



中国科学院大学
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Supermassive primordial black holes for nano-Hertz gravitational waves

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University of Chinese Academy of Sciences, China

Based on: [Huang H L, Piao Y S. arXiv:2312.11982,](#)
[Huang H L, Cai Y, Jiang J Q, et al. arXiv:2306.17577](#)

New Horizons in Primordial Black Hole Physics, Edinburgh 20th June 2024

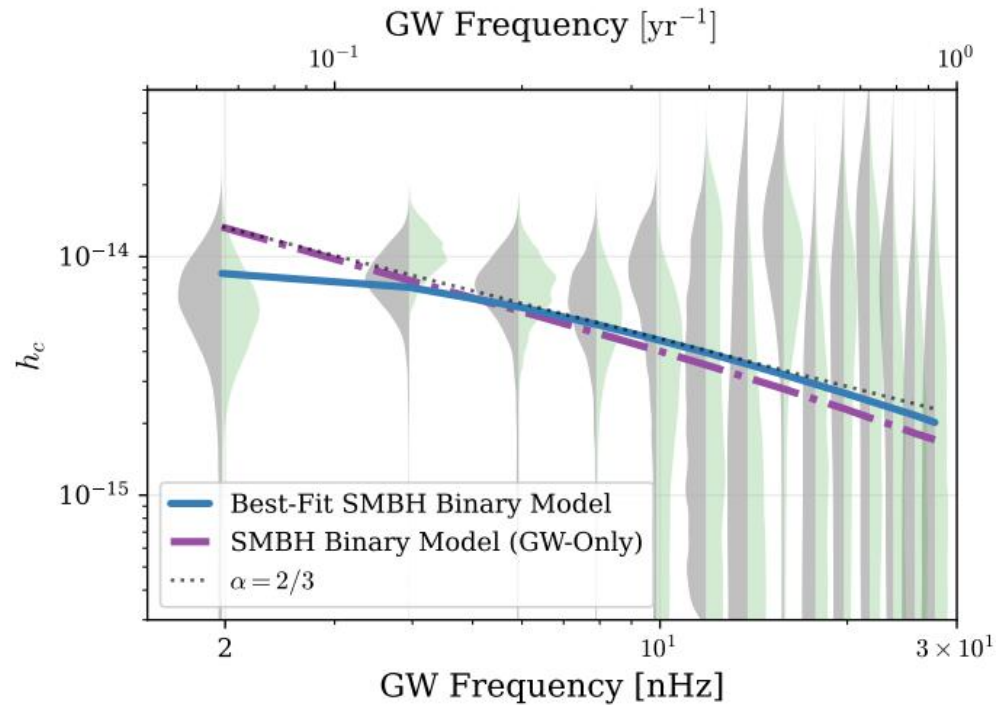
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The **nano-Hertz stochastic GW background** found with PTA might be interpreted with a population of SMBH binaries

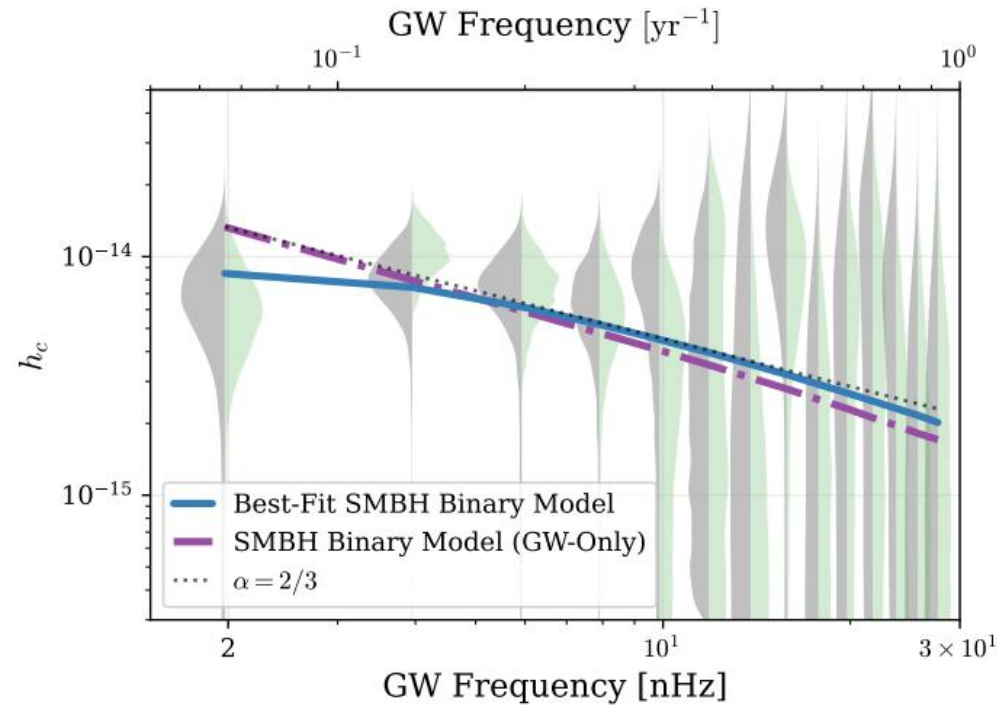


[NANOGrav Collaboration. (2023)]

