Sebastien Clesse
Université Libre de Bruxelles (ULB)
Mostly preliminary work in collaboration with:
Simon Biot, Ioanna Stamou, Vincent Juste, Virgile Dandoy,
Sonali Verma, Nicolas Esser, Marine Prunier, Guillaume Poth, others…

Can PBHs explain LIGO/Virgo observations ?

Towards accurate merger rates of PBHs with broad mass distributions

NEHOP Workshop - Edinburgh, 17-20 June 2024

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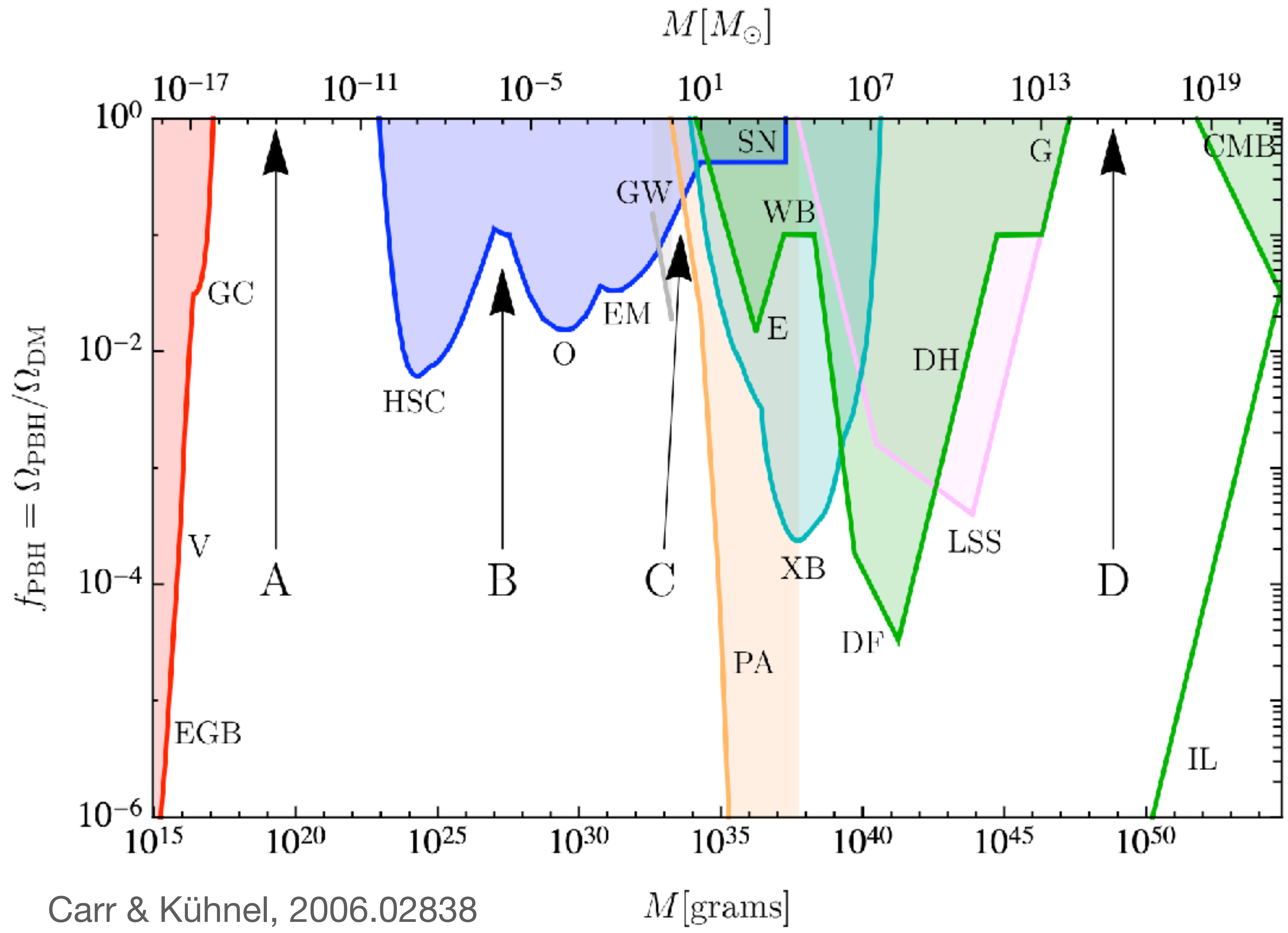
NEHOP Workshop - Edinburgh, 17-20 June 2024

1. Is Dark Matter made of PBHs?

Limits or clues: a question of point of view



Carr & Kühnel, 2006.02838

Hawking radiation
Microlensing
LIGO-Virgo
CMB
Dynamical effects
Accretion
Large scale structures

- ✓ Solar mass region excluded by several probes
- ✓ No limit on asteroid-masses
- ✓ If PBHs + WIMPs (or particle DM) => stronger limits, e.g. [Serpico+20] [Carr+20] [Byrnes+] [Boudaud+21]

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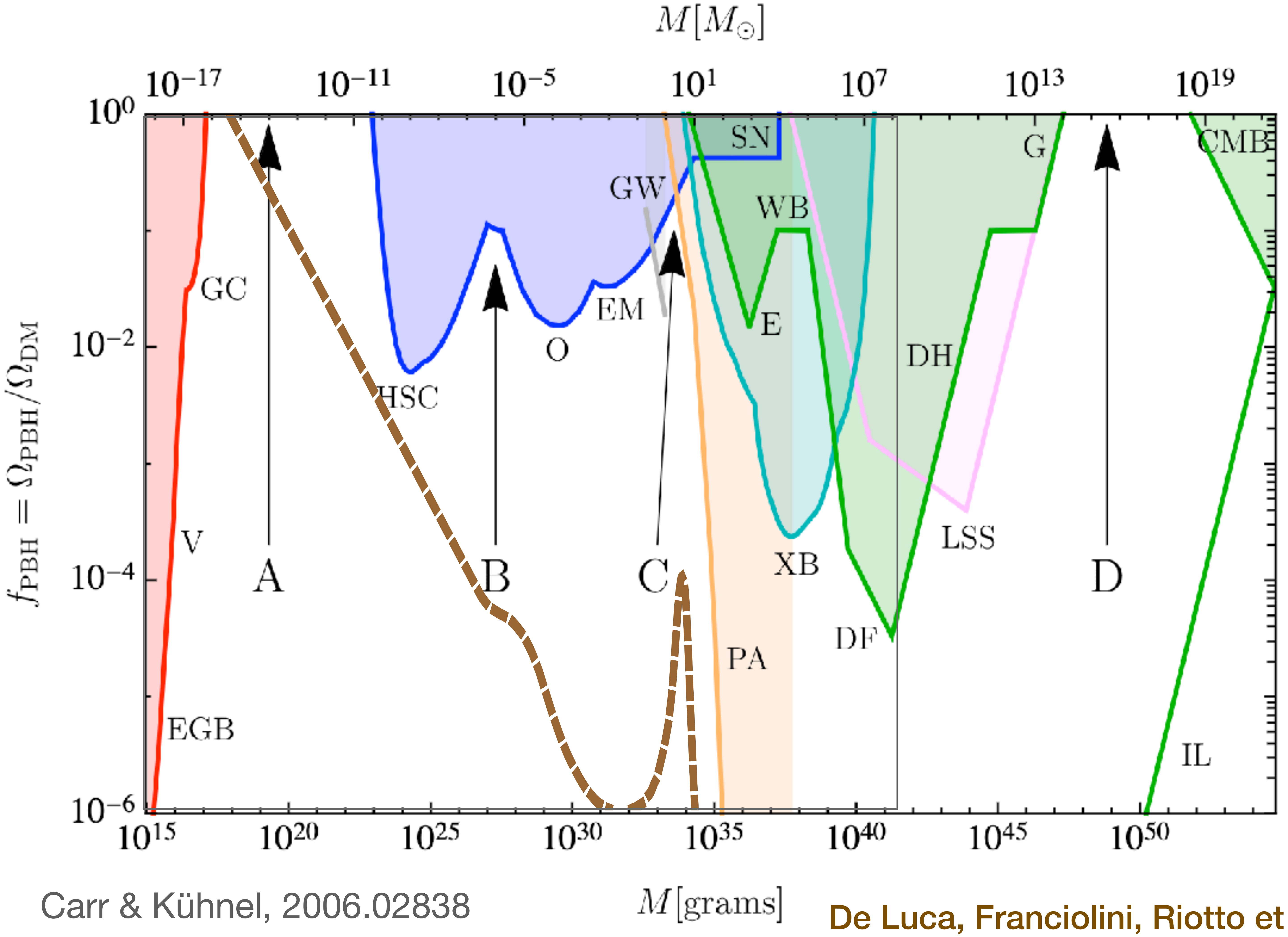

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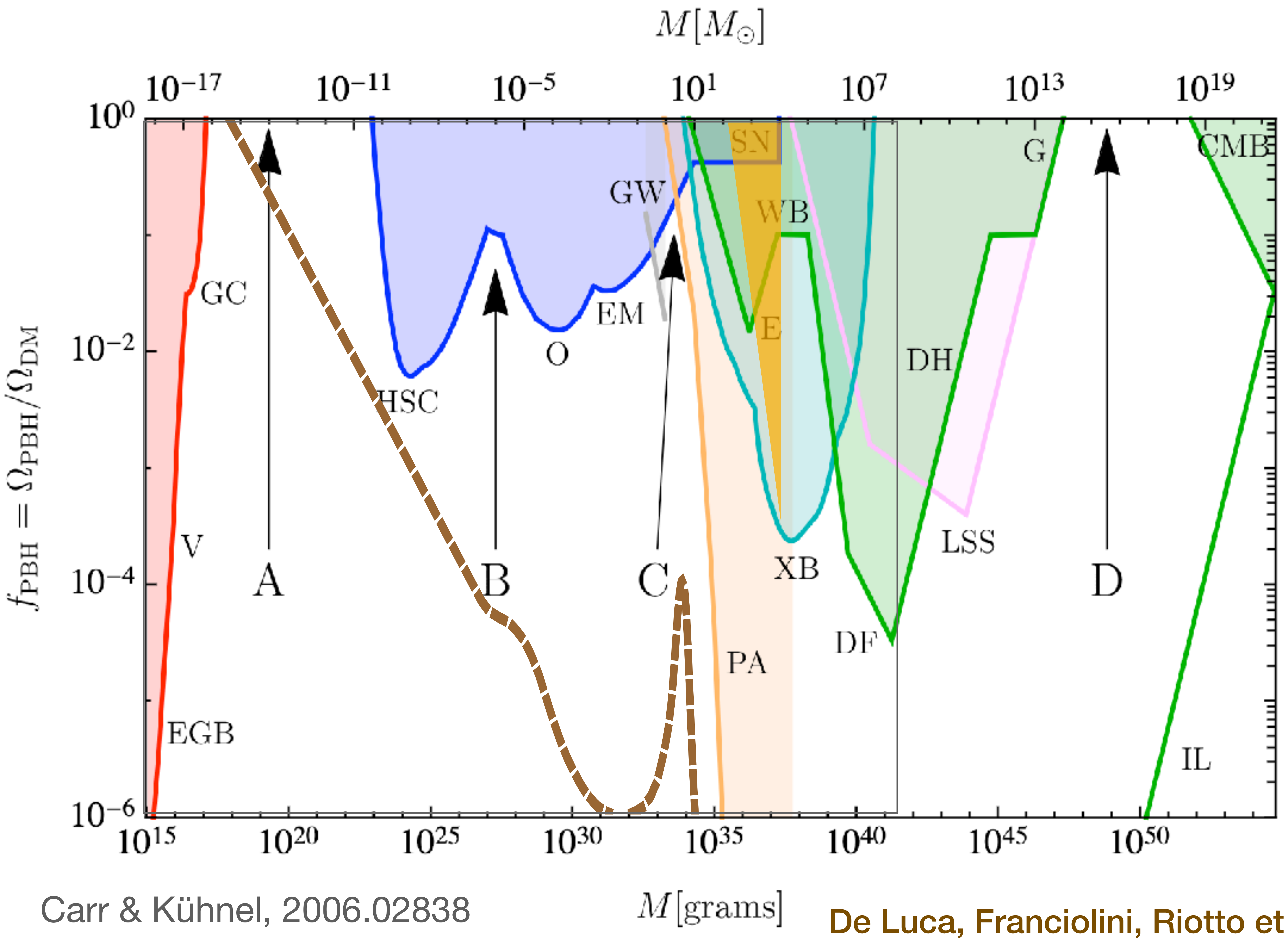

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◉ Lot of uncertainties: astrophysical assumption
Exemple: CMB limits relaxed in Parkes-Ricotti accretion model (G. Facchinetti, M. Lucca, S.C, 2212.07968), role of local ionization (Agius et al, 2403.18895)

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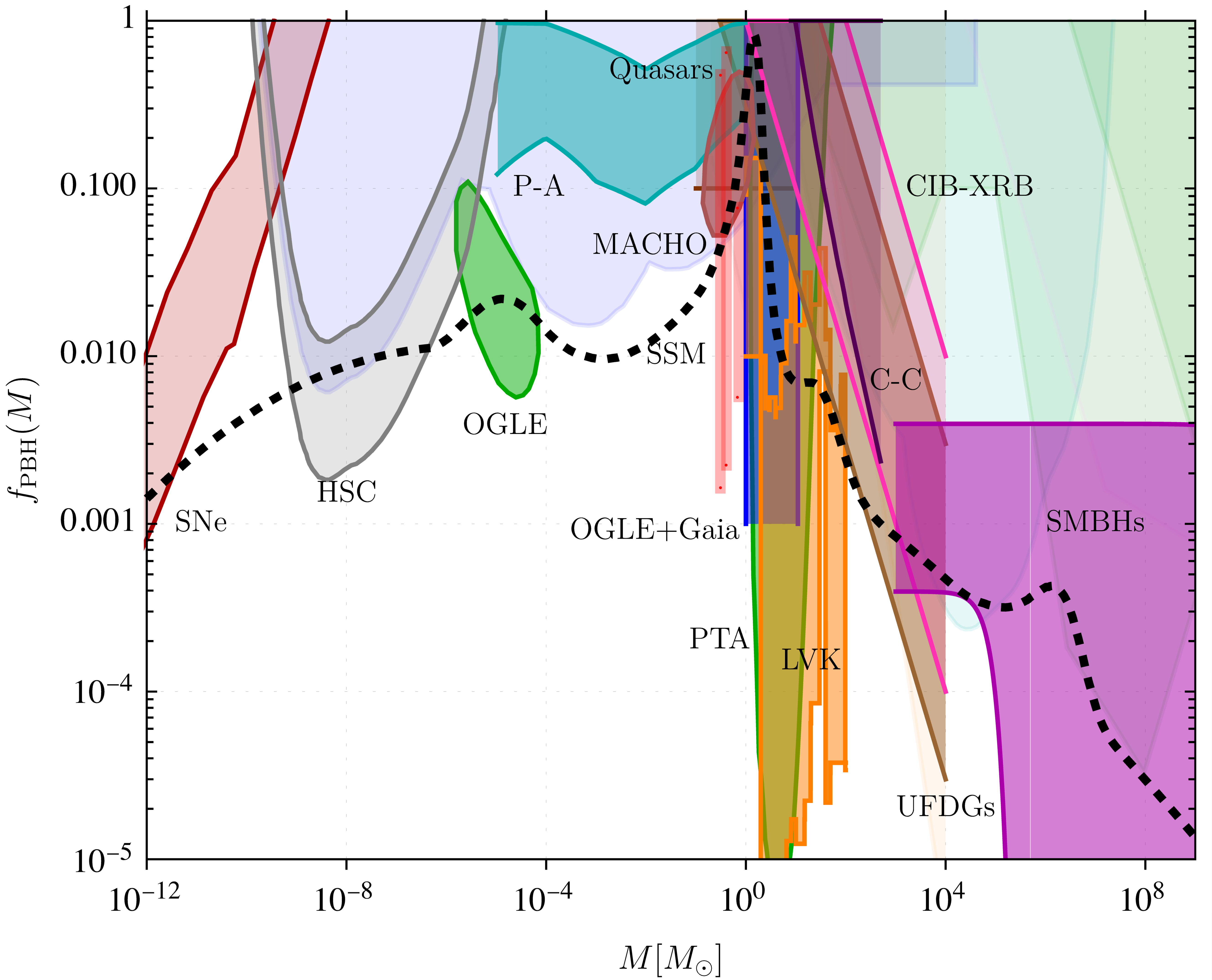

B. Carr, S.C., J. Garcìa-Bellido, M. Hawkins, F. Kühnel
arXiv:2306.03903

- ✓ Many constraints are also possible evidence
- ✓~20 possible observational evidence for PBHs
- ✓ Most of them point to the stellar-mass region
- ✓ Support for $f_{\rm PBH}$ > 0.1 at solar-mass scale
- ✓ Unified scenario possible

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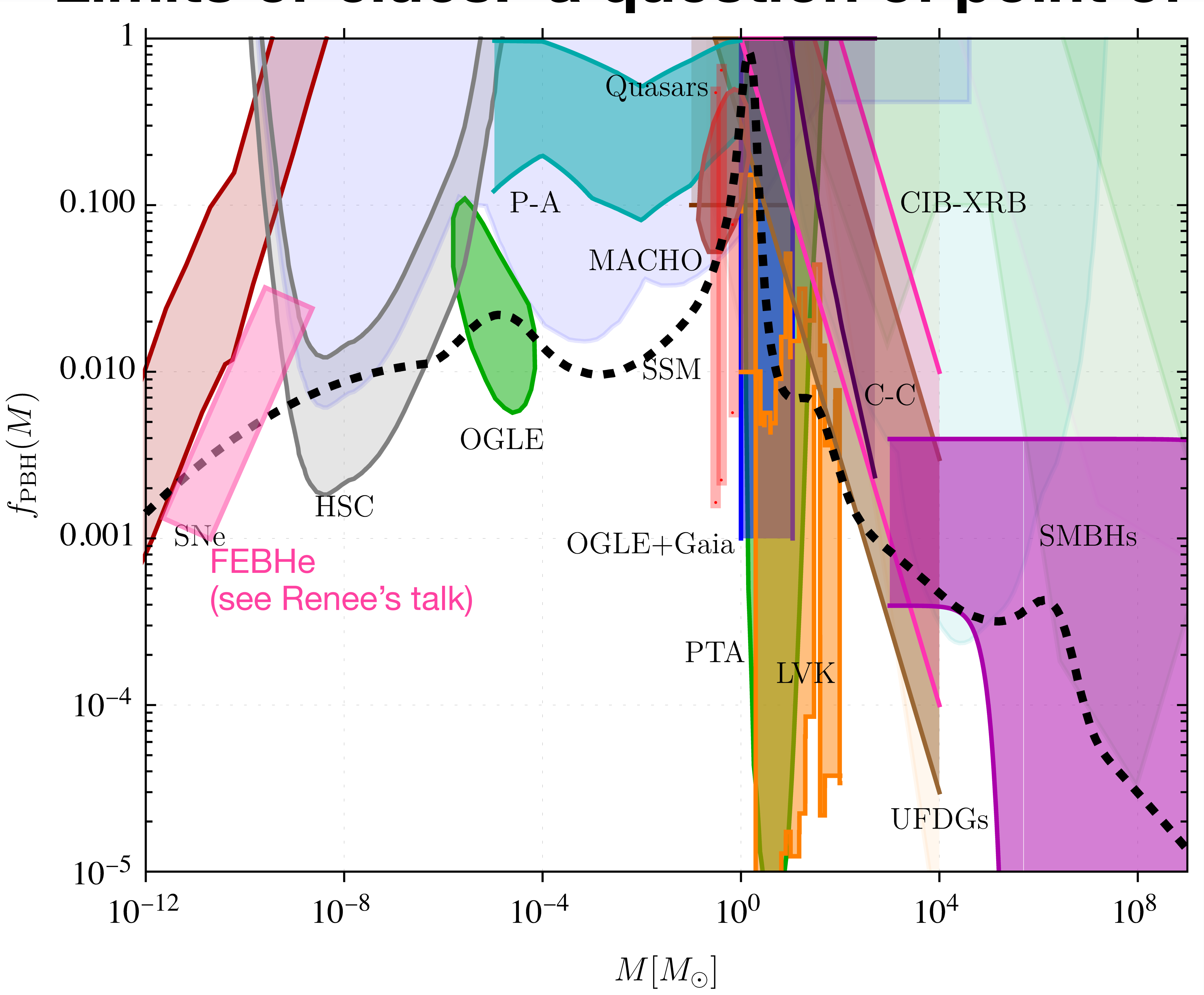

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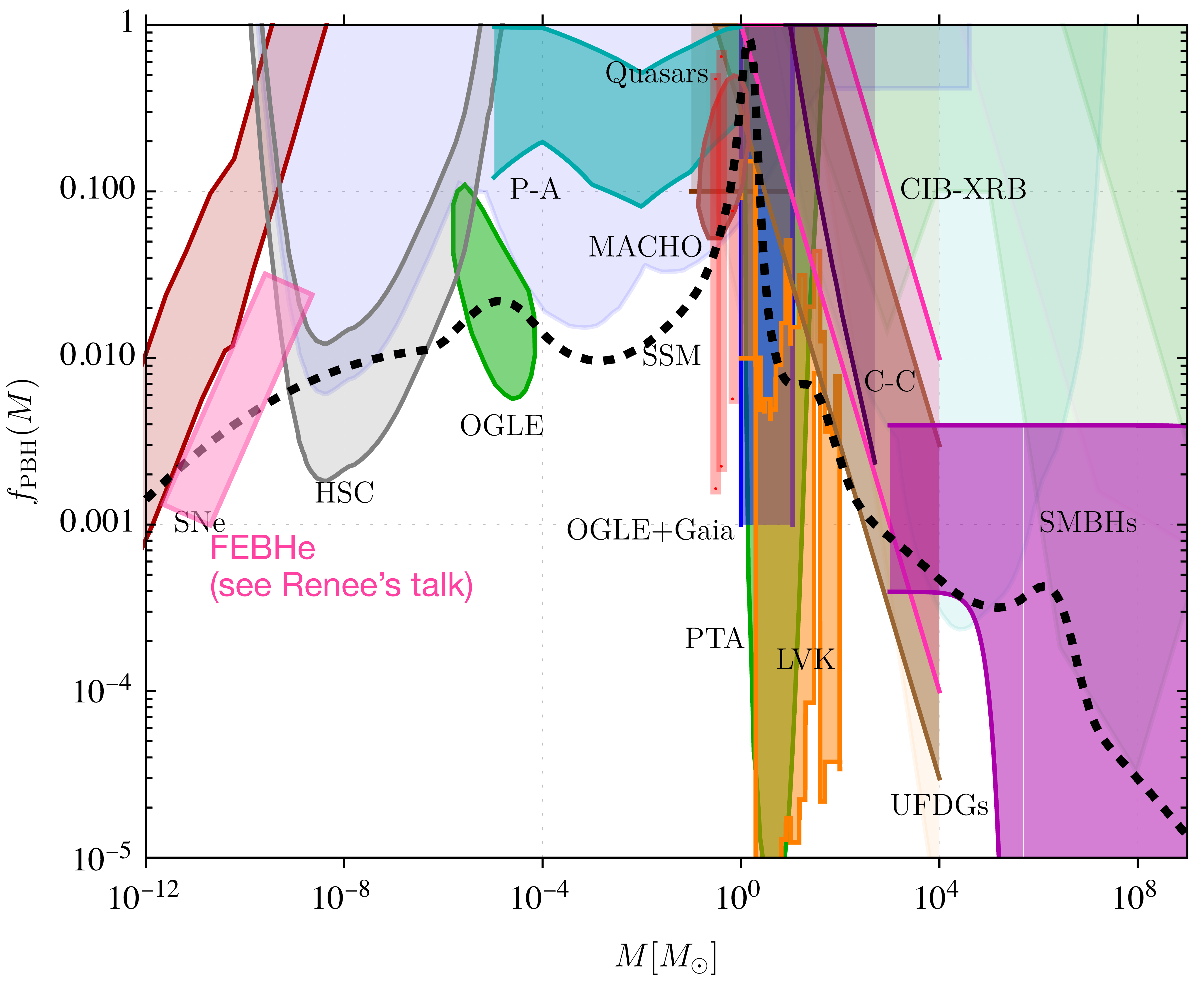

