Numerical First Order QED calculations of Hawking Radiation from Asteroid Mass PBHs

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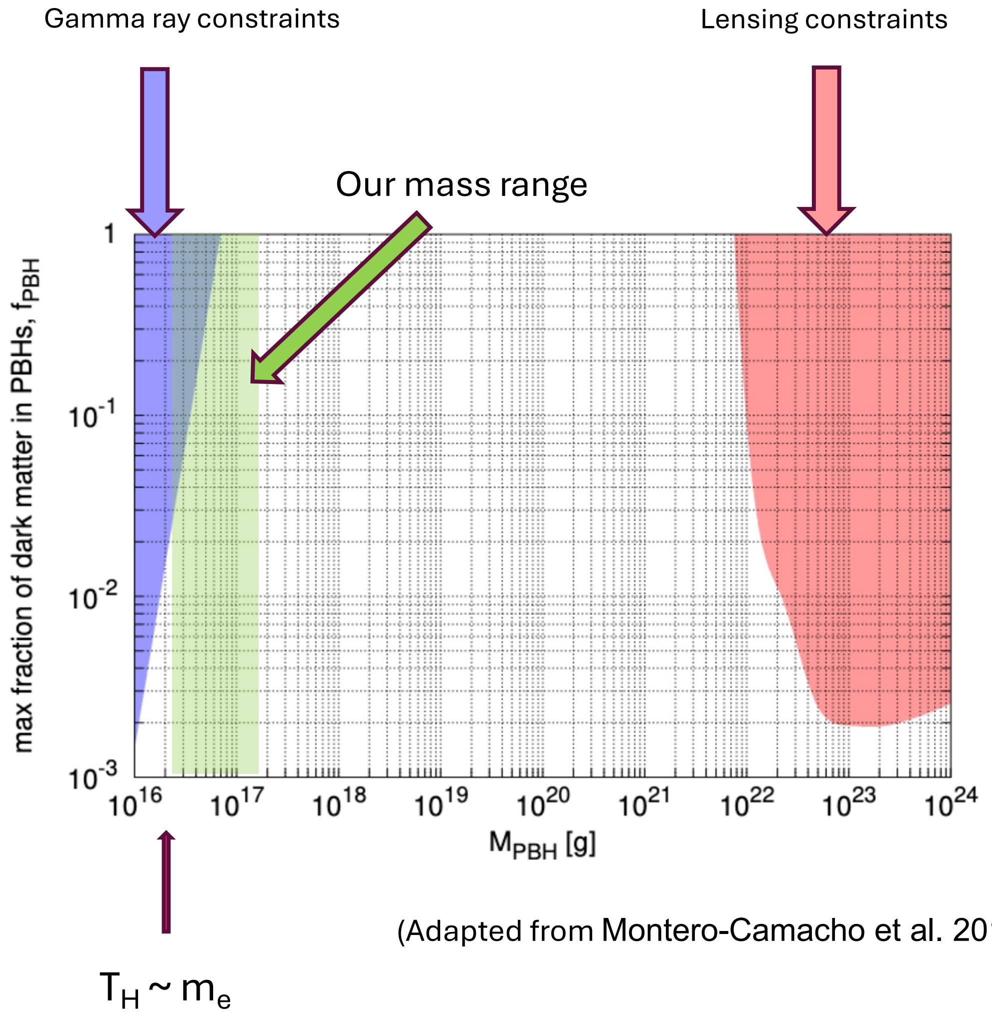
Hawking radiation at Asteroid Masses

