Characterising spacetime during cosmological collapse

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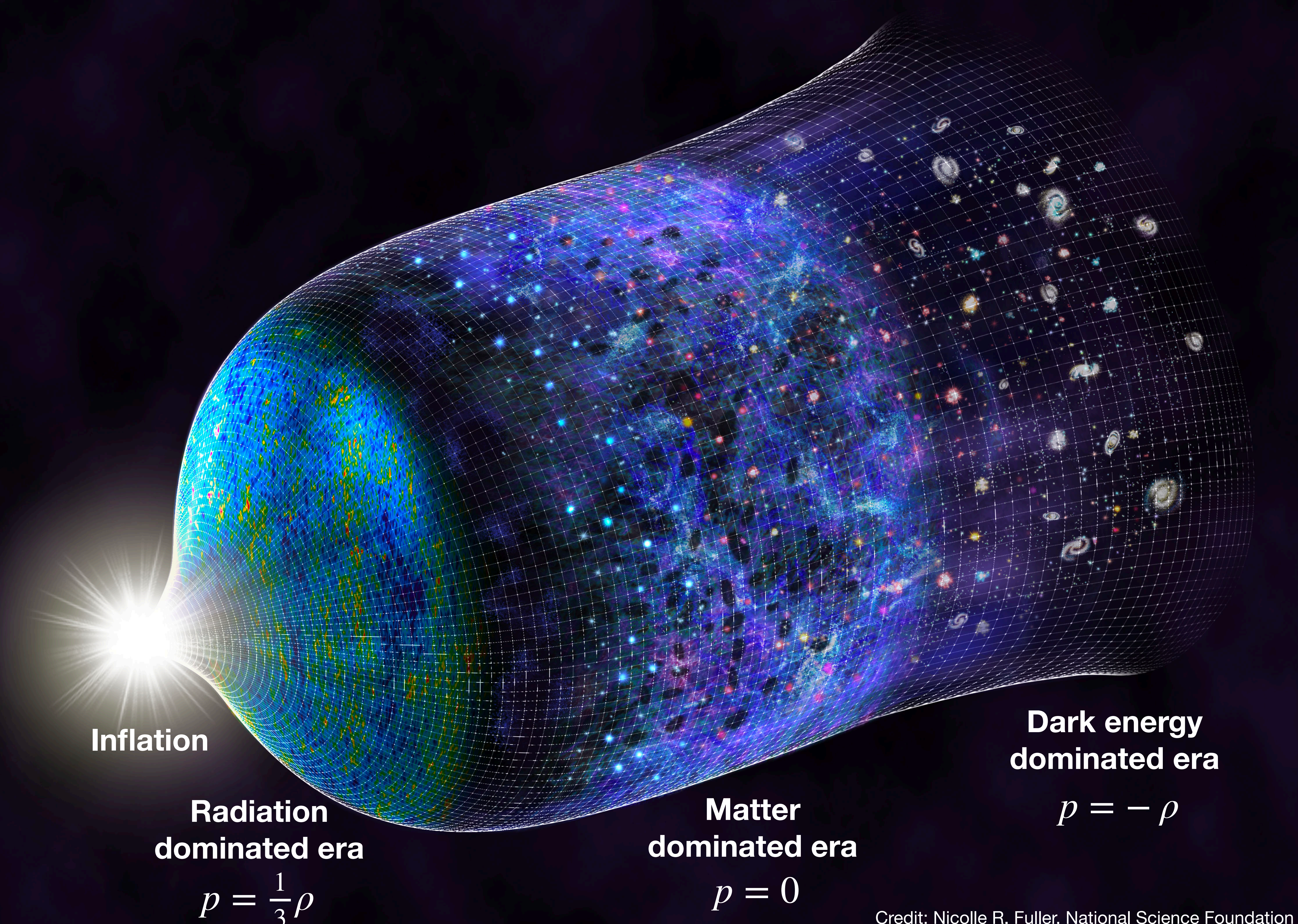
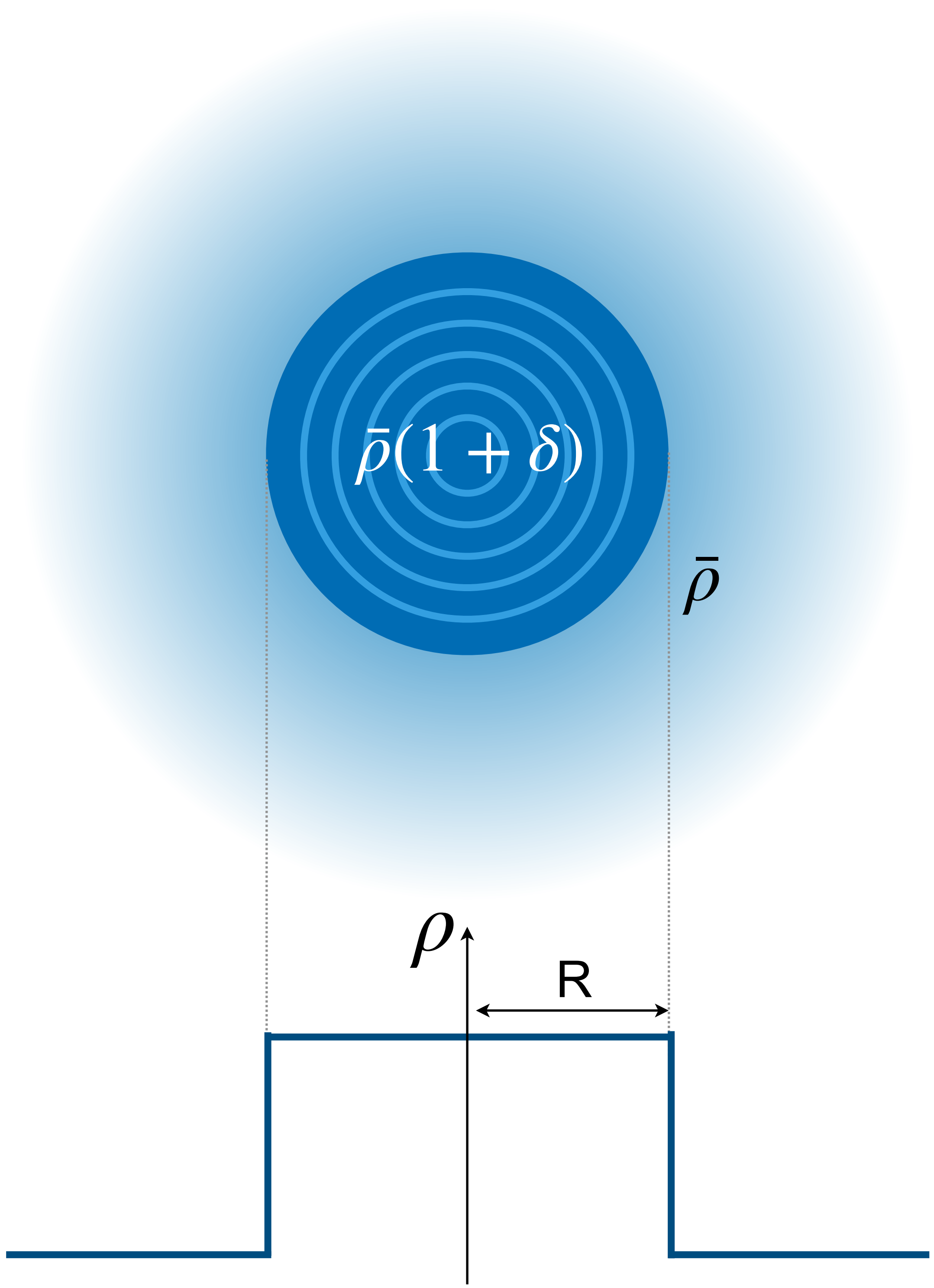
Robyn L. Munoz
Based on **2211.08133** and **2302.09033**
With Marco Bruni

