Primordial Black Holes and Induced Gravitational Waves from a Smooth Crossover beyond Standard Model

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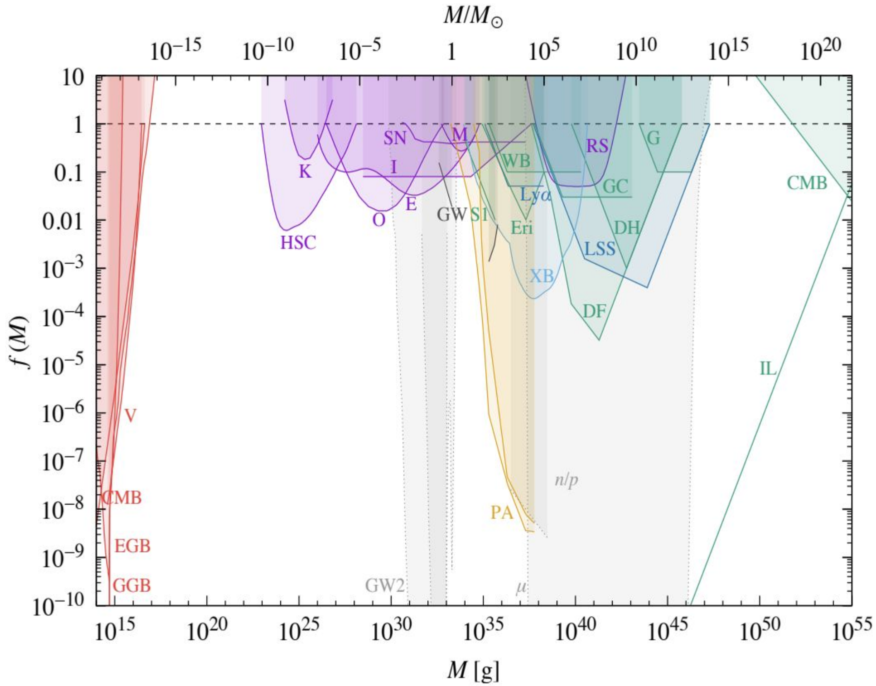
Based mainly on: A. Escrivà, Y. Tada and C.M Yoo. ArXiv:2311.17760

What is dark matter? Maybe PBHs ("maybe"....)

Please, let me advertise our PBH book chapter (200 pages, ~800 references)

Review on PBHs: **Escriva, Kuhnel, Tada. ArXiv:2211.05767**



B. Carr, K. Kohri, Y. Sendouda, J. Yokoyama. ArXiv:2002.12778

But please, also check other reviews

M. Sasaki, T. Suyama, T. Tanaka, S. Yokoyama. ArXiv:1801.05235 (grav. Waves perspectives)

B. Carr, K. Kohri, Y. Sendouda, J.Yokoyama. ArXiv:2002.12778 (PBH constraints)

M. Sasaki, T. Suyama, T. Tanaka, S. Yokoyama. ArXiv:1801.05235 (grav. Waves perspectives)

B. Carr, F. Kuhnel. ArXiv:2006.02838 (PBH constraints)

Anne M. Green, Bradley J. Kavanagh. ArXiv:2007.10722 (PBH constraints and phenomenology)

A.Escrivà. ArXiv:2111.12693 (numerical simulations)

Chul-Moon Yoo. ArXiv:2211.13512 (statistics)

Shi Pi. arXiv:2404.06151 (NGs)

ETC (please check more in Arxiv)

Motivation

In general, we focus on the radiation-dominated era, assuming a radiation-perfect fluid w=⅓

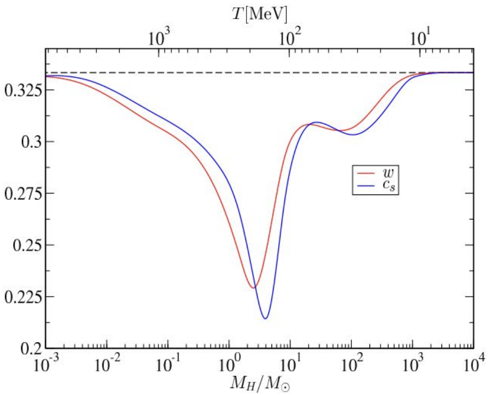
But we could have , for instance, a time-dependent equation of state

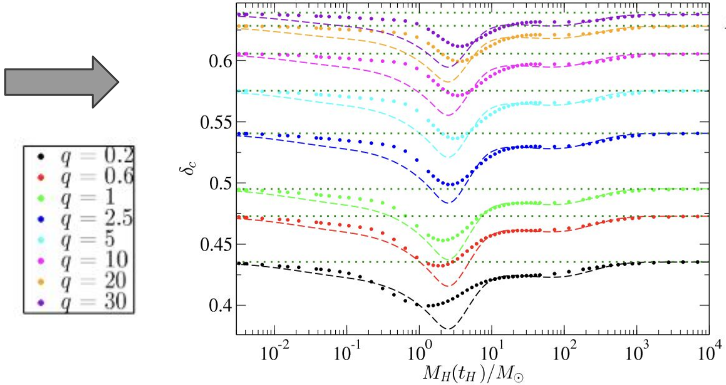
Pressure gradients depends on the equation of state and the shape of the curvature profile

$$w \downarrow \; \Rightarrow p \downarrow \; \Rightarrow \delta_c \downarrow \; \Rightarrow \beta \uparrow$$

enhances the probability of forming PBHs at a given specific mass scale

Very well known example: the QCD crossover (several works on the topic, please check)





A.Escriva, E. Bagui and S.Clesse. arXiv:2209.06196

$$K_{\rm pol}(r) = \frac{3}{2}\frac{\delta_{\rm m}}{r_{\rm m}^2}\frac{1+1/q}{1+\frac{1}{q}\left(\frac{r}{r_{\rm m}}\right)^{2(q+1)}}$$

Relating profiles with the same curvature around the peak of the compaction function (universal threshold)

$$q \equiv -\frac{\mathcal{C}''(r_{\rm m})r_{\rm m}^2}{4\mathcal{C}(r_{\rm m})}$$

Discovered, and introduced for the first time in the literature in:

