

# Curvature Perturbations Protected Against One Loop

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# Main message

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Superhorizon curvature perturbations are constant.

Separate Universe picture is valid.

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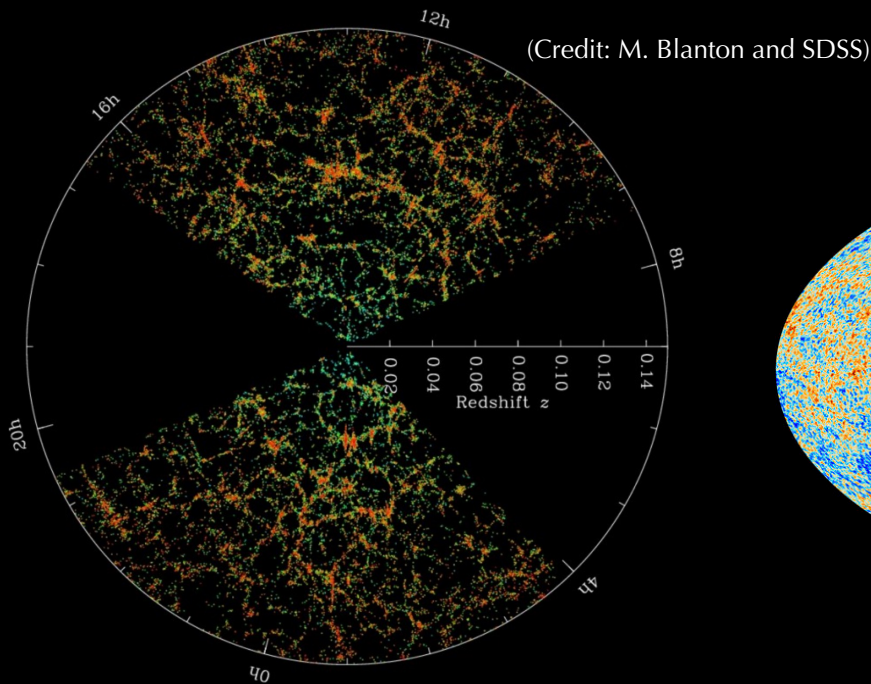
**Even at one loop level**

# Outline

- Introduction of curvature perturbations
- Recent claim: Curvature is not conserved?
- Conservation of curvature at one loop
- Summary

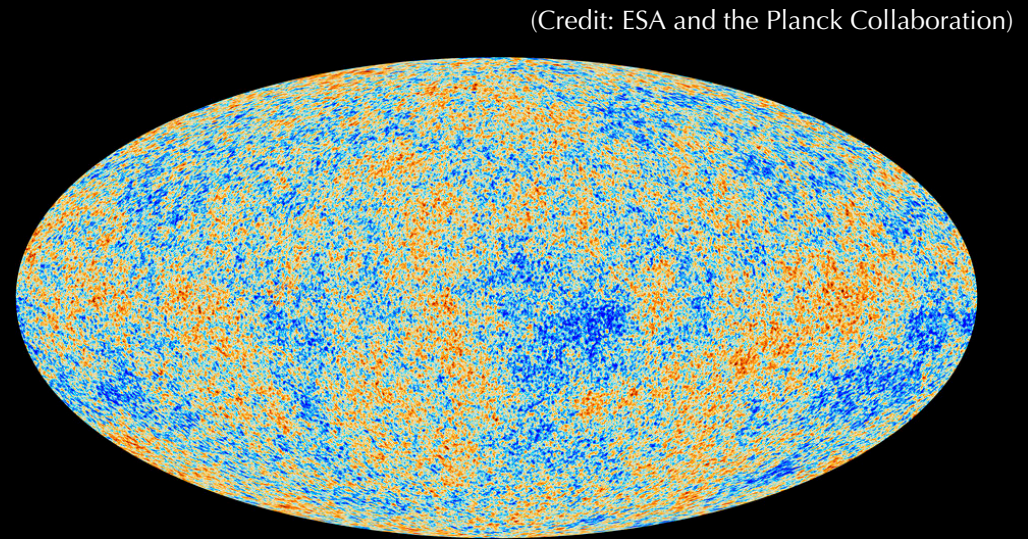
# Cosmological perturbations

We have observed cosmological density perturbations in the Universe.



Large Scale Structure

(Galaxies are gathered in the bright regions.)



CMB anisotropies

(red: hot = dense, blue: cold = sparse)


$$\mathcal{P}_\zeta = 2.1 \times 10^{-9} \text{ (Planck 2018)}$$

$$\rightarrow \delta\rho/\bar{\rho} \simeq 10^{-5}$$

$\zeta$ : curvature perturbation

# What is curvature perturbation?

$$d^2s = g_{00}d\eta^2 + 2g_{0i}d\eta dx^i + a^2 e^{-2\psi} \delta_{ij} dx^i dx^j$$

curvature perturbation 

(In the gauge where  $g_{ij|i \neq j} = 0$ )

In the isotropic and homogeneous Universe (FLRW metric),

$$a^2 e^{-2\psi} \delta_{ij} dx^i dx^j \rightarrow a^2 \left( \frac{d^2 r}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\Omega^2) \right)$$

$$\nearrow R_3 = \frac{6K}{a^2} = 4\nabla^2 \psi$$

3-dim. Ricci scalar

# What people call curvature perturbation

$$d^2s = g_{00}d\eta^2 + 2g_{0i}d\eta dx^i + a^2 e^{-2\psi} \delta_{ij} dx^i dx^j$$

$\psi$  itself is gauge dependent (depends on the coordinate choice).

