Inflation and primordial black holes production in Starobinsky-like supergravity

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Motivation

Goal of research: describe viable inflation with efficient PBHs production in Starobinsky-like supergravity

Method of research: realize Starobinsky inflation in 4D, N=1 supergravity, consistently with CMB measurements, and extend it to produce PBHs via enhancement of the power spectrum of scalar perturbations and gravitational collapse of **large** perturbations.

Main problem to solve: low value of n_s tilt of CMB power spectrum of scalar perturbations in the presence of PBHs production during inflation.

Starobinsky inflation

The action of the Starobinsky inflation model is ($\hbar = c = 1$)

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left(R + \frac{R^2}{6M^2}\right), \quad M_{\rm Pl} = 1/\sqrt{8\pi G_N} \tag{1}$$

with the mass parameter $M \sim 10^{-5} M_{\rm Pl}$ (from known CMB amplitude).
After Legendre-Weyl transformation to the quintessence picture, the scalar potential of canonical inflaton field φ is given by

$$V_{\rm star.}(\varphi) = \frac{3}{4}M^2 M_{\rm Pl}^2\left(1 - e^{-\sqrt{2/3}\frac{\varphi}{M_{\rm Pl}}}\right)^2. \tag{2}$$

The observable values of n_s and r are given by (Planck, BICEP/Keck)

$$n_s = 0.9649 \pm 0.0042, \;\; r < 0.032. \tag{3}$$

For Starobinsky inflation, these observables in terms of the e-folding number N are predicted as (with best fit $N \approx 55$)

$$n_s \approx 1 - \frac{2}{N} \approx 0.9636, \quad r \approx 3(1-n_s)^2 \approx \frac{12}{N^2} \approx 0.003. \tag{4}$$

Pole inflation

Pole inflation allows one to essentially describe inflation by a non-canonical kinetic term having a pole, with a related unification of inflationary models into the universality classes. The Starobinsky inflaton φ can be redefined as $\varphi = -\sqrt{3/2}\ln(1-\sqrt{2/3}\phi)$, that leads to the quintessence action of the non-canonical field ϕ as $(M_{\rm Pl}=1)$

$$S_{\rm Star.} = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{3(\partial_\mu\phi)^2}{4(1-\sqrt{2/3}\phi)^2} - \frac{1}{2}M^2\phi^2\right], \qquad (5)$$

where the mass term can be replaced by any regular (at ϕ=0) potential $V(\phi)$, for instance, leading to the α-attractors (Kallosh, Linde, Röst, 2013).

Starobinsky inflation in supergravity and its modification

Our supergravity extension of the Starobinsky model of inflation is described by the following Kähler potential and the superpotential (cf. Cecotti, Kallosh, 2014):

$$K = -3\ln\left(T + \bar{T} - |C|^2 + \zeta\frac{|C|^4}{T + \bar{T}}\right), \tag{6}$$

$$W = MC(T-1) + g(T), \quad M_{\rm Pl} = 1, \tag{7}$$

in terms of the inflaton superfield T and the goldstino superfield C also needed as the stabilizer of inflation near $C = 0$. We have added a new analytic function $g(T)$ to the superpotential, which is needed for PBHs production.

The bosonic part of the Lagrangian reads (when $C = 0$)

$$\mathcal{L} = -\frac{3}{(T+\bar{T})^2}\partial_\mu T \partial^\mu \bar{T} - V, \tag{8}$$

$$V = \frac{1}{3}\frac{M^2|T-1|^2}{(T+\bar{T})^2} + \frac{1}{3}\frac{1}{(T+\bar{T})}\left|\frac{dg}{dT}\right|^2 - \frac{\frac{dg}{dT}\bar{g} + \frac{d\bar{g}}{d\bar{T}}g}{(T+\bar{T})^2}. \tag{9}$$

The effective scalar Lagrangian

It is convenient to parametrize equations (8) and (9) by a canonical scalar field φ (*scalaron*) and another scalar field θ (*sinflaton*) as

$$Z = re^{i\theta} = \tanh\frac{\varphi}{\sqrt{6}}\,e^{i\theta}, \quad \text{where} \quad T = \frac{1+Z}{1-Z}. \tag{10}$$

Then the kinetic term of Z can be rewritten to

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{3}{4}\sinh^2\frac{2\varphi}{\sqrt{6}}(\partial_\mu\theta)^2. \tag{11}$$

In the case of $g(Z) = 0$, the scalar potential (9) reduces to

$$V = \frac{M^2}{12}\sinh^2\frac{2\varphi}{\sqrt{6}}\left(1 + \tanh^2\frac{\varphi}{\sqrt{6}} - 2\tanh\frac{\varphi}{\sqrt{6}}\cos\theta\right), \tag{12}$$

the θ-field is stabilized ($m_\theta \gtrsim H$) and its dynamics can be ignored. Then the effective single-field potential for inflation is close to the potential of the Starobinsky model.

