Primordial black hole formation from self-resonant preheating?

Based on 2406.09122, with Guillermo Ballesteros, Joaquim Iguaz Juan, and Marco Taoso

Pasquale Dario Serpico (Annecy, France)
NEHOP, 18/06/24, Edinburgh

On PBH formation pathways: General motivation

Schematically, two types of mechanisms in the literature

Set $\delta > \delta_c$ at super-horizon scales → collapse soon after its main Fourier mode re-enters the horizon → requires non-standard inflation, departing from slow-roll and/or single field; high degree of tuning is typical

In particular, see talk by Andrew Gow tomorrow

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Collapse of large sub-horizon inhomogeneities (e.g. bubble collisions in 1st order PTs, dynamics of topological defects…) → non-generic early universe physics (fields & symmetry groups, breaking pattern, supercooling…)

Any sub-horizon mechanism generically associated with inflation?

PBH from (metric) self-resonant preheating

Self-resonant growth in the post-inflationary era due to the metric feedback, computed in a quadratic approximation of the (reheating-stage) inflaton potential→ typically collapse into PBH!

K. Jedamzik, M. Lemoine and J. Martin, 1002.3039, J. Martin, T. Papanikolaou and V. Vennin, 1907.04236 ...

Contrarily to alternative pathways, this would be (largely) independent of the inflaton potential, of the presence of other fields and their couplings with the inflaton!

Plan

1. Review the proposed mechanism: parametric resonance
(Explaining the jargon in the previous slide…experts can take a nap)
2. Arguments advanced to foresee PBH formation: An appraisal
3. More realistic models: a 'new new hope' via anharmonic terms?
4. Lattice studies
5. Conclusions

I. Review of the proposed mechanism

linear perturbation theory @ end of inflation

Inflaton condensate ϕ

determines background expansion

$$H^2 = \frac{1}{3M_\mathrm{P}^2}\left(\frac{\dot\phi^2}{2} + V\right),\quad \dot H = -\frac{\dot\phi^2}{2M_\mathrm{P}^2}$$

$$\ddot\phi + 3H\dot\phi + \mathrm{d}V/\mathrm{d}\phi = 0$$

$\epsilon = -\dot H/H^2$ Hubble (first) slow-roll parameter

$\epsilon_V \equiv M_\mathrm{P}^2(V_{,\phi}/V)^2/2$ Potential slow-roll parameter

$\mathrm{d}N = \mathrm{d}\phi/(\sqrt{2\epsilon}M_\mathrm{P})$ Number of e-folds

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Fluctuations, described by a

single (curvature) perturbation $\mathcal{R}$ or

$$\tilde v = \sqrt{2\epsilon}\, M_\mathrm{P}\, a^{3/2}\, \mathcal{R}$$

e.g. in newtonian gauge

$$\mathrm{d}s^2 = (1+2\Psi)\mathrm{d}t^2 - a^2(t)(1-2\Phi)\mathrm{d}\mathbf{x}^2$$

$$\mathcal{R} = -\Phi - H\delta\phi/\dot\phi$$

Mukhanov-Sasaki eq (in Fourier space)

$$\ddot{\tilde v} + \omega^2(k)\tilde v = 0$$

$$\omega^2 = \frac{\mathrm{d}^2V}{\mathrm{d}\phi^2} + \frac{k^2}{a^2} + \frac{2}{M_\mathrm{P}^2}\frac{\mathrm{d}V}{\mathrm{d}\phi}\frac{\dot\phi}{H} + \frac{3\dot\phi^2}{M_\mathrm{P}^2} - \frac{\dot\phi^4}{2H^2M_\mathrm{P}^4} + \frac{3}{4M_\mathrm{P}^2}P$$

Standard solution and related approximations

- Quadratic potential $$V = \frac{1}{2}m^2\phi^2$$

- Potential slow-roll condition for end of inflation

$$\ddot{a} = 0 \Leftrightarrow \epsilon = 1 \qquad \epsilon_V \simeq 1 \qquad \Rightarrow \qquad \begin{aligned} \phi_{\rm end} &\simeq \sqrt{2}M_{\rm P} \\ H_{\rm end} &\simeq m/\sqrt{2} \end{aligned}$$

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- WKB approximation, $H \ll m$ (certainly ok at late times)

$$\ddot{\phi} + 3H\dot{\phi} + {\rm d}V/{\rm d}\phi = 0 \rightarrow \phi(t) = \phi_{\rm end} \left(\frac{a_{\rm end}}{a} \right)^{3/2} \cos[m\,(t - t_{\rm end})]$$