In Cosmology, the following gauge-invariant quantities are often used:

$$\zeta = -\psi + \frac{\delta\rho}{3(\rho + P)}$$

$\zeta$  coincides the curvature  
in uniform density gauge,  $\delta\rho = 0$ .

$$\mathcal{R} = -\psi - \frac{H\delta\phi}{\dot{\phi}}$$

$\mathcal{R}$  coincides the curvature  
in comoving gauge,  $\delta\phi = 0$ .

In the superhorizon limit,

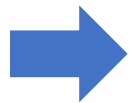
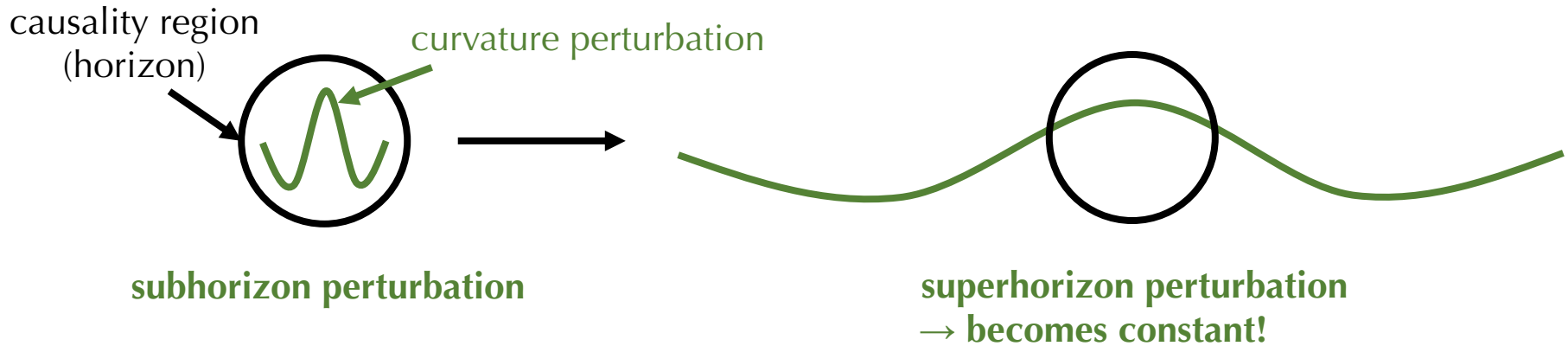
$$\zeta = \mathcal{R}$$

People often call  $\zeta$  and  $\mathcal{R}$   
curvature perturbations.

# Why is curvature perturbation used?

$\zeta$  and  $\mathcal{R}$  are conserved (constant) on superhorizon scales in single field inflation models.

**During inflation**



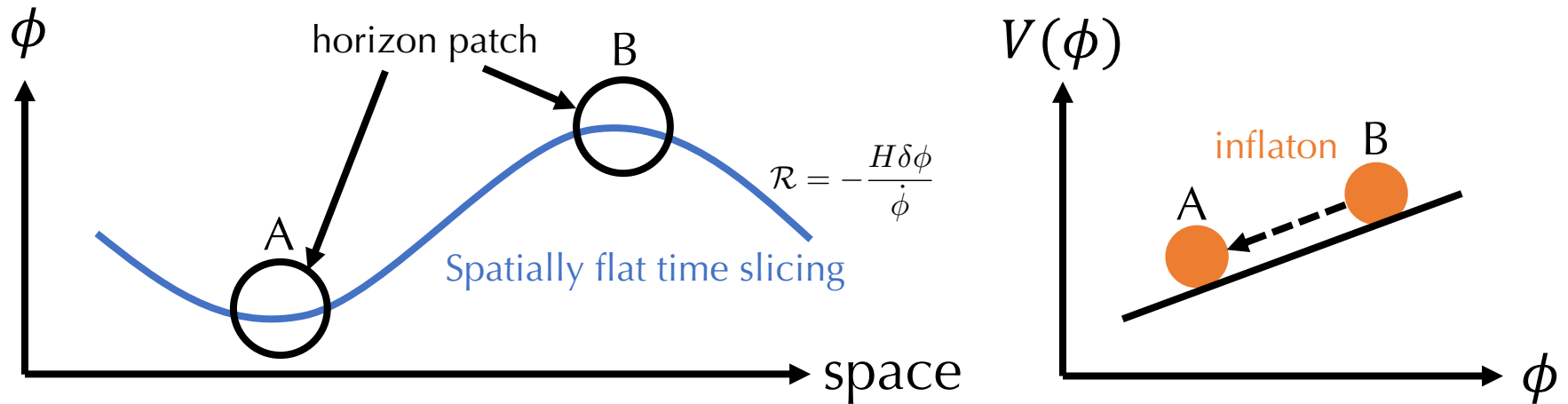
Useful in characterizing the amplitude of cosmological perturbations.

$$\mathcal{P}_\zeta(k < \mathcal{O}(1) \text{ Mpc}^{-1}) \simeq 2.1 \times 10^{-9} \text{ (Planck 2018)}$$



# Separate Universe and curvature conservation

Focus on superhorizon-limit curvature perturbations. (neglect  $\mathcal{O}((k/aH)^2)$  contributions)



## Separate Universe picture:

For local Universes, superhorizon perturbations can be regarded as the background.

A local observer inside a horizon patch **cannot recognize the existence of the superhorizon-limit curvature perturbations**. To be consistent with this, curvature must be constant.

Q: What if superhorizon curvature is time dependent?