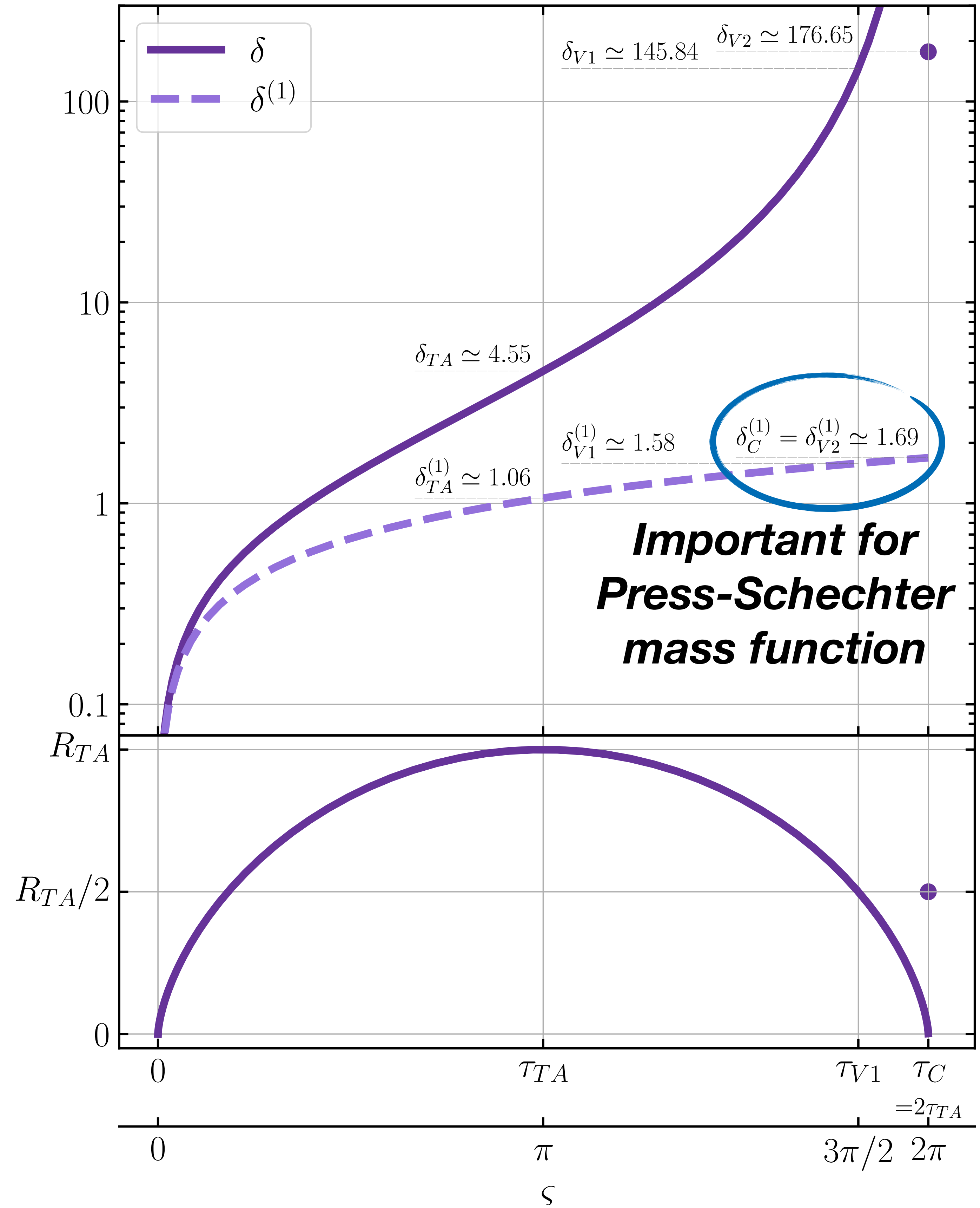


Credit: Nicolle R. Fuller, National Science Foundation

Top-Hat Spherical Collapse Model

J.E.Gunn and J.R.Gott (1972)





Important for Press-Schechter mass function

Objective:

Explore the top-hat spherical collapse model in full GR, even beyond spherical symmetry, and see how the spacetime responds.

Initial Conditions

Background:

- Flat FLRW metric,
- ΛCDM with pressureless perfect fluid,
- Matter-dominated era.

$$^{(3)}R^{(1)} = \frac{1}{a^2}\delta^{ij}\partial_i\partial_j\mathcal{R}_c$$

- $\mathcal{R}_c$ and $\zeta^{(1)}$ are used to quantify perturbations created during inflation
- $\mathcal{R}_c$ is gauge invariant at first order
- $\dot{\mathcal{R}}_c = 0$

Inhomogeneity:

- Synchronous and comoving gauge
- Scalar perturbations,

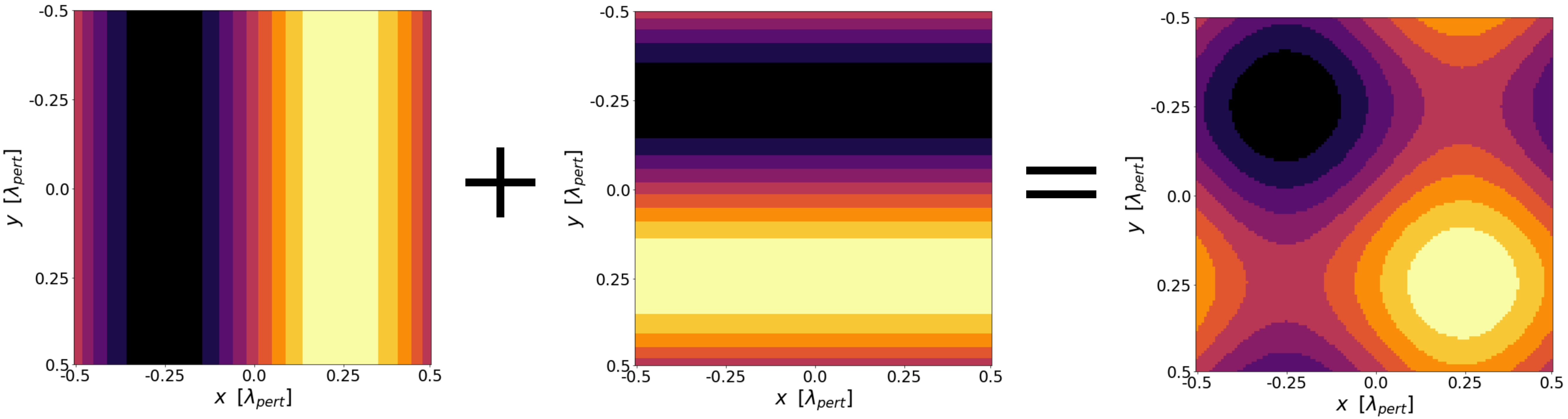
M.Bruni, J.C.Hidalgo, N.Meures and D.Wands (2014) **1307.1478**

Comoving Curvature Perturbation: $\mathcal{R}_c$ → γ_{ij} → K_{ij} and $^{(3)}R$ → ρ

- γ_{ij}: Spatial metric (Perturbed up to 1st order)
- K_{ij}: Extrinsic curvature
- $^{(3)}R$: 3 Ricci scalar
- ρ: Energy density (From the Hamiltonian constraint equation)

Initial Conditions

$$\mathscr{R}_c = A_{pert}\left[\sin\left(\frac{2\pi x}{\lambda_{pert}}\right) + \sin\left(\frac{2\pi y}{\lambda_{pert}}\right) + \sin\left(\frac{2\pi z}{\lambda_{pert}}\right)\right]$$



$$\sin\left(\frac{2\pi x}{\lambda_{pert}}\right)$$

$$\sin\left(\frac{2\pi y}{\lambda_{pert}}\right)$$

Initial Conditions

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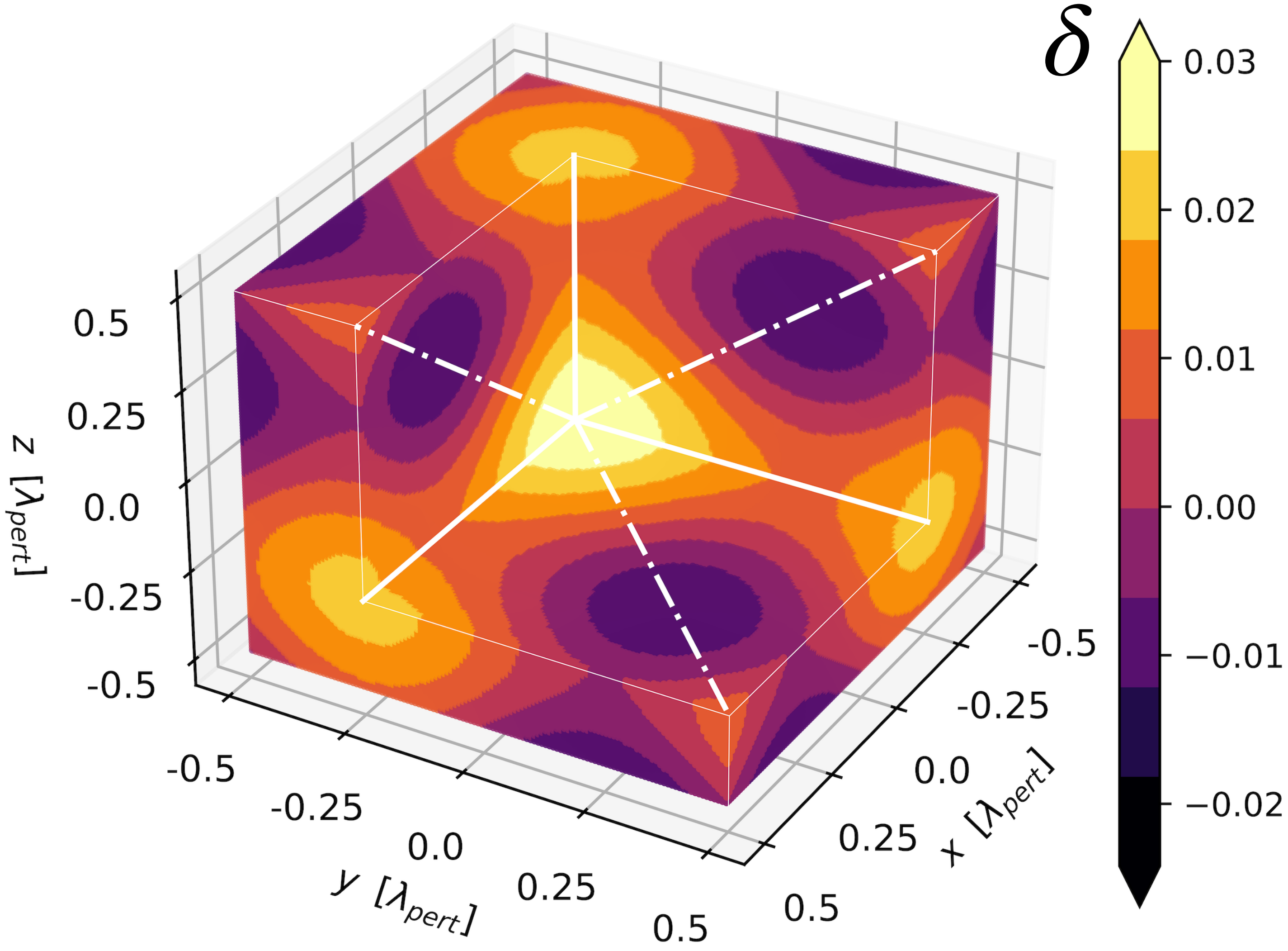

