Memory Burden Effect in Solitons: Consequences for primordial black holes

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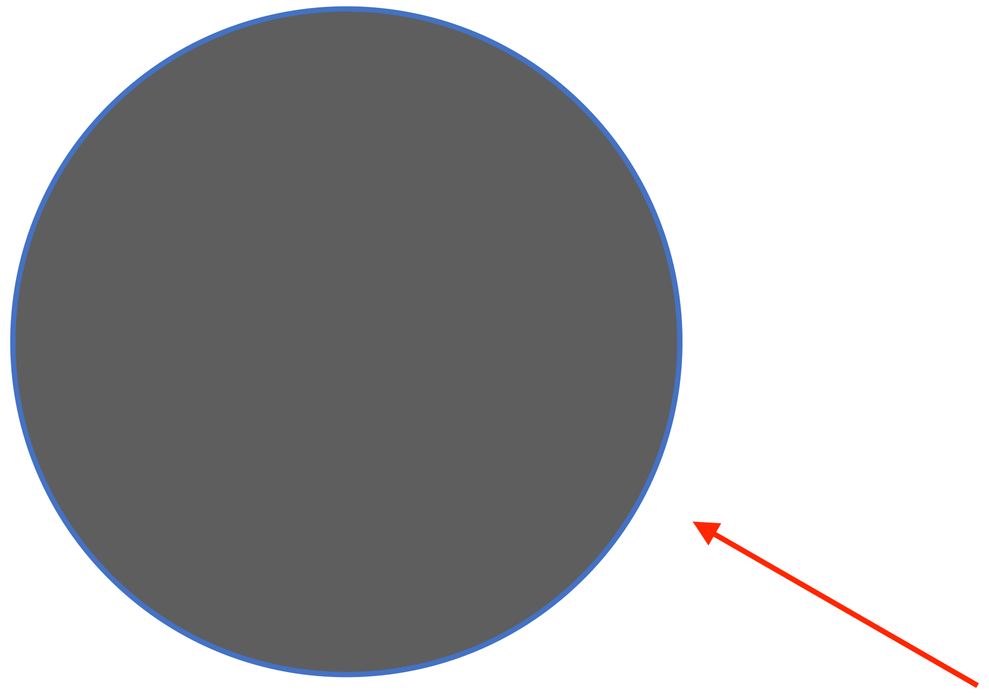
NEHOP 24

Work in collaboration with G. Dvali (MPP & LMU, Munich), J.S. Bermudez (IFAE, Barcelona), F. Kühnel (MPP, Munich), O. Kaikov (MPP, Munich)

Memory burden effect

G. Dvali, arXiv:1810.02336 [hep-th]; G. Dvali, L. Eisemann, M. Michel, S. Zell, arXiv:2006.00011 [hep-th]

Information carried by an object stabilizes it.



The modes outside are highly gapped.
Due to this, information stored in memory modes cannot escape for a long time. This leads to backrection on the configuration itself.

Gapless ''memory modes'' (Goldstones) live only inside

This is the essence of the memory burden effect.

Memory burden is prominent in systems with high information-storage capacity such as **soltions, black holes** etc. This has consequences for PBHs DM.

G. Dvali, J. Bermudez, **MZ,** arXiv:2405.13117 [hep-th]

Vacuum bubble with high information-storage capacity

- $d = 3 + 1$
- ϕ in the adjoint representation of $SU(N)$ global symmetry
- $N \gg 1$
- Theory is renormalizable

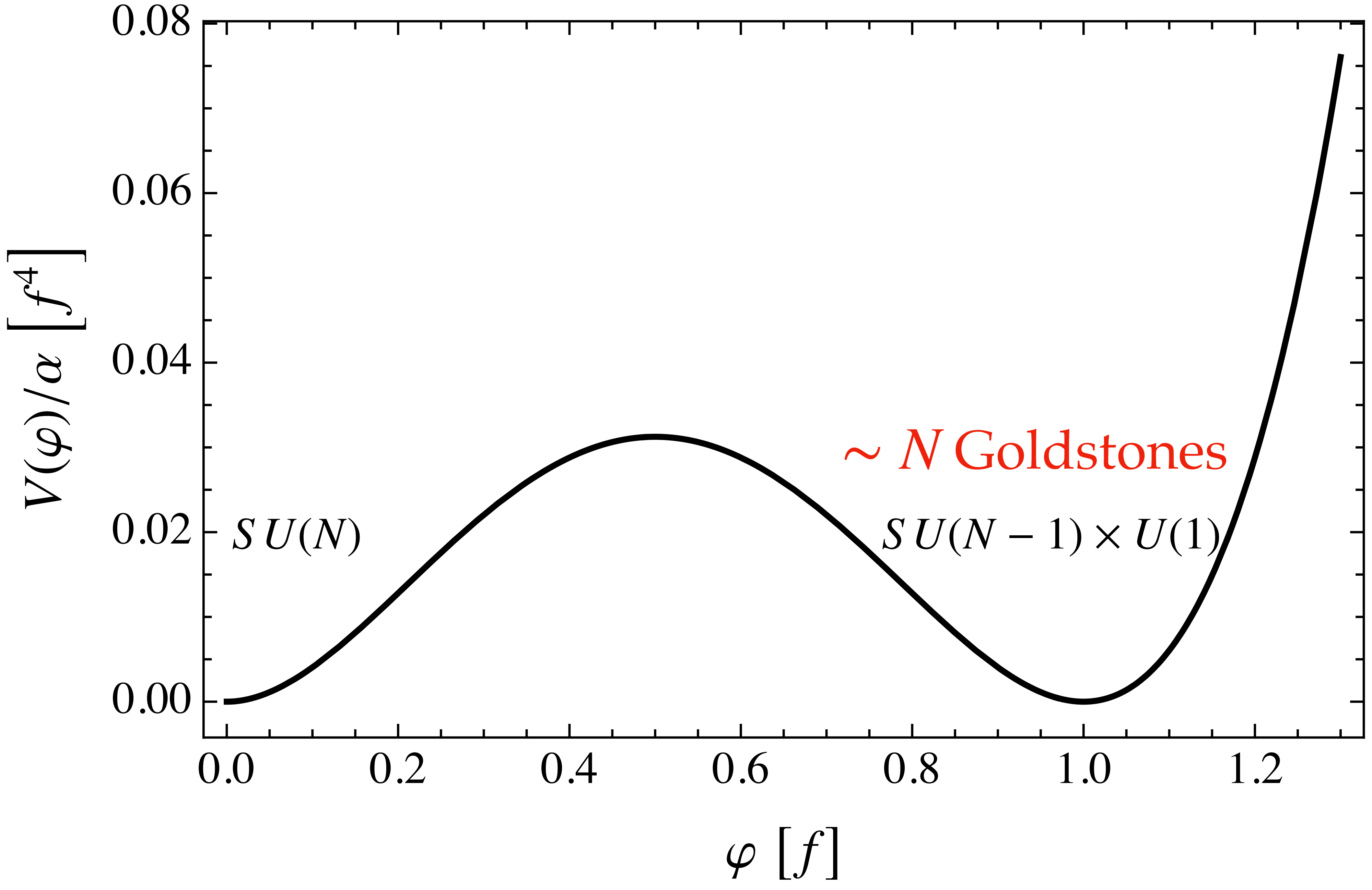
$$\mathscr{L} = \frac{1}{2}\mathrm{Tr}\left[(\partial_\mu \phi)(\partial^\mu \phi)\right] - V[\phi]$$

$$V[\phi] = \frac{\alpha}{2}\mathrm{Tr}\left[\left(f\phi - \phi^2 + \frac{I}{N}\mathrm{Tr}[\phi^2]\right)\right]^2$$

Unitarity requires: $\alpha N \leqslant 1$

↑

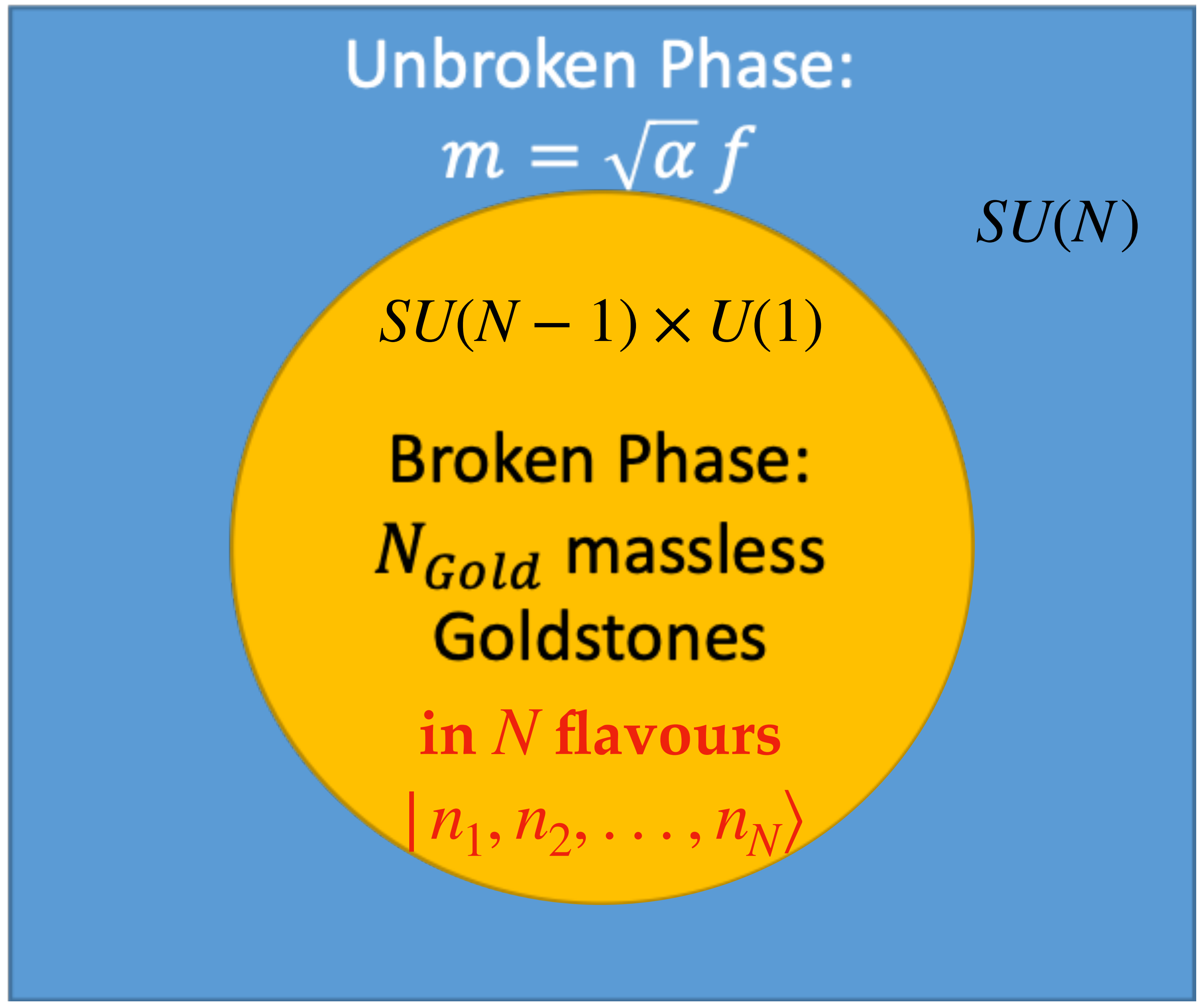
Validity domain of QFT description in terms of ϕ



$$\varphi^2 = \mathrm{Tr}\left[\phi^2\right]$$

$$V(\varphi) \sim \frac{\alpha}{2}\varphi^2(f - \varphi)^2$$

Vacuum bubble with high information-storage capacity



Unbroken Phase:

$m = \sqrt{\alpha}\, f$

$SU(N)$

$SU(N-1) \times U(1)$

Broken Phase:

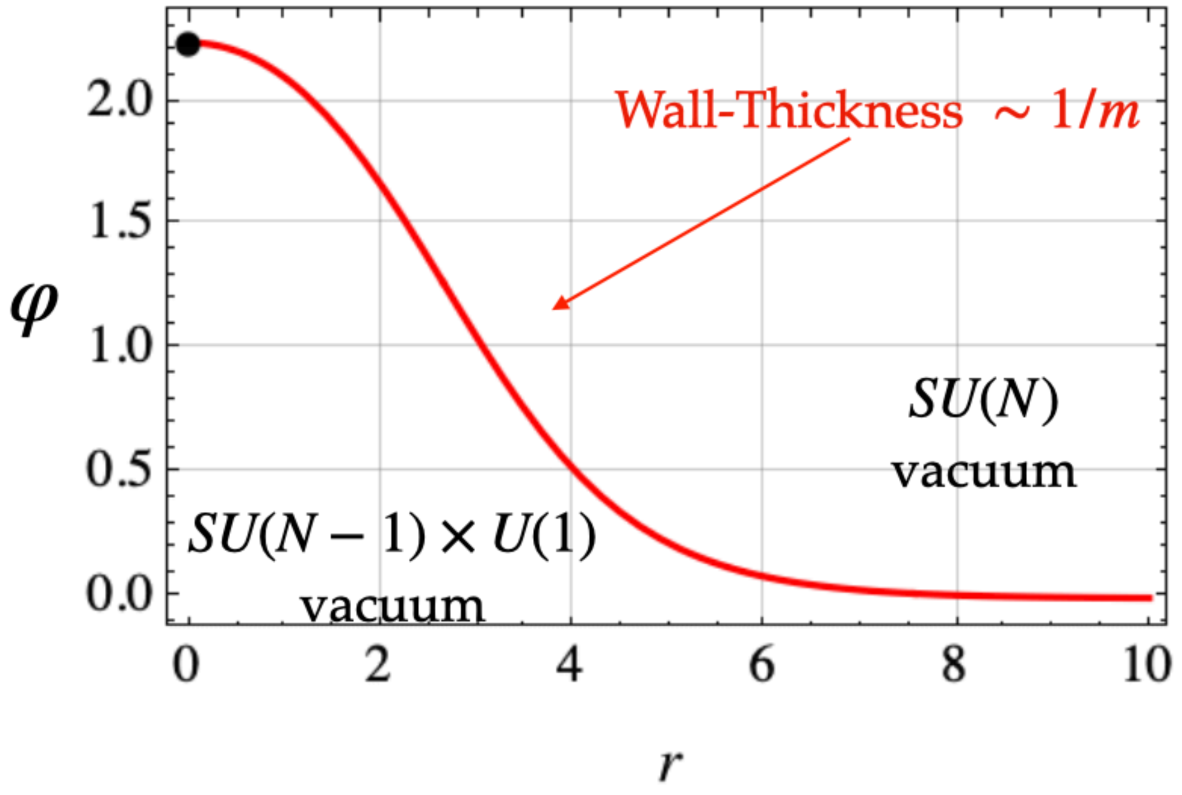
N_{Gold} massless

Goldstones

in N flavours

$|n_1, n_2, \ldots, n_N\rangle$

$$\Phi_{\mathrm{D}} \propto \varphi(r)$$



Vacuum bubbles:

$$\phi = U^\dagger \Phi_{\mathrm{D}} U$$

- $U = \exp\left[-i\,\theta\, T\right]$
- T corresponds to broken generator

$$\theta = \omega\, t$$

- $\varphi(r)$ is order parameter localizing bubble (Goldstone) region
- Their number correspond to charge $N_{\mathrm{Gold}} = Q$

Some similarities with $U(1)$ Q-balls construction

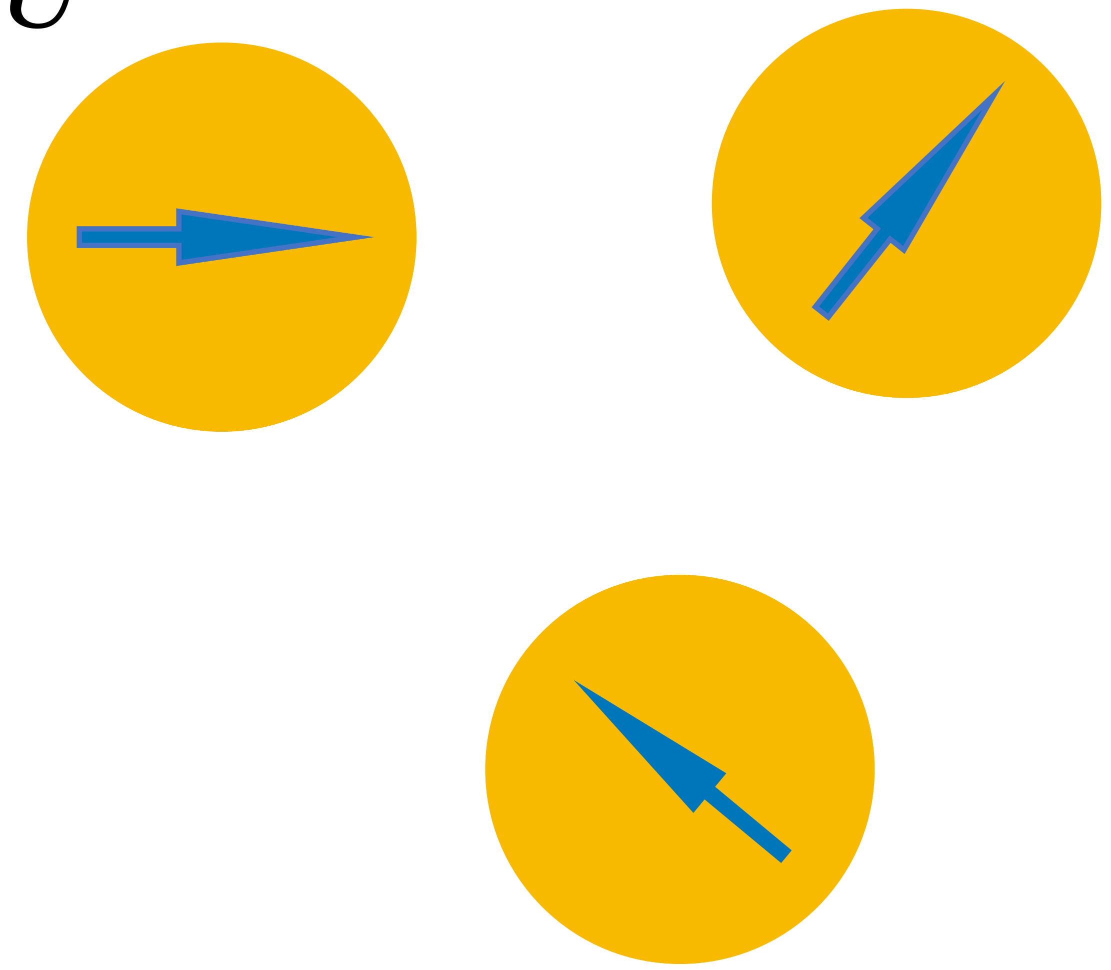
Vacuum bubble with high information-storage capacity

G.Dvali, O. Kaikov, J.S. Valbuena-Bermudez, '21; G.Dvali, J.S. Valbuena-Bermudez, **MZ** '24.