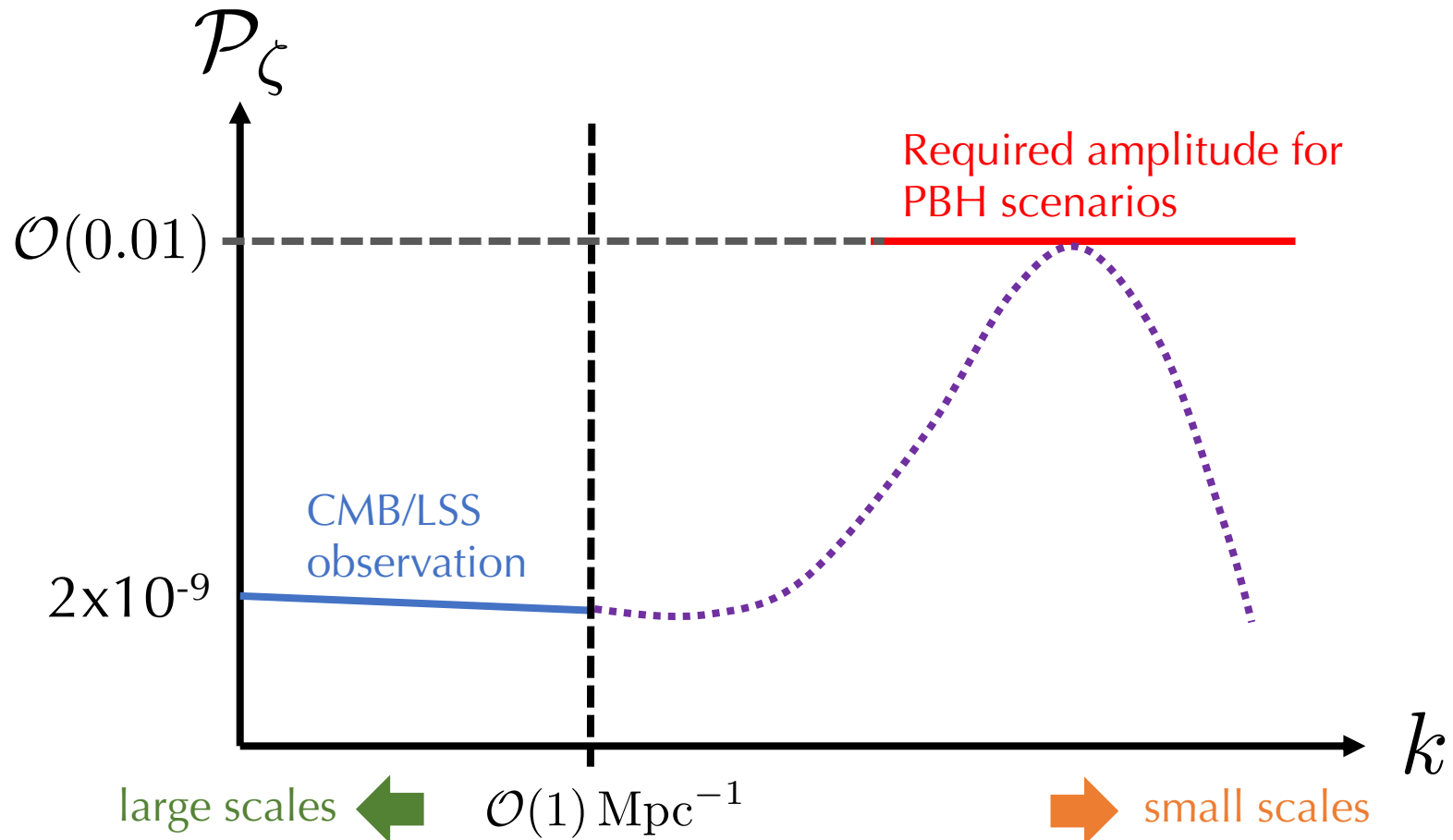
A: The local observer can recognize its existence through  $e^{2\zeta(\eta)} a^2 dx^2$

