One-loop power spectrum in ultra slow-roll inflation and implications for primordial black hole dark matter

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based on

arXiv:2404.07196, with G. Ballesteros

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Introduction

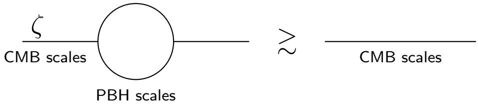
- PBH as a DM candidate:

$$f_{PBH} \sim 1 \;\Rightarrow\; \mathcal{P}_\zeta \big|_{\text{PBH scales}} \sim 10^{-2} \gg \mathcal{P}_\zeta \big|_{\text{CMB scales}} \sim 10^{-9}$$

Enhancement of $\mathcal{P}_\zeta \Rightarrow$ USR

- If ζ is large, is perturbation theory still valid?

Question highlighted by [J. Kristiano & J. Yokoyama '22]

If 

$\Rightarrow$ PBHs are ruled out

- Very active topic:

[A. Riotto '23, H. Firouzjahi '23, G. Franciolini et al. '23, Y. Tada et al. '23, K. Inomata '24, ...]

- Contradictory results
- What about renormalization?

- In our work:

- All the interactions + counterterms: $\mathcal{P}_\zeta$ finite
- The validity Perturbation Theory depends on the width of the transitions between SR and USR

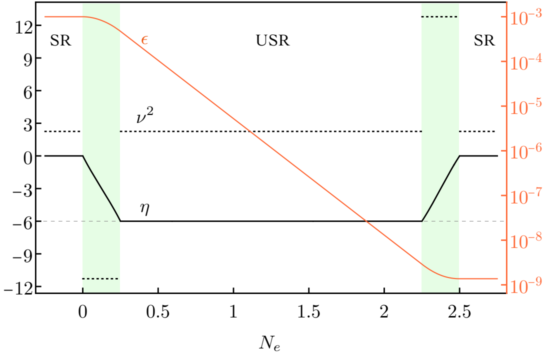
Inflationary model

- Background evolution

Three phases: SR - USR (ΔN) - SR

Smooth transitions between them (δN)

[G. Ballesteros et al. '20, G. Franciolini et al. '23]



Inflationary model

- Scalar fluctuations: $\delta\phi$-gauge

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + a^2\left((1+2\cancel{\zeta})\delta_{ij} + \partial_{ij}\cancel{E}\right) \times (N_i\mathrm{d}t + \mathrm{d}x_i)(N_j\mathrm{d}t + \mathrm{d}x_j)$$

$$\phi(x) = \phi_0(t) + \delta\phi(x)$$

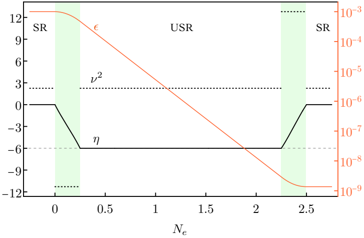
- In this gauge, if $\epsilon = -\dot{H}/H^2 \ll 1$ and other SR parameters are not:

$$S_{\rm int} = -\int a^3\mathrm{d}^4x\,\frac{1}{3!}V_3\delta\phi^3 + \frac{1}{4!}V_4\delta\phi^4 \supset -\int a^3\mathrm{d}^4x\,V(\phi)$$

Interactions coming from $g_{\mu\nu}$ are ϵ suppressed

- In the literature, work is often done in the ζ-gauge

Inflationary model: Shape of the interactions



- We describe the model with a new parameter:

$$\nu^2 \equiv \frac{9}{4} + \frac{1}{2}\left(3\,\eta + \frac{\eta^2}{2} + \frac{\eta'}{aH}\right)\,, \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}$$

Use of ν^2 to describe the inflationary phases:

Continuity on η and $\nu = \text{const.} \Rightarrow \nu = \nu(\delta N)$

- In the limit of instantaneous transitions: $\nu^2 \simeq \frac{\eta'}{2aH} \sim \frac{\Delta\eta}{\Delta t} \sim 1/\delta N$

The case studied in [Kristiano et al. '22] is recovered

- Derivatives of the potential

$$V_2 = -H^2(\nu^2 - 9/4)\,,\ V_n \sim \frac{\mathrm{d}^{(n-2)}\nu^2}{\mathrm{d}\tau^{(n-2)}} \;\Rightarrow\; V_{3,4} \sim \Delta\nu^2\delta(\tau-\tau_*) \sim 1/\delta N\,\delta(\tau-\tau_*)$$