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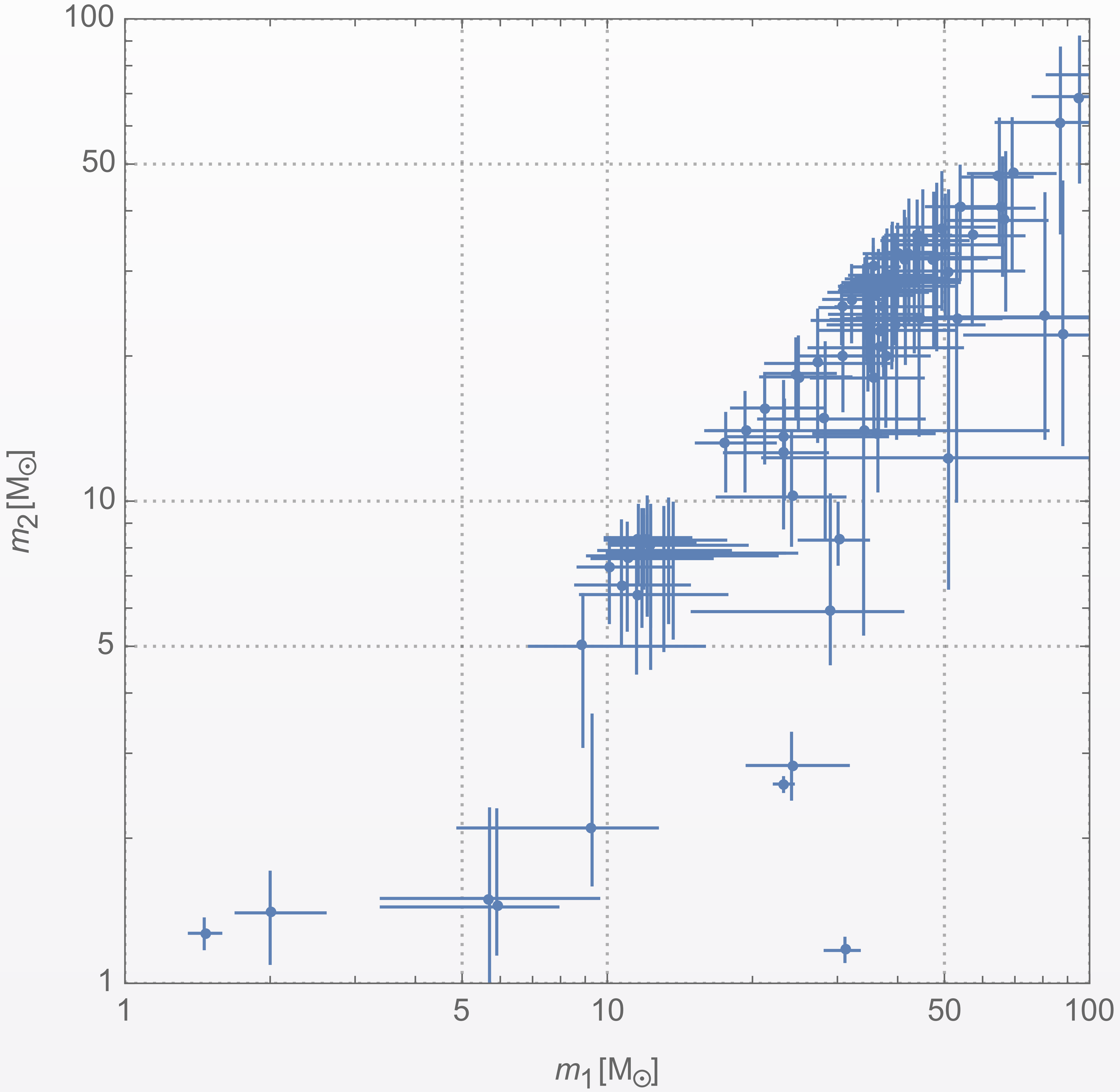
- ◉ Lot of uncertainties, astrophysical assumptions
- ◉ All of these observations can have a non-PBH origin
- ◉ Limits and clues cohabit in the stellar-mass regions…
- ◉ CMB distortion and PTA limits are a challenge, maybe pointing to fully non-Gaussian models

2. Do GWs come from PBHs?

Intriguing features:

- Initially, **black hole masses** ~30 solar mass
- Low **effective spins**
- Black holes in **pair-instability mass gap**, with no sign of cut-off
- Mergers with **low mass ratios** (ex: GW190814) and **low primary component spin**
- **Dynamical channel** challenging (velocity kicks, mergers with similar masses, single black hole population…)
- **Subsolar-mass candidates** (e.g. SSM200308)



Maybe all mergers come from PBHs ?

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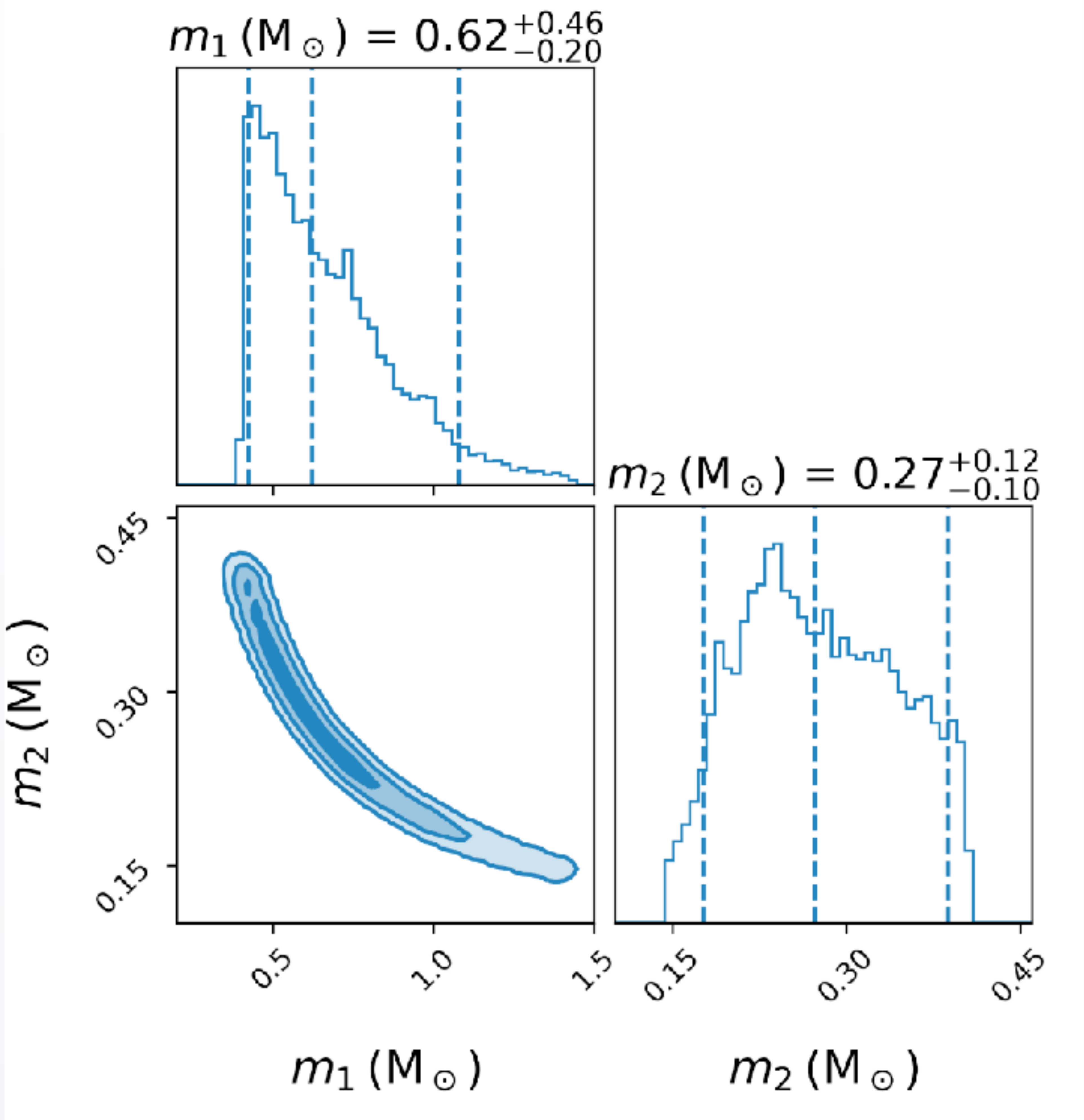
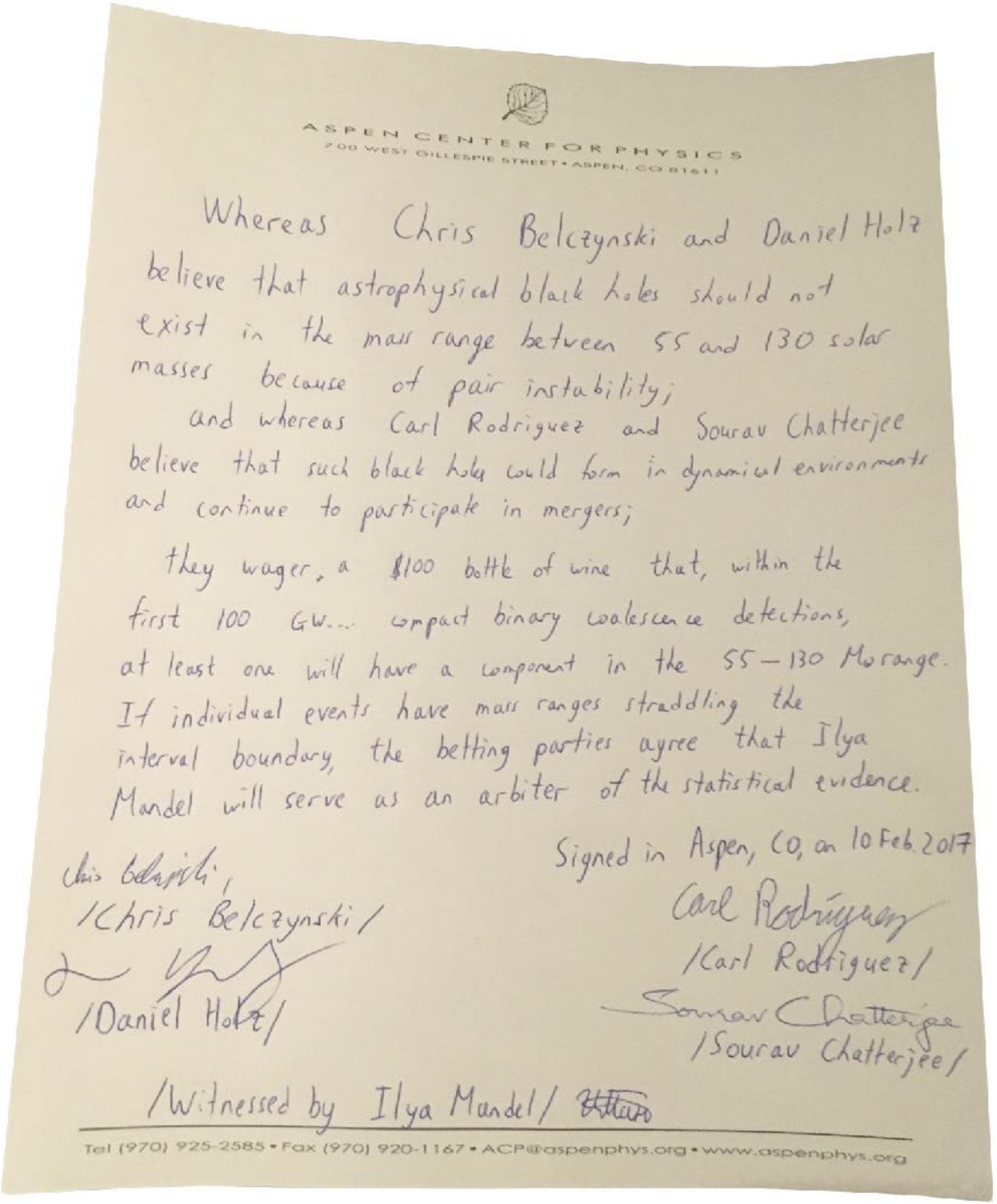


Parameter estimation for SSM200308
M. Prunier, SC et al, 2311.16085



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Whereas Chris Belczynski and Daniel Holz believe that astrophysical black holes should not exist in the mass range between 55 and 130 solar masses because of pair instability;
and whereas Carl Rodriguez and Sourav Chatterjee believe that such black holes could form in dynamical environments and continue to participate in mergers;
they wager a $100 bottle of wine that, within the first 100 GW... compact binary coalescence detections, at least one will have a component in the 55–130 M☉ range. If individual events have mass ranges straddling the interval boundary, the betting parties agree that Ilya Mandel will serve as an arbiter of the statistical evidence.

Signed in Aspen, CO, on 10 Feb 2017

Chris Belczynski
/Chris Belczynski/

Daniel Holz
/Daniel Holz/

Carl Rodriguez
/Carl Rodriguez/

Sourav Chatterjee
/Sourav Chatterjee/

/Witnessed by Ilya Mandel/ Ilya Mandel

Tel (970) 925-2585 • Fax (970) 920-1167 • ACP@aspenphys.org • www.aspenphys.org

https://www.quantamagazine.org/possible-detection-of-a-black-hole-so-big-it-should-not-exist-20190828/

GW190521 (and a few others…)

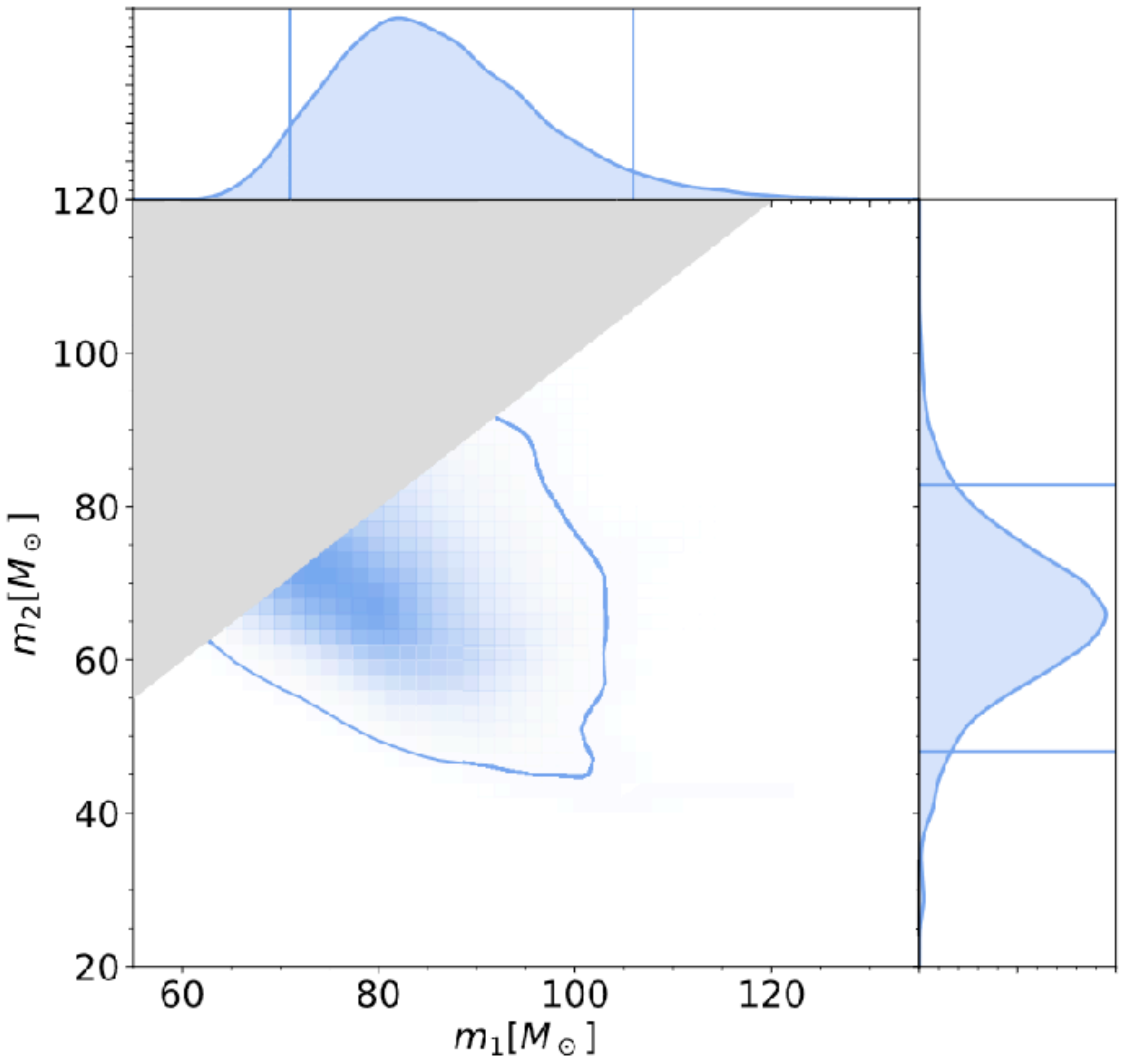


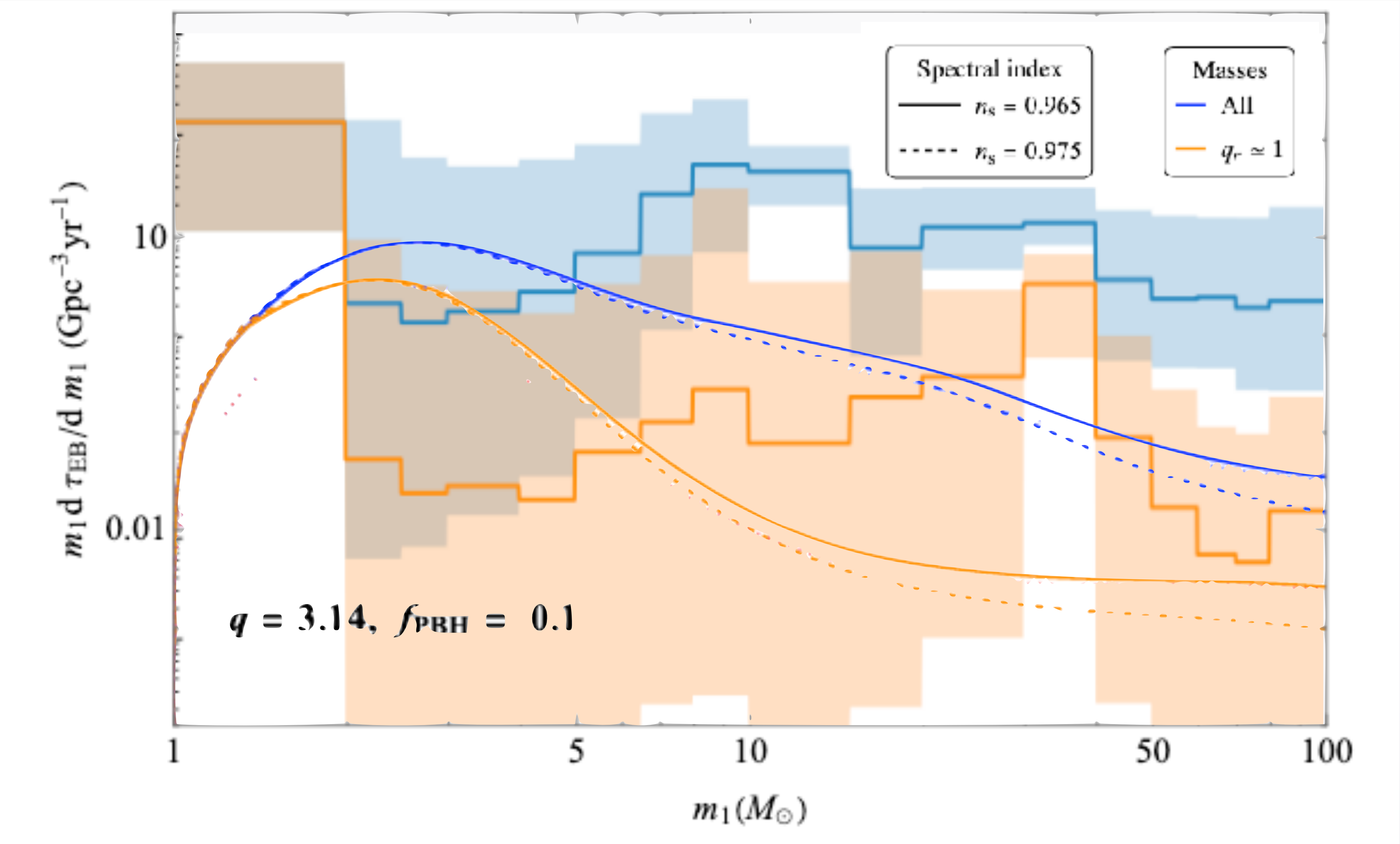
FIG. 2. Posterior distributions for the progenitor masses of GW190521 according to the NRSur7dq4 waveform model. The 90% credible regions are indicated by the solid contour in the joint distribution and by solid vertical and horizontal lines in the marginalized distributions.