$$|\text{Pattern}\rangle = |n_1, n_2, \ldots, n_N\rangle \qquad \text{With: } \sum_a^N n_a = N_{\text{Gold}} = Q$$

This stabilizes the bubble

Bubbles rotated by relative $SU(N)$ transformations: $\Phi \rightarrow U^\dagger \Phi U$



Due to Goldstone modes, each bubble exhibits an exponentially high microstate degeneracy

$$n_{\text{states}} \simeq \left(1 + \frac{2N}{N_{\text{Gold}}}\right)^{N_{\text{Gold}}} \left(1 + \frac{N_{\text{Gold}}}{2N}\right)^{N} \qquad S = \log n_{\text{states}}$$

Vacuum bubble as a black hole prototype

These stationary objects display properties analogous to black holes in a certain parameter space region

$$N_{\text{Gold}} \sim \frac{1}{\alpha}\left(\frac{m}{\omega}\right)^5 \qquad R \sim \frac{m}{\omega^2} \qquad M \sim \omega\, N_{\text{Gold}}$$

$$n_{\text{states}} \simeq \left(1+\frac{2N}{n_{\text{Gold}}}\right)^{n_{\text{Gold}}}\left(1+\frac{n_{\text{Gold}}}{2N}\right)^{N} \qquad S = \log n_{\text{states}}$$

In particular, for $\boxed{\omega \sim m \quad \text{and} \quad N_{\text{Gold}} \sim N \sim 1/\alpha}$

$$\downarrow$$

$$S = R^2 f^2 = M\,R = \frac{1}{\alpha} = N = N_{\text{Gold}}$$

- Identifying $f \leftrightarrow M_{\text{Pl}}$ entropy area-law for black hole is reproduced $\alpha_{\text{gr}} = (q/M_{\text{Pl}})^2 = 1/(RM_{\text{Pl}})^2$
- The above entropy saturates the entropy bound imposed by unitarity $\alpha N \sim \alpha N_{\text{Gold}} \lesssim 1$
- These objects are termed saturons [G. Dvali, '21] . Black holes belong to this class
- Saturons are objects with high-information storage capacity

Universal properties of saturated configurations

It has been observed that striking similarities between black holes and saturons of renormalizable QFTs extend to their other key properties:

- Existence of information horizon in the semi-classical theory
- Thermal-like evaporation at initial stages of the decay (Hawking)

G. Dvali, ’21; G.Dvali, O. Kaikov, J. Bermudez, ’21, + **MZ** ‘24

- Minimal time-scale of the start of the information-retrieval, $t \sim S\,R$, which is identical to the Page's time in black holes

G. Dvali, ’21 + Sakhelashvili ’21, + Venugopalan ’21 + …

- Relation between the maximal spin and the entropy

G. Dvali, F. Kühnel, **MZ,** ’22,
G. Dvali, O. Kaikov, J. Bermudez, F. Kühnel, **MZ,** ‘24

↓

- Black hole properties are not specific to gravity and can be understood within calculability domains of renormalizable QFTs - useful theoretical laboratories
- Predict new features

Vorticity in saturons

G. Dvali, F. Kühnel, **MZ,** PRL 129 (2022) 6, 061302

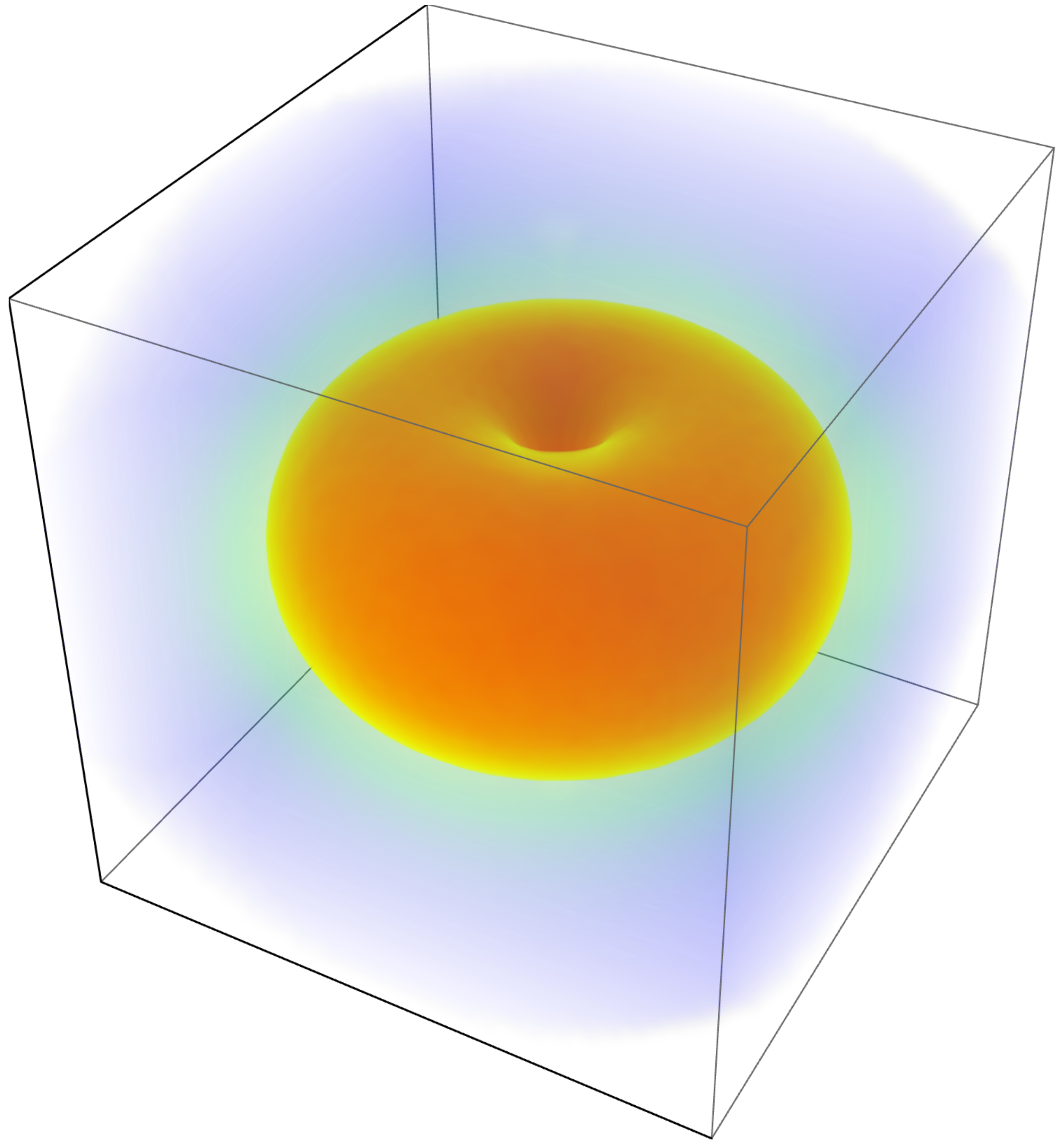
There is a way to spin a saturon bubble in an axial-symmetric way: Vorticity

$$\theta = \omega t + n\,\varphi$$

winding number $= n = 0, \pm 1, \pm 2, \ldots$ $\varphi =$ polar angle

Angular momentum $J = n\,N_{\text{Gold}} = S$ at saturation $(n \sim \mathcal{O}(1))$

Profile for $n = 1$



Construction has similarities with spinning U(1) Q-ball see Volkov, Wohnert '02

Vorticity in saturons

G. Dvali, F. Kühnel, **MZ,** PRL 129 (2022) 6, 061302

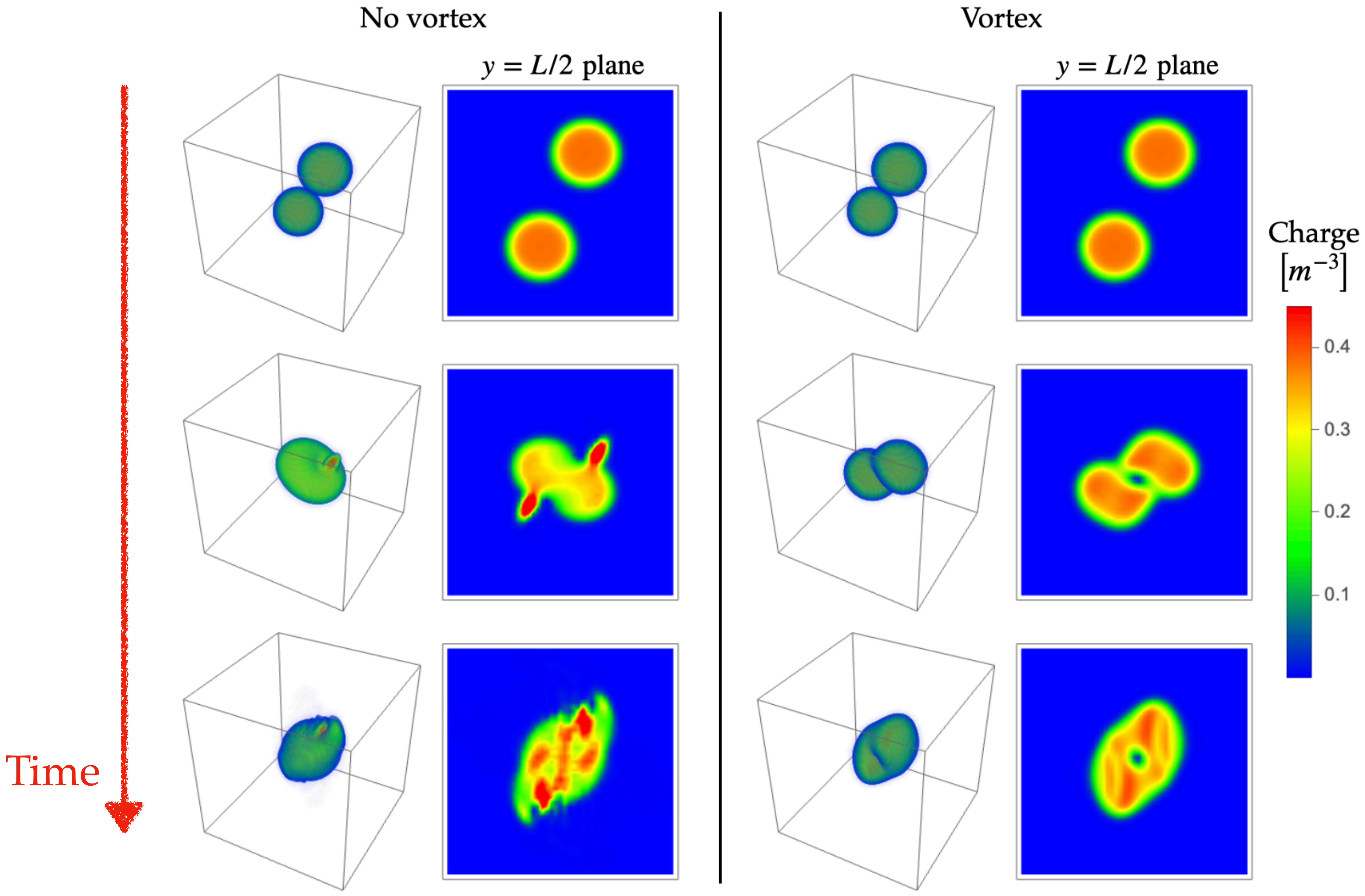
	Saturon bubble	Black hole
Maximal spin	S $n \sim \mathcal{O}(1)$	S_{BH}

- Saturon of a renormalizable QFT and black holes obey the same bound on spin
- Microscopic interpretation of extremality bound in terms of vorticity
- Topological explanation of the absence of Hawking (soft) radiation due to macroscopic integer nature of the vortex
- Can vorticity be a property manifesting in highly spinning black holes? If so, pheno consequences?

Vorticity in saturons

G. Dvali, F. Kühnel, **MZ**, PRL 129 (2022) 6, 061302 + G. Dvali, O. Kaikov, J. Bermudez, F. Kühnel, **MZ**, PRL 132 (2024) 15, 151402

Example: Study the impact of vorticity in saturated configurations. Could similar features emerge in black hole mergers?



- Due to the integer nature of the vortex, its emergence leads to macroscopic deviations in the emitted radiation.

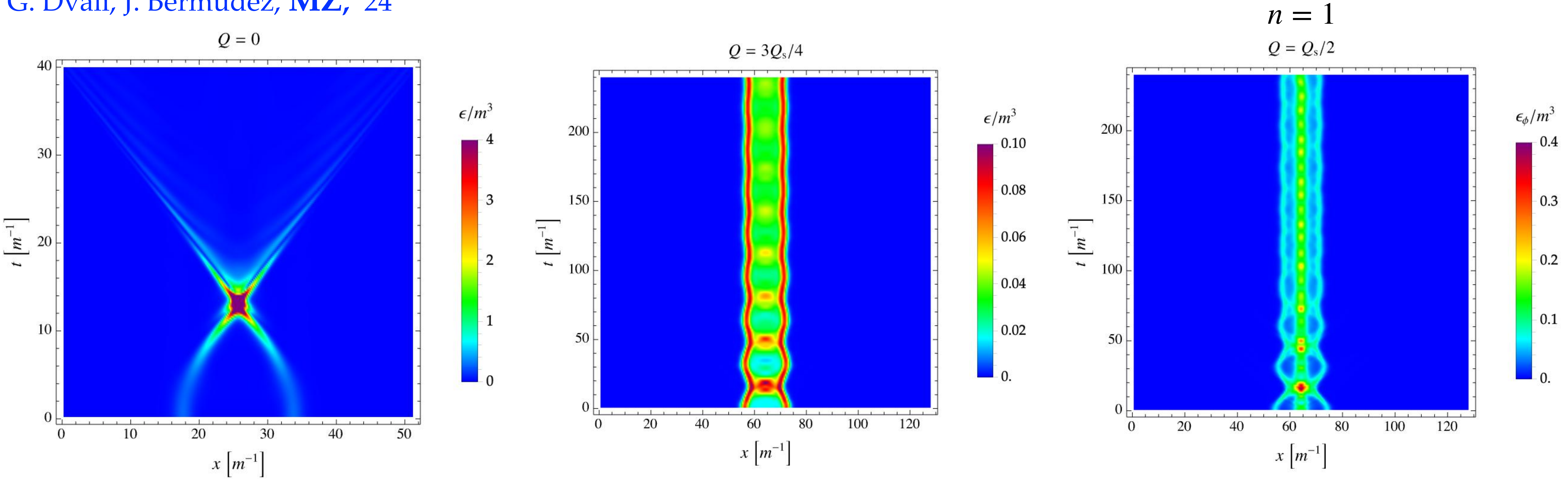
1. Similar behaviours are expected in black hole mergers if vorticity localizes in the intermediate configuration.

Dynamics

Example of bubble dynamically stabilized by memory.

Dynamics

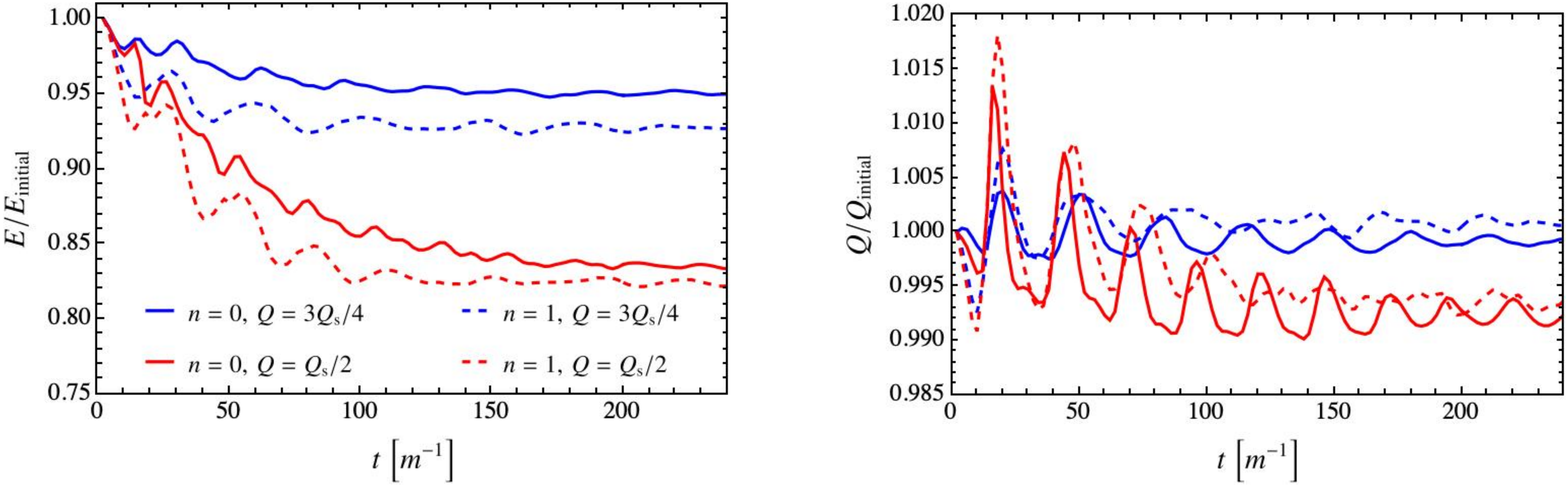
G. Dvali, J. Bermudez, **MZ,** ‘24



- The bubble without memory (left panel) simply annihilates (zero charge).
- The bubble with charge - central and right panel - is stabilized.
- The stabilisation is independent on other details such as vorticity, width of the bubble wall or couplings to new degrees of freedom.

Dynamics

G. Dvali, J. Bermudez, **MZ,** '24



- Charge is conserved within 1%. These configurations are, in fact, highly efficient at storing information
- Presence of information horizon in Goldstone flavour space
- Different asymptotic energies depending on the initial information stored in the configuration

Implications for black holes

G. Dvali, J. Bermudez, **MZ,** ‘24

- Memory burden effects seems inevitable in all configurations with a high-information storage capacity, such as saturons.

- Black holes are indeed saturated configurations.

- It is therefore expected that black holes experience the memory burden effect.

This is also supported by the so-called black hole N-portrait (Dvali, Gomez ’11), where black holes are described in terms of condensate of marginally bounded gravitons

Implications for black holes

G. Dvali, J. Bermudez, **MZ,** ‘24

The detailed analysis shows that the memory burden effect sets-in latest by half-decay. After this point, the evaporation/decay is slowed down so that the lifetime is extended as:

$$t \sim R\, S^{n+1}$$

$n =$ positive integer. This is because the decay rate is analytic both in number of memory modes as well as in the coupling, which scale as:

$$\alpha \sim 1/S\,, \quad N_{\mathrm{Gold}} \sim S$$

Already for $n = 1$ - seemingly the most motivated value - new window for PBHs DM opens up for masses $10^3\mathrm{g} \lesssim M_{\mathrm{PBH}} \lesssim 10^{14}\mathrm{g}$.