Instantaneous transitions: the interactions diverge $\Rightarrow$ importance of smooth transitions

Computing the power spectrum

- To calculate the power spectrum ζ, we have to change the gauge

$$\zeta = -\frac{\delta\phi}{\sqrt{2\epsilon}M_P} + \frac{\eta}{4}\left(\frac{\delta\phi}{\sqrt{2\epsilon}M_P}\right)^2 + \ldots, \quad \eta = 0 \text{ during SR}$$

$$\mathcal{P}_\zeta(\tau, k) = \int \frac{\mathrm{d}^3\mathrm{r}}{(2\pi)^3} e^{-ik\mathrm{r}} \frac{4\pi k^3}{2\epsilon(\tau)M_P^2} \langle \delta\phi(\mathrm{x}+\mathrm{r})\delta\phi(\mathrm{x})\rangle \text{ at the end of inflation}$$

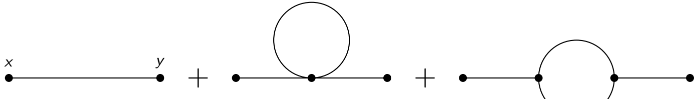
- In-in formalism

$$\langle \delta\phi(\mathrm{x})\delta\phi(\mathrm{y})\rangle = \langle 0| \left(F(t, -\infty^-)\right)^\dagger \delta\phi(\mathrm{x})\delta\phi(\mathrm{y})F(t, -\infty^-)|0\rangle$$

$$F(t, -\infty^-) = T\exp\left(-i\int_{-\infty(1-i\omega)}^{t} \mathrm{d}t' H_I(t')\right)$$

The damping in the lower limit is crucial for convergence

- Expansion in loops



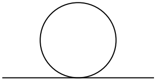
$\mathcal{O}(H_I^0)$ $\mathcal{O}(H_I^1)$ $\mathcal{O}(H_I^2)$

Computing the power spectrum: Renormalization

- The loop integrals diverge ⇒ we need to regularize

We use a cutoff that modifies both momentum and time integrals

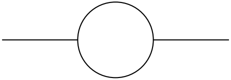
Quartic contribution:



We only need to regularize the momentum integral:

$$\int^{\infty} \mathrm{d}p \to \int^{\Lambda} \mathrm{d}p$$

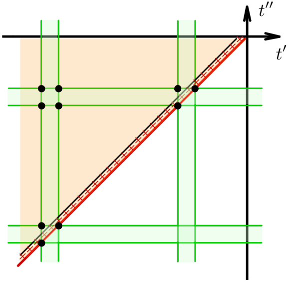
Cubic contribution:



Momentum integral:

$$\int^{\infty} \mathrm{d}p \to \int^{\Lambda} \mathrm{d}p$$

Time integral:



- Using the $-\infty(1 - i\omega)$ prescription: the cubic contribution is finite

Computing the power spectrum: Counterterms

- To absorb the dependence on Λ, we include the counterterms

The cts arise from the original action $\Rightarrow$ we will focus on those coming from $V(\phi)$

We tune $V(\phi)$ to get the background we want

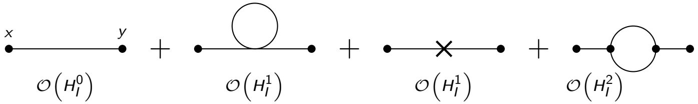
$$V(\phi) = C_0 + C_1\phi + \cdots \to (C_0^R + \delta C_0) + (C_1^R + \delta C_1)\phi + \cdots \equiv V^R(\phi) + \delta V(\phi)$$

At second order:

$$S \supset -\int \sqrt{-g}\mathrm{d}^4\, V(\phi) = \cdots - \int \sqrt{-g}\mathrm{d}^4x\, \tfrac{1}{2}\left(V_2^R + \delta V_2\right)\delta\phi^2 + \ldots$$

δV_2: arbitrary function of time

- The counterterms introduce a new diagram



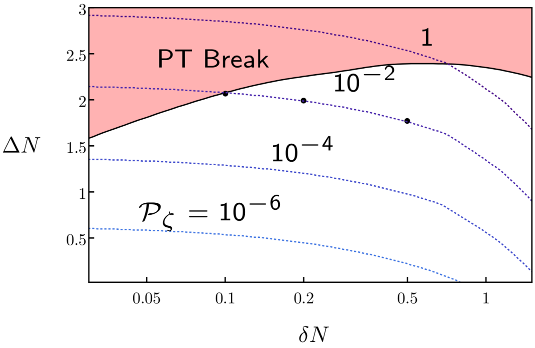
The quartic contribution can be absorbed completely by the cts