LVK collaboration, 2009.01075

2. Do GWs come from PBHs?

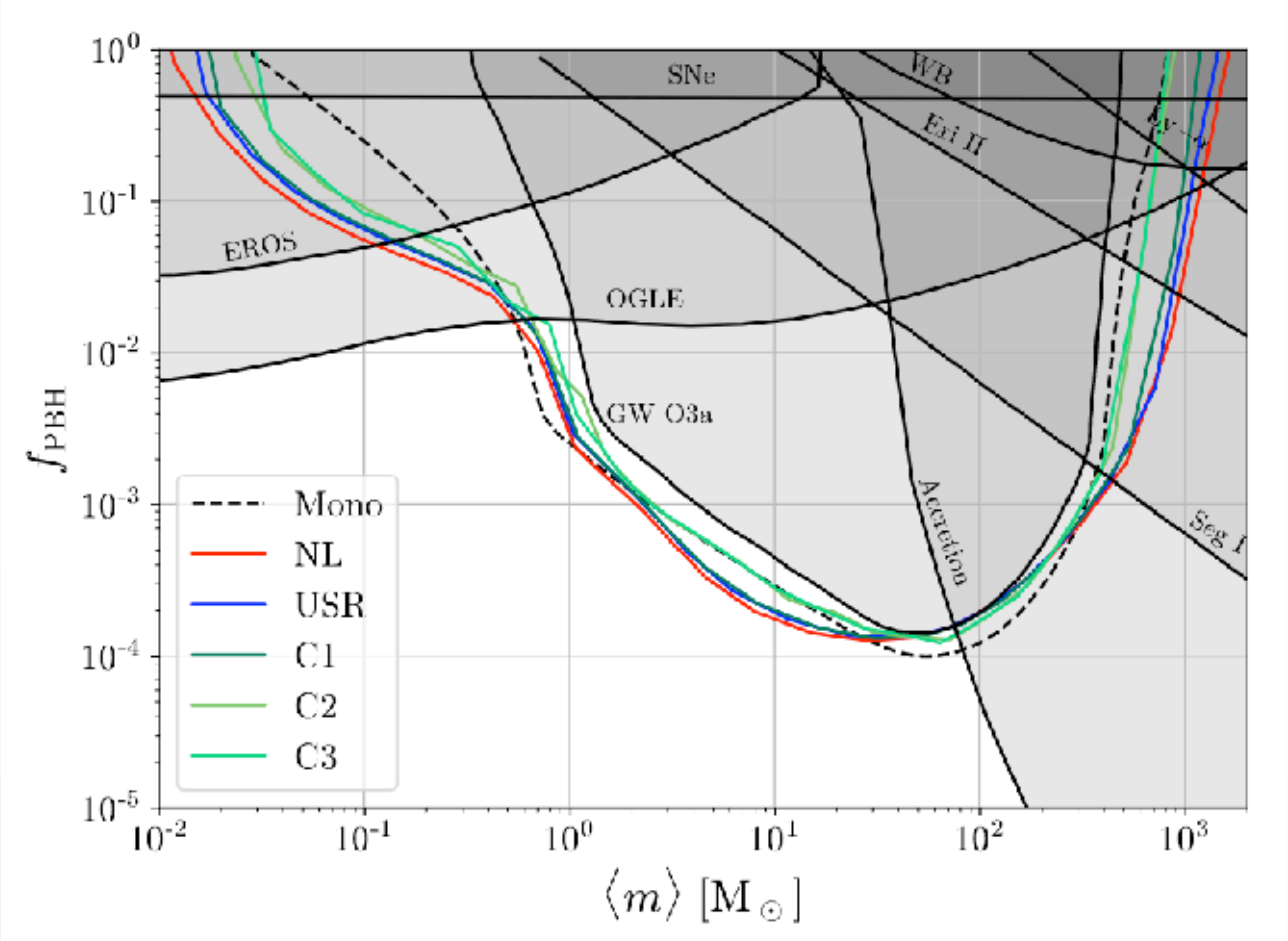
Based on rates and event distributions

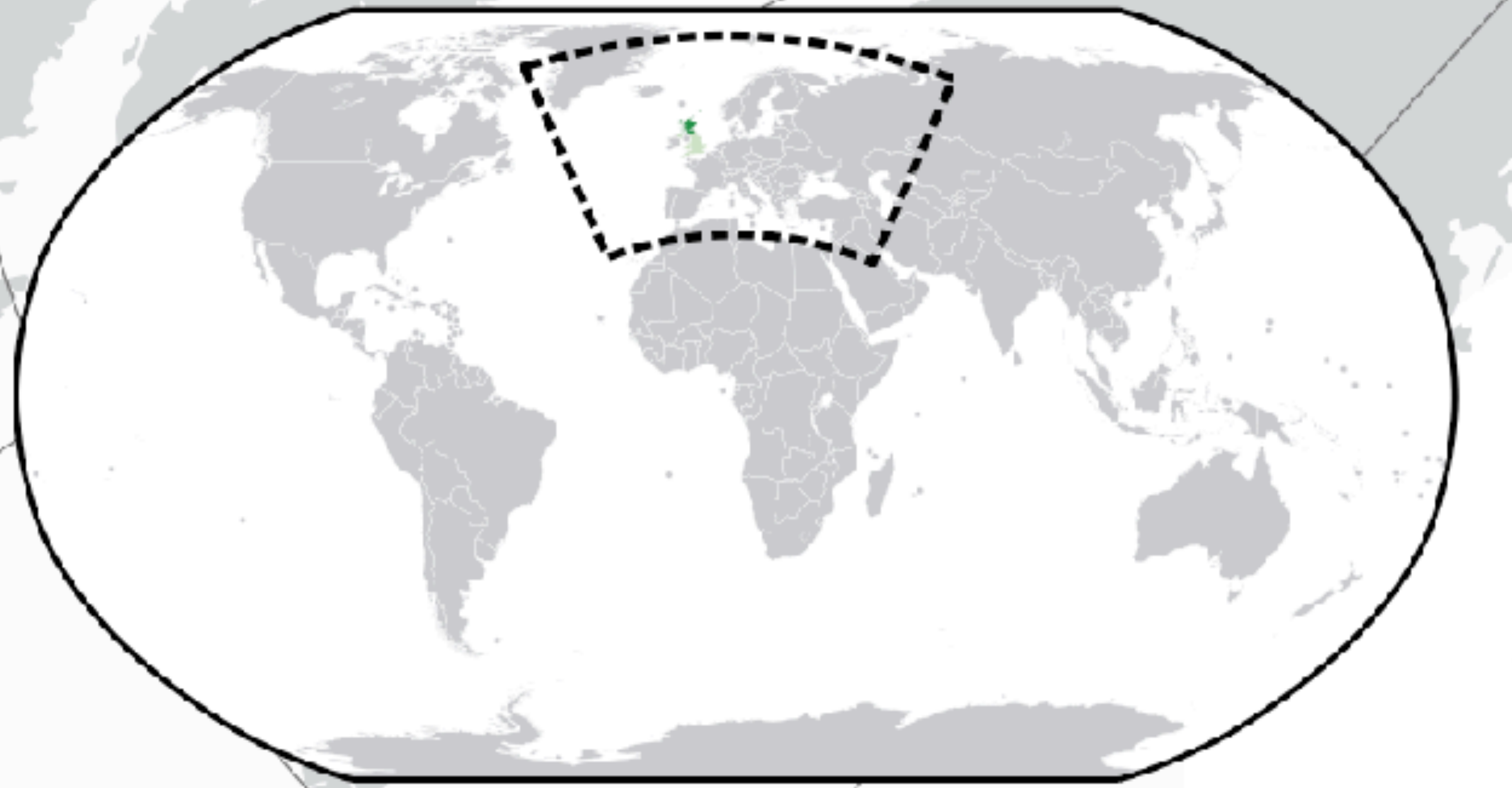
- QCD features, for early binaries and late binaries: [Bagui, SC, Escriva 2022]
- [+ Jedamzik 2020], SC & Garcia-Bellido, Carr, SC, Garcia-Bellido, Kuhnel, others]



Bayesian population analysis

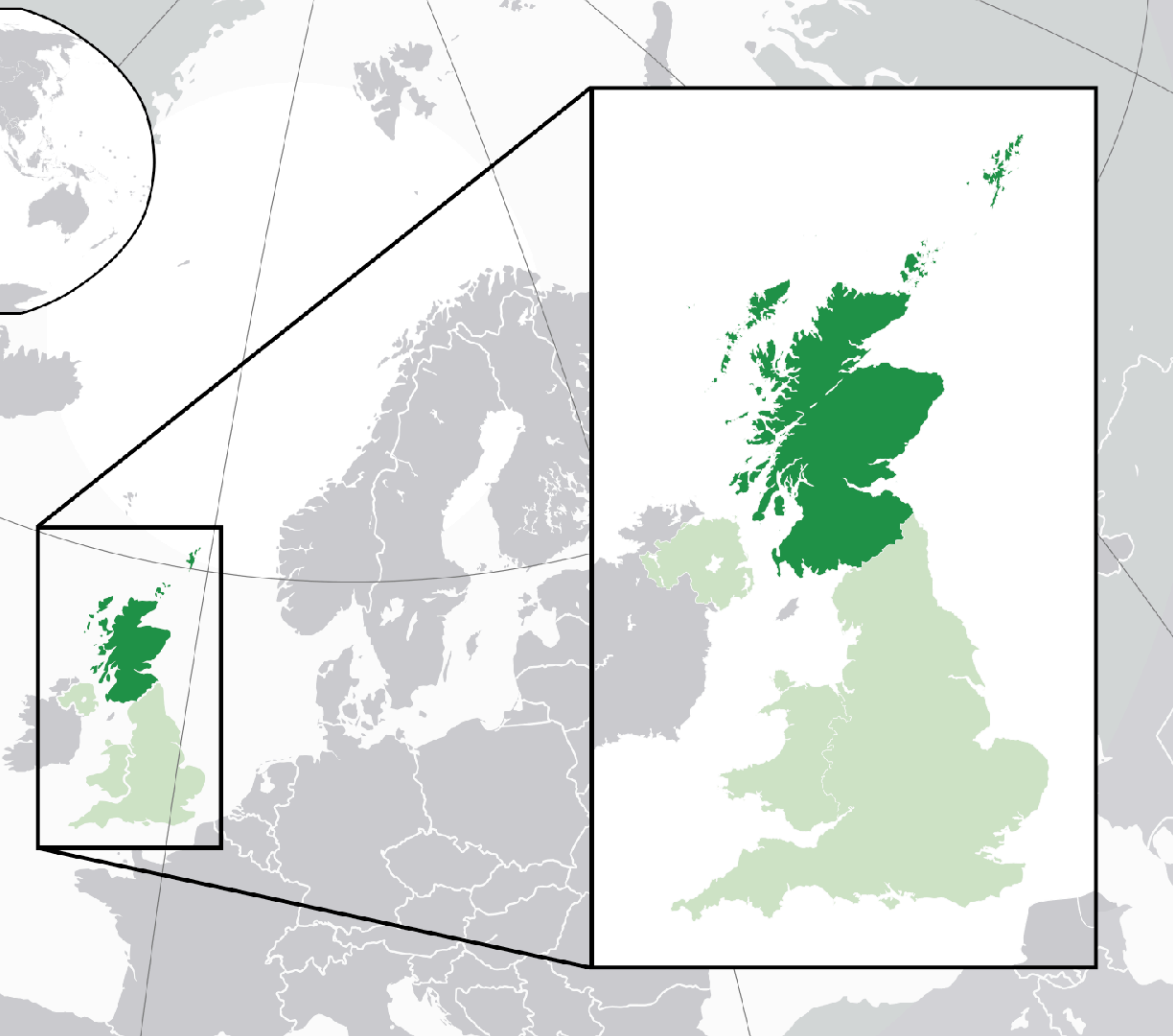
- QCD features, for early binaries only: [Franciolini et al. 2022] [Andrès-Carcasona et al. 2024]



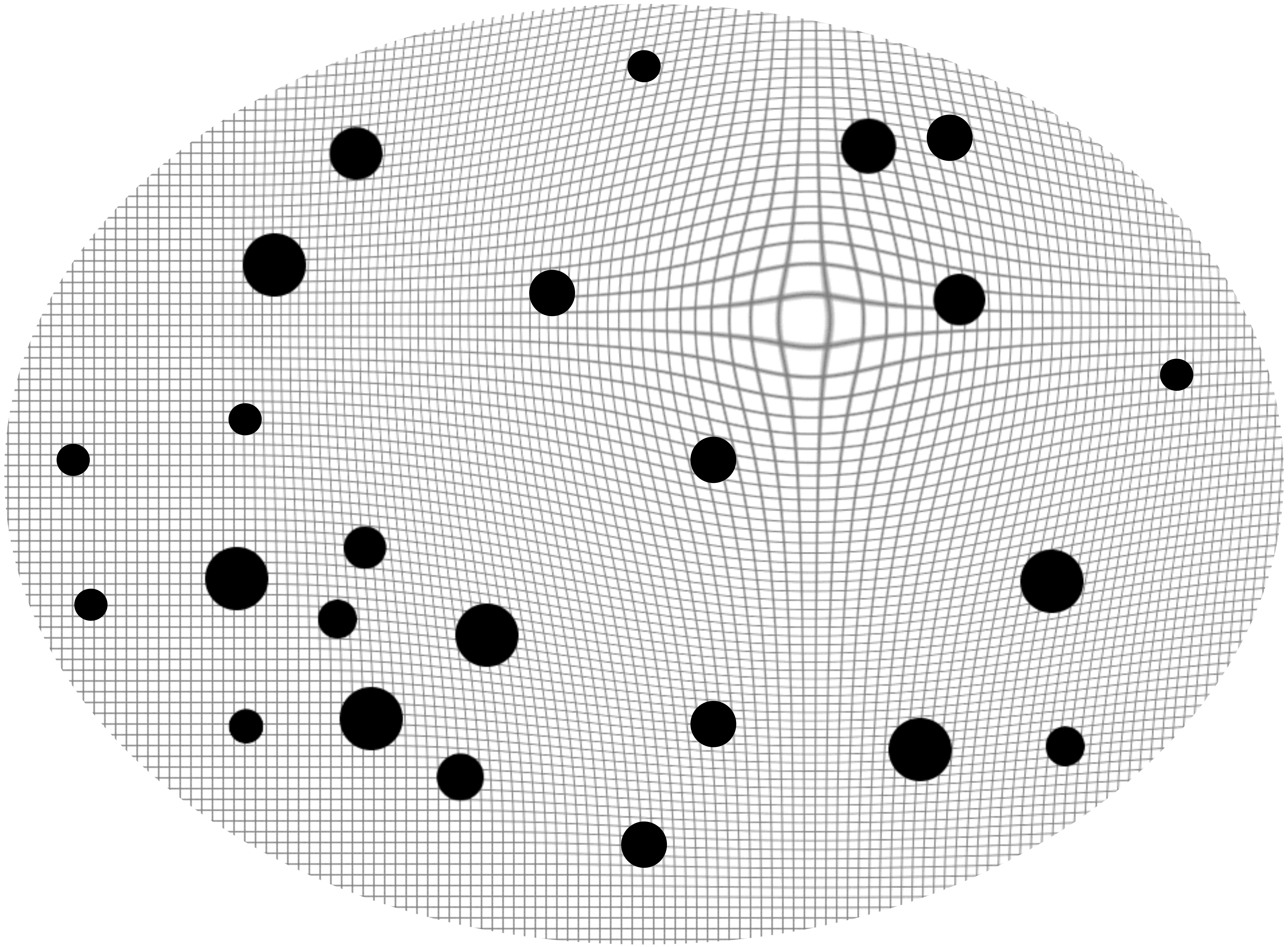


Outcome of binaries:

- Formation ?
- Merger ?
- Dirsrupted ?
- Member of a cluster ?
- 3-body interactions: perturbed or disrupted by a smaller ?



3. PBH binary formation

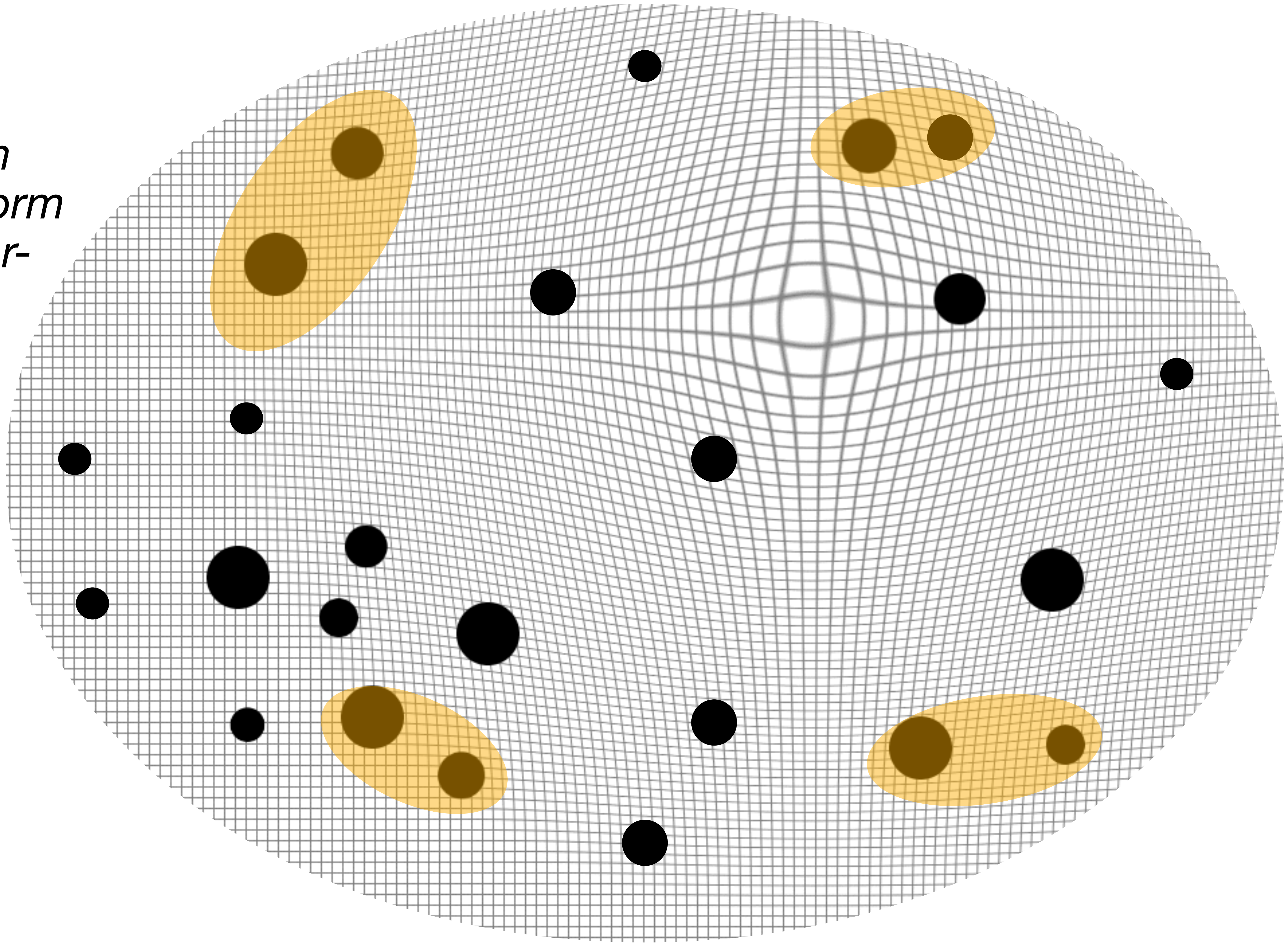


3. PBH binary formation

Early binaries:

When two PBHs form sufficiently close to form a binary before matter-radiation equality

(exaggerated picture)



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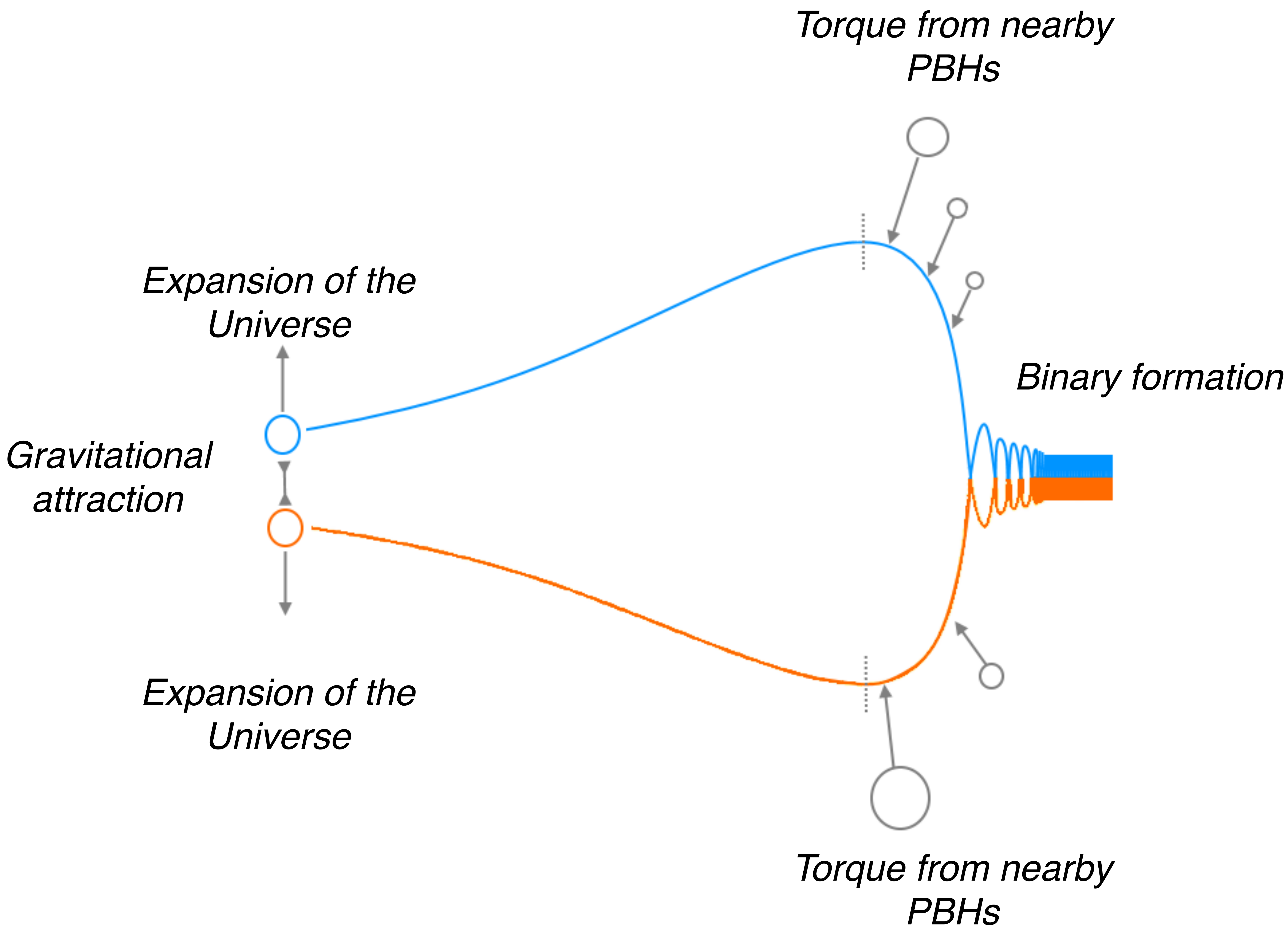
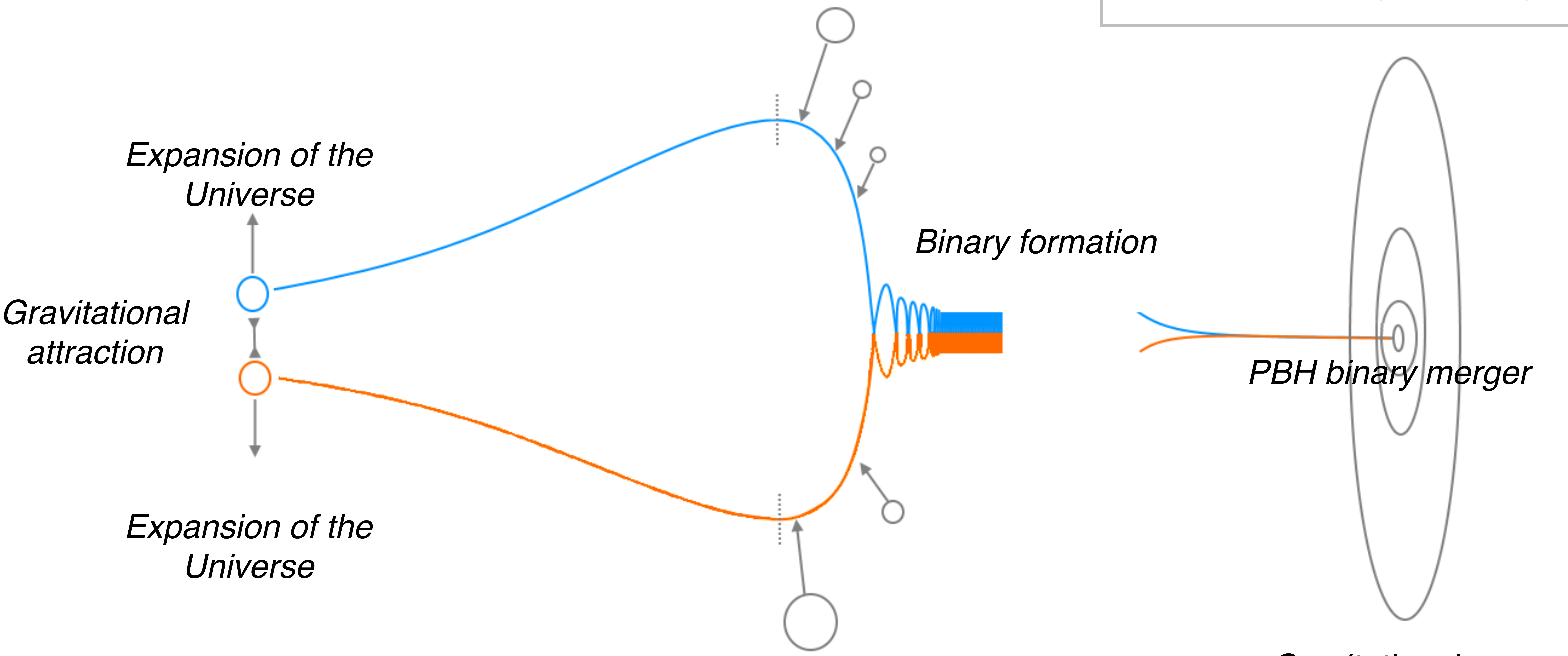