Equations of motion and effective potential, $g(Z) \neq 0$

When the function $g(Z) \neq 0$, both scalars φ and θ get dynamics. The corresponding equations of motion in flat Friedman universe read

$$0 = \ddot{\varphi} + 3H\dot{\varphi} - \sqrt{\tfrac{3}{8}} \sinh\left(\sqrt{\tfrac{8}{3}}\varphi\right)\dot{\theta}^2 + \partial_\varphi V \quad , \tag{13}$$

$$0 = \ddot{\theta} + 3H\dot{\theta} + 2\sqrt{\tfrac{2}{3}}\coth\left(\sqrt{\tfrac{2}{3}}\varphi\right)\dot{\varphi}\dot{\theta} + \tfrac{2}{3}\mathrm{csch}^2\left(\sqrt{\tfrac{2}{3}}\varphi\right)\partial_\theta V \quad , \tag{14}$$

$$0 = \tfrac{1}{2}\dot{\varphi}^2 + \tfrac{3}{4}\sinh^2\left(\sqrt{\tfrac{2}{3}}\varphi\right)\dot{\theta}^2 - 3H^2 + V \quad , \tag{15}$$

with the potential

$$\begin{aligned} V =& \frac{M^2}{3}\frac{r^2(1-2r\cos\theta+r^2)}{(1-r^2)^2} + \frac{1}{24}\frac{(1-2r\cos\theta+r^2)^3}{1-r^2}\left|\frac{\mathrm{d}g}{\mathrm{d}Z}\right|^2 \\ &- \frac{1}{8}\frac{(1-2r\cos\theta+r^2)^2}{(1-r^2)^2}\left[(1-re^{i\theta})^2\frac{\mathrm{d}g}{\mathrm{d}Z}\bar{g} + (1-re^{-i\theta})^2\frac{\mathrm{d}\bar{g}}{\mathrm{d}\bar{Z}}g\right] \quad . \end{aligned} \tag{16}$$

Scan of $g(Z)$ functions for PBHs production

Below is the list of $g(Z)$ functions we considered. In a polynomial case, one of the parameters (g_0) is fixed by the condition that the minimum of the potential is Minkowski.

- **Linear** functions $g(Z) = M(g_0 + g_1 Z)$: any value of g_1 does not give the second plateau for the Hubble function.
- **Quadratic** functions $g(Z) = M(g_0 + g_1 Z + g_2 Z^2)$: PBHs production is possible but with small masses of PBHs, well below the Hawking evaporation limit of 10^{15}g.
- **Kachru-Kallosh-Linde-Trivedi (KKLT)** functions $g(Z) = M(C + Ae^{-aZ})$: the θ-direction is stable but cannot lead to a tachyonic instability needed for PBHs production.
- **Cubic** functions $g(Z) = M(g_0 + g_1 Z + g_2 Z^2 + g_3 Z^3)$: possible to produce PBHs with masses larger than the Hawking evaporation limit, and fit the CMB observational results within 1σ accuracy. ✓

Inflationary dynamics and observables

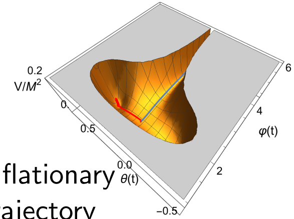
The inflationary dynamics in our model has three phases (SR-USR-SR):

1. The first slow-roll (SR) stage of inflation is driven by φ, while sinflaton θ is suppressed.
2. During the ultra slow-roll (USR) phase, near the saddle point, the effective mass squared of sinflaton becomes negative (tachyonic instability), which leads to large iso-curvature perturbations fueling scalar perturbations, and the subsequent enhancement on the power spectrum of scalar perturbations signalling PBHs production.
3. The second SR stage of inflation is driven by sinflaton θ.

The CMB tilts (the scalar spectrum index n_s and the tensor-to-scalar ratio r) are numerically derived by using the transport method and the dedicated code (Dias, Frazer, Seery, 2015).

