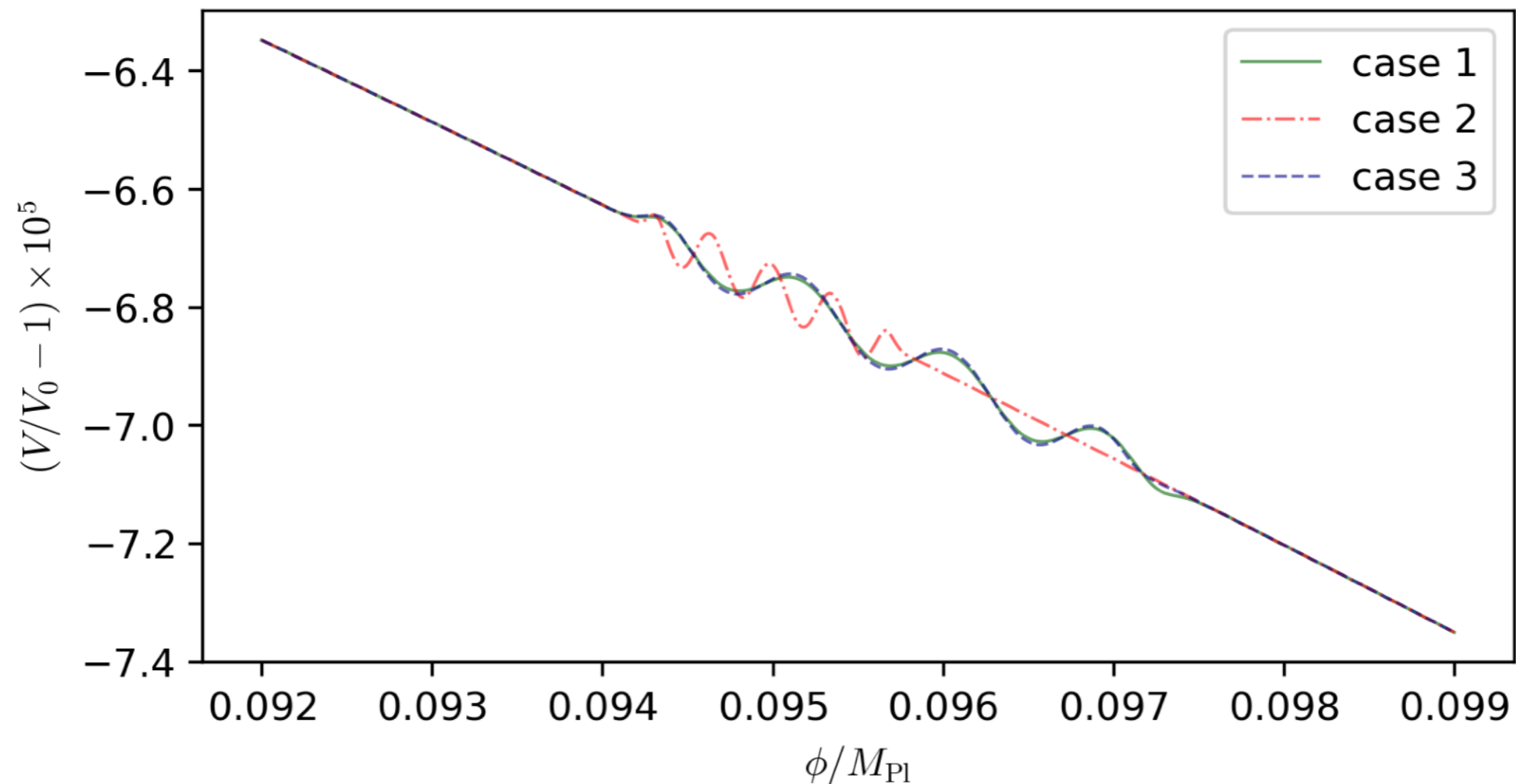


exponential growth
of perturbations

Oscillatory potential

$$V(\phi) = V_{\text{sr}}(\phi) + \Lambda^4 W(\phi) \left[\cos \left(\frac{\phi - \phi_0}{f} \right) - 1 \right]$$

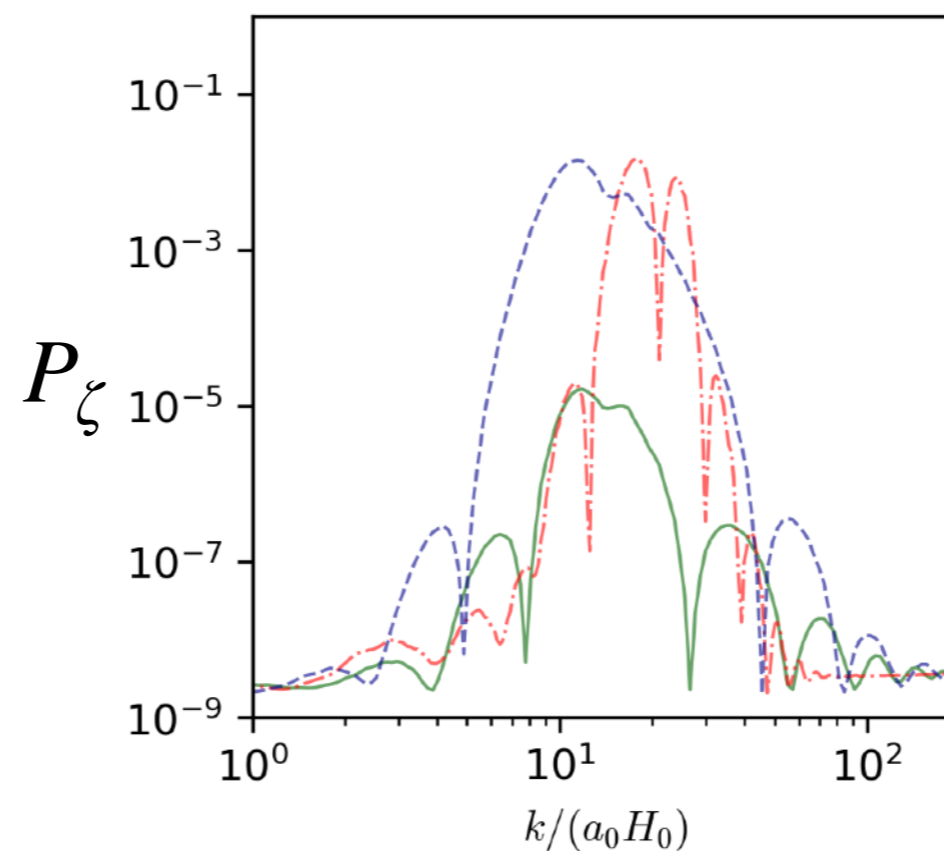
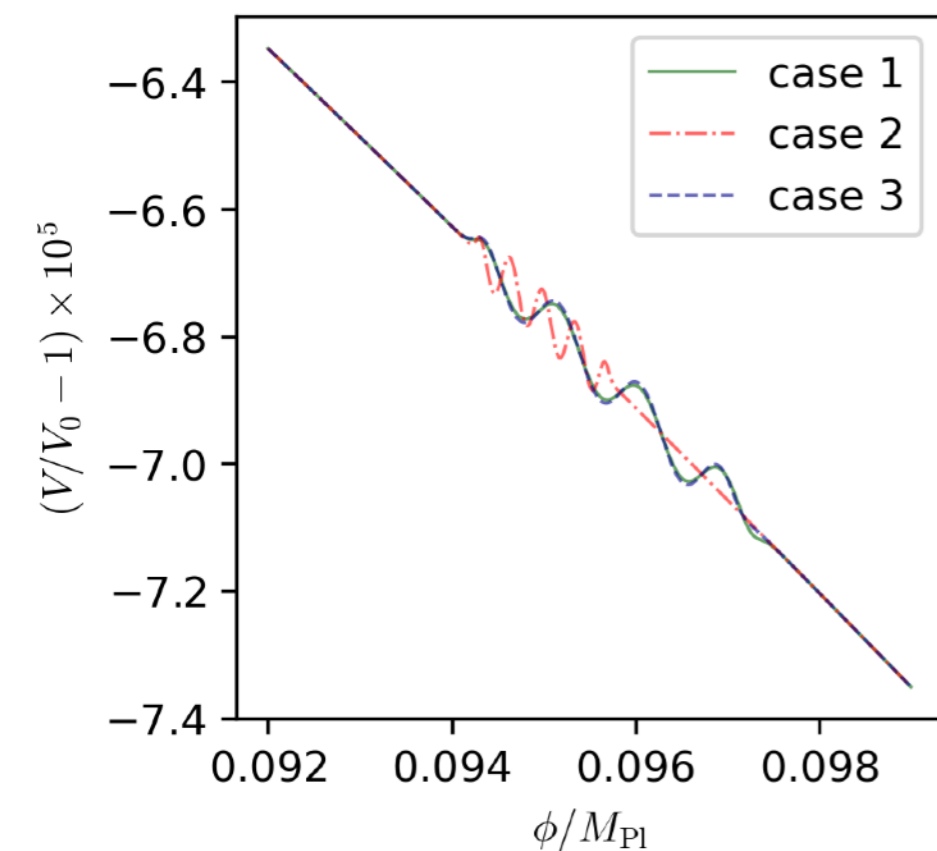
Let's consider the following three cases:



Oscillatory potential

$$V(\phi) = V_{\text{sr}}(\phi) + \Lambda^4 W(\phi) \left[\cos \left(\frac{\phi - \phi_0}{f} \right) - 1 \right]$$

The feature induces a growth of the power spectrum:



Case 1: $P_\zeta \simeq 10^{-5}$

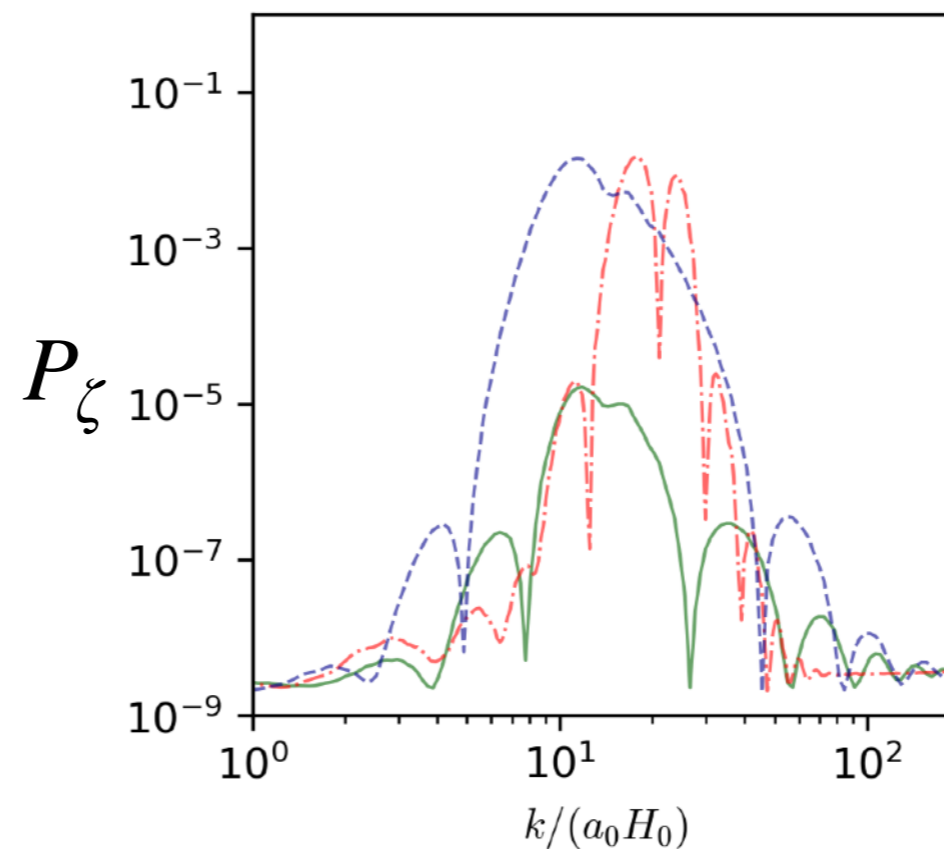
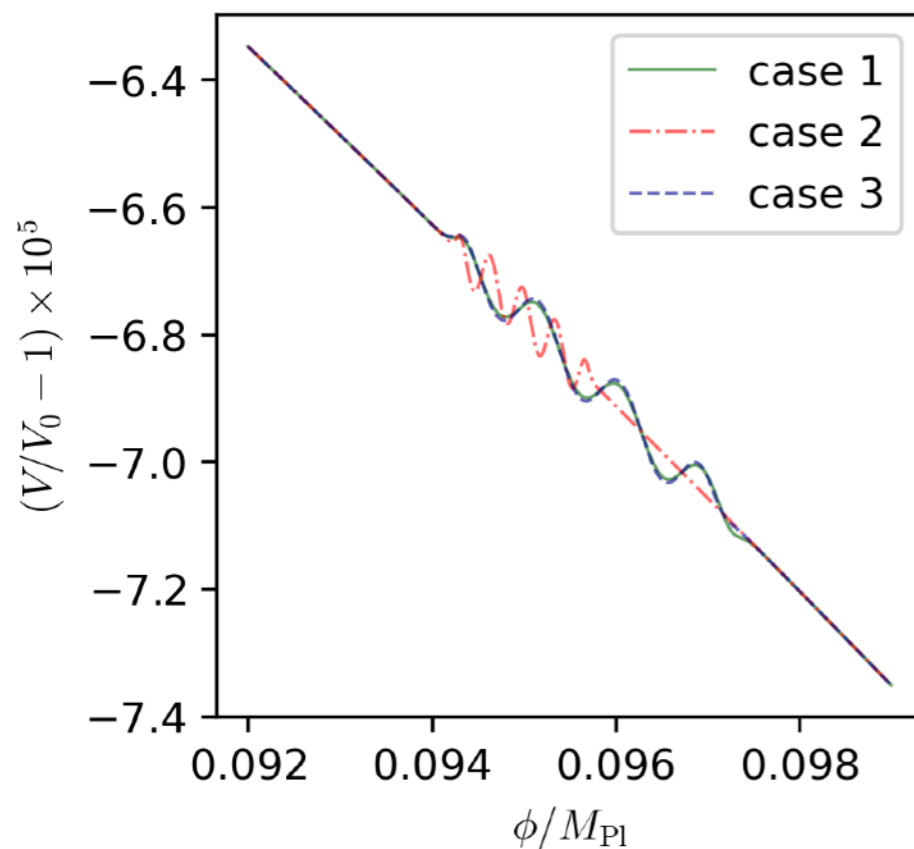
Case 2: $P_\zeta \simeq 10^{-2}$

Case 3: $P_\zeta \simeq 10^{-2}$

Oscillatory potential

$$V(\phi) = V_{\text{sr}}(\phi) + \Lambda^4 W(\phi) \left[\cos \left(\frac{\phi - \phi_0}{f} \right) - 1 \right]$$

The feature induces a growth of the power spectrum:



Case 1: $P_\zeta \simeq 10^{-5}$
 Case 2: $P_\zeta \simeq 10^{-2}$
 Case 3: $P_\zeta \simeq 10^{-2}$

[K. Inomata, M. Braglia, X. Chen, S. Renaux-Petel 2211.02586]

