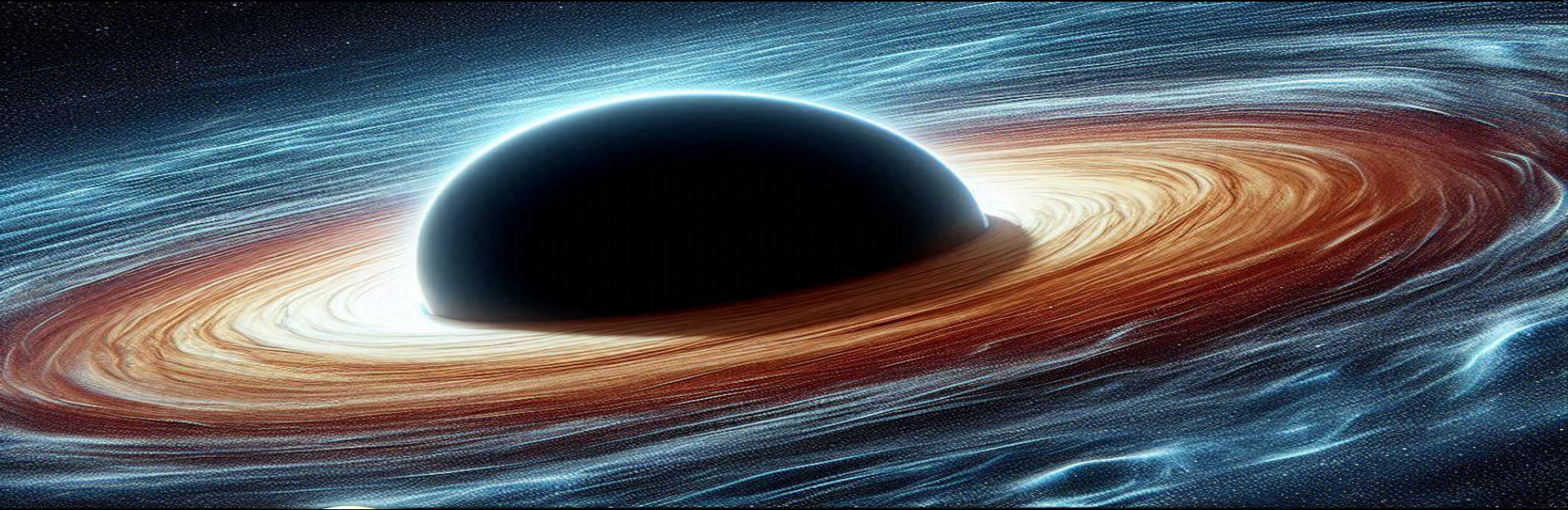


Antonio Junior Iovino



Timekeepers of the Universe:

The recent gravitational wave observation by PTA and PBH



SAPIENZA
UNIVERSITÀ DI ROMA

NEHOP
NEW HORIZONS IN
PRIMORDIAL BLACK HOLE PHYSICS



**UNIVERSITÉ
DE GENÈVE**

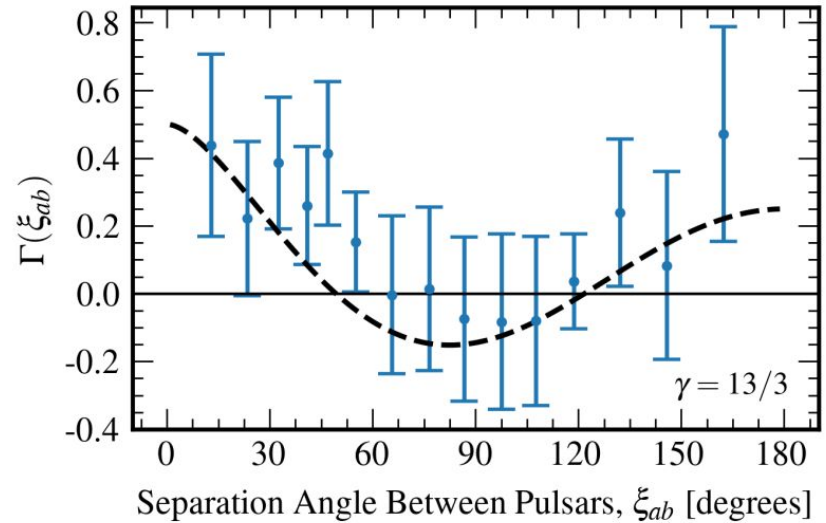
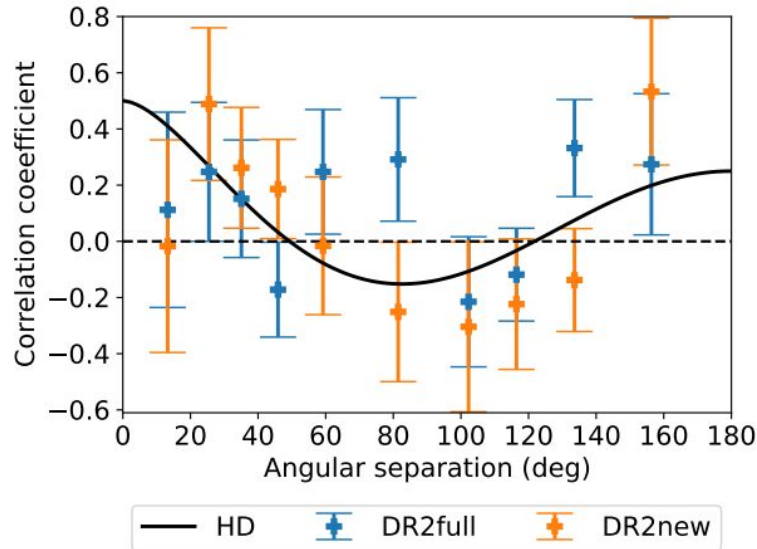
PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