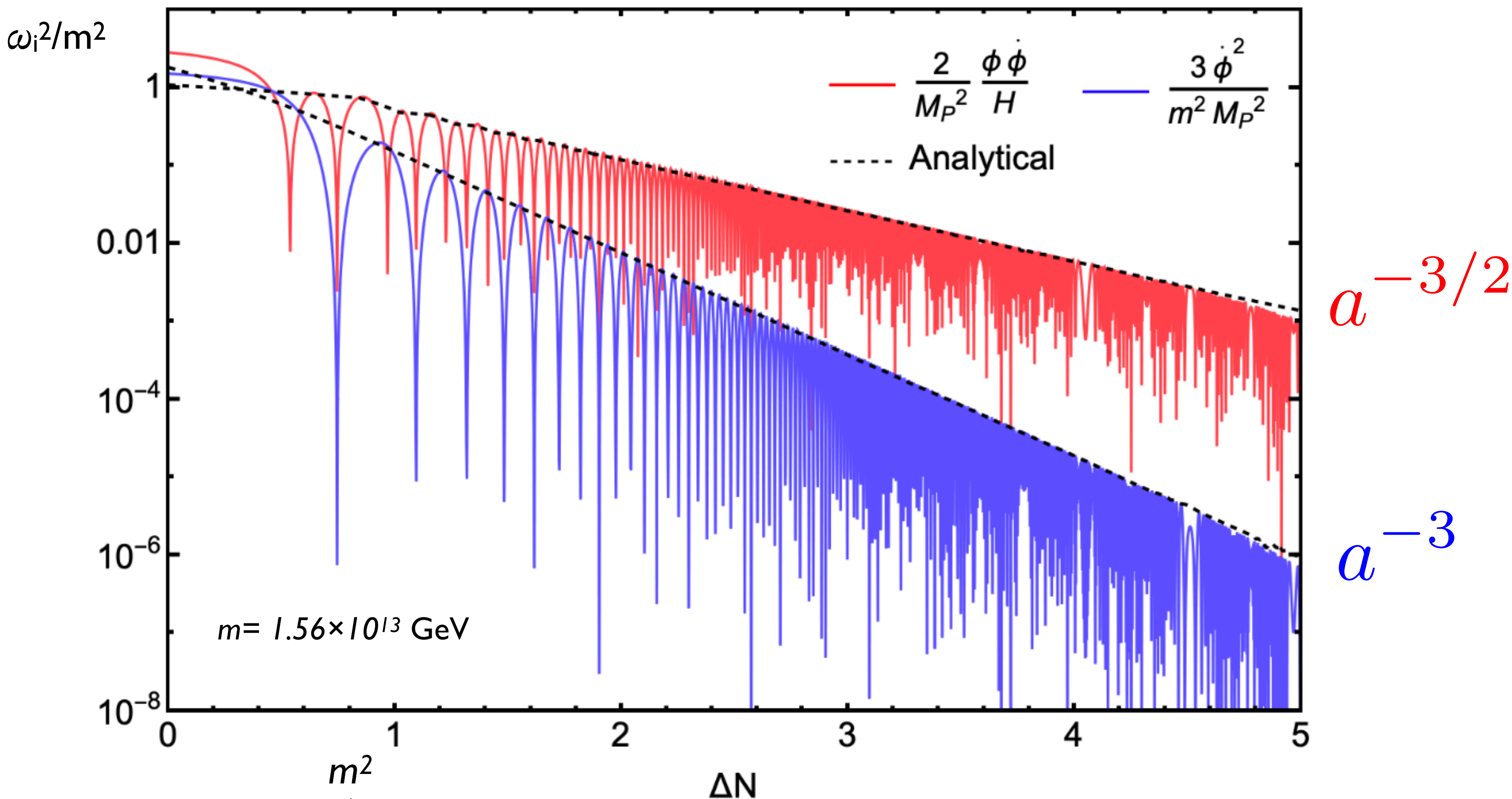
- matter-dominated expansion

(leading order in H/m,

average over fast scale $1/m$)

$$\bar{\rho}(t) \equiv m^2 \frac{\phi_{\rm end}^2}{2} \left(\frac{a_{\rm end}}{a} \right)^3 \qquad \bar{P} \simeq 0$$