$$P_{\zeta,1\text{-loop}} \gtrsim P_{\zeta,\text{tree}}$$

In case 3 and 2, but not 1

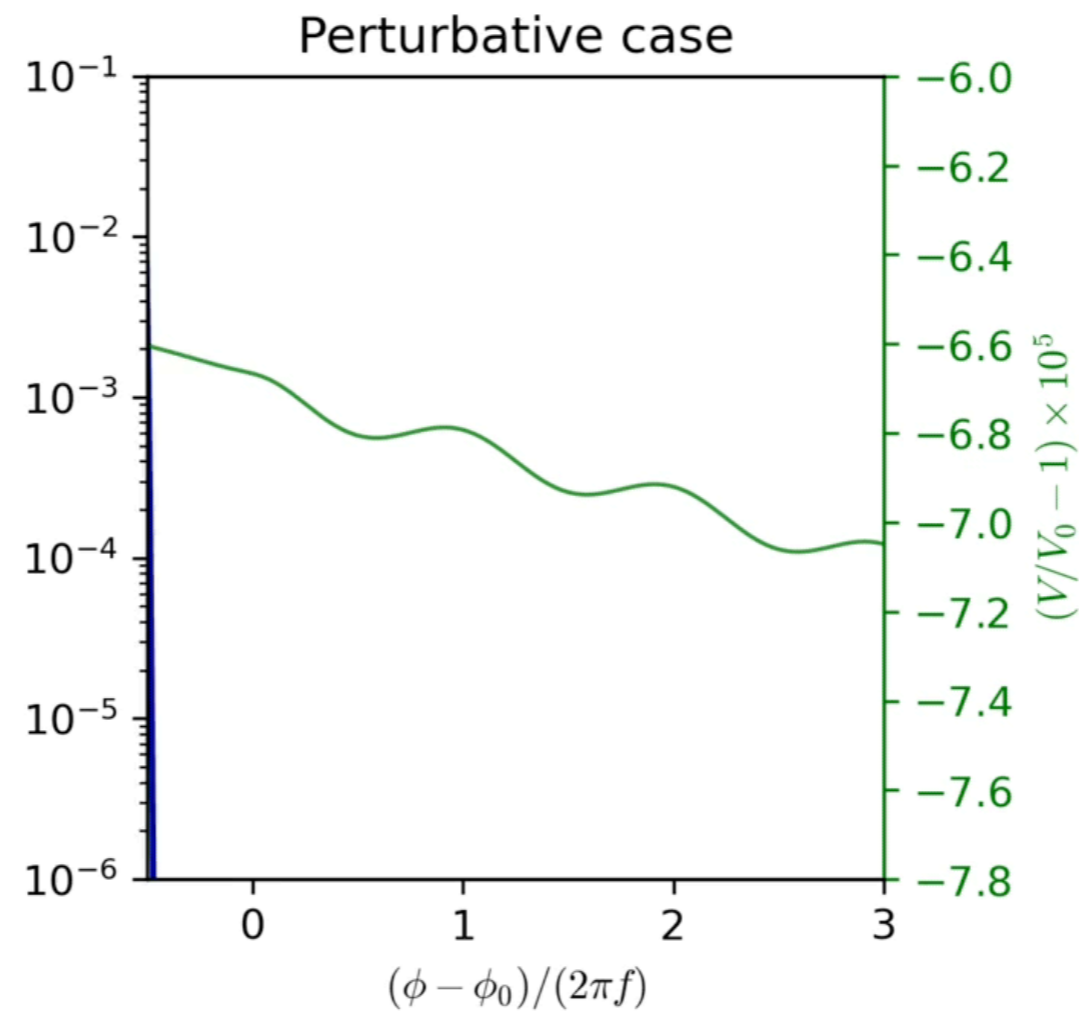
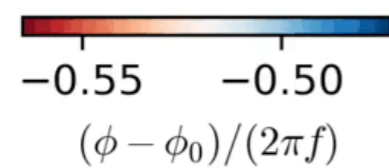
Case 1. ($P_\xi \sim 10^{-5}$)

Case 1 is perturbative

Animations:
(link is also in the paper)



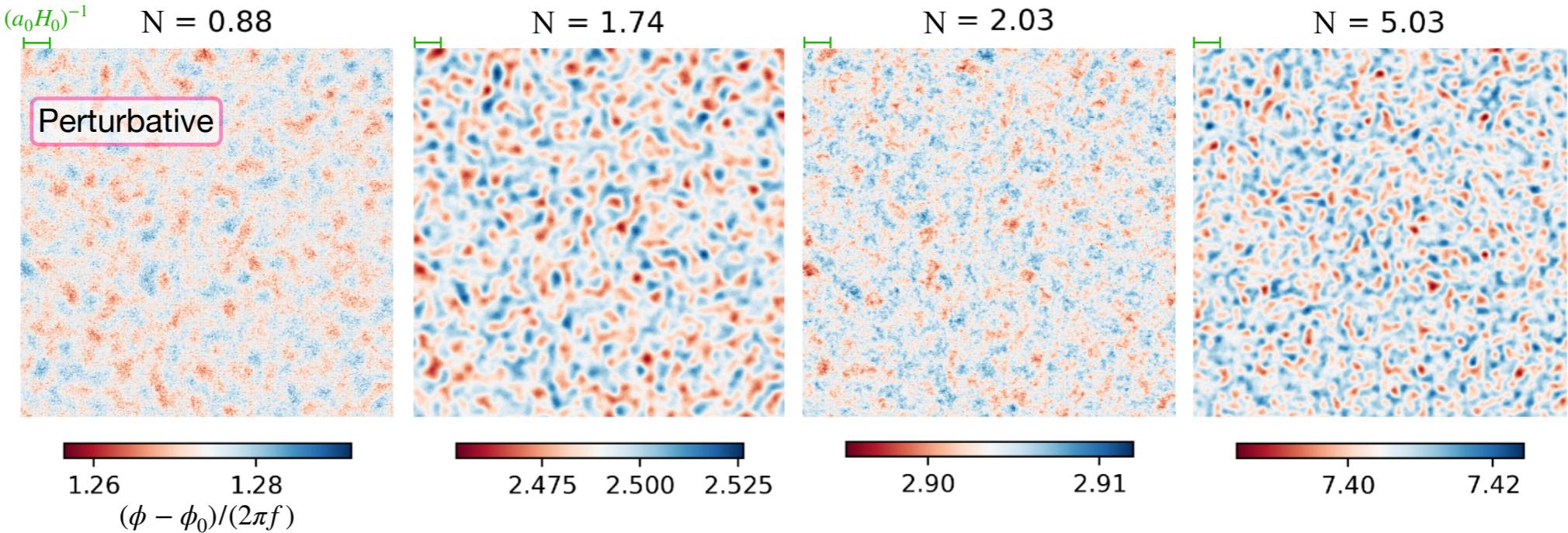
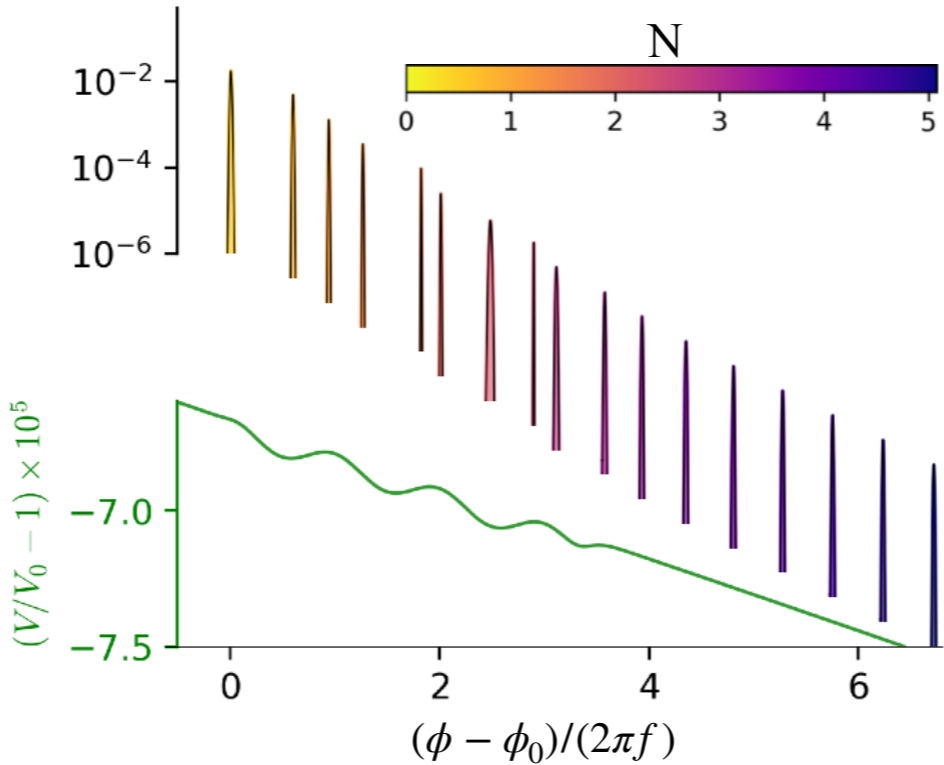
$N = -0.3$



Case 1. ($P_\zeta \sim 10^{-5}$)