EPTA – arXiv:2306.16214

NANOGrav – arXiv:2306.16213

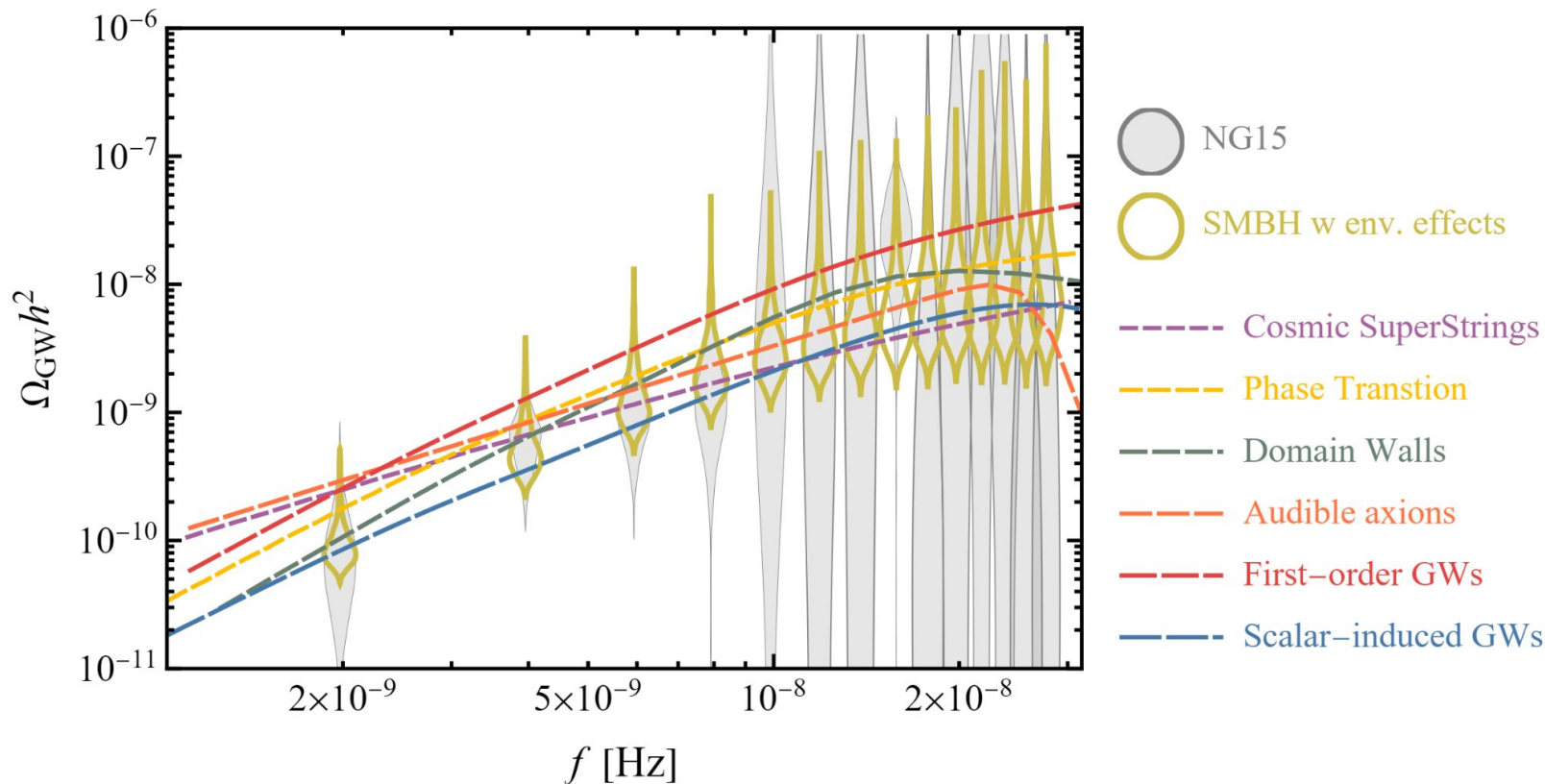
arXiv:2306.16219



What is the PTA source?

arXiv:2308.08546 J. Ellis [A.J.L. et al \(PRD\)](#)

All the PTA possible sources: Astro vs Cosmo



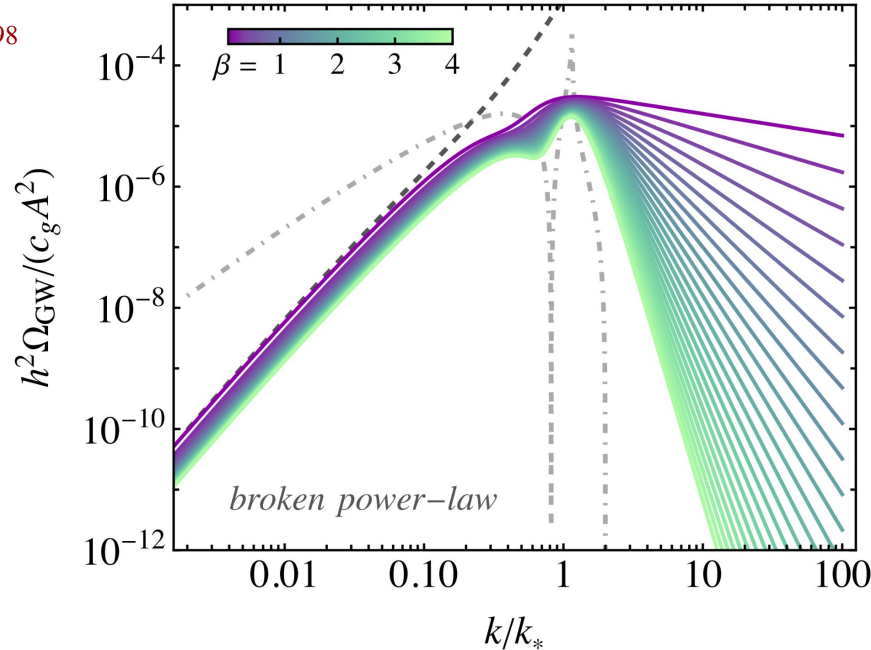
PBH and SGWB

SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

$$h^2 \Omega_{\text{GW}}(k) = \frac{h^2 \Omega_r}{24} \left(\frac{g_*}{g_*^0} \right) \left(\frac{g_{*s}}{g_{*s}^0} \right)^{-\frac{4}{3}} \mathcal{P}_h(k)$$

