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# Primordial black hole formation from a nonspherical density profile with a misaligned deformation tensor

Chulmoon Yoo(Nagoya Univ.)

CY arXiv: 2403.11147

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# DYNAMICS OF PRIMORDIAL BLACK HOLE FORMATION II

@NAGOYA, JAPAN

\*Please beware of a phishing scam pretending to be a travel agency arranging accommodation in Nagoya. We do not have any contract with a travel agency.

## OBJECTIVES

In 2015, a black hole binary was confirmed by gravitational waves, and since it was pointed out that this may be a binary of primordial black holes, many researchers, not only in cosmology and gravity theory, but also in elementary particles and astrophysics, have come to realize utility of primordial black holes. As a result, primordial black hole research has rapidly developed and expanded in the last 10 years. The motivation for organizing this workshop is to take this opportunity to promote international mutual understanding, build cooperative relationships, and lead to future constructive and cutting-edge international joint research.

This workshop invites researchers who are conducting cutting-edge research on various aspects of primordial black holes, and provides a forum for intensive discussion, allowing researchers to interact with each other at a depth.



**\*To access the participants and invited speakers list page, you will need the following password:**

**dpbhf2**

# Nagoya?



# Why you should attend this workshop

## ©Objectives

To take this opportunity to promote international mutual understanding, build cooperative relationships, and lead to future constructive and cutting-edge international joint research.

©It should be a good enough reason, but you may have additional reasons, e.g., ...

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# Schedule

OCTOBER 2024						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
29	30	1	2	3	4	5
6	7	8	9	10	11	12
	← workshop →					
13	14	15	16	17	18	19
20	21	22	23	24	25	26
	← COSMO'24@Kyoto →					
27	28	29	30	31	1	2

Main organizers

**Albert Escrivà**

(Nagoya)

**Tomohiro Harada**

(Rikkyo)

**Yuichiro Tada**

(Nagoya)

**Takahiro Terada**

(KMI, Nagoya),

**Shuichiro Yokoyama**

(KMI, Nagoya)

**Chulmoon Yoo**

(Nagoya)

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# Primordial black hole formation from a nonspherical density profile with a misaligned deformation tensor

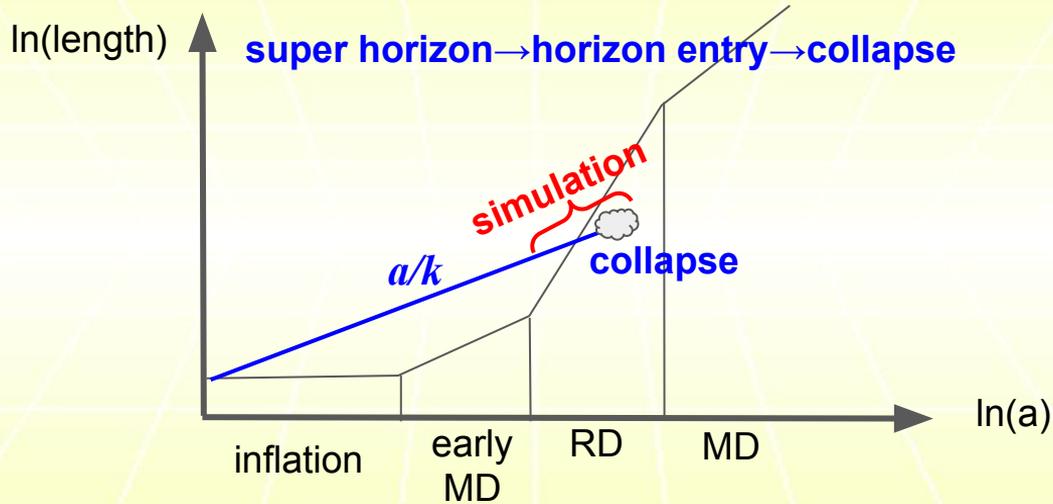
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# PBH formation