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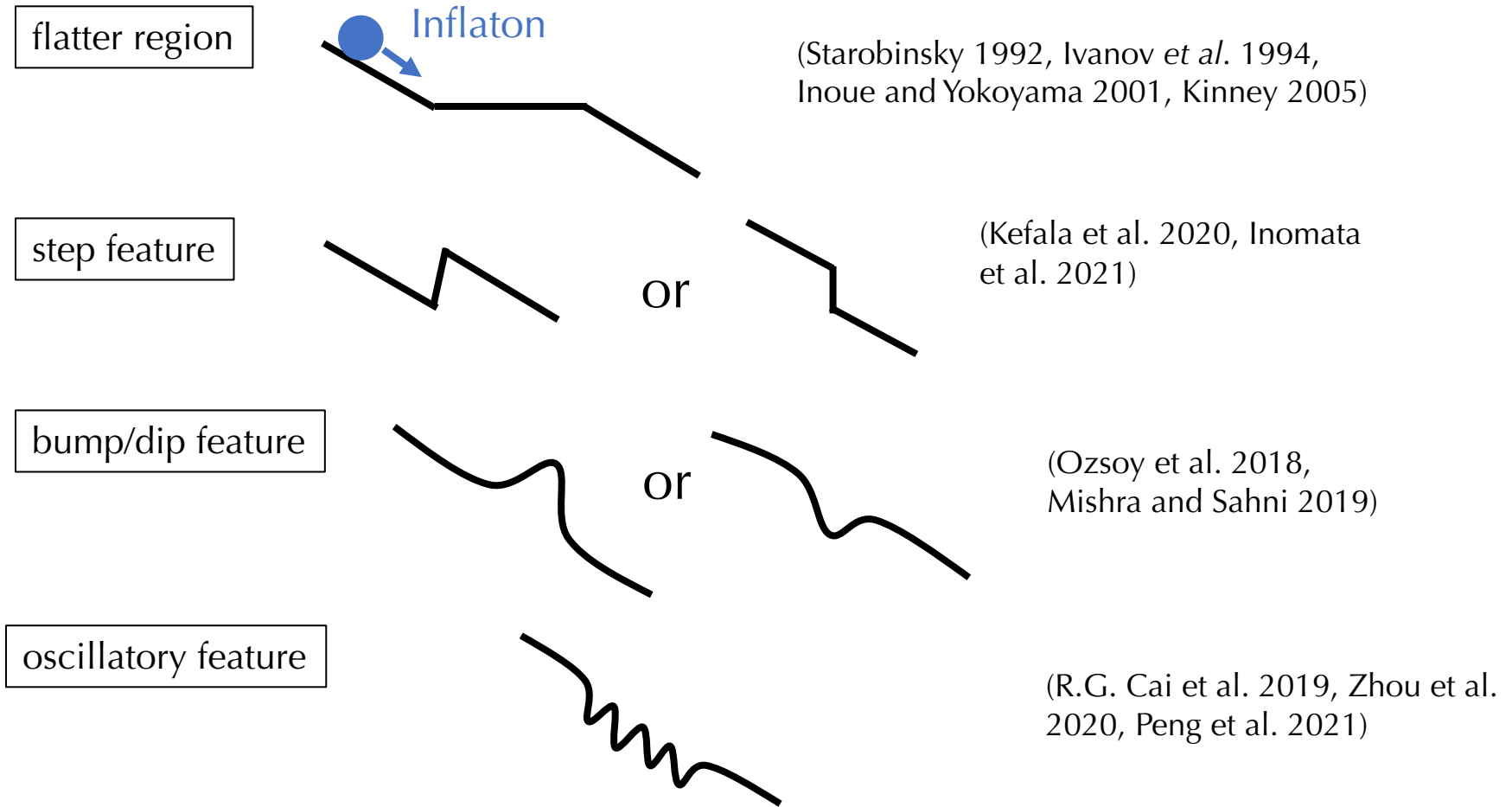
# Large perturbations for PBH scenarios

Primordial black holes are candidates of DM and BHs detected by LIGO-Virgo-KAGRA collaborations.



# Inflaton potentials for large amplification

Single field models for large amplification of density perturbations:



# One loop corrections

Lagrangian: 
$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$$

E.o.m. for the inflaton fluctuations: (slow-roll-parameter suppressed terms neglected)  $(V_{(n)} \equiv \partial^n V / \partial \phi^n)$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

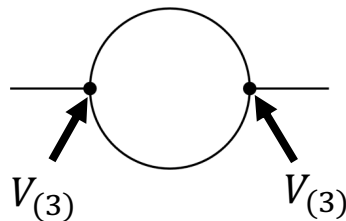
beyond linear order corrections

**In-in formalism:** (Jordan 1986, Calzetta and Hu 1987, Weinberg 2005)

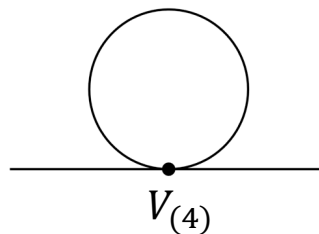
$$\langle\delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\rangle = \langle 0 | \left( T e^{-i\int_{-\infty}^{\eta} d\eta' H_{\text{int}}(\eta')} \right)^\dagger \delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta) \left( T e^{-i\int_{-\infty}^{\eta} d\eta'' H_{\text{int}}(\eta'')} \right) | 0 \rangle$$

$$\left( H_{\text{int},n} \equiv \int d^3x a^4 \mathcal{H}_n, \mathcal{H}_{n(>2)} = \frac{1}{n!} V^{(n)}(\phi)\delta\phi^n \right)$$

two vertices



one vertex



The lowest order corrections to linear power spectrum appear as one loops.

# Superhorizon curvature evolves?

(Note: the conservation of linear  $\zeta$  is well known.)

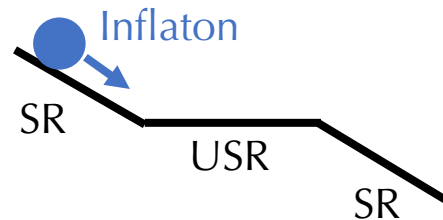
## Recent claim:

Superhorizon-limit curvature perturbations **are not conserved at one-loop level** in the transitions of slow-roll (SR)  $\rightarrow$  ultra-slow roll (USR)  $\rightarrow$  SR.