Formation mechanism in new mass window, based on confinement of heavy quarks

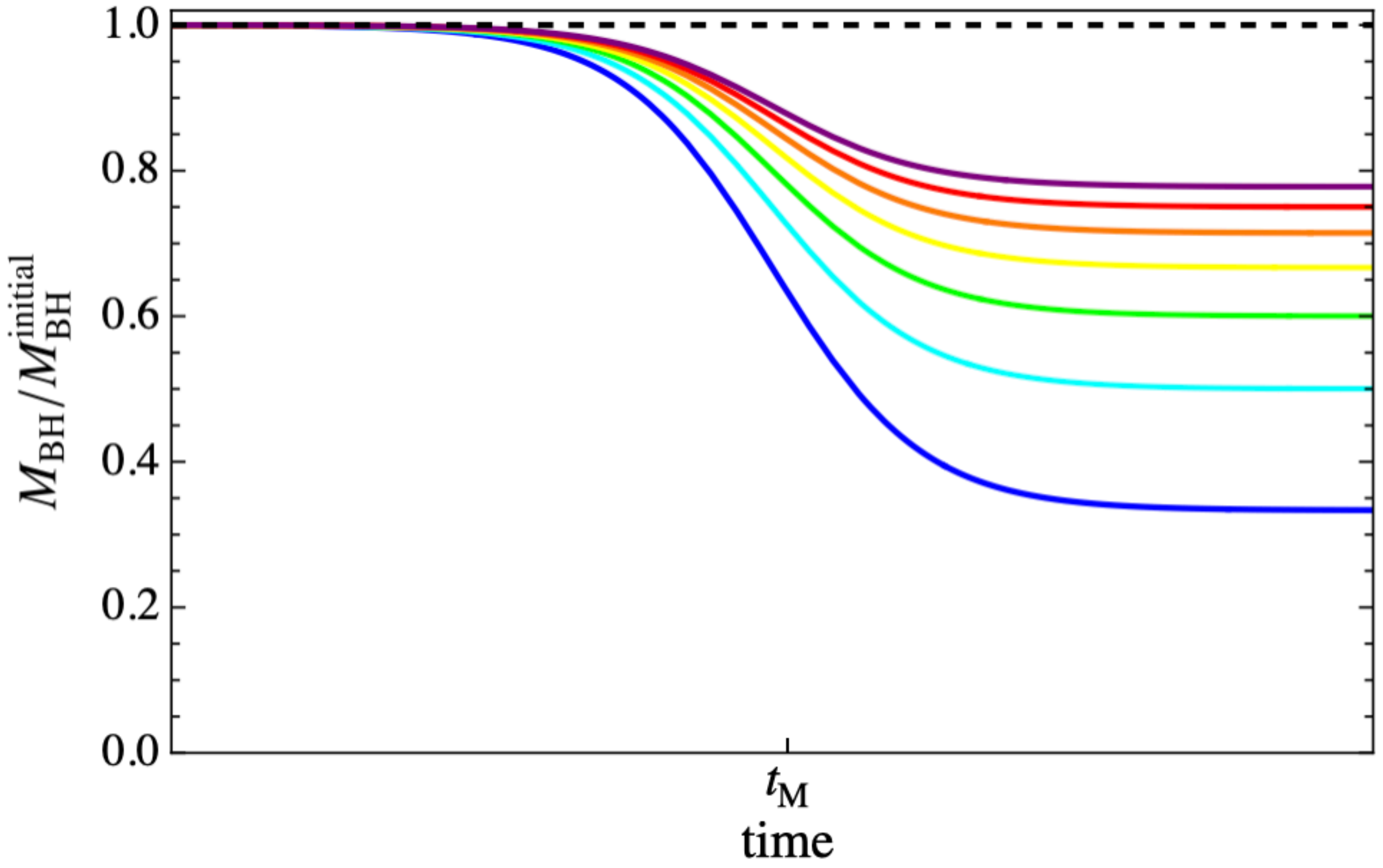
Dvali, Eisemann, Michel, Zell, ’21; Dvali, Kühnel, **MZ**, ’21
Alexandre, Dvali, Koutsangelas, '24; Thoss, Burkert, Kohri, ’24; Dvali, Bermudez, **MZ,** ‘24

Implications for black holes

G. Dvali, J. Bermudez, **MZ,** ‘24

$$|\text{Pattern}\rangle = |n_1, n_2, \ldots, n_S\rangle \quad \text{With: } \sum_a^S n_a = N_G$$

The spread in masses of stabilised remnants is determined by the statistical distribution of N_G among the initial black holes. Assuming no energy bias



$$\mathscr{P}_{N_\mathrm{G}} = 2^{-S}\frac{S!}{(S-N_\mathrm{G})!N_\mathrm{G}!}$$

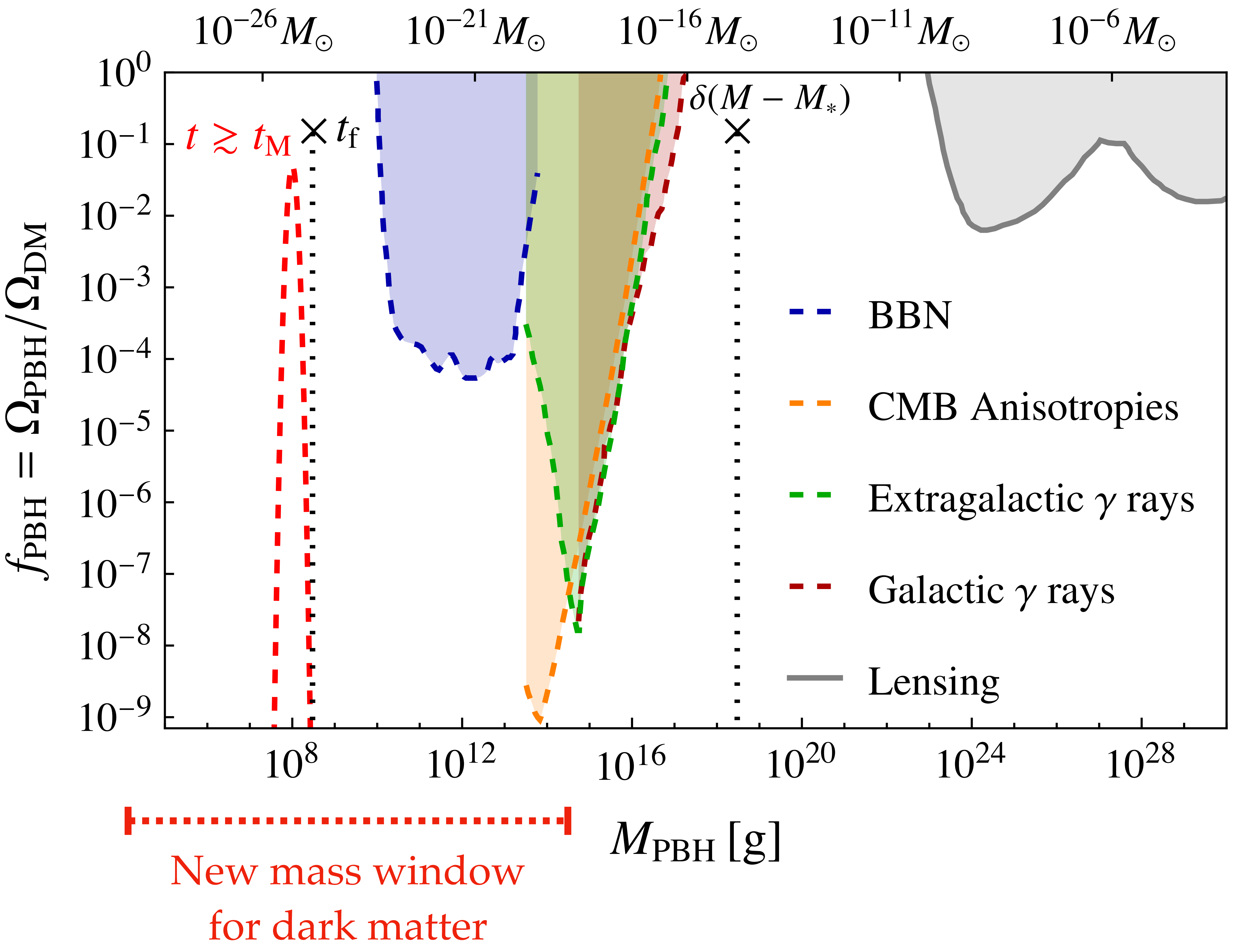
which is maximal for $N_\mathrm{G} \simeq S/2$, $\mathscr{P}_{S/2} \sim 1/\sqrt{S}$, while the width is $\sqrt{S}/2$.

On the other hand, for $N_\mathrm{G} \ll S$, the probability is exponentially suppressed as,

$$\mathscr{P}_{N_\mathrm{G}\ll S} \sim 2^{-S}\left(\frac{S\,\mathrm{e}}{N_\mathrm{G}}\right)^{N_\mathrm{G}}.$$

Consequences for black holes as dark matter

G. Dvali, J. Bermudez, **MZ,** ‘24



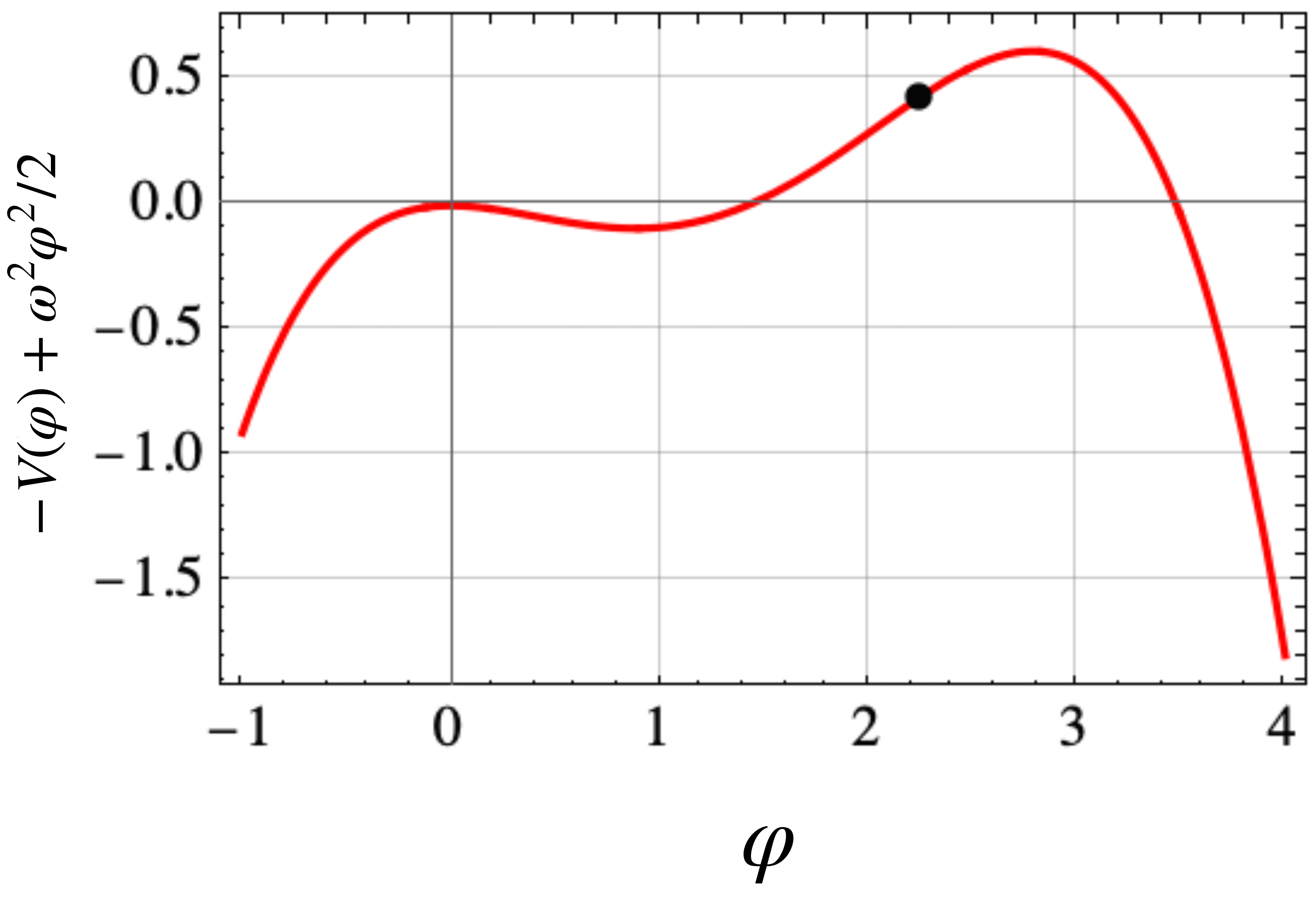
- A new mass window for dark-matter PBHs stabilised by their memory opens up below 10^{14}g
- In such mass widnow, PBHs are normally discarded as potential dark matter candidates because they are naively too-short lived. However memory burden effect stabilizes them.
- Novelty of our work: any initial distribution in such region will observe a natural spread, of width of order initial mass, on time scales longer than the memory burden back reaction time $t_{\rm M}$
- An example of this is depicted in the figure. Shaded areas denote existing constraints (See Thoss Talk on Tuesday for in depth discussion), while dotted lines correspond to two monochromatic distribution of mass M_* formed at time $t_{\rm f}$
- The distribution in the light mass region, is stabilised and spread by the memory burden effect, resulting in the pictorially denoted dashed-red distribution at late times

Backup

Vacuum bubble with high information-storage capacity

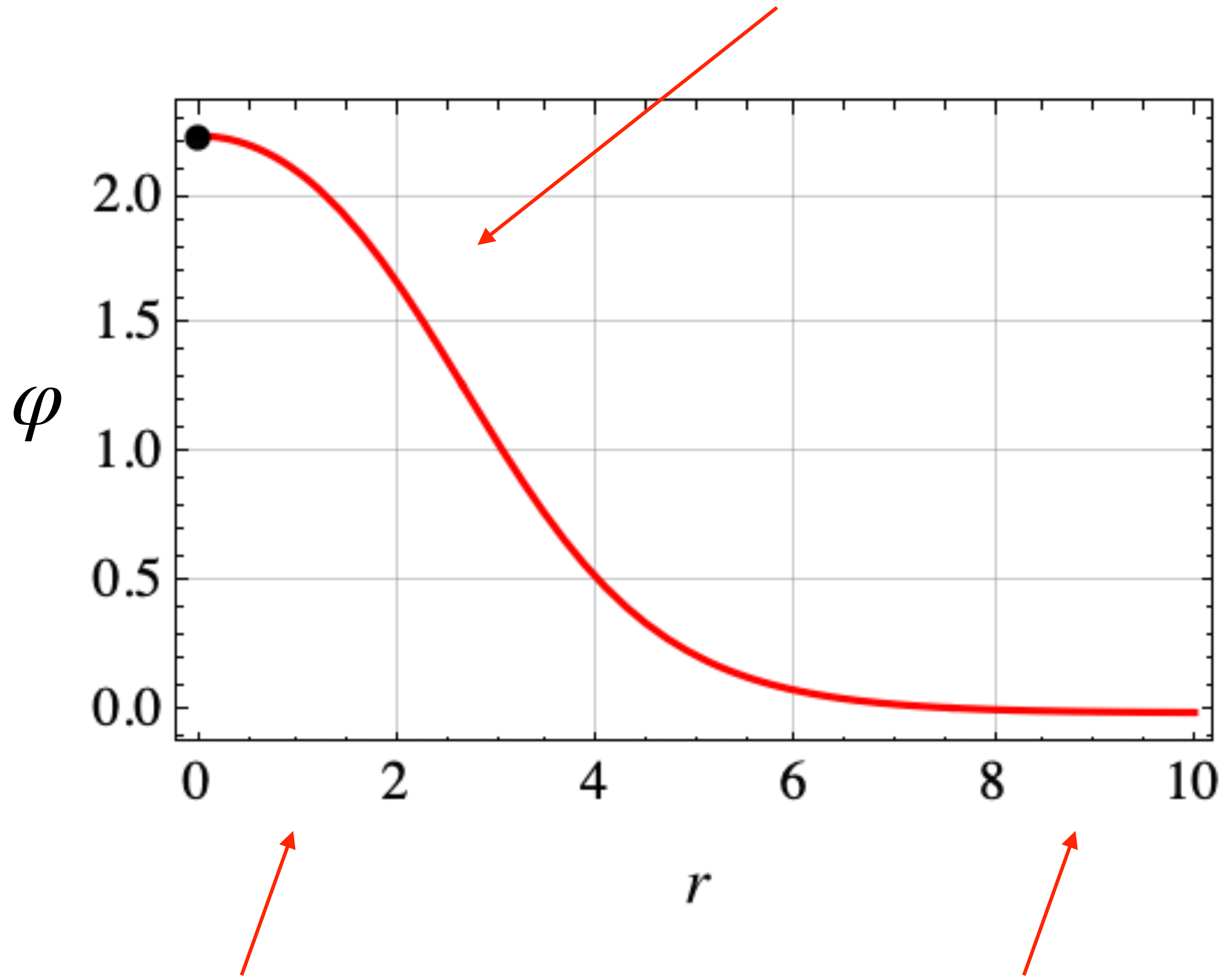
$n = 0$ stationary solution example with charge Q_s:

$$\partial_r^2\varphi(r) + \frac{2}{r}\partial_r\varphi(r) + \omega^2\varphi(r) - \frac{\partial V[\varphi]}{\partial\varphi} = 0$$



$\Phi_D \propto \varphi(r)\,\mathrm{diag}\,(N-1,-1,-1,\ldots,-1)$

Wall-Thickness $\sim 1/m$



$SU(N-1)\times U(1)$
vacuum

$SU(N)$
vacuum

Memory burden in many body approach

G. Dvali, arXiv:1810.02336 [hep-th]; G. Dvali, L. Eisemann, M. Michel, S. Zell, arXiv:2006.00011 [hep-th];
G. Dvali, J. Bermudez, **MZ**, arXiv:2405.13117 [hep-th];

Consider the following simple prototype Hamiltonian

Positive factor

$$\hat{H} = m_\phi \hat{n}_\phi + \left(1 + \frac{\hat{n}_\phi}{N_\phi}\right)^q \sum_j m_j \hat{n}_j + \frac{\tilde{m}}{\sqrt{N_\phi}} \hat{b}^\dagger \hat{a}_\phi + \frac{\tilde{m}^*}{\sqrt{N_\phi}} \hat{a}_\phi^\dagger \hat{b} + m_\phi \hat{n}_b$$

Memory modes - Charge of the bubble

Master mode - profile mode "$|\varphi(r)|$"

Hawking emission - added dof for relaxation

- Memory is stored in a pattern $|n_1, n_2, \ldots, n_N\rangle =$ "$|1,0,0,\ldots,1,0\rangle$".
- The effective gap of memory mode is $\omega_j \equiv \left(1 - \frac{n_\phi}{N_\phi}\right)^q m_j$, gapless for initial condition $\langle \hat{n}_\phi \rangle = N_\phi$
- As n_ϕ decreases, the memory modes backrest on the master modes, stopping its mixing with $\hat{b}$ modes
- Mapping of above system to bubble case: $q = 2/3, \ldots$

Implications for black holes

The universality of memory burden effect shows that it must be shared by all systems of enhanced capacity of information storage (saturons), including black holes.