A. Escrivà, C. Germani, R. K. Sheth. ArXiv:1907.13311

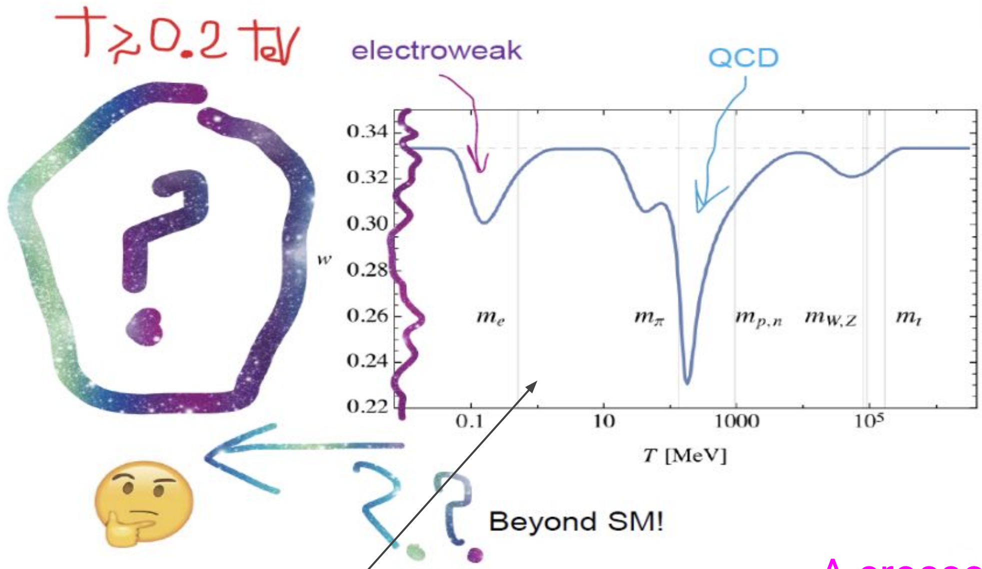
Motivation

Usually phase transitions beyond SM are considered, but not crossovers

But, what about if we consider a crossover beyond SM?

$$w \downarrow \Rightarrow p \downarrow \Rightarrow \delta_c \downarrow \Rightarrow \beta \uparrow$$

A. Escrivà, J. G. Subils. ArXiv:2211.15674



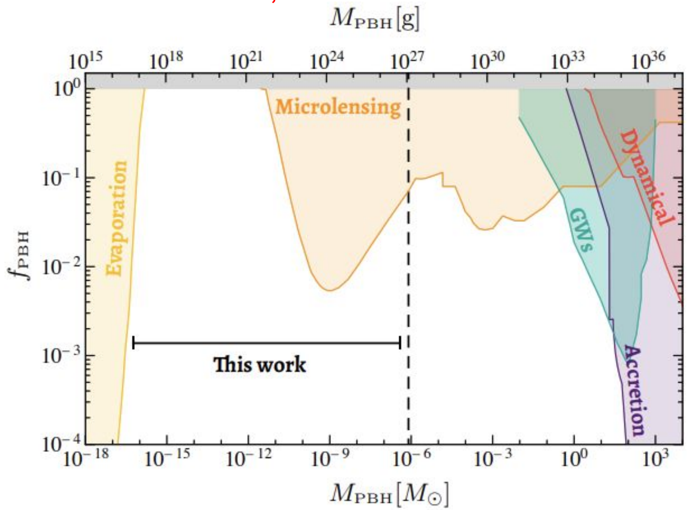


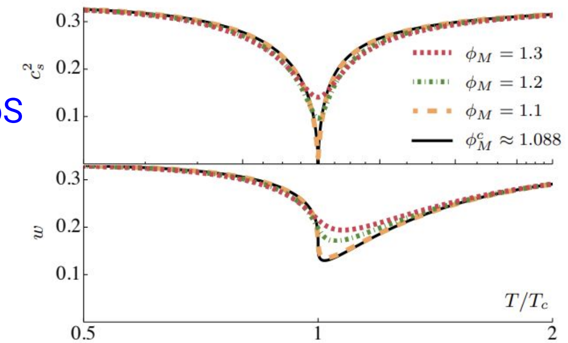
Diagram modified from
B. Carr, S. Clesse, J.G. Bellido, F. Kuhnel. ArXiv:1906.08217

A crossover (softening of the EoS) can also be realized beyond SM theories

Motivation

A. Escrivà, J. G. Subils. ArXiv:2211.15674

In our work, we considered an holographic model to modulate the softening of the EoS



$c_s^2(T = T_c) = 0 \quad \phi_M = \phi_M^c$

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^{[5]}}\left(R^{[5]} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

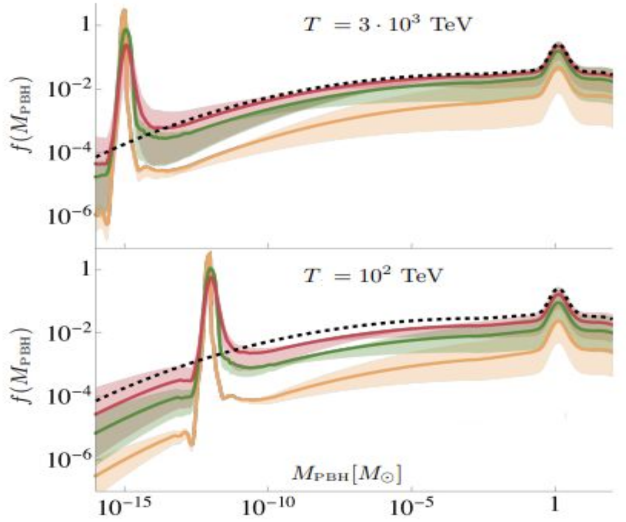
$$W(\phi) = -\frac{3}{2} - \frac{\phi^2}{8} - \frac{\phi^4}{64\phi_M^2} + \frac{\phi^6}{64\phi_Q}$$

$$V(\phi) = -\frac{16}{3}W(\phi)^2 + 8W'(\phi)^2$$

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} = \frac{s}{T}\frac{\mathrm{d}T}{\mathrm{d}s} \qquad p(T) = \int_0^T s(T')\mathrm{d}T' \qquad w = \frac{p}{\rho}$$

Maldacena. Arxiv:hep th /9711200
Bea, Solana,Giannakopoulos , Jansen, Mateos, Garitaonandia, Zilhão . ArXiv:2202.10503

In the PBH scenario, actually can have important implications!



See also the work of P. Lu, V.Takhistov, G. M. Fuller. ArXiv:2212.00156

Motivation

A lot of literature on the topic

Currently, we have a very important connexion between

Gravitational waves <—> Primordial Black Holes

- Induced GWs-> can be a direct probe of the existence of primordial scalar curvature fluctuations, in particular at much smaller scales than the cosmic microwave background (CMB) scale.