$$\mathcal{P}_h(k) \propto \mathcal{P}_\zeta^2(k)$$

REVIEW G. Domenech—arXiv:2109.01398



PBH and SGWB

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Log-likelihood analysis

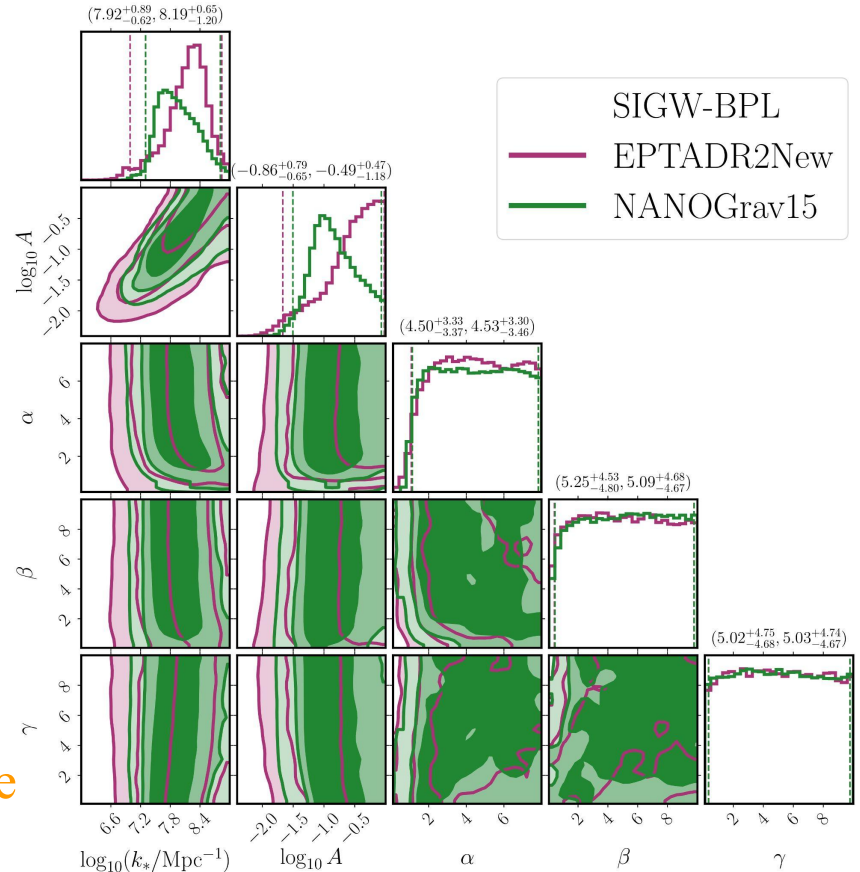
Fitting the posterior distributions

$$\mathcal{P}_\zeta^{\text{BPL}}(k) = A \frac{(\alpha + \beta)^\gamma}{\left(\beta (k/k_*)^{-\alpha/\gamma} + \alpha (k/k_*)^{\beta/\gamma}\right)^\gamma}$$

$$\mathcal{P}_\zeta^{\text{LN}}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{1}{2\Delta^2} \ln^2(k/k_*)\right)$$

arXiv:2306.17149 (PRL)

G.Franciolini, [A.J.I.](#), V. Vaskonen, H. Veermae



PBH and SGWB

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Fitting the posterior distributions

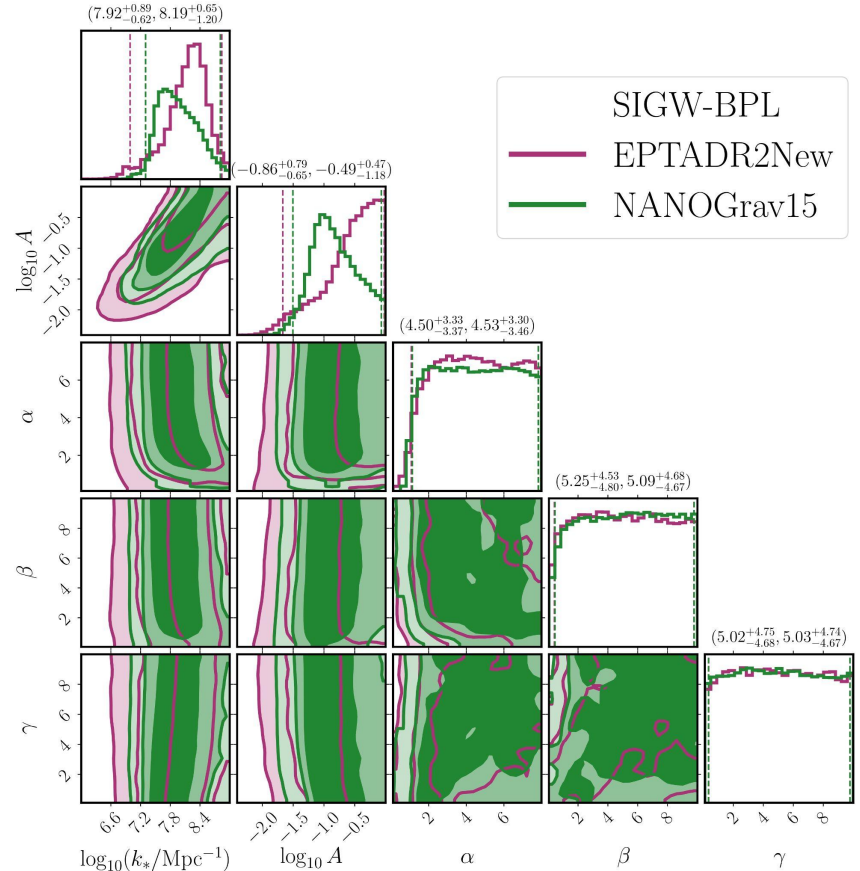
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Results:

The causality tail is not good:

$$\Omega_{\text{GW}}(k \ll k_*) \propto k^3 (1 + \tilde{A} \ln^2(k/\tilde{k}))$$



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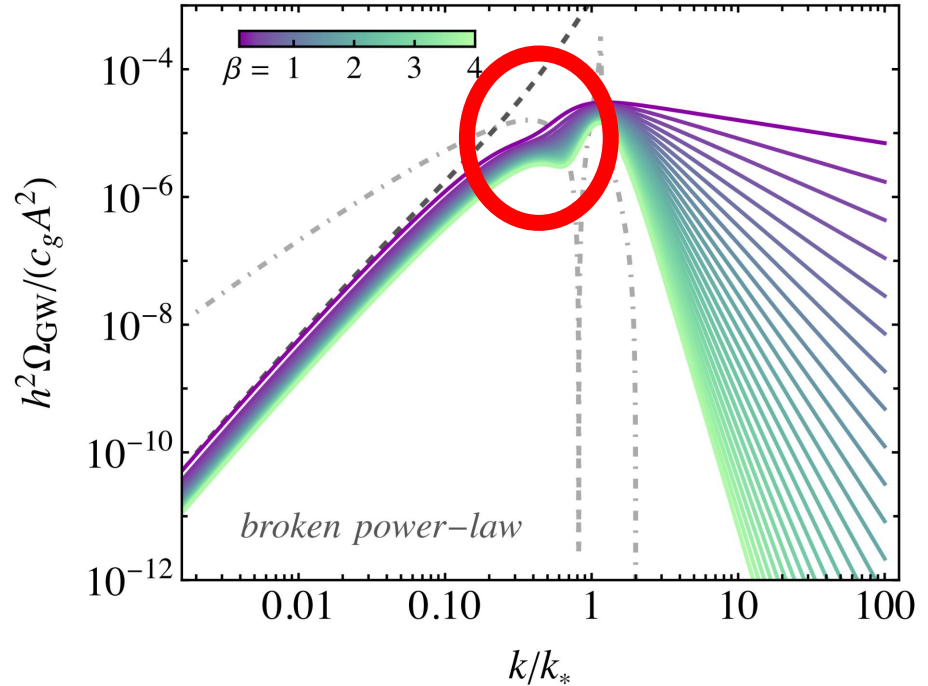
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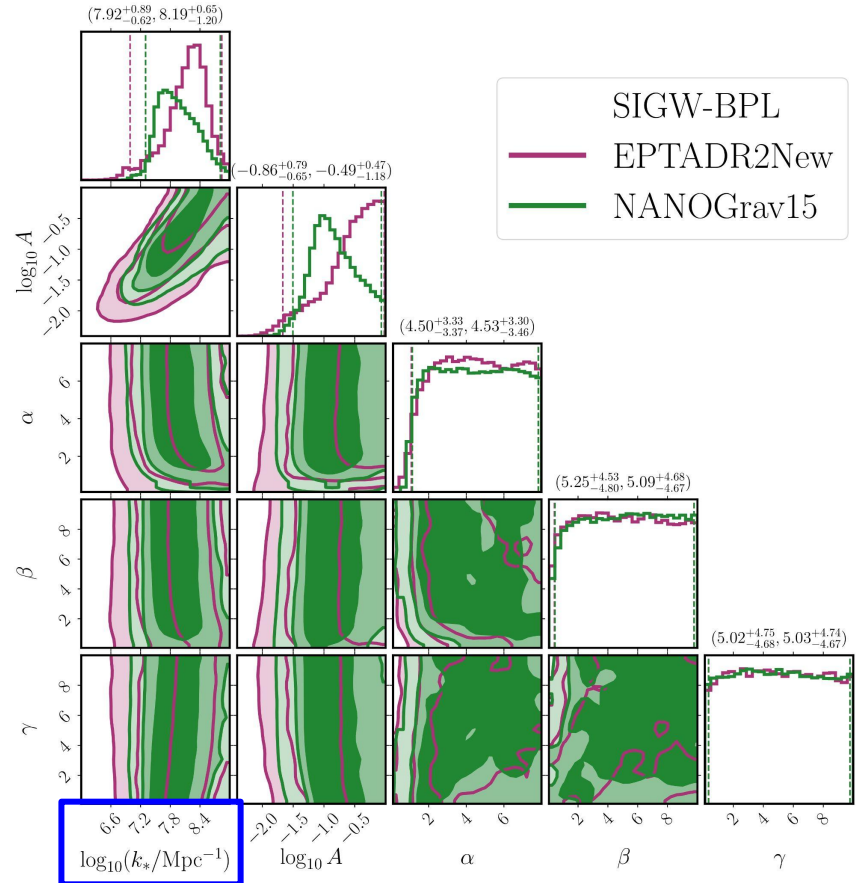
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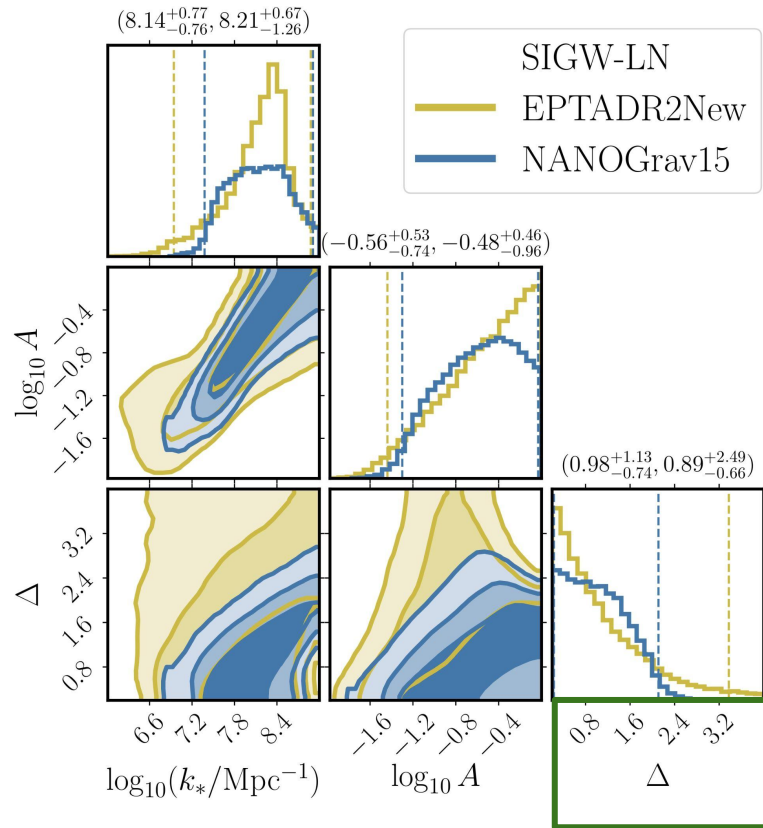
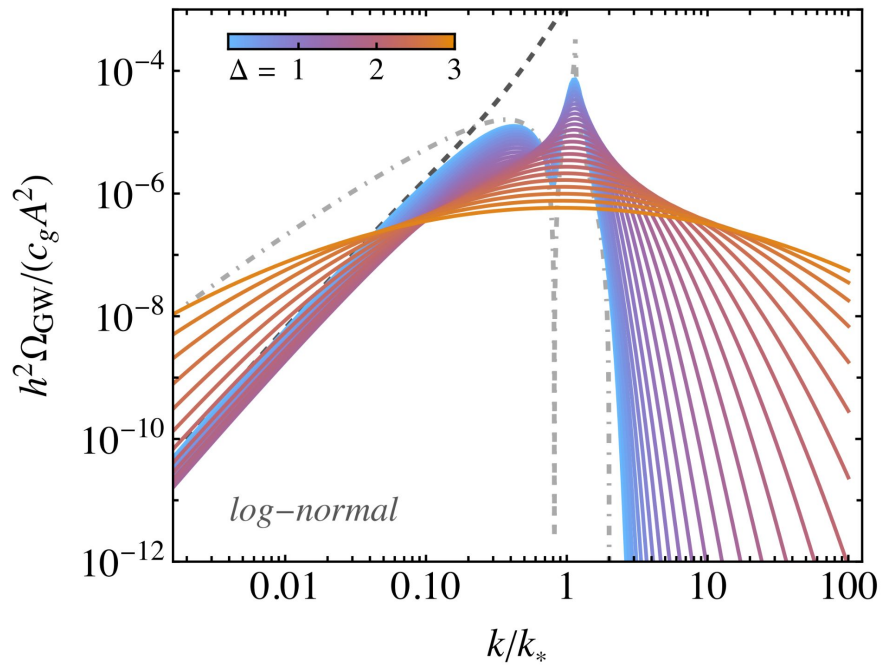
$$\mathcal{P}_{\zeta}^{\text{LN}}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{1}{2\Delta^2} \ln^2(k/k_*)\right)$$

Results:

Position of the peak at higher frequencies.

Broad spectrum does not fit so well.





$$\mathcal{P}_{\zeta}^{\text{LN}}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{1}{2\Delta^2} \ln^2(k/k_*)\right)$$

Improvement respect to NANOGrav analysis.

NANOGrav collaboration
arXiv:2306.16219

Power spectrum \leftrightarrow *Abundance* \leftrightarrow *GWs*

- Non-Gaussianities in the abundance.
- Dependency of the PBH formation parameters on the PS shape.
- QCD impact on threshold.

Abundance of PBHs: The role of NGs.

