Exploring Dark Matter: Primordial Black Holes from Inflationary Models Beyond Fine-Tuning

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Presentation at the
New Horizons in Primordial Black Hole Physics
Edinburgh, Scotland

Structure of the presentation

The layout of this presentation:

- Introduction
- Mechanisms from producing PBHs
- PBHs from a spectator field
- Gravitational Waves & PBHs
- Conclusions

Introduction

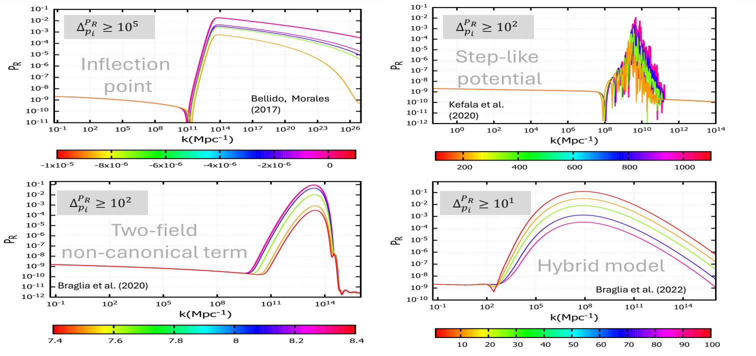
Introduction

- The detection of Gravitational Waves (GWs) by the merger of binary black holes by LIGO/VIRGO has reignited interest in the study of Primordial Black Holes (PBHs). PBHs can explain a fraction of Dark Matter (DM) in the Universe.
- The generation of PBHs can be explained in the framework of inflation. Specifically, a substantial amplification in the scalar power spectrum can provide an explanation to PBHs.
- There are many mechanisms in the framework of inflation which can lead to a significant enhancement in scalar power spectrum: inflection point, step-like potential, waterfall trajectory, two-field models etc.
- Mechanisms of a light quantum stochastic spectator scalar field during inflation can bypass fine-tuning.

Mechanisms from Production PBHs

Mechanisms from Production PBHs

There are several mechanism for producing of PBHs from inflation.



A drawback of models from inflation is the fine-tuning in order to obtain such amplification in power spectrum:

$$\Delta^{P_R}_{p_i} \equiv \mathrm{Max}\left(\frac{\partial \log \mathrm{P_R}}{\partial \log \mathrm{p_i}}\right),$$

pi: parameters of each model. [I.S.] (arXiV: 2404.14321)

PBHs from a spectator field

Spectator field for PBHs

IDEA OF SPECTATOR FIELD:

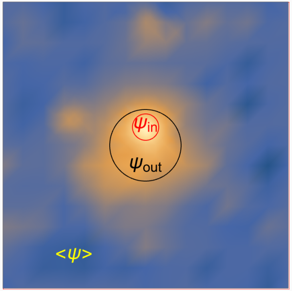
- A light stochastic spectator scalar field during inflation acquires different mean values in different current Hubble patches.
- There are huge number of these patches, so these necessarily exist some in which these fluctuations leave the horizon at values required to form PBHs.

  [B. Carr, S. Clesse, J.Garcia-Bellido].

Schematic representation of the spectator field fluctuations:
[S. Clesse, I.S.], Phys.Rev.D (2023)

- Rare fluctuation leading at ψ_{out} to exit the horizon.
- Smaller fluctuation at ψ_{in} becomes super-Hubble at time $N_{\rm inf}$.

$$\delta\psi_{\rm in} \equiv \psi_{\rm in}(x, N_{\rm inf}) - \psi_{\rm out}(x, N_{\rm inf} - 1)$$
$$\delta\psi_{\rm out} \equiv \psi_{\rm out}(x, N_{\rm inf} - 1) - \langle\psi\rangle$$



➤ **During inflation:** The inflaton field drives the inflation & the spectator field remains frozen.