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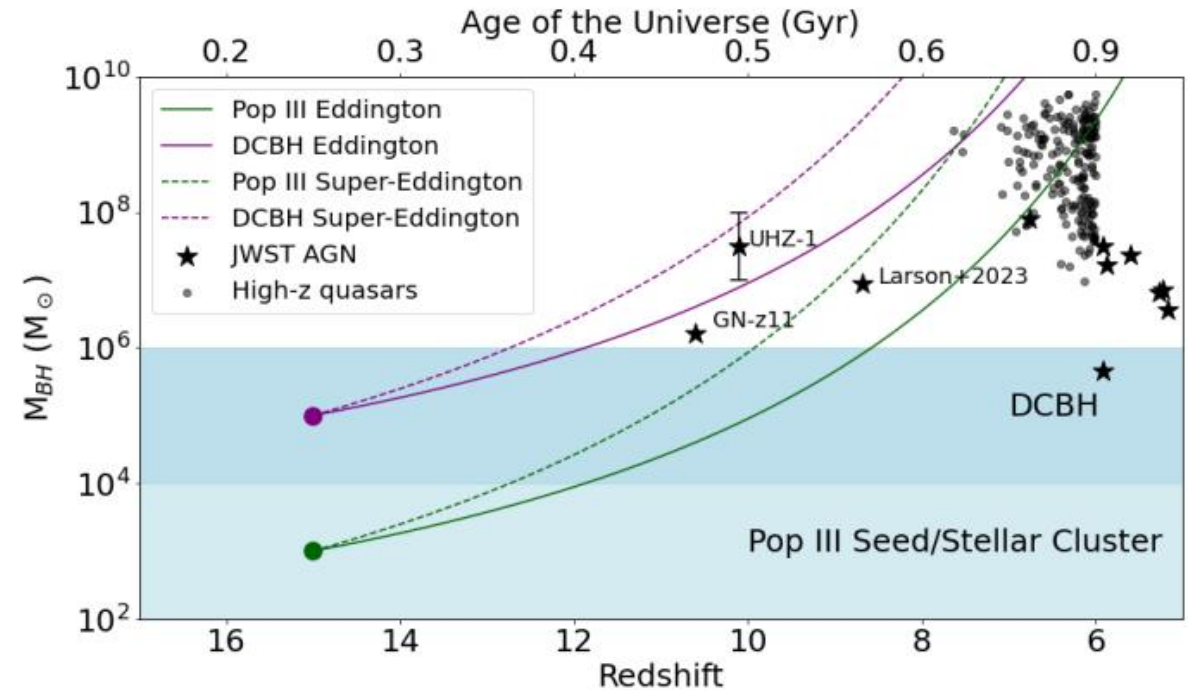
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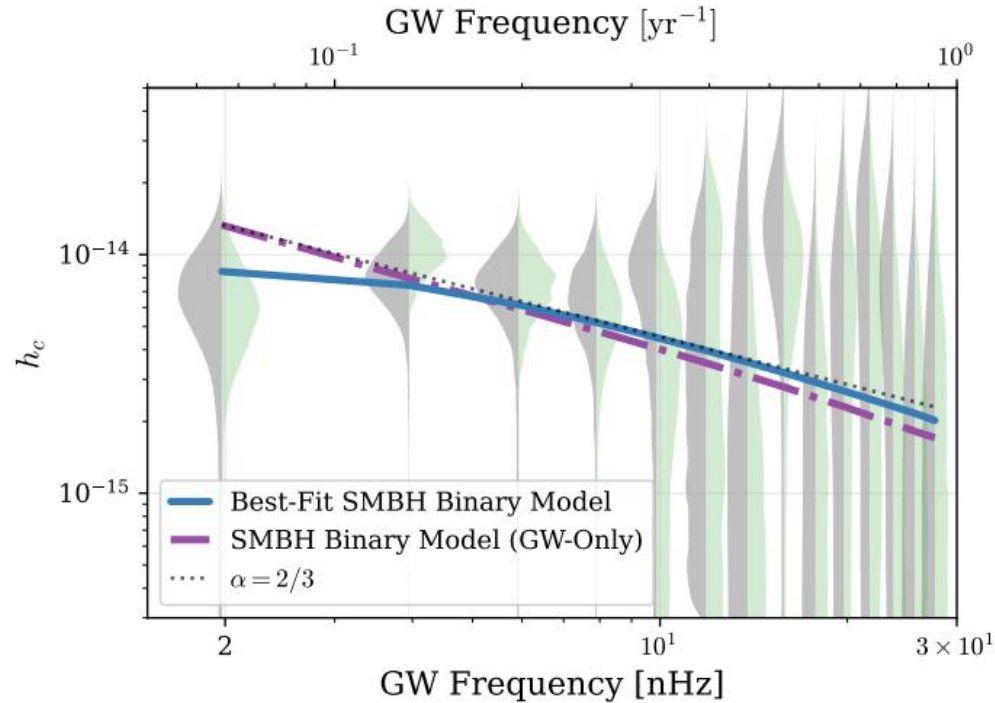