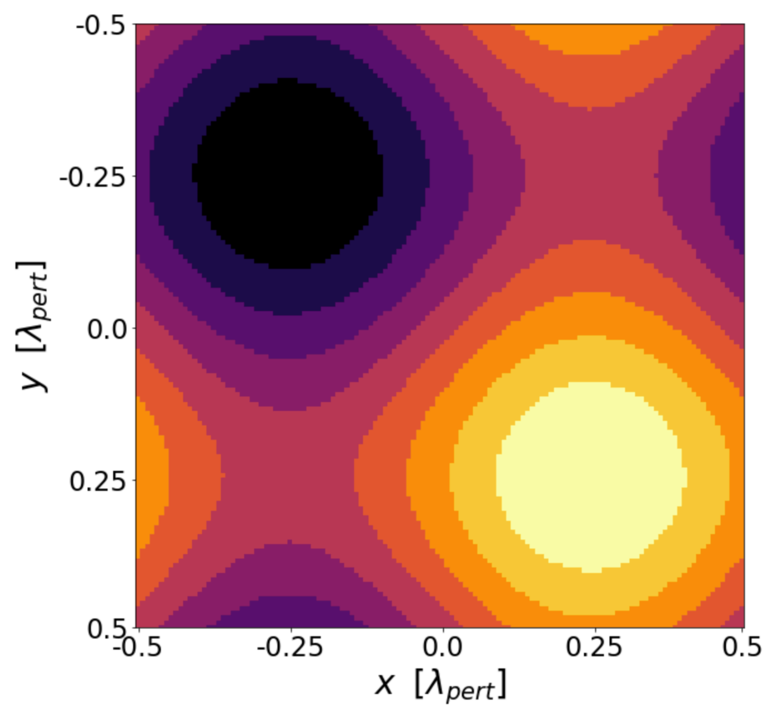

Einstein Toolkit

einstein toolkit

Initial conditions
in Einstein Toolkit

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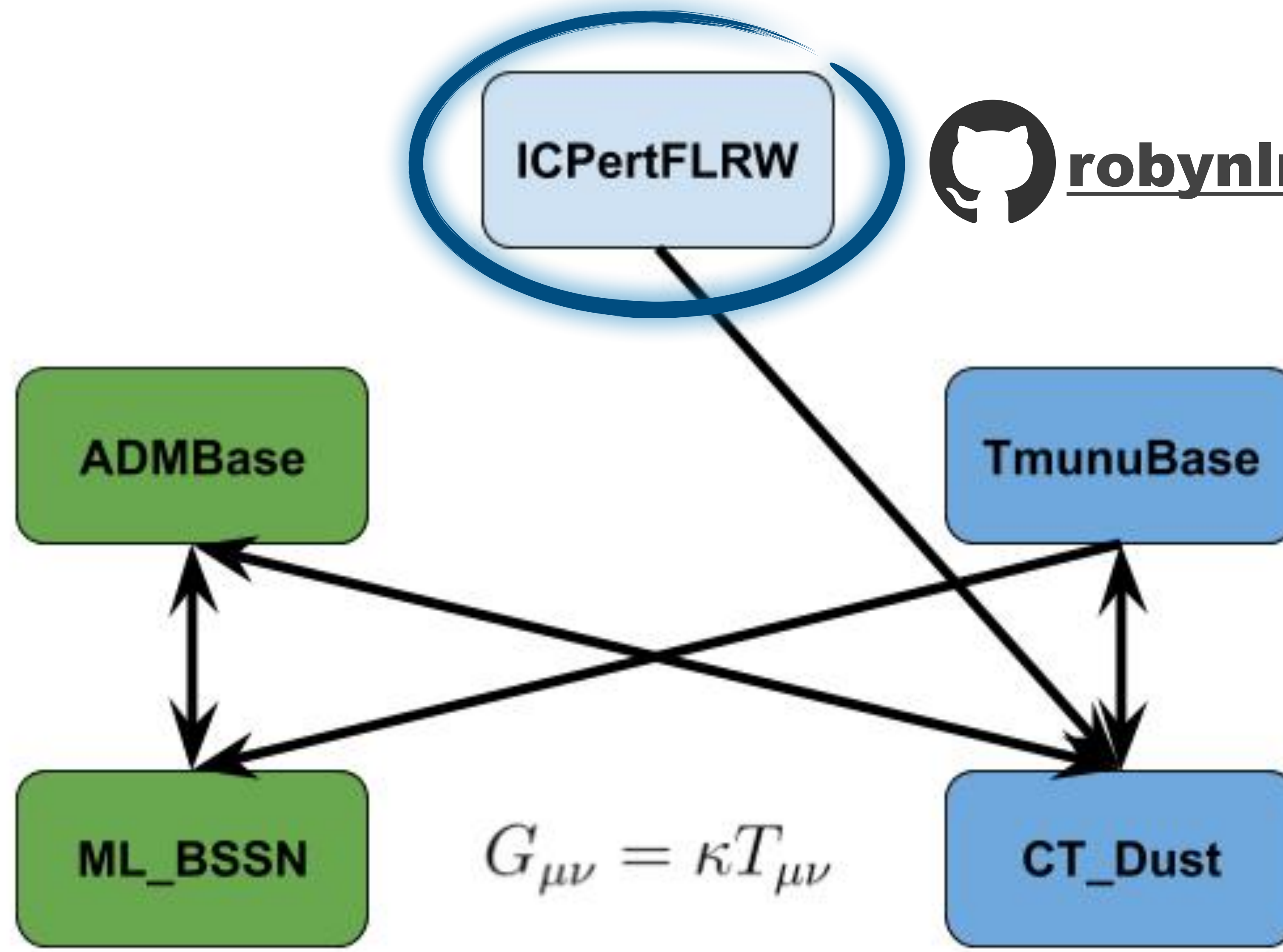
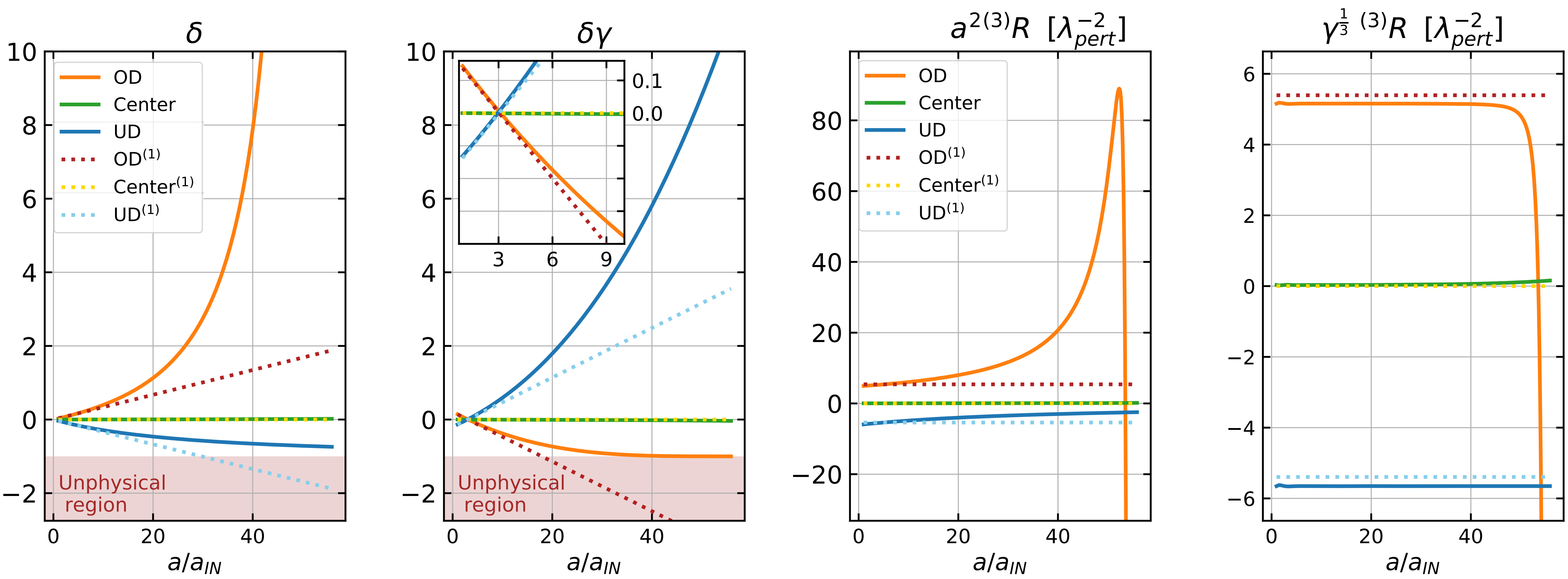


$p = 0$

Simulation evolution

$\delta_{OD,\,IN} = 0.03 \quad L = 4/\left(a_{IN}H_{IN}\right)\text{Mpc} \quad z_{IN} = 302.5$