This statement is independent of a particular microscopic theory of a black hole.
However it is always useful to have one, in order to make things explicit.

Such a framework is provided by ``black hole's quantum N-portrait" (Dvali, Gomez '11).

This theory describes a black hole of radius R as saturated coherent state of soft gravitons of wavelengths R and occupation number:

$$N_g = (RM_P)^2$$

These gravitons are analogous to the radial (profile) mode - ''master mode" - localizing the symmetry broken region in the bubble case.
This picture makes the origin of memory modes explicit: They represent Goldstone-type excitations of very short wavelength gravitons. In black hole background, they are gapless in the way very similar to Goldstones of a vacuum bubble.

Dvali, Gomez '12,'13; Dvali, Franca, Gomez, Wintergerst, '13; Averin, Dvali, Gomez, Lust, '15, Dvali '17,'18,…

More on memory modes in black holes

The number of flavours of memory mode should be of order

$$N \sim S_{\mathrm{BH}}.$$

A suitable candidates are the graviton modes corresponding to various spherical harmonics $Y_{l,m}$.

All modes up to the cutoff M_{Pl} ought to be included in the counting. Notice that these correspond to modes of angular momentum of order RM_{Pl}.

Their multiplicity is therefore the needed one

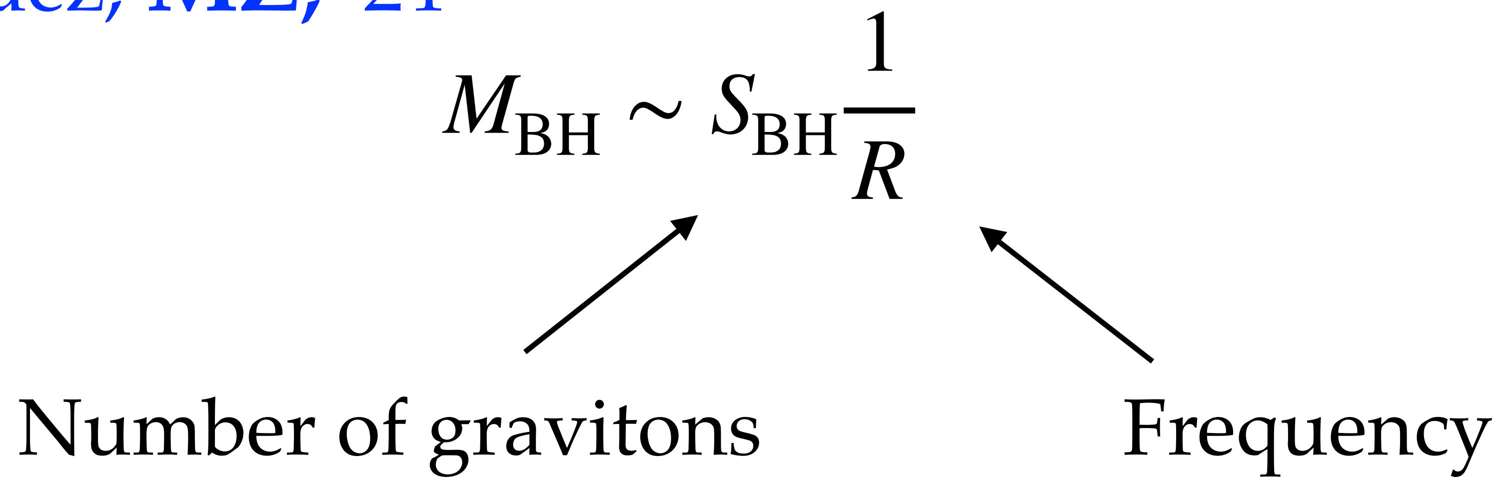
$$N \sim l^2 \sim (RM_{\mathrm{Pl}})^2 \simeq S_{\mathrm{BH}}$$

These modes have a gap of order M_{Pl}. However, they are rendered gapless by the black hole master mode with occupation number N_g (order parameter). These are effectively the Goldstones associated to the Poincaré symmetry broken by the black hole itself.

Averin, Dvali, Gomez, Lust, ’15, Dvali ‘18

Classical vs quantum memory

G. Dvali, C. Gomez ’11; G. Dvali, J. Bermudez, **MZ,** ‘24



$$M_{\rm BH} \sim S_{\rm BH}\frac{1}{R}$$

Number of gravitons

Frequency

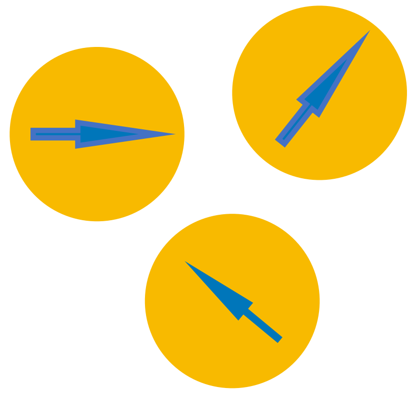
The occupation number of the master graviton sets the upper bound on other occupation numbers.
That is, for a black hole the occupation numbers of master modes satisfy the bound

$$n_{\text{any mode}} \leqslant S_{\rm BH}$$

When the above bound is saturated in black hole, it reaches extremality. This corresponds to presence of some long-range classical hair. The quantum pattern plays no role.
However, the extremal black holes of the same electric charge and mass can carry information patterns of very different content.

In contrast, when a black hole is stabilized by a quantum memory burden, no classical charge associated with a long-range gauge field is required. Of course, a macroscopic ``hair'' emerges in form of a memory burden parameter, $S_{\rm BH}$.
This parameter is in principle measurable by a scattering experiment, but there is no inconsistency with the no-hair properties: it is a quantum object.

Bubble unifies these features as the memory burden corresponds to macroscopic occupation of memory mode.
However, it connects to quantum states via $SU(N)$ rotation to other degenerate states.



Hawking emission vs retrieval of memory

G. Dvali, J. Bermudez, **MZ,** '24

Black hole can emit master modes - initially - reproducing Hawking entropy. The coupling suppression is compensated by the large occupation number

$$\Gamma_{\text{master mode}} \sim \alpha_{\text{gr}}^2 N_{\text{g}}^2 \frac{1}{R} \sim \frac{1}{R}$$

Quantum memory modes $|n_1, \ldots, n_S\rangle$ do not have large occupation number, i.e., no compensation. Their annihilation is very rare:

$$\Gamma_{\text{memory}} \sim \alpha_{\text{gr}}^2 \frac{1}{R} \sim \frac{1}{S^2\, R}$$

As a consequence, it is natural to expect that, due to memory burden effect, the lifetime of the black hole is enhanced as

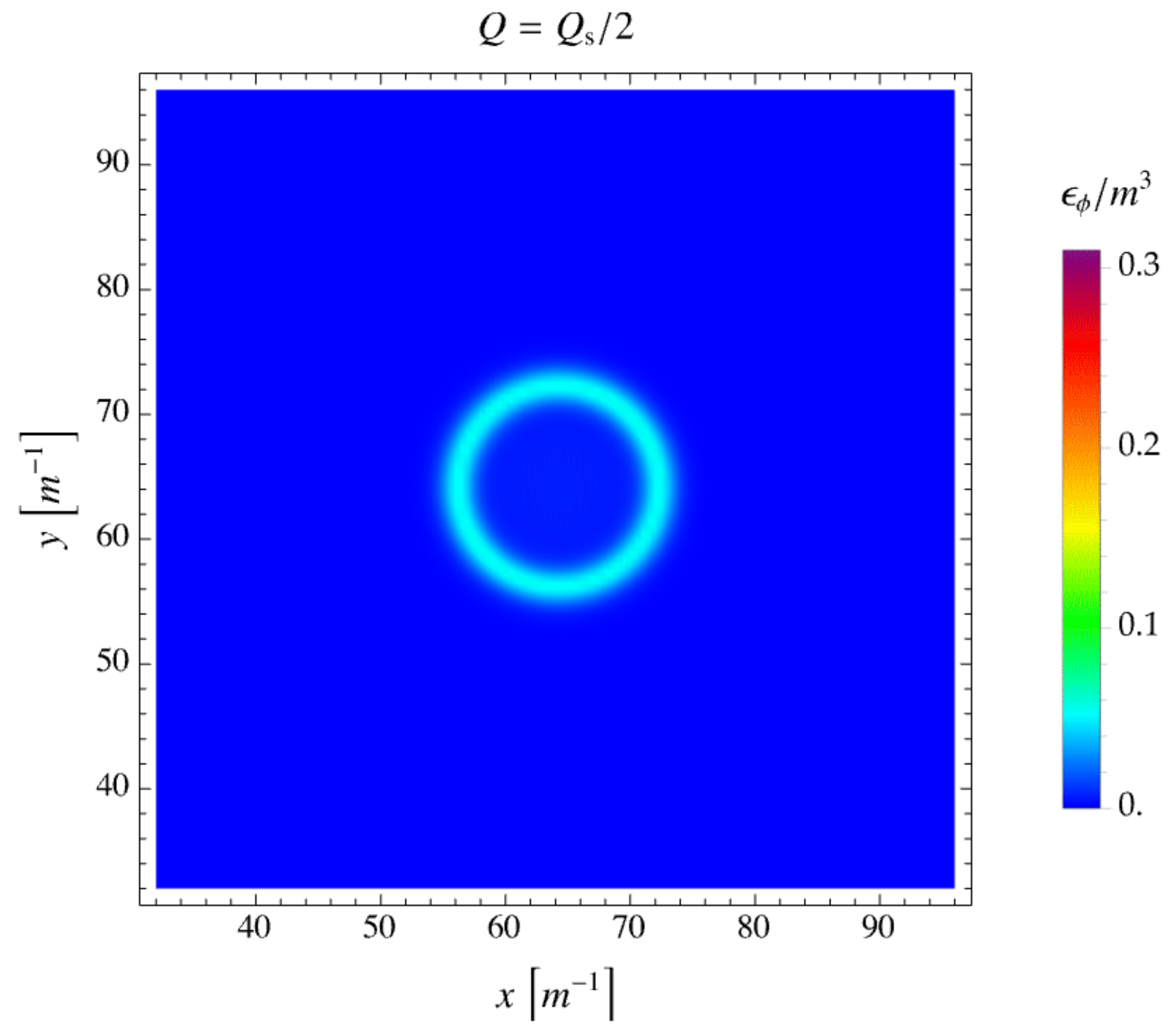
$$\tau_{\text{semiclassical}} \sim R\, S \to \tau \gtrsim \tau_{\text{semiclassical}}\, S^2$$

Model dependent different scalings are possible

Dynamics

2+1 dimensional perspective

- Analogous dynamics is observed if a $SU(N)$ -singlet χ is derivatively coupled to the bubble with interaction $\chi \mathrm{Tr}\left[(\partial_\mu \phi)(\partial^\mu \phi)\right]$


$Q = Q_s/2$
ϵ_ϕ/m^3
0.3
0.2
0.1
0.
90
80
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$y\ [m^{-1}]$
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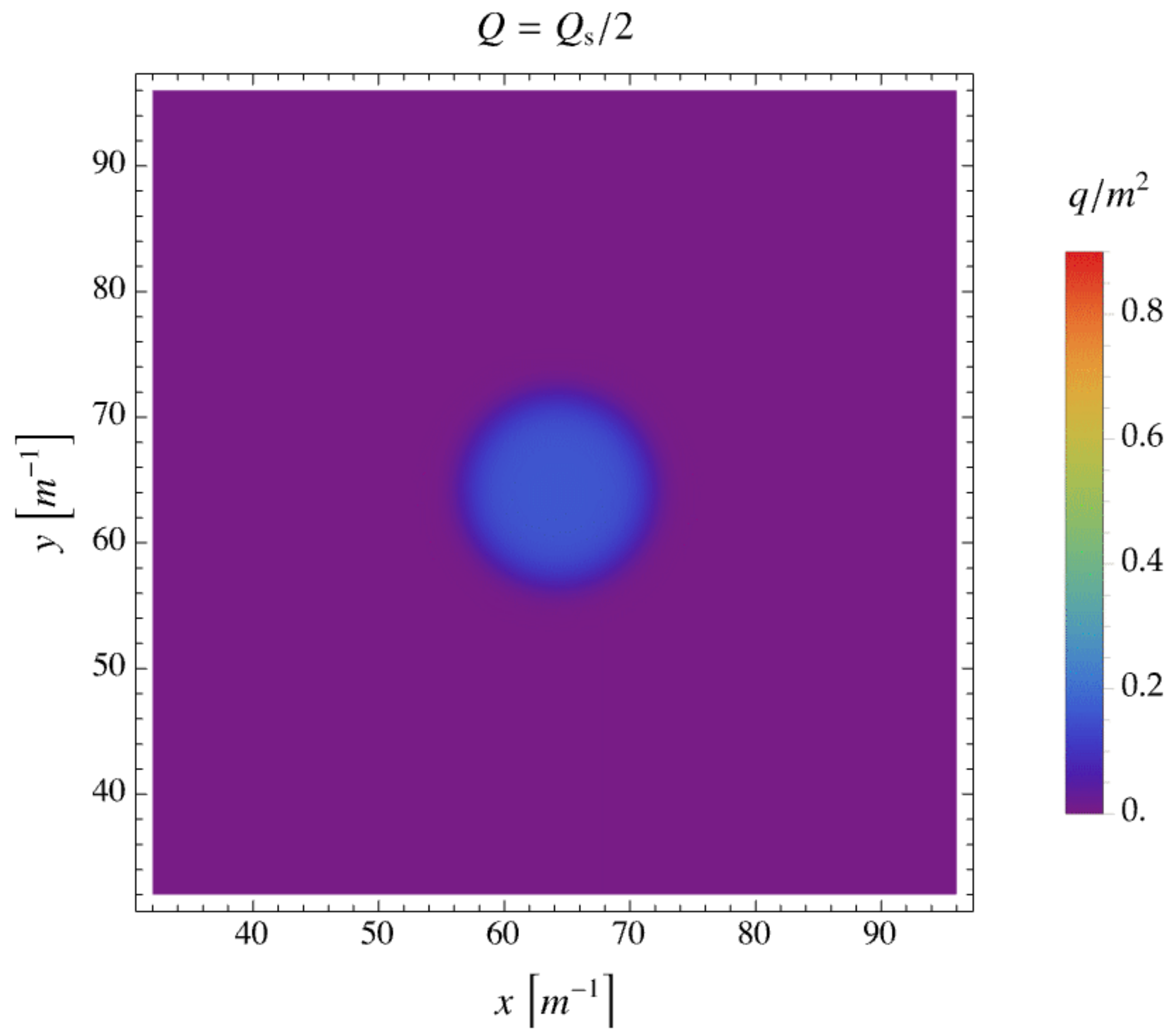
$x\ [m^{-1}]$