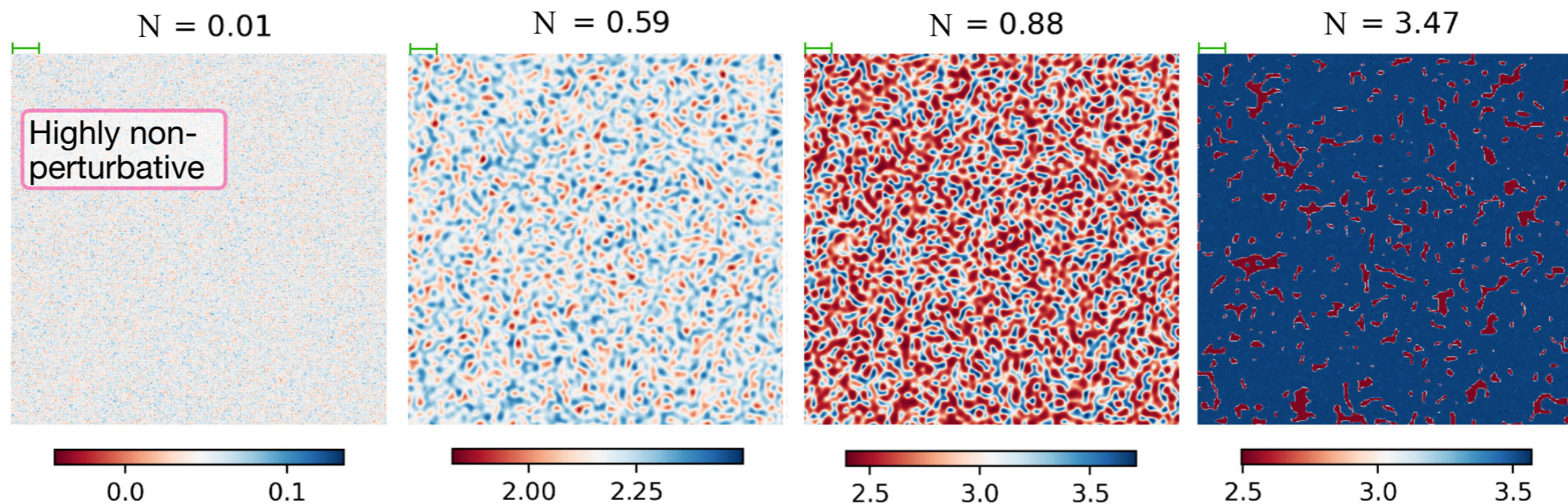
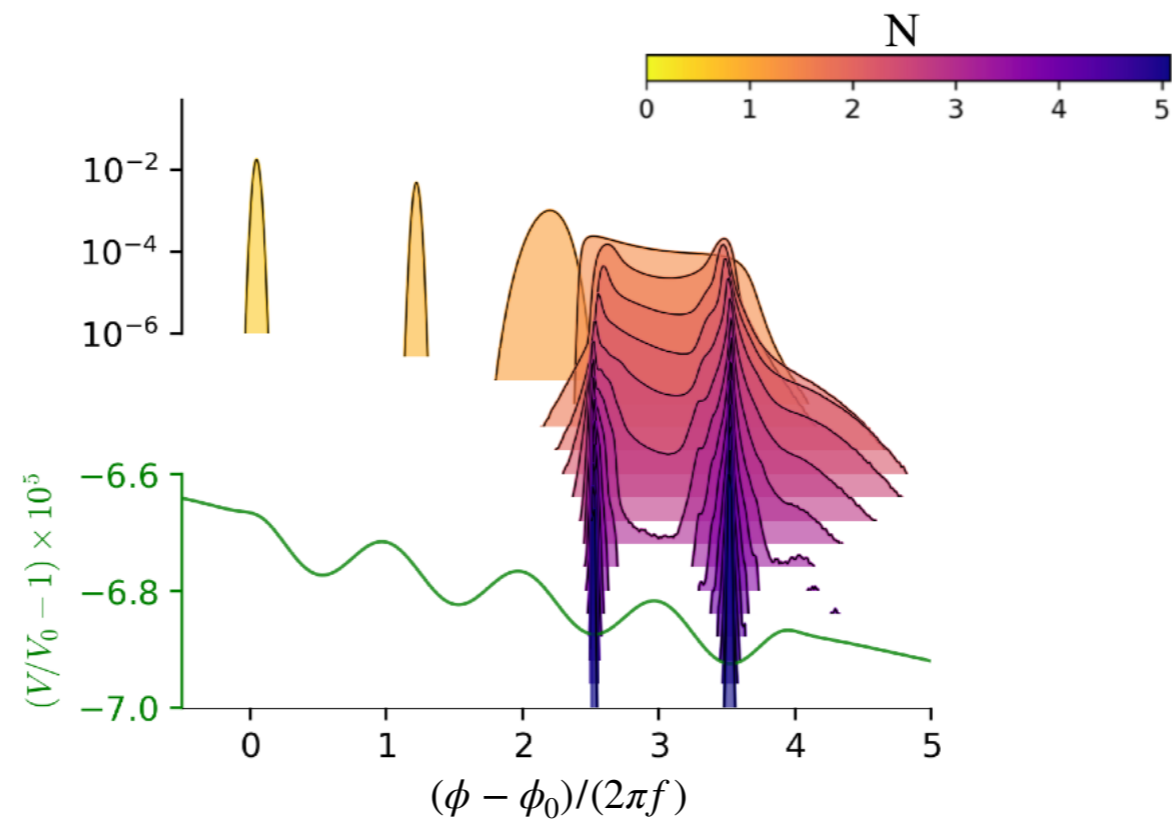


Case 1 is perturbative



Case 2. ($P_\zeta \sim 10^{-2}$)

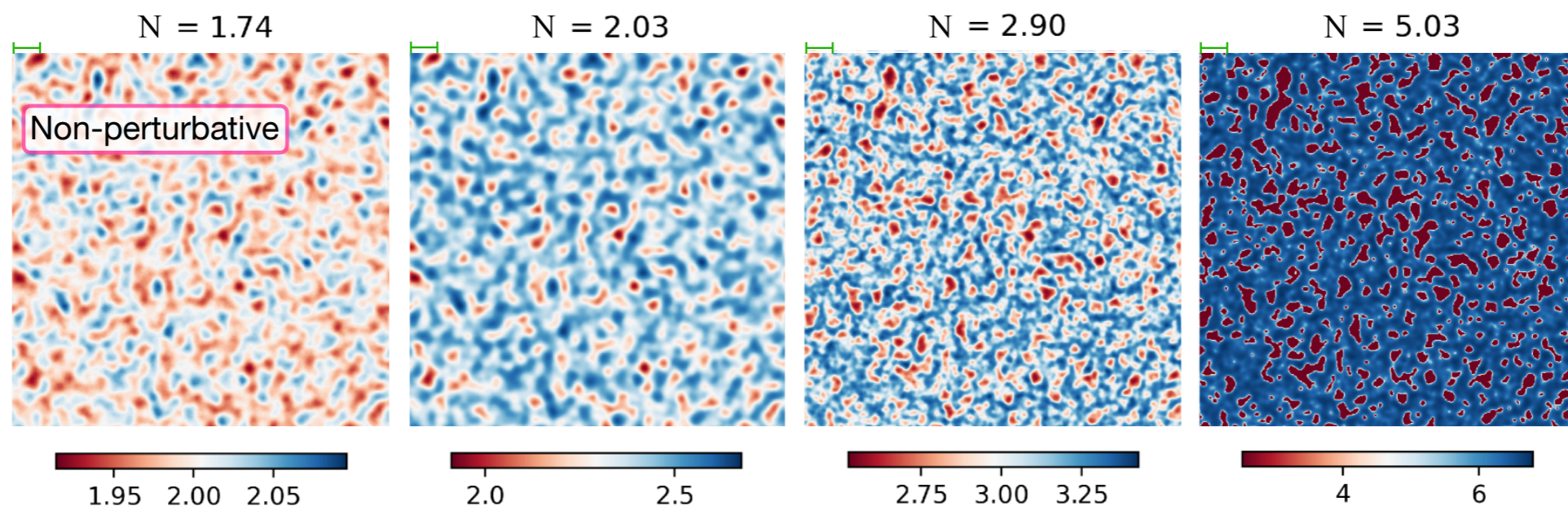
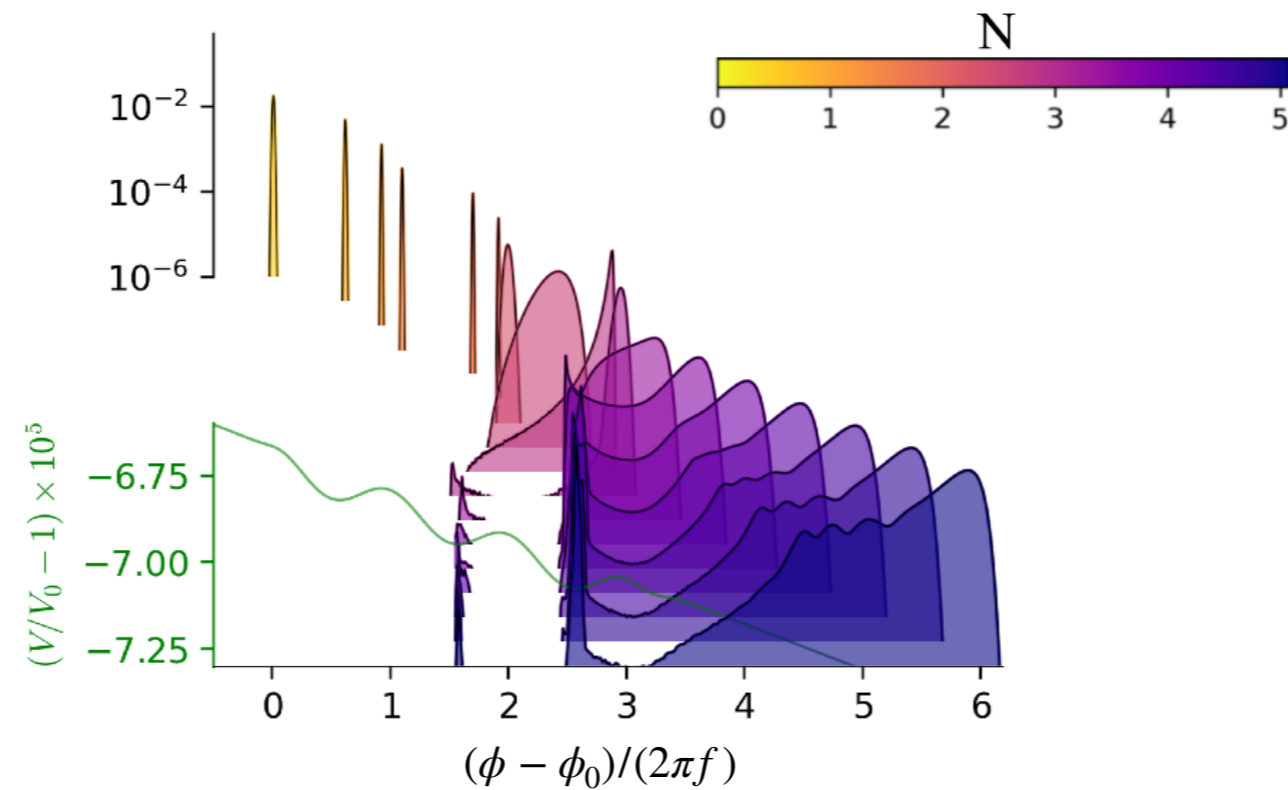
Case 2: Inflaton is stuck inside the oscillatory potential