The slow-roll parameters and the amplitude, $\mathrm{A_s}$, in slow-roll approximation is given:

$$\epsilon_1 \equiv -\frac{\mathrm{d}\ln \mathrm{H}}{\mathrm{d}N}, \quad \epsilon_2 \equiv \frac{\mathrm{d}\ln \epsilon_1}{\mathrm{d}N} \qquad \& \quad A_\mathrm{s} = 2.1\times 10^{-9} \simeq \frac{H_*^2}{8\pi^2\epsilon_{1*}M_\mathrm{P}^2}. \tag{1}$$

The spectral index, $\mathrm{n_s}$, and the tensor-to-scalar ratio, r, are given:

$$n_\mathrm{s} = 0.9649 \pm 0.0042 \simeq 1 - 2\epsilon_{1*} - \epsilon_{2*}, \quad r = 16\epsilon_{1*} < 0.07 \tag{2}$$

Model	ϵ_{1*}	ϵ_{2*}	$\mathrm{H_*(M_P)}$	r
1	0.00507	0.0207	2.9×10^{-5}	0.08115
2	0.00020	0.0351	5.8×10^{-6}	0.00325

.

- The quantum fluctuations of ψ produced during one e-fold in a Hubble-sized region

$$\langle\delta\psi_\mathrm{in}^2(N_\mathrm{inf})\rangle \simeq \frac{H_*^2}{4\pi^2}\exp\left\{-2\frac{\epsilon_{1*}}{\epsilon_{2*}}\left[e^{\epsilon_{2*}(N_\mathrm{inf}-N_*)}-1\right]\right\},$$

$$\langle\delta\psi_\mathrm{out}^2(N_\mathrm{inf})\rangle \simeq \frac{H_*^2}{8\pi^2\epsilon_{1*}}\left[1-\exp(-2\epsilon_{1*}(N_\mathrm{inf}-N_*))\right] \simeq \frac{H_*^2(N_\mathrm{inf}-N_*)}{4\pi^2}.$$

➤ **After inflation**: The spectator field starts to dominate the Universe.

The equations of the spectator field ψ (cosmic time):

$$\ddot{\psi} + 3H\dot{\psi} + \frac{\partial V}{\partial \psi} = 0\ , \quad \dot{N} = H = \sqrt{\frac{\rho}{3M_P^2}}, \tag{3}$$

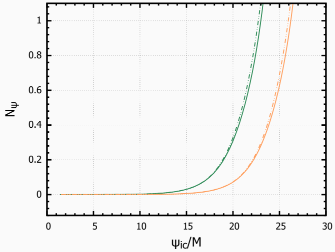
where $\rho = \rho_{\mathrm{m,r}} e^{-\kappa N} + V(\psi)$.

• In some regions of the Universe, if the spectator field rests in a particular flat part of its potential, an extra expansion occurs. This triggers a curvature fluctuation, leading to the formation of PBHs. These are formed later when these fluctuations re-enter the Hubble radius.

• We consider the potential:

$$V(\psi) = \Lambda^4 \left(1 - \exp\left[-\frac{\psi}{M}\right]\right)$$

An extra expansion can occur. & An extra expansion can occur.
$M(M_{\mathrm{P}})=\{4 \times 10^{-6}, 8 \times 10^{-7}\}$



➤ Probability distribution

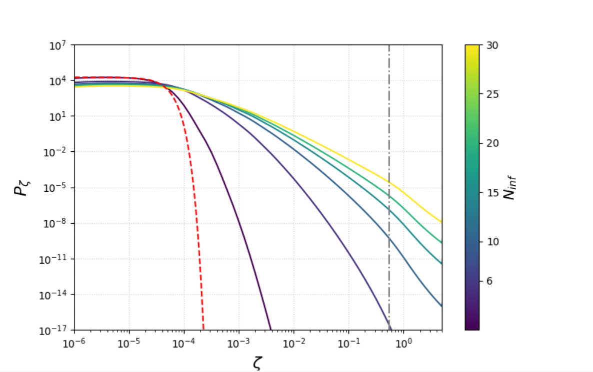
- The probability distribution as a function of ζ:

$$P(\zeta_{\rm in} - \zeta_{\rm out}) = \int \mathrm{d}\delta\psi_{\rm out} P(\delta\psi_{\rm in}) P(\delta\psi_{\rm out}) \left.\frac{\mathrm{d}\psi}{\mathrm{d}N}\right|_{\psi_{\rm out}}$$

where:

$$P(\delta\psi_{\rm in}) = \frac{1}{\sqrt{2\pi\langle\delta\psi^2_{\rm in}(N_{\rm inf})\rangle}} \exp\left[\frac{-\delta\psi^2_{\rm in}(N_{\rm inf})}{2\langle\delta\psi^2_{\rm in}(N_{\rm inf})\rangle}\right]$$

$$P(\delta\psi_{\rm out}) = \frac{1}{\sqrt{2\pi\langle\delta\psi^2_{\rm out}(N_{\rm inf}-1)\rangle}} \times \exp\left[\frac{-\delta\psi^2_{\rm out}(N_{\rm inf}-1)}{2\langle\delta\psi^2_{\rm out}(N_{\rm inf}-1)\rangle}\right].$$



Inflaton $[\mathrm{H}_*(\mathrm{M_P}), \mathrm{r}]$	Spectator $[\mathrm{M}(M_\mathrm{P})]$
2.9×10^{-5}, 0.08115	4×10^{-6}
5.8×10^{-6}, 0.00325	8×10^{-7}

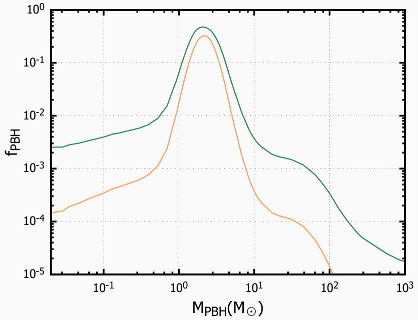
➤ PBH formation

The mass fraction of PBH, $\beta(M_{\rm PBH})$, is connected to the probability distribution P:

$$\beta(M_{\rm PBH}) \equiv \frac{{\rm d}\rho}{{\rm d}\ln M_{\rm PBH}} = \int_{\zeta_{\rm cr}}^{\infty} P(\zeta){\rm d}\zeta\,. \tag{4}$$

The fractional abundance of PBHs, $f_{\rm PBH}(M_{\rm PBH})$, is given as:

$$f_{\rm PBH}(M_{\rm PBH}) \approx 2.4\beta(M_{\rm PBH})\left(\frac{2.8\times 10^{17}M_\odot}{M_{\rm PBH}}\right)^{1/2}\,. \tag{5}$$



- Production of PBHs with a peak at range of $[2-5]{\rm M}\odot$.
- Consistency with CMB data: The power spectrum evaluated from the fluctuation of spectator field respect the constraints at CMB scales

$$\mathcal{P}_\zeta^\psi(k) = \frac{H_*^2(k)}{4\pi^2}\left(\frac{{\rm d}N}{{\rm d}\psi}\bigg|_{\langle N\rangle}\right)^2 \ll A_{\rm s}\,.$$

- PBHs formation without fine-tuning: PBHs form with a spectator field avoiding the need of fine-tuning:

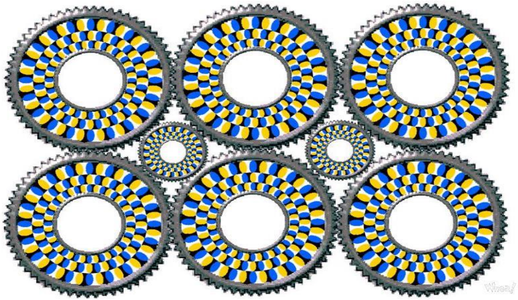
$$\Delta_p^{f_{PBH}} \equiv Max\left(\frac{{\rm d}\log f_{PBH}}{{\rm d}\log M}\right) \sim \mathcal{O}(1)\,.$$

Applications

How can this mechanism be applied to models?