$Q = Q_s/2$
q/m^2
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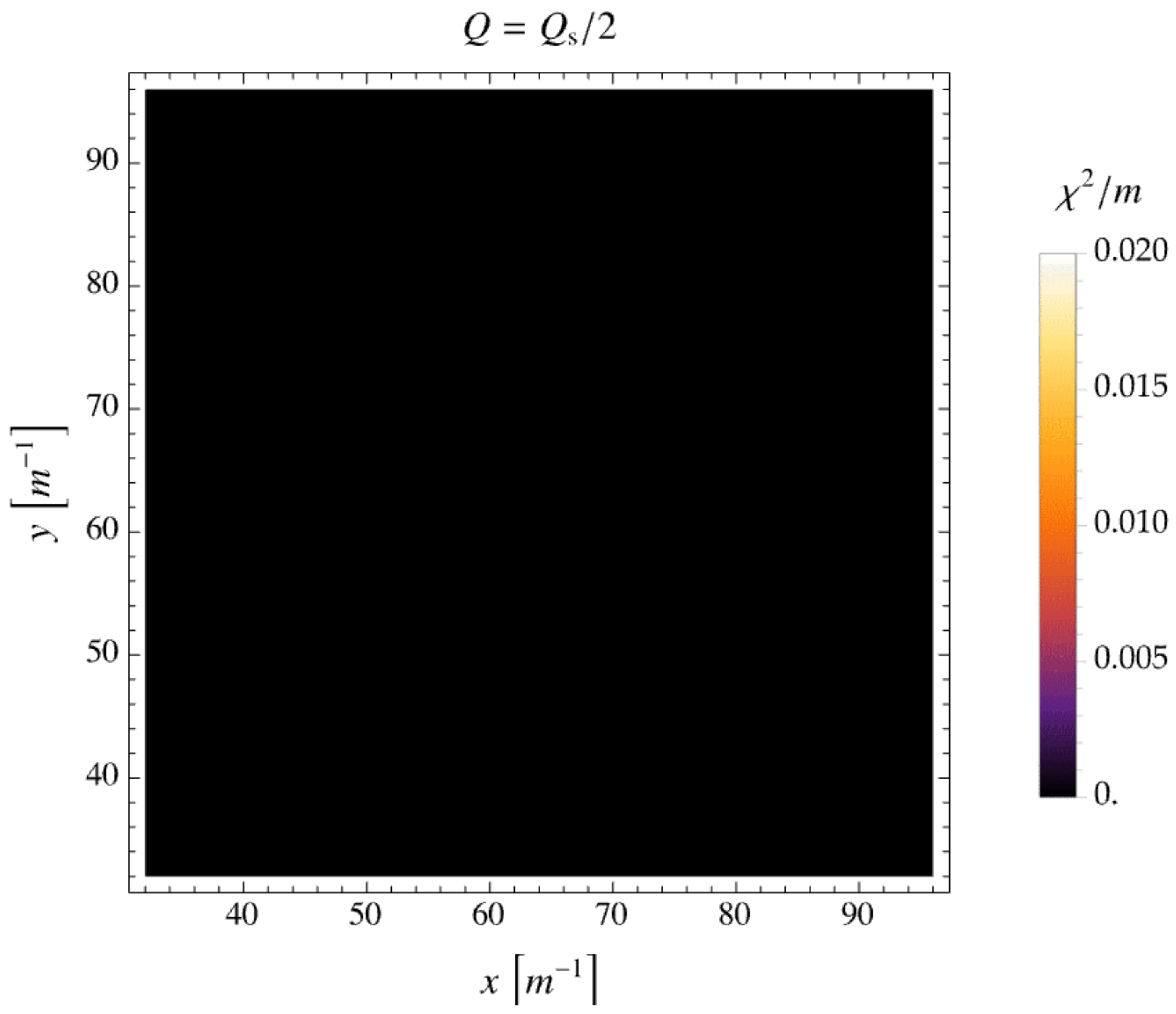
$Q = Q_s/2$
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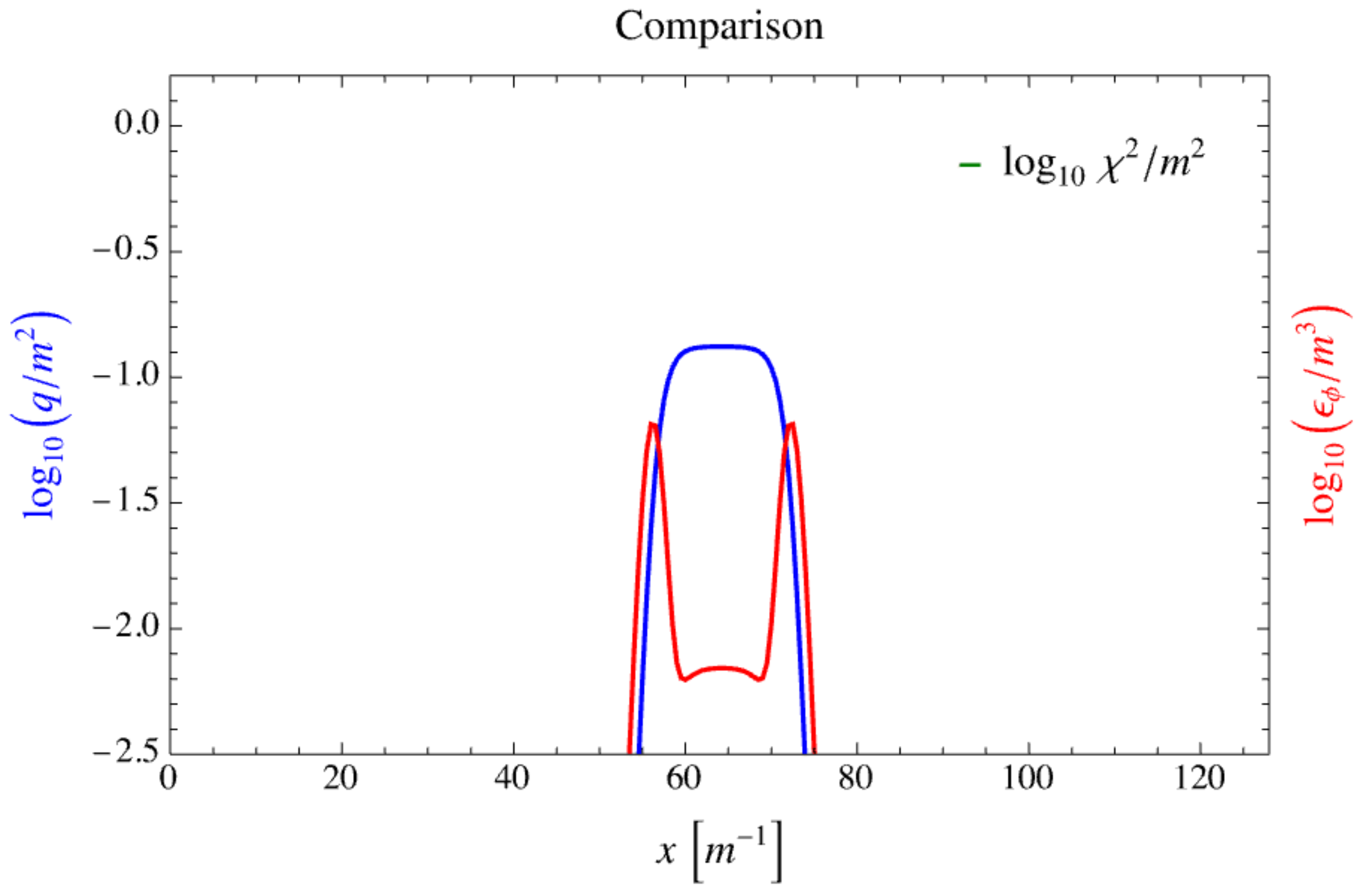
Comparison
$\log_{10} \chi^2/m^2$
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−2.5
$\log_{10}(q/m^2)$
$\log_{10}(\epsilon_\phi/m^3)$
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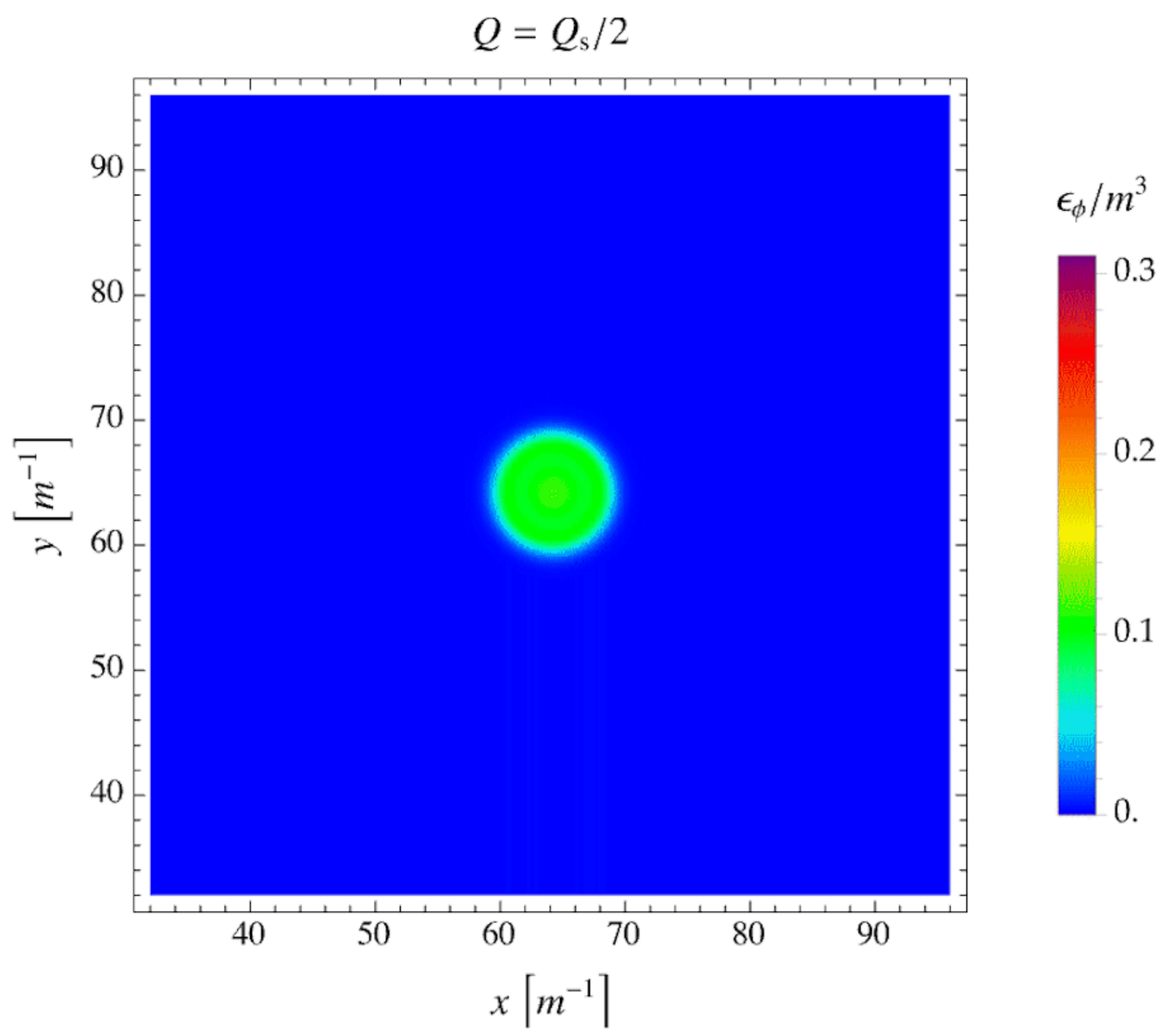
60
80
100
120
$x\ [m^{-1}]$


$Q = Q_s/2$
ϵ_ϕ/m^3
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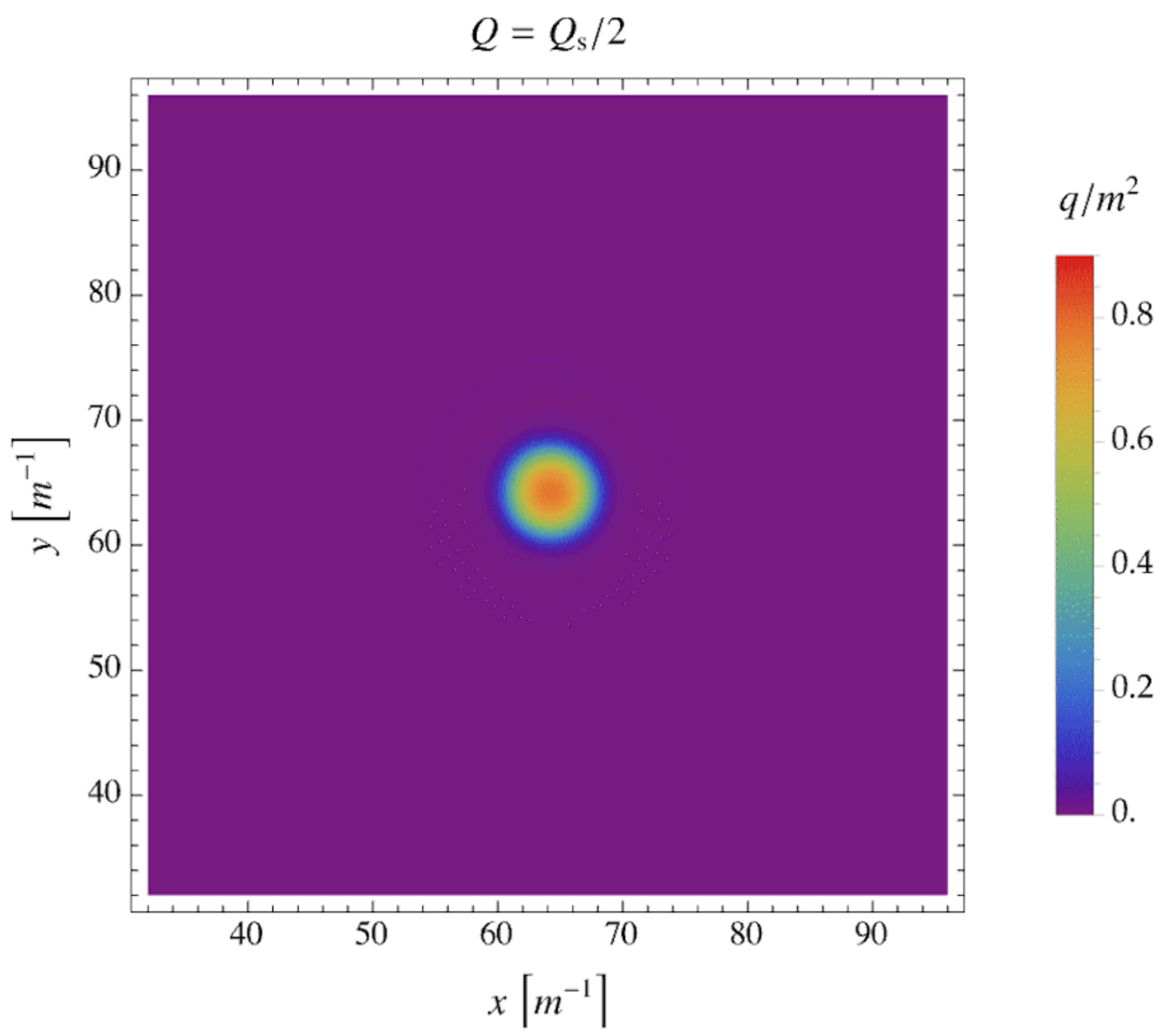
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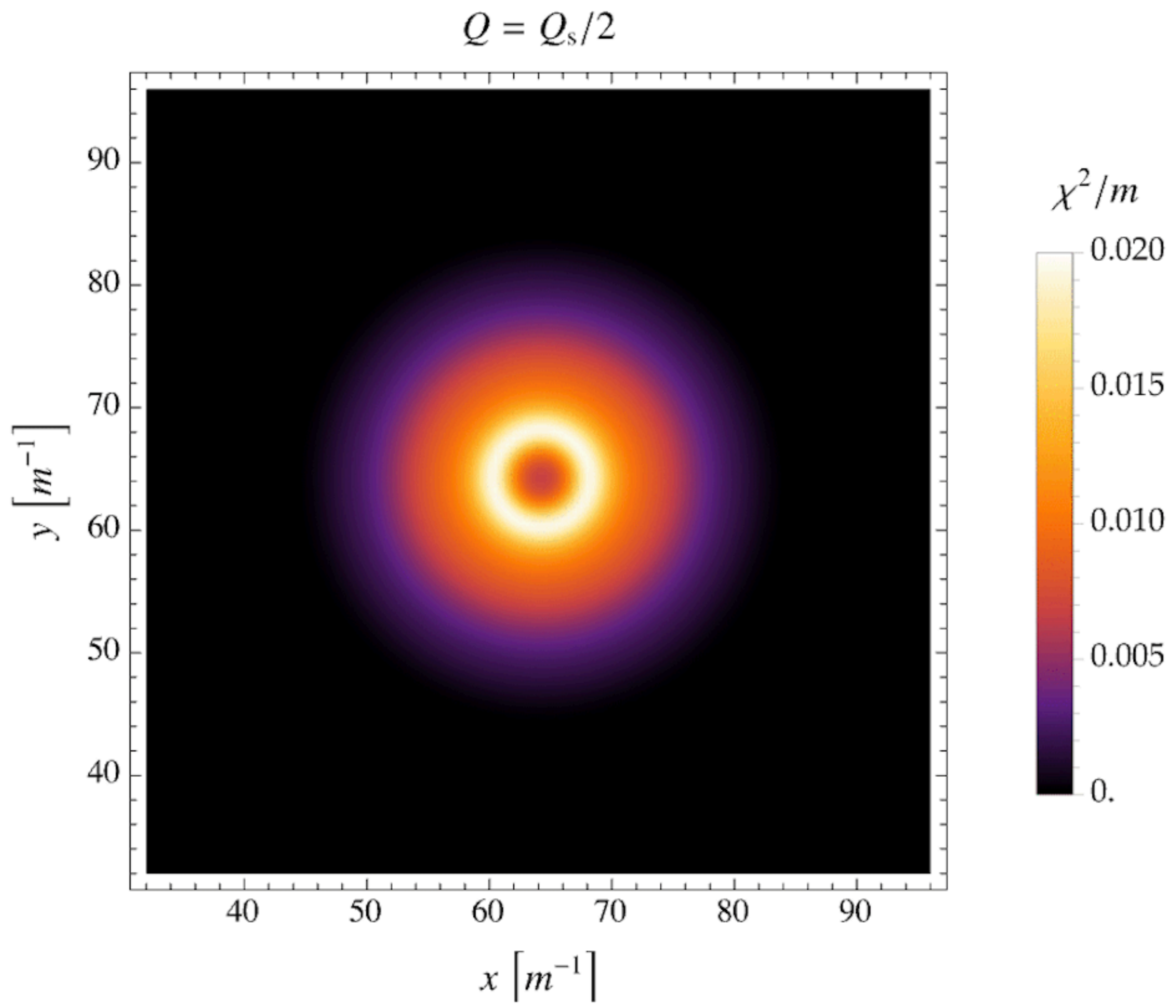
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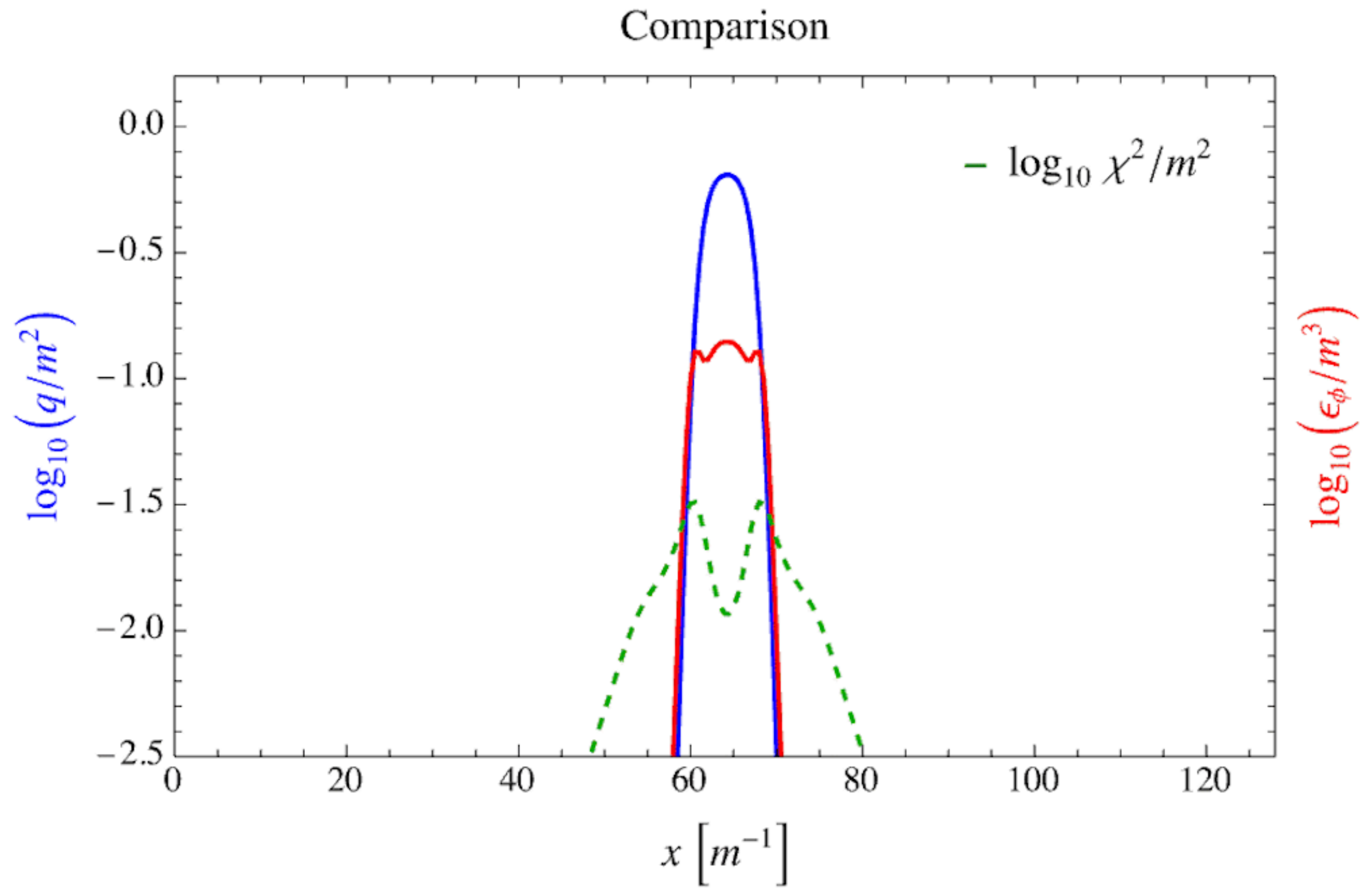
Comparison
$\log_{10}\chi^2/m^2$
$\log_{10}(q/m^2)$
$\log_{10}(\epsilon_\phi/m^3)$
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0
20
40
60
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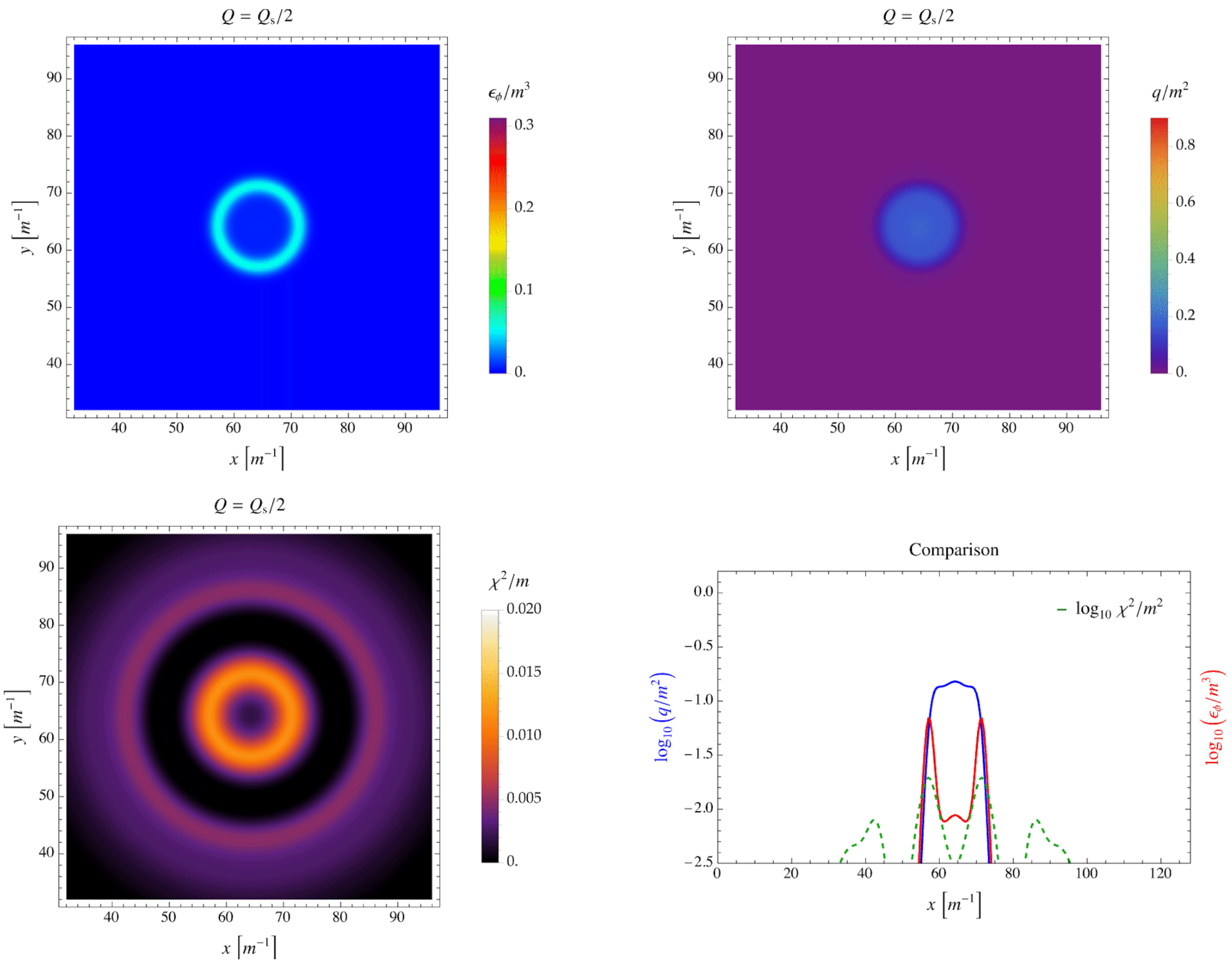
100
120
$x\ [m^{-1}]$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ϵ_ϕ/m^3
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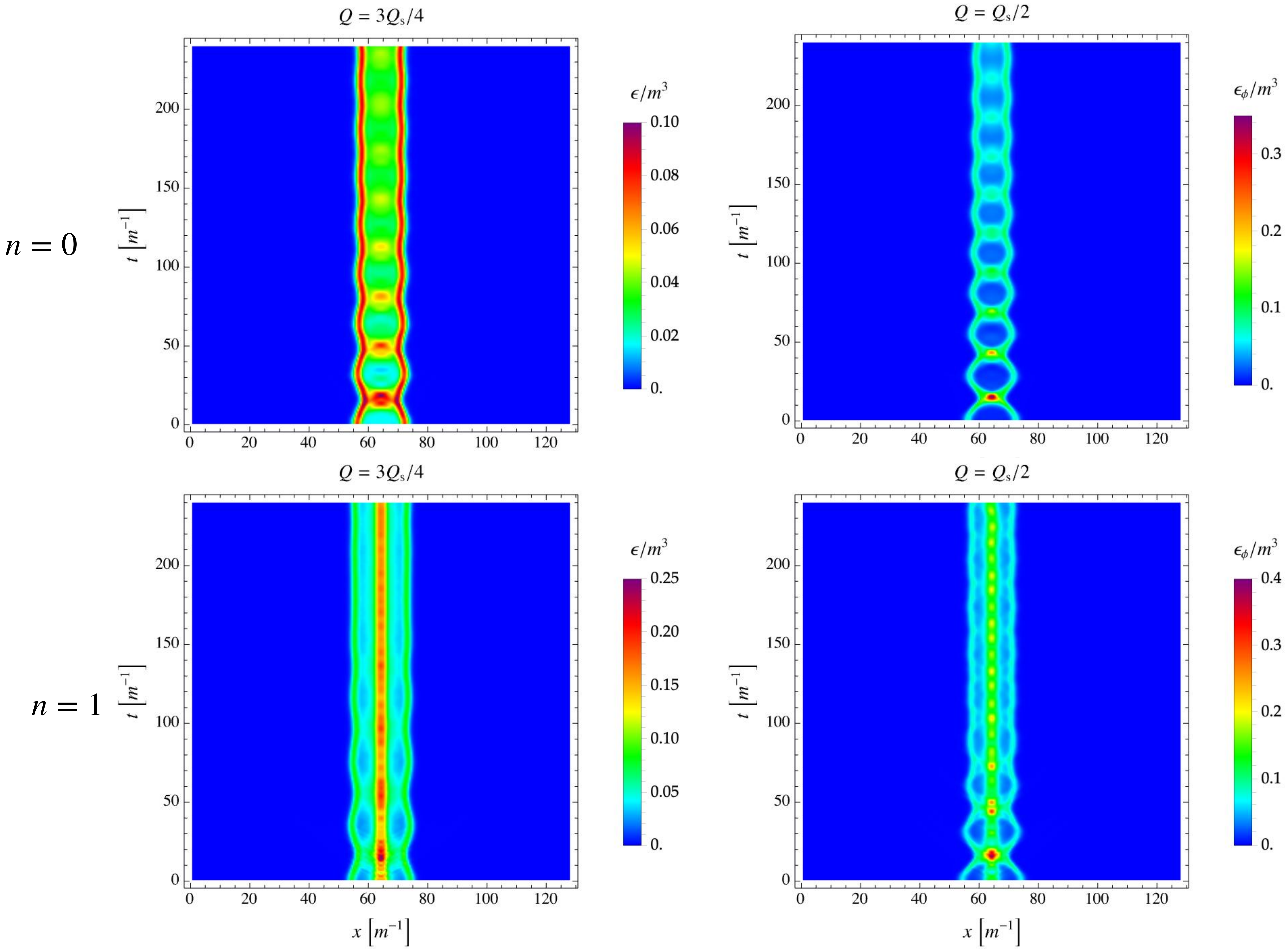
0.2
0.1
0.
$y\ [m^{-1}]$
$x\ [m^{-1}]$
40
50
60
70
80
90
$Q = Q_s/2$
q/m^2
0.8
0.6
0.4
0.2
0.
$Q = Q_s/2$
χ^2/m
0.020
0.015
0.010
0.005
0.
Comparison
$\log_{10}\chi^2/m^2$
$\log_{10}(q/m^2)$
$\log_{10}(\epsilon_\phi/m^3)$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
20
40
60
80
100
120