Validity of perturbation theory

- Once $\mathcal{P}_\zeta$ is finite and does not depend on Λ:

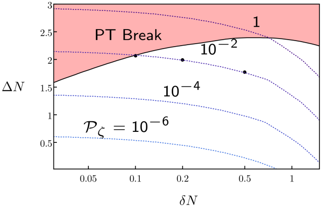
  Is perturbation theory valid? $\Rightarrow$ We need to compare $\mathcal{P}_\zeta^{1l}$ with $\mathcal{P}_\zeta^{tl}$

  (Natural) assumption: the finite contribution of the cts $\sim \mathcal{P}_\zeta^{1l}$

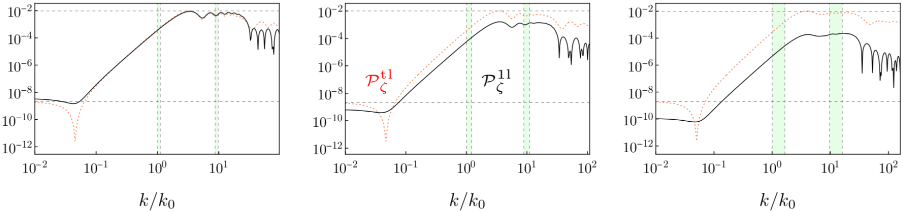


- The validity of PT depends on the values chosen for $\{\Delta N\,,\delta N\}$

Validity of perturbation theory: Behavior in $\delta N \to 0$



- The validity of PT depends on the values chosen for $\{\Delta N\,, \delta N\}$



- We note that the contribution at one-loop increases as $\delta N \to 0$

Expected result, the coupling of interactions is $V_{3,4} \sim \Delta\nu^2 \sim 1/\delta N$ and diverges in this limit

Origin of the dependence on δN in the ζ-gauge

- Our result shows a divergence in $\delta N \to 0$ that had not been previously detected

In the $\delta\phi$-gauge, the origin of the divergence is clear

The calculation using the $\delta\phi$ and ζ gauges must be consistent: what is the origin of this divergence in the ζ-gauge?

- The dominant cubic interaction in the ζ-gauge is

$$S \supset \int \mathrm{d}^4x \, M_P^2 \, a^2 \, \epsilon \, \frac{\eta'}{2} \, \zeta' \, \zeta^2$$

All other interactions are either ϵ suppressed or canceled at the end of inflation

This interaction depends on $\zeta' \;\Rightarrow\; H_I$ will have a cubic and a quartic contribution:

$$H_I(\tau) = \int \mathrm{d}^3\mathrm{x} \, M_P^2 a^2 \epsilon \left(-\frac{\eta'}{2} \zeta' \zeta^2 + \frac{(\eta')^2}{16} \zeta^4 \right)$$

See e.g. [X. Chen et al. '06, H. Firouzjahi '23]

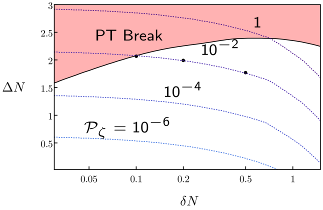
Origin of the dependence on δN in the ζ-gauge

$$H_I(\tau) = \int \mathrm{d}^3x \, M_P^2 a^2 \epsilon \left(-\frac{\eta'}{2}\zeta'\zeta^2 + \frac{(\eta')^2}{16}\zeta^4 \right)$$

- The induced quartic term is usually neglected, and is responsible for the divergence in the limit $\delta N \to 0$

Taking into account this contribution, $\mathcal{P}_\zeta$ calculated in both gauges coincides exactly

Conclusions



- By including all interactions and counterterms, we make $\mathcal{P}_\zeta$ finite

PT remains valid over a wide parameters space $\{\Delta N\,,\delta N\}$

- A divergence in the limit of instantaneous transitions ($\delta N \to 0$) has been detected

Origin of the difference from previous results in the ζ-gauge: derivation of the H_I

- Convenience of working in the $\delta\phi$-gauge in a model such as USR

The set of interactions becomes simpler: these come from the potential