We studied two applications:

- Higgs Standard Model
- No-scale SUGRA model



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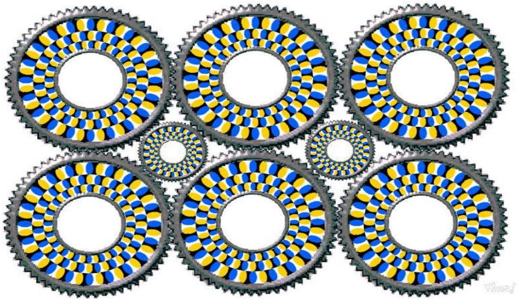


Applications: Higgs Standard Model

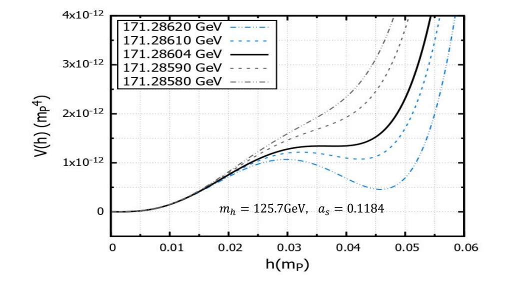
➤ Higgs Standard Model

The effective potential of the SM Higgs is given by

$$V(h) = \frac{\lambda(h)}{4} h^4.$$

and the self-coupling $\lambda(h)$ is determined by the β function,

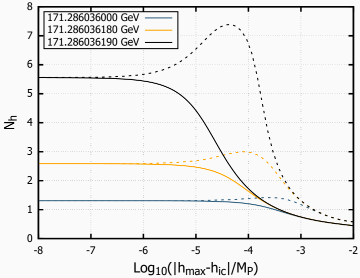
$$\beta_\lambda = \frac{d\lambda}{d\ln\mu}.$$

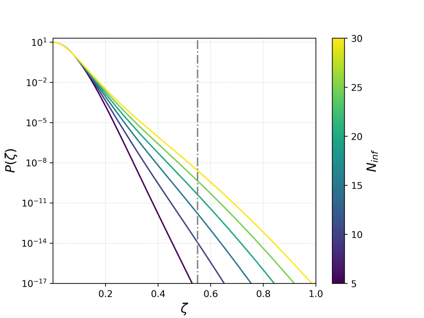


• If the BEH field lies exactly at the transition between metastability and stability, the potential exhibits an inflection point.

Applications: Higgs Standard Model

[S. Clesse, I.S.] Phys.Rev.D (2024)





[✓] The SM Higgs as spectator field leads to significant PBHs abundances without the need of extra parameters.

[✘] The prediction for the power spectrum at CMB scales, which is evaluated from the mean value of the spectator field can be preserved?

Applications

How can this mechanism be applied to models?

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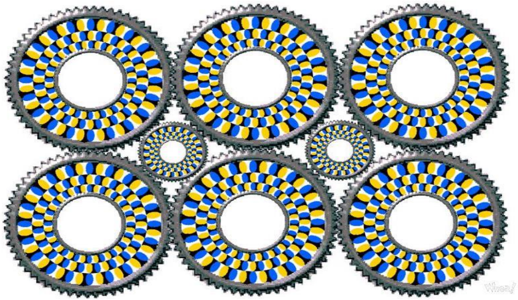