© Comoving scale of an inhomogeneity  $\sim 1/k$



© GR simulation starting from a **super-horizon non-linear initial data**

© We assume the linear equation of state  $p=w\rho$  with  $w=1/3$  and  $1/5$

# Super-horizon scale metric

◎ Leading order metric with gradient expansion

$$ds^2 = -dt^2 + a^2 \Psi^4 f_{ij} dx^i dx^j$$

◎ The next-leading order perturbations

$$\psi(t, x^k) = \Psi(x^k)(1 + \xi(t, x^k)), \quad \delta = \frac{\rho - \rho_b}{\rho_b}, \quad h_{ij} = \tilde{\gamma}_{ij} - f_{ij}, \quad \chi = \alpha - 1, \quad \kappa = \frac{K - K_b}{K_b}, \quad \tilde{A}_{ij}$$

$$\xi = -\frac{1}{6w+6} f\left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$$\chi = -\frac{3w+1}{3w+3} f\left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4) \quad f(x^k) := -\frac{4}{3} \frac{\bar{\Delta}\Psi}{\Psi^5}$$

$$\delta = f\left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4) \quad p_{ij}(x^k) := \frac{1}{\Psi^4} \left[ -\frac{2}{\Psi} (\bar{\mathcal{D}}_i \bar{\mathcal{D}}_j \Psi - \frac{1}{3} f_{ij} \bar{\Delta}\Psi) + \frac{6}{\Psi^2} (\bar{\mathcal{D}}_i \Psi \bar{\mathcal{D}}_j \Psi - \frac{1}{3} f_{ij} \bar{\mathcal{D}}^k \Psi \bar{\mathcal{D}}_k \Psi) \right]$$

$$h_{ij} = -\frac{4}{(3w+5)(3w-1)} p_{ij} \left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$$\tilde{A}_{ij} = \frac{2}{3w+5} p_{ij} H_b \left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$\Psi(x^k)$  fixes everything



# Initial condition

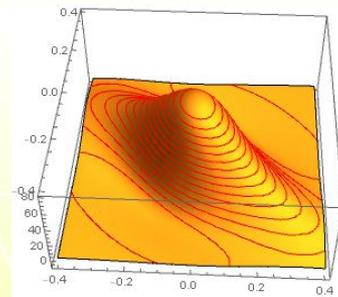
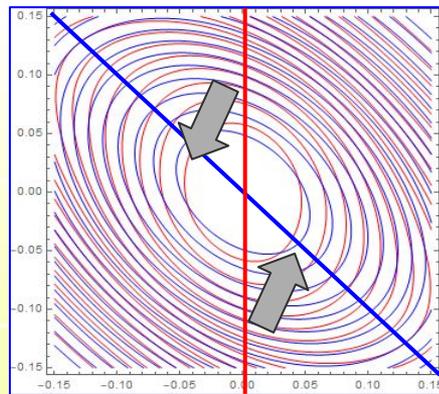
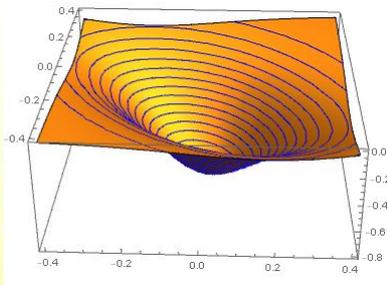
## ©Initial curvature perturbation

$$\frac{\zeta}{\mu} := -\frac{2}{\mu} \ln \Psi \simeq -1 + \frac{1}{2} (k_1^2 (x+y)^2 / 2 + k_2^2 (x-y)^2 / 2 + k_3^2 z^2) + \mathcal{O}(r^4)$$

$$\frac{\Delta \zeta}{\mu k^2} \simeq 1 - \frac{1}{2} (\kappa_1^2 x^2 + \kappa_2^2 y^2 + \kappa_3^2 z^2) + \mathcal{O}(r^4)$$

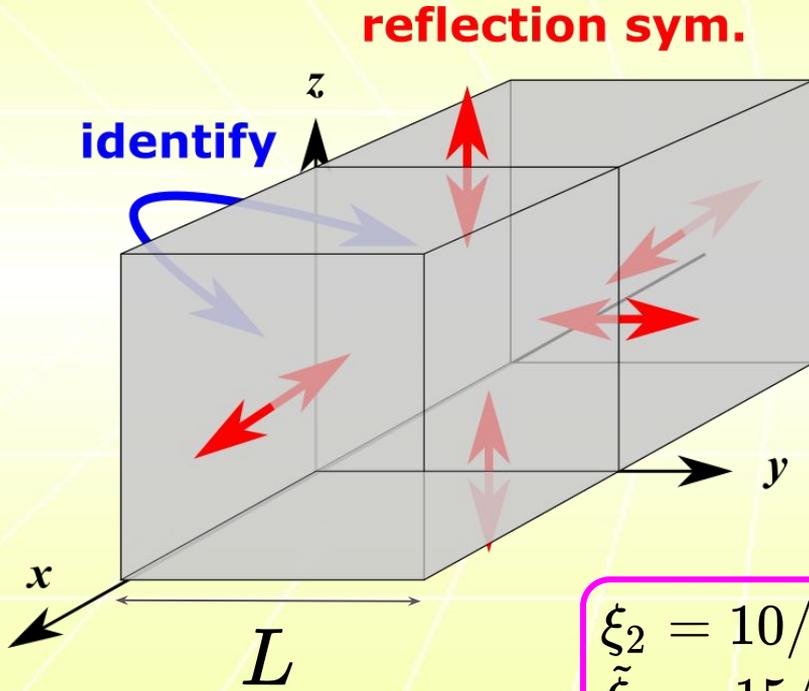
$\zeta \sim$  gravitational potential on (x,y) plane