Case 3. ($P_\xi \sim 10^{-2}$)

Case 3: Only **some patches** are stuck in the resonant potential!

The **rest** continues slow-rolling



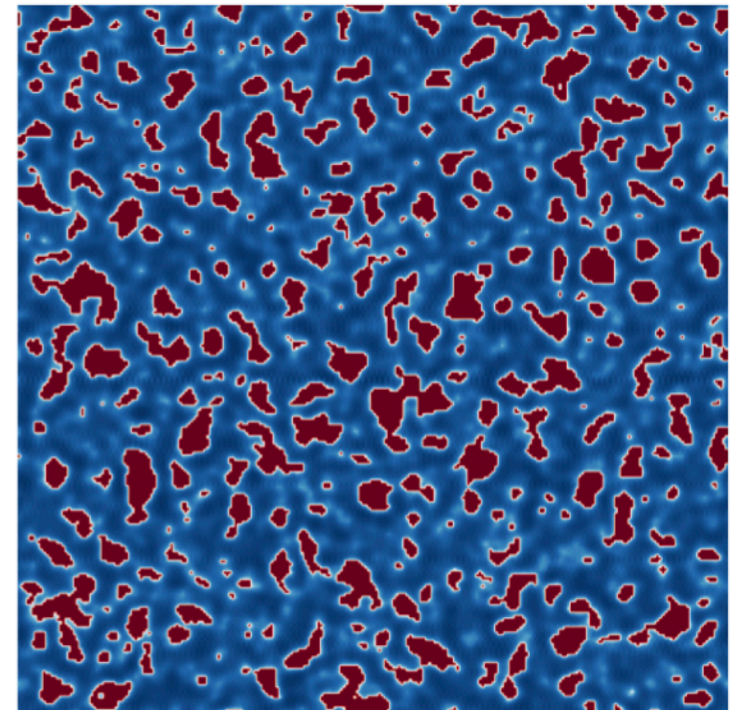
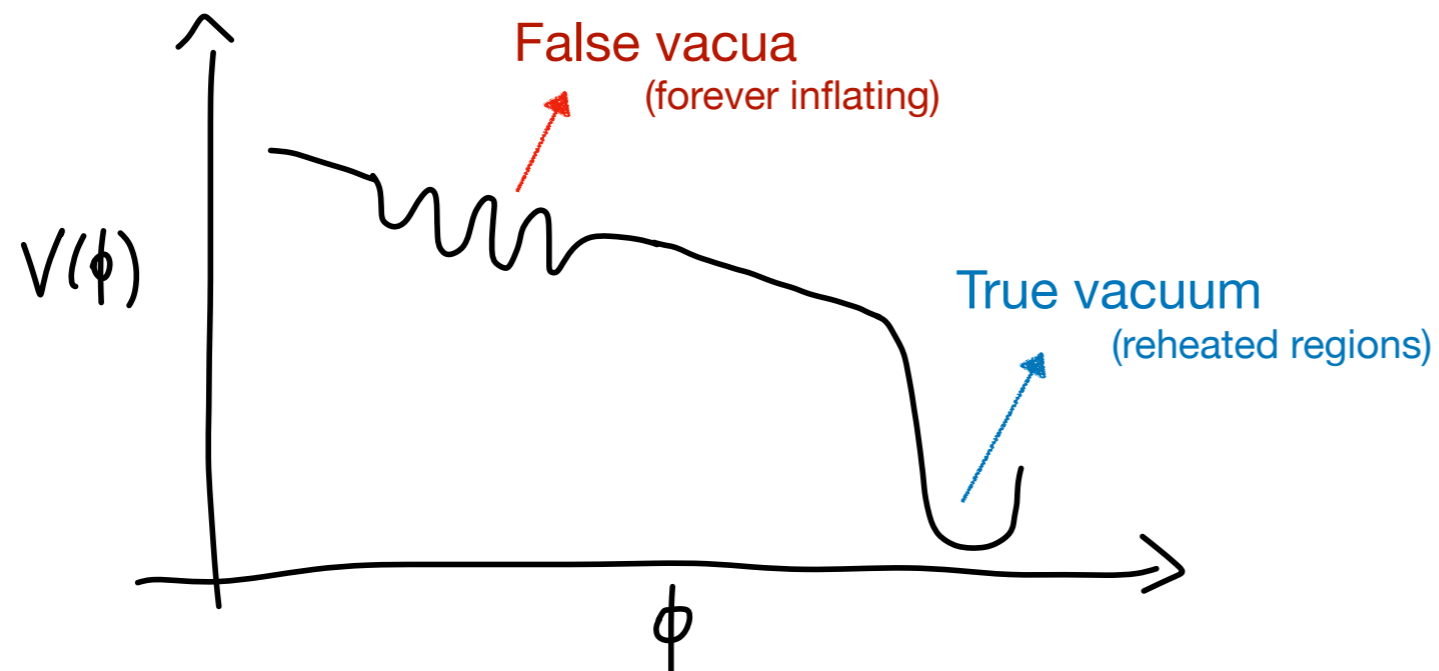
Case 3: inflaton trapping



Case 3:

What happens to the trapped regions at the end of inflation?

Their fate is analogous to false vacuum trapping.



Inflaton trapping and PBHs

Case 3:

What happens to the trapped regions at the end of inflation?

Their fate is analogous to false vacuum trapping.