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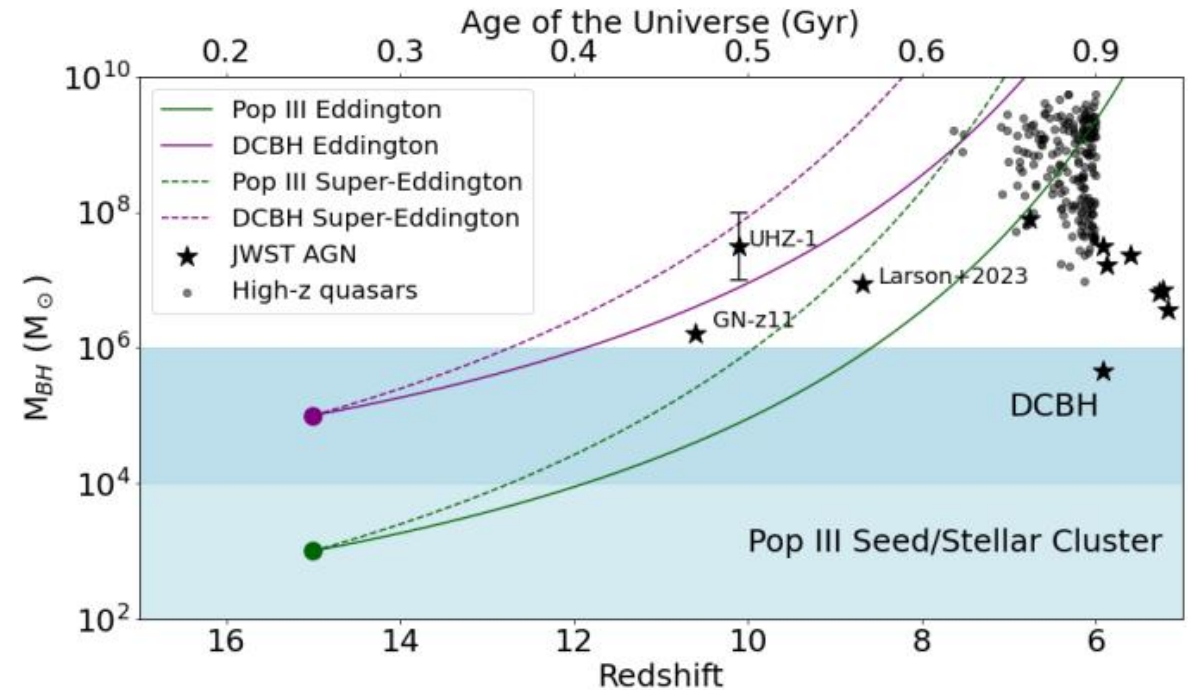
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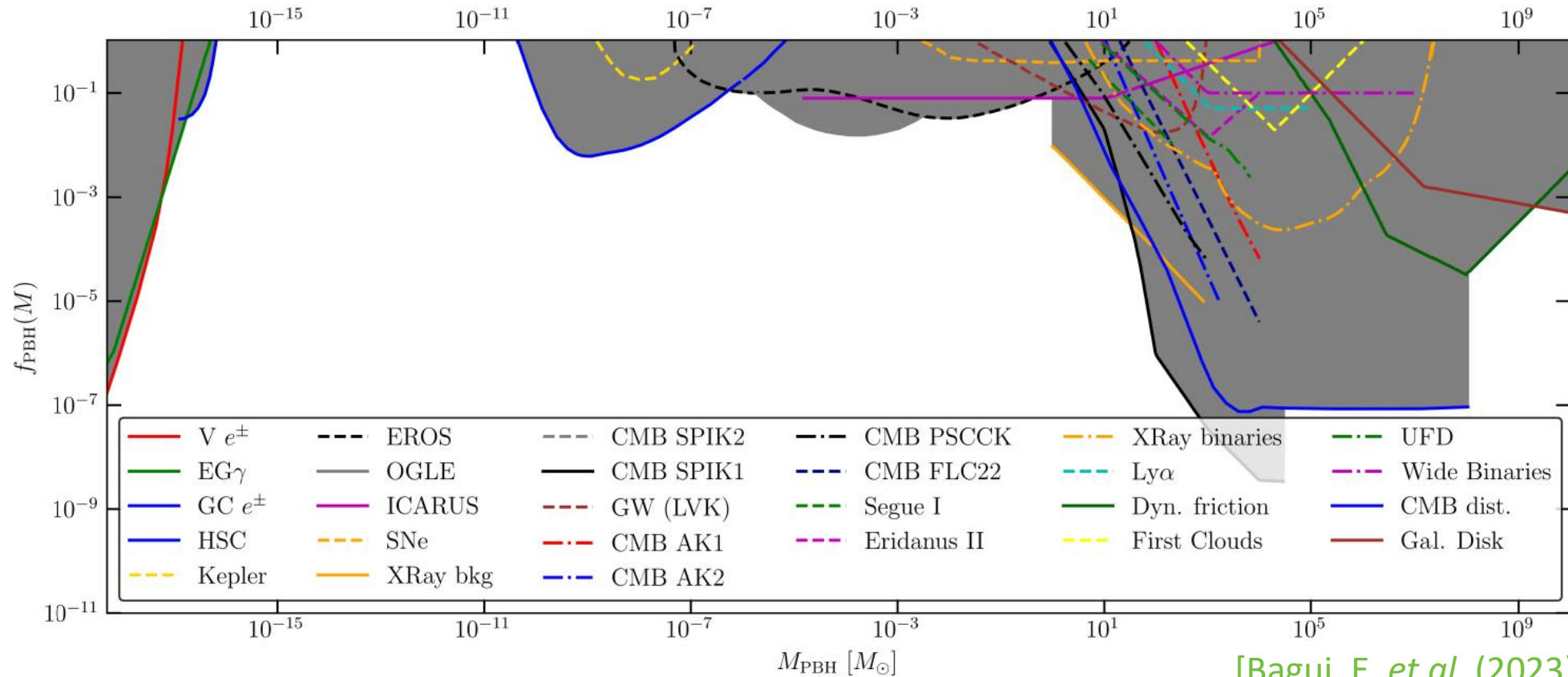


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➤ Could the SMBHs be primordial?