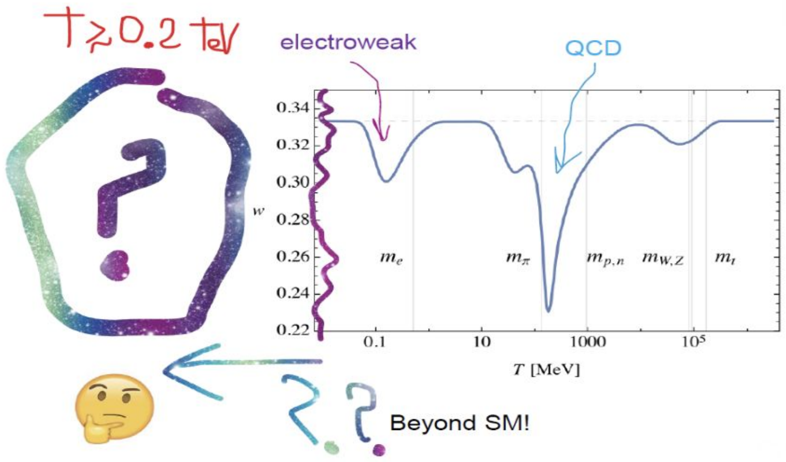
- But also, it can be an indirect probe of the existence of primordial black holes (scenario of PBH formation from the collapse of super-horizon curvature fluctuations).

  + give a quantification of the quantity of dark matter from PBHs

Motivation

In this work, basically,

We want to obtain an observational signature in GWs (which could be tested with future space-based GW interferometers) from a hypothetical crossover beyond SM and make the corresponding connection with the PBH scenario.



Let's consider a flat power spectrum

The model

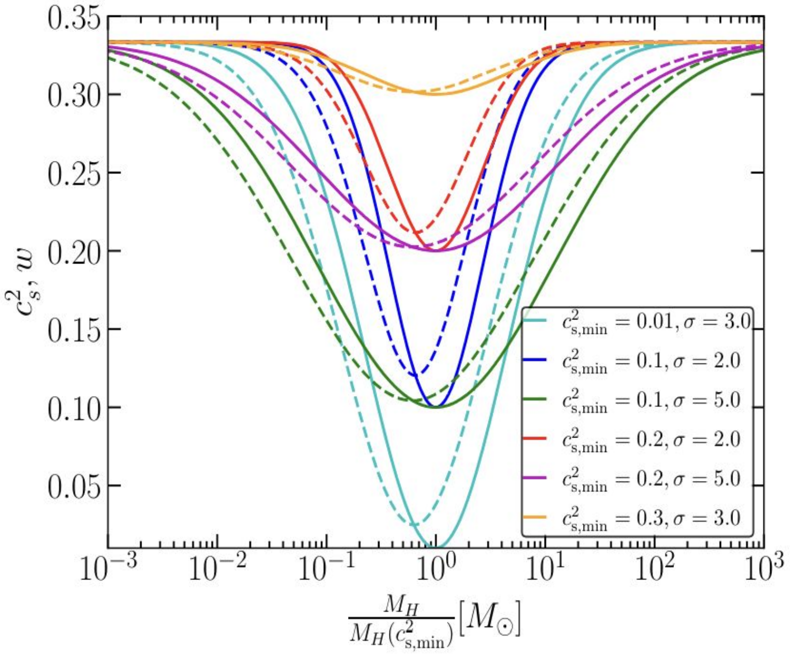
- Consider a hypothetical softening beyond SM with the following crossover template:

Model with two parameters

$$c_s^2(\rho) = w_0 - (w_0 - c_{s,\min}^2)\exp\left[\frac{-(\ln\tilde{\rho})^2}{2\sigma^2}\right]$$

$$w(\rho) = w_0 - \frac{\sigma}{\tilde{\rho}}\sqrt{\frac{\pi}{2}}e^{\sigma^2/2}(w_0 - c_{s,\min}^2)\,\mathrm{erfc}\left[\frac{\sigma^2-\ln(\tilde{\rho})}{\sqrt{2}\sigma}\right]$$

$$c_s^2(\rho) = \frac{\partial w(\rho)}{\partial\rho}\rho + w(\rho) \Leftrightarrow w(\rho) = \frac{1}{\rho}\int_{\bar{\rho}=0}^{\bar{\rho}=\rho} c_s^2(\bar{\rho})d\bar{\rho}.$$



BKG dynamics

$$\rho'(\eta) = -\sqrt{24\pi}(1+w(\rho))a(\eta)\rho^{3/2}(\eta),$$

$$a'(\eta) = \sqrt{\frac{8\pi\rho(\eta)}{3}}a^2(\eta),$$

Gravitational collapse of super-horizon adiabatic curvature fluctuations

Sufficiently large fluctuations generated during inflation (very rare events) will collapse during the radiation epoch after they reenter the cosmological horizon.

(at super-horizon scales) $$ds^2 = -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2\left(d\theta^2 + \sin^2(\theta)\, d\phi^2\right)\right]$$

M. Shibata, M. Sasaki. Arxiv:gr-qc/9905064

$(\nu = \mu/\sigma \gg 1)$

$$\zeta(r) = \mu\, g(r) \quad \zeta \Rightarrow \frac{\delta\rho}{\rho}$$

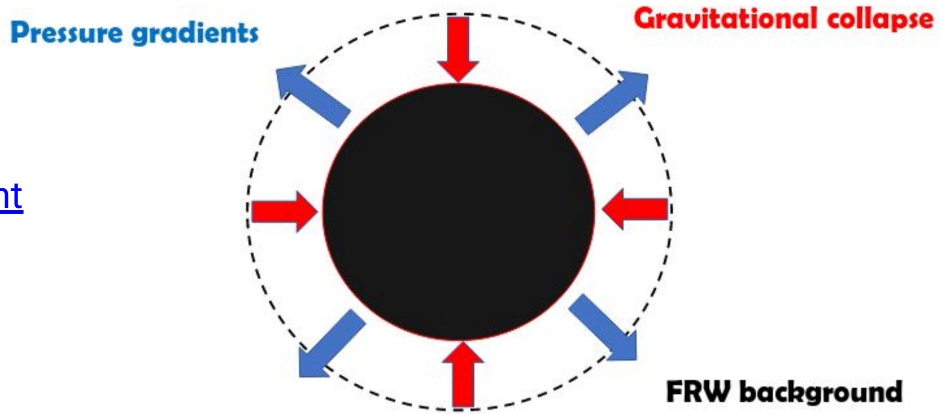
Large peaks ⟶ we assume approximately spherically symmetric (BBKS paper. *Astrophys.J.* 304 (1986) 15-61)

$\mu > \mu_c$ Collapse of the fluctuations

$\mu < \mu_c$ Dispersion of the fluctuations

We need relativistic numerical simulations to determine the threshold value (most important quantity for estimating the PBH abundance)

We use A. Escrivà. ArXiv:1907.13065



PBH threshold and abundance estimation

C-M. Yoo, T. Harada, J. Garriga, K. Kohri. ArXiv:1805.03946
C.M Yoo, T. Harada, S. Hirano, K. Kohri. Arxiv:2008.02425