$\Delta \zeta \sim$  energy density on (x,y) plane



tidal torque  $\Rightarrow$  angular momentum transfer  $\Rightarrow$  spinning PBH

# Numerical domain and parameter setting



$$\zeta = -\mu \left[ 1 + \frac{1}{2} (k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2) + \frac{1}{4} (k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2)^2 + \frac{1}{280} k^2 r^2 (9\kappa_1^2 - \kappa_2^2 - \kappa_3^2) x^2 + (\kappa_1^2 - 9\kappa_2^2 + \kappa_3^2) y^2 + (\kappa_1^2 + \kappa_2^2 - 9\kappa_3^2) z^2 \right]^{-1} \exp \left[ -\frac{1}{2880} k^6 r^6 \right]$$

$$k_1^2 = \frac{1}{3} (\xi_1 + 3\xi_2 + \xi_3) \quad k_2^2 = \frac{1}{3} (\xi_1 - 3\xi_2 + \xi_3)$$

$$k_3^2 = \frac{1}{3} (\xi_1 - 2\xi_3)$$

$$\kappa_1^2 = \frac{1}{3} (\tilde{\xi}_1 + 3\tilde{\xi}_2 + \tilde{\xi}_3) \quad \kappa_2^2 = \frac{1}{3} (\tilde{\xi}_1 - 3\tilde{\xi}_2 + \tilde{\xi}_3)$$

$$\kappa_3^2 = \frac{1}{3} (\tilde{\xi}_1 - 2\tilde{\xi}_3)$$

$$\xi_1 = \tilde{\xi}_1 = 100/L^2 \quad \xi_3 = \tilde{\xi}_3 = 0$$

the most probable values

$$\xi_2 = 10/L^2$$

$$\tilde{\xi}_2 = 15/L^2$$

too large to be statistically expected  
too much idealized for spin generation

# About the numerical code

©Originally provided by Hirotada Okawa (for E-eqs and real scalar field w/ periodic BC)

©COSMOS(秋桜) code by C++ [CY, Hirotada Okawa, Ken-ichi Nakao(1306.1389),  
Hirotada Okawa, Helvi Witek, Vitor Cardoso(1401.1548)]

©Basically follows the SACRA(桜) code by Fortran [Tetsuro Yamamoto, Masaru Shibata, Keisuke Taniguchi(arXiv:0806.4007)]

©Independently developed by CY and Hirotada Okawa

©In the CY side, it is mainly dedicated to PBH formation as follows

- Inhomogeneous coordinate system has been implemented [CY, Taishi Ikeda,Hirotada Okawa(arXiv:1811.00762)]
- Fluid evolution code has been implemented [CY, Tomohiro Harada,Hirotada Okawa(arXiv:2004.01042)]
- 1+1 code for spherical systems has been developed based on COSMOS [CY, Harada, Hirano, Okawa, Sasaki(arXiv:2112.12335)]
- Recently, a mesh refinement procedure has been implemented

# Summary of resolution difference

©Resolution in the previous simulation 2004.01042 CY, Tomohiro Harada, Hirotada Okawa

- Scale-up reference coordinates  $x^i$  related to the Cartesian coord.  $X^i$  by

$$X^i = x^i - \frac{S}{1+S} \frac{L}{\pi} \sin\left(\frac{\pi}{L} x^i\right) \text{ with } S = 15$$

- Resolution at the center ( $\Delta x = L/100$ )