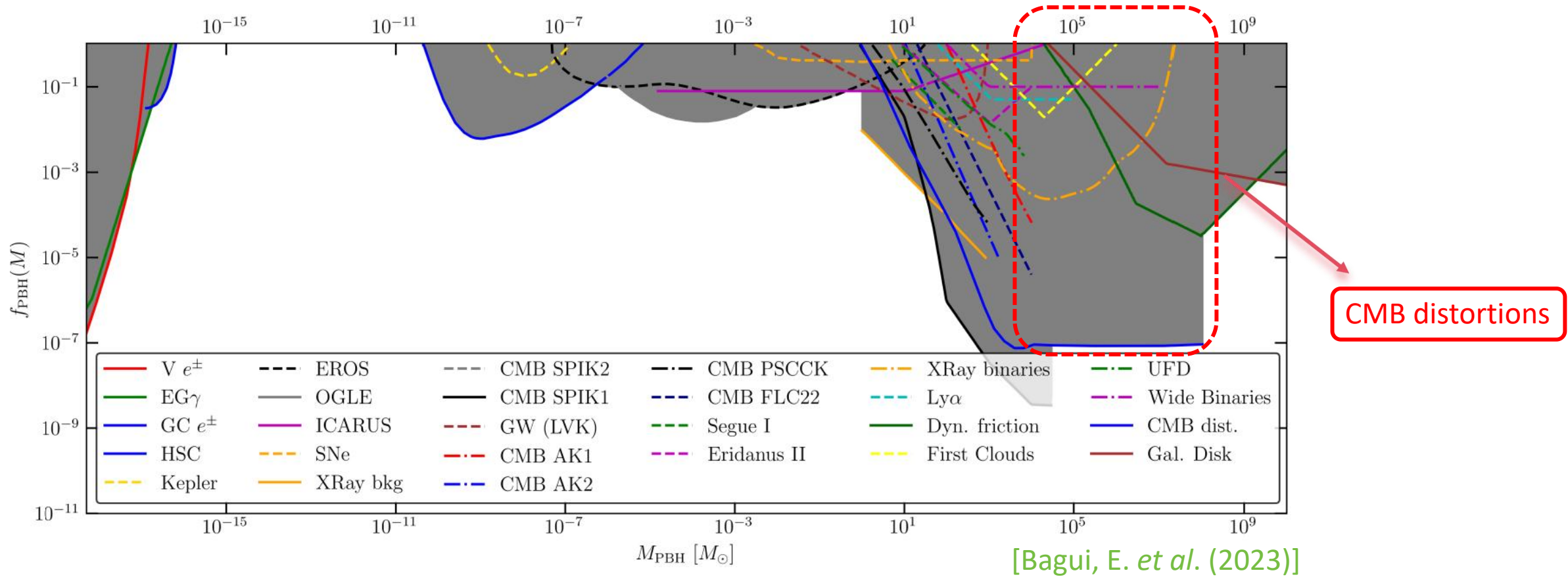
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- Explaining SMBHs via supermassive primordial black holes (SMPBHs) is challenging, primarily because of tight cosmic μ -distortion constraints.



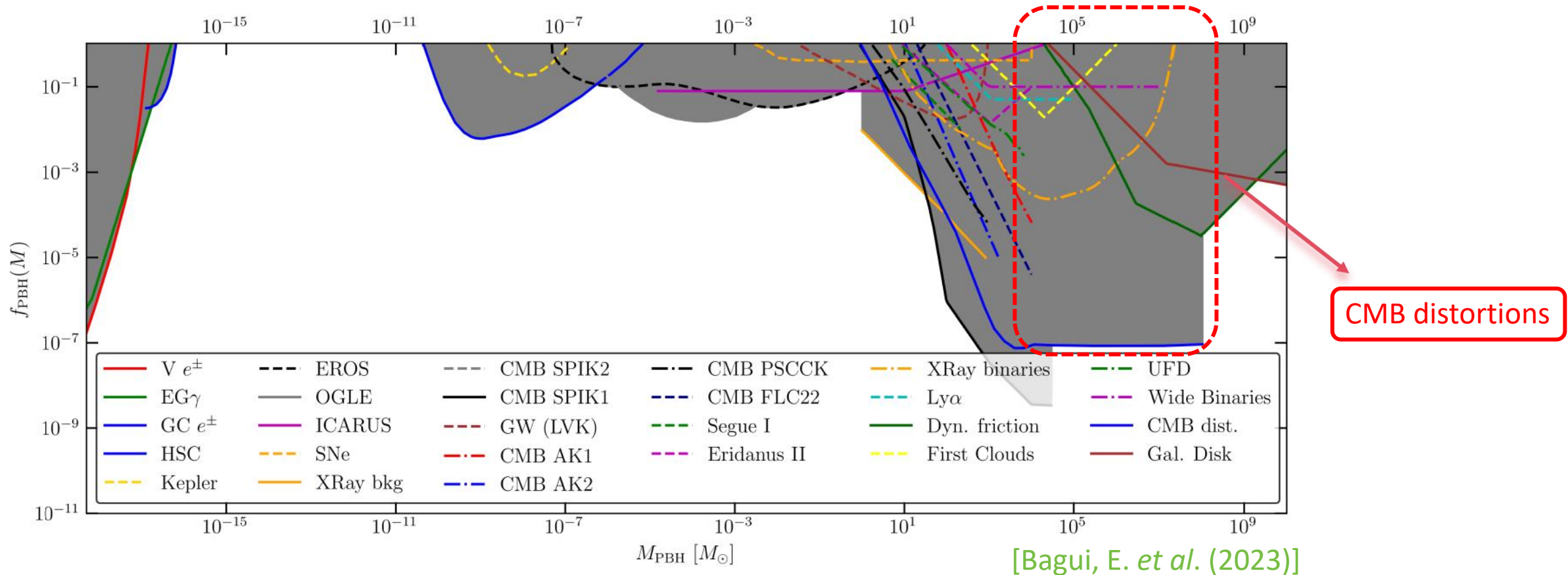
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- non-Gaussian perturbations & other PBH formation mechanisms?