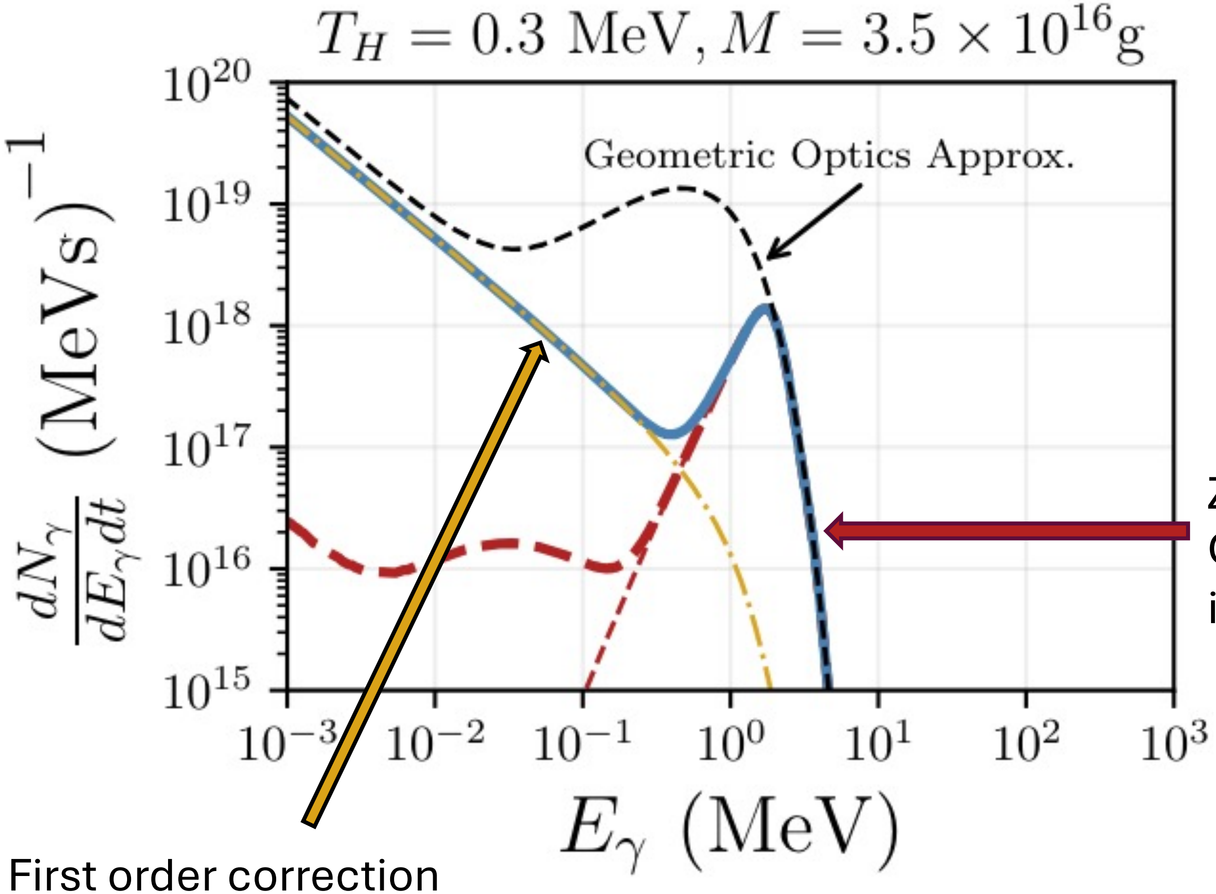
- Currently the asteroid mass PBHS ($M \sim 10^{16}$ - 10^{21} g) are still a viable dark matter candidate
- $T_{Hawking} = 1/(8\pi M) \sim 100$ keV for low asteroid mass PBHs
- Quantum electrodynamic processes are dominant in this energy range



(Adapted from Montero-Camacho et al. 2019)

First Order Interactions

- First order in α_{em}~1/137
- Page, Carr, & MacGibbon (2008) show at low frequencies first order inner bremsstrahlung interactions are dominant
- Coogan et al (2021) show first order interactions are the dominant contribution to low energy spectra
- Secondary interactions (e.g. final state radiation) are based on special relativity



Zeroth Order in α_{em}

First order correction from secondary interactions

(Adapted from Coogan et al. 2021)

Dissipative Spectrum Calculation

Corrections to Hawking radiation from asteroid mass primordial black holes: Formalism of dissipative interactions in quantum electrodynamics

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$$
\begin{aligned}
\left.\frac{dN_\gamma^{(1)}}{dt\,d\omega}\right|_{\rm diss} &= \frac{1}{2\pi}\sum_{\ell m_\gamma p}\frac{d}{dt}\langle \hat a^\dagger_{{\rm out},\ell m_\gamma \omega(p)}\hat a_{{\rm out},\ell m_\gamma\omega(p)}\rangle_{\rm diss}\\
&= \frac{e^2}{2\pi}\sum_{\ell=1}^{\infty}\sum_p\int\frac{dh}{2\pi}\sum_{kk'}\Delta(j,j',\ell)\delta_{ss'(-1)^{k+k'+\ell},(-1)^p}\\
&\quad\times\Bigg[|R_{1,\ell,\omega}|^2\Bigg\{\frac{2}{e^{8\pi M(\omega+h)}+1}|\llbracket I^{-+}_{{\rm in},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)\rrbracket|^2\\
&\quad+\frac{1}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega-h)}+1)}|\llbracket I^{++}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)\rrbracket|^2\\
&\quad+\frac{2e^{8\pi Mh}}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega+h)}+1)}|\llbracket I^{-+}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)\rrbracket|^2\Bigg\}\\
&\quad+\frac{|T_{1,\ell,\omega}|^2}{e^{8\pi M\omega}-1}\Bigg\{-|\llbracket I^{++}_{{\rm in},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)\rrbracket|^2+\frac{2e^{8\pi M\omega}}{e^{8\pi M(\omega+h)}+1}|\llbracket I^{-+}_{{\rm in},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)\rrbracket|^2\\
&\quad-\frac{2}{e^{8\pi Mh}+1}\left(e^{8\pi Mh}|\llbracket I^{++}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)\rrbracket|^2+|\llbracket I^{-+}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)\rrbracket|^2\right)\Bigg\}+\\
&\quad+{\rm Re}\,T^*_{1,\ell,\omega}R_{1,\ell,\omega}\Bigg[\frac{2(2e^{8\pi M\omega}-1)}{(e^{8\pi M(\omega+h)}+1)(e^{8\pi M\omega}-1)}\llbracket I^{-+}_{{\rm in},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)\rrbracket\llbracket I^{-+*}_{{\rm in},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)\rrbracket\\
&\quad+\frac{1}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega-h)}+1)}\llbracket I^{++}_{{\rm up},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)\rrbracket\llbracket I^{++*}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)\rrbracket\\
&\quad+\frac{2e^{8\pi Mh}}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega+h)}+1)}\llbracket I^{-+}_{{\rm up},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)\rrbracket\llbracket I^{-+*}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)\rrbracket\\
&\quad-\frac{1}{e^{8\pi M\omega}-1}\Bigg(\llbracket I^{++}_{{\rm in},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)\rrbracket\llbracket I^{++*}_{{\rm in},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)\rrbracket\\
&\quad+\frac{2e^{8\pi Mh}}{e^{8\pi Mh}+1}\llbracket I^{++}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)\rrbracket\llbracket I^{++*}_{{\rm up},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)\rrbracket\\
&\quad+\frac{2}{e^{8\pi Mh}+1}\llbracket I^{-+}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)\rrbracket\llbracket I^{-+*}_{{\rm up},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)\rrbracket\Bigg)\Bigg]\Bigg]
\end{aligned}
$$