Further approximation: only retain leading term @ $t \gg t_{end}$



m^2

$$\omega^2 = \frac{\mathrm{d}^2V}{\mathrm{d}\phi^2} + \frac{k^2}{a^2} + \frac{2}{M_\mathrm{P}^2}\frac{\mathrm{d}V}{\mathrm{d}\phi}\frac{\dot{\phi}}{H} + \frac{3\dot{\phi}^2}{M_\mathrm{P}^2} - \frac{\dot{\phi}^4}{2H^2M_\mathrm{P}^4} + \frac{3}{4M_\mathrm{P}^2}P$$

Mathieu equation: Parametric resonance

change time variable $s \equiv m(t - t_{\rm end}) + 3\pi/4$

$$\frac{\mathrm{d}^2\tilde{v}}{\mathrm{d}s^2} + \Big[A(k) - 2\,q\cos(2\,s)\Big]\tilde{v} = 0$$

$$A(k) = 1 + \frac{k^2}{m^2 a^2} \qquad q = \frac{\phi_{\rm end}}{M_{\rm Pl}}\left(\frac{a_{\rm end}}{a}\right)^{3/2}$$

solutions exhibit ***parametric resonance***:

complex-valued "Floquet exponent"

$$\tilde{v}_k = e^{\mu_k s}\mathcal{P}_{k+}(s) + e^{-\mu_k s}\mathcal{P}_{k-}(s)$$

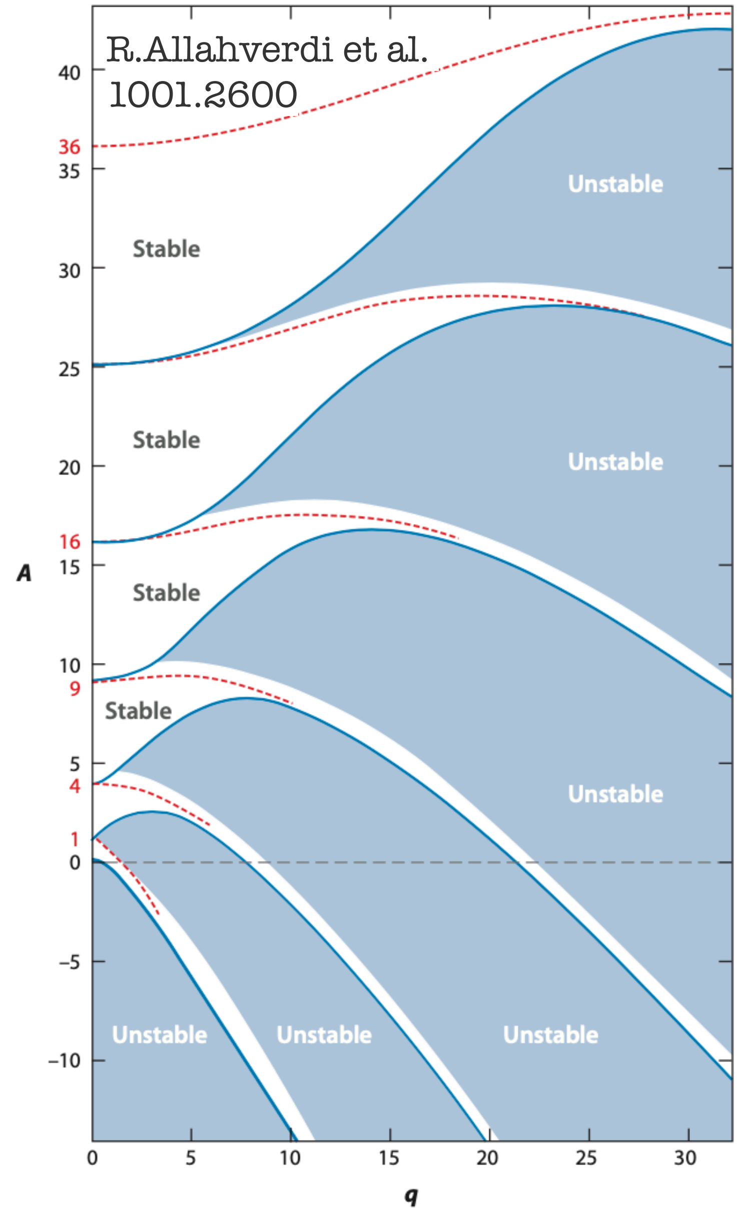
$\mathcal{P}_{k\pm}$ periodic functions of s, with period of the oscillating condensate, T=2π/m