Image credit: Simon Biot

3. PBH binary formation

Coalescence time:

$$\tau = \frac{3}{85}\frac{1}{G^3 m_1 m_2 (m_1+m_2)}(1-e^2)^{7/2} a^4 c^5$$



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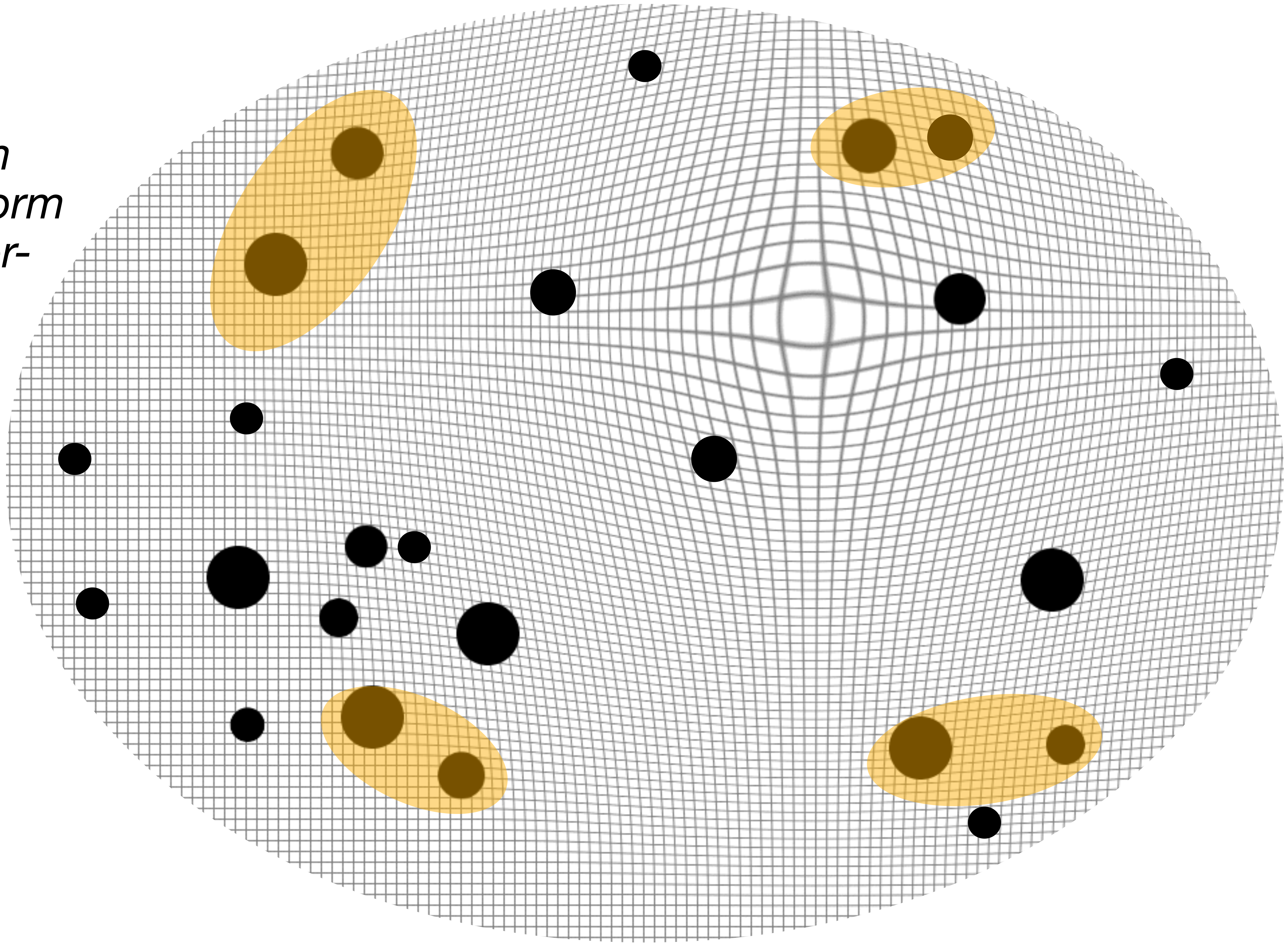

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$$R_{\rm EB} = \frac{1.6\times10^6}{\rm Gpc^3 yr} \times f_{\rm sup}(m_1, m_2, f_{\rm PBH}) f_{\rm PBH}^{53/37} f(m_1) f(m_2) \times \left(\frac{t}{t_0}\right)^{-34/37} \left(\frac{m_1+m_2}{M_\odot}\right)^{-32/37} \left[\frac{m_1 m_2}{(m_1+m_2)^2}\right]^{-34/37}$$

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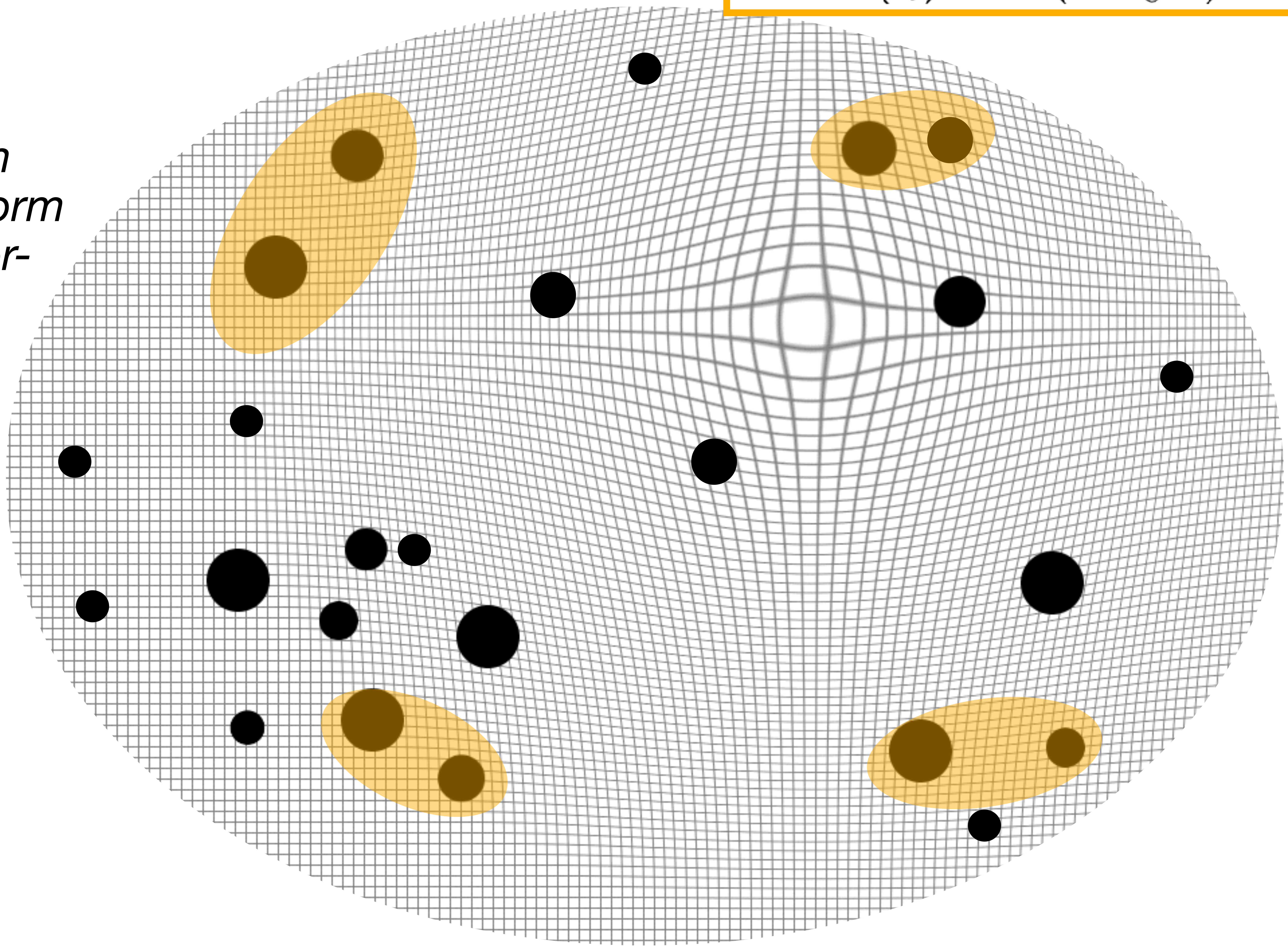

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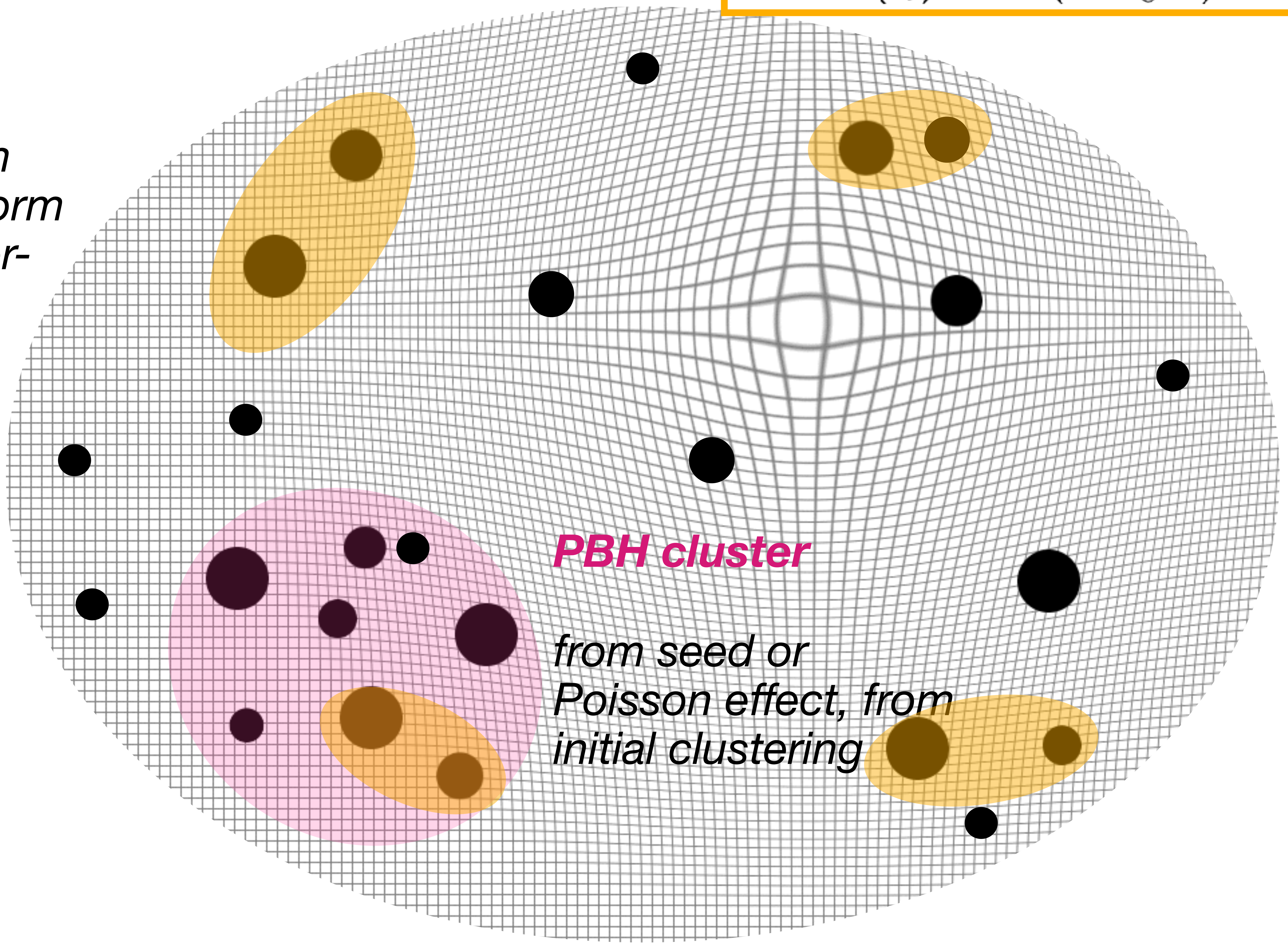

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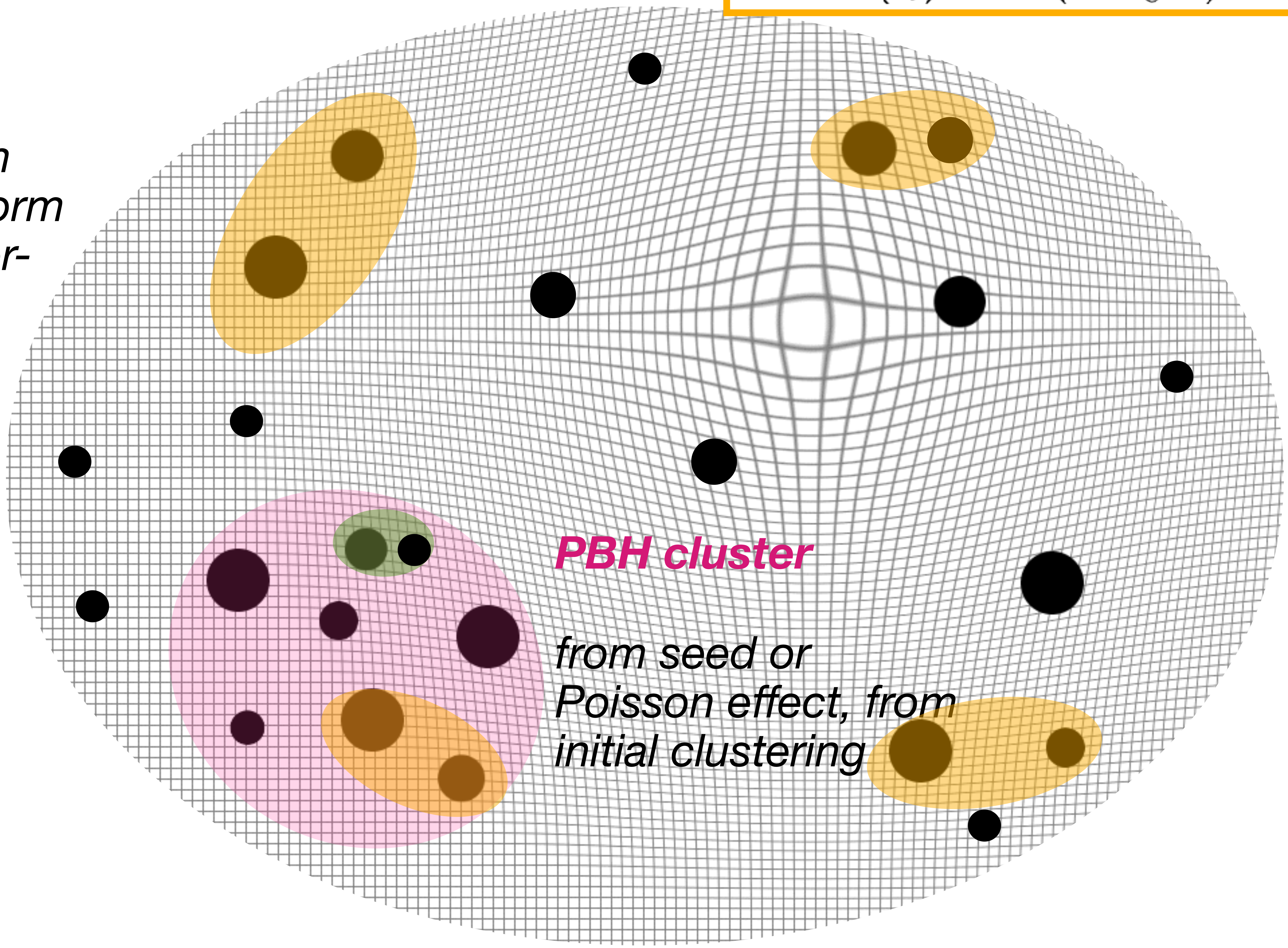

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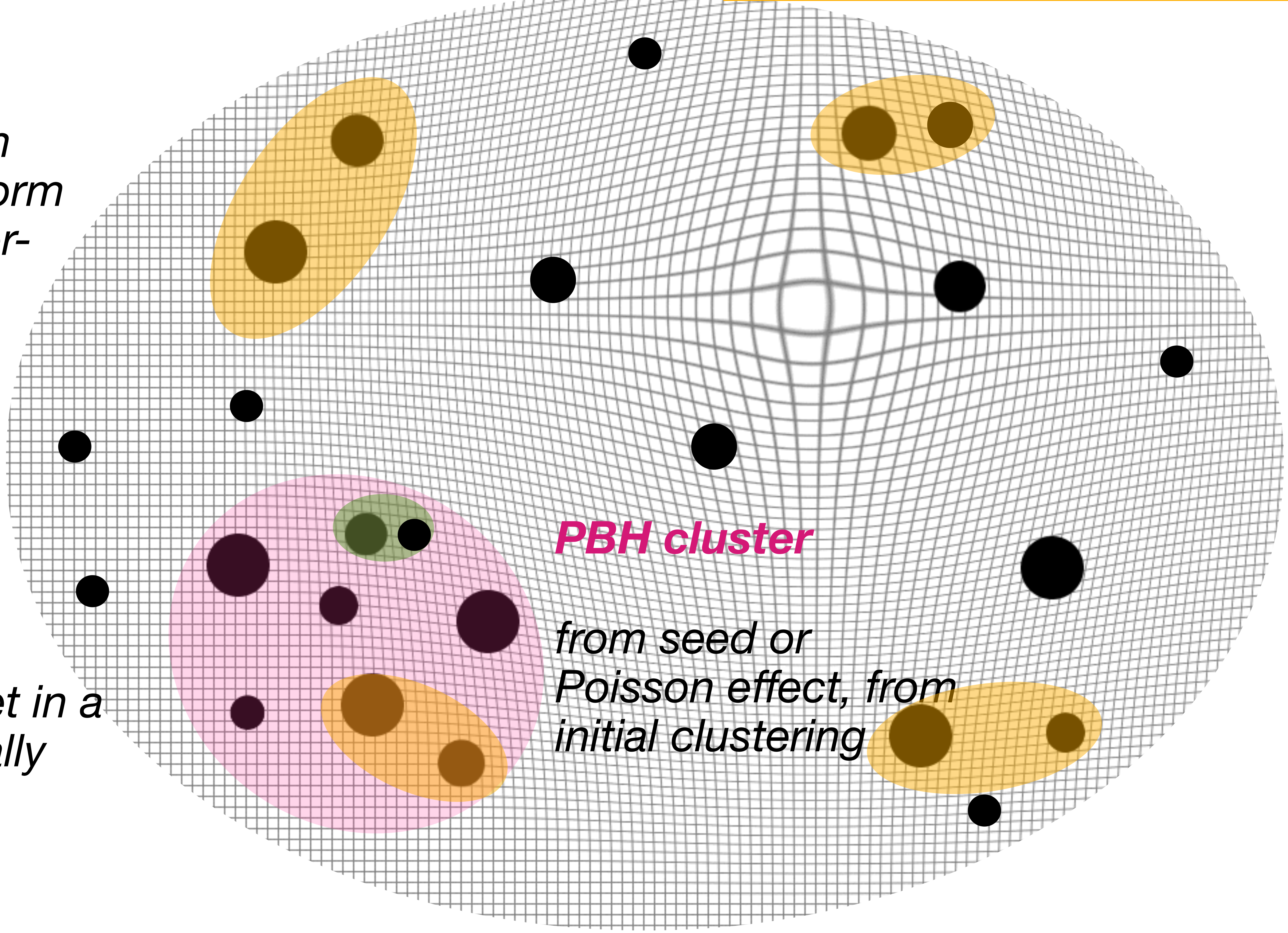
Early binaries:

When two PBHs form sufficiently close to form a binary before matter-radiation equality

Late binaries:

When two PBHs meet in a cluster and dynamically form a binary



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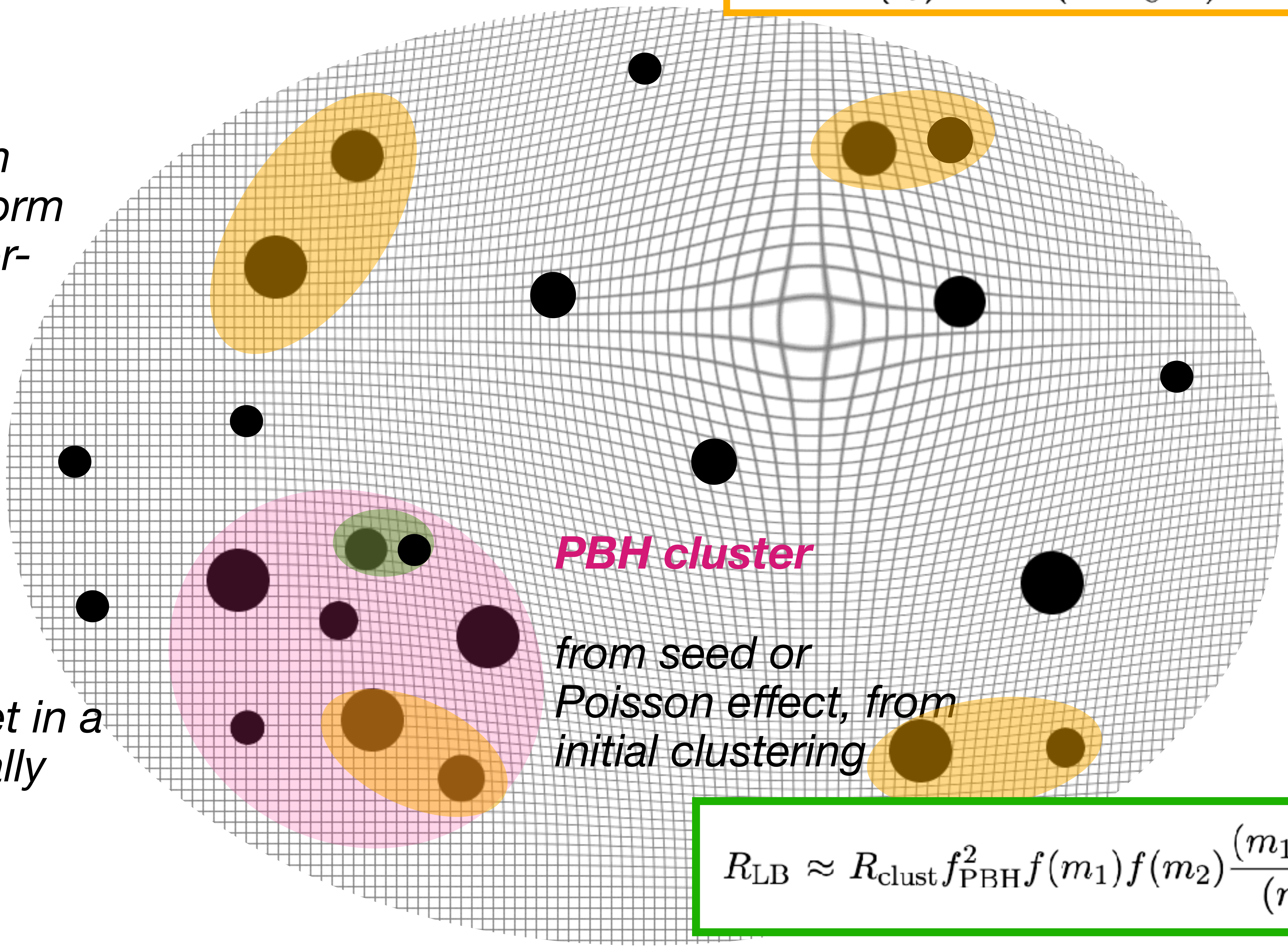
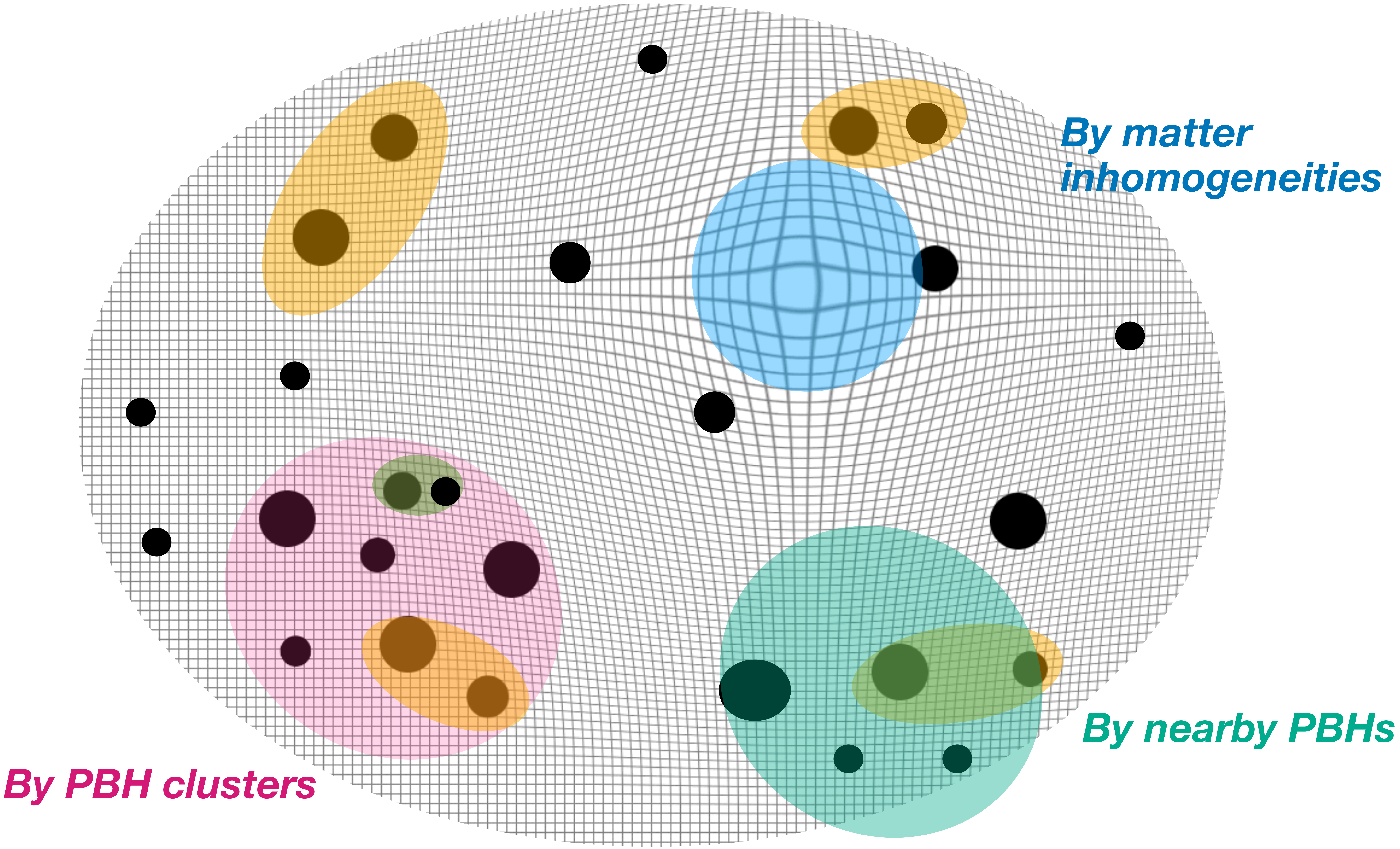


Late binaries:

When two PBHs meet in a cluster and dynamically form a binary

$$R_{\rm LB} \approx R_{\rm clust} f_{\rm PBH}^{2} f(m_1) f(m_2) \frac{(m_1+m_2)^{10/7}}{(m_1 m_2)^{5/7}} {\rm yr^{-1} Gpc^{-3}}$$

4. PBH binary perturbations/dirsuption



4. Perturbation/disruption by nearby PBHs

Depends on the number of PBHs $\bar{N}$ in the « sphere of influence » of the binary

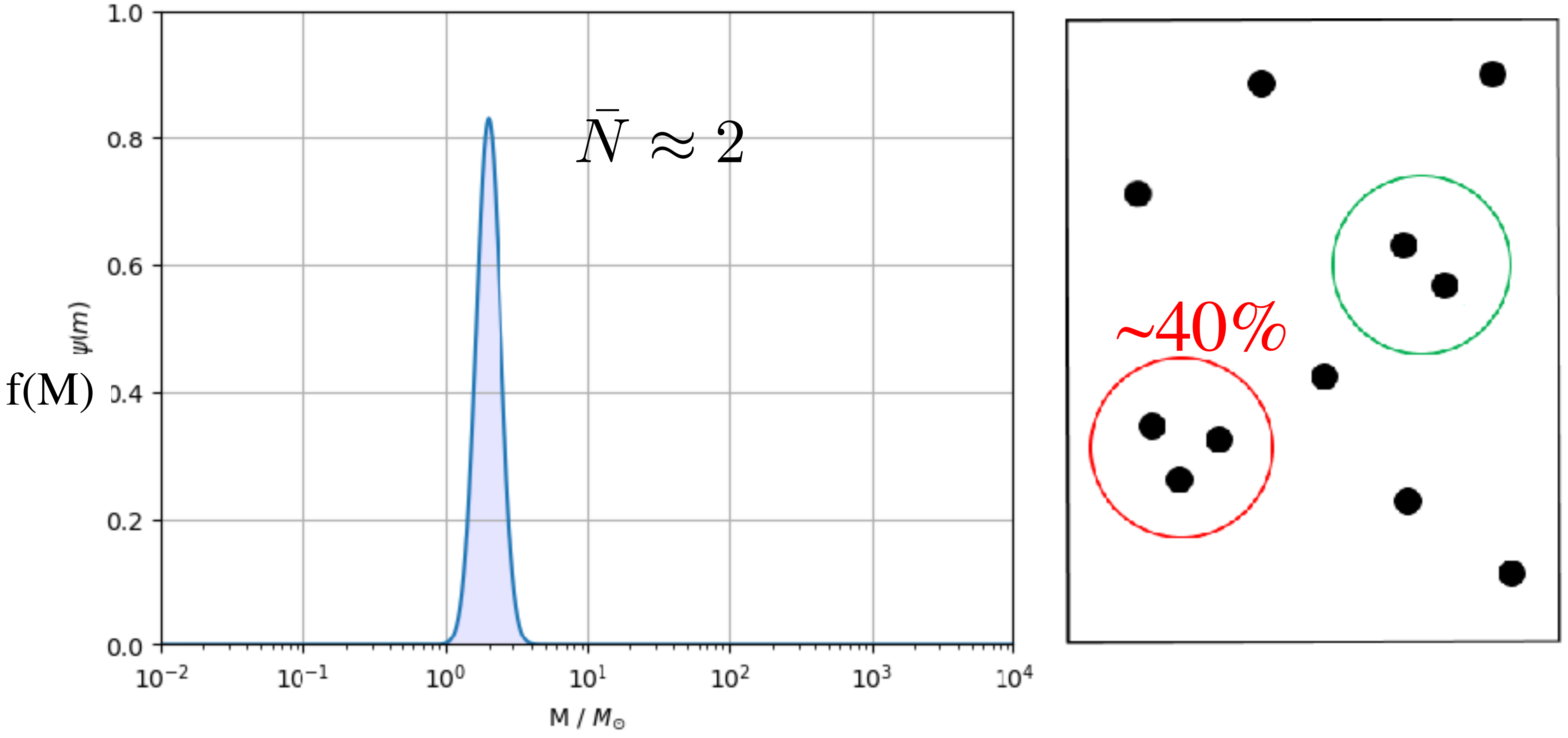
$$\bar{N} \approx \frac{m_1 + m_2}{\langle m_{\rm PBH} \rangle}$$

« Standard » prescription [Raidal et al. 2018] :
every binary with a nearby PBH is removed

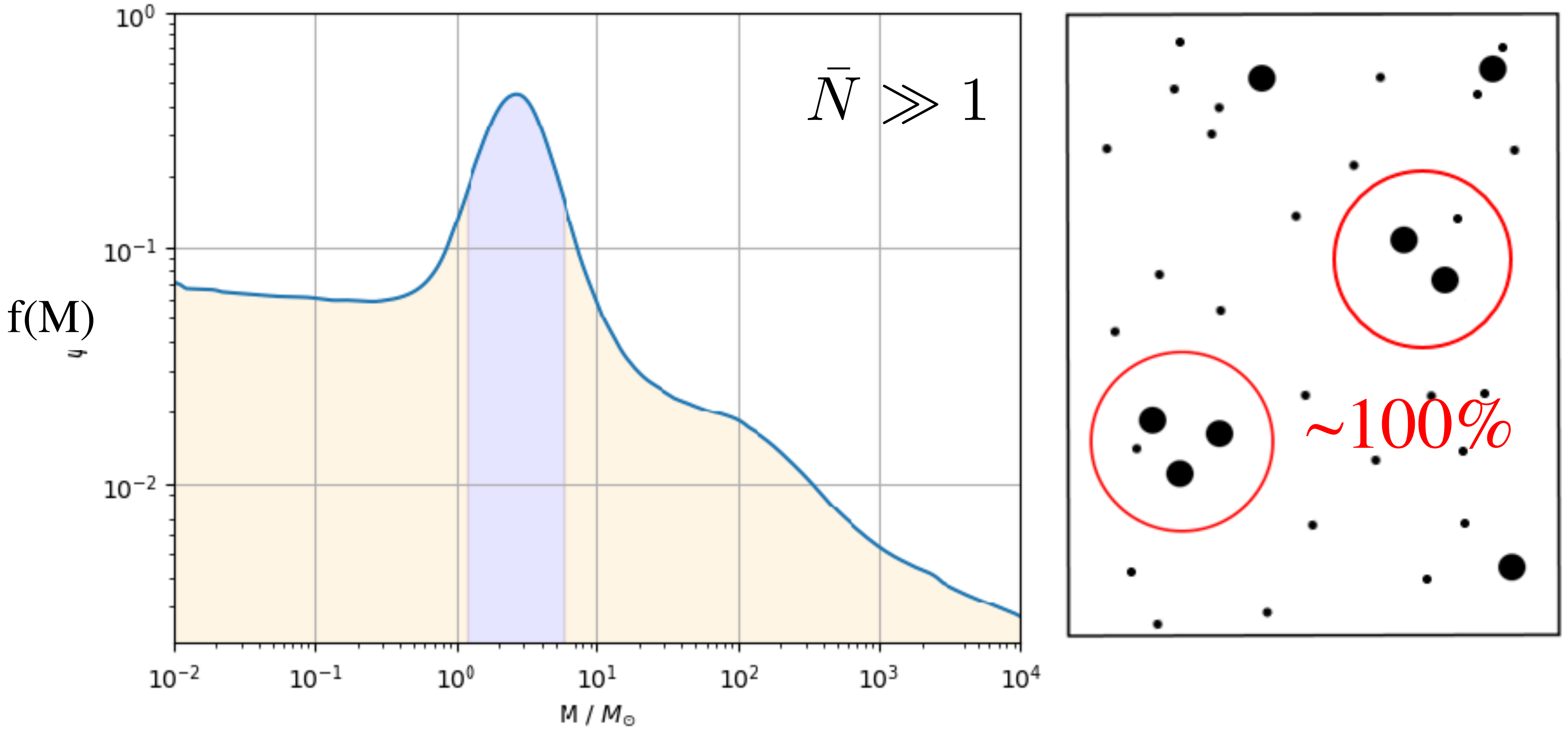
$$f_{\rm sup} \approx 1.42 \left[\frac{\langle m_{\rm PBH}^2 \rangle / \langle m_{\rm PBH} \rangle^2}{\bar{N}} \right]^{-21/74} e^{-\bar{N}}$$

But probably becomes wrong for a very light perturber !
What is the **realistic value of $\bar{N}$** for a broad mass distribution ?

Log-normal



Broad - QCD features



If we strictly follow this prescription: no early PBH binaries => no constrain from LIGO/Virgo !

4. Perturbation/disruption by nearby PBHs

What is « hidden » under the kilt ?

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What is « hidden » under the kilt ?

- Kill all perturbed binaries
- Subtle differences in calculation of $\bar{N}$: [Franciolini et al. 2022]
- Select a scalar spectral index that avoid too many light PBHs [Andrès-Carcasona et al. 2024]
- Include the merger rate of perturbed binaries but again, assumes a peaked distribution [Vaskonen+Veermäe 2019]

$$f_{\mathrm{PBH}} \lesssim \mathcal{O}(10^{-3})$$

- Realistic case ?

?

- Do like if you had a peak: $\bar{N} = 2$ [Bagui, SC., Escrivà, 2022]

Only consider perturbations by the peak of the PBHs

Neglect perturbations by intruders with $m_3 \ll m_1 + m_2$

$$f_{\mathrm{PBH}} \lesssim \mathcal{O}(0.1 - 1)$$

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$$f_{\mathrm{PBH}} \lesssim \mathcal{O}(10^{-3})$$

- Realistic case ?
- Quantification of the perturbation in the coalescence time:

$$\frac{\Delta\tau}{\tau} = \left(\frac{\Delta L}{L} + 1\right)^7 \cdot \left(\frac{\Delta E_b}{E_b} + 1\right)^{-1/2} - 1$$

- Mass condition for the intruder:

$$\frac{\Delta E_b}{E_b} = 2 \cdot C \cdot \frac{m_3}{m_1 + m_2} \qquad \frac{\Delta L}{L} = \beta \cdot \frac{\Delta E_b}{E_b}$$

$$\left|\left(\beta \cdot \frac{2 \cdot C \cdot m_3}{m_1 + m_2} + 1\right)^7 \cdot \left(2 \cdot C \cdot \frac{m_3}{m_1 + m_2} + 1\right)^{-1/2} - 1\right| = 0.073$$

- Numerical simulation of synthetic PBH population

- Do like if you had a peak: $\bar{N} = 2$ [Bagui, SC., Escrivà, 2022]