Perturbation in:

Matter density:

$\delta = \rho/\bar{\rho} - 1$

Determinant of spatial metric:

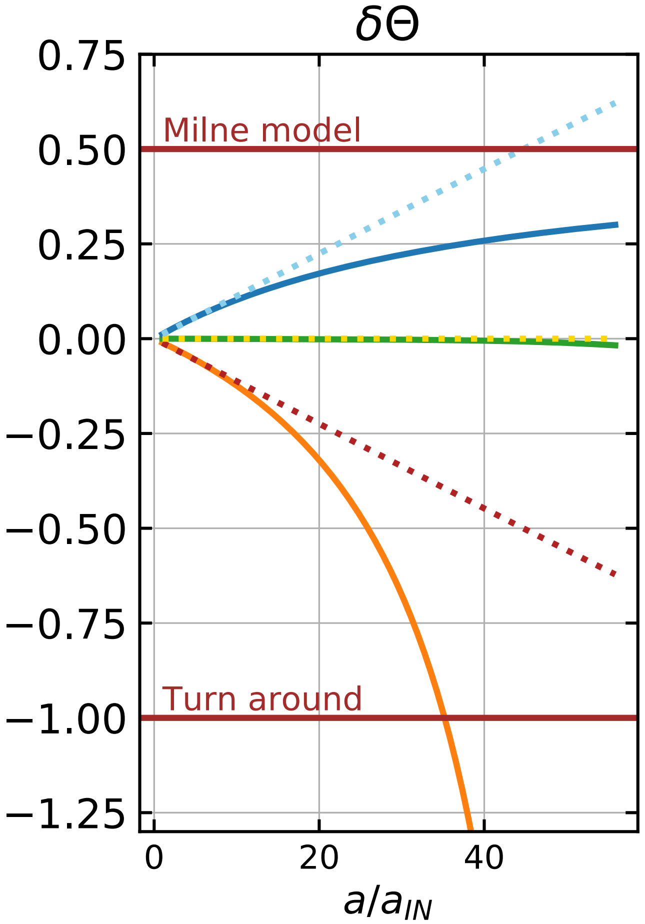
$\delta\gamma = \gamma/\bar{\gamma} - 1$

Conformal 3 Ricci scalar:

$a^{2\,(3)}R$

Local conformal 3 Ricci scalar:

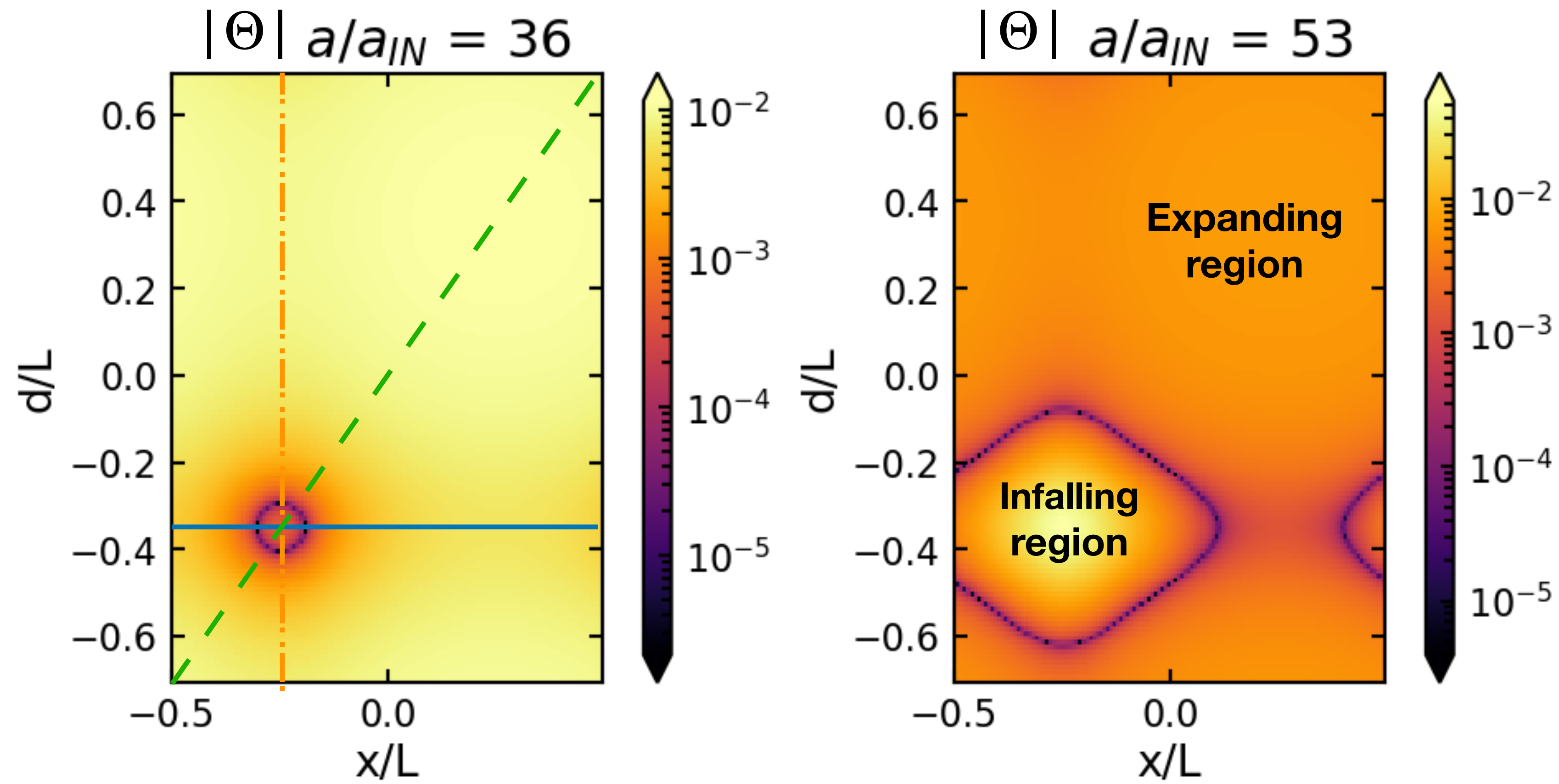
$\gamma^{\frac{1}{3}\ (3)}R$



Perturbation in:
Trace of expansion:

$$\delta\Theta = \Theta/\bar{\Theta} - 1$$

Turn-around boundary $\Theta = 0$



The expansion of the turn-around boundary depends on the initial distribution.

The directions going through under-dense regions eventually stop expanding their infalling region and reduce in size.

Simulation vs Top-Hat model

$\delta_{OD,\,IN} = 0.03$ $L = 4/\left(a_{IN}H_{IN}\right)$		Top Hat	Here 2302.09033	E.Bentivegna & M.Bruni (2016) 1511.05124	W.East et al (2018) 1711.06681
Turn Around $\Theta = 0$	$\delta^{(1)}_{OD}$	1.062 41	1.057 34 ± 2e-6	1.8 *Correction	
	δ_{OD}	4.551 65	4.551 64 ± 1e-5		
Collapse / Crash	$\delta^{(1)}_{OD}$	1.686 47	1.678 ± 3e-3	2.88	1.686

At the peak of the over-density the top-hat spherical collapse model is an excellent approximation.

Simulation vs Top-Hat model

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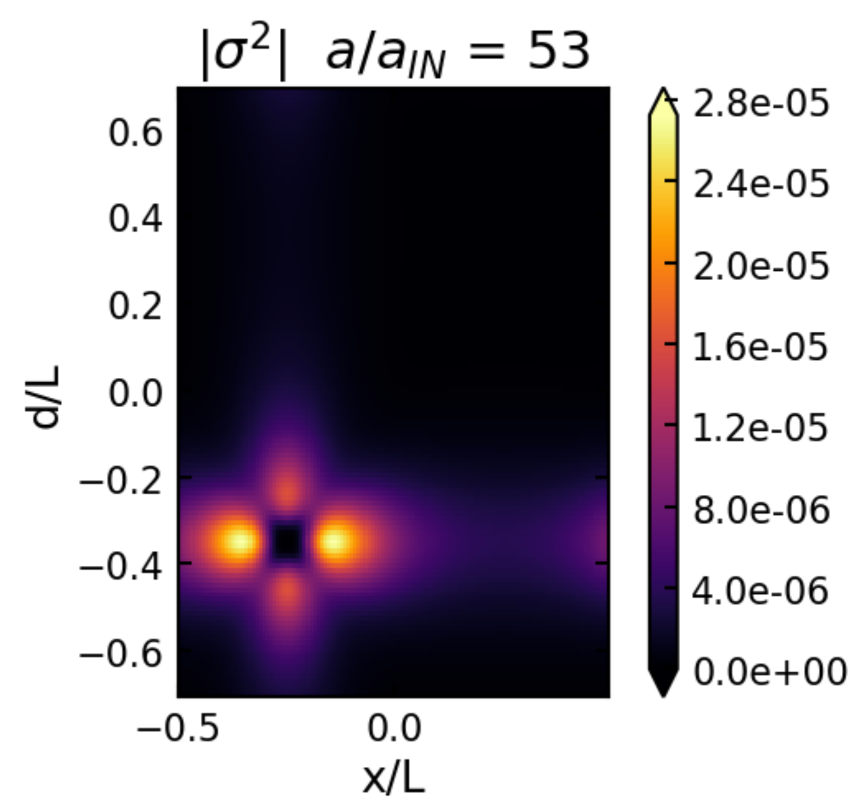
At the peak of the over-density the top-hat spherical collapse model is an excellent approximation.

