SCALAR-INDUCED GRAVITATIONAL WAVES

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What can we observe with 3rd generation detectors?

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Primordial Black Holes

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Primordial Black Holes

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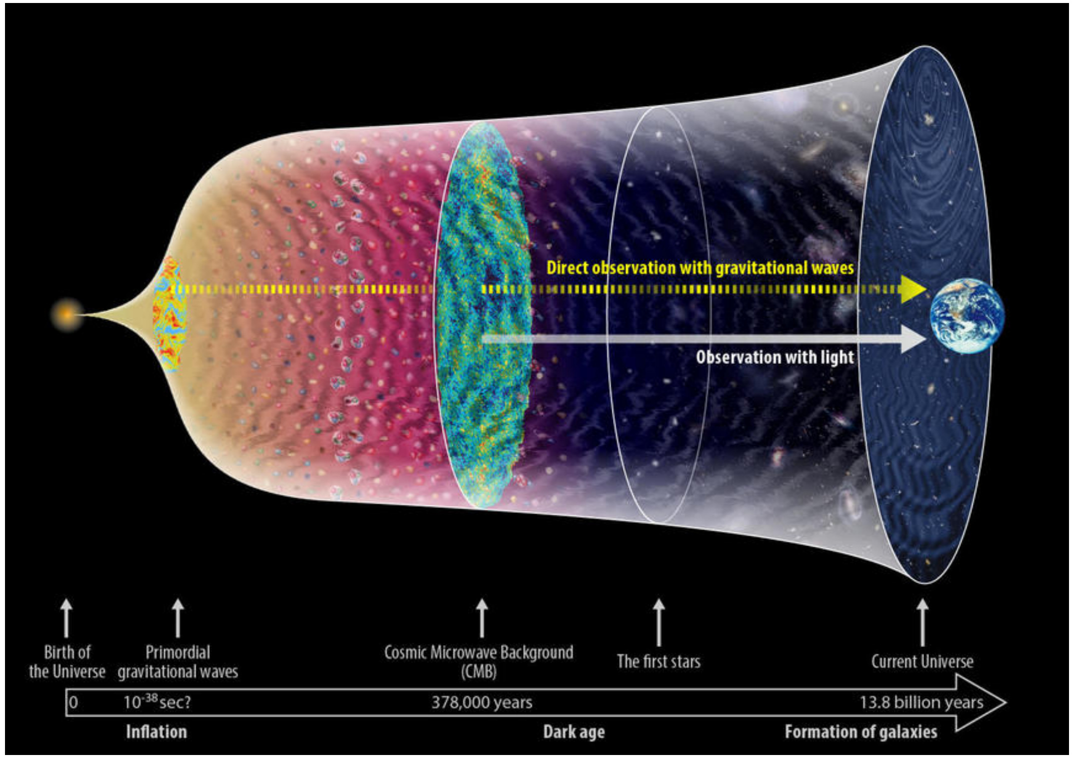
Scalar-Induced Gravitational Waves

DETECTION

Characteristic frequencies fall in the band of current and future GW interferometers



Pulsar Timing Arrays (PTA) observation providing strong evidence for a SGWB & compatibility with “scalar-induced” background

Depends on modeling assumptions highlighting that more data and analysis are needed to discern between cosmological or astrophysical origin

References: arXiv:2307.02399v2

WHAT ARE SCALAR-INDUCED GRAVITATIONAL WAVES?