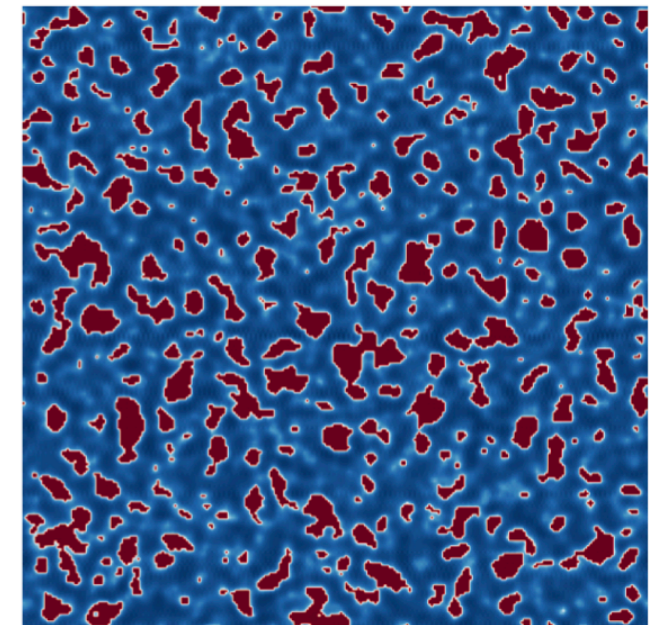
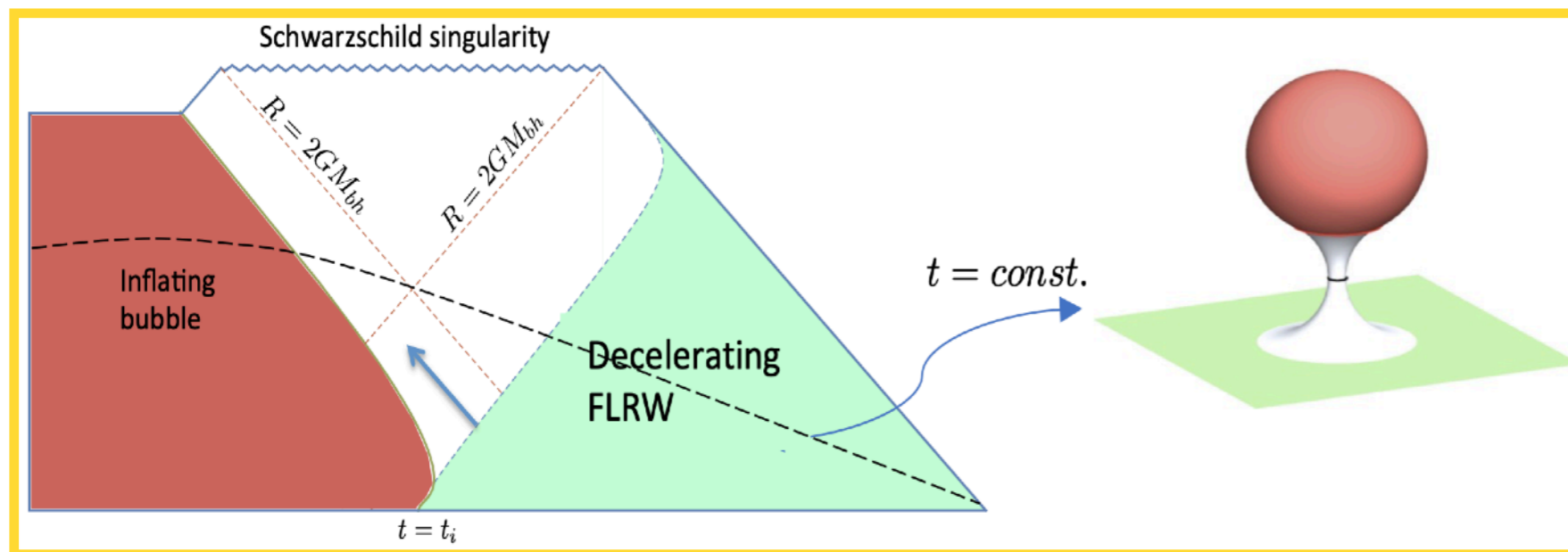


Figure credit:

[J. Garriga, A. Vilenkin, J. Zhang arXiv:1512.01819]

The **trapped regions** become PBHs at the end of inflation! (in the form of baby universes)

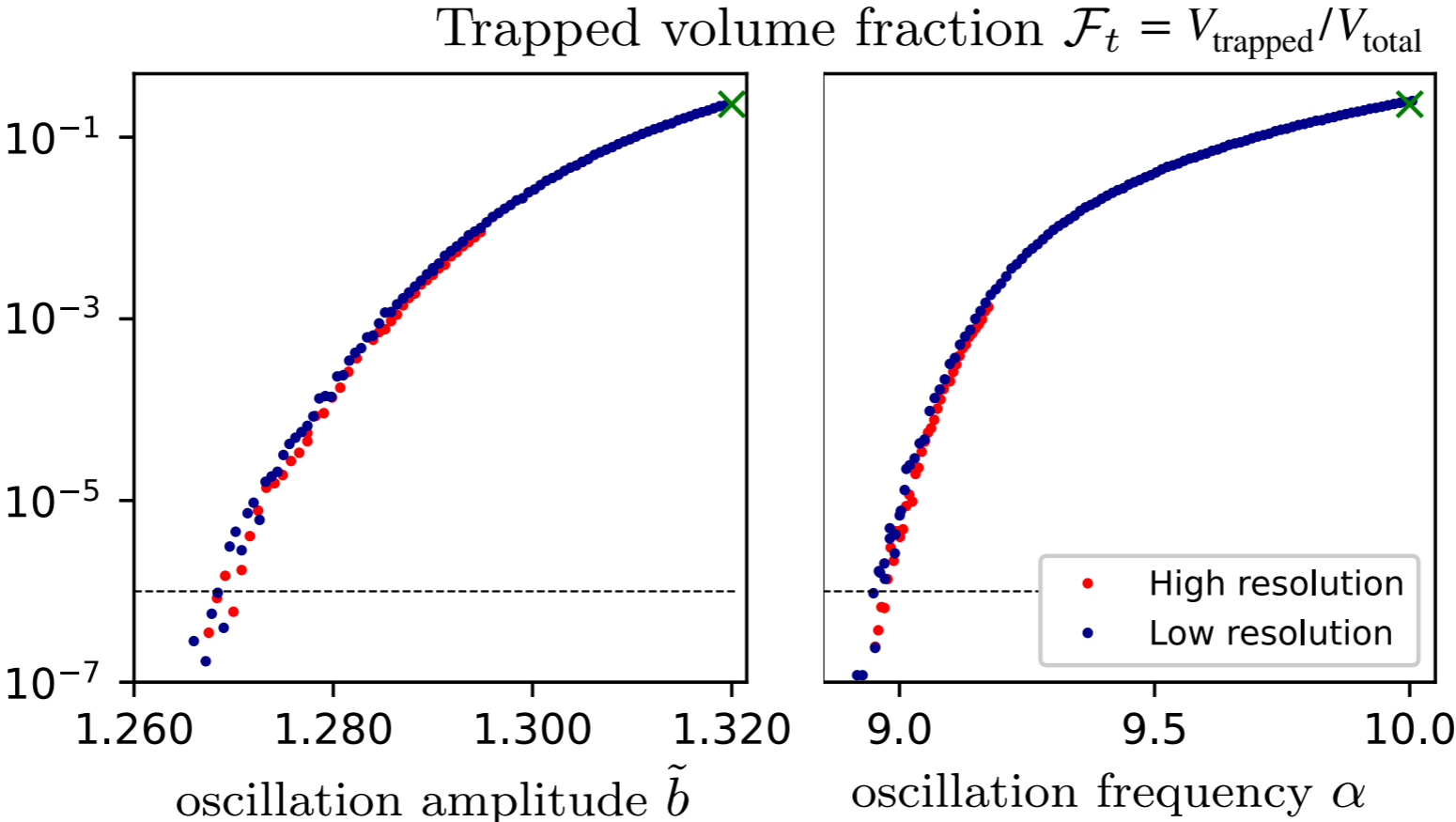
PBH abundance

Case 3:

The trapped regions become PBHs at the end of inflation!

How many PBHs?

Mass fraction in PBHs at the time of formation



500 lattice simulations in this plot

Inflationary Butterfly Effect



Lorenz (1972):

“Can the Flap of a Butterfly’s Wings in Brazil Set Off a Tornado in Texas?” [1]

Can tiny, small-scale quantum fluctuations affect the dynamics of the entire Universe?