This is because we find that the shear is locally negligible. Then, neglecting the shear in the Raychaudhuri equation gives the spherical collapse model.

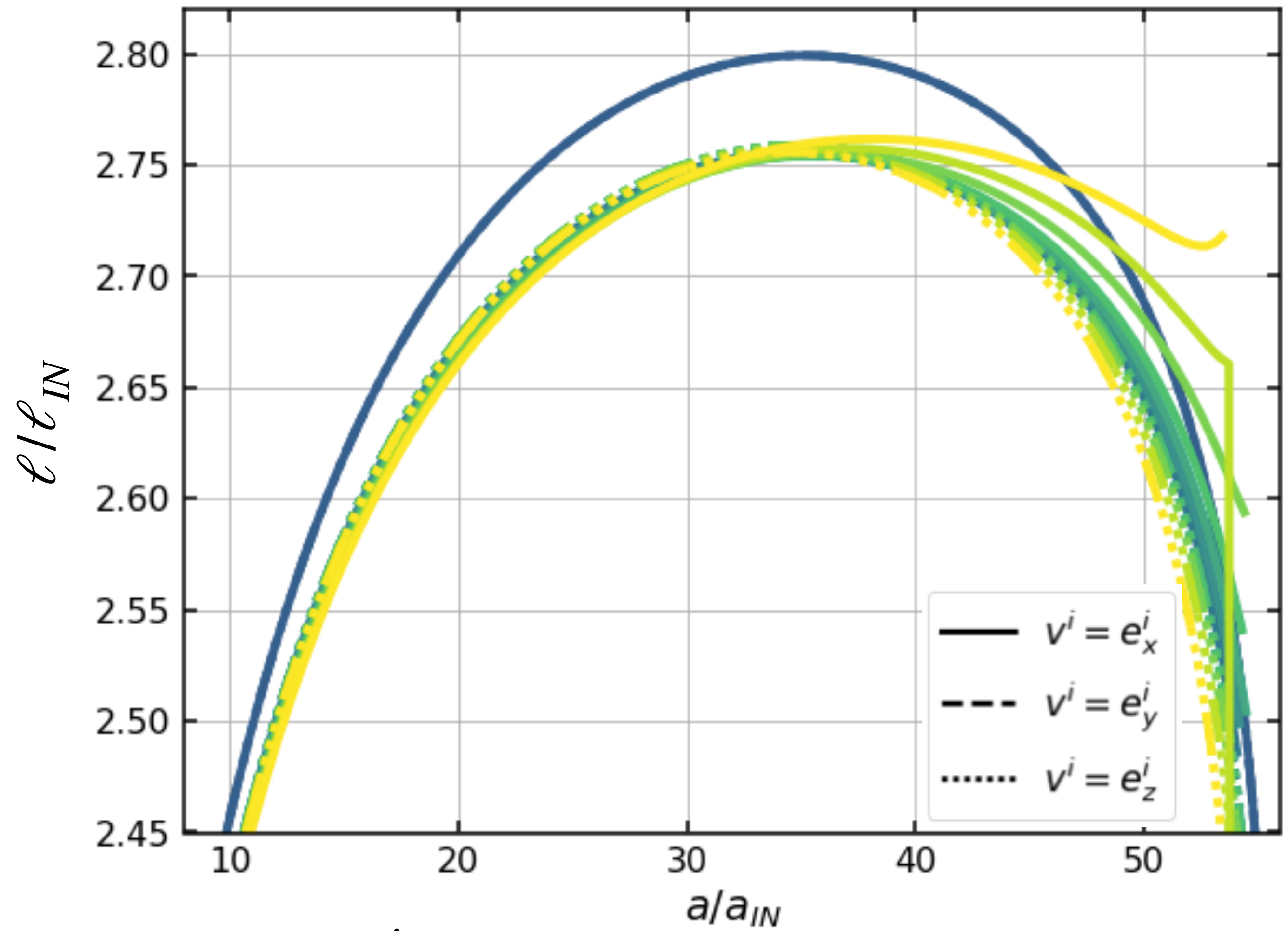


Simulation vs Top-Hat model

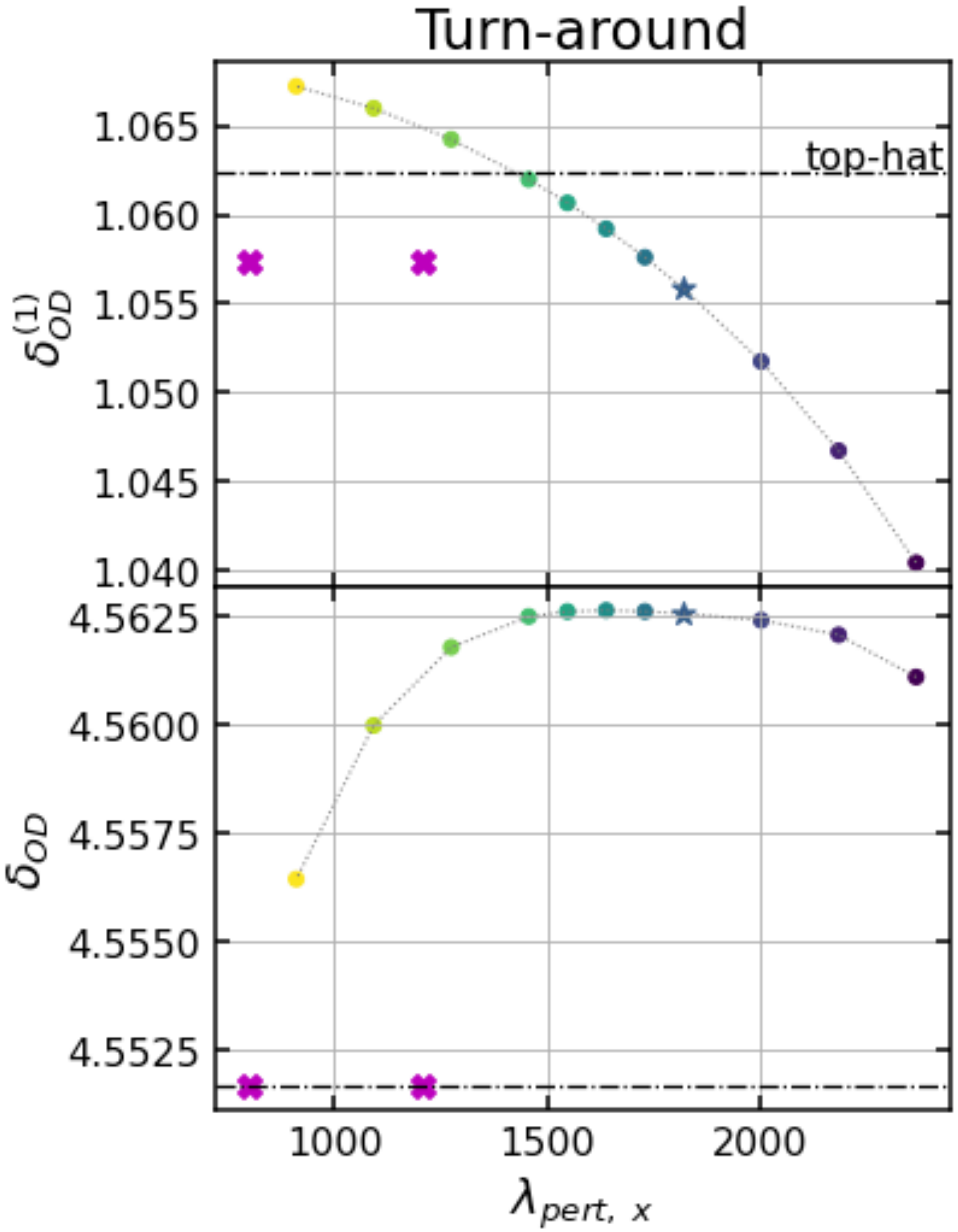
PRELIMINARY

Varying the wavelength in the x direction while keeping $\delta_{OD,\,IN} = 0.03$

$$\mathscr{R}_c = A_{pert}\left[\sin\left(\frac{2\pi x}{\lambda_{pert,\,x}}\right) + \sin\left(\frac{2\pi y}{\lambda_{pert,\,y}}\right) + \sin\left(\frac{2\pi z}{\lambda_{pert,\,z}}\right)\right]$$