METRIC PERTURBATIONS

Linear-order

— perturbations are decoupled

$$ds^2 = a^2(\eta)\left[-\left(1+2\phi^{(1)}+\phi^{(2)}\right)d\eta^2 + \left(\partial_i w^{(1)} + \frac{1}{2}\partial_i w^{(2)} + \frac{1}{2}w_i^{(2)}\right)d\eta dx^i\right.$$
$$\left.\left\{\left(1-2\psi^{(1)}-\psi^{(2)}\right)\delta_{ij} + D_{ij}\left(\chi^{(1)}+\frac{1}{2}\chi^{(2)}\right) + \frac{1}{2}\left(\partial_i\chi_j^{(2)} + \partial_j\chi_i^{(2)} + \chi_{ij}^{(2)}\right)\right\}dx^i dx^j\right]$$

METRIC PERTURBATIONS

Linear-order

— perturbations are decoupled

Higher-order

— perturbations can be sourced by the coupling of lower-order perturbations

$$ds^2 = a^2(\eta)\left[-\left(1+2\phi^{(1)}+\phi^{(2)}\right)d\eta^2+\left(\partial_i w^{(1)}+\frac{1}{2}\partial_i w^{(2)}+\frac{1}{2}w_i^{(2)}\right)d\eta dx^i\right.$$
$$\left.\left\{\left(1-2\psi^{(1)}-\psi^{(2)}\right)\delta_{ij}+D_{ij}\left(\chi^{(1)}+\frac{1}{2}\chi^{(2)}\right)+\frac{1}{2}\left(\partial_i\chi_j^{(2)}+\partial_j\chi_i^{(2)}+\chi_{ij}^{(2)}\right)\right\}dx^i dx^j\right]$$

SCALAR-INDUCED GW

Secondary tensor modes produced due to the coupling of first-order scalar fluctuations

Tensor perturbation

Scalar perturbation

$$\chi_j^{i\,(2)\prime\prime} + 2\mathcal{H}\chi_j^{i\,(2)\prime} - \nabla^2\chi_j^{i\,(2)} = \partial^i\phi^{(1)}\partial_j\phi^{(1)} + \ \ldots$$

Scalar perturbations

Observed in CMB temperature anisotropies and density fluctuations

Tensor perturbations

At second-order, observed scalar spectrum sources the generation of GWs

SIGW E.o.M

$$\chi_j^{i(2)''} + 2\mathcal{H}\chi_j^{i(2)'} - \nabla^2\chi_j^{i(2)} = \boxed{-4P_{jm}^{li}S_l^m}$$

For more references:

- Matarrese, Mollerach and Bruni, 1997 (arXiv:astro-ph/9707278)
- Andanda, Clarkson and Wands, 2007 (arXiv:gr-qc/0612013)
- Baulmann et al. 2007 (arXiv:heb-th/0703290)

SOURCE TERM IN POISSON GAUGE

$$
\begin{aligned}
S^i_j = {} & \partial^i \phi^{(1)} \partial_j \phi^{(1)} + 2\phi^{(1)} \partial^i \partial_j \phi^{(1)} - 2\psi^{(1)} \partial^i \partial_j \phi^{(1)} - \partial_j \phi^{(1)} \partial^i \psi^{(1)} - \partial^i \phi^{(1)} \partial_j \psi^{(1)} \\
& + 3\partial^i \psi^{(1)} \partial_j \psi^{(1)} + 4\psi^{(1)} \partial^i \partial_j \psi^{(1)} \\
& - \frac{4}{3\mathcal{H}^2 \left(1+w\right)} \left[\partial^i \left(\psi^{(1)\prime} + \mathcal{H}\phi^{(1)}\right) \partial_j \left(\psi^{(1)\prime} + \mathcal{H}\phi^{(1)}\right)\right] \\
& + 4\psi^{(1)} \delta^{ik} \boxed{\left[\left(\partial_j \partial_k - \frac{1}{3}\nabla^2 \delta_{jk}\right)\left(\phi^{(1)} - \psi^{(1)}\right)\right]}
\end{aligned}
$$

Anisotropic stress

ANISOTROPIC STRESS

Standard GR

- Contribution from free-streaming photon and neutrinos — negligible
- At linear-order, scalar potentials are set to be equal to one another, $\phi^{(1)} = \psi^{(1)}$

SIGW source term

Evidence of coupling between scalar potential and anisotropic stress

$$4\psi^{(1)}\delta^{ik}\left[\left(\partial_j\partial_k - \frac{1}{3}\nabla^2\delta_{jk}\right)\left(\phi^{(1)} - \psi^{(1)}\right)\right]$$