Let's consider an accurate approach in peak theory to statistically estimate the PBH abundance

$$\zeta = \mu_2 \, g(r, k_\bullet) \qquad \mu_2 = -\left(\frac{\sigma_1}{\sigma_2}\right)^2 \Delta\zeta\Big|_{r=0} \qquad k_\bullet = -\frac{\Delta\Delta\zeta\big|_{r=0}}{\Delta\zeta\big|_{r=0}}$$

$$g(r;k_\bullet) = \frac{1}{1-\gamma_3^2}\left(\psi_1 + \frac{1}{3}R_3^2\Delta\psi_1\right) - \frac{k_\bullet^2}{\gamma_3(1-\gamma_3^2)}\frac{\sigma_2}{\sigma_4}\left(\gamma_3^2\psi_1 + \frac{1}{3}R_3^2\Delta\psi_1\right)$$

"Typical profile" (this is not the mean profile in general) of the curvature fluctuation

$$\sigma_n^2 = \int \frac{dk}{k} k^{2n} \mathcal{P}_g(k), \quad \psi_n(r) = \frac{1}{\sigma_n^2}\int \frac{dk}{k} k^{2n} \frac{\sin(kr)}{kr}\mathcal{P}_g(k),$$
$$\gamma_n = \frac{\sigma_n^2}{\sigma_{n-1}\sigma_{n+1}}, \qquad R_n = \frac{\sqrt{3}\sigma_n}{\sigma_{n+1}} \quad \text{for odd } n.$$

$$\mu_2 = \frac{-1+\sqrt{1-3\mathcal{C}(r_m)/2}}{r_m \, \partial_r g_m(r;k_\bullet)\big|_{r=r_m}}$$

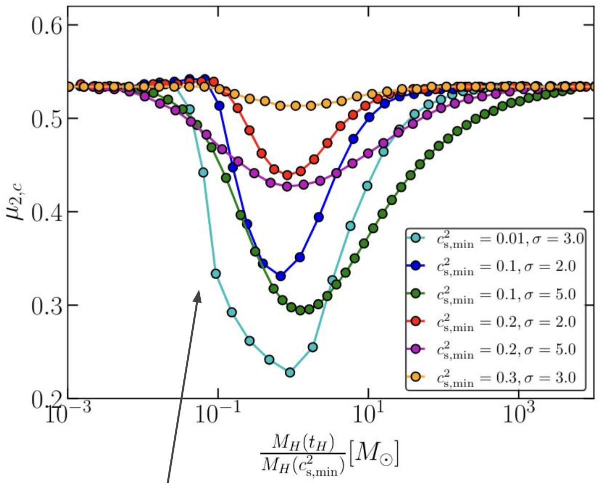
← **Connexion with the compaction function**

$$P_1^{(n)}(\nu,\xi) = \frac{1}{2\pi\sqrt{1-\gamma_n^2}} \exp\left[-\frac{1}{2}\left(\nu^2 + \frac{(\xi-\gamma\nu)^2}{1-\gamma_n^2}\right)\right]$$

$$\beta_0 \, d\ln M = \frac{M n_{\rm PBH}}{\rho a^3} d\ln M = \frac{4\pi}{3} n_{\rm PBH} k_{\rm eq}^{-3}\left(\frac{M}{M_{\rm eq}}\right)^{3/2} d\ln M$$
$$= \frac{2\times 3^{-5/2} k_{\rm eq}^{-3}}{(2\pi)^{1/2}} \frac{\sigma_4^2\sigma_2^3}{\sigma_1^4\sigma_3^3}\left(\frac{M}{M_{\rm eq}}\right)^{3/2}\left[\int_{\mu_{2,c}}^{\infty} d\mu_2 \, \mu_2 k_\bullet f\left(\frac{\sigma_2^2}{\sigma_1^2\sigma_4}\mu_2 k_\bullet^2\right)\right.$$
$$\left. \times P_1\left(\frac{\sigma_2}{\sigma_1^2}\mu_2, \mu_2 k_\bullet^2 \frac{\sigma_2^2}{\sigma_1^2\sigma_4}\right)\left|\frac{d}{dk_\bullet}\ln r_m + \mu_2 \frac{d}{dk_\bullet} g_m\right|^{-1}\right] d\ln M$$

PBH abundance calculation

PBH threshold and abundance estimation



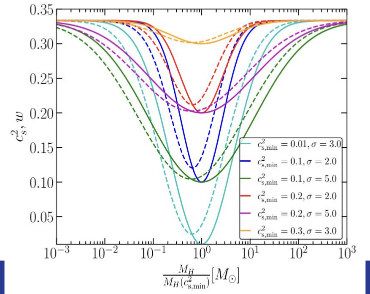
Numerical thresholds for PBH formation

"Typical profile" of the curvature fluctuation for flat spectrum

$$g(x;\kappa) = \frac{6}{x^4\lambda^4}\Big(-12\kappa^2 + x^2\lambda^2(3-4\kappa^2) + 4\left(3\kappa^2-2\right)x\lambda\sin(x\lambda) + 8 + \left[12\kappa^2 - 8 + x^2\lambda^2(1-2\kappa^2)\right]\cos(x\lambda)\Big)$$

PBH abundance

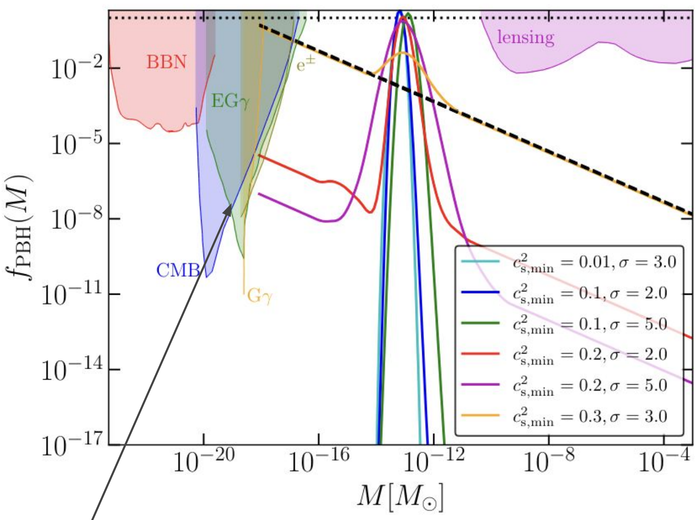
$$\beta_{0,\rm max}^{\rm approx} := \left[\frac{3\kappa\lambda^3}{4\sqrt{\pi}(6\kappa^4-8\kappa^2+3)} e^{3\mu_2 g(1;\kappa)} \times f\left(\sqrt{\frac{2}{A}}\kappa^2\mu_2\right) P_1\left(\frac{\mu_2}{\sqrt{A}}, \sqrt{\frac{2}{A}}\kappa^2\mu_2\right) \times \left|\frac{\mathrm{d}}{\mathrm{d}\kappa}\ln\lambda + \mu_2\frac{\mathrm{d}}{\mathrm{d}\kappa}g_{\rm m}\right|^{-1}\right]_{\kappa=\kappa_{\rm t},\ \mu_2=\mu_{2,c}(k_{\rm t})}$$

