Primordial black hole formation: Type II B

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This talk is mainly based on the following collaborations:

TH, C. M. Yoo and Y. Koga, PRD**108** (2023) 043515 [arXiv:2304.13284].

K. Uehara, A. Escrivà, TH, D. Saito and C. M. Yoo, [arXiv:2401.06329].

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PBH mass, fraction and formation probability

- The smaller, the older (the earlier)! [Carr & Hawking (1974)]

$$M \simeq M_H(t_f) \simeq \frac{c^3}{G} t_f \simeq 1 M_\odot \left(\frac{t_f}{10^{-5}\ \mathrm{s}} \right)$$

- Hawking evaporation only important if $M \lesssim 10^{15}$ g [Hawking (1974)]

$$T_H \simeq 100\ \mathrm{MeV} \left(\frac{M}{10^{15}\mathrm{g}} \right)^{-1}, \quad t_{\mathrm{ev}} \simeq 10\ \mathrm{Gyr} \left(\frac{M}{10^{15}\mathrm{g}} \right)^{3}.$$

- $f(M)$ can be observationally constrained.

$$f(M) = \left. \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{CDM}}} \right|_{t=t_0}$$

- $\beta(M)$: the fraction of the Universe which have gone to PBHs of M

$$\beta(M) \simeq 2 \times 10^{-18} \left(\frac{M}{10^{15}\ \mathrm{g}} \right)^{1/2} f(M)$$

for $M > 10^{15}$ g due to concentration during the RD.

Formation studies of PBHs

- Can we predict $\beta(M)$ and other observables for a given cosmological scenario?
- How do they form? Can we find any new phenomena?
- Possible formation mechanism
  - “Inevitable scenario”: fluctuations generated by inflation
  - Other scenarios: collapse of domain walls, bubble nucleation, collision of bubbles, phase transitions, ...
- We here focus on the formation from fluctuations generated by inflation.

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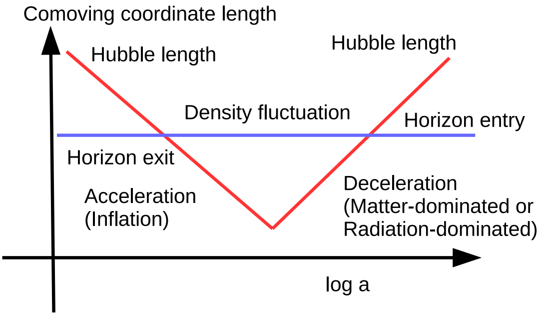
Fluctuation generated in inflation

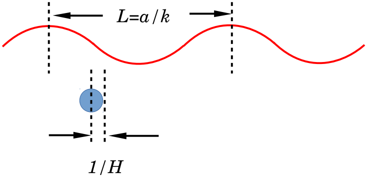
- The scales of perturbations of super-horizon scale generated in inflation enter the horizon in the decelerated expansion.



- Inflation gives the power spectrum $P_\zeta(k)$ and the statistics of curvature perturbations ζ [See e.g. Sasaki et al. (2018)].

Large-amplitude long wavelength solutions

- Gradient expansion in powers of $\epsilon = k/(aH) \ll 1$ [Tomita (1972), Shibata & Sasaki (1999), Lyth et al. (2005), Polnarev & Musco (2007)]



- $3+1$ decomposition of spacetime

$$ds^2 = -\alpha^2 dt^2 + e^{2\zeta} a^2(t) \tilde{\gamma}_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

where $\zeta = \zeta(\mathbf{x}) = O(1)$ generates the LWL solution and is necessary for PBH formation for RD.

- The density perturbation $\delta = O(\epsilon^2)$ [TH, Yoo, Nakama & Koga (2015)]

$$\delta_{\mathrm{CMC}} \simeq -\frac{4}{3a^2H^2} e^{-\frac{5\zeta}{2}} \Delta_{\mathrm{flat}} e^{\frac{\zeta}{2}}$$

Formation threshold for RD

- PBH formation needs numerical relativity. The threshold condition in terms of the LWL solutions is very useful if such a condition exists.
- Formation threshold in radiation domination
  - $\delta_H \simeq 0.45$, where δ_H is the averaged density perturbation δ_H in the comoving slice [Musco et al. (2005), ...].
  - $C_{\rm SS}(r) \simeq 0.4$, where $C_{\rm SS}(r)$ is the Shibata-Sasaki compaction function [Shibata & Sasaki (1999), Musco & Miller (2013), TH, Yoo, Nakama & Koga (2015)]
  - $\overline{C}_{\rm com} \simeq 2/5$, where $\overline{C}_{\rm com}$ is the spatial average of $C_{\rm com} := 2\delta M/R$ in the comoving slice over the central region surrounded by its maximum. Universal over different profiles. [Escriva et al. (2020)].

Revisiting compaction functions

TH, C. M. Yoo and Y. Koga, PRD108 (2023) 043515 [arXiv:2304.13284].

There was some confusion about the compaction function.

- $C_{\rm SS}$ was (intended to be) defined as [Shibata & Sasaki 1999]

$$C_{\rm SS}(t,r) =? \frac{\delta M(t,r)}{R(t,r)}$$

in the constant-mean-curvature (CMC) slice, where δM and R are the Kodama mass excess and the areal radius.

- However, the expression they actually used (and we use here) is

$$C_{\rm SS}(t,r) = \frac{1}{R}\int dV \rho_b \delta$$

in the CMC slice. The velocity perturbation's contribution is omitted.

Revisiting compaction functions

TH, C. M. Yoo and Y. Koga, PRD108 (2023) 043515 [arXiv:2304.13284].