Threshold statistics on the compaction function

G.Ferrante, G. Franciolini, A.J.I. A.Urbano.–arXiv:2211.01728
A.Gow *et al*– arXiv:2211:08348

NON-LINEARITIES (NL)

$$\delta(r, t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(r)} \left[\zeta''(r) + \frac{2}{r}\zeta'(r) + \frac{1}{2}\zeta'(r)^2\right]$$

T. Harada, C. M. Yoo, T. Nakama
and Y. Koga,– arXiv:1503.03934

PRIMORDIAL NG IN $\zeta=F(\zeta_G)$

$$\zeta = \log [X(r_{\text{dec}}, \zeta_G)]$$

curvaton case

$$\zeta = -\frac{2}{\beta} \log \left(1 - \frac{\beta}{2}\zeta_G\right)$$

Inflection-point (IP or USR) case

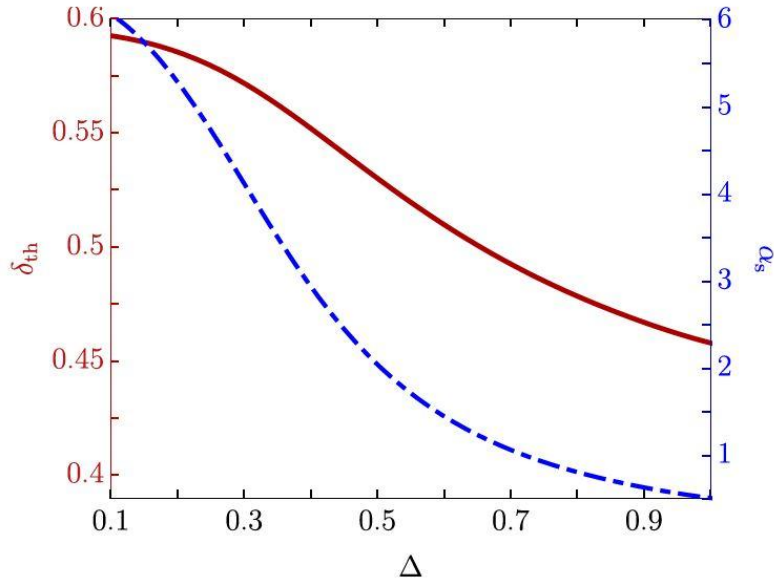
$$\zeta = \zeta_G + \frac{3}{5} f_{\text{NL}} \zeta_G^2$$

Quadratic approx.

Abundance of PBHs: *See Musco's Talk*

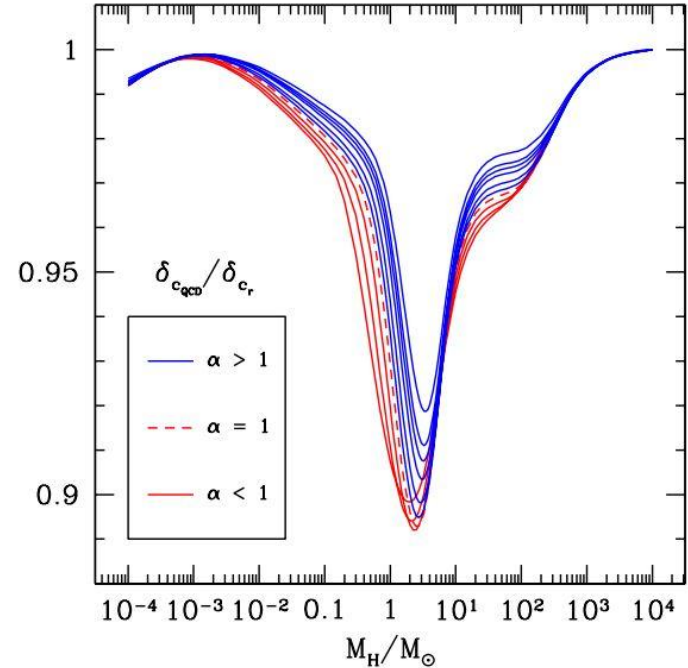
Shape dependencies

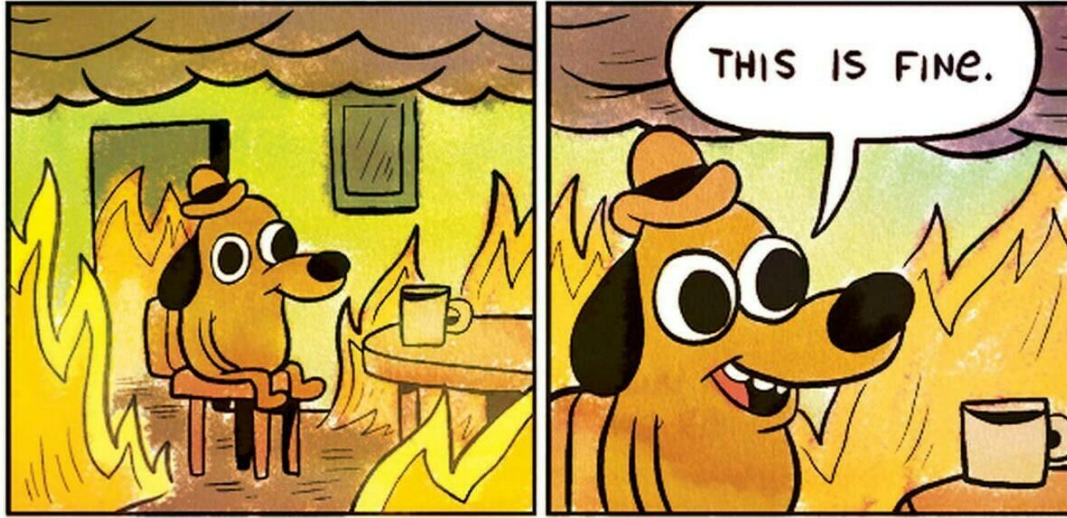
I. Musco, V. De Luca, G. Franciolini, A. Riotto. – arXiv:2011.03014



QCD phase transitions

I. Musco, K. Jedamzik, S. Young. – arXiv:2303.07980





A potential issue

When we compute the threshold using the average value for the compaction, non-linear effects not included in the linear transfer function lead to different super-horizon threshold conditions.