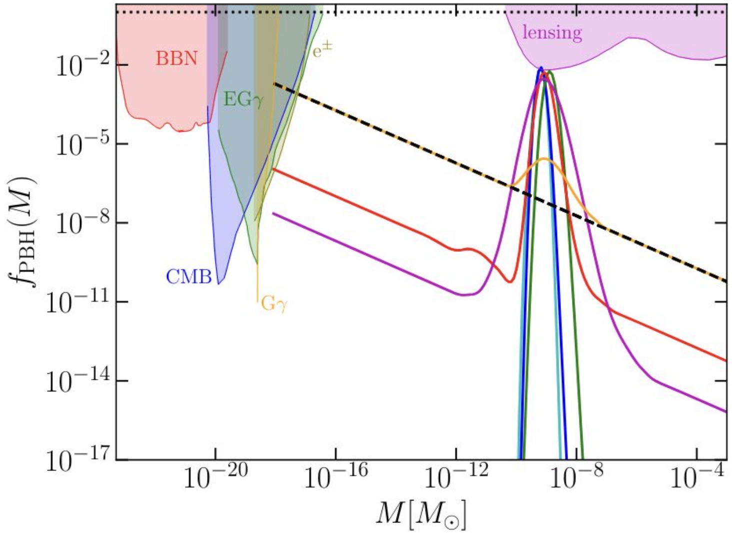


We make some simplifications

PBH mass function

Mass function highly peaked at the mass scale of the crossover



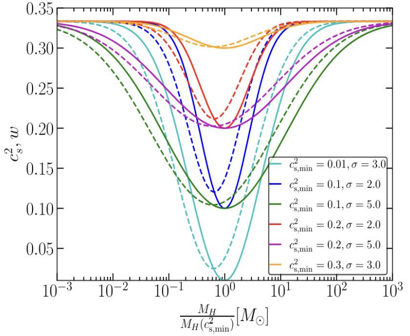


Cut-off scale

Model (A)-> $M_H \approx 10^{-13} M_\odot \, , \rho^{1/4} \approx 1.7 \cdot 10^6 \mathrm{GeV}$

Model (B)-> $M_H \approx 10^{-9} M_\odot \, , \rho^{1/4} \approx 17 \, \mathrm{TeV}$

$c^2_{s,\min}$	σ	$\mathcal{A}^{\mathbf{A}}/10^{-3}$	$\mathcal{A}^{\mathbf{B}}/10^{-3}$
0.1	2.0	1.972	1.920
0.1	5.0	1.539	1.504
0.2	2.0	3.393	3.320
0.2	5.0	3.156	3.086
0.3	3.0	4.268	3.840



Computation of the induced GWs

New code using "Julia" language

QCD case-> K. T. Abe, Y. Tada, I. Ueda. ArXiv:2010.06193

The numerical computation is actually quite hard...

$$\Omega_{\rm GW}(k,\eta_0)h^2 = \Omega_{r,0}h^2 \left(\frac{a_{\rm sh}\mathcal{H}_{\rm sh}}{a_{\rm f}\mathcal{H}_{\rm f}}\right)^2 \frac{1}{24}\left(\frac{k}{\mathcal{H}_{\rm sh}}\right)^2 \overline{\mathcal{P}_h(k,\eta_{\rm sh})}.$$

$$\mathcal{P}_h(k,\eta) = \frac{64}{81a^2(\eta)}\int_{|k_1-k_2|\le k\le k_1+k_2} \mathrm{d}\ln k_1\,\mathrm{d}\ln k_2\, I^2(k,k_1,k_2,\eta) \times \frac{\left(k_1^2-(k^2-k_2^2+k_1^2)^2/(4k^2)\right)^2}{k_1k_2k^2}\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2), \quad (11)$$

$$I(k,k_1,k_2,\eta) = k^2\int_0^\eta \mathrm{d}\tilde\eta\; a(\tilde\eta)G_k(\eta,\tilde\eta)\Big[2\Phi_{k_1}(\tilde\eta)\Phi_{k_2}(\tilde\eta) + \frac{4}{3(1+w(\tilde\eta))}\left(\Phi_{k_1}(\tilde\eta)+\frac{\Phi'_{k_1}(\tilde\eta)}{\mathcal{H}(\tilde\eta)}\right)\left(\Phi_{k_2}(\tilde\eta)+\frac{\Phi'_{k_2}(\tilde\eta)}{\mathcal{H}(\tilde\eta)}\right)\Big]$$

Barden equation

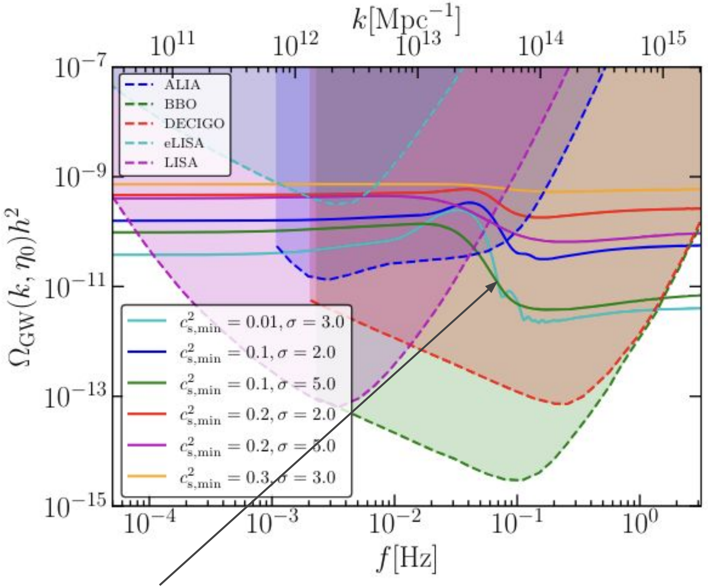
$$\Phi''_k(\eta) + 3\mathcal{H}(1+c_s^2)\Phi'_k(\eta) + \left[c_s^2k^2 + 3\mathcal{H}^2(c_s^2-w)\right]\Phi_k(\eta) = 0,$$