Our analytic result for dissipative order $\mathcal{O}(\alpha_{\rm EM})$ correction for photon hawking radiation spectra

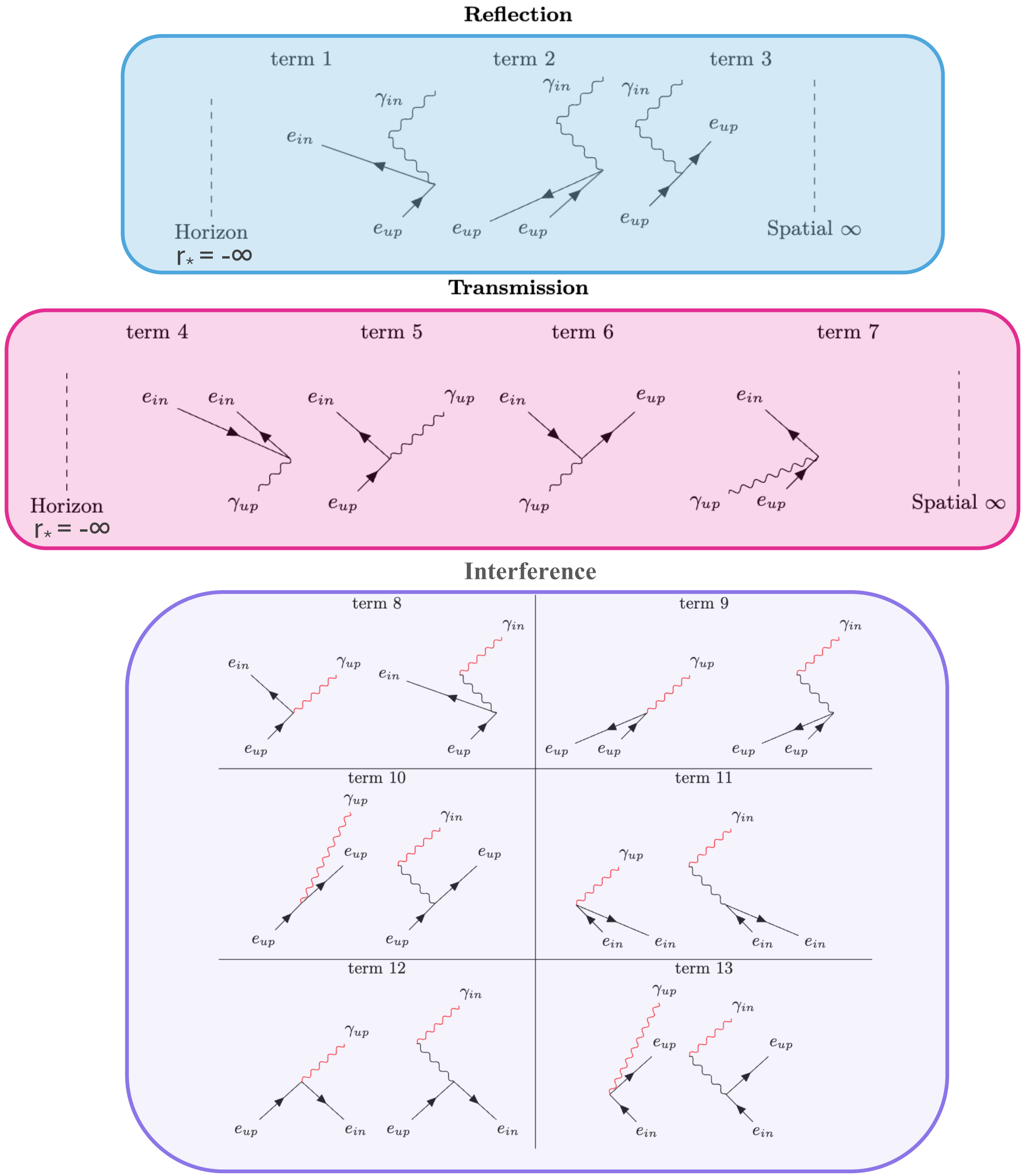
Dissipative: terms where the number of particles is changing