Applications: A no-scale SUGRA model

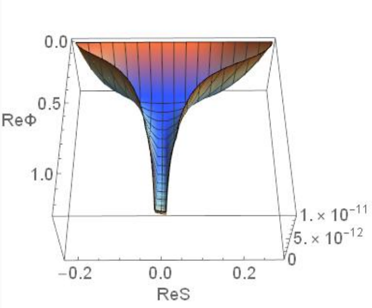
We consider the following Kähler potential and superpotential [I.S.] (Phys.Lett.B(2024)):

$$K = -3\ln\left(1 - \frac{|\Phi|^2}{3} - \frac{|S|^2}{3}\right), \tag{6}$$

and

$$W = M_{\rm inf} S\left(1 - e^{b_{\rm inf}\Phi}\right)^2 (3 - \Phi^2) + M_{\rm S}\Phi\left(1 - e^{b_{\rm S} S}\right)^2 (3 - S^2) \tag{7}$$

where Φ is the inflaton field and S is the modulo (spectator field). [$M_{\rm inf} \gg M_{\rm S}$]



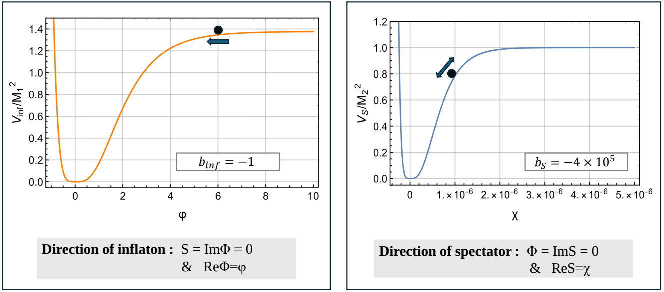
Applications: A no-scale SUGRA model

- Inflationary direction: $\mathrm{S} = \mathrm{Im}\Phi = 0, \quad \mathrm{Re}\Phi = \varphi.$

$$V_{\text{inf}} = M_1^2 \left(1 - \exp\left[\sqrt{3} b_{\text{inf}} \tanh\left(\varphi/\sqrt{6}\right)\right]\right)^4 \tag{8}$$

- Spectator direction: $\Phi = \mathrm{ImS} = 0, \quad \mathrm{ReS} = \chi.$

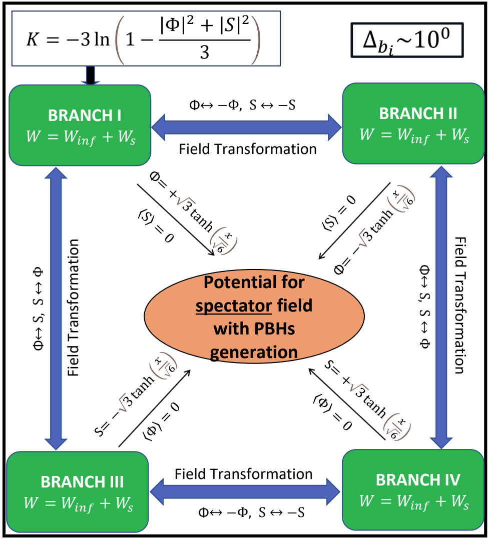
$$V_S = M_2^2 \left(1 - \exp\left[-\sqrt{3} b_S \tanh\left(\chi/\sqrt{6}\right)\right]\right)^4 \tag{9}$$



$H_* = 5.36 \times 10^{-6}$ Mp, $n_s = 0.9690$, r= 0.0027

$\langle \chi \rangle = 10^{-6}$Mp

Applications: A no-scale SUGRA model



- We respect the symmetry of the coset $\mathrm{SU}(2,1)/\mathrm{SU}(2) \times \mathrm{U}(1)$:

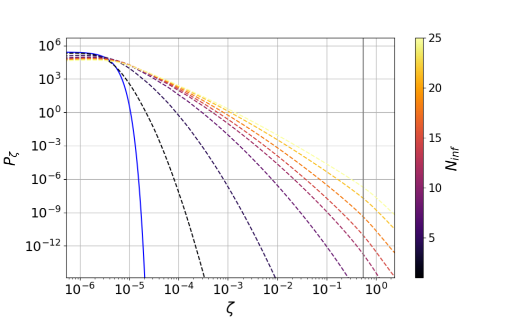
$$\Phi \to S,\ S \to \Phi$$
$$\Phi \to -\Phi, \quad S \to -S$$