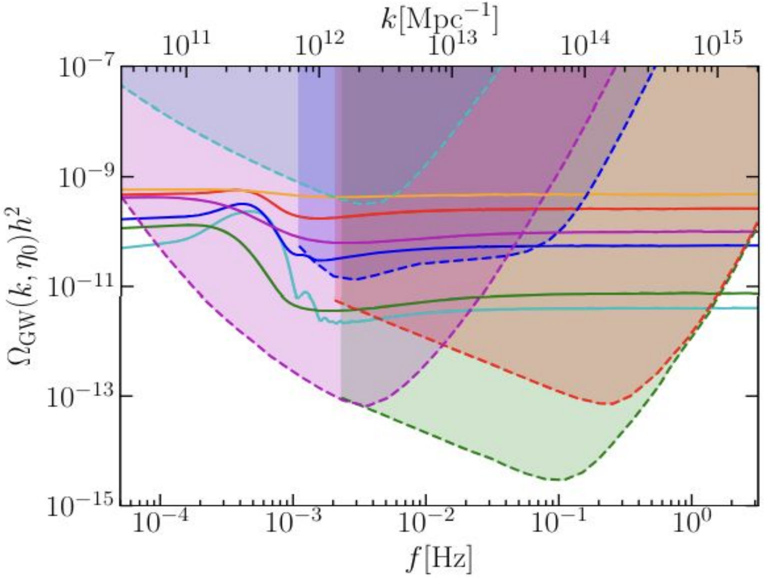
Green function

$$G_k(\eta,\tilde\eta) = \frac{1}{\mathcal{N}_k}\left[g_{1k}(\eta)g_{2k}(\tilde\eta) - g_{1k}(\tilde\eta)g_{2k}(\eta)\right]\Theta(\eta-\tilde\eta)$$

$$\left(\partial_\eta^2 + k^2 - \frac{1-3w(\eta)}{2}\mathcal{H}^2(\eta)\right)g_{jk}(\eta) = 0.$$

Gravitational Wave signature

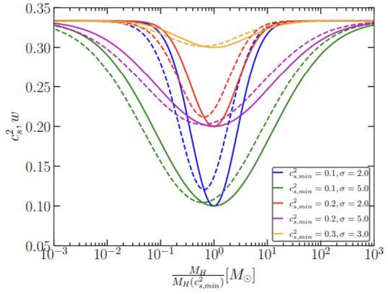




The “jump” between the two levels is caused by the crossover

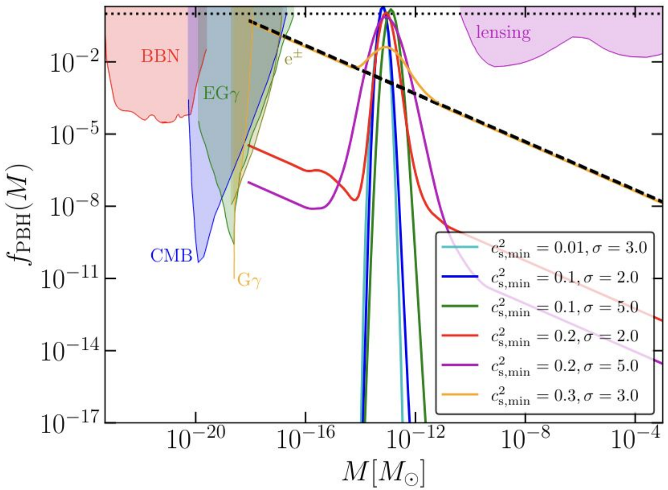
$$a_{\mathrm{sh}}\mathcal{H}_{\mathrm{sh}}/a_{\mathrm{f}}\mathcal{H}_{\mathrm{f}}$$

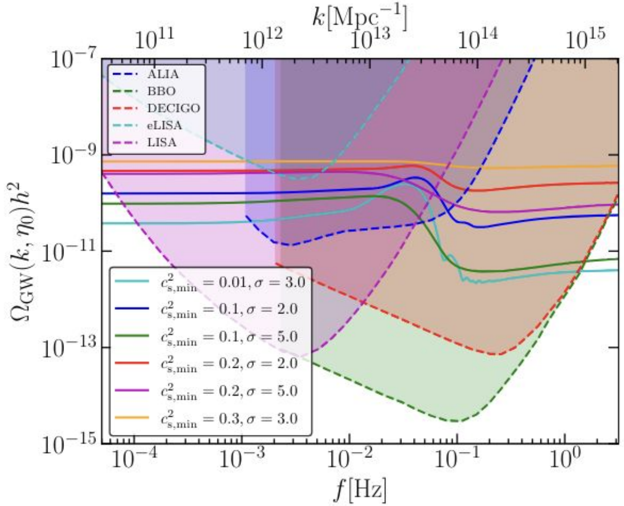
Difference between the two levels



Solving the degeneracy

Our results show that the GW signal can be used to resolve the existing degeneracy of sharply peaked mass function caused by peaked power spectrums and broad ones in the presence of softening crossovers.





Theoretical prediction

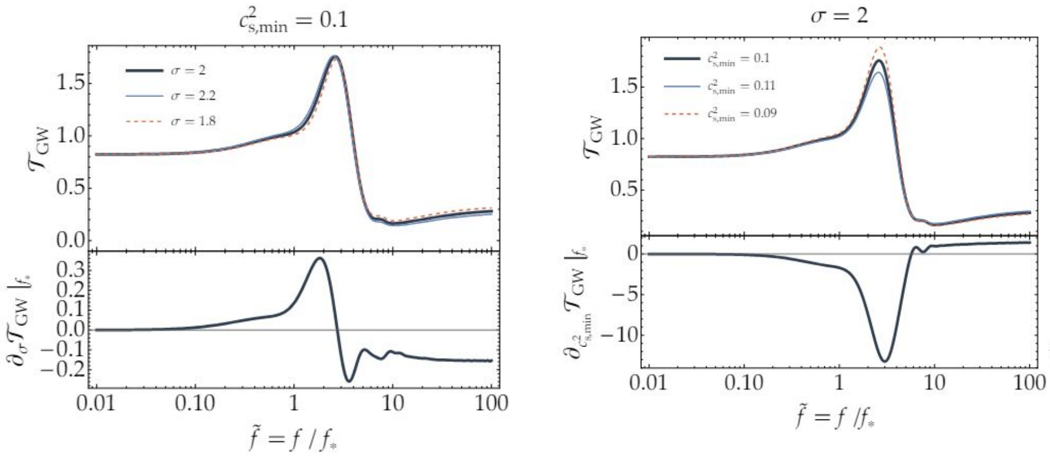
Observational signature

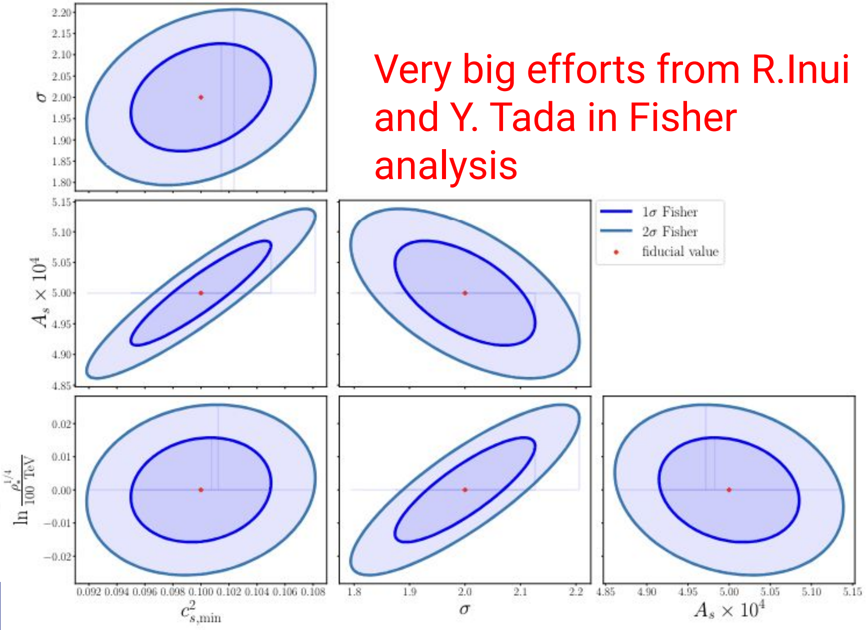
Detectability of the SC?