Bands…& effects of expansion

Regions in $\{A,q\}$ where $\Re(\mu_k) \neq 0$:

exponential growth of perturbations at the expense of the condensate oscillating at the bottom of the potential

|q|≪1 **(Narrow resonance)**	$\Re(\mu_k) = \|q\|/2$
First instability band	$1 - \|q\| < A(k) < 1 + \|q\|$
i.e. growing modes	$\frac{k}{a} < \sqrt{3Hm}$



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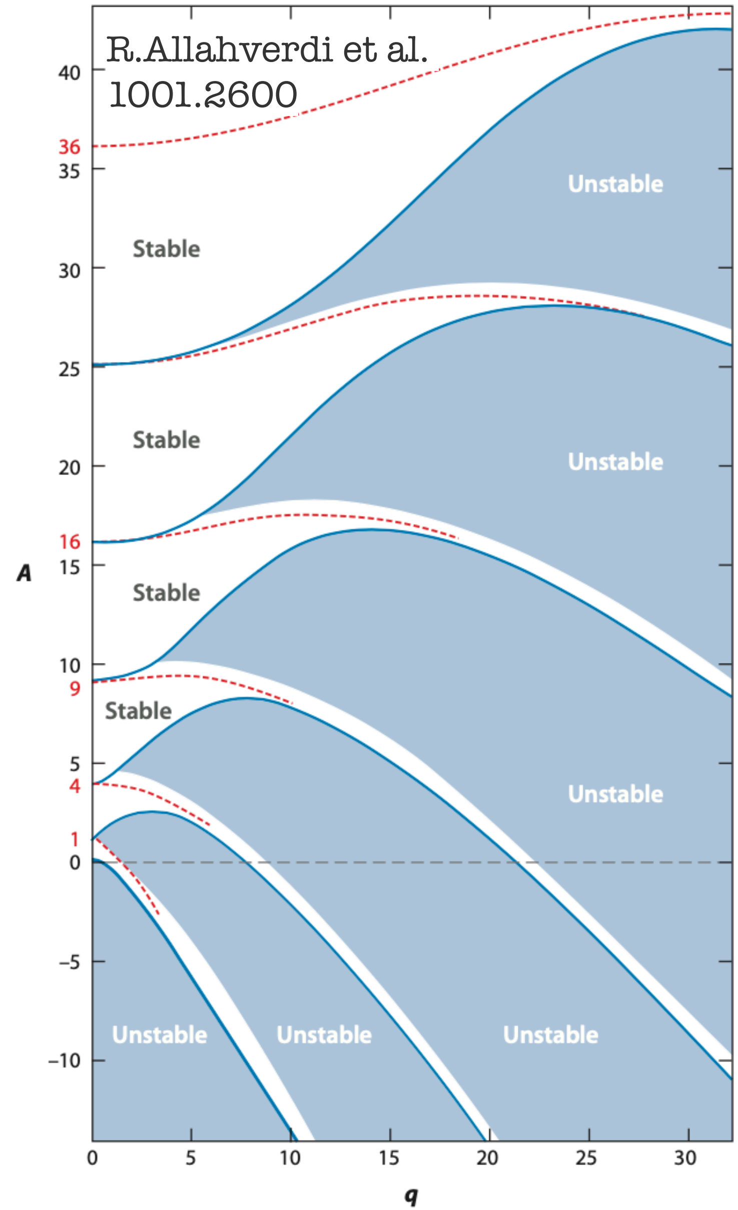
i.e. growing modes	$\frac{k}{a} < \sqrt{3Hm}$

Accounting for the Universe expansion:

- modes redshift at low k/a
- instability growth damped (not exp.)