Conservative contributions still in progress

Dissipative Spectrum Calculation

Our work is a perturbative QED calculation canonically quantized on Schwarzchild background

13 terms that can be grouped into reflected ingoing radiation terms, transmitted upgoing radiation terms, and interference terms

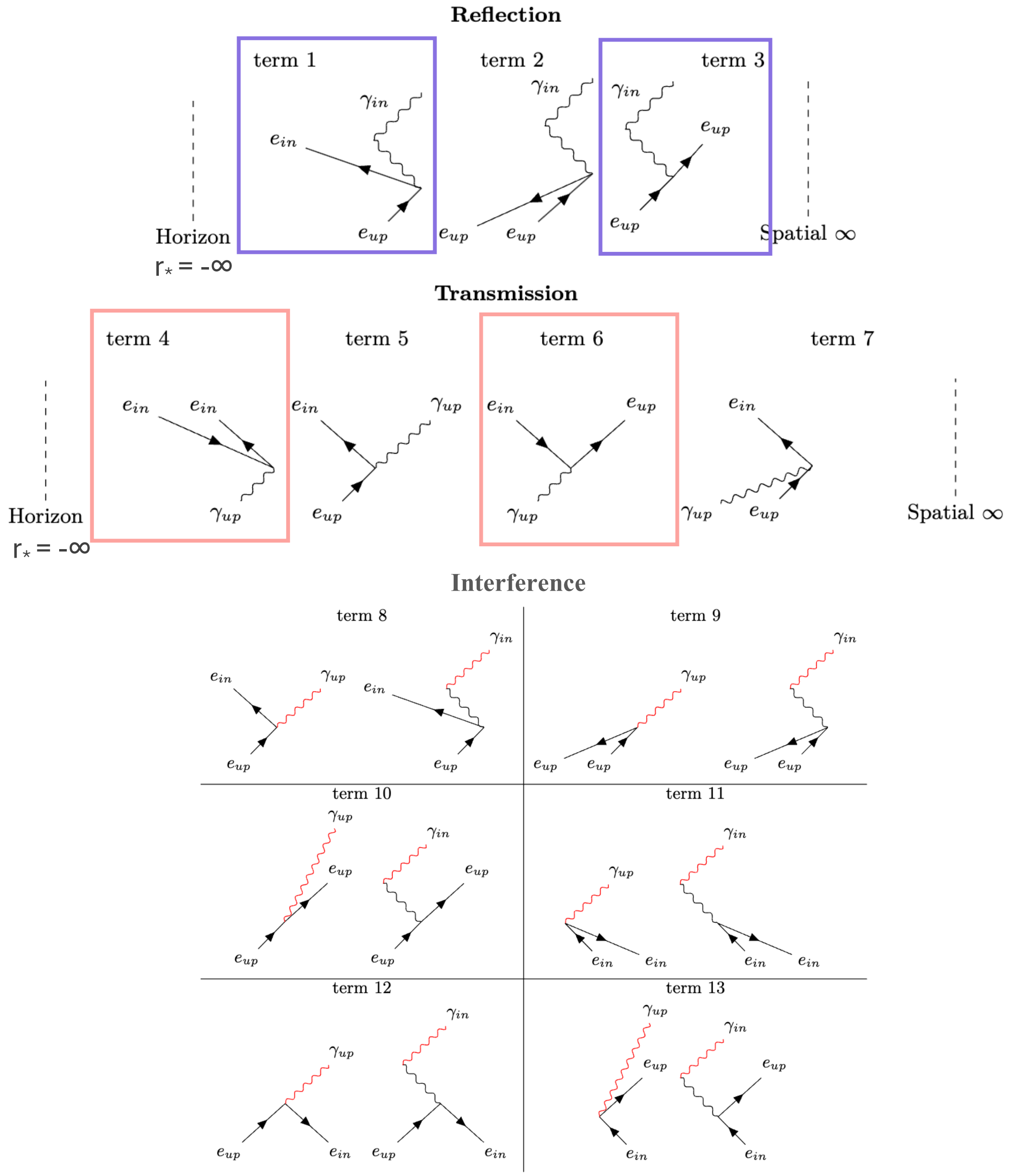


Dissipative Spectrum Calculation

Our work is a perturbative QED calculation canonically quantized on Schwarzchild background