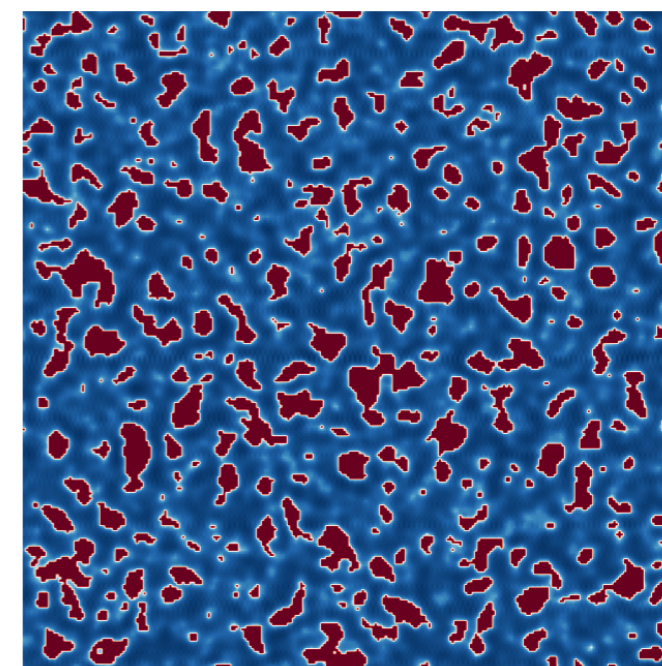
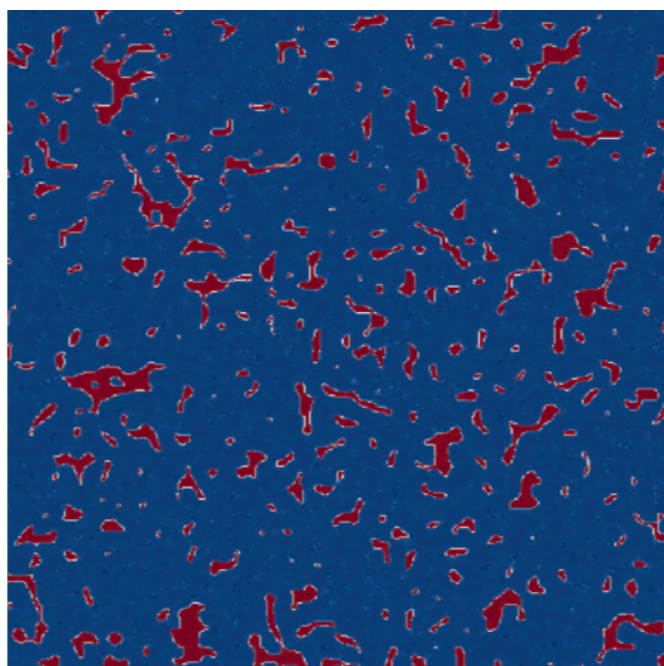
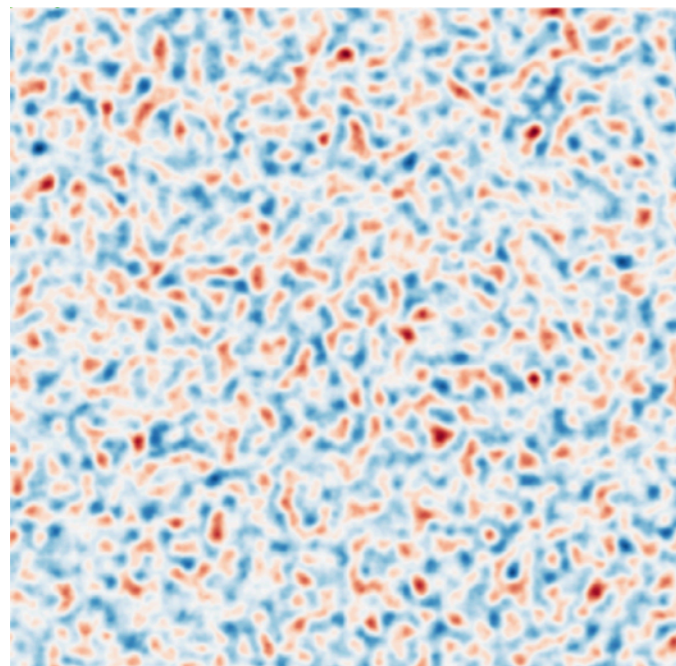
[1]: E. N. Lorenz, American Association for the Advancement of Science (1972).

Inflationary Butterfly Effect



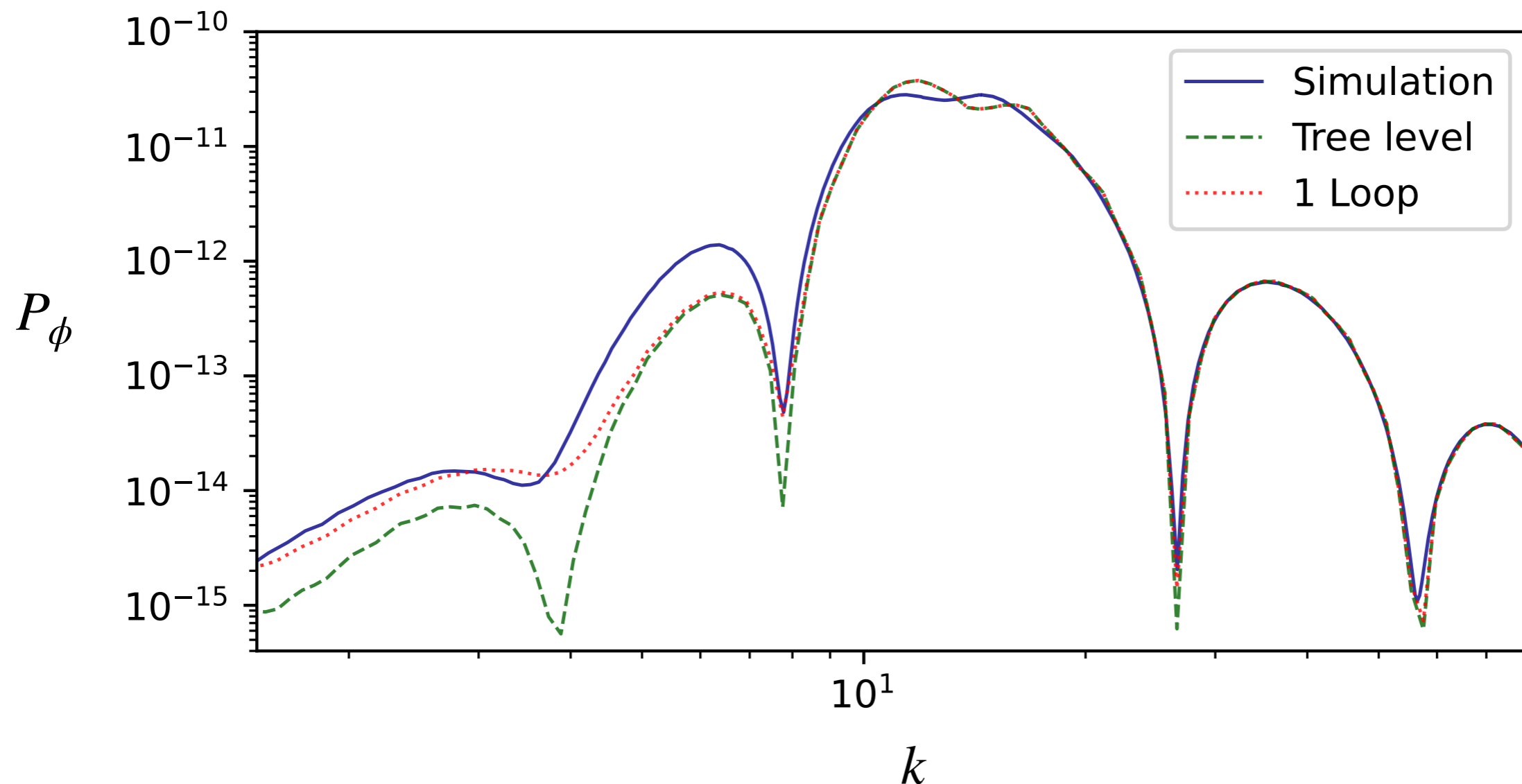
Main lesson:

Non-perturbative physics at small scales can have drastic effects on the inflationary dynamics when $\mathcal{P}_\zeta \sim 10^{-2}$