Fostering growth requires large $\Re(\mu_k)m/H$



If instead small, adiabatic approx

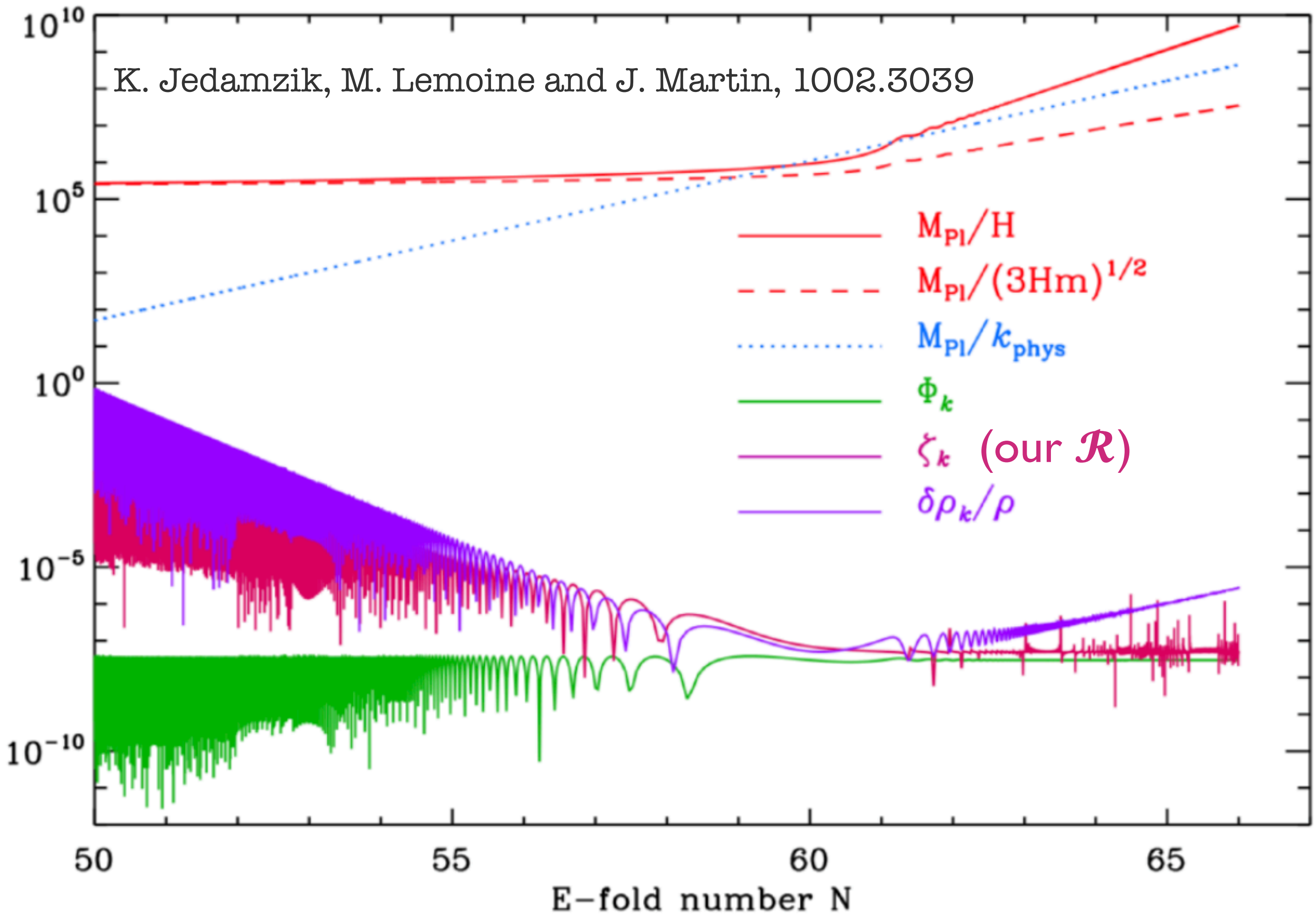
$$\tilde{v}_k \propto \exp\left(\mathrm{d}s\int\mu_k\right) \propto a^{3/2}$$

$\delta \propto a$ as in MD

& conserved $\mathcal{R}$

II. Forming PBHs…or not

Suggestion of PBH formation from linear evolution...



...but why should these curves suggest PBH formation?

When does one form a PBH?

Hoop conjecture e.g C. W. Misner, K. S. Thorne, and J. A. Wheeler, "Gravitation", pp. 867- 868

"the entirety of the object's mass must be compressed to the point that it can fit within a sphere whose radius is equal to that object's Schwarzschild radius."