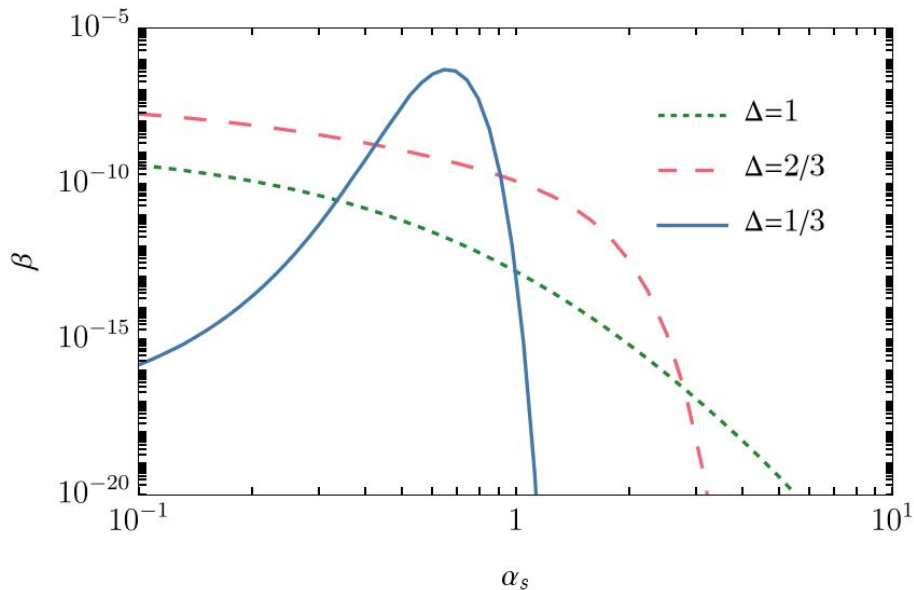
V. De Luca, A. Kehagias, A. Riotto.– [arXiv:2307.13633](https://arxiv.org/abs/2307.13633)

Going beyond the average profile

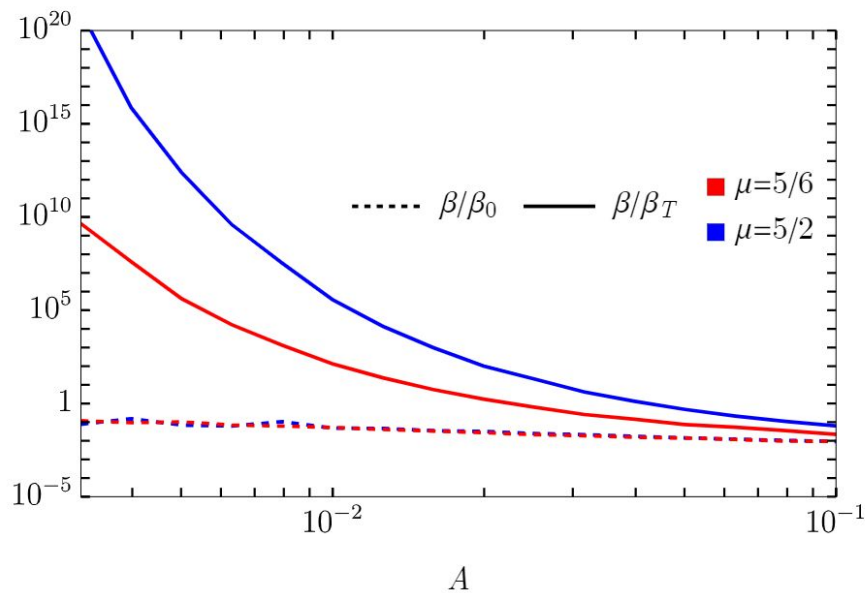
arXiv:2402.11033 A.Iannicari, [A.J.L.](#), A. Kehagias, D. Perrone, A. Riotto (PRD)

In realistic cases, threshold is determined by the broadest possible compaction function.

$$\mathcal{P}_g(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left[-\ln^2(k/k_\star)/2\Delta^2\right]$$



$$\zeta(\mathbf{x}) = -\mu_\star \ln\left(1 - \frac{\zeta_g(\mathbf{x})}{\mu_\star}\right)$$



We need to reduce the amplitude of a factor $O(2-3)$, so PTA-PBH tension is exacerbated 15

Conclusions

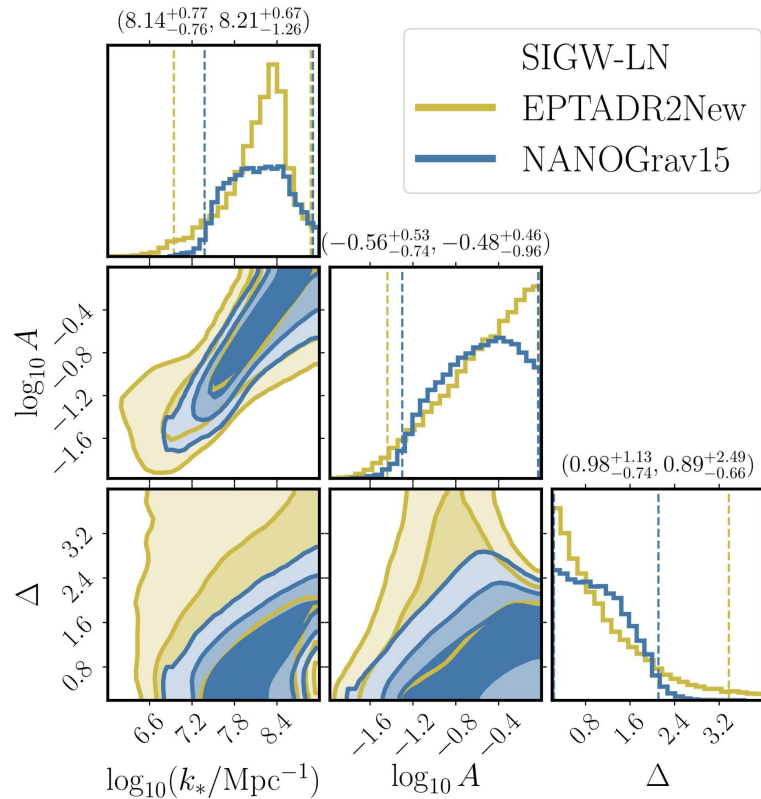
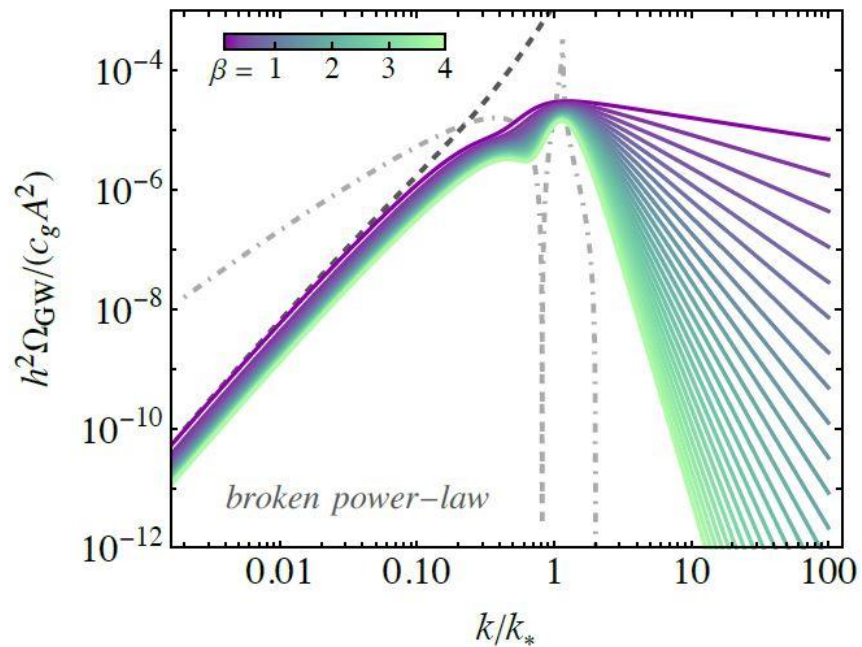
Negative NGs to alleviate the tension between PTA and PBH overproduction.