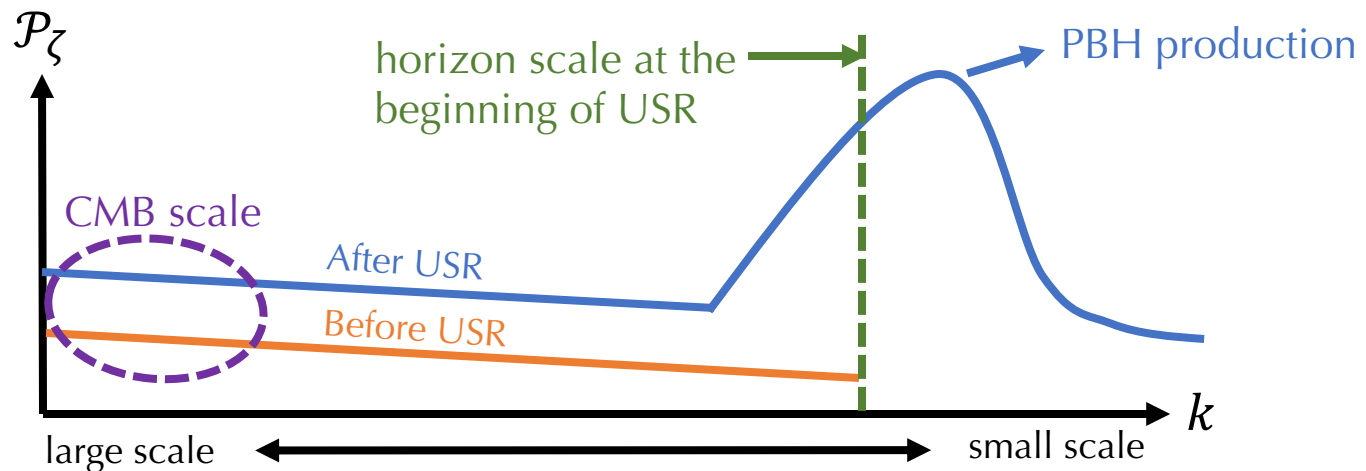
e.o.m.

$$3H\dot{\phi} + V'(\phi) = 0 \text{ (SR)}$$

$$\ddot{\phi} + 3H\dot{\phi} = 0 \text{ (USR)}$$



Kristiano & Yokoyama (2022), followed by Riotto, Choudhury et al., Firouzahahi, Motohashi & Tada, Franciolini et al., Gianmassimo, Cheng et al., Maity et al., Davies et al. (2023), Saburov & Ketov, Ballesteros & Egea (2024)



The one loop corrections can be comparable to the tree-level power spectrum in some PBH models.  $\rightarrow$  Break down of perturbation theory?  $\rightarrow$  models constrained?

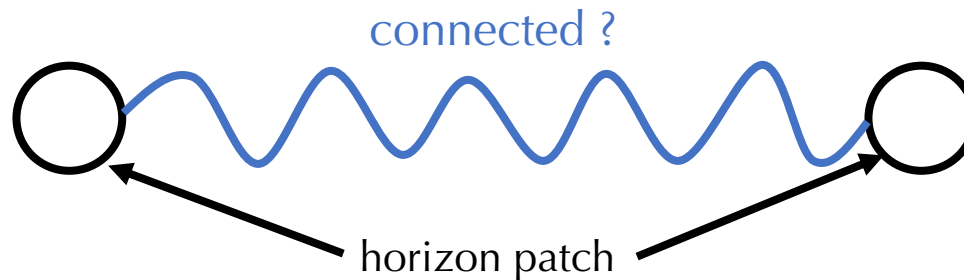
**However, this is inconsistent with the separate Universe picture...**

# Universes connected?

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The violation of the separate Universe means the Universes connected through the distance larger than the horizon.

→ The causality is violated?



**I am going to show the conservation of superhorizon curvature at one loop level.**

**(separate universe picture is valid, causality is satisfied)**

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# Ongoing debates

The higher order action is needed for one loop calculation.

**Comoving gauge ( $\delta\phi=0$ )** is often taken, where  $\zeta$  appears as a metric perturbation.

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right]$$

astro-ph/0210603, Maldacena

$$S_3 = \int \frac{1}{4} \frac{\dot{\phi}^4}{\dot{\rho}^4} [e^{3\rho} \dot{\zeta}^2 \zeta + e^\rho (\partial\zeta)^2 \zeta] - \frac{\dot{\phi}^2}{\dot{\rho}^2} e^{3\rho} \dot{\zeta} \partial_i \chi \partial_i \zeta +$$

$$- \frac{1}{16} \frac{\dot{\phi}^6}{\dot{\rho}^6} e^{3\rho} \dot{\zeta}^2 \zeta + \frac{\dot{\phi}^2}{\dot{\rho}^2} e^{3\rho} \dot{\zeta} \zeta^2 \frac{d}{dt} \left[ \frac{1}{2} \frac{\ddot{\phi}}{\dot{\rho}} + \frac{1}{4} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right] + \frac{1}{4} \frac{\dot{\phi}^2}{\dot{\rho}^2} e^{3\rho} \partial_i \partial_j \chi \partial_i \partial_j \zeta$$

$$+ f(\zeta) \left. \frac{\delta L}{\delta \zeta} \right|_1$$

arXiv:0709.2708, Jarnhus & Sloth

$$S^{(4)} = \frac{1}{2} \int dt d^3x a^3 \left\{ -\frac{1}{3} \zeta^3 \partial^2 \zeta - 2\alpha^{(1)} (\zeta \partial_i \zeta \partial^i \zeta + \zeta^2 \partial^2 \zeta) + \dot{\phi}_e^2 \alpha^{(1)2} \left[ \frac{9}{2} \zeta^2 - 3\zeta \alpha^{(1)} + \alpha^{(1)2} \right] \right.$$

$$\left. \left[ \frac{1}{2} \zeta^2 + \zeta \alpha^{(1)} + \alpha^{(1)2} \right] [\partial_i \partial_j \chi^{(1)} \partial^i \partial^j \chi^{(1)} - \partial^2 \chi^{(1)} \partial^2 \chi^{(1)}] + (6H^2 - \dot{\phi}^2) \alpha^{(2)2} \right.$$