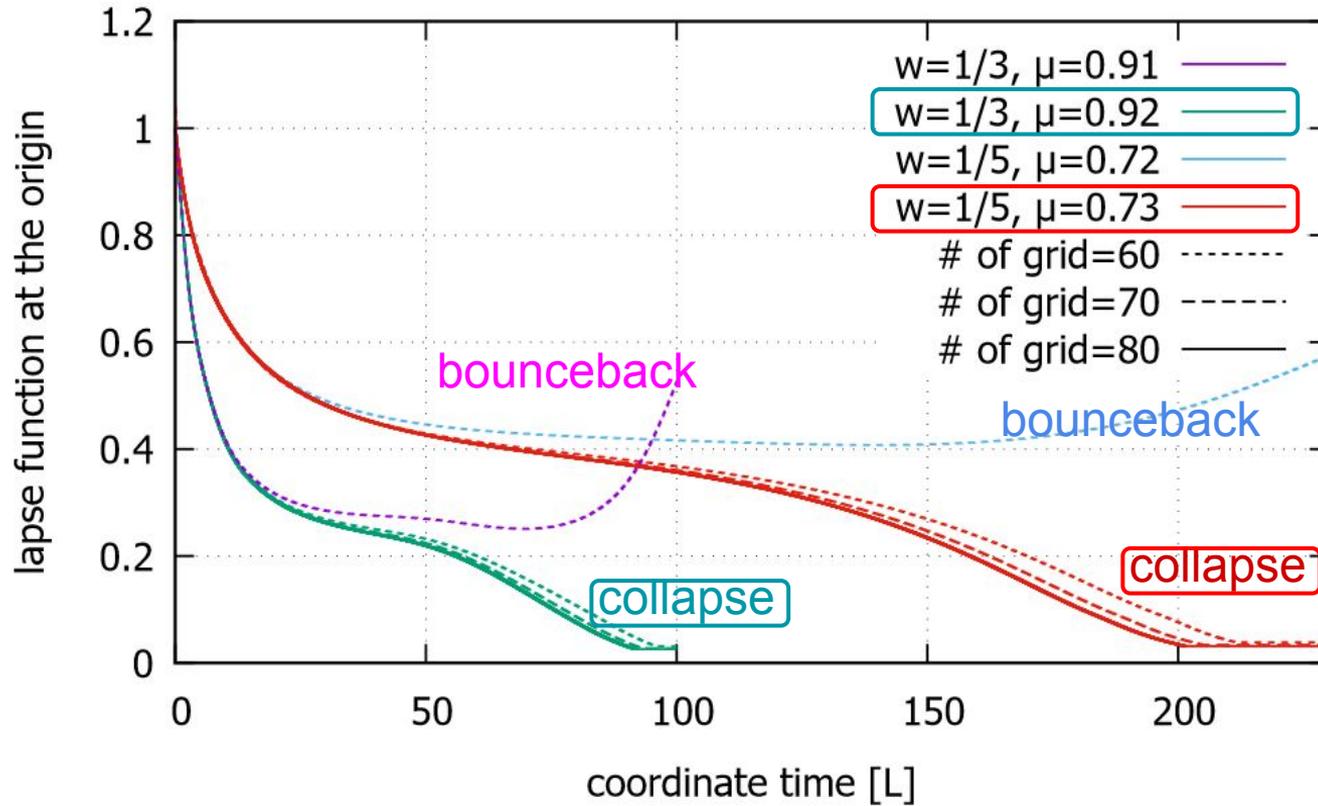
$$\Delta X|_{\text{center}} = \frac{1}{1+S} \Delta x = \frac{1}{16} \frac{L}{100} = \frac{L}{1600}$$