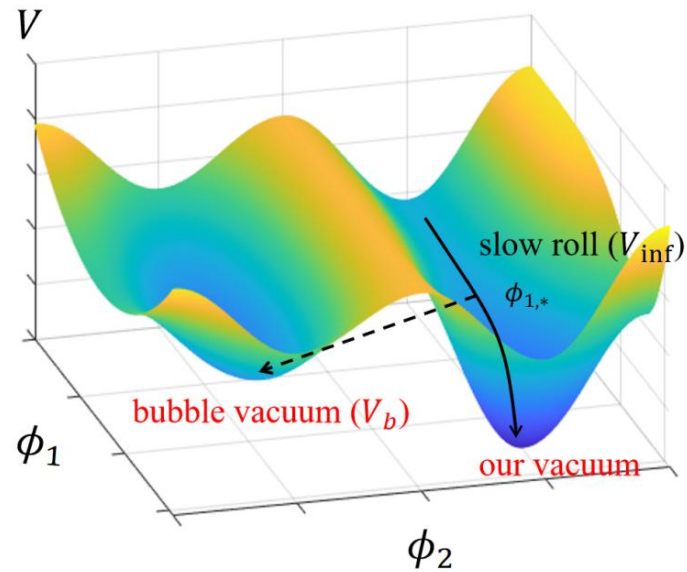
SMPBHs sourced from supercritical bubbles

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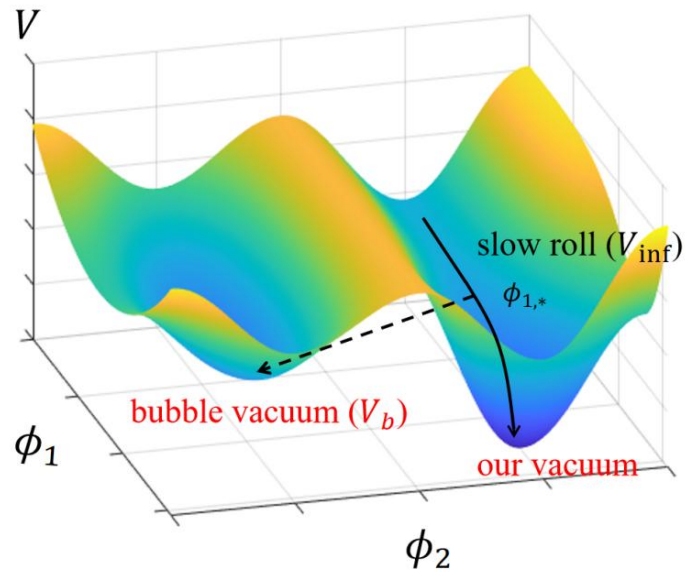
During inflation, bubble nucleation may occur by quantum tunneling



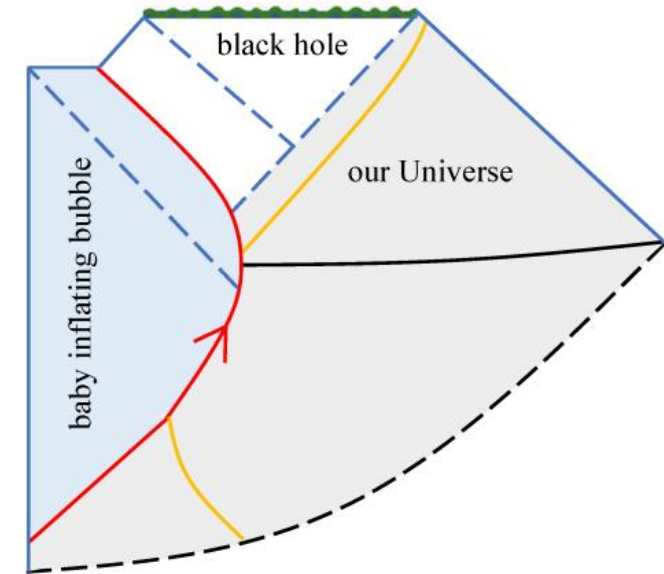
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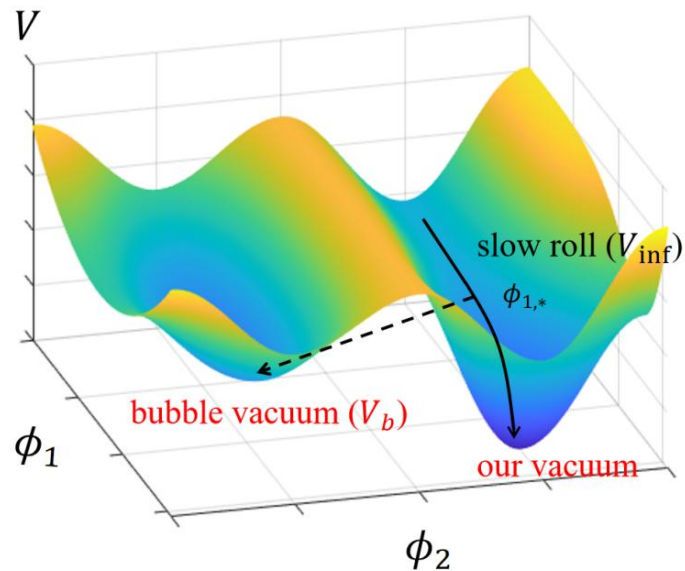
These supercritical bubbles (with an inflating baby universe inside it) evolve into PBHs in our observable Universe