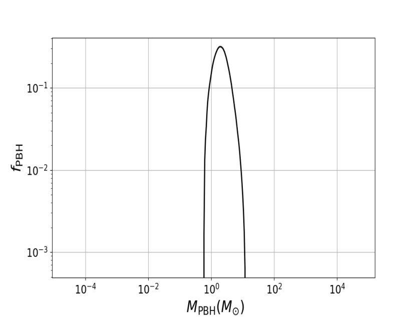
- We can avoid the need of the fine-tuning:

$$\Delta_{bi}^{f_{PBH}} = \mathcal{O}(10^0)$$

Applications: A no-scale SUGRA model

Results of the probability distribution and the fractional abundances to PBHs.





The no-scale SUGRA model we propose:

- leads to significant PBHs abundances.
- fulfils both the condition on CMB scales and get PBH formation.
- conserves the fields' transformation law of the coset $\mathrm{SU}(2,1)/\mathrm{SU}(2) \times \mathrm{U}(1)$.

Gravitational Waves & PBHs

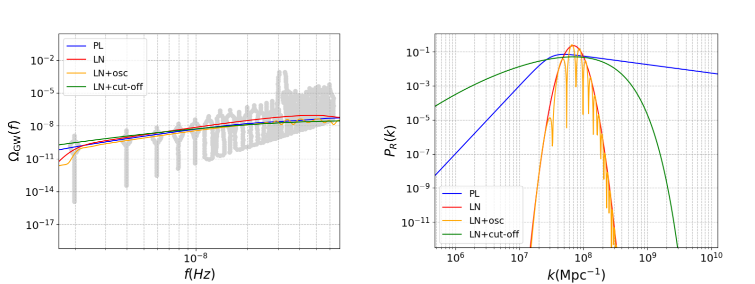
Gravitational Waves & PBHs

The energy density of PBHs can be connected with the power spectrum:

$$\Omega_{GW} \approx \int \int dk dk' \left(\int dt f(k,k',t) \right) P_R(k) P_R(k'). \quad (10)$$

In **[F. Kuhnel, I.S.]** (arXiV:2404.06547):

- Reconstruct four schemes of power spectra (PL,LN,LN+osc,LN+cut off) for given datasets of GWs. .



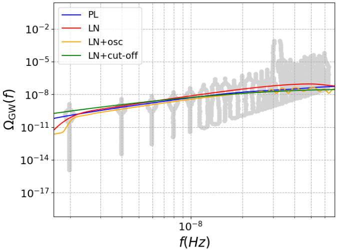
Gravitational Waves & PBHs

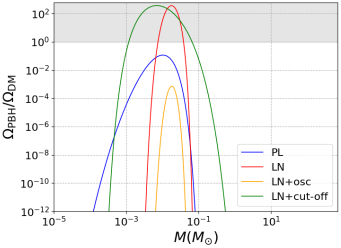
The abundances of PBHs can be evaluated from the scalar power spectra:

$$\frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}[M(k)] = 1.52\times 10^{8}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g_*}{106.75}\right)^{-1/4}\times\left(\frac{M(k)}{M_\odot}\right)^{-1/2}\beta[M(k)]\,,$$

PBHs production from given energy densities of GWs :

- Evaluating the fractional abundance of PBHs in order to explain the PTA signal.
- Taking into consideration the evaluation of critical thresholds for each scenario.





Conclusion

Conclusions

➤ In this presentation:

- We provide a novel mechanism of PBHs formation with the following advantages:
  1) avoiding the need of fine-tuning,
  2) consistency with constraints of CMB and explain the PBHs,
  3) applicable to other models.

- We present two applications of this mechanism in Higgs Standard Model and in model based on no-scale theory. We show that this mechanism can explain the production of PBHs.