$$- 2[\zeta + \alpha^{(1)}] [\partial_i \partial_j \chi^{(1)} \partial^i \partial^j \chi^{(2)} - \partial^2 \chi^{(1)} \partial^2 \chi^{(2)} - 2\partial_i \partial_j \chi^{(1)} \partial^i \chi^{(1)} \partial^j \zeta]$$

$$- 2 [2\partial_i \partial_j \chi^{(2)} \partial^i \chi^{(1)} \partial^j \zeta + 2\partial_i \partial_j \chi^{(1)} \partial^i \chi^{(2)} \partial^j \zeta - \partial_j \chi^{(1)} \partial_i \zeta \partial^i \chi^{(1)} \partial^j \zeta]$$

$$\left. + \frac{1}{2} \partial_i \beta_j^{(2)} \partial^i \beta^{j(2)} - 2\alpha^{(1)} \partial_i \partial_j \chi^{(1)} \partial^i \beta^{j(2)} \right\}$$

However, these expressions neglect the boundary terms.

$$\text{e.g. } \int dt A \dot{B} = - \int dt \dot{A} B + \int dt \frac{d}{dt} (AB)$$

boundary term

Ongoing debates: the missing boundary terms lead to the curvature conservation?

Fumagalli (2023), Tada et al. (2023), Firouzahai (2023), Braglia & Pinol (2024), Kawaguchi et al. (2024)

# Strategy of this work

In this work, **spatially-flat gauge** ( $\psi = \mathbf{0}$ ) is taken, where  $\delta\phi$  is the basic quantity.

The metric perturbations are suppressed by slow-roll parameter  $\epsilon$ , compared to  $V_{(n)}$  terms.

( $\epsilon \rightarrow 0$  is known as the decoupling limit in effective field theory of inflation.)

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right]$$

$$\downarrow \quad \left( V_{(n)} \equiv \frac{\partial^n V(\phi)}{\partial \phi^n} \right)$$

$$S_n = - \int d^4x a^4 \frac{V_{(n)}(\bar{\phi})}{n!} \delta\phi^n \quad \text{Simple!}$$

**Advantage:** The higher order action can be easily obtained.  
 → no need to worry about boundary terms!

**Strategy:** We first calculate the one-loop power spectrum of  $\delta\phi$ .  
 Then, we connect it to the one loop-power spectrum of  $\zeta$ .

# One loop calculation

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$$(\delta\phi = \delta\phi^{(1)} + \delta\phi^{(2)} + \delta\phi^{(3)} + \dots)$$



$$\hat{\mathcal{N}}_k\delta\phi_{\mathbf{k}}^{(1)} = 0,$$

$$\hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2V_{(2)}(\bar{\phi})$$

$$\hat{\mathcal{N}}_k\delta\phi_{\mathbf{k}}^{(2)} = -\frac{a^2}{2}V_{(3)}\int\frac{d^3p}{(2\pi)^3}\delta\phi_{\mathbf{k}-\mathbf{p}}^{(1)}\delta\phi_{\mathbf{p}}^{(1)},$$

$$\hat{\mathcal{N}}_k\delta\phi_{\mathbf{k}}^{(3)} = -\frac{a^2}{2}V_{(3)}\int\frac{d^3p}{(2\pi)^3}(\delta\phi_{\mathbf{k}-\mathbf{p}}^{(1)}\delta\phi_{\mathbf{p}}^{(2)} + \delta\phi_{\mathbf{k}-\mathbf{p}}^{(2)}\delta\phi_{\mathbf{p}}^{(1)}) - \frac{a^2}{6}V_{(4)}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3p'}{(2\pi)^3}\delta\phi_{\mathbf{p}}^{(1)}\delta\phi_{\mathbf{p}'}^{(1)}\delta\phi_{\mathbf{k}-\mathbf{p}-\mathbf{p}'}^{(1)}$$

**In-in formalism = equation of motion approach:** (Musso (2013), Inomata et al. (2022))

$$\begin{aligned} \langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle &= \langle 0| \left( T e^{-i\int_{-\infty}^{\eta} d\eta' H_{\text{int}}(\eta')} \right)^{\dagger} \delta\phi_{\mathbf{k}}^{(1)}\delta\phi_{\mathbf{k}'}^{(1)} \left( T e^{-i\int_{-\infty}^{\eta} d\eta'' H_{\text{int}}(\eta'')} \right) |0\rangle \\ &= \dots \\ &= \langle 0|\delta\phi_{\mathbf{k}}^{(1)}\delta\phi_{\mathbf{k}'}^{(1)}|0\rangle + \langle 0|\delta\phi_{\mathbf{k}}^{(2)}\delta\phi_{\mathbf{k}'}^{(2)}|0\rangle + \langle 0|\delta\phi_{\mathbf{k}}^{(1)}\delta\phi_{\mathbf{k}'}^{(3)}|0\rangle + \langle 0|\delta\phi_{\mathbf{k}}^{(3)}\delta\phi_{\mathbf{k}'}^{(1)}|0\rangle \end{aligned}$$

$(H_{\text{int},n} \equiv \int d^3x a^4 \mathcal{H}_n, \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n)$