©New simulation with mesh refinement

- $S = 10, \Delta x = L/80$
- Two additional layers for the mesh refinement

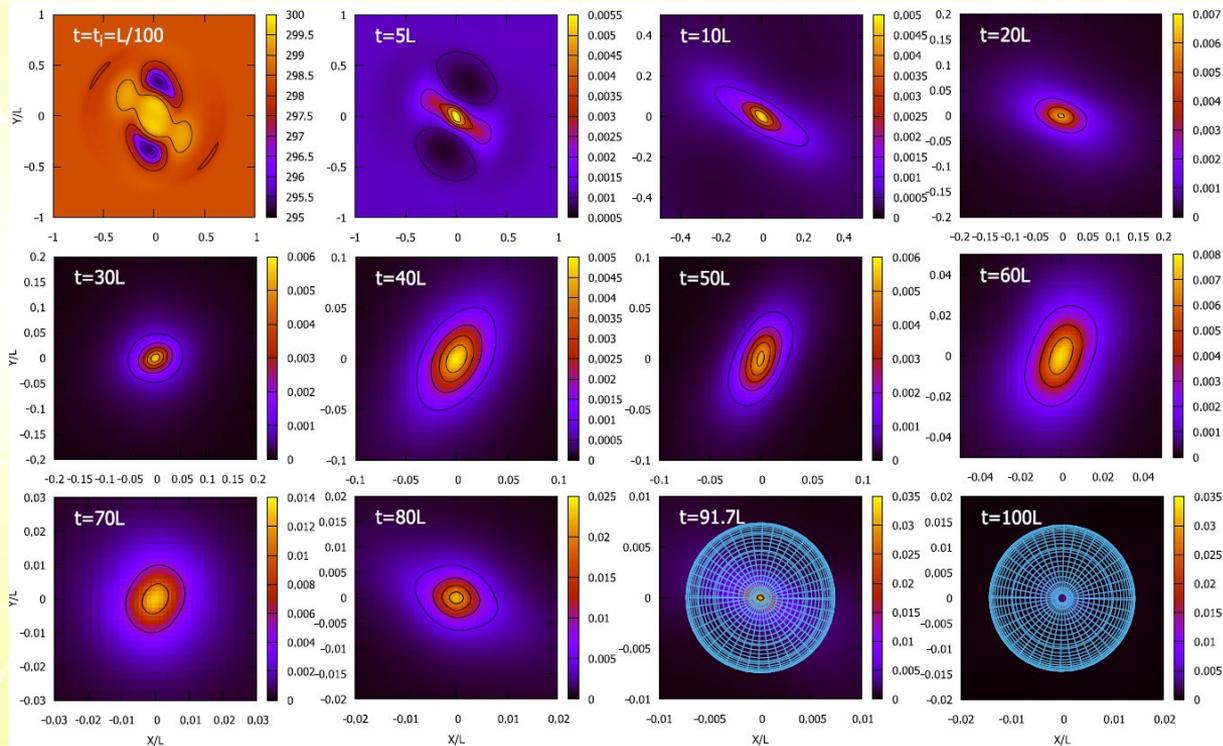
$$\Delta X|_{\text{center}} = \frac{1}{1+S} \times \frac{1}{2^2} \times \Delta x = \frac{1}{44} \times \frac{L}{80} = \frac{L}{3520}$$