13 terms that can be grouped into reflected ingoing radiation terms, transmitted upgoing radiation terms, and interference terms

See interactions that refer to inner bremsstrahlung radiation and to pair production/annihilation



Numerical methods

Parameters: M = [2,4,8.5,17]*10^{16} g
$r_\star$ = [-70,2000] M
ℓ = [1-5]
k= [-10,10]
ω and h = [0.01,20] T_H

Wave Functions

Use unperturbed Schwarzschild background mode functions

(vs. plane waves in Minkowski)

$$\Psi_{\mathrm{in},\ell,\omega}(r_\star) \to \begin{cases} T_{1,\ell,\omega} e^{-i\omega r_\star} & r_\star \to -\infty \\ e^{-i\omega r_\star} + R_{1,\ell,\omega} e^{i\omega r_\star} & r_\star \to \infty \end{cases}$$

$$\begin{pmatrix} F_{\mathrm{up}} \\ G_{\mathrm{up}} \end{pmatrix} \to \begin{cases} \begin{pmatrix} \sqrt{h} \\ -i\sqrt{h} \end{pmatrix} e^{ihr_\star} - R^*_{\frac{1}{2},k,h} e^{2i \arg T_{1/2,k,h}} \begin{pmatrix} \sqrt{h} \\ i\sqrt{h} \end{pmatrix} e^{-ihr_\star} & r_\star \to -\infty \\ T_{\frac{1}{2},k,h} v^{-1/2} \begin{pmatrix} \sqrt{h+\mu} \\ -i\sqrt{h-\mu} \end{pmatrix} e^{i\zeta \ln(r_\star/2M)} e^{i\sqrt{h^2-\mu^2}\, r_\star} & r_\star \to +\infty \end{cases}$$

Interaction Integrals

Describe “vertices” between photons and fermion interactions

Forms are 3-mode integrals

Heaviest computational load

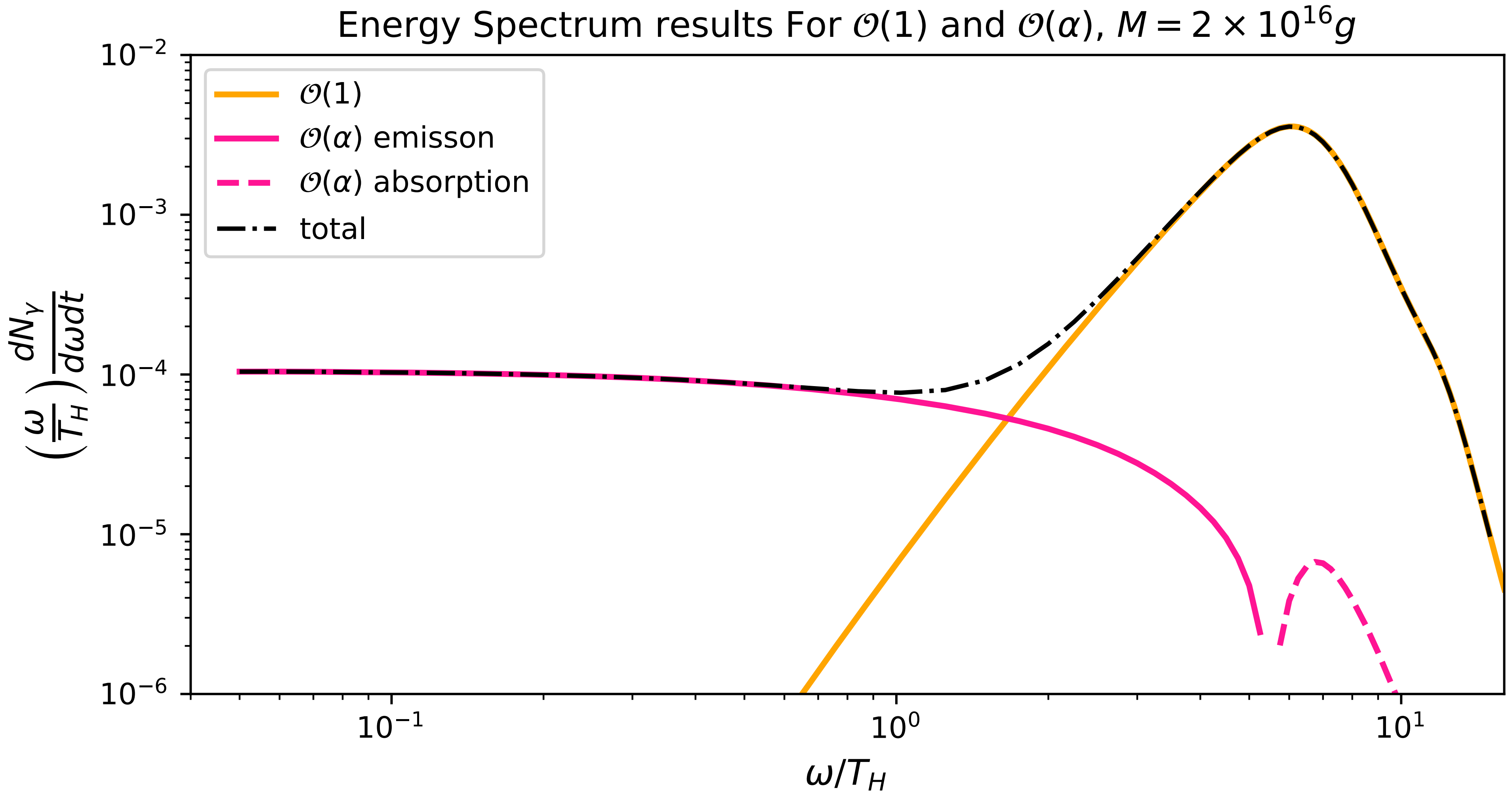
$$I^{++}_{Xkm,X'k'm',X_\gamma \ell m_\gamma (e)}(h,h',\omega) =$$