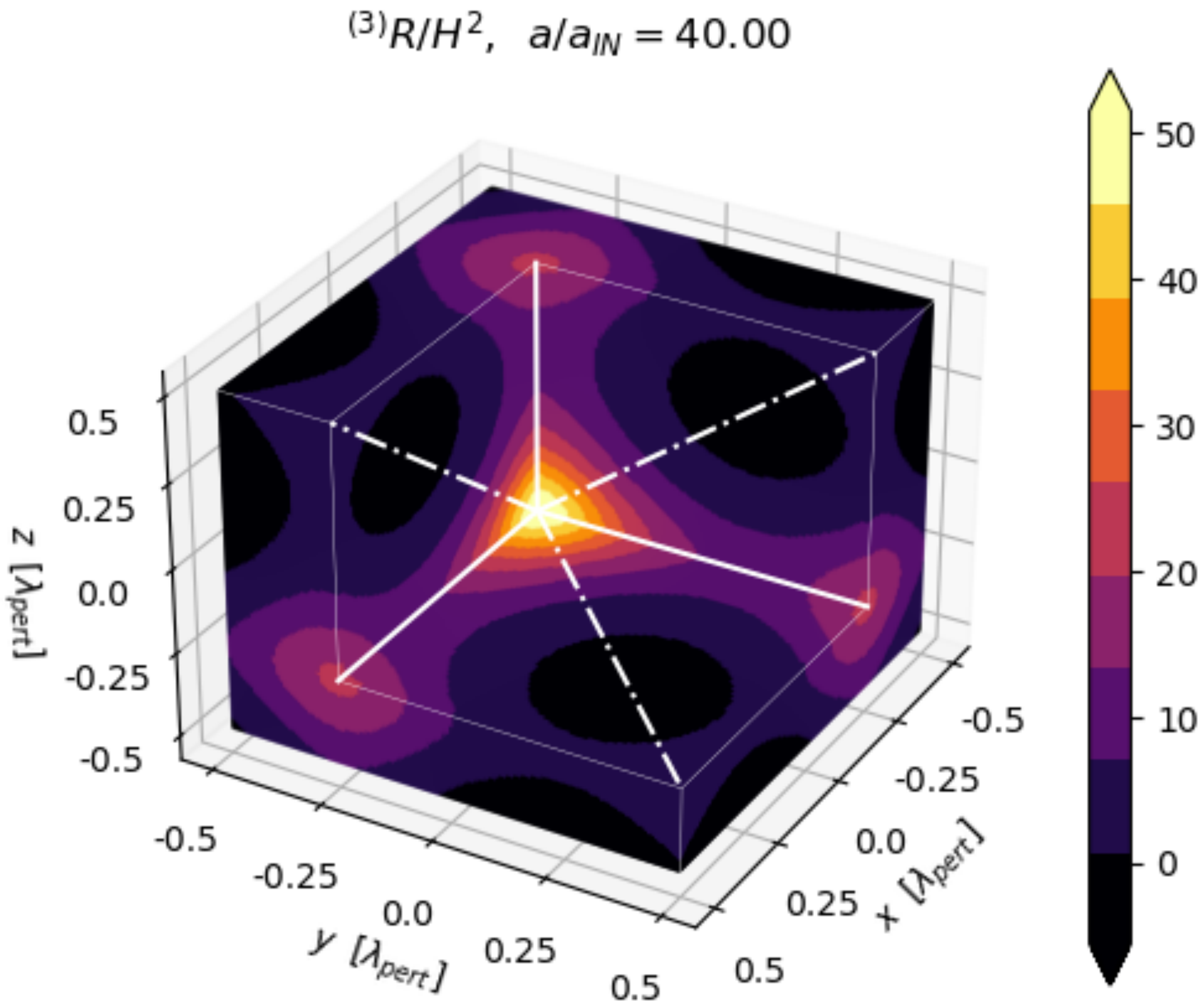
From integrating $\frac{\delta \dot{l}^{\{e\}}}{\delta l^{\{e\}}} = \frac{1}{3}\Theta + \sigma_{\alpha\beta}e^{\alpha}e^{\beta}$



The Top-Hat model is still a very good approximation.

Gravito-electromagnetism

Riemann tensor: $R_{\alpha\beta\mu\nu}$ —*Trace*→ **Ricci Tensor:** $R_{\beta\nu}$ —*Trace*→ **Ricci Scalar:** R



Gravito-electromagnetism

Riemann tensor: $R_{\alpha\beta\mu\nu}$ — *Traceless part* → **Weyl tensor:** $C_{\alpha\beta\mu\nu}$ — *Projection along the fluid* →

Electric part: $E_{\alpha\mu} = u^{\beta} u^{\nu} C_{\alpha\beta\mu\nu}$

Magnetic part: $B_{\beta\nu} = u^{\alpha} u^{\mu} C^{*}_{\alpha\beta\mu\nu}$

- **Gauge invariant at 1st order**

  Following from the Stewart-Walker lemma

- **Physically meaningful**
  - ***At first order***
  - In terms of the ***fluid***
  - With the complex ***Weyl scalars*** $\Psi_{0...4}$

Gravitational pull

$$E^{\alpha\beta} = \Re(\Psi_2) e_C^{\alpha\beta} + \frac{1}{2}\Re(\Psi_0 + \Psi_4) e_{T+}^{\alpha\beta} + \frac{1}{2}\Im(\Psi_0 - \Psi_4) e_{T\times}^{\alpha\beta} - 2\Re(\Psi_1 - \Psi_3) e_1^{(\alpha} e_2^{\beta)} - 2\Im(\Psi_1 + \Psi_3) e_1^{(\alpha} e_3^{\beta)}$$

$$B^{\alpha\beta} = -\Im(\Psi_2) e_C^{\alpha\beta} - \frac{1}{2}\Im(\Psi_0 + \Psi_4) e_{T+}^{\alpha\beta} + \frac{1}{2}\Re(\Psi_0 - \Psi_4) e_{T\times}^{\alpha\beta} + 2\Im(\Psi_1 - \Psi_3) e_1^{(\alpha} e_2^{\beta)} - 2\Re(\Psi_1 + \Psi_3) e_1^{(\alpha} e_3^{\beta)}$$

Frame Dragging **Gravitational Waves**

Gravito-electromagnetism with EBWeyl

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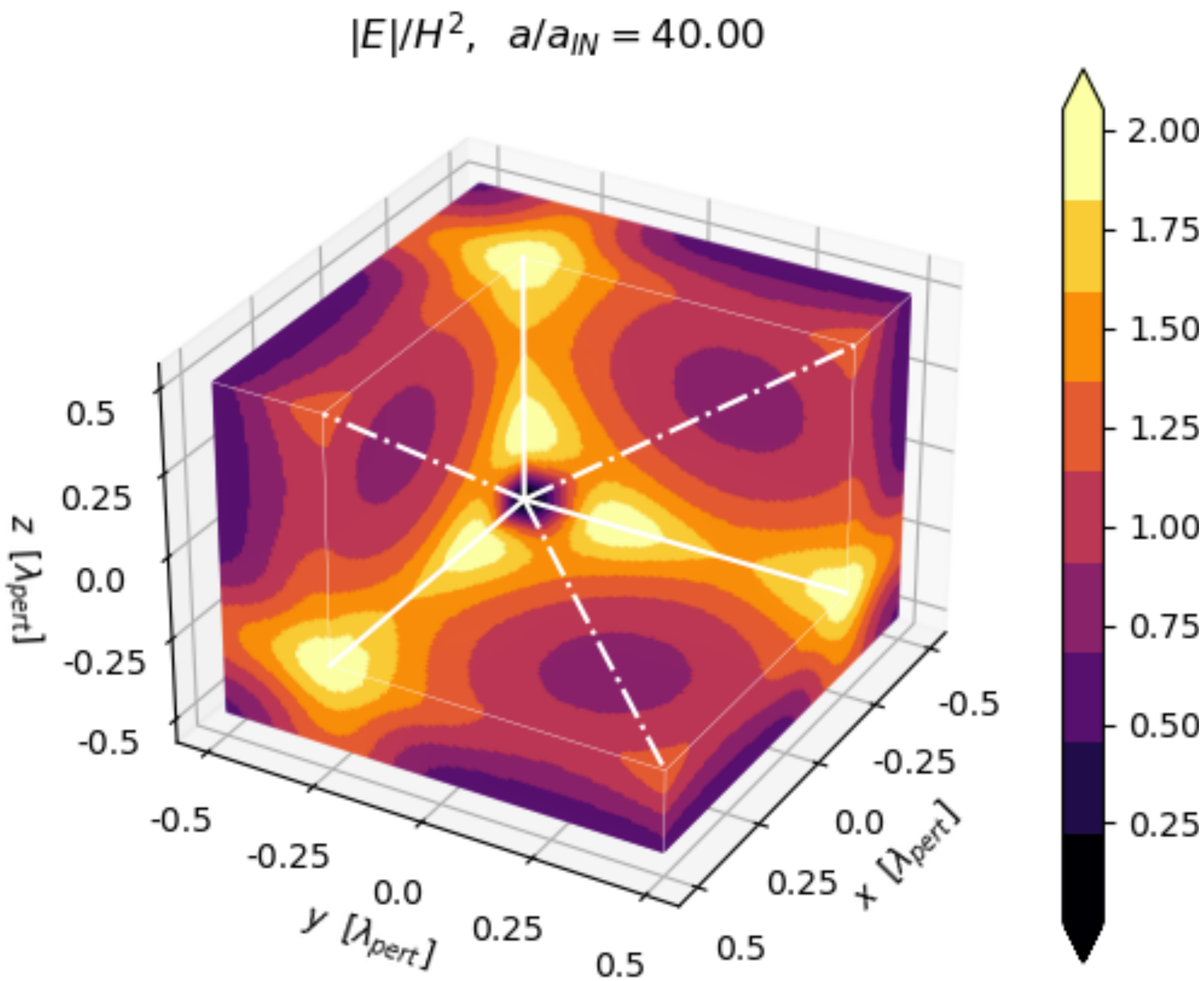
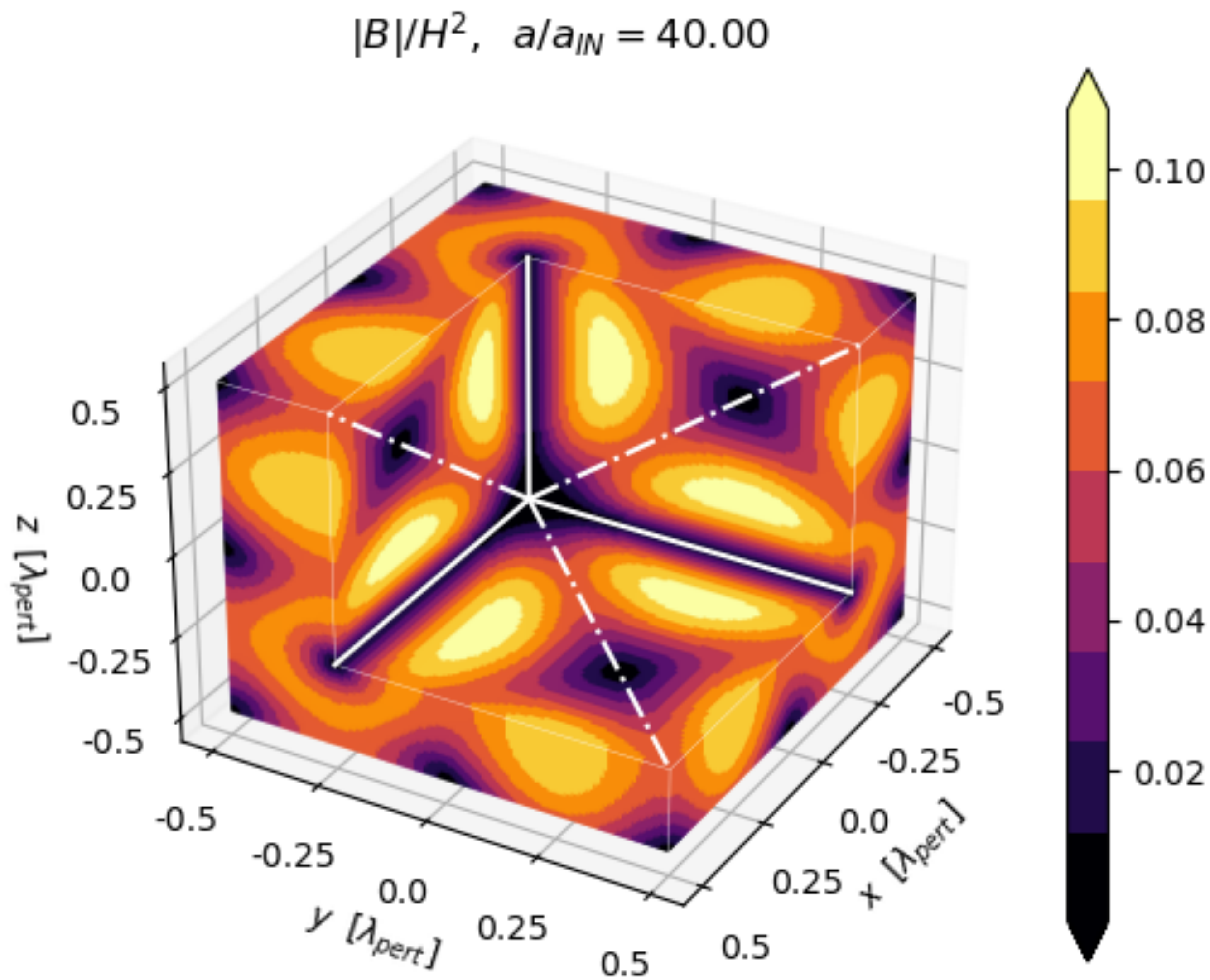
Riemann tensor: $R_{\alpha\beta\mu\nu}$ — *Traceless part* → **Weyl tensor:** $C_{\alpha\beta\mu\nu}$ — *Projection along the fluid* →