Only consider perturbations by the peak of the PBHs

Neglect perturbations by intruders with $m_3 \ll m_1 + m_2$

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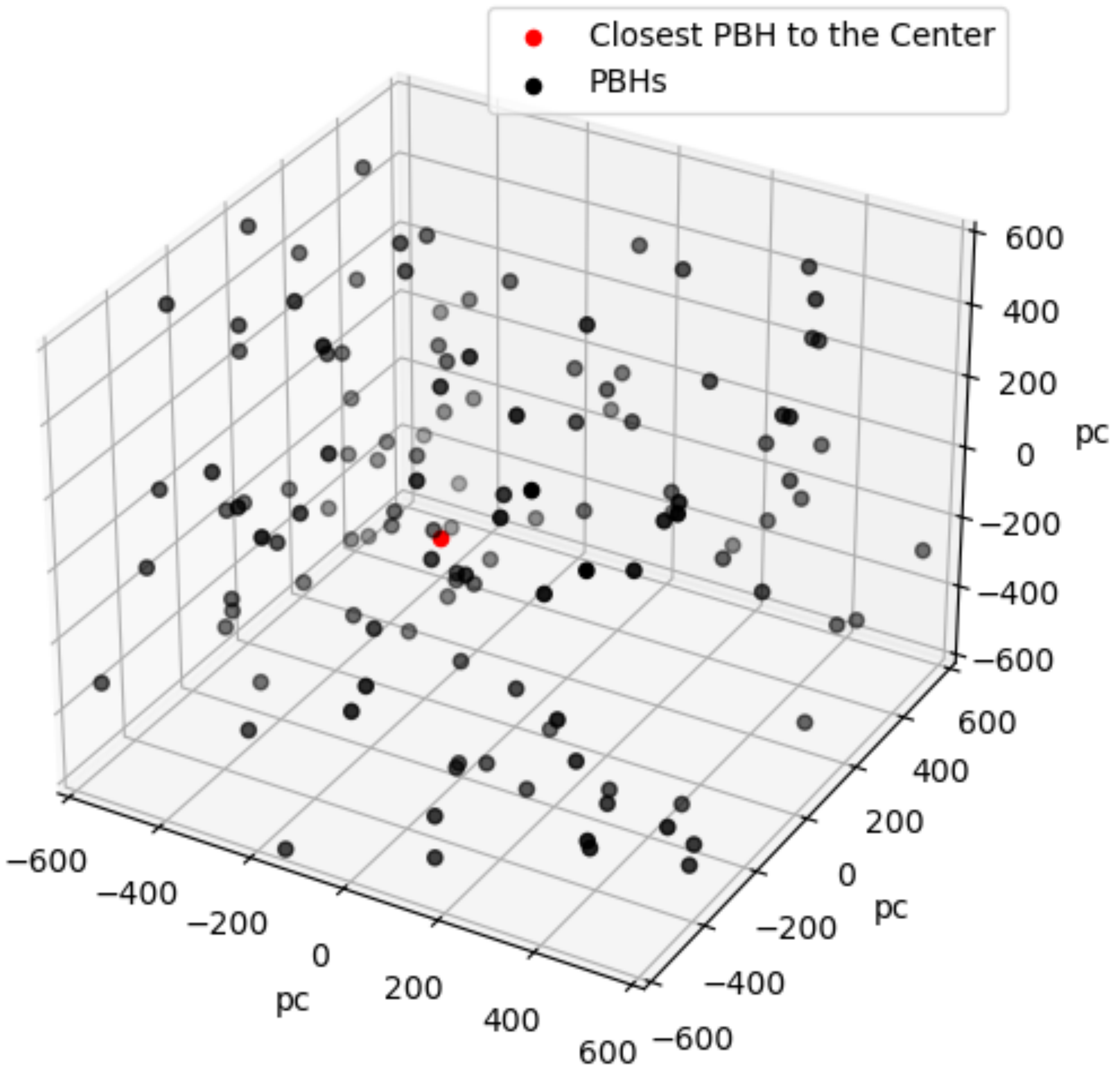
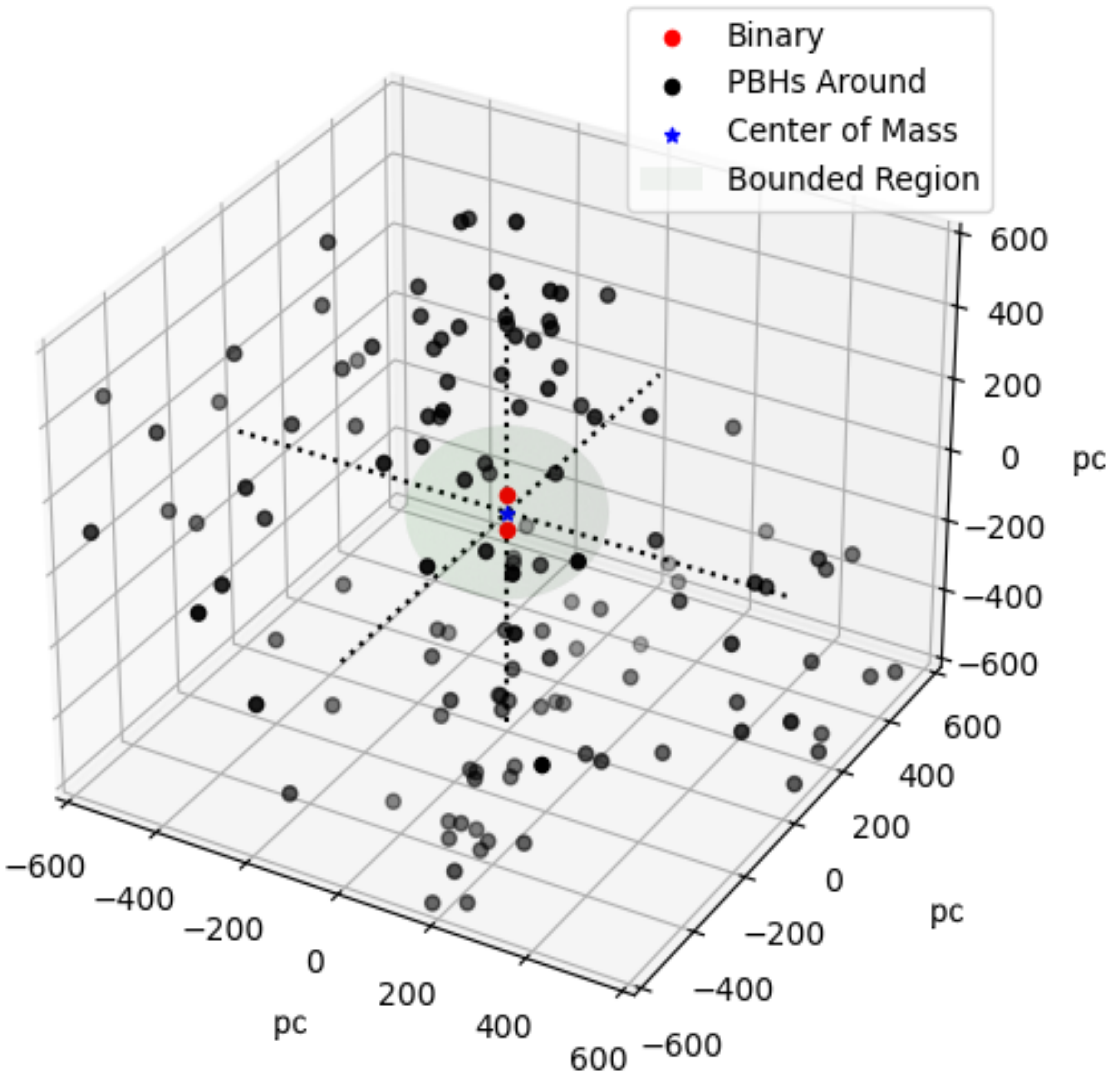
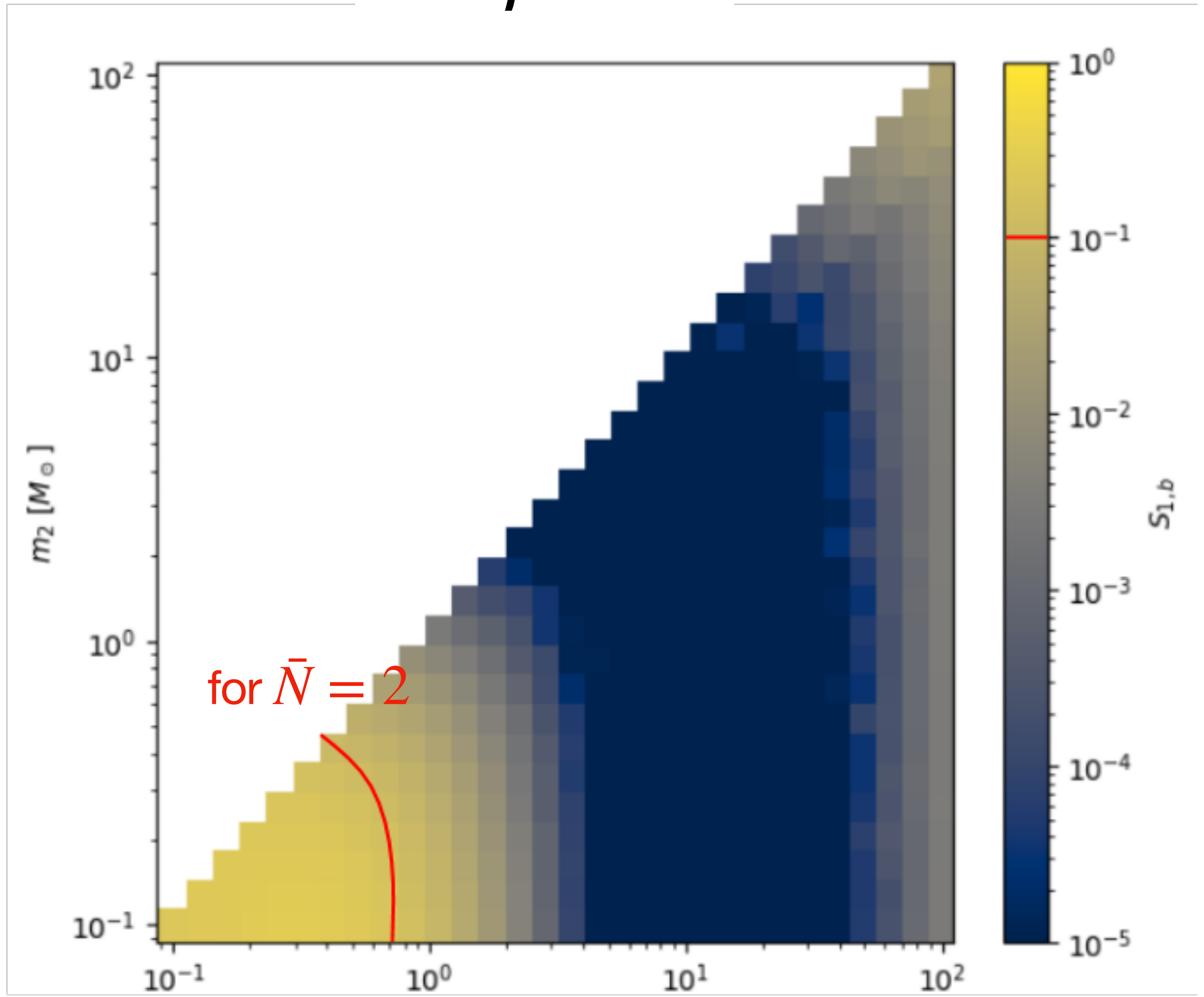




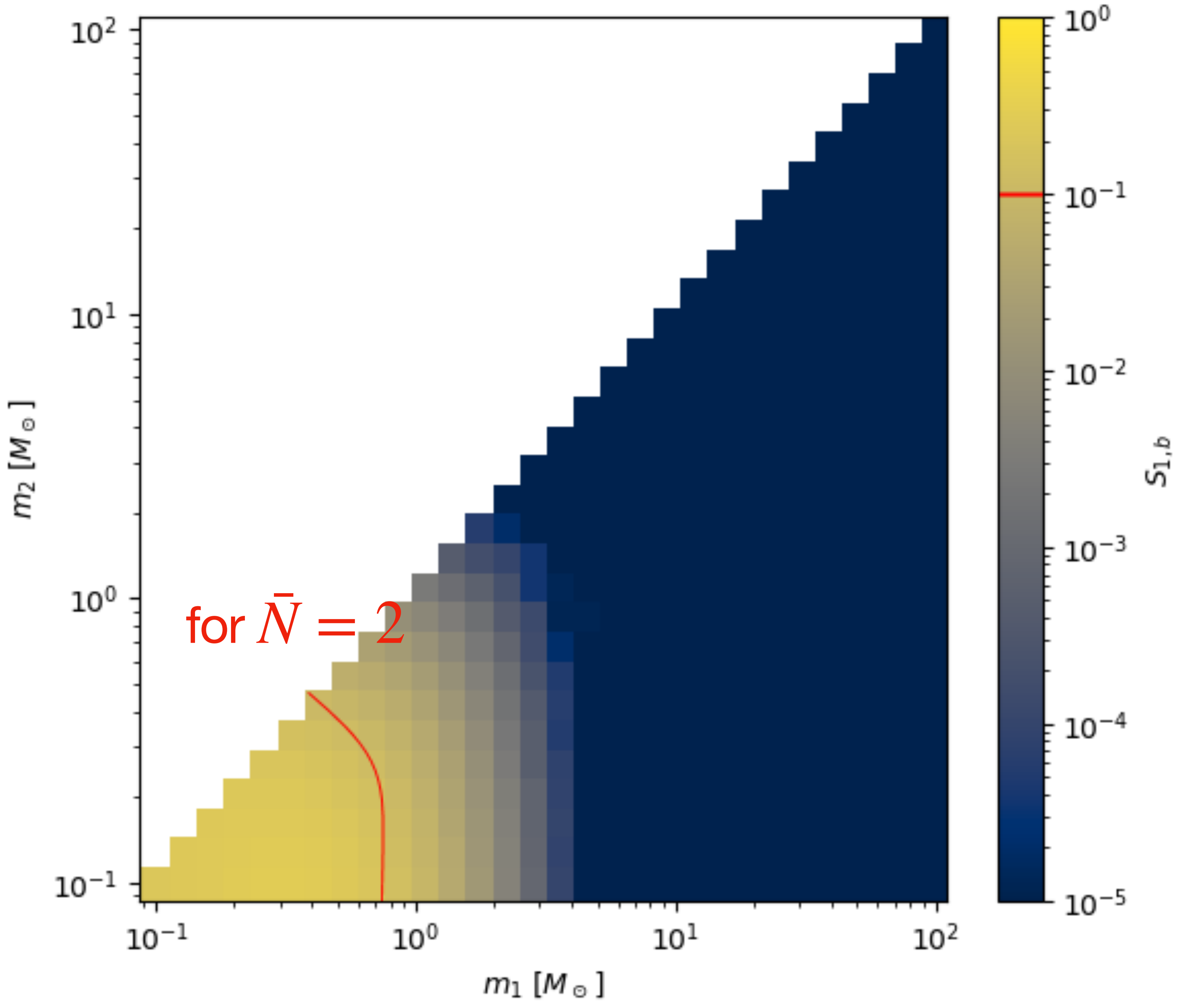
4. Perturbation/disruption by nearby PBHs

New rate suppression factor for $f_{\rm PBH} = 1$

$\beta = 1$

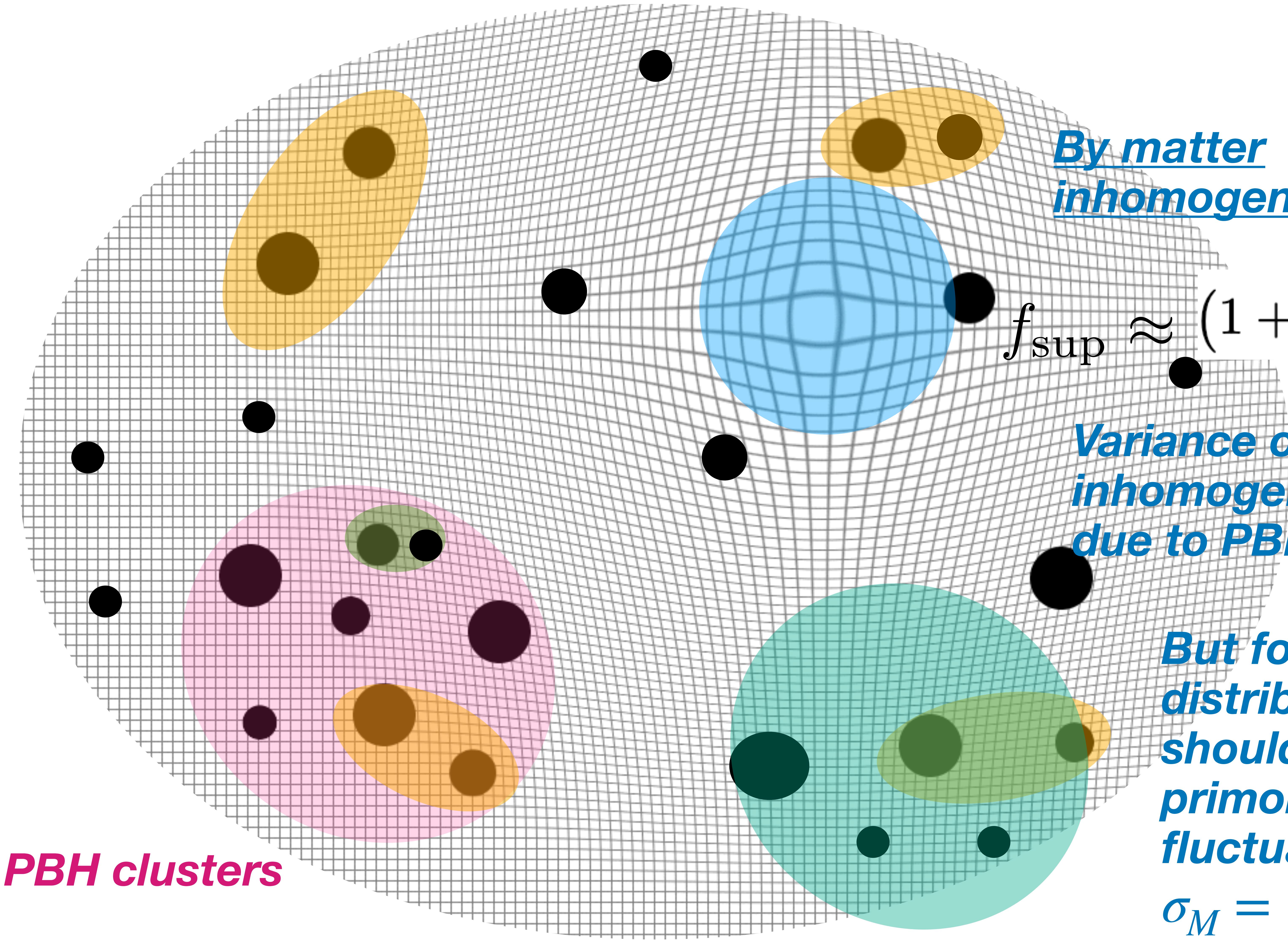


$\beta \to \infty$



Early binaries become subdominant, except for subsolar-mass PBHs !

4. PBH binary perturbations/dirsuption



By PBH clusters

By matter inhomogeneities

$$f_{\rm sup} \approx \left(1+\sigma_{\rm M}^2/f_{\rm PBH}^2\right)^{-21/74}$$

Variance of matter inhomogeneities due to PBHs : $\sigma_{\rm M} \simeq 0.004$

But for broad mass distributions, one should also have primordial matter fluctuations with $\sigma_M = \mathcal{O}(0.1)$

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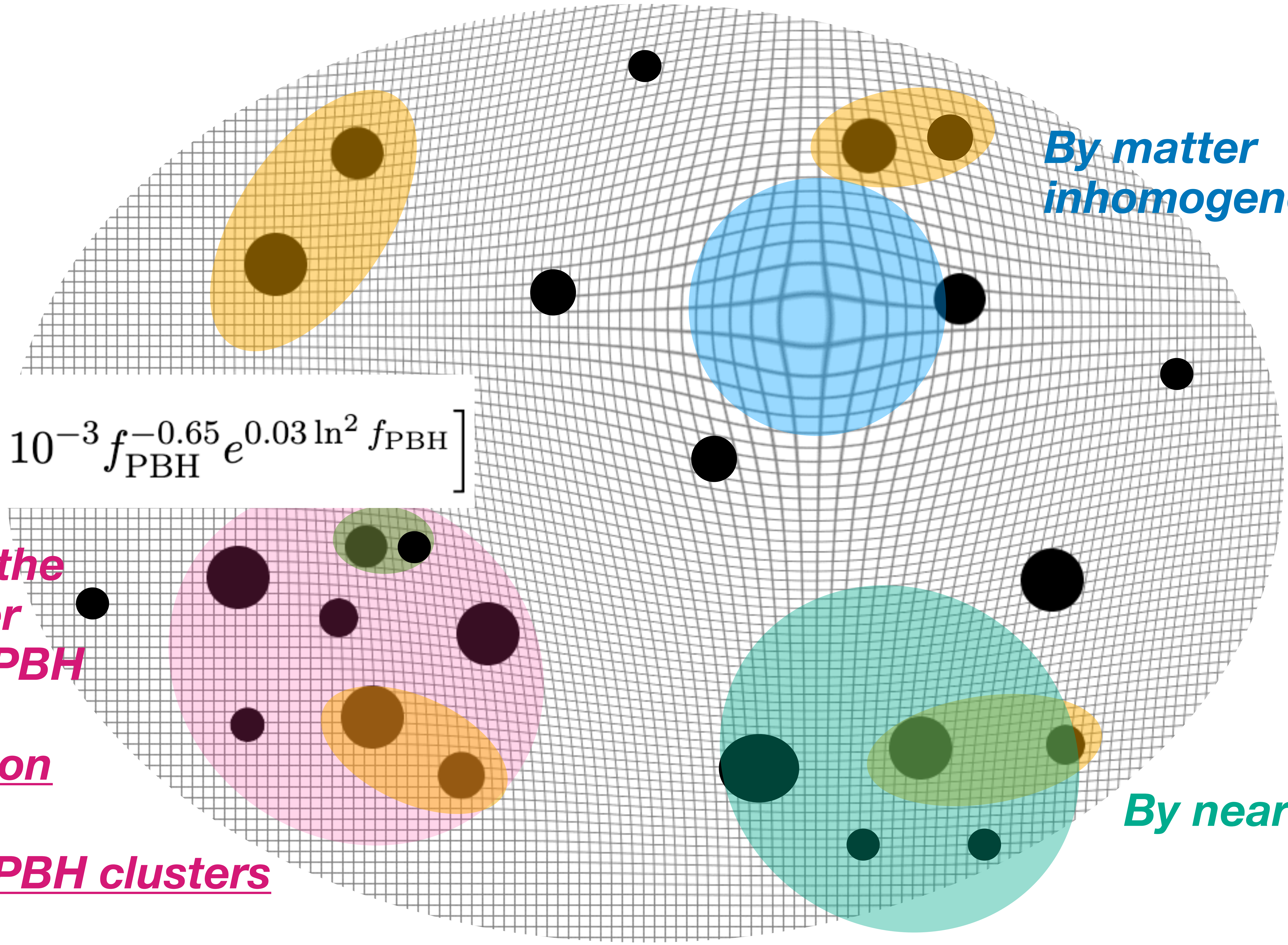


By matter inhomogeneities

$$f_{\rm sup} \approx \min\left[1,\, 9.6\times10^{-3} f_{\rm PBH}^{-0.65} e^{0.03\ln^2 f_{\rm PBH}}\right]$$

Takes into account the probability of cluster formation and the PBH crossing time, <u>for a peaked mass function</u>

By nearby PBHs

<u>By PBH clusters</u>

4. PBH binary perturbations/dirsuption

Size r_c and mass M_c of clusters seeded by Poisson fluctuations, impacted by dynamical heating

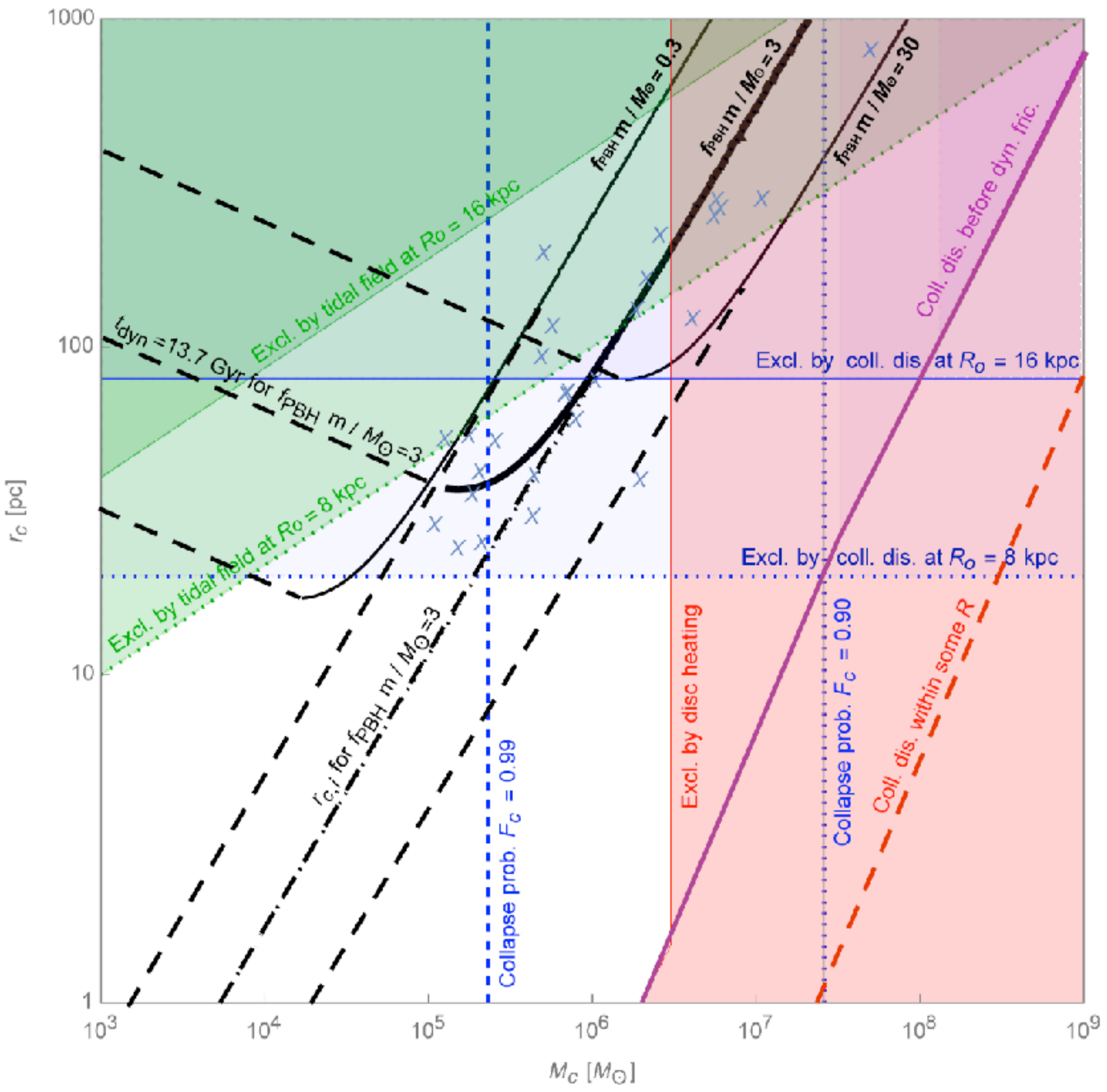
Broad mass function:

$$f_{PBH} m_{\rm PBH} \leftrightarrow f_{\rm PBH} \langle f \times m_{\rm PBH} \rangle$$