MODIFIED GRAVITY THEORIES

A non-standard relation between the potentials can arise due to modification in geometric part of Einstein equation

Background quantity

Anisotropic stress

$$\psi^{(1)} - \phi^{(1)} = \sigma(a)\Pi + \pi_m$$

Combination of linear perturbations

$f(R)$ theory

- $f(R)$ gravity, Lagrangian density f is a function of Ricci scalar
- Modified evolution equation:

$$F(R)G^{\lambda}_{\mu} = g^{\lambda\nu}g_{\mu\nu}\frac{(f(R) - RF(R))}{2} + g^{\lambda\nu}\nabla_{\mu}\nabla_{\nu}F(R) - g^{\lambda\nu}g_{\mu\nu}\Box F(R) + \kappa^2 T^{\lambda}_{\mu}$$

- Model considered: $f(R) = R + \alpha R^2$
- Concentrating on the first-order corrections, i.e. $\mathcal{O}(\alpha)$

TRACE-FREE FIRST-ORDER FIELD EQUATION

$$
\begin{aligned}
&-\left(1+12\alpha a^{-2}\left[\mathcal{H}'+\mathcal{H}^2\right]\right)\left(\partial^i\partial_j-\frac{1}{3}\nabla^2\delta^i_j\right)\left(\phi^{(1)}-\psi^{(1)}\right)\\
&\quad = 2\alpha a^{-2}\boxed{\left(\partial^i\partial_j-\frac{1}{3}\nabla^2\delta^i_j\right)\left(-6\psi^{(1)\prime\prime}-6\mathcal{H}\phi^{(1)\prime}-18\mathcal{H}\psi^{(1)\prime}-12\left[\mathcal{H}'+\mathcal{H}^2\right]\phi^{(1)}-2\nabla^2\phi^{(1)}+4\nabla^2\psi^{(1)}\right)}\\
&\quad + \kappa^2\pi^{(1)i}_j
\end{aligned}
$$

- **NO MODIFICATION** to the fluid description — anisotropic stress contribution neglected
- Non-standard relation at first-order $\mathcal{O}(\alpha)$ correction

SIGW E.o.M in $f(R)$ gravity

$$\left(1+12a^{-2}\alpha\left[\mathcal{H}'+\mathcal{H}^2\right]\right)\left(\chi_j^{i(2)''}+2\mathcal{H}\delta\chi_j^{i(2)'}+k^2\delta\chi_j^{i(2)}\right)+12a^{-2}\alpha\left[\mathcal{H}''+2\mathcal{H}\mathcal{H}'\right]\chi_j^{i(2)'}$$
$$=S+\alpha\delta S$$

Standard GR

First-order modification

Future plans

1. Applying the first-order solutions to the scalar potentials in the source term and finding the spectral density of induced GW both in standard GR and modified gravity
2. Studying detectability and observational effects

Kugarajh, Maselli, Matarrese and Ricciardone (in preparation)

PRIMORDIAL BLACK HOLES

Which density fluctuations are the source of induced GWs?

Large fluctuations re-entering horizon can collapse and form PBHs

After formation, gas of PBHs can be treated as fluid with density fluctuations, GW produced via gravitational potential of gas of PBHs

Scalar-induced gravitational waves can provide constraints on abundance of PBHs

References: arXiv:1612.06264v2 & arXiv:2012.08151v2

Conclusion

- Scalar-induced gravitational waves can open-up a new door for cosmology
- Detectability
    - Probes of early universe
    - Distinguish between early universe models, in GR and beyond GR
- Understanding PBHs and constraining abundance

Back up slides

Cosmological Perturbation Theory

Small inhomogeneous perturbations, produced by quantum vacuum fluctuations, on homogenous and isotropic FRW background

$$\mathbf{T}(\eta, x^i) = \mathbf{T}_0(\eta) + \delta\mathbf{T}(\eta, x^i)\,.$$