Let's resolve the confusion!

- It turns out that

$$C_{\rm SS}(t,r) \neq \frac{\delta M(t,r)}{R(t,r)}.$$

- The correct relation in the LWL limit is for the EOS $p = w\rho$

$$C_{\rm SS}(r) \simeq \frac{1}{2}\left[1-(1+r\zeta')^2\right] \simeq \frac{3w+5}{3(1+w)}\frac{\delta M_{\rm com}}{R}.$$

- It has been empirically established that $C_{\rm SS}$ (or $C_{\rm com}$) is very robust to give a threshold for PBH formation. This feature is not affected by this confusion.
- So, you can use the compaction function with no problem but should be careful when you write the equation $C_{\rm SS} = \delta M/R$.

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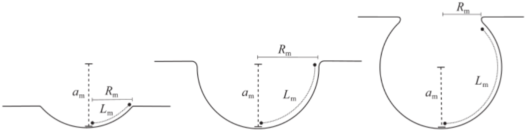
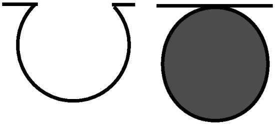
Positive curvature region and type II perturbation

- The overdense adiabatic perturbation can be described by a locally positive-curvature region in the flat FLRW universe.
- This implies the following configurations [Carr (1975), Kopp et al. (2011), TH & Carr (2015)]:





  - Type I, marginal, Type II and “separate universe”
  - They are not time evolution but the spatial geometries of different sets of initial data.
  - Type II is usually rarer than type I, but can be dominant in a particular model [Escriva et al. (2023)].

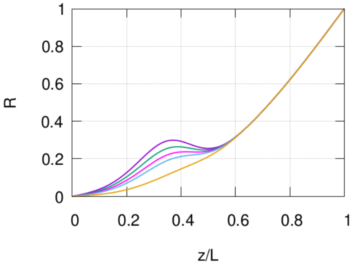
Type II perturbation [Uehara, Escriva, TH, Saito & Yoo (2024)]

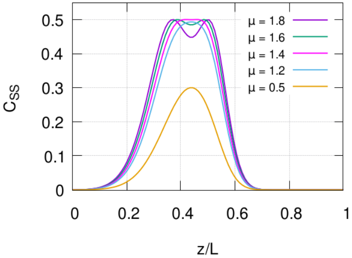
- The initial data for simulation is a LWL solution generated by $\zeta(r)$. We choose the following function

$$\zeta(r) = \mu e^{-(1/2)k^2 r^2} W(r),$$

where $W(r)$ is an appropriate window function.

  - μ beyond the critical value entails throat structure.





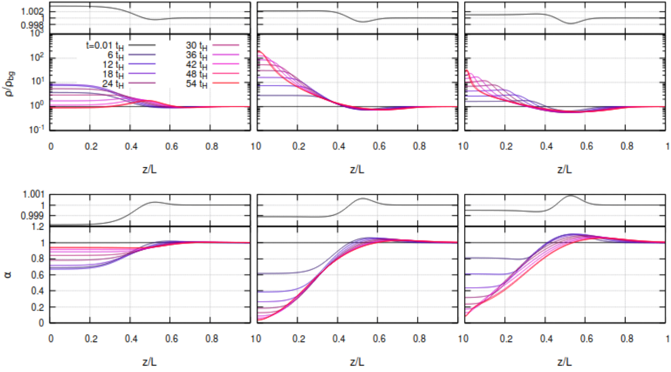
  - Left: The areal radius R shows the throat's feature for $\mu \gtrsim 1.4$.
  - Right: We can prove that $C_{SS}(r)$ has two peaks with $1/2$ and a minimum between them for type II.

Time evolution of Type II perturbation in RD

[Uehara, Escriva, TH, Saito & Yoo (2024)]

We have implemented 1D NR simulations for pure radiation.

- A Type I perturbation results in a BH only beyond the threshold.
- A Type II perturbation necessarily results in a BH.

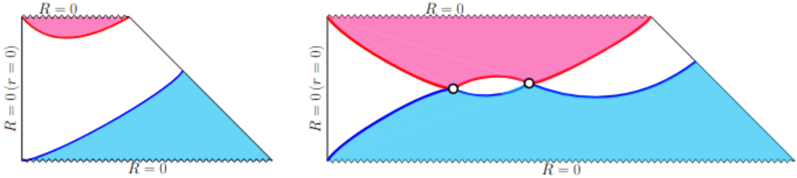


(a) $\mu = 0.5$ (b) $\mu = 1.2$ (c) $\mu = 1.8$

Type B horizon structure

[Uehara, Escriva, TH, Saito & Yoo (2024)]

- Two distinct structures of trapping horizons (≃ apparent horizons) are inferred from the numerical results.



Left: type A, Right: type B [Red: future trapped, Blue: past trapped]

- We call the horizon structure on the right type B, featured with two Bifurcating ($\theta_+ = \theta_- = 0$) trapping horizons.
- The type I perturbation results in a type A, while the type II does not always in a type B for radiation.
- The type II B may be related to a baby universe, although the expansion never happens for pure radiation.

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Summary

- PBH formation
    - Can we predict $\beta(M)$ and other observables for a given cosmological scenario?
    - We can understand the formation process and possibly find new phenomena.
    - Basic concepts: inflation-generated fluctuation, LWL solutions, threshold
- We have resolved a long-standing (24-year!) confusion about the compaction function.
- Type II B PBH: The time evolution of a type II perturbation can entail Bifurcating trapping horizons.