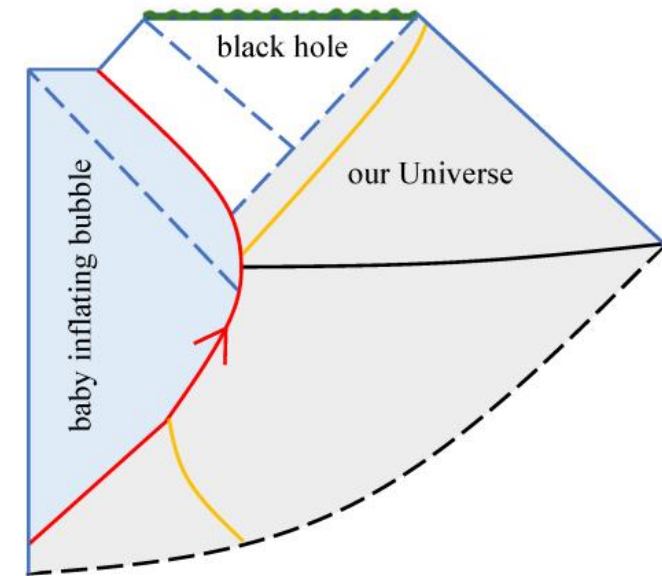
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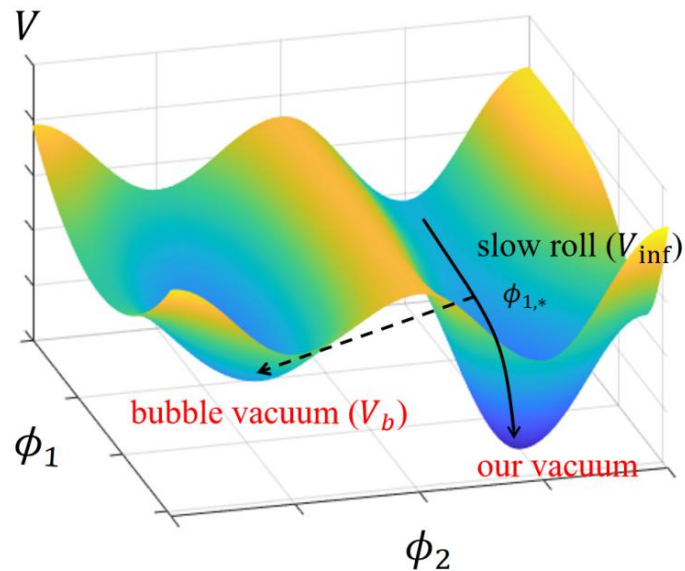
➤ Assuming that tunneling happens with nearly constant probability per unit time and volume: $\lambda \sim \text{const}$

the PBH mass $M \sim M_{\text{Pl}}^2 r_H$

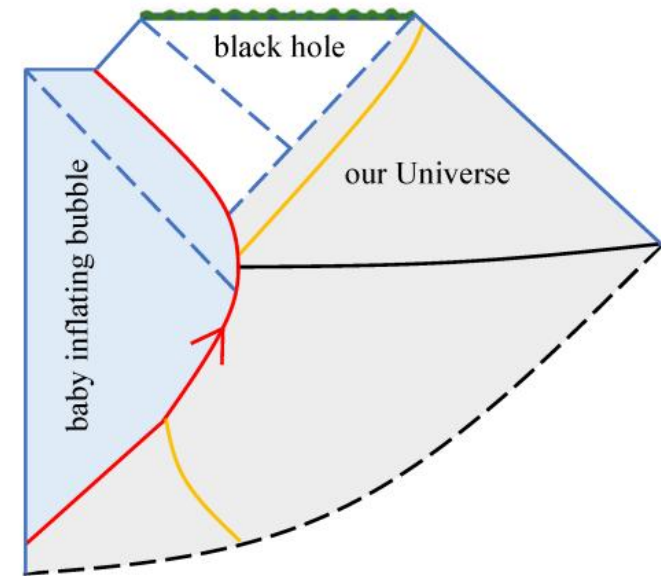
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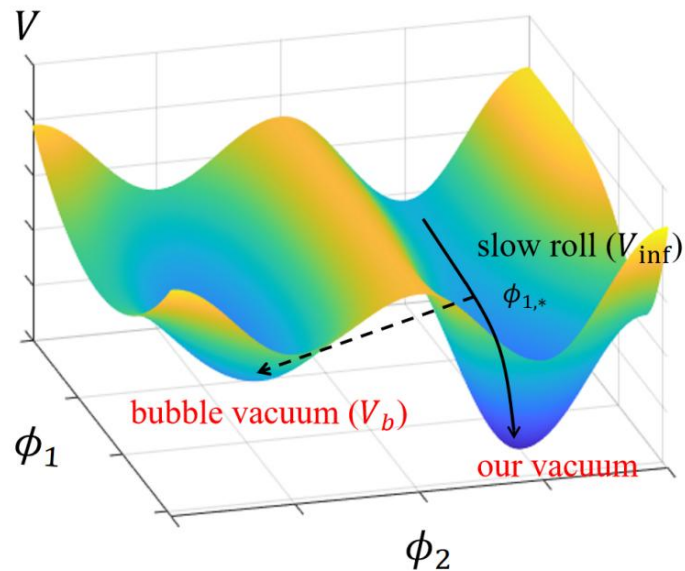
the PBH mass $M \sim M_{\text{Pl}}^2 r_H$ \longrightarrow the PBH mass function