A broad mass function with seeds of supermassive black holes $m_{\rm PBH} \sim 10^3 M_\odot$ and $f_{\rm PBH} \sim 3 \times 10^{-3}$ lead to the same clustering than with $m_{\rm PBH} \sim 3 M_\odot$ and $f_{\rm PBH} \sim 1$

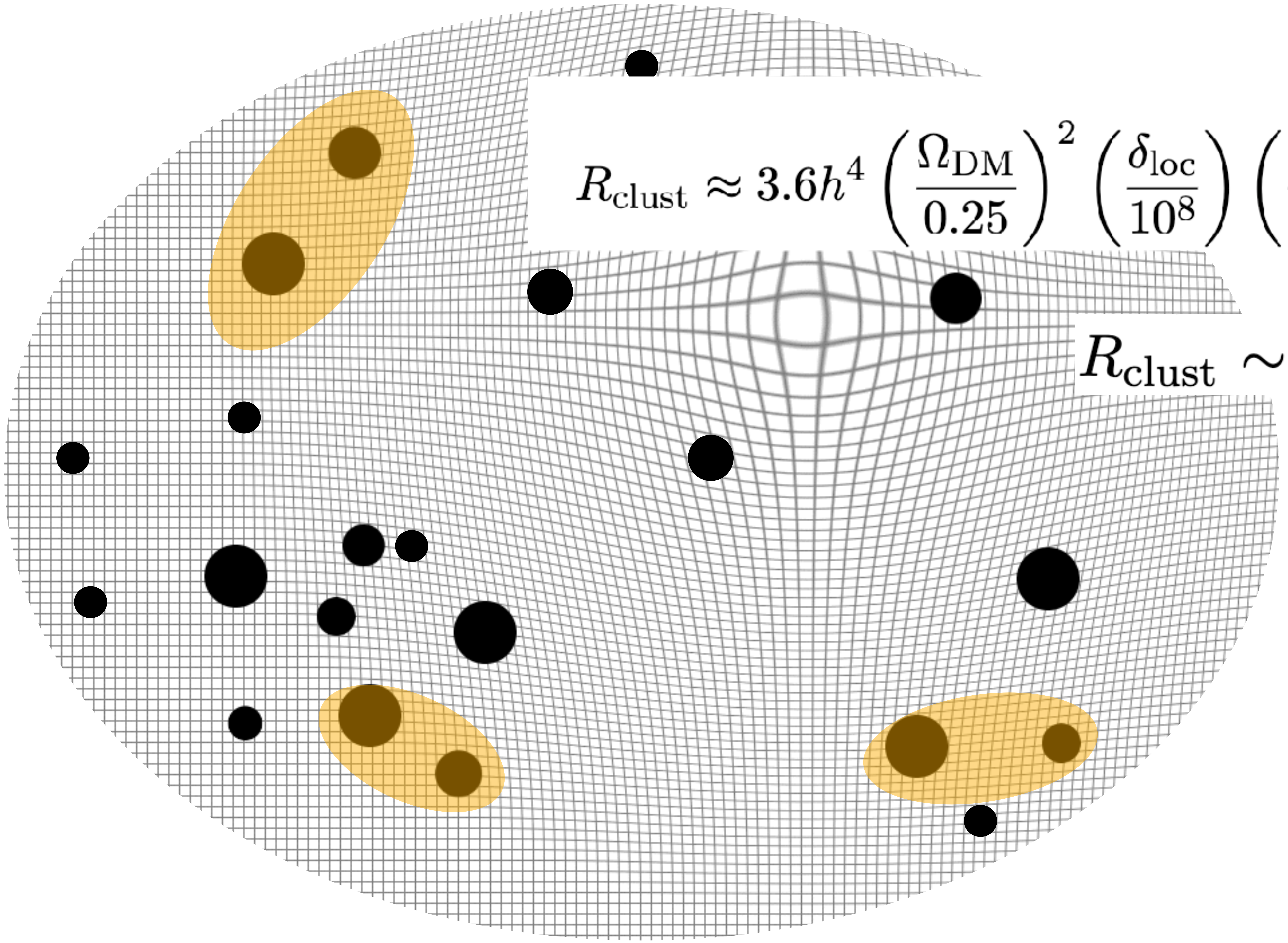
In this case, the rate of stellar-mass early binaries should also be heavily suppressed by Poisson clusters

Implications for the rate of late binaries…



B. Carr, S.C., J. Garcìa-Bellido, M. Hawkins, F. Kühnel arXiv:2306.03903

5. Late binary formation



$$R_{\rm clust} \approx 3.6 h^4 \left(\frac{\Omega_{\rm DM}}{0.25}\right)^2 \left(\frac{\delta_{\rm loc}}{10^8}\right) \left(\frac{v_0}{10\ {\rm km/s}}\right)^{-11/7}$$

$$R_{\rm clust} \sim 10^2 - 10^3.$$

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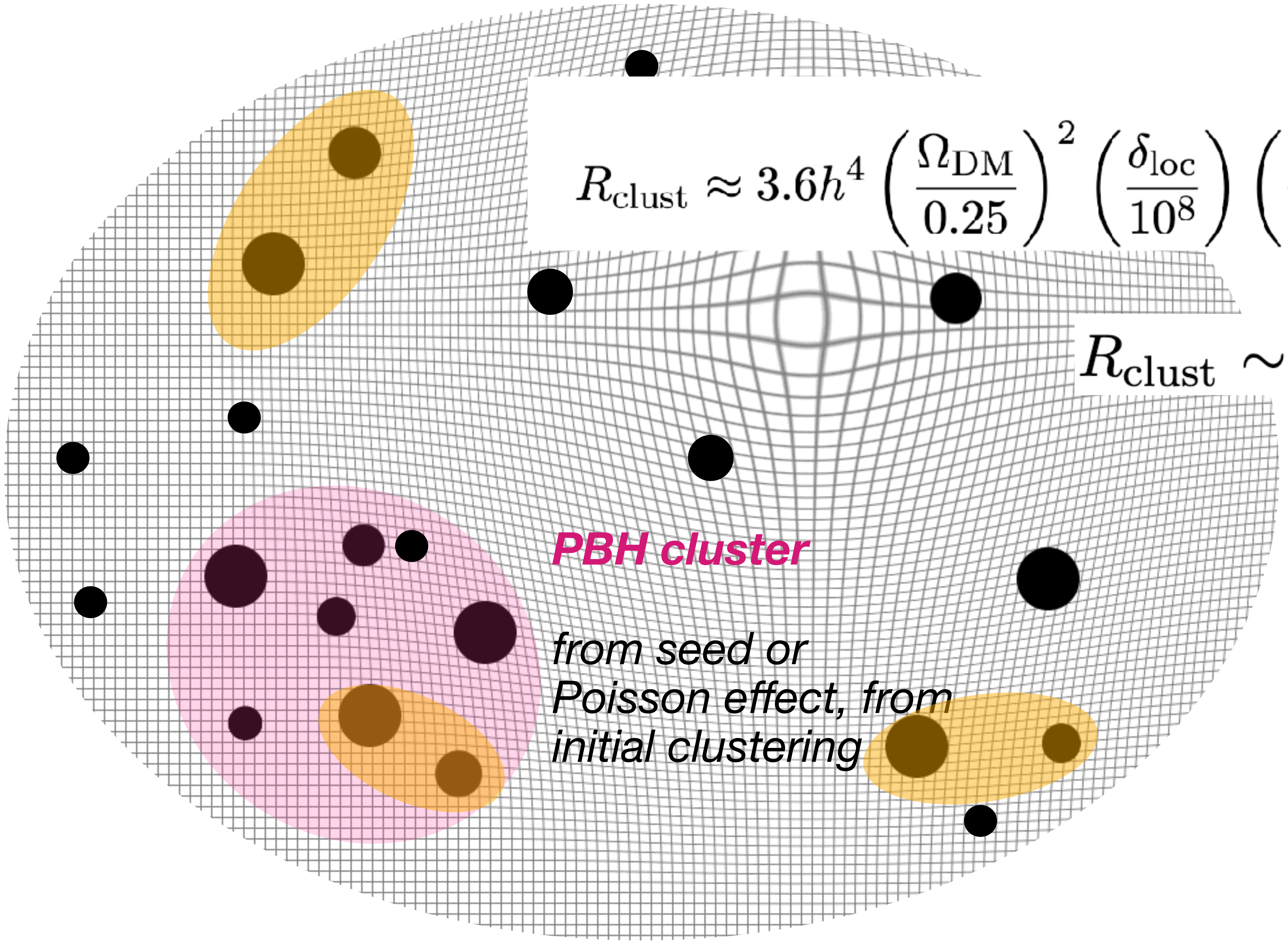

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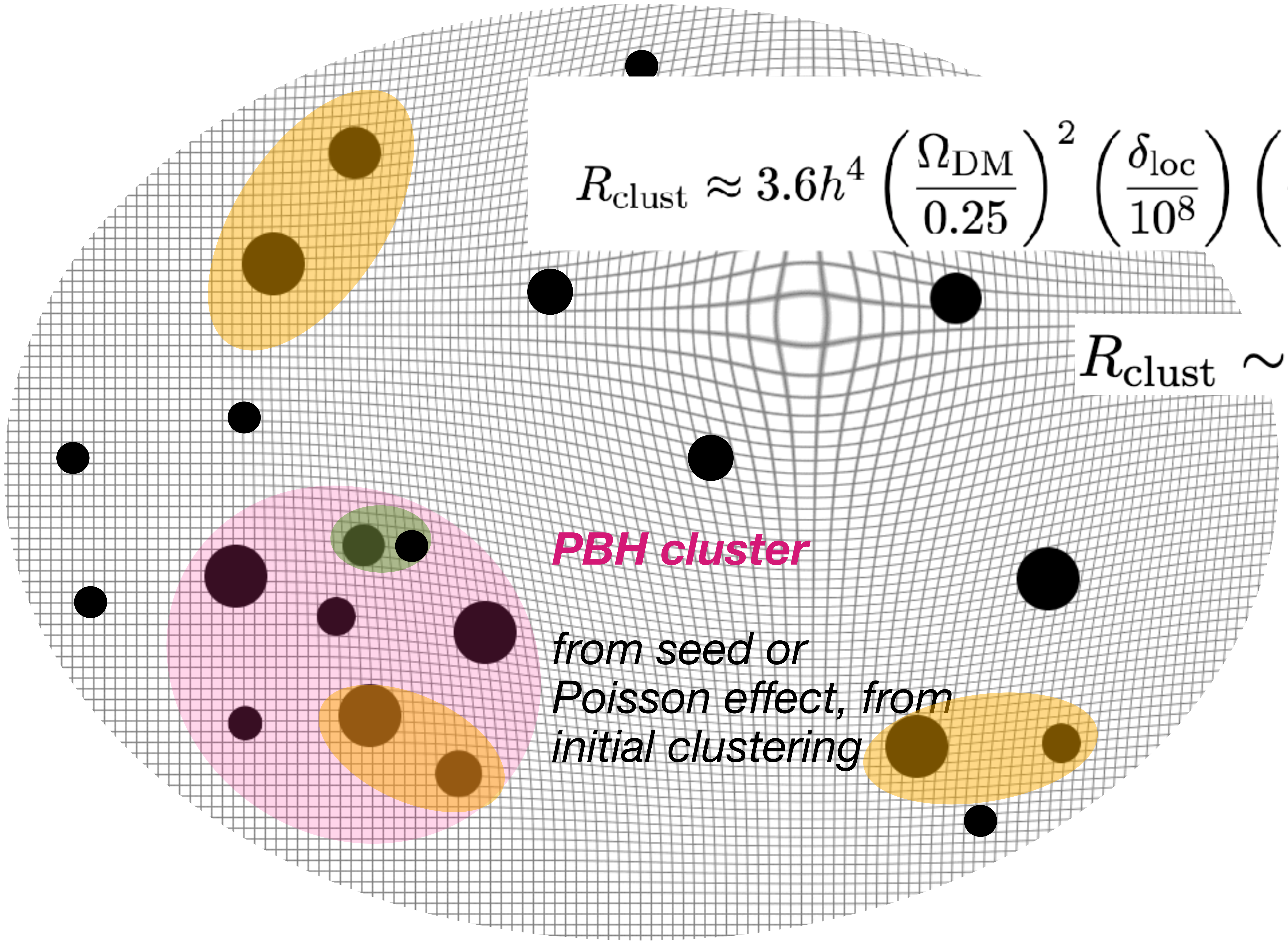

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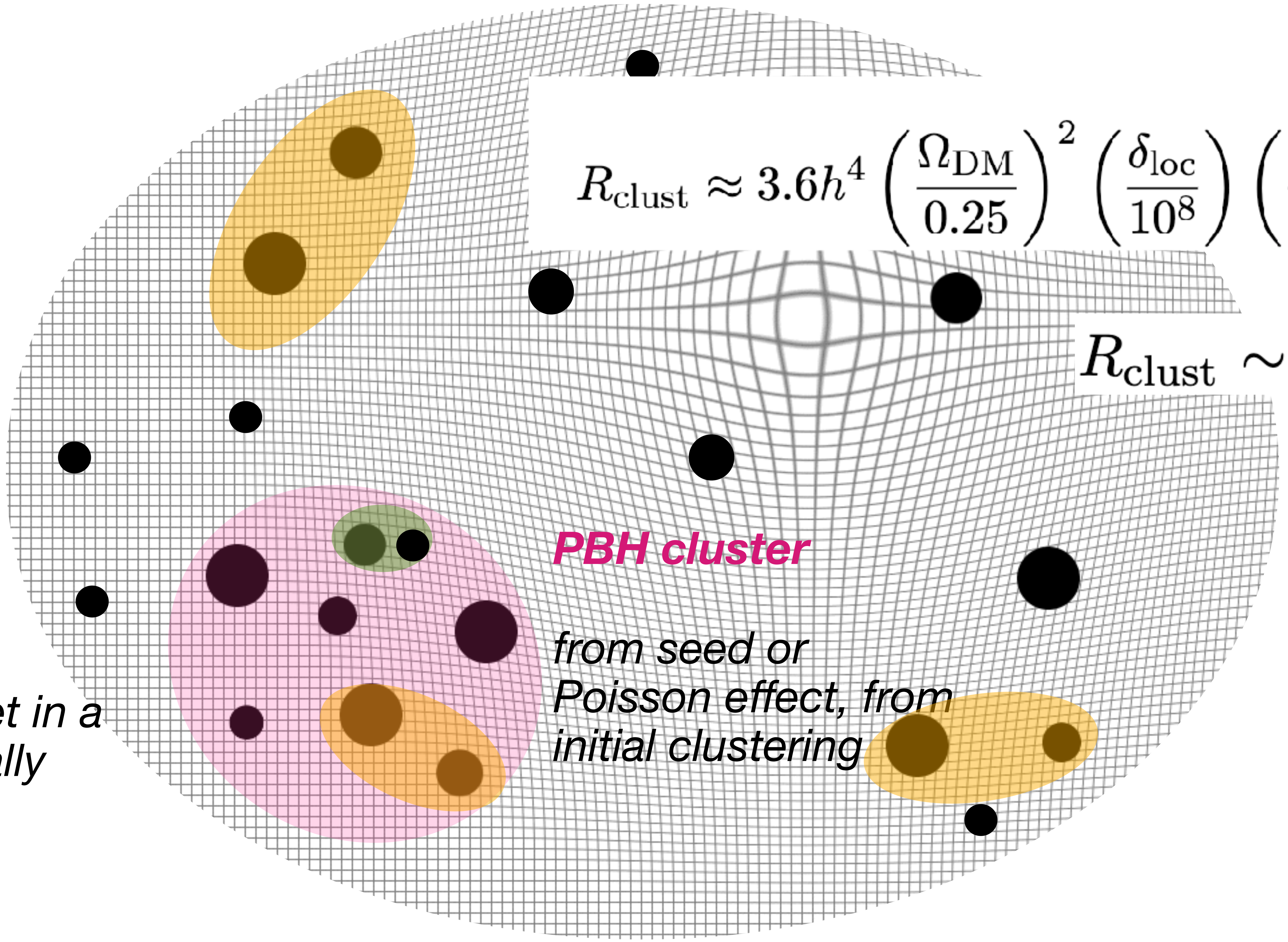

Late binaries:

When two PBHs meet in a cluster and dynamically form a binary

5. Late binary formation

$$R_{\rm LB} \approx R_{\rm clust} f_{\rm PBH}^2 f(m_1) f(m_2) \frac{(m_1+m_2)^{10/7}}{(m_1 m_2)^{5/7}} {\rm yr}^{-1}{\rm Gpc}^{-3}$$

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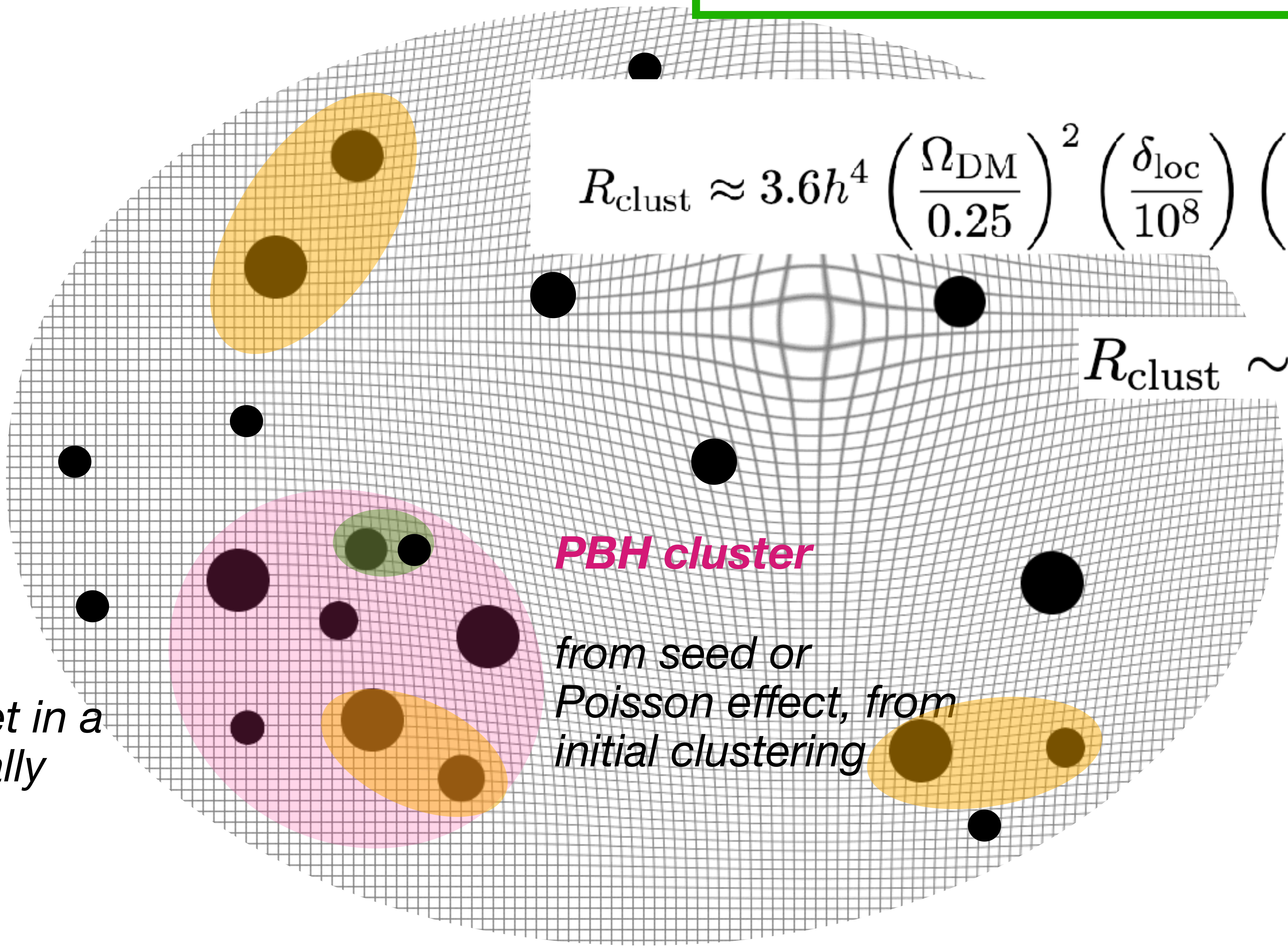

