Classical and stochastic δN formalisms

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Outline

- Classical and stochastic δN approaches to the PDF of ζ
- Validity of standard approximations
- Classical vs stochastic comparison

☞ Details in 2406.02417 with G. Ballesteros, T. Konstandin, M. Pierre and J. Rey

Non-linear relation between ζ and $\delta\phi$, $\delta\pi = \delta\phi'$ (e.g. [Sugiyama, Komatsu, Futamase '13])

$$\boxed{\zeta = \delta N \equiv N\left[\{\bar{\phi}(N_i) + \delta\phi, \bar{\pi}(N_i) + \delta\pi\} \to \bar{\phi}(N_f)\right] - N\left[\{\bar{\phi}(N_i), \bar{\pi}(N_i)\} \to \bar{\phi}(N_f)\right] \ ,}$$

- Only valid for N_i when perturbations have frozen (necessarily after horizon crossing) ☞ *Separate universe approach*
- The functions $\bar{\phi}$, $\bar{\pi}$, N are unperturbed solutions of

$$\phi' = \pi$$

$$\pi' = -(3-\epsilon)[\phi + (\log V)_{,\phi}]$$

Slow roll parameters: $\epsilon = \pi^2/2$, $\eta = \epsilon - (\log\epsilon)'/2$

Two approaches to compute $P(\zeta) = P(\delta N)$

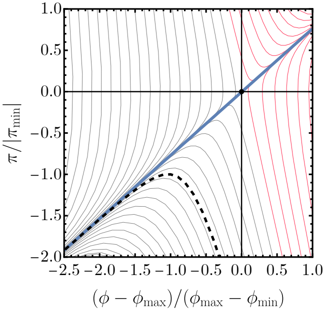
- Classical δN: simple change of variables: $P(\zeta) = P(\delta N) = P(\delta\phi)\left|\frac{d\delta\phi}{d\delta N}\right|$.
  For consistency, $\delta\pi = (\pi'/\pi)|_{N_i}\,\delta\phi$.
- Stochastic δN: solve stochastic inflation

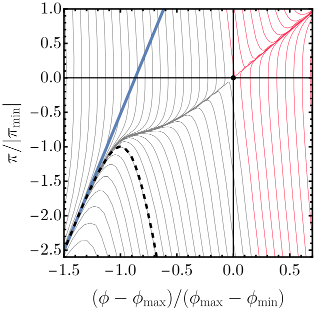
$$\phi' = \pi + \xi_\phi\,,$$
$$\pi' = -\left(3 - \pi'^2/2\right)\left[\pi + (\log V)_{,\phi}\right] + \xi_\pi\,,$$
$$\text{with } \langle \xi_{\phi,\pi}(N)\,\xi_{\phi,\pi}(\tilde{N})\rangle = \mathcal{P}_{\delta\phi,\delta\pi}[k \ll a(N)H(N)]\,\delta(N-\tilde{N})$$

and compute δN w.r.t. unperturbed solution with $\xi_{\phi,\pi} = 0$.
For consistency, $\xi_\pi = (\pi'/\pi)|_N\,\xi_\phi$.

2. Case of study

Two models of *single-field inflation* with an *USR phase* (local maximum & minimum), followed by *CR phase* ($\eta = \text{cst} < 0$). Details ☞ [2406.02417]





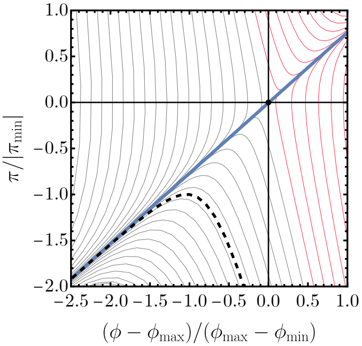
Blue line: $(\pi'/\pi)|_{N_i}$ (direction perturbations)

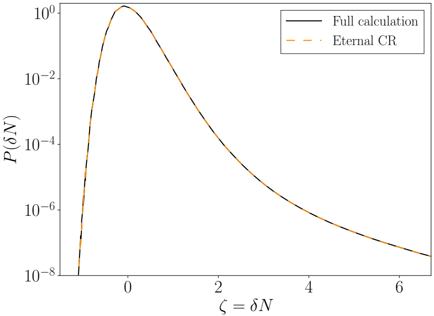
Dashed line: unperturbed solution

η parameter: slope of tangent to trajectories

Momentum perturbations such that ϕ sticks to a trajectory of constant η. Allows for analytical calculation of $P(\zeta)$ [Tomberg '23], [Pi, Sasaki '23].

Good approximation for models with **pure CR** attractor after local maximum





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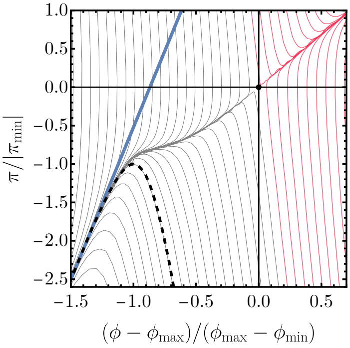
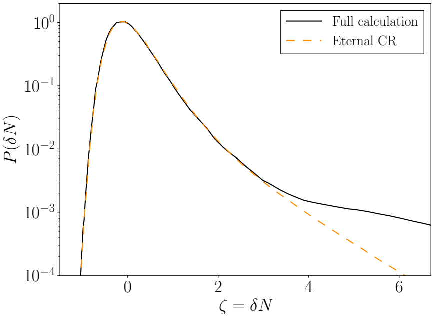
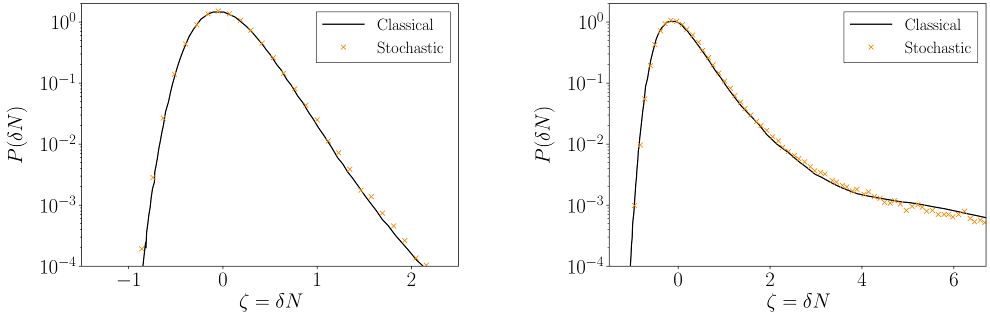




4. Classical vs stochastic

Equivalence of classical and stochastic δN: **exact** for **pure CR** phase [Tomberg '23], **effective** for **non-constant** slope attractor ☞ [2406.02417]

NB Non-CR attractor ($\eta \neq$ cst) $\implies$ non-constant exponential tail.



Conclusions

- **Classical and stochastic** δN predict the **same non-Gaussianity** for USR models
- **Classical** δN allows for **very cheap numerical** calculation of $P(\zeta)$
- Eternal CR approximation (allows for **analytical calculation**): only works for **certain models** (with $\eta = \text{cst}$ attractor)

See [2406.02417] for more details.