$$\psi(m) \equiv \frac{m}{\rho_{\text{pbh}}} \frac{dn_{\text{pbh}}}{dm} \propto \lambda m^{-\frac{3}{2}}$$

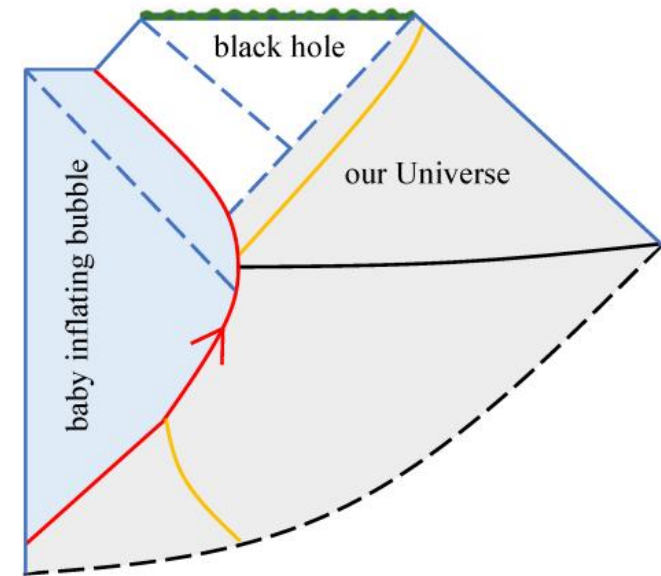
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negligible at supermassive band!

SMPBHs sourced by supercritical bubbles

➤ The nucleation rate of the CDL bubble: $\lambda \sim e^{-B}$

➤ In the thin wall limit, assuming that the radius of the nucleating bubble $R \sim H_i^{-1}$: $B = \frac{2\pi^2\sigma}{H_i^3}$

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→ a multi-peaks spectrum

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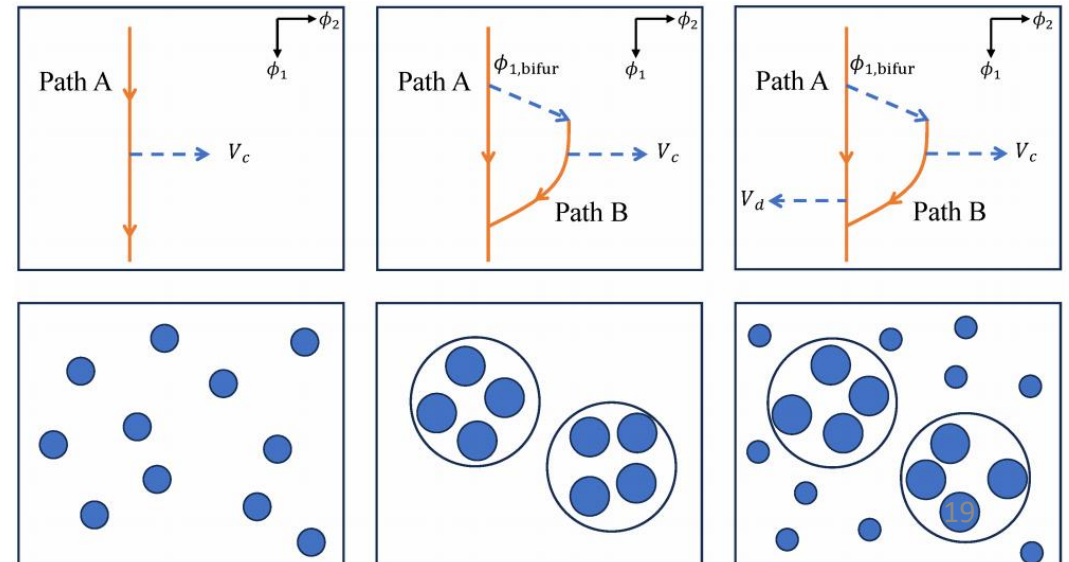
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➤ Multiple paths → cluster initially at different levels

$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}}{\rho_{\text{DM}}} = \lambda_B \frac{\rho_{\text{PBH}}^B}{\rho_{\text{DM}}} = \lambda_B f_{\text{PBH}}^B$$