# One loop calculation

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$$(\delta\phi = \delta\phi^{(1)} + \delta\phi^{(2)} + \delta\phi^{(3)} + \dots)$$



$$\hat{\mathcal{N}}_k\delta\phi_{\mathbf{k}}^{(1)} = 0,$$

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$$\hat{\mathcal{N}}_k\delta\phi_{\mathbf{k}}^{(3)} = -\frac{a^2}{2}V_{(3)}\int\frac{d^3p}{(2\pi)^3}(\delta\phi_{\mathbf{k}-\mathbf{p}}^{(1)}\delta\phi_{\mathbf{p}}^{(2)} + \delta\phi_{\mathbf{k}-\mathbf{p}}^{(2)}\delta\phi_{\mathbf{p}}^{(1)}) - \frac{a^2}{6}V_{(4)}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3p'}{(2\pi)^3}\delta\phi_{\mathbf{p}}^{(1)}\delta\phi_{\mathbf{p}'}^{(1)}\delta\phi_{\mathbf{k}-\mathbf{p}-\mathbf{p}'}^{(1)}$$

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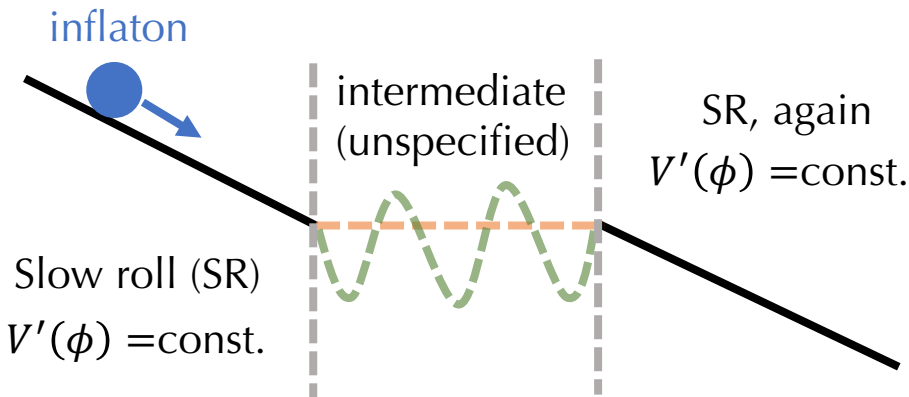
$$\begin{aligned} \langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle &= \langle 0 | \left( T e^{-i\int_{-\infty}^{\eta} d\eta' H_{\text{int}}(\eta')} \right)^\dagger \delta\phi_{\mathbf{k}}^{(1)} \delta\phi_{\mathbf{k}'}^{(1)} \left( T e^{-i\int_{-\infty}^{\eta} d\eta'' H_{\text{int}}(\eta'')} \right) | 0 \rangle \\ &= \dots \\ &= \underbrace{\langle 0 | \delta\phi_{\mathbf{k}}^{(1)} \delta\phi_{\mathbf{k}'}^{(1)} | 0 \rangle}_{\text{tree}} + \underbrace{\langle 0 | \delta\phi_{\mathbf{k}}^{(2)} \delta\phi_{\mathbf{k}'}^{(2)} | 0 \rangle}_{\text{Poisson fluctuations}} + \underbrace{\langle 0 | \delta\phi_{\mathbf{k}}^{(1)} \delta\phi_{\mathbf{k}'}^{(3)} | 0 \rangle + \langle 0 | \delta\phi_{\mathbf{k}}^{(3)} \delta\phi_{\mathbf{k}'}^{(1)} | 0 \rangle}_{\text{nonzero even in superhorizon limit}} \\ &\quad \rightarrow 0 \text{ in superhorizon limit} \end{aligned}$$

Superhorizon curvature perturbations evolve at one loop? → **No. There is a trick!**

# Conservation of curvature

The trick lies in the relation between  $\delta\phi$  and  $\zeta$ .

Consider the simplest case:



During the intermediate period,  $\zeta$  is enhanced.  
 $\rightarrow$  Peak power spectrum

We assume the separate Universe satisfied at least during the SR periods (**do not assume that during the intermediate period**).

From  $\delta N$  formalism, curvature perturbations during the SR periods are:

$$-\zeta|_{\leq 1\text{-loop}} = \left. \frac{H\delta\phi}{\dot{\phi}} \right|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\phi}^{(0)} + \dot{\phi}^{(2)}} \quad (\text{SR})$$

On the other hand,  $\frac{H\delta\phi}{\dot{\phi}}$  is **always** constant:  $\left. \frac{H\delta\phi}{\dot{\phi}} \right|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\phi}^{(0)} + \dot{\phi}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\phi}^{(0)}} = \text{const.}$

**Point:**  $\dot{\phi}$  gets the one-loop backreaction,  $\dot{\phi}^{(2)}$ , which cancels  $\delta\phi^{(3)}$ .

$\zeta$  during the first and the second SR periods coincide.  $\rightarrow \zeta$  is conserved!

# One loop backreaction

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V_{(1)}(\bar{\phi}) = -\frac{1}{2}V_{(3)}(\bar{\phi}) \langle (\delta\phi^{(1)})^2 \rangle$$



Take time derivative

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(0)} = 0,$$

$$\left( \bar{\Pi} \equiv \dot{\bar{\phi}}, \hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial \eta^2} + 2\mathcal{H} \frac{\partial}{\partial \eta} + k^2 + a^2 V_{(2)}(\bar{\phi}) \right)$$

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(2)} = -\frac{a^2}{2} \left( V_{(3)} \langle \delta\phi^2 \rangle' + V_{(4)} \langle \delta\phi^2 \rangle \bar{\Pi}^{(0)} \right)$$