Dynamics

The same mechanism of stabilisation takes place in the case of bubble endowed with vorticity, characterised by integer winding number $n = 1$

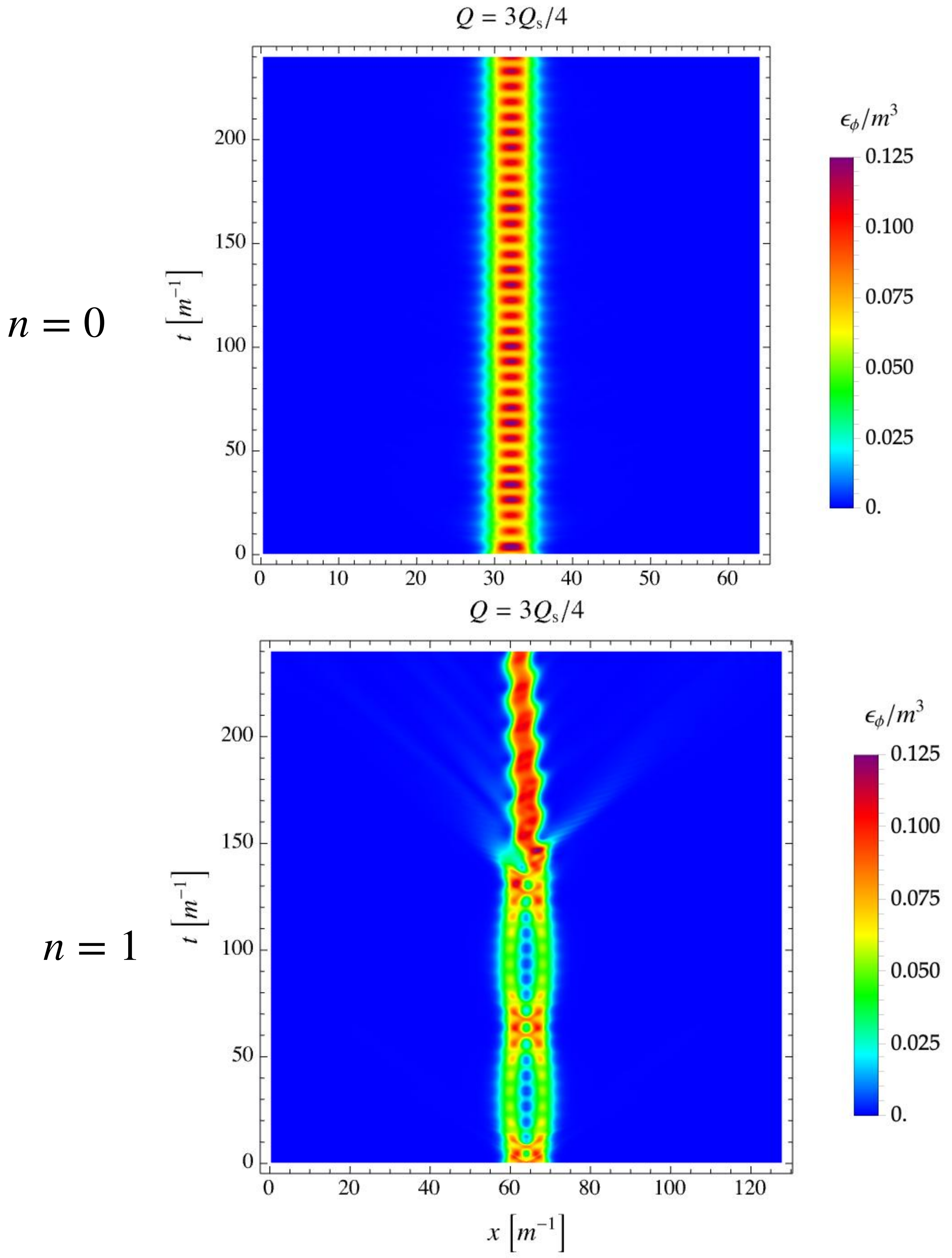
Dynamics



Energy density as a function of time for different initial memories, $Q = 3Q_s/4$ (left column) and $Q = Q_s/2$ (right column), for winding number $n = 0$ (top row) and $n = 1$ (bottom row). Energy is emitted in χ quanta, relaxing the configuration.

It carries no information.

Dynamics



- Analogous dynamics is observed for bubbles in thick-wall regime

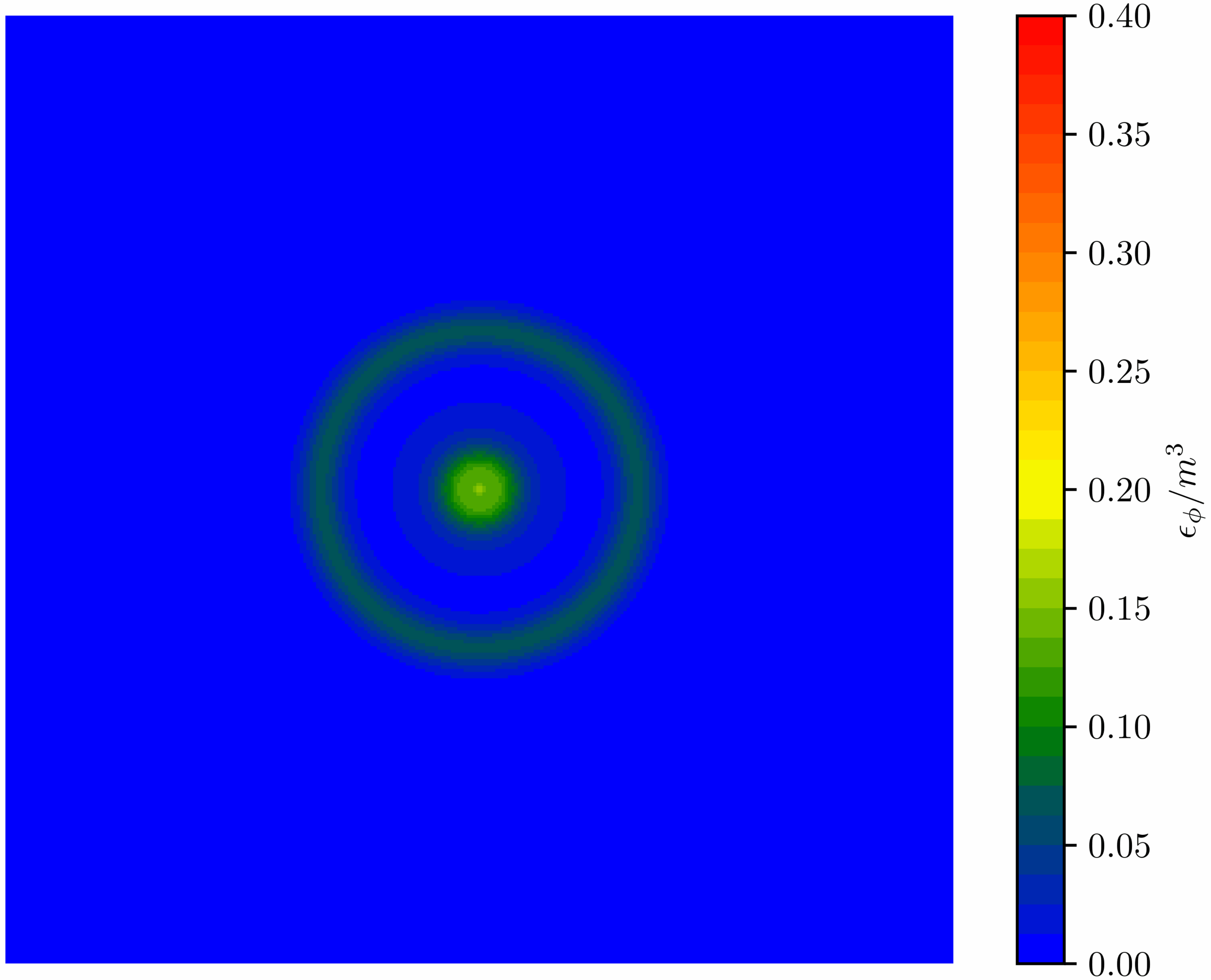
- In the case of vorticity, at late times the bubble sometimes ejects the vortex (bottom panel) or fragments

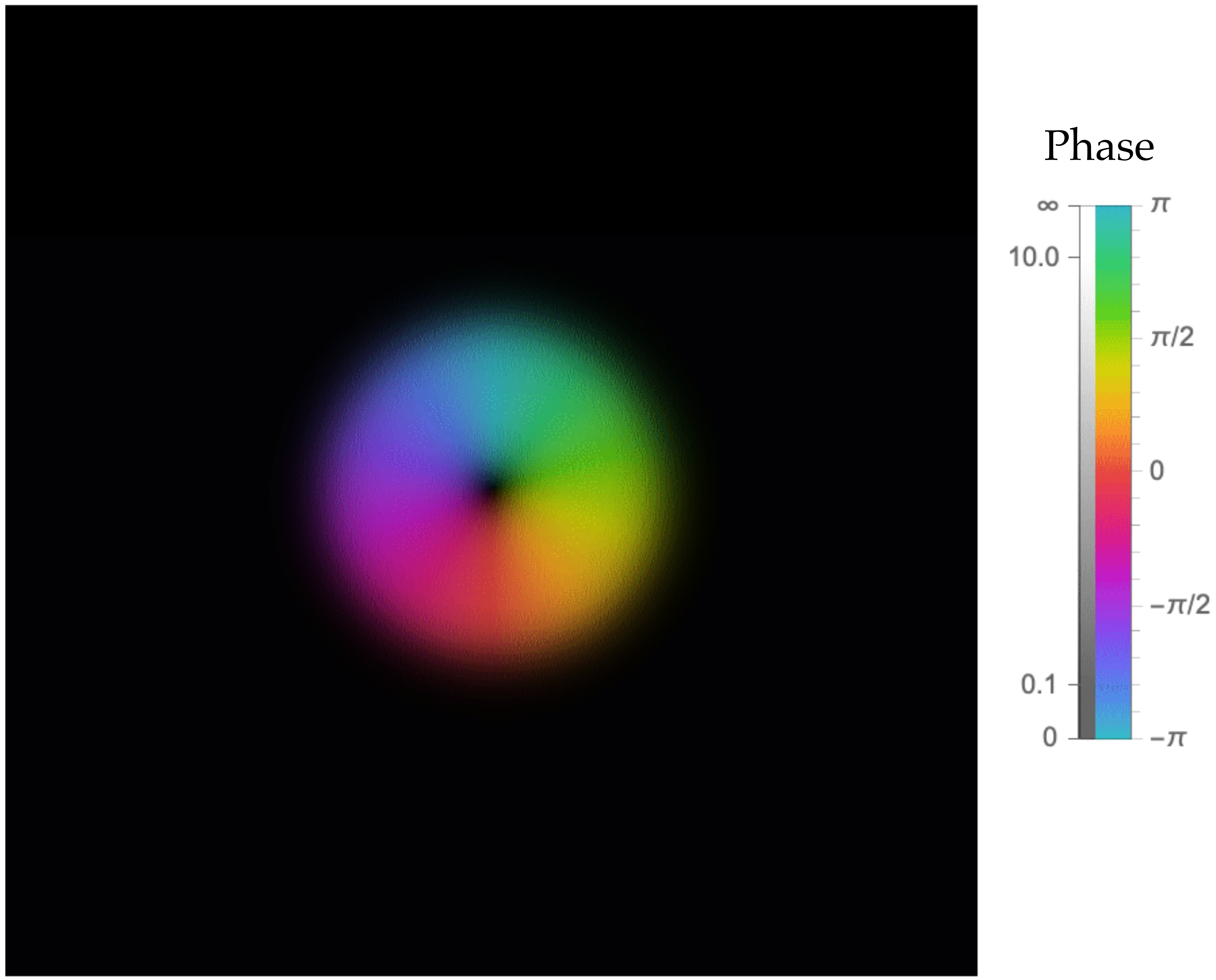
- In the former case, the residual configuration is left with vanishingly small angular momentum - we characterised this phenomenon already in arXiv:2310.02288 - Dvali, Kaikov, Kühnel, Valbuena Bermúdez, Zantedeschi, *Phys. Rev. Lett.* 132 (2024) 15, 151402