# Thresholds



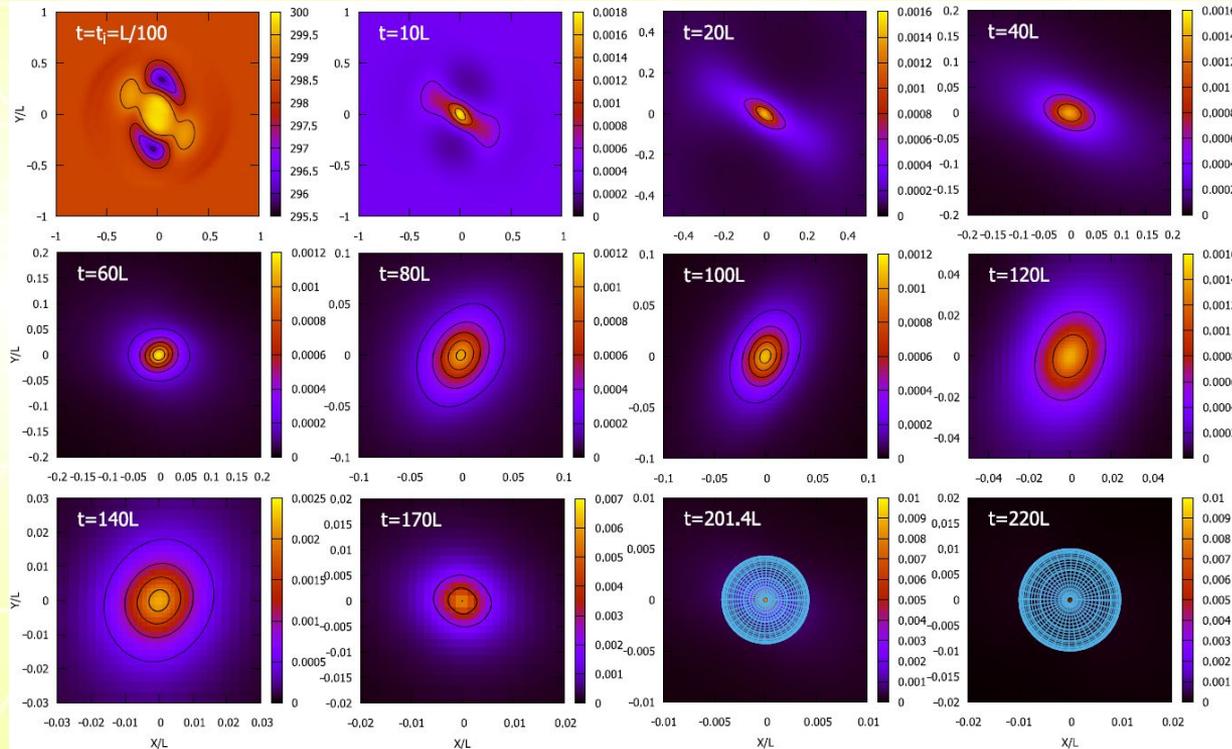
# Effective spin parameter

⊙  $w=1/3$ ,  $\mu=0.92$  case



# Softer equation of state

© $w=1/5$ ,  $\mu=0.73$  case



# Effective spin parameter

## ©Kerr black hole case

$$\text{area : } A_{\text{Kerr}} = 8\pi(M^2 + \sqrt{M^4 - a^2 M^2})$$

$$\text{equatorial circumference : } d_{\text{Kerr}} = 4\pi M$$

## ©Effective dimensionless spin

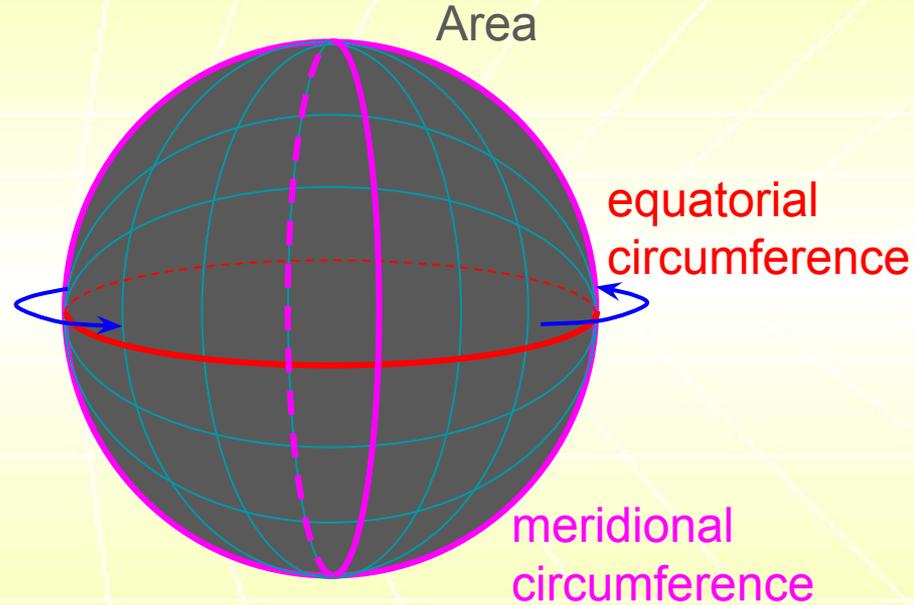
$$\Rightarrow s^2 := \left( \frac{a^2}{M^2} \right)_{\text{eff}} = \frac{4\pi A(d^2 - \pi A)}{d^4}$$