Electric part: $E_{\alpha\mu} = u^{\beta}u^{\nu}C_{\alpha\beta\mu\nu}$

Magnetic part: $B_{\beta\nu} = u^{\alpha}u^{\mu}C^{*}_{\alpha\beta\mu\nu}$

$$|E| = \sqrt{E_{\alpha\beta}E^{\alpha\beta}}$$





Conclusion

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- At the peak of the over-density, the **<u>spherical collapse model is an excellent approximation</u>**.

- Non-negligeable gravito-magnetic effects arise surrounding “gravitational currents”.

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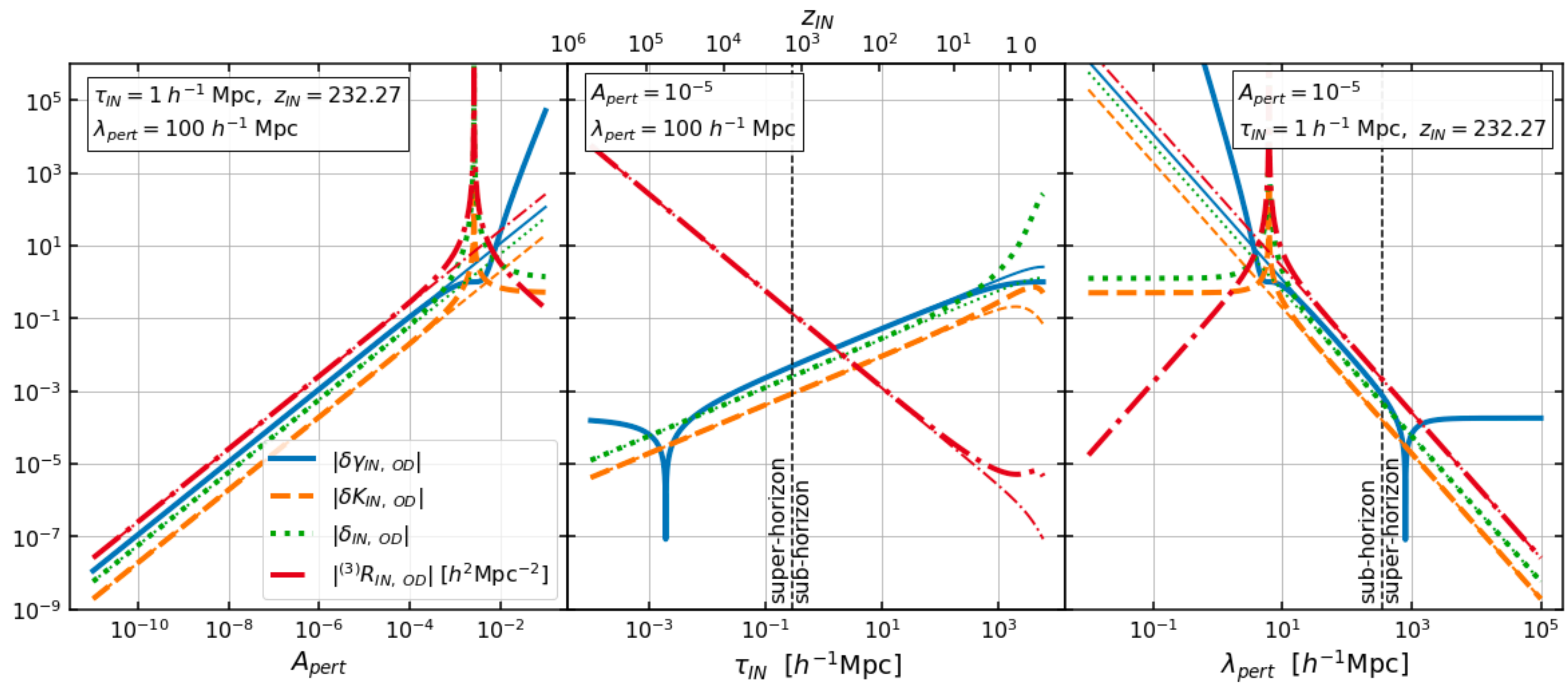
Backup slides

Initial Conditions



Perturbation in:

Matter density:

$$\delta = \rho/\bar{\rho} - 1$$

$$\delta^{(1)} \propto a$$

$$\delta^{(1)}_{IN} \propto A_{pert}\lambda_{pert}^{-2}(1 + z_{IN})^{-1}$$

Determinant of spatial metric:

$$\delta\gamma = \gamma/\bar{\gamma} - 1$$

$$\delta\gamma^{(1)} = -6\left(\delta^{(1)}/3 + \mathscr{R}_c\right)$$

$\mathscr{R}_c$ **dominant when**

$$\lambda_{phy} > 2\pi/H\sqrt{3(f_1 + 3\Omega_m/2)}$$

Trace of extrinsic curvature:

$$\delta K = K/\bar{K} - 1$$

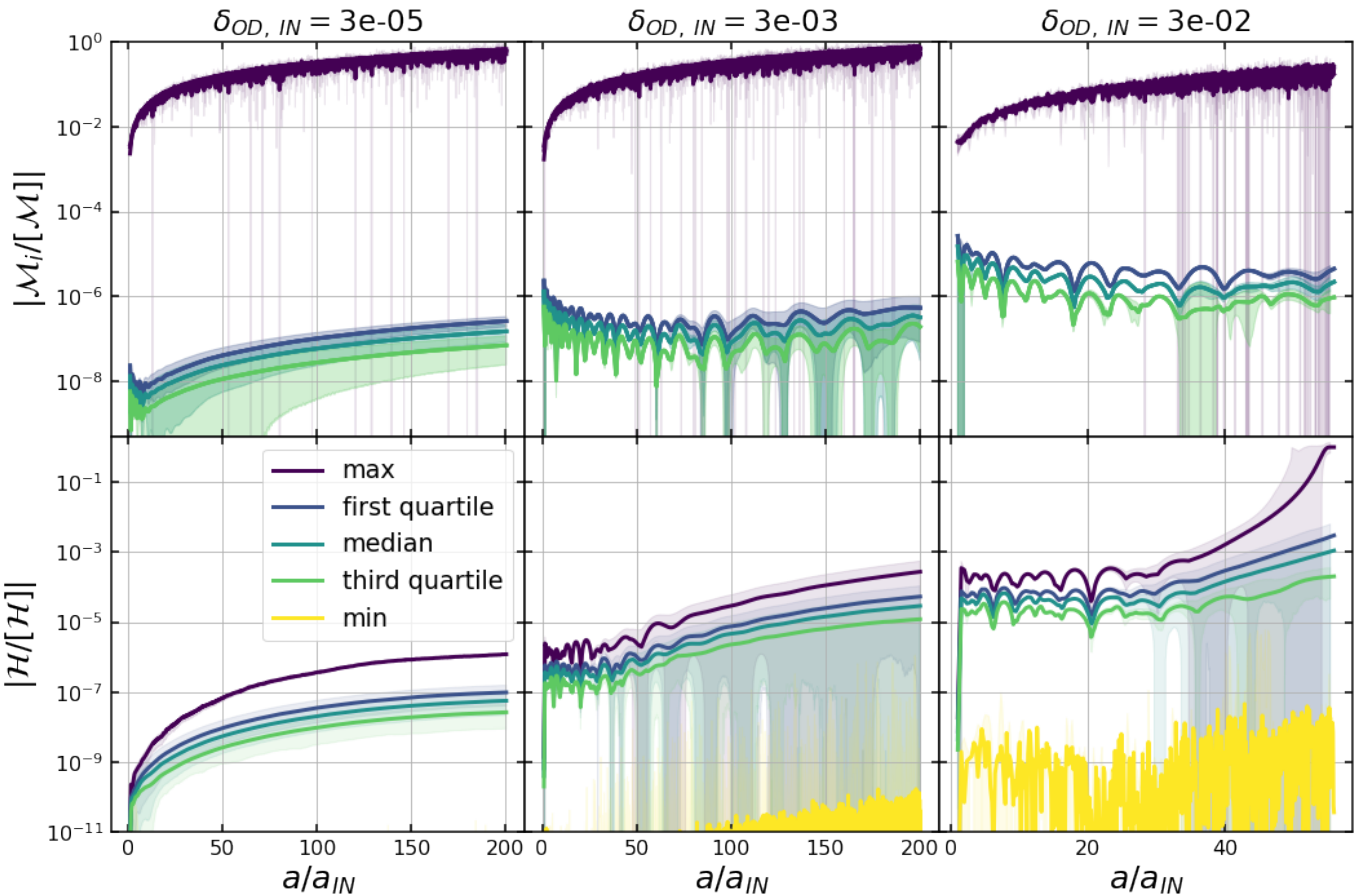
$$\delta K^{(1)} = -f_1\delta^{(1)}/3$$

3-Ricci scalar:

$$^{(3)}R^{(1)} = 4\nabla^2\mathscr{R}_c$$

$$^{(3)}R^{(1)} \propto a^{-2}$$

Constrainst



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Electric part: $E_{\alpha\mu} = u^\beta u^\nu C_{\alpha\beta\mu\nu}$

Magnetic part: $B_{\beta\nu} = u^\alpha u^\mu C^*_{\alpha\beta\mu\nu}$

What EBWeyl does:

Instead of computing the Weyl tensor in full from the metric, you can use the extrinsic curvature.

$$E^{\{n\}}_{ij} = {}^{(3)}R_{ij} + K_{ij}K - K^k_i K_{kj} - \frac{1}{3}\gamma_{ij}\left({}^{(3)}R + K^2 - K^{kl}K_{kl}\right) - \frac{\kappa}{2}\left(S_{ij} - \frac{1}{3}\gamma_{ij}S\right)$$

$$B^{\{n\}}_{ij} = \epsilon^{kl}{}_j\left(D_k K_{li} + \frac{1}{2}\gamma_{ik}\left(D_l K - D_m K^m_l\right)\right)$$

However, E and B have been projected along n^μ and not u^μ.

$$C_{\alpha\beta\mu\nu} = 2\left(l_{\alpha[\mu}E^{\{n\}}_{\nu]\beta} - l_{\beta[\mu}E^{\{n\}}_{\nu]\alpha} - n_{[\mu}B^{\{n\}}_{\nu]\lambda}\epsilon^\lambda_{\alpha\beta} - n_{[\alpha}B^{\{n\}}_{\beta]\lambda}\epsilon^\lambda_{\mu\nu}\right) \text{ with } l_{\mu\nu} = g_{\mu\nu} + 2n_\mu n_\nu$$

Classification of Petrov type with EBWeyl

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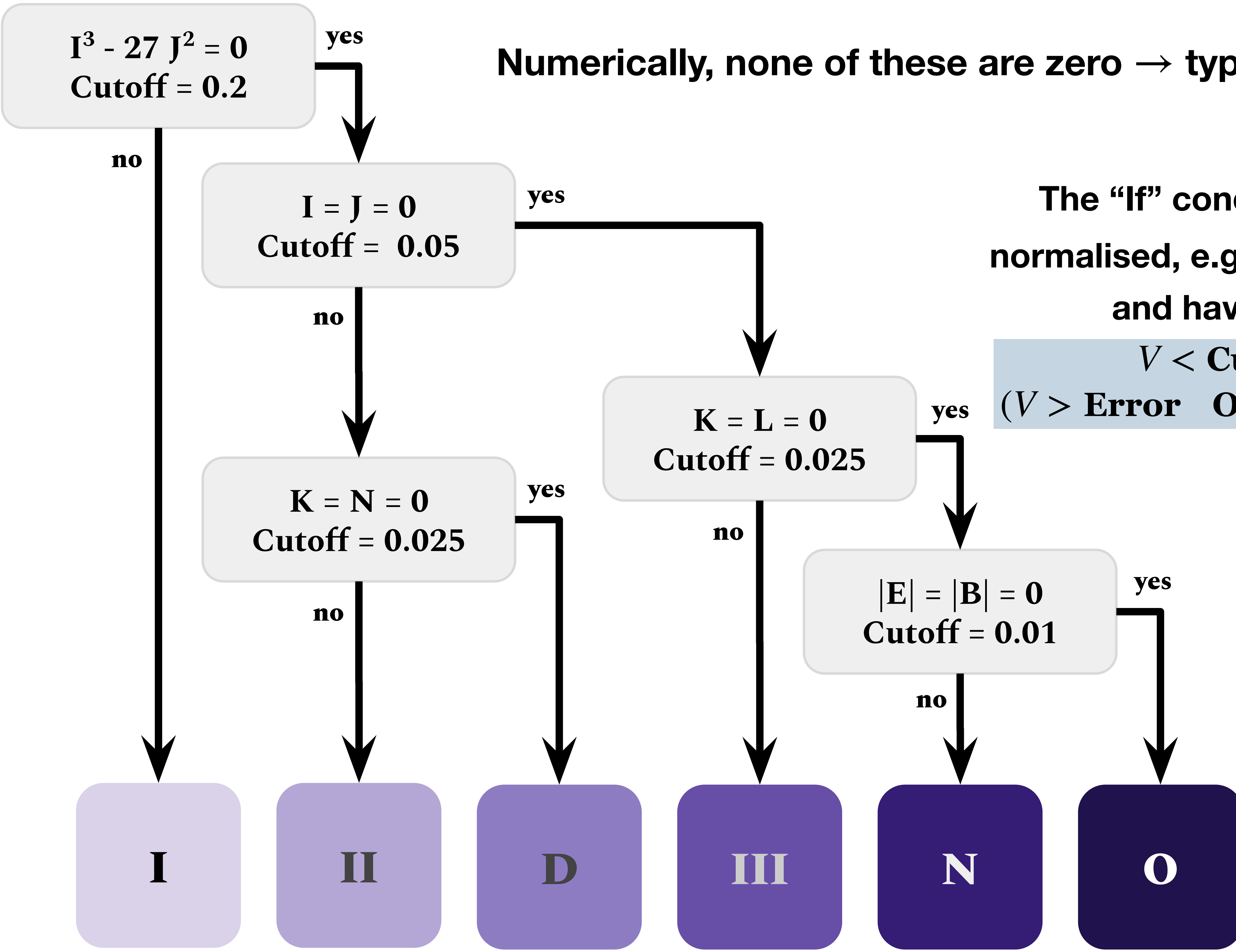
Numerically, none of these are zero → type I everywhere



The "If" condition needs to be normalised, e.g. : $V = |Re(\mathbf{I}^{1/2})|/H^2$ **and have a threshold:**

$$V < \text{Cutoff} \quad \text{AND}$$
$$(V > \text{Error} \quad \text{OR} \quad \text{Cutoff} > \text{Error})$$

Classification of Petrov type with EBWeyl

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- Transition from

  $$\mathbf{O} \rightarrow \mathbf{N} \rightarrow \mathbf{D} \rightarrow \mathbf{II} \rightarrow \mathbf{I}.$$

- Strong presence of type **N**, that of gravitational wave spacetimes.

- In the very centre of the over-density, it is type **O**. This is consistent with the spherical collapse model.

- Mostly **D** along the filaments.

- **O** remains in the under-density as it is conformally flat.