$$\frac{-i}{\sqrt{4hh'}} \Delta^{kk'l}_{mm'm_\gamma} \int_{-\infty}^{\infty} \Bigg[(F^*_{Xkh} F^*_{X'-k'h'} - G^*_{Xkh} G^*_{X'-k'h'}) \Psi_{X_\gamma \ell\omega} \sqrt{\ell(\ell+1)} \frac{1-2M/r}{r^2\sqrt{2\omega^3}}$$
$$+ (F^*_{Xkh} F^*_{X'-k'h'} + G^*_{Xkh} G^*_{X'-k'h'}) \frac{k-k'}{\sqrt{\ell(\ell+1)}} \frac{1}{\omega} \Psi'_{X_\gamma \ell\omega} \frac{\sqrt{1-2M/r}}{r\sqrt{2\omega}} \Bigg] dr_\star$$

Spectrum Corrections

Calculate 13 terms independently for each photon mode and split by parity, then sum for total result

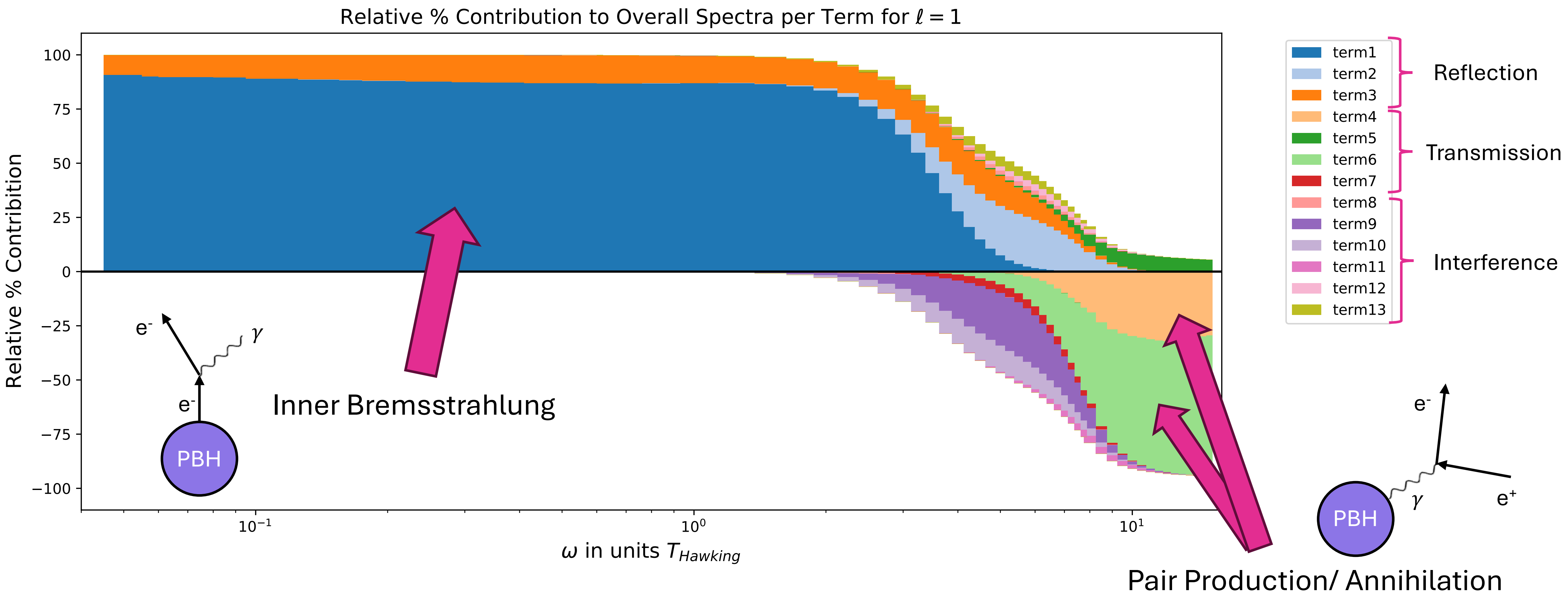
Corrected Spectrum Results



We see the order alpha corrections dominate at low energies

There are regions with extra emission and others with extra absorption

Investigation of Term Contributions



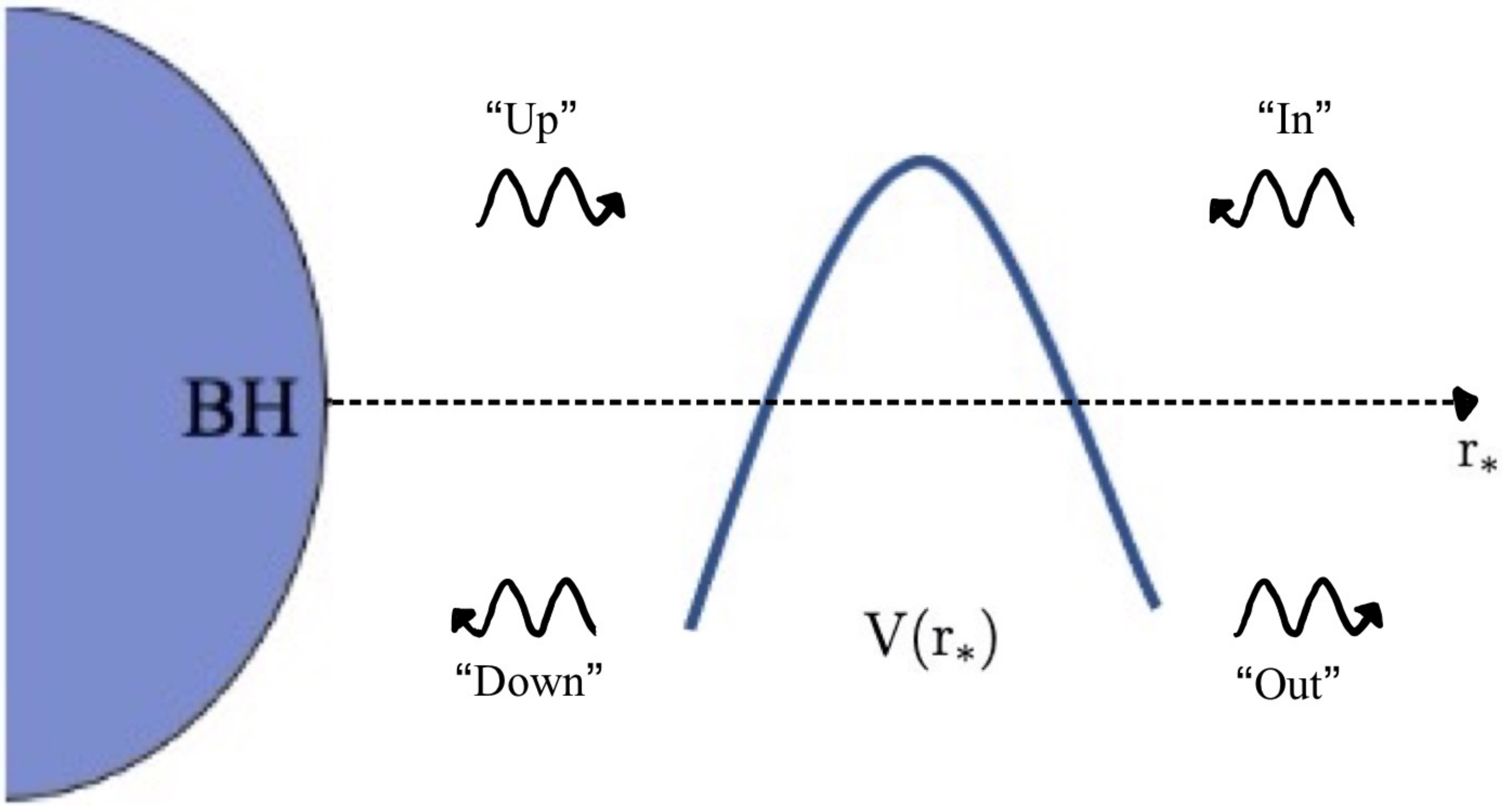
Takeaways & Next Steps

More Questions? Find me later or email at koivu.1@osu.edu

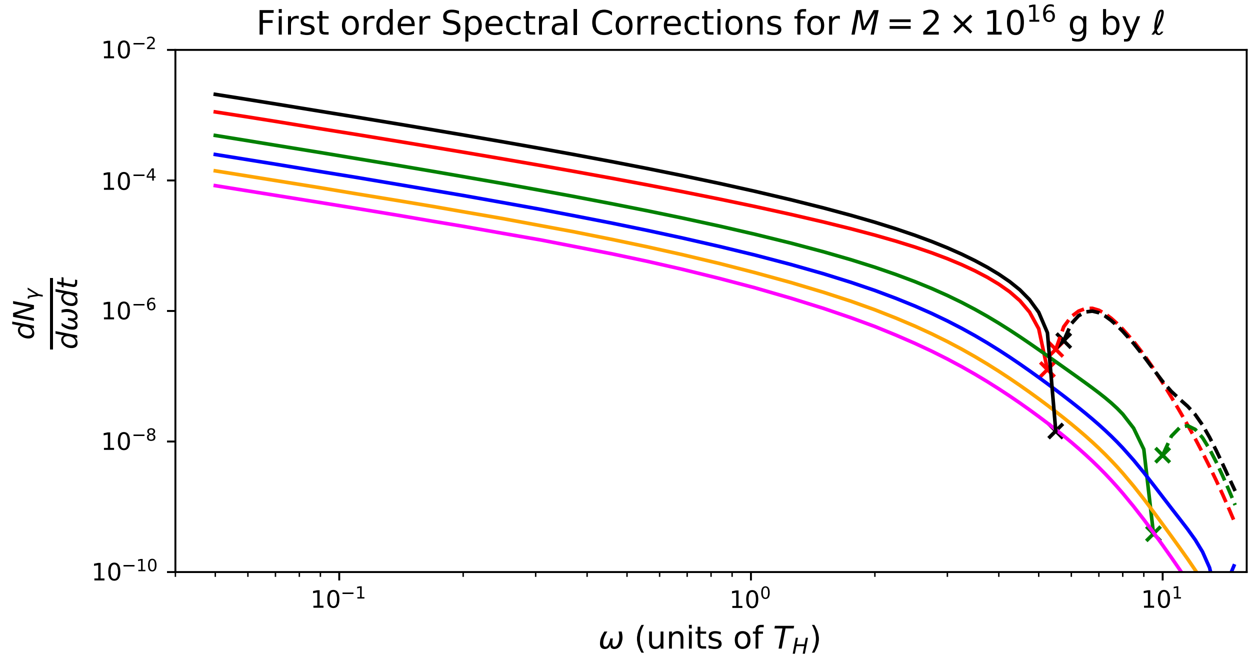
- We have successfully computed a dissipative order alpha hawking radiation spectra correction!