Dynamics

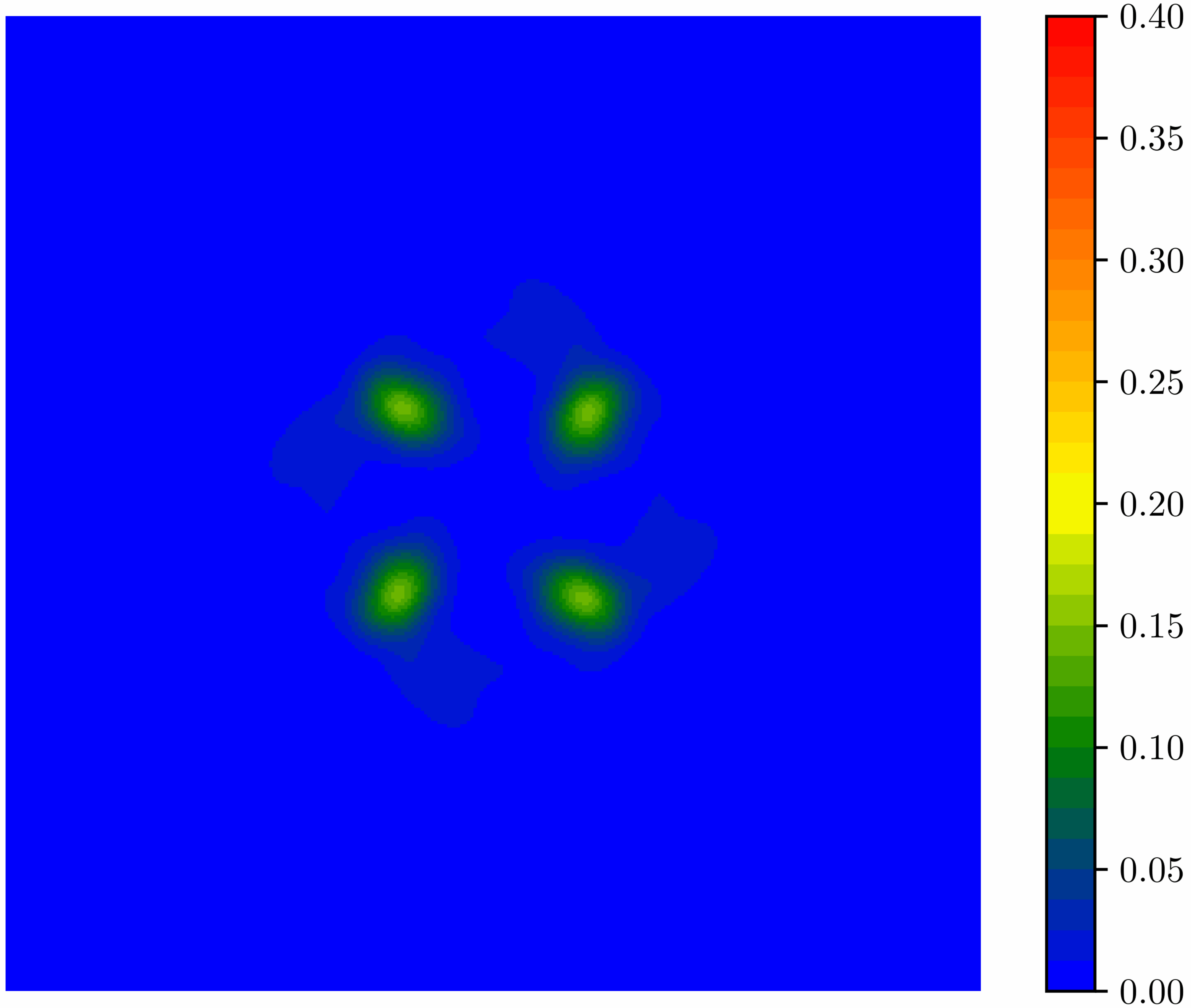
Example of fragmentation

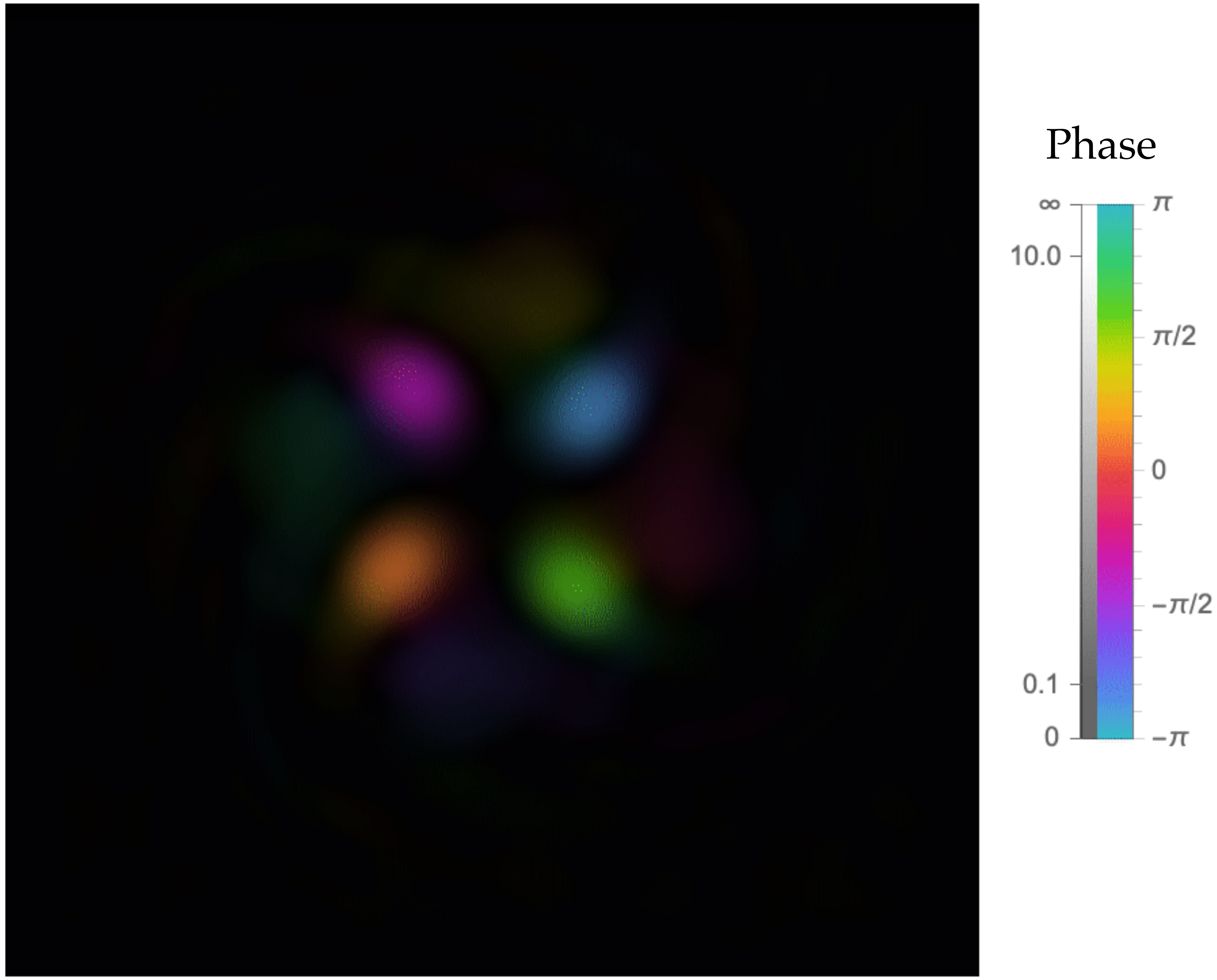
Dynamics


0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
ϵ_ϕ/m^3


Phase
∞
10.0
0.1
0
π
$\pi/2$
0
$-\pi/2$
$-\pi$

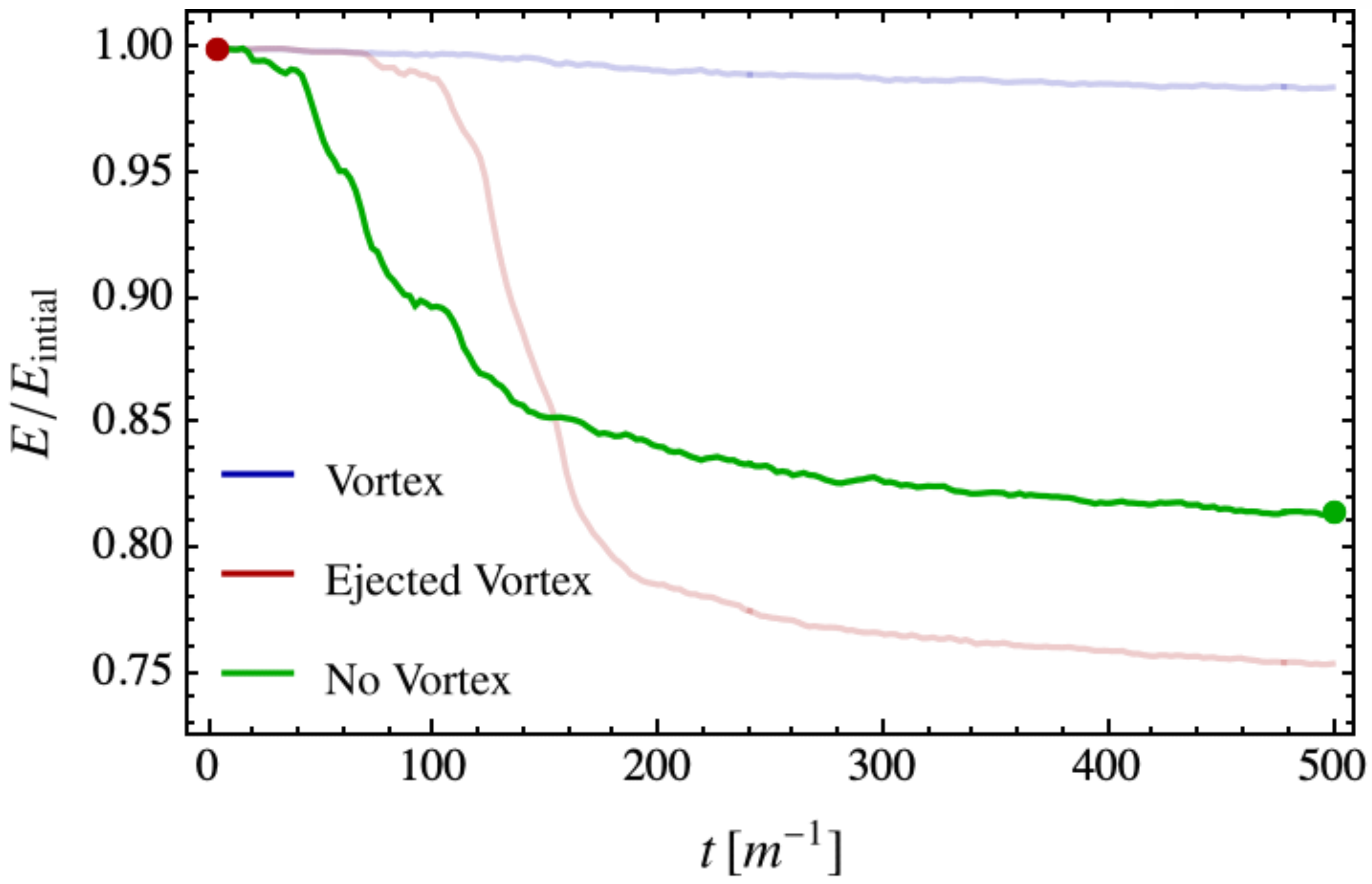
Dynamics

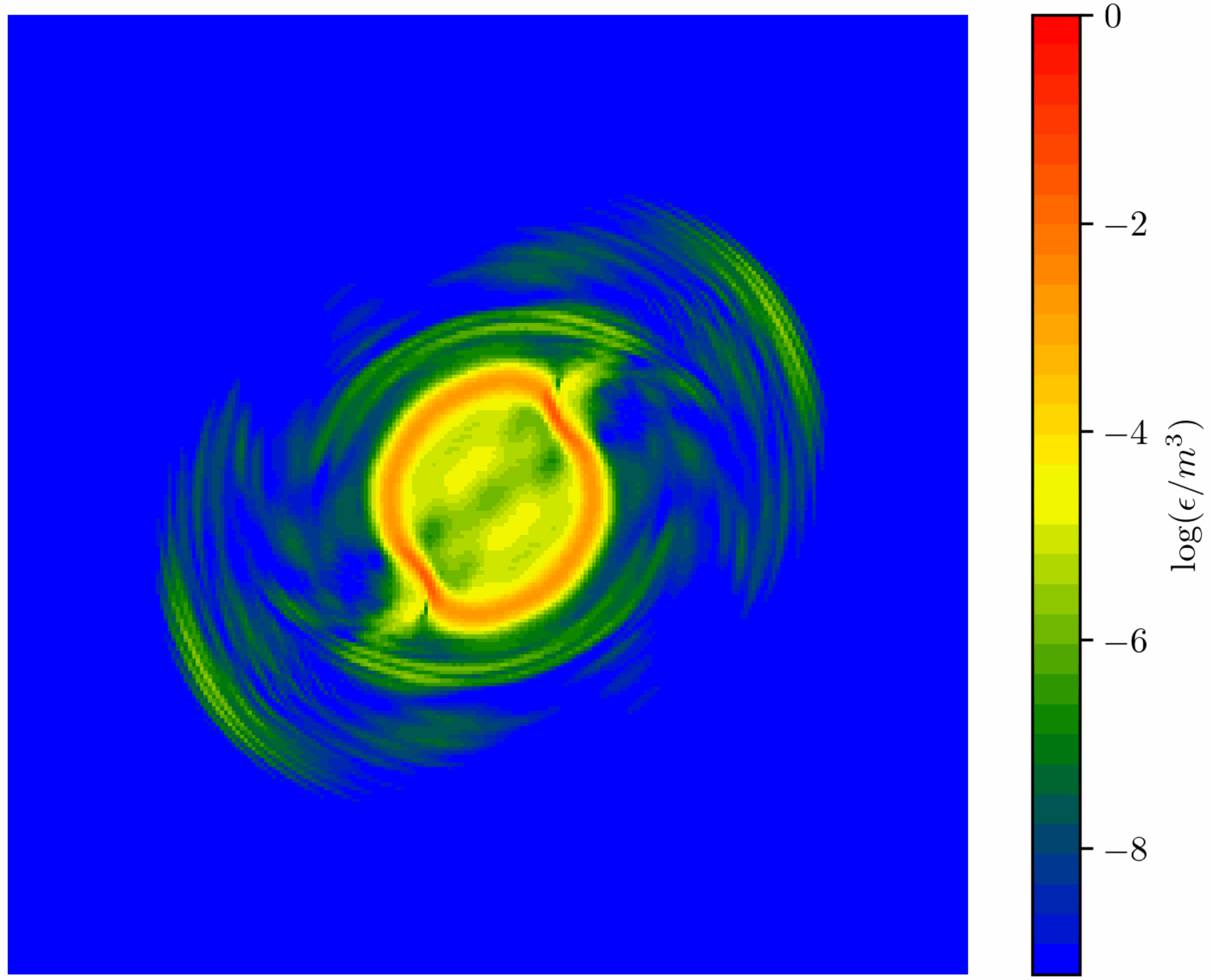

0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
ϵ_ϕ/m^3


Phase
∞
10.0
0.1
0
π
π/2
0
−π/2
−π

2+1D perspective

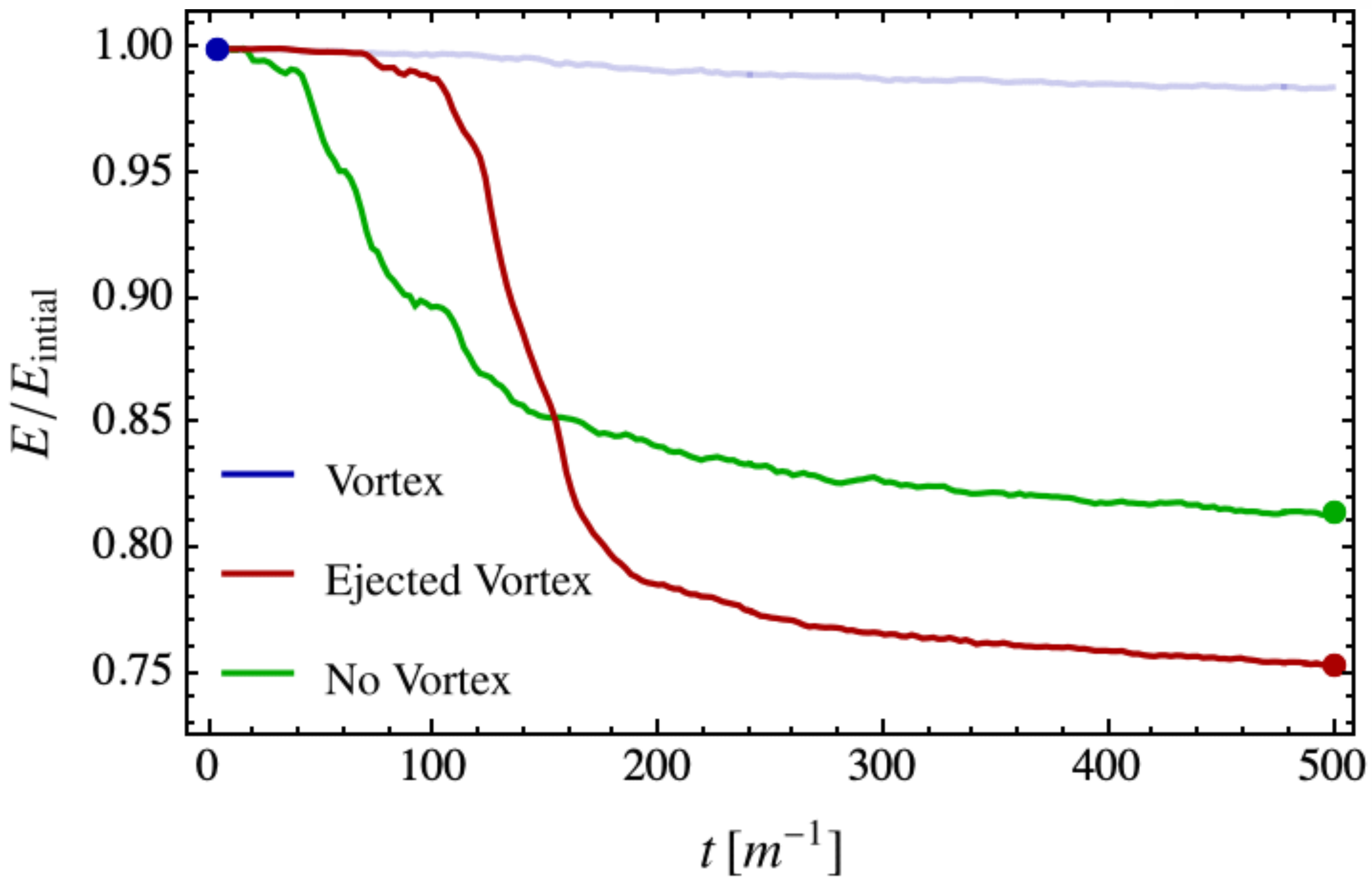
No-Vortex Case

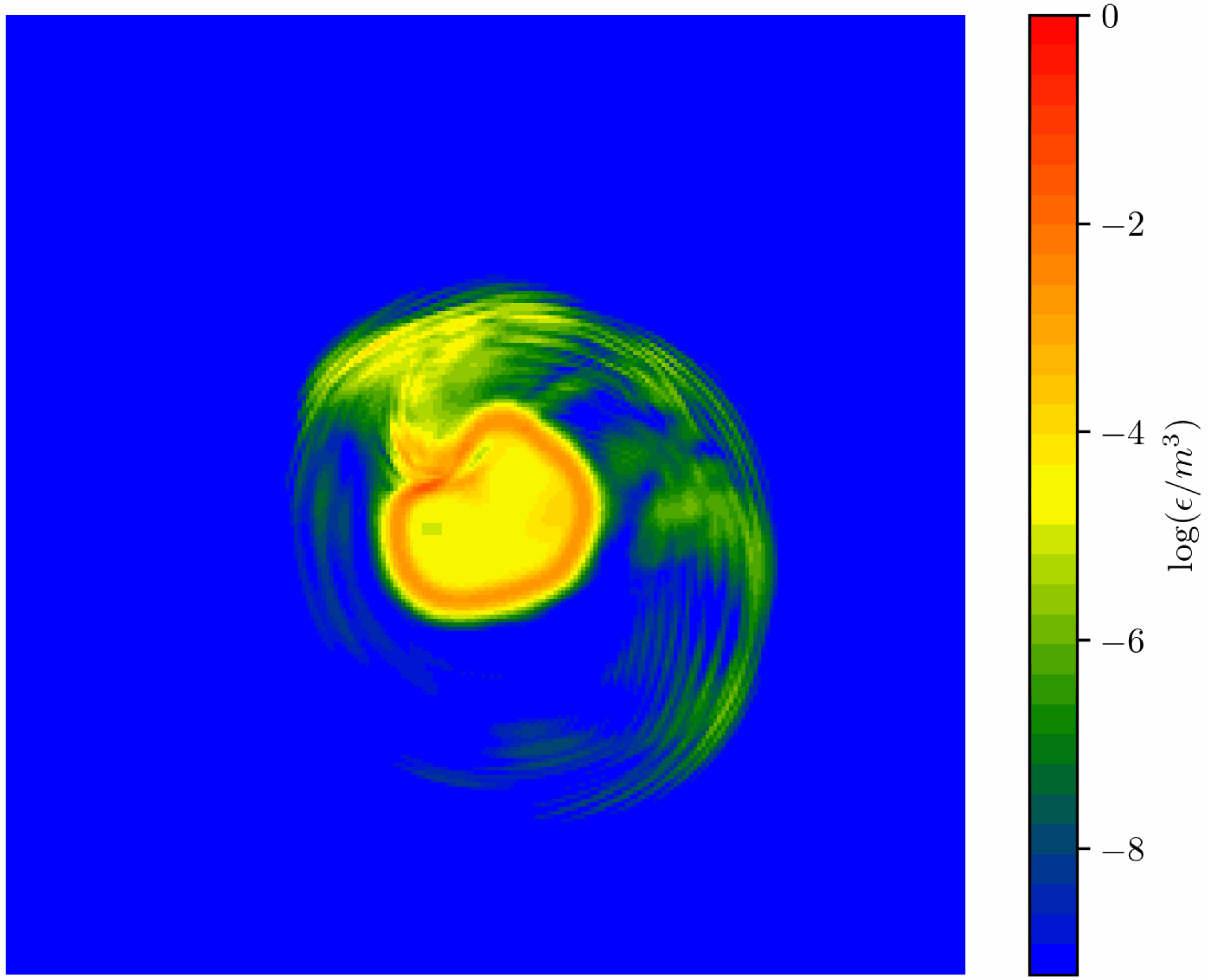




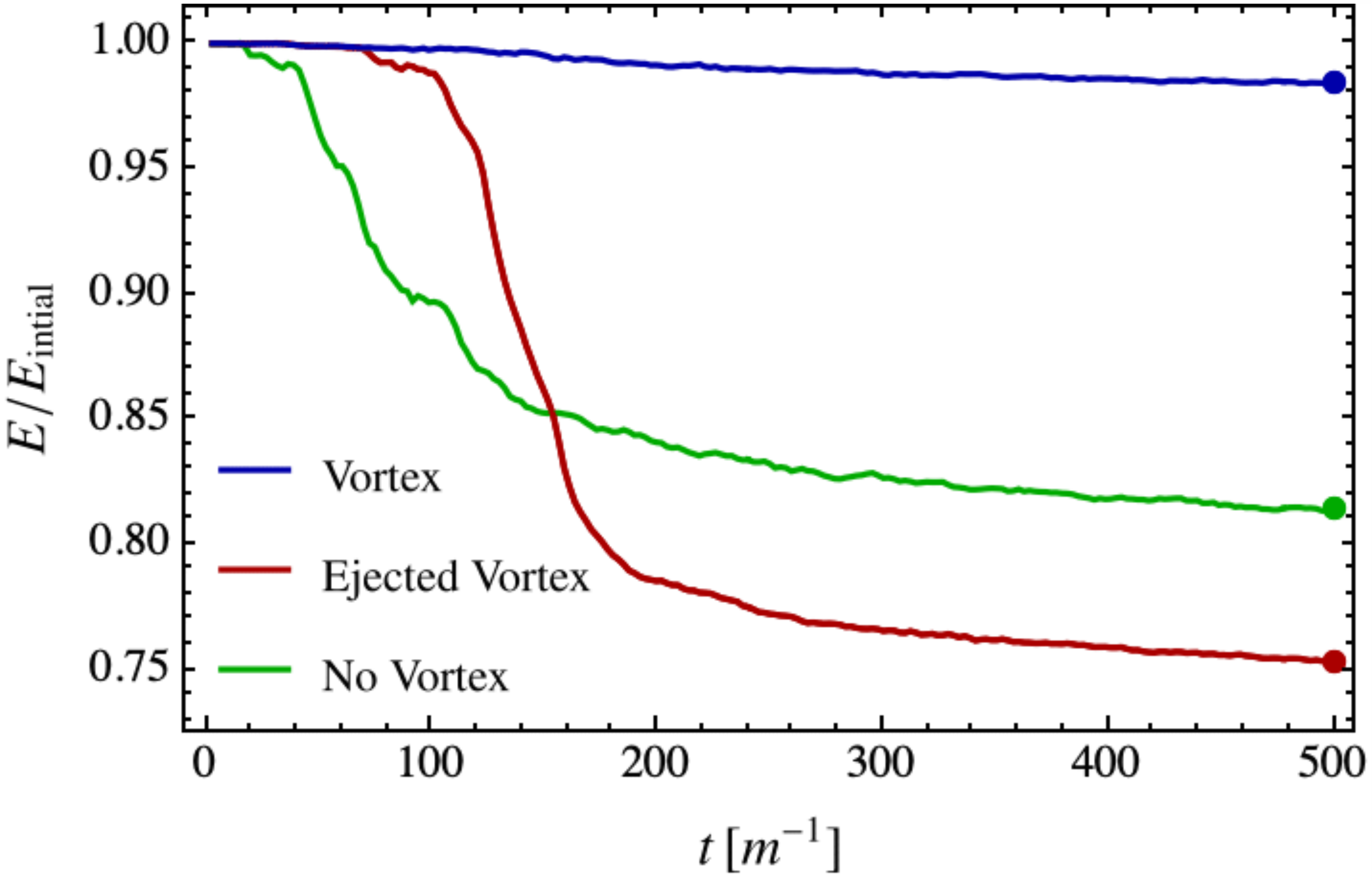
2+1D perspective