A. Escrivà, R. Inui, Y. Tada, C-M Yoo. ArXiv:2404.12591

The smooth crossover can be detectable in LISA if $\mathcal{A} > 5 \cdot 10^{-4}$

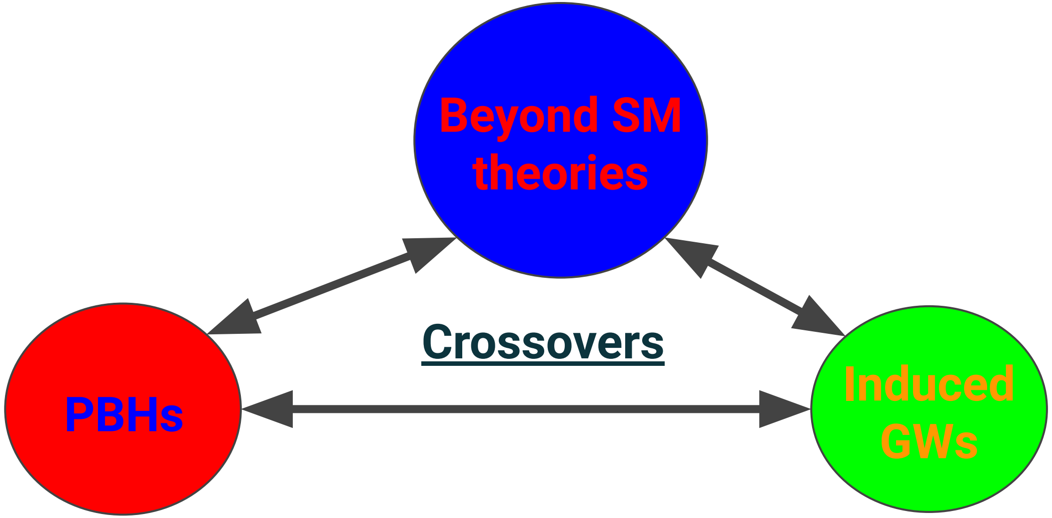
To make the analysis is needed very careful precise computation of the induced GWs (with IGWsfSC code)





Very big efforts from R.Inui and Y. Tada in Fisher analysis

Conclusions and messages to take home:



- Crossovers softenings beyond SM are usually not considered, but they can have important implications!
- We provide a potential observational signature for detecting a crossover beyond SM in future space based GW interferometers
- Such scenario can allow PBHs to be the dark matter, with a mass function specifically peaked at the mass scale of the minimum of the crossover.
- We solve the degeneracy of peaked mass functions caused by peaked power spectrums and broad ones in the presence of crossover softenings.

Extra slides 1

$$\mathrm{d}s^2 = -A(r,t)^2\,\mathrm{d}t^2 + B(r,t)^2\,\mathrm{d}r^2 + R(r,t)^2\left(\mathrm{d}\theta^2 + \sin^2(\theta)\,\mathrm{d}\phi^2\right)$$

$$\dot{U} = -A\left[\frac{c_s^2(\rho)}{1+w(\rho)}\frac{\Gamma^2}{\rho}\frac{\rho'}{R'} + \frac{M}{R^2} + 4\pi R w(\rho)\rho\right],$$