SMPBH binaries for nano-Hertz GWs

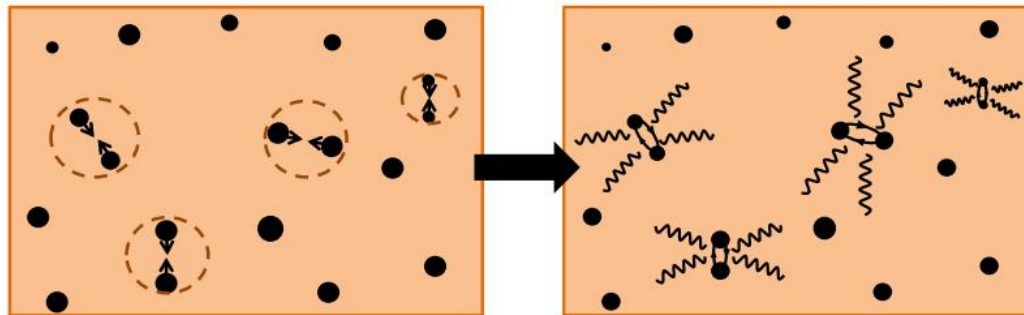
➤ GWs from SMPBH binary merges:

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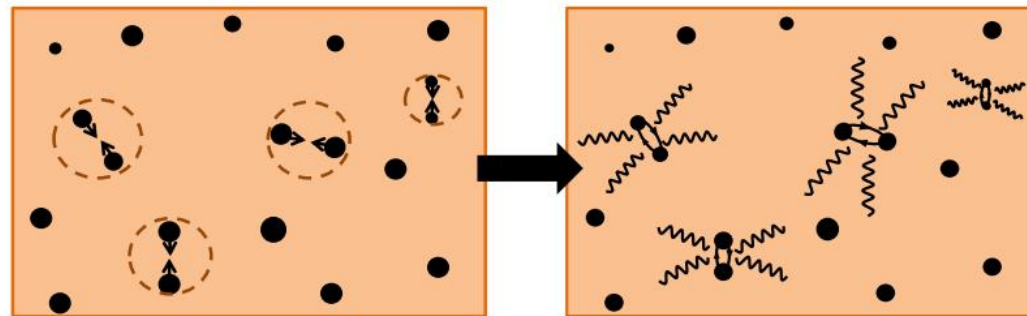


[Sasaki *et al.* (2018)]

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- The differential merger rate per unit volume at the time t

$$\mathcal{R}(m_i, m_j, t) \approx \frac{1.02 \times 10^8}{\text{Gpc}^3 \text{yr}} f^2 \left(\frac{M_\odot}{m_i} \right) \left(\frac{M_\odot}{m_j} \right) \left(\frac{m_i + m_j}{M_\odot} \right) \left(\frac{t_0}{t} \right) Y(y(m_i, m_j, t)) \psi(m_i) \psi(m_j)$$

[Huang, H. L., Jiang, J. Q., & Piao, Y. S. (2024)]

SMPBH binaries for nano-Hertz GWs

➤ The spectral density of GWs from unresolved SMPBH binaries :

$$h^2 \Omega_{\text{GW}}(f) = \frac{v}{\frac{\rho_c}{h^2}} \int \frac{dR(m_i, m_j, z) dz}{(1+z)H(z)} \frac{dE_{\text{GW}}(f')}{df'} \Big|_{f'=(1+z)f}$$

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The emitted energy in GWs per binary and per frequency bin

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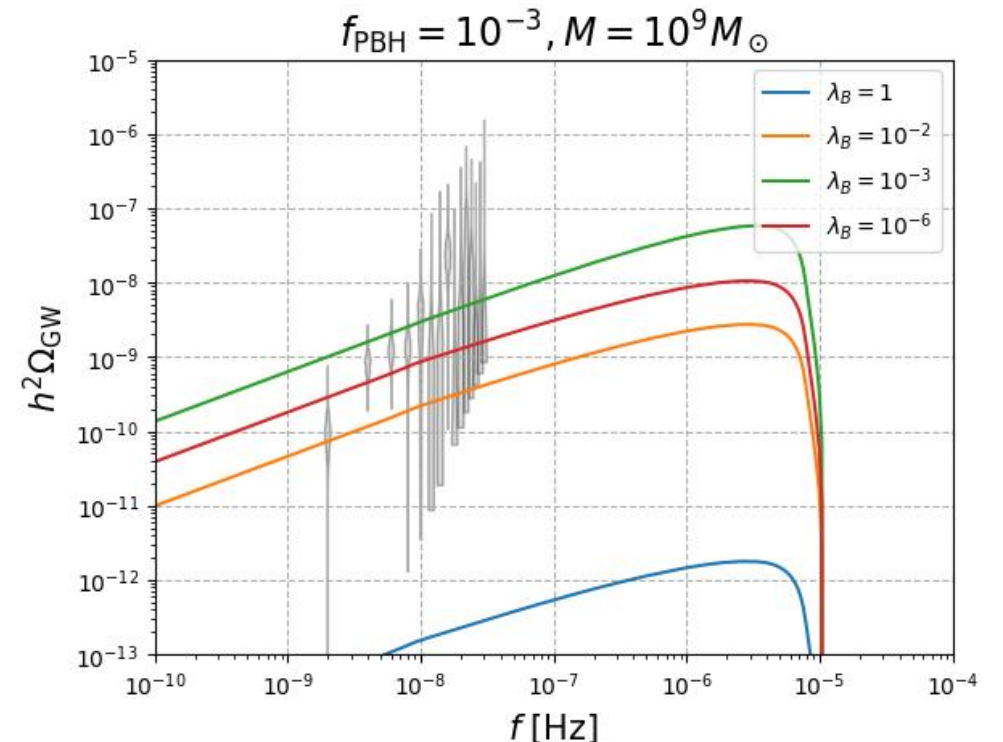
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Conclusions

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