There is still a lot of uncertainties in the way we compute the PBH abundance and on the possibility that they maybe the PTA explanation .*

**A.J.L., G.Perna, A. Riotto and H. Veermae to appear*

Thank you

Back-up slides



Threshold statistics on the Compaction: Mathematical formulation

By integrating δ over the radial coordinate r we get the compaction function \mathcal{C}

$$\mathcal{C}(r) = -2\Phi r \zeta'(r) \left[1 + \frac{r}{2} \zeta'(r) \right] = \mathcal{C}_1(r) - \frac{1}{4\Phi} \mathcal{C}_1(r)^2, \quad \mathcal{C}_1(r) := -2\Phi r \zeta'(r)$$

In the presence of NG \mathcal{C}_l takes the form

$$\mathcal{C}_l(r) = -2\Phi r \zeta'_G(r) \frac{dF}{d\zeta_G} = \mathcal{C}_G(r) \frac{dF}{d\zeta_G}, \quad \text{with } \mathcal{C}_G(r) := -2\Phi r \zeta'_G(r)$$

From the two-dimensional joint PDF of ζ_G and \mathcal{C}_G , called P_G

NG PBH mass fraction-distribution adopting threshold statistics on the compaction function

$$\beta_{\text{NG}} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C} - \mathcal{C}_{\text{th}})^\gamma P_G(\mathcal{C}_G, \zeta_G) d\mathcal{C}_G d\zeta_G,$$

$$\mathcal{D} = \{ \mathcal{C}_G, \zeta_G \in \mathbb{R} : \mathcal{C}(\mathcal{C}_G, \zeta_G) > \mathcal{C}_{\text{th}} \wedge \mathcal{C}_1(\mathcal{C}_G, \zeta_G) < 2\Phi \},$$

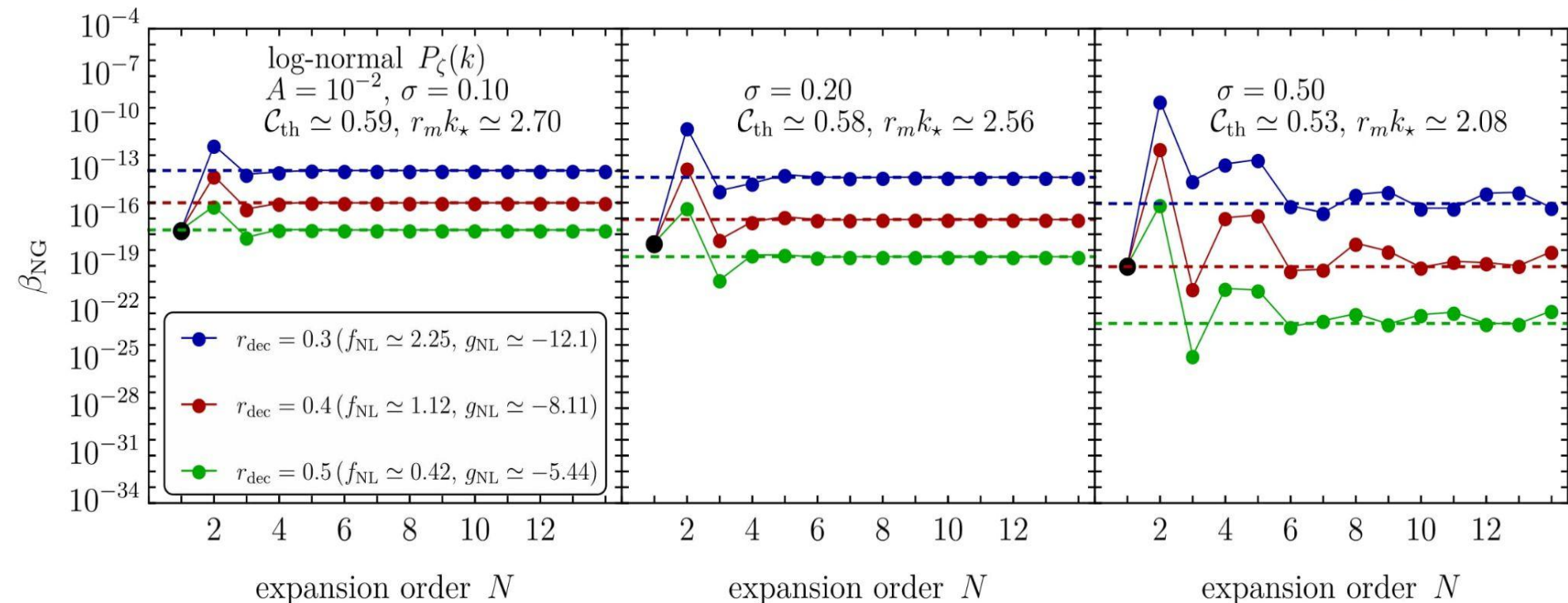
$$f_{\text{PBH}}(M_{\text{PBH}}) = \frac{1}{\Omega_{\text{DM}}} \int_{M_{\text{H}}^{\text{min}}(M_{\text{PBH}})} d \log M_{\text{H}} \left(\frac{M_{\text{eq}}}{M_{\text{H}}} \right)^{1/2} \left[1 - \frac{\mathcal{C}_{\text{th}}}{\Phi} - \frac{1}{\Phi} \left(\frac{M_{\text{PBH}}}{\mathcal{K} M_{\text{H}}} \right)^{1/\gamma} \right]^{-1/2} \frac{\mathcal{K}}{\gamma} \left(\frac{M_{\text{PBH}}}{\mathcal{K} M_{\text{H}}} \right)^{\frac{1+\gamma}{\gamma}} \\ \times \int d\zeta_G P_G(\mathcal{C}_G(M_{\text{PBH}}, \zeta_G), \zeta_G | M_{\text{H}}) \left(\frac{dF}{d\zeta_G} \right)^{-1}.$$