$$\dot{R} = AU,$$

$$\dot{\rho} = -A\rho\,[1+w(\rho)]\left(2\frac{U}{R} + \frac{U'}{R'}\right),$$

$$\dot{M} = -4\pi A w(\rho)\rho U R^2,$$

$$A' = -A\frac{\rho'}{\rho}\frac{c_s^2(\rho)}{1+w(\rho)},$$

$$M' = 4\pi\rho R^2 R',$$

$$A(r,t) = 1+\epsilon^2(t),$$

$$R(r,t) = a(t)re^{\zeta(r)}(1+\epsilon^2(t)\tilde{R}),$$

$$U(r,t) = H(t)R(r,t)(1+\epsilon^2(t)\tilde{U}),$$

$$\rho(r,t) = \rho_b(t)(1+\epsilon^2(t)\tilde{\rho}),$$

$$M(r,t) = \frac{4\pi}{3}\rho_b(t)R(r,t)^3(1+\epsilon^2(t)\tilde{M}),$$

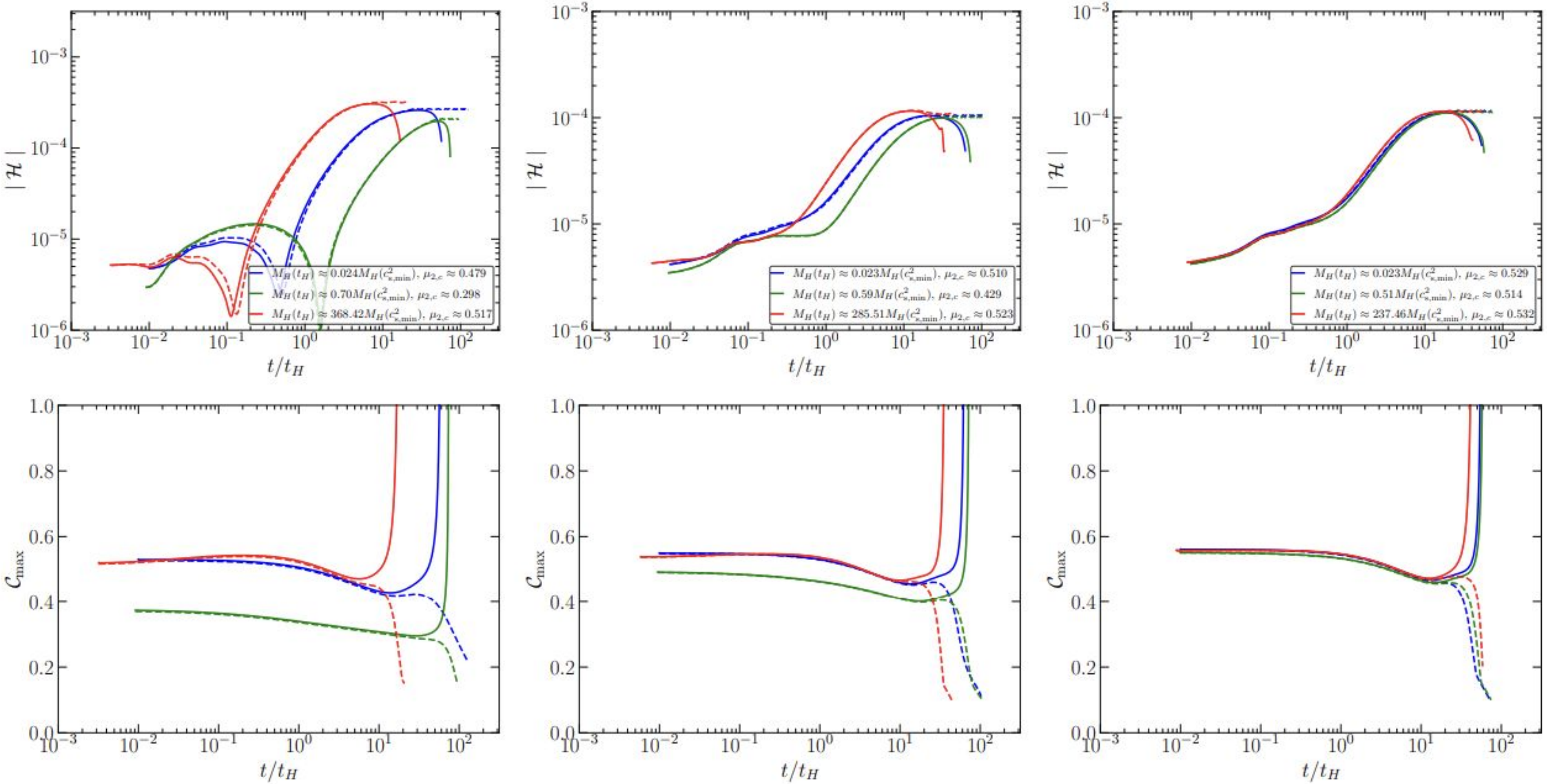
$$\phi_\rho(r) = -\frac{1}{3r}e^{-2\zeta(r)}\left[2r\zeta''(r) + \zeta'(r)(4+r\zeta'(r))\right],$$

$$\phi_U(r) = -\frac{1}{r}e^{-2\zeta(r)}\zeta'(r)\left[2+r\zeta'(r)\right],$$

Extra slides 2

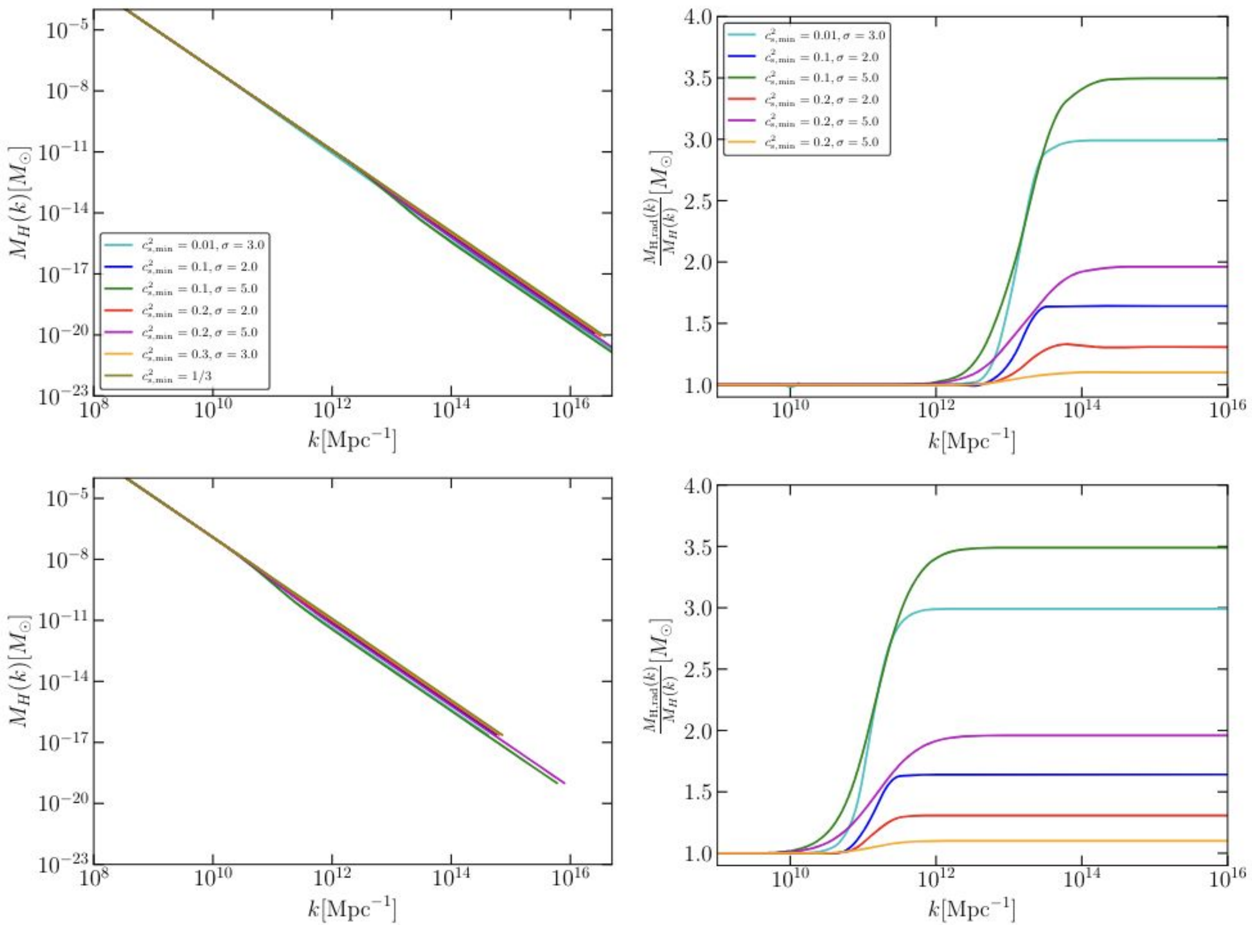
$$\mathcal{H} \equiv \frac{M'_{\rm num} - M'_{\rm def}}{M'_{\rm def}} = \frac{M'_{\rm num}/R'_{\rm num}}{4\pi\rho_{\rm num}R^2_{\rm num}} - 1,$$

$$|\mathcal{H}| \equiv \frac{1}{N_{\rm cheb}}\sqrt{\sum_{i=1}^{N}\left(\frac{M'_i/R'_i}{4\pi\rho_i R_i^2} - 1\right)^2}.$$



Extra slides 3

$$a^2(t_H)\rho(t_H)\tau^2 = \rho_b(t_0),$$



Extra slides 4