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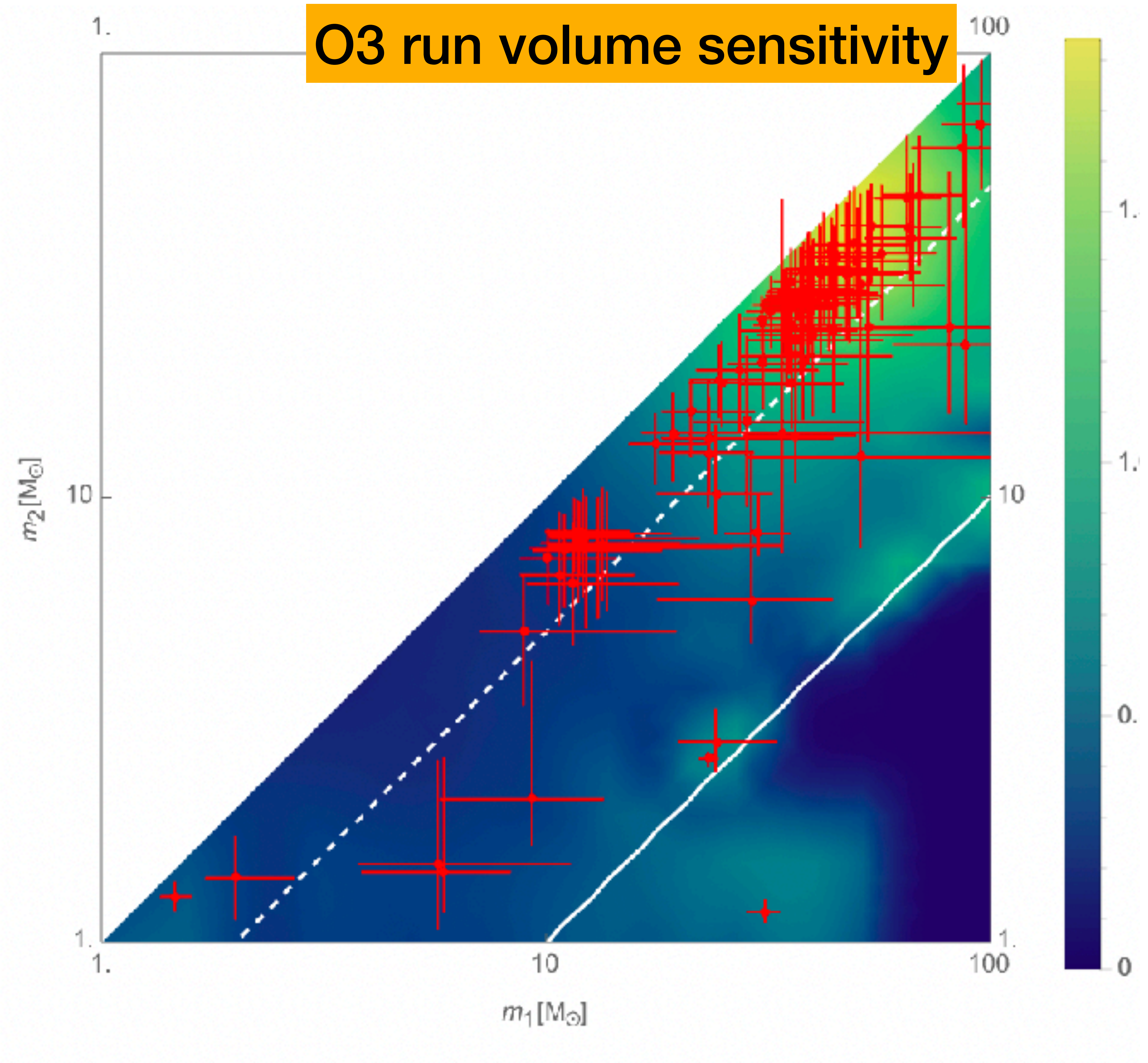
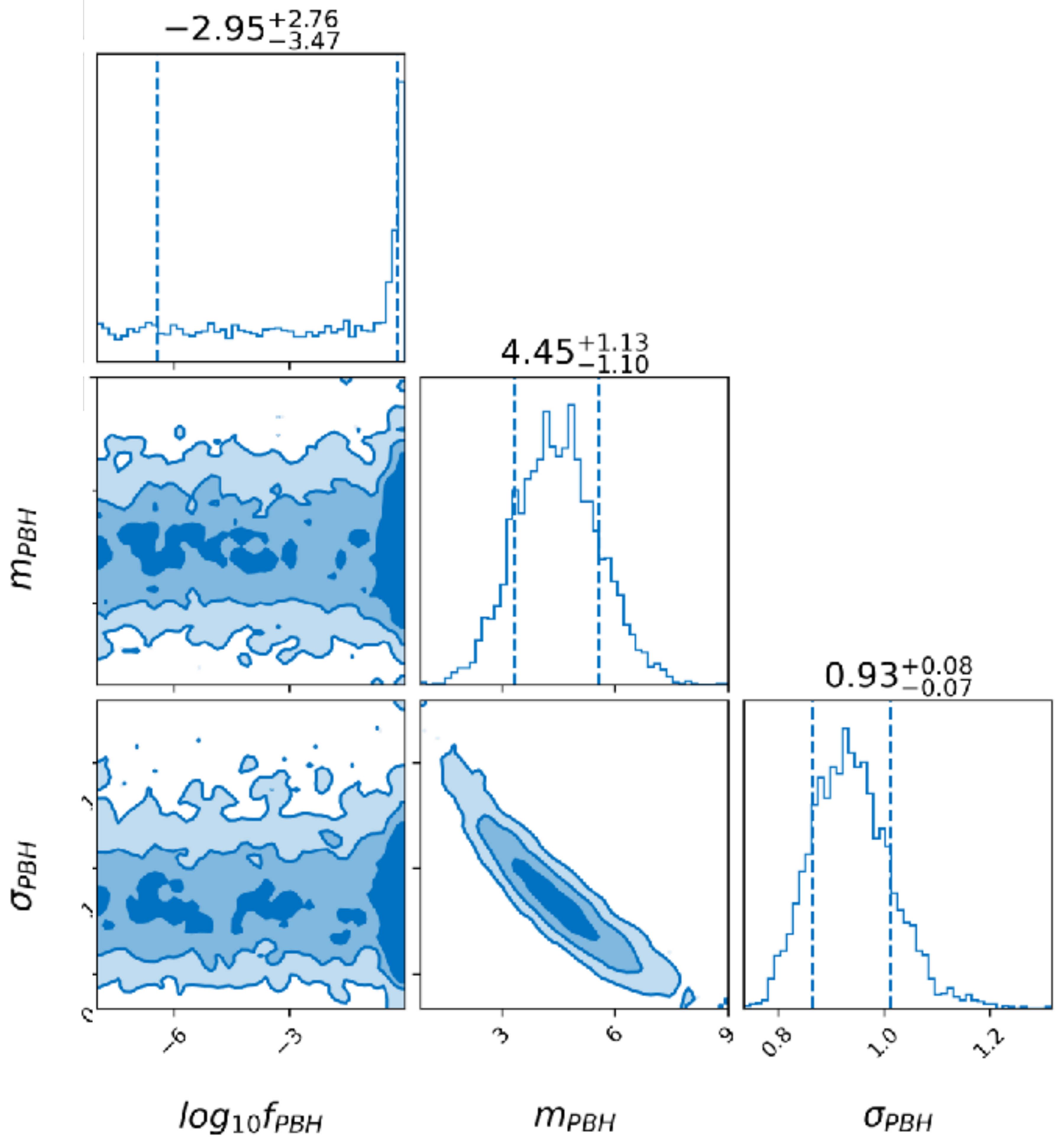
$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} \right\rangle = \langle VT \rangle \times \left(\frac{\mathrm{d}\tau_{\mathrm{clust}}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} + \frac{\mathrm{d}\tau_{\mathrm{prim}}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} \right)$$

O3 run volume sensitivity





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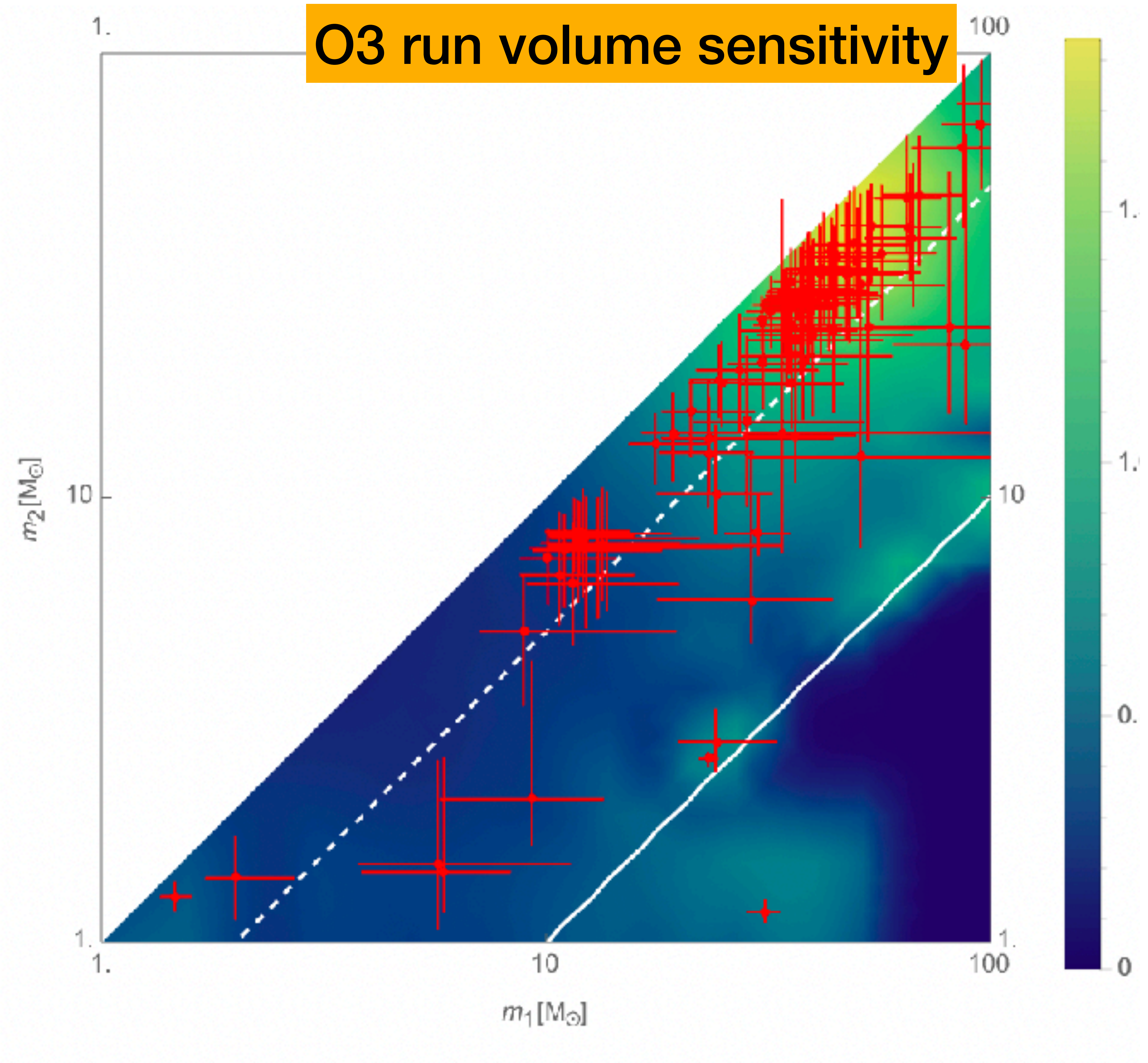

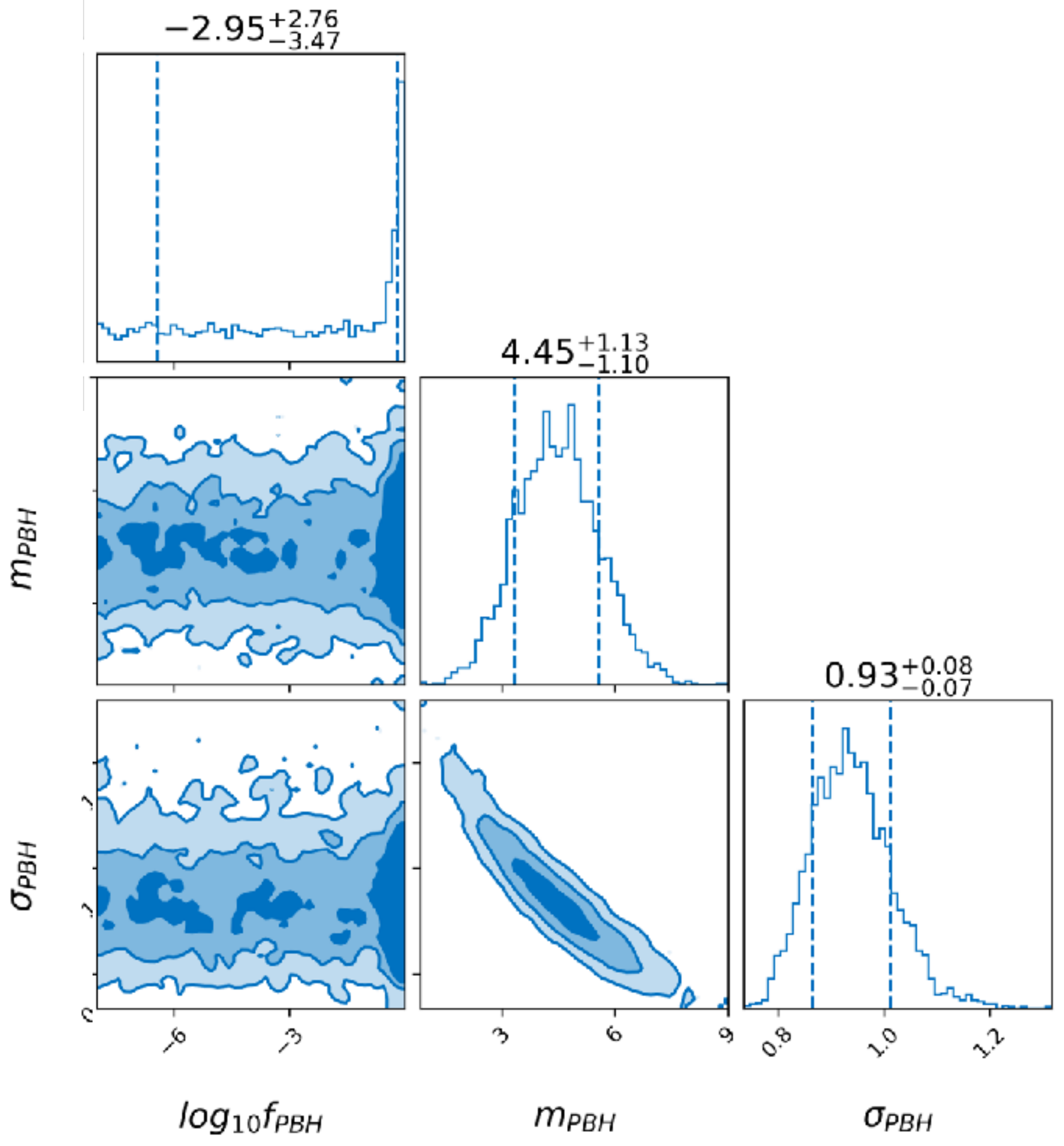

Bayesian population analysis of late binaries with GWPopulation, for a log-normal mass distribution. Credit: G. Poth

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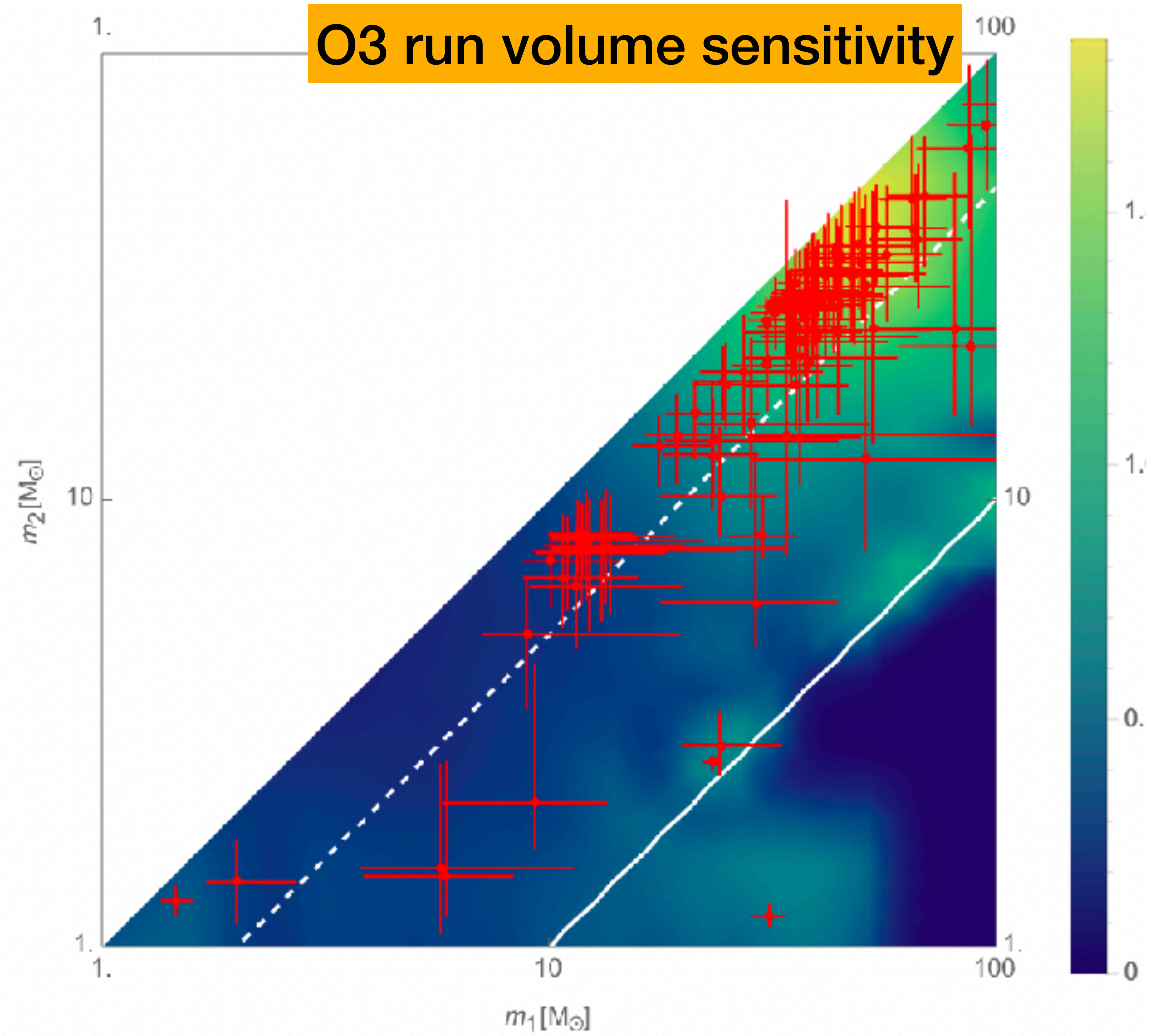
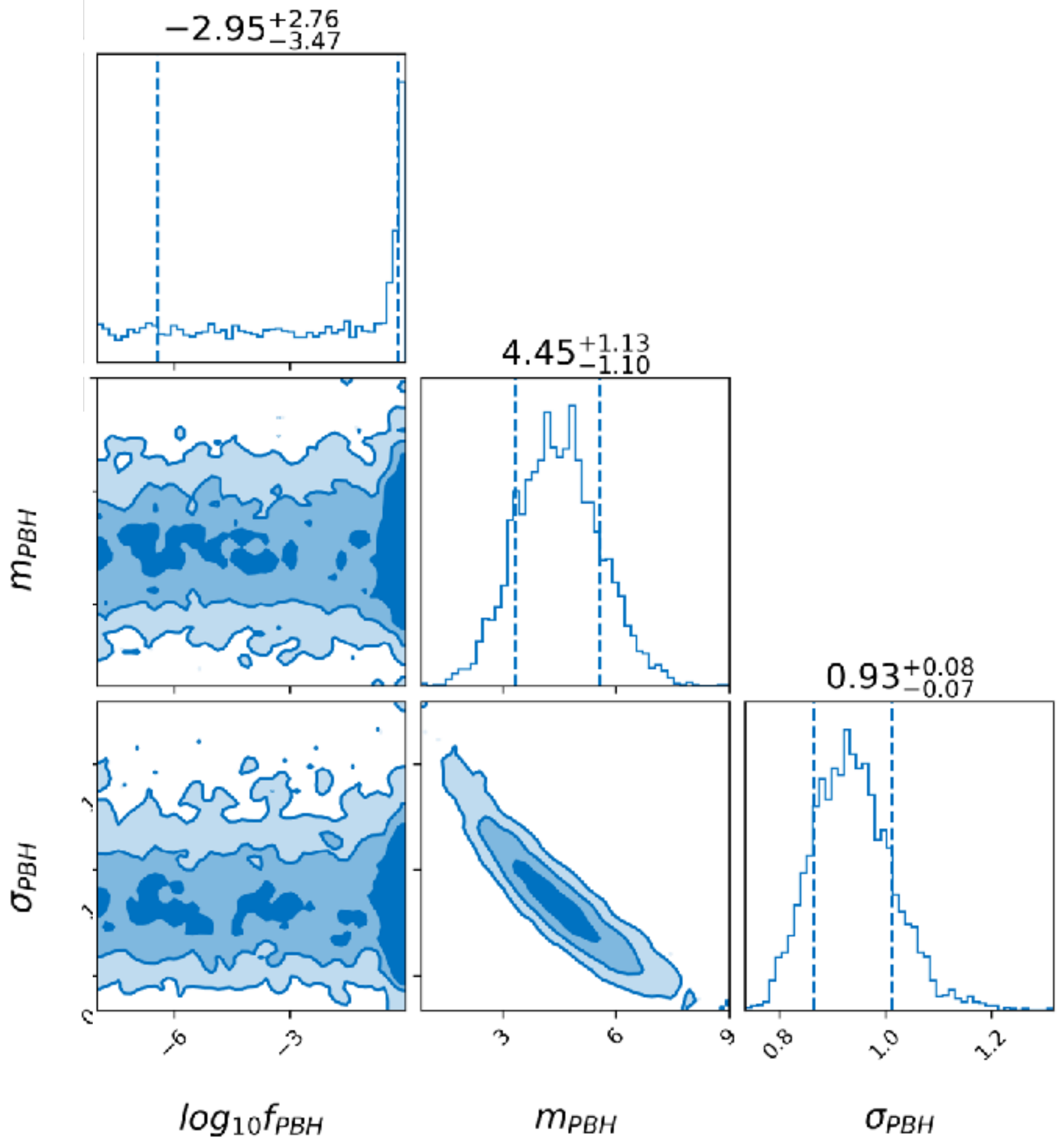


But still uncertainties in the rates: e.g. due to mass segregation (energy equipartition ?) cluster mass function and evolution…



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- **Ongoing work**: Refined calculations, Bayesian population analysis, re-estimation of subsolar-mass rates…
- **For some well-motivated mass distributions (QCD features), LIGO/Virgo do not exclude** $f_{\rm PBH} = 1$

Escrivà et al [76] Clesse et al.* [74] Bagui et al. [439]	Raidal et al. [65] Franciolini et al.* [87] Hall et al. [71]	Hutsi et al. [75]	Kocsis et al. [437]
$f \equiv \frac{1}{\rho_{\rm PBH}} \frac{{\rm d}\rho_{\rm PBH}}{{\rm d}\ln m}$	$\psi_1 \equiv \frac{1}{\rho_{\rm PBH}} \frac{{\rm d}\rho_{\rm PBH}}{{\rm d}m}$	$\psi_2 \equiv \frac{1}{n_{\rm PBH}} \frac{{\rm d}n_{\rm PBH}}{{\rm d}\ln m}$	$\psi_3 \equiv \frac{1}{n_{\rm PBH}} \frac{{\rm d}n_{\rm PBH}}{{\rm d}m}$
$f = m\psi_1$ $= m\psi_2/\langle m\rangle$ $= m^2\psi_3/\langle m\rangle$	$\psi_1 = f/m$ $= \psi_2/\langle m\rangle$ $= m\psi_3/\langle m\rangle$	$\psi_2 = f\langle m\rangle/m$ $= \langle m\rangle\psi_1$ $= m\psi_3$	$\psi_3 = \langle m\rangle f/m^2$ $= \langle m\rangle\psi_1/m$ $= \psi_2/m$
$\int f{\rm d}\ln m = 1$	$\int \psi_1{\rm d}m = 1$	$\int \psi_2{\rm d}\ln m = 1$	$\int \psi_3{\rm d}m = 1$
$\langle m\rangle = \left(\int \frac{f}{m}{\rm d}\ln m\right)^{-1}$	$\left(\int \frac{\psi_1}{m}{\rm d}m\right)^{-1}$	$\int m\psi_2{\rm d}\ln m$	$\int m\psi_3{\rm d}m$
$\langle m^2\rangle = \langle m\rangle \int mf{\rm d}\ln m$	$\langle m\rangle \int m\psi_1{\rm d}m$	$\int m^2\psi_2{\rm d}\ln m$	$\int m^2\psi_3{\rm d}m$

Table 2. Different definitions of the normalized PBH mass distribution proposed in various references with their conversion, their normalisation rule and the corresponding $\langle m\rangle$ and $\langle m^2\rangle$. The asterisk denotes the references in which an inconsistency has been found (see details in the text). In this review, we considered both f and $\psi_1 \equiv \psi$ with our notations. Table adapted from [76].