Ejected-Vortex Case

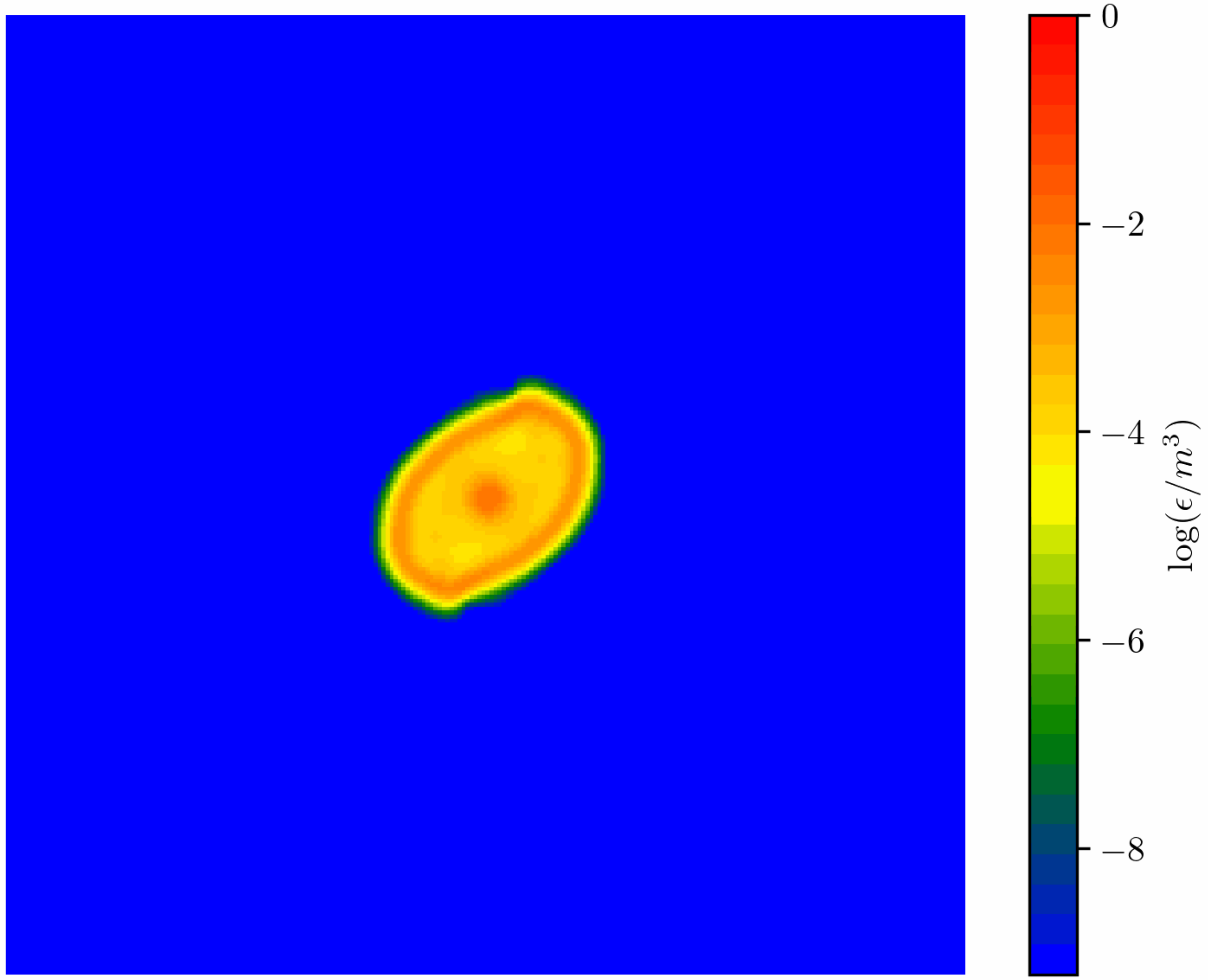




2+1D perspective

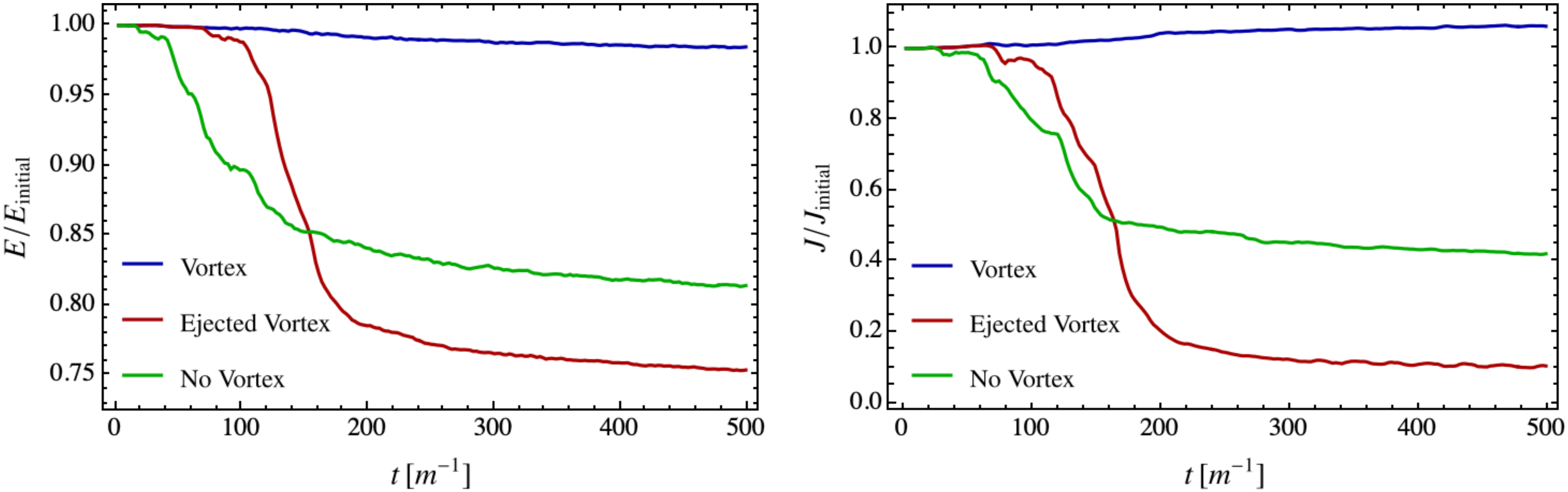


Vortex Case



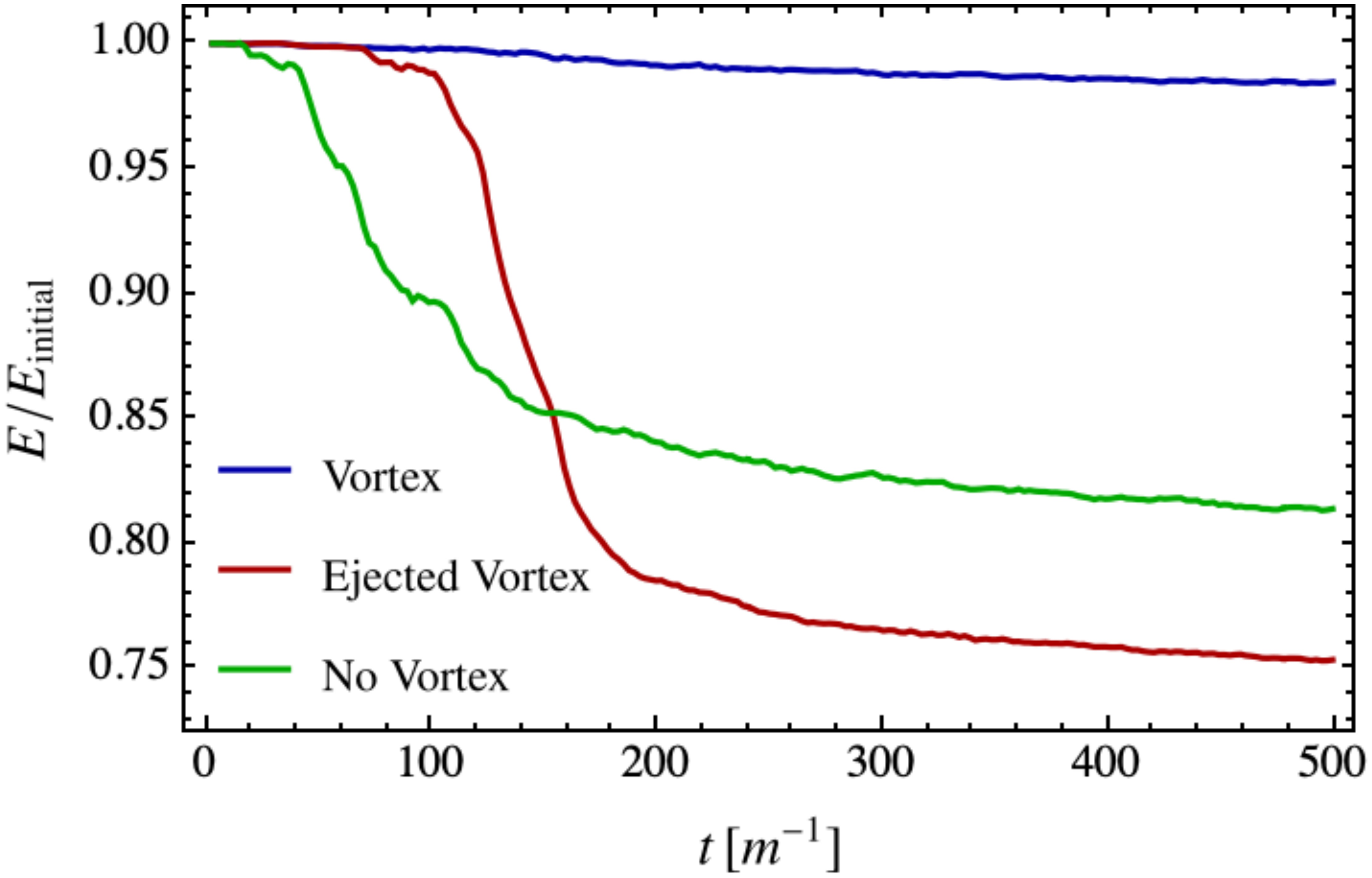
2+1D perspective

Energy and Spin evolve in a similar manner



2+1D perspective

Energy and Spin evolve in a similar manner



- No Vortex: the solitons simply merge
- Ejected Vortex: the resulting soliton possesses a vortex for a while. Eventually it is ejected resulting in a close-to-zero spin configuration
- Vortex: almost no emission takes place in this case. The energy and angular momentum are invested in vortex formation