Recap:

$$\hat{\mathcal{N}}_k \delta\phi_{\mathbf{k}}^{(1)} = 0,$$

$$\hat{\mathcal{N}}_k \delta\phi_{\mathbf{k}}^{(3)} = -\frac{a^2}{2} V_{(3)} \int \frac{d^3 p}{(2\pi)^3} (\delta\phi_{\mathbf{k}-\mathbf{p}}^{(1)} \delta\phi_{\mathbf{p}}^{(2)} + \delta\phi_{\mathbf{k}-\mathbf{p}}^{(2)} \delta\phi_{\mathbf{p}}^{(1)}) - \frac{a^2}{6} V_{(4)} \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 p'}{(2\pi)^3} \delta\phi_{\mathbf{p}}^{(1)} \delta\phi_{\mathbf{p}'}^{(1)} \delta\phi_{\mathbf{k}-\mathbf{p}-\mathbf{p}'}^{(1)}$$

After some (easy) calculation, we find

$$\hat{\mathcal{N}}_q \delta\phi_{\mathbf{q}}^{(3)} \Big|_{q \rightarrow 0} = -\frac{a^2}{2} \left( V_{(3)} \langle \delta\phi^2 \rangle' + V_{(4)} \langle \delta\phi^2 \rangle \bar{\Pi}^{(0)} \right) \frac{\delta\phi_{\mathbf{q}}^{(1)}}{\bar{\Pi}^{(0)}}$$

We finally obtain

$$\delta\phi_{\mathbf{q}}^{(3)} = \frac{\dot{\bar{\phi}}^{(2)}}{\dot{\bar{\phi}}^{(0)}} \delta\phi_{\mathbf{q}}^{(1)} \quad \longrightarrow \quad \frac{H\delta\phi}{\dot{\bar{\phi}}} \Big|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$

**This relation is always satisfied.**

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Non-conservation of superhorizon curvature perturbations at one-loop level has recently been claimed.

However, the claim is inconsistent with the separate Universe picture.

I have taken the spatially-flat gauge and focus on  $\delta\phi$  evolution at one loop level.

I have found that the superhorizon curvature is conserved if we carefully consider the one-loop backreaction.



# Main message

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Superhorizon curvature perturbations are constant.

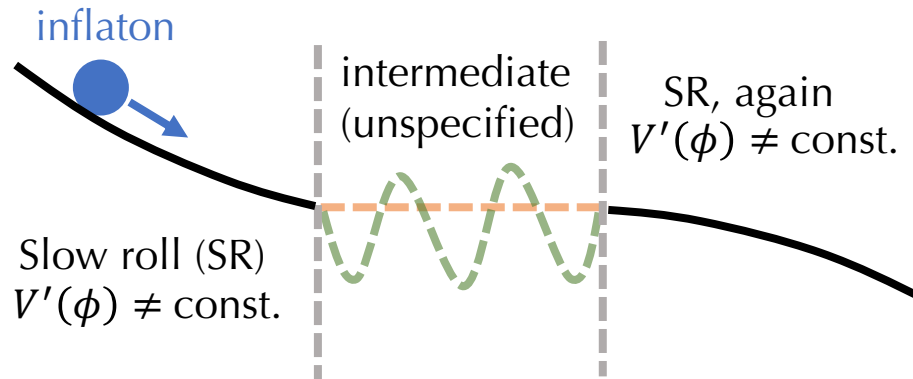
Separate Universe picture is valid.

**Even at one loop level**

# Backup



# In general SR potentials



Again, we assume the separate Universe satisfied at least during the SR periods **(do not assume that during the intermediate period)**.

When the separate Universe is satisfied, the curvature perturbations are conserved even at non-perturbative (including one-loop) level. (Lyth, Malik, and Sasaki, 2004)

This means that, if we find one concrete SR potential for the conservation of  $\zeta$ , the conservation is secured for any types of SR potential.

**One concrete example:** the SR potentials that have region of  $V'(\phi) = \text{const.}$

# Renormalization

In general, the loop contributions have divergence, which must be cancelled out by counter terms.

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(2)} = -\frac{a^2}{2} \left( V_{(3)} \langle \delta\phi^2 \rangle \cdot + V_{(4)} \langle \delta\phi^2 \rangle \bar{\Pi}^{(0)} \right)$$

$$\hat{\mathcal{N}}_q \delta\phi_{\mathbf{q}}^{(3)} \Big|_{q \rightarrow 0} = -\frac{a^2}{2} \left( V_{(3)} \langle \delta\phi^2 \rangle \cdot + V_{(4)} \langle \delta\phi^2 \rangle \bar{\Pi}^{(0)} \right) \frac{\delta\phi_{\mathbf{q}}^{(1)}}{\bar{\Pi}^{(0)}}$$

Counter terms are introduced in the same way for  $\bar{\Pi}^{(2)} (= \dot{\bar{\phi}}^{(2)})$  and  $\delta\phi_{\mathbf{q}}^{(3)}$  through  $\hat{\mathcal{N}}_0$ .

$$\hat{\mathcal{N}}_q \equiv \frac{\partial^2}{\partial \eta^2} + 2\mathcal{H} \frac{\partial}{\partial \eta} + q^2 + a^2 V_{(2)}(\bar{\phi}) + \underline{a^2 m_{\text{ct}}^2}$$

counter term

The introduction of the counter terms does not break the following relations:

$$\delta\phi_{\mathbf{q}}^{(3)} = \frac{\dot{\bar{\phi}}^{(2)}}{\dot{\bar{\phi}}^{(0)}} \delta\phi_{\mathbf{q}}^{(1)}$$

$$-\zeta = \frac{H\delta\phi}{\dot{\bar{\phi}}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$