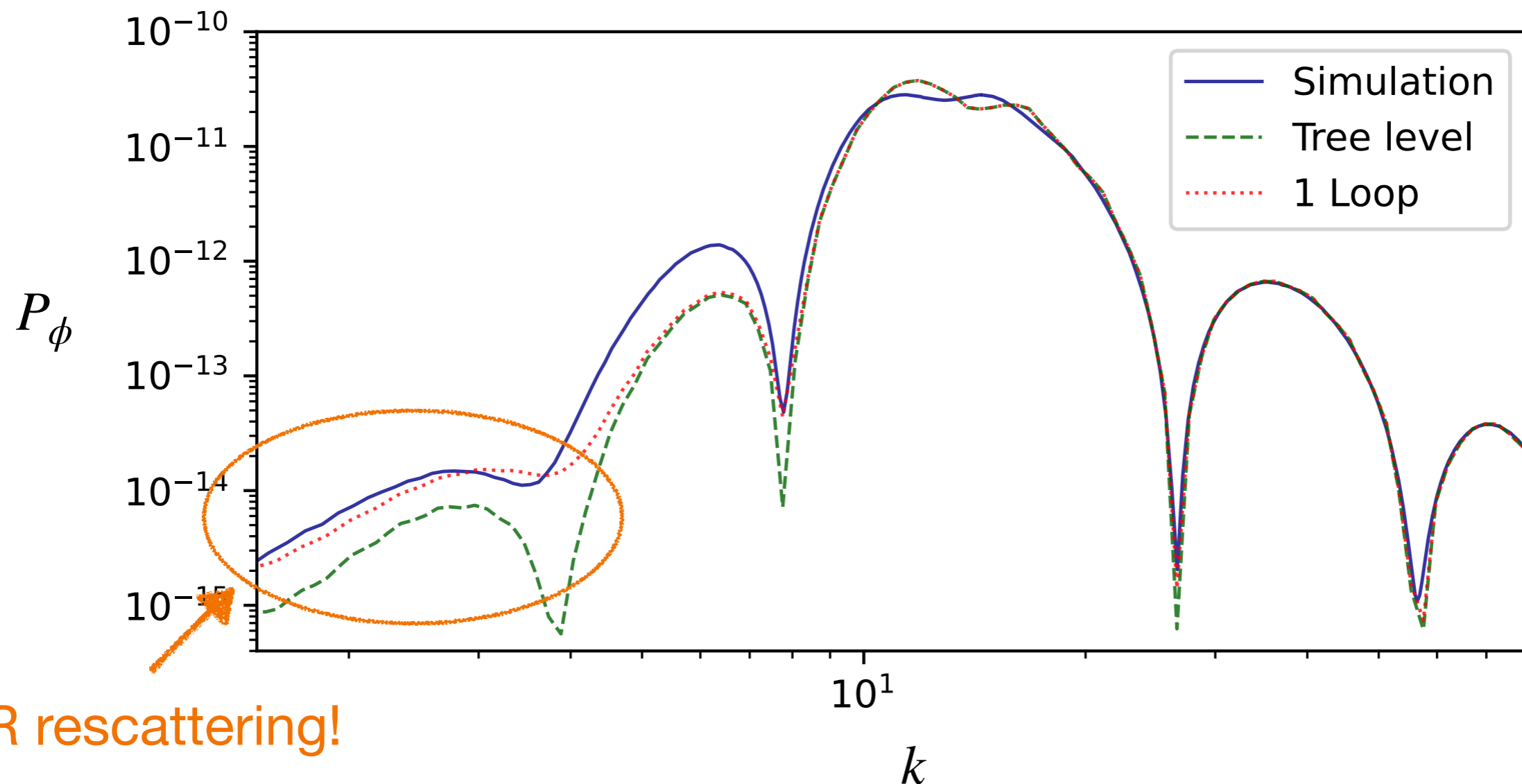
Loop effects

In the perturbative setup,
first **quantitative comparison** between full nonlinear, tree-level and 1-loop



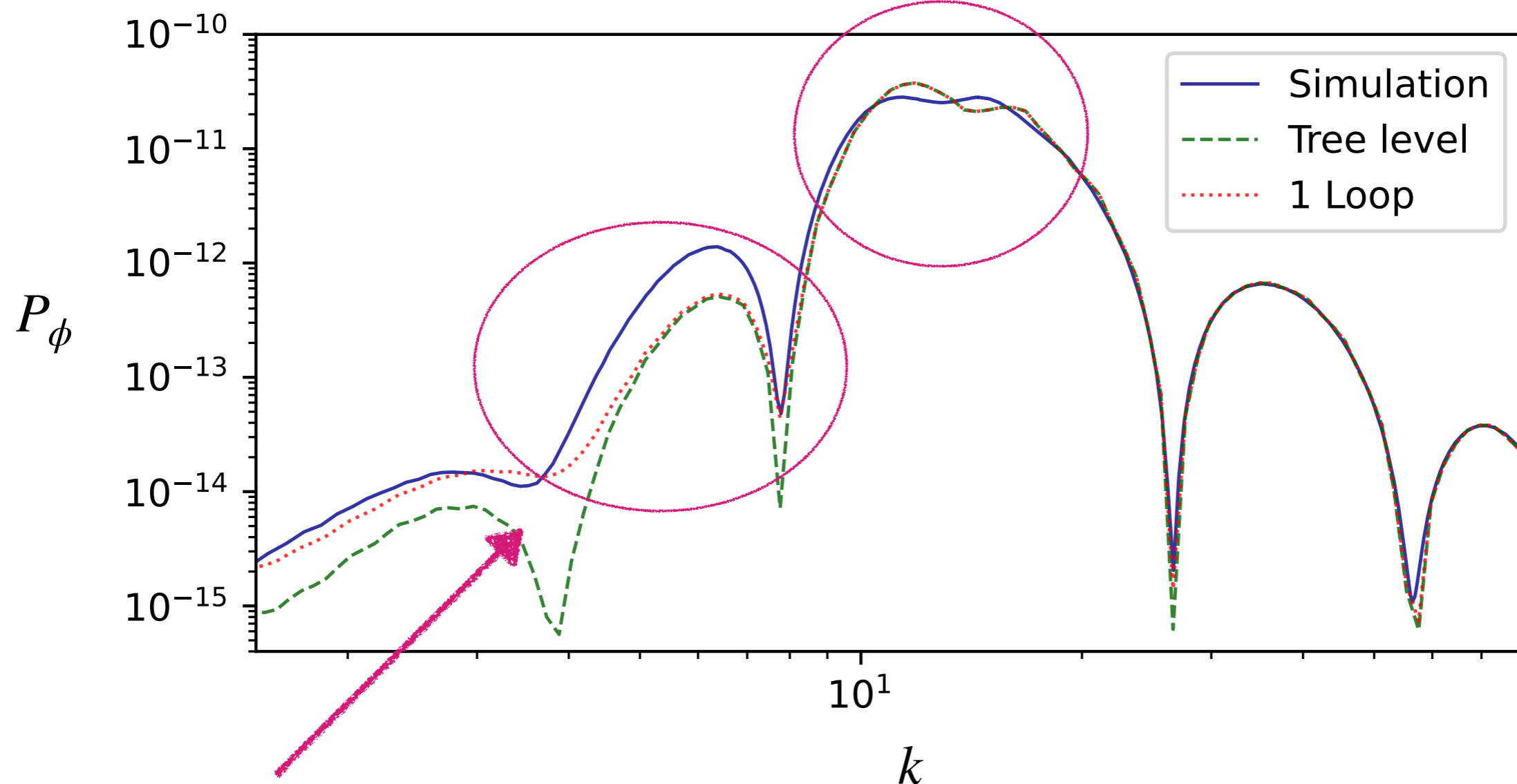
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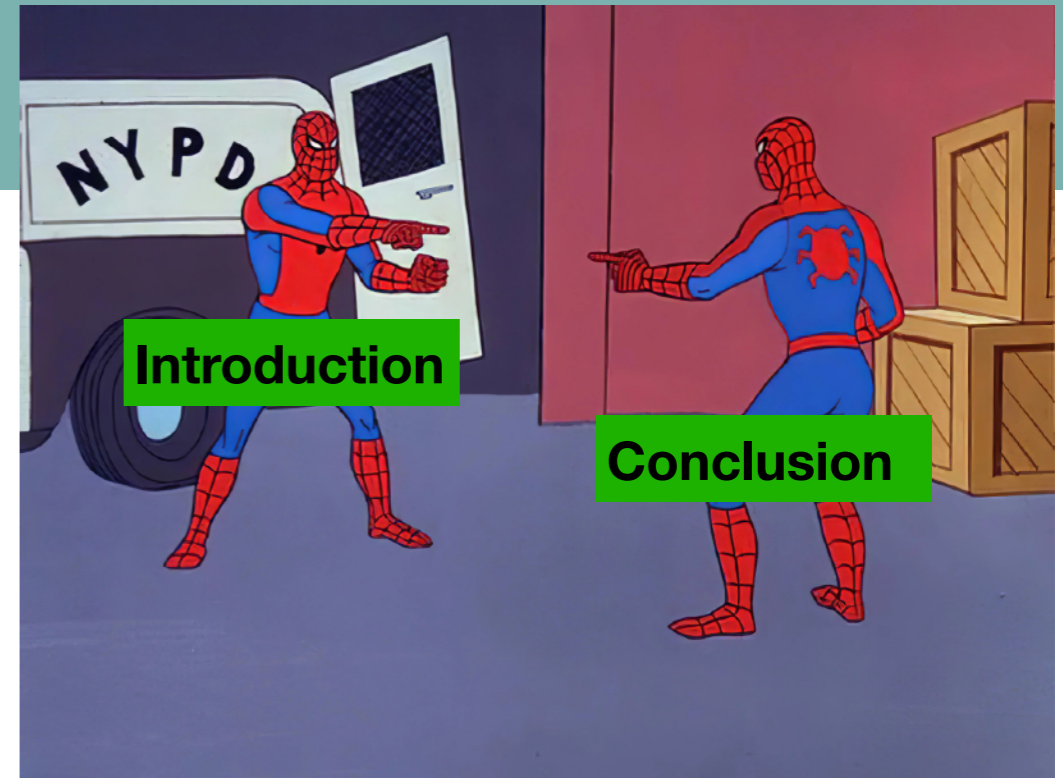
Loop effects

In the perturbative setup,
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Beyond 1-loop

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