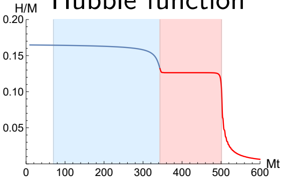
Our results with cubic $g(Z)$ function, I

The model with a cubic $g(Z)$ function has double inflation and the large (up to 10^{-2}) enhancement (peak) in its power spectrum, which leads to the formation of PBHs with masses larger than the Hawking evaporation limit. The most successful case, $(g_1, g_2, g_3) = (-21, 0, 16)$, is described by

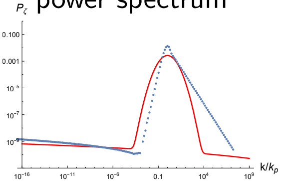


inflationary trajectory

Hubble function



power spectrum



The log-normal fit for the power spectrum of scalar perturbations is

$$\mathcal{P}_\zeta(k) = \frac{\mathcal{A}_\zeta}{\sqrt{2\pi}\Delta}\exp\left[-\frac{\ln^2(k/k_p)}{2\Delta^2}\right], \quad \text{where} \tag{17}$$

$\mathcal{P}_\zeta(k_p) \approx 10^{-2} \sim 10^{-3}$, Peak position $k_p \approx 10^{14}\mathrm{Mpc}^{-1}$,
$\mathcal{P}_{\rm CMB} \approx 10^{-9}$, Pivot scale (CMB): $k_* = 0.05\mathrm{Mpc}^{-1}$.

Our results with cubic $g(Z)$ function, II

The dependence of our solutions upon initial conditions on φ was found to be weak, but the dependence upon the velocity θ' at the saddle point was found to be strong. The values of the parameter θ' we took randomly, like a quantum 'kick' caused by quantum diffusion. We find

g_1	g_3	g_0	n_s	r	θ'	$M_{\rm PBH}$ (g)	$\mathcal{P}_{\rm peak}$
-20.6	15.6	14.2	0.96081	0.0041	5.5×10^{-6}	2.2×10^{17}	1×10^{-3}
-20.8	15.8	14.2	0.96092	0.0040	2.9×10^{-6}	1.8×10^{17}	4×10^{-3}
-21	16	14.3	0.96089	0.0040	1.4×10^{-6}	2.2×10^{17}	2×10^{-2}

where $g_2 = 0$ in all cases, and the duration of the second SR stage of inflation is about 20 e-folds. $M_{\rm PBH}$ (g) is the PBH mass calculated from (Pi, Zhang, Huang, Sasaki, 2017)

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H(t_*)}\exp\left[2(N_{\rm end}-N_*)+\int_{t_*}^{t_{\rm exit}}\epsilon(t)H(t)dt\right]\ , \tag{18}$$

$$\text{or, roughly,}\ \ M_{\rm PBH}(k_p)\approx 10^{-16}\left(\frac{k_p}{10^{14}{\rm Mpc}^{-1}}\right)^{-2}\ . \tag{19}$$

PBHs production induced gravitational waves (GW)

Numerically calculating the energy density fraction of the PBHs-induced GWs in the second order with respect to perturbations (Espinosa, Racco, Riotto, 2018), with the power spectrum in our model yields

$$\Omega_{\rm GW}(k) \approx 3 \times 10^{-5} \mathcal{P}_\zeta^2(k). \tag{20}$$

The induced GW frequencies are related to the PBHs masses as (De Luca, Franciolini, Riotto, 2021)

$$f_p \approx 5.7 \left(\frac{M_\odot}{M_{\rm PBH}} \right)^{1/2} 10^{-9} {\rm Hz}, \tag{21}$$

where $M_\odot \approx 2 \times 10^{33} g$ is the mass of the Sun. In our model, $M_{\rm PBH}$ are about $10^{17} g$ and, therefore, the resulting GW frequencies are $f_p \approx 0.6$ Hz, cf. the GW frequencies detected by NANOGrav between 3 and 400 nHz.

Conclusion

- Our model of inflation in Starobinsky-like (modified) supergravity agrees (1σ) with the current CMB observational results and can produce PBHs with the masses larger than the Hawking evaporation limit. Therefore, these PBHs can be a candidate for dark matter in current Universe. It is achieved by adopting the cubic Wess-Zumino-type superpotential for $g(Z)$.
- Demanding the large enhancement of the power spectrum of scalar perturbations during inflation is usually in tension with the observed value of n_s. In our model, we solved this problem by fine-tuning of the parameters in $g(Z)$. The fine-tuning may not be a problem itself because both inflation and PBHs production, if any, were unique events in the Universe.
- The PBHs production-induced GWs in our model have the broad spectrum, $\Omega_{\rm GW} \sim 10^{-5}\mathcal{P}_\zeta^2(k)$, with the predicted frequency ≈ 0.6 Hz.

Thank you for your attention!