➤ Perspectives:

- Solving the full Fokker–Planck equation and re-examine the Higgs as a spectator field for the production of PBHs **[S.Clesse, I.S.]**.

Thank you!

Additional slides

Definitions:

1. Characteristics wave-numbers:

- the scale of the observable Universe $2.3 \times 10^{-4}\mathrm{Mpc}^{-1}$
- the CMB pivot scale $0.05\mathrm{Mpc}^{-1}$
- the PBH/QCD scale $10^{6}\mathrm{Mpc}^{-1}$

2. The δN formalism relates the curvature perturbation, ζ, to the perturbation in the number of e-folds, N, by expressing ζ at any point in space as the fluctuation in the number of e-folds from an initial flat hypersurface to a final uniform-density hypersurface:

$$\zeta = \delta N.$$

Energy density of GWs

The energy density of the GWs in terms of scalar power spectrum is given: [Espinosa, Racco, Riotto (2019)]

$$\Omega_{GW}(k) = \frac{c_g \Omega_r}{36} \int_0^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2 - 1/3)(d^2 - 1/3)}{s^2 + d^2} \right]^2 \times P_R(kx) P_R(ky) (I_c^2 + I_s^2) \qquad (11)$$

where the radiation density $\Omega_r \approx 5.4 \times 10^{-5}$.

The variables x and y are:

$$x = \frac{\sqrt{3}}{2}(s + d), \quad y = \frac{\sqrt{3}}{2}(s - d).$$

Finally, the functions I_c and I_s are given:

$$I_c = -36\pi \frac{(s^2 + d^2 - 2)^2}{(s^2 - d^2)^3} \Theta(s - 1)$$

$$I_s = -36 \frac{(s^2 + d^2 - 2)^2}{(s^2 - d^2)^2} \left[\frac{(s^2 + d^2 - 2)}{(s^2 - d^2)} \log \left| \frac{d^2 - 1}{s^2 - 1} \right| + 2 \right]$$

Evaluating the production of PBHs

The present abundance of PBH is given by the integral:

$$f_{PBH} = \int d\ln M \frac{\Omega_{PBH}}{\Omega_{DM}}$$

$$\frac{\Omega_{PBH}}{\Omega_{DM}} = \frac{\beta(M_{PBH}(k))}{8\times 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g}{106.75}\right)^{-1/4}\left(\frac{M_{PBH}(k)}{10^{-18}\text{gram}}\right)^{-1/2}. \tag{12}$$

The mass is given as a function of k mode:

$$M_{PBH}(k) = 10^{18}\left(\frac{\gamma}{0.2}\right)\left(\frac{g}{106.75}\right)^{-1/6}\left(\frac{k}{7\times 10^{13}Mpc^{-1}}\right)^{-2}\text{gram}.$$

The mass fraction β is given by:

$$\beta(M_{PBH}) = \frac{1}{\sqrt{2\pi\sigma^2(M_{PBH}(k))}}\int_{\delta_c}^{\infty} d\delta\ e^{\frac{-\delta^2}{2\sigma^2(M_{PBH}(k))}} = \frac{\Gamma\left(\frac{1}{2},\frac{\delta_c^2}{2\sigma^2}\right)}{2\sqrt{\pi}}.$$

The variance of curvature perturbation, σ, is related to the power spectrum:

$$\sigma^2(M_{PBH}(k)) = \frac{4(1+w)^2}{(5+3w)^2}\int\frac{dk'}{k'}\left(\frac{k'}{k}\right)^4 P_R(k')\tilde{W}^2\left(\frac{k'}{k}\right)$$

where w is the barotropic index (in radiation dominated epoch is $w=1/3$). $\tilde{W}$ is the Gaussian distribution.