## ©Asphericity parameters

$$\text{meridional circumference : } \ell_{x=0}, \ell_{y=0}$$

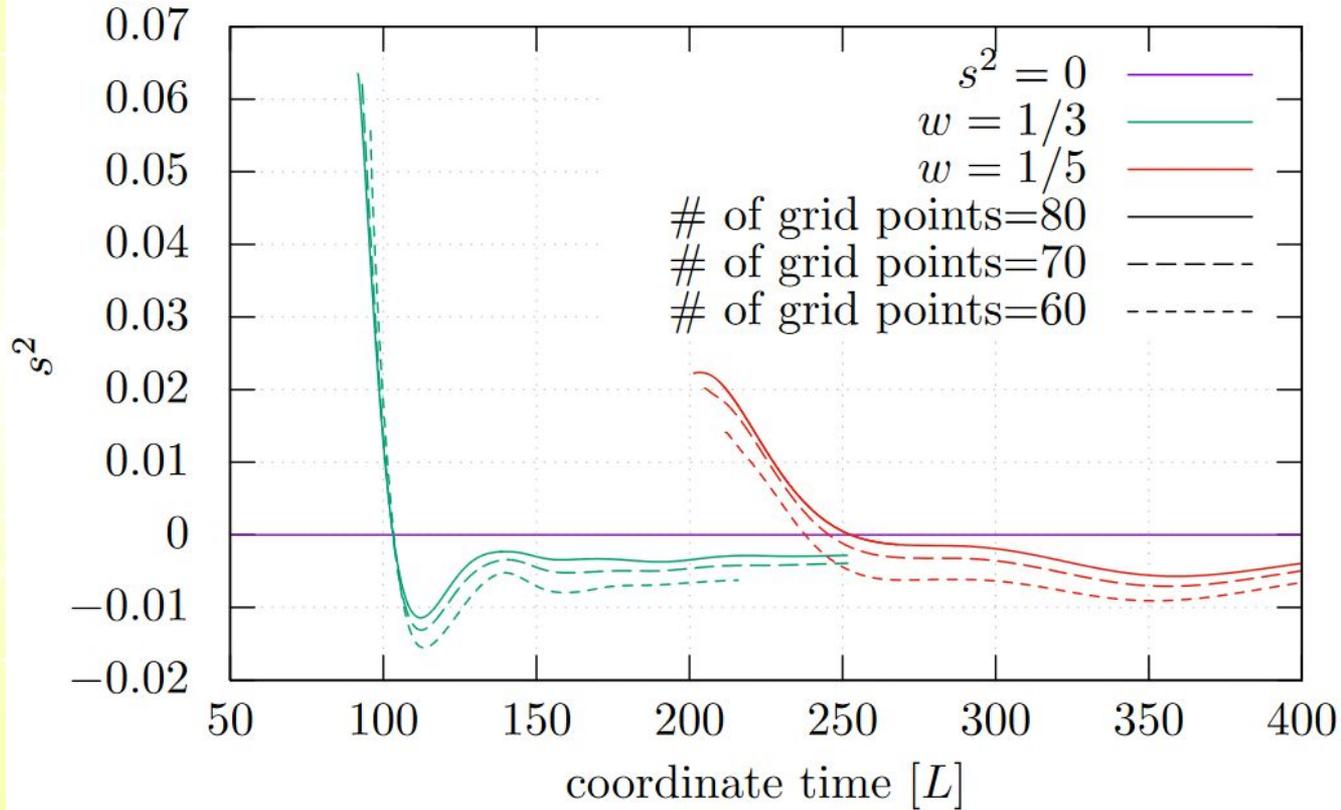
$$\alpha := \frac{\ell_{x=0}}{d}$$

$$\beta := \frac{\ell_{y=0}}{d}$$





# Effective spin parameter



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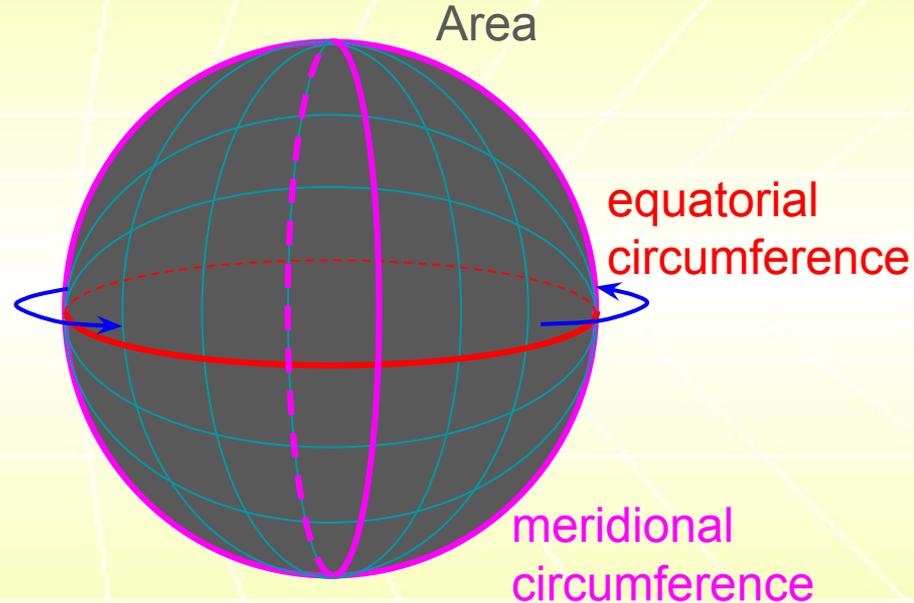