- Order alpha calculation is dominant at low energies
- Inner Bremsstrahlung radiation is present

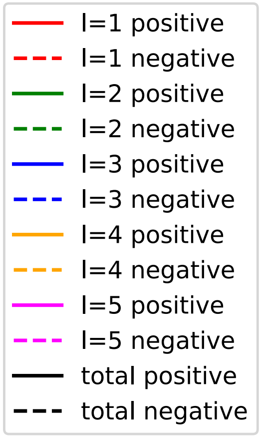
- Next steps:
  - Conservative piece (requires renormalization)
  - $e^{\pm}$ spectra
  - Other special topics related to this idea (impacts of inner bremsstrahlung, resonances, stochastic charge, formalism applications to echoes from astrophysical black holes, ...)

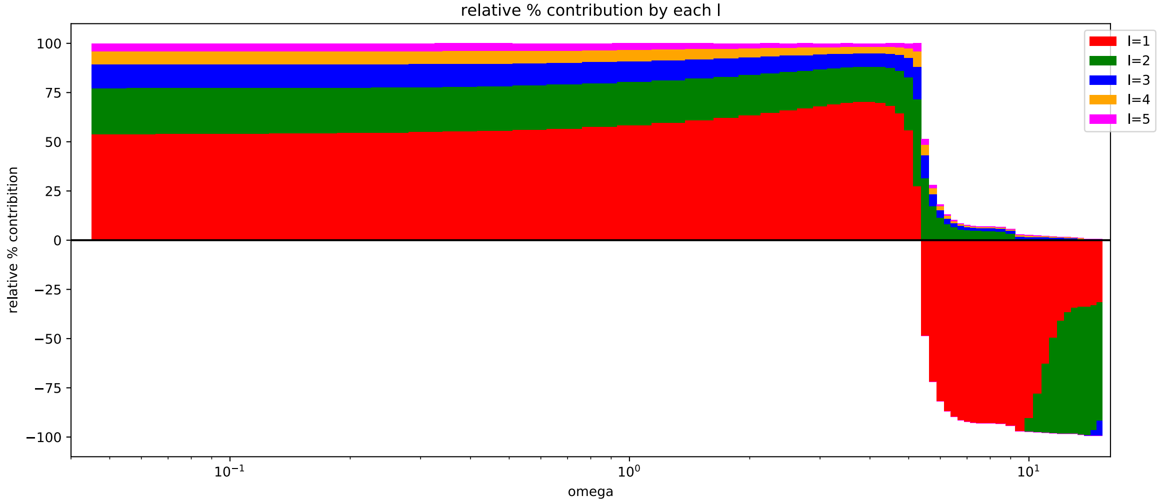
Basis for Calculations



Convergence of l values







Different Masses