Quadratic gives wrong result

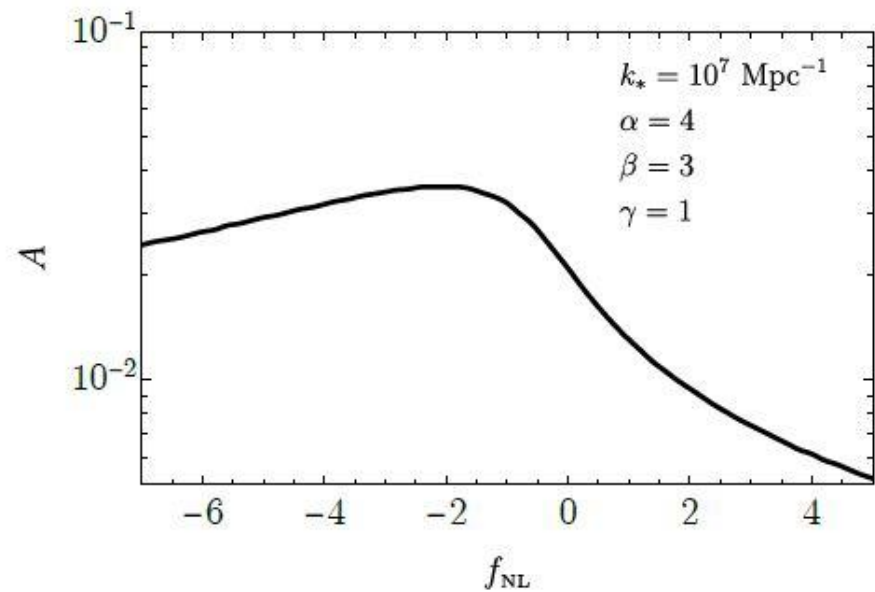
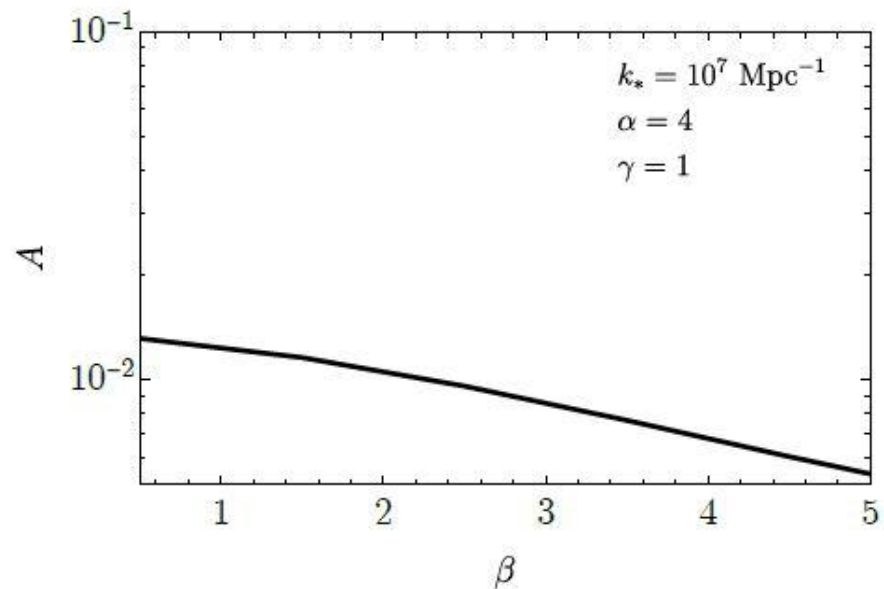
Failure of the perturbative approach (Narrow)

--- $\zeta = \log [X(r_{\text{dec}}, \zeta_G)]$

● $\zeta_N = \sum_{n=1}^N c_n(r_{\text{dec}}) \zeta_G^n$



NG generic features



NG generic features

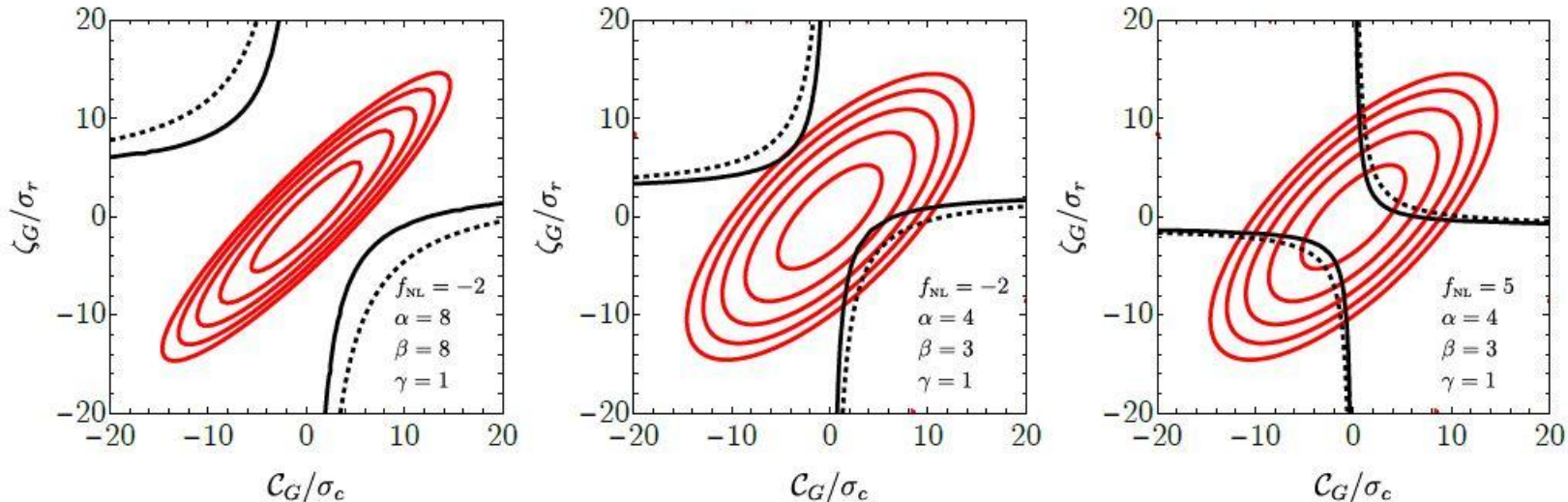


FIG. S4. Two dimensional PDF as a function of (C_G, ζ_G) compared to the over-threshold condition $C > C_{th}$. In all panels, we considered the BPL power spectrum with an amplitude $A = 0.05$. The red lines indicates the contour lines corresponding to $\log_{10}(P_G) = -45, -35, -25, -15, -5$. The collapse of type-I PBHs take place between the black solid and dashed lines (see more details in Ref. [195]). *Left panel:* Example of a very narrow power spectrum with $\alpha = \beta = 8$. The abundance is suppressed in the presence of negative f_{NL} by the strong correlation between C_G and ζ_G obtained for narrow spectra. *Center panel:* Example of negative non-Gaussianity and representative BPL spectrum. The PBH formation is sourced by regions of small ζ_G and positive C_G or both negative C_G and ζ_G . *Right panel:* Example with positive f_{NL} , showing the region producing PBHs populates the correlated quadrants of the plot, at odds with that is found in the other panels.

PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

IPTA – arXiv:2309.00693

NANOGrav – arXiv:2306.16213
arXiv:2306.16219