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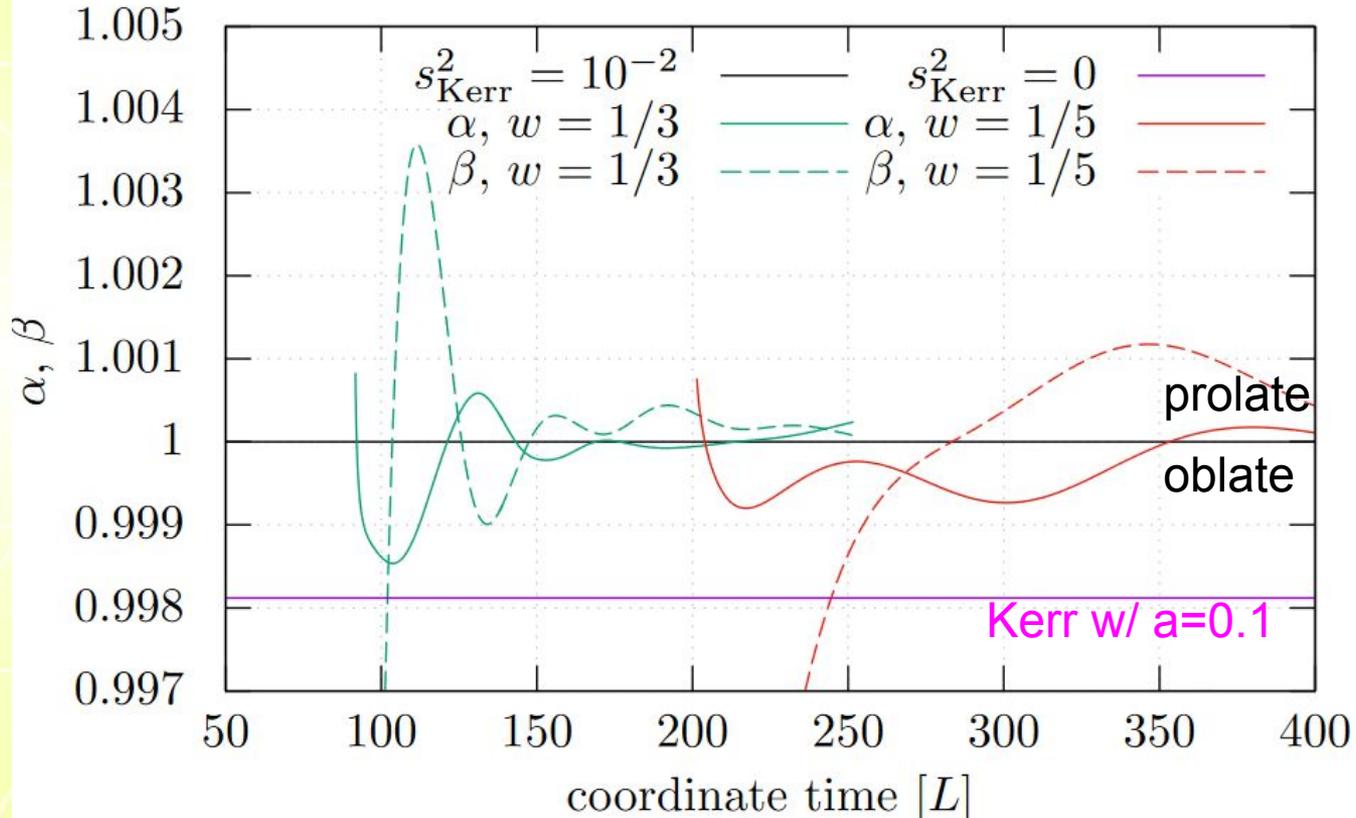
$$\text{meridional circumference : } \ell_{x=0}, \ell_{y=0}$$

$$\alpha := \frac{\ell_{x=0}}{d}$$

$$\beta := \frac{\ell_{y=0}}{d}$$



# Non-sphericity



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# Spin of PBH is very small

for the equation of states  $p=w\rho$  with  $w>1/5$

Thank you for your attention