Press-Schecter approach

In PS approach the mass fraction β_{PS} is given by the probability that the overdensity δ is above a certain threshold of collapse, denoted as δ_c:

$$\beta_{PS}(M_{PBH}) = \frac{1}{\sqrt{2\pi\sigma^2(M)}} \int_{\delta_c}^{\infty} d\delta \; e^{\frac{-\delta^2}{2\sigma^2(M)}} = \frac{\Gamma\left(\frac{1}{2}, \frac{\delta_c^2}{2\sigma^2}\right)}{2\sqrt{\pi}}. \tag{13}$$

where the variance of curvature perturbation σ is related to the power spectrum:

$$\sigma^2(M_{PBH}(k)) = \frac{4(1+wa)^2}{(5+3w)^2} \int \frac{dk'}{k'} \left(\frac{k'}{k}\right)^4 P_R(k') \tilde{W}^2\left(\frac{k'}{k}\right) \tag{14}$$

where w is the parameter of the equation of state (in radiation dominated epoch is $w = 1/3$) and $\tilde{W}$ is a window function. We will use the Gaussian distribution for this function:

$$\tilde{W}(x) = e^{-x^2/2}.$$

Peak theory

The number density of peaks above a threshold given:

$$n_{peaks} = \frac{1}{(2\pi)^2} \left(\frac{\langle k^2 \rangle}{3} \right)^{3/2} \left(\left(\frac{\delta_c}{\sigma} \right)^2 - 1 \right) exp\left(-\frac{\delta_c^2}{2\sigma^2} \right) \quad (15)$$

where $\langle k^2 \rangle$ reads:

$$\langle k^2 \rangle = \frac{1}{\sigma^2} \int_0^\infty \frac{dk'}{k'} k'^2 \tilde{W} \left(\frac{k'}{k} \right)^2 P_\Delta(k') \quad (16)$$

where σ is the variance of curvature perturbation and $\tilde{W}$ is a window function, as before. The density power spectrum is defined as:

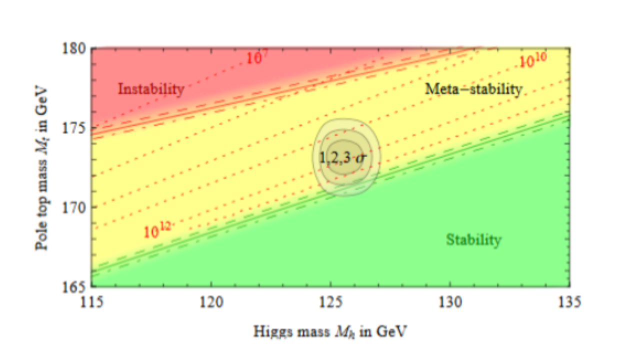
$$P_\Delta = \frac{4(1+\omega)^2}{(5+3\omega)^2} \left(\frac{k'}{k} \right)^4 P_R(k'). \quad (17)$$

The mass fraction β_{PT} is given by:

$$\beta_{PT} = n_{peaks} (2\pi)^{3/2} (1/k)^3. \quad (18)$$

Stability- Metastability

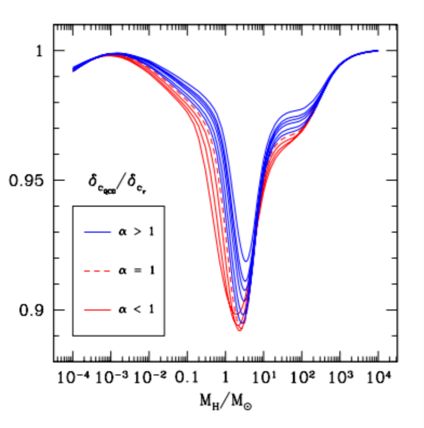
- Regions of absolute stability, meta-stability and instability of the SM vacuum in the m_t–m_h plane:



[Degrassia,Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia (2013)]

The threshold for PBHs during the QCD transition.

- . The threshold δc during the QCD transition for different values of the shape parameter:



[Musco, Jedamzik, Young (2023)]