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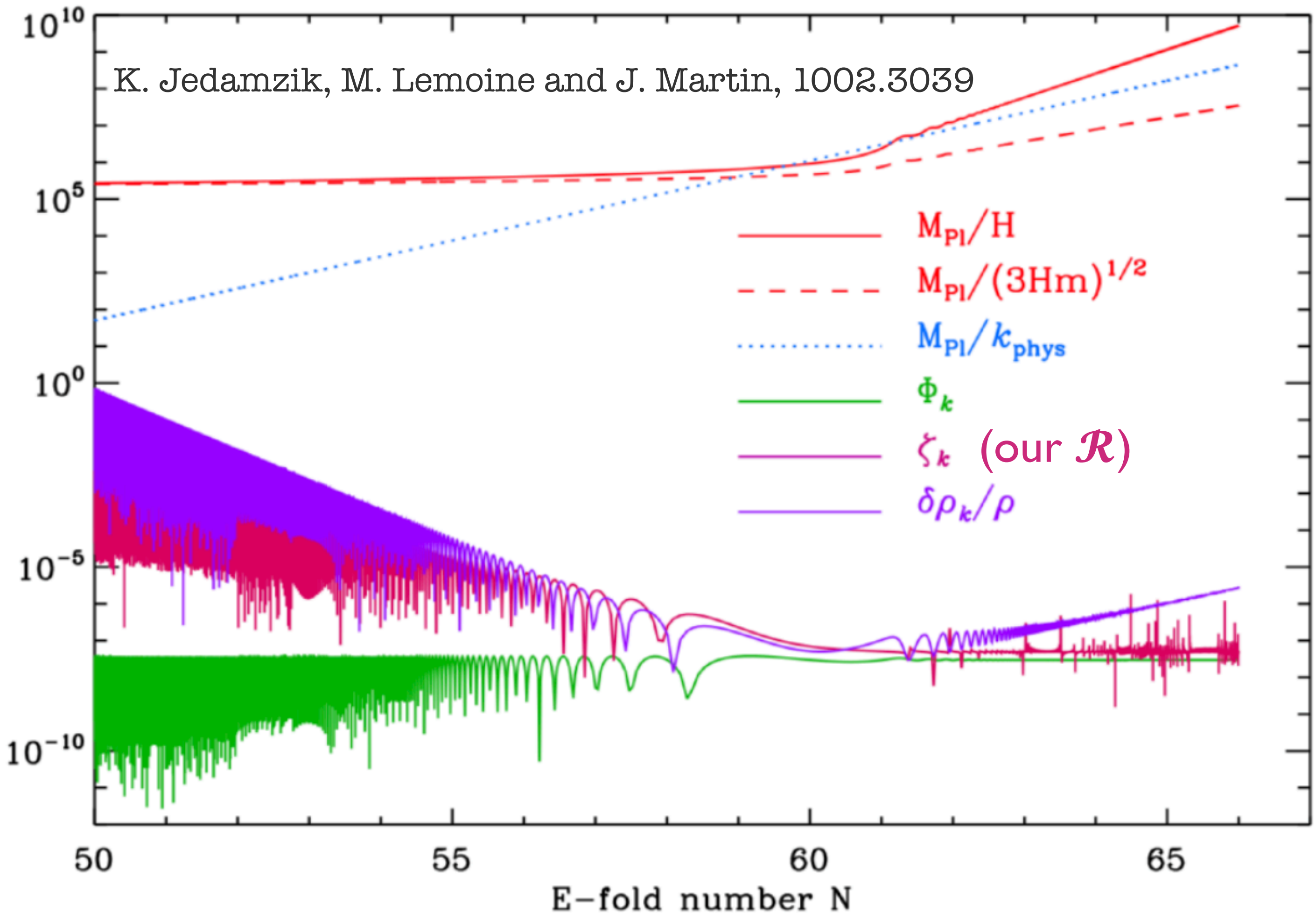
For horizon-scale perturbation, link analogue condition on the curvature to the density contrast at horizon-scale, hence criteria in the literature like δ_c~0.4 in RD or less restrictive threshold in MD, influenced by asphericity, angular momentum, etc.

In the general, sub-horizon case, one has to follow the evolution in the non-linear regime to establish if PBH form (no simple criterion exists…)

Suggestion of PBH formation from linear evolution???



Not possible from a linear analysis, of course.

Yet, linear analysis indicates $\mathcal{R}$~const. Not inching any closer to PBH formation.

PBH form according to an analytical, non-linear model

Generic formation of PBHs supported by solving the non-linear evolution equation of a spherically symmetric fluctuation of the energy density of the scalar field, evolving in the homogeneous background of the scalar field itself.

J. Martin, T. Papanikolaou and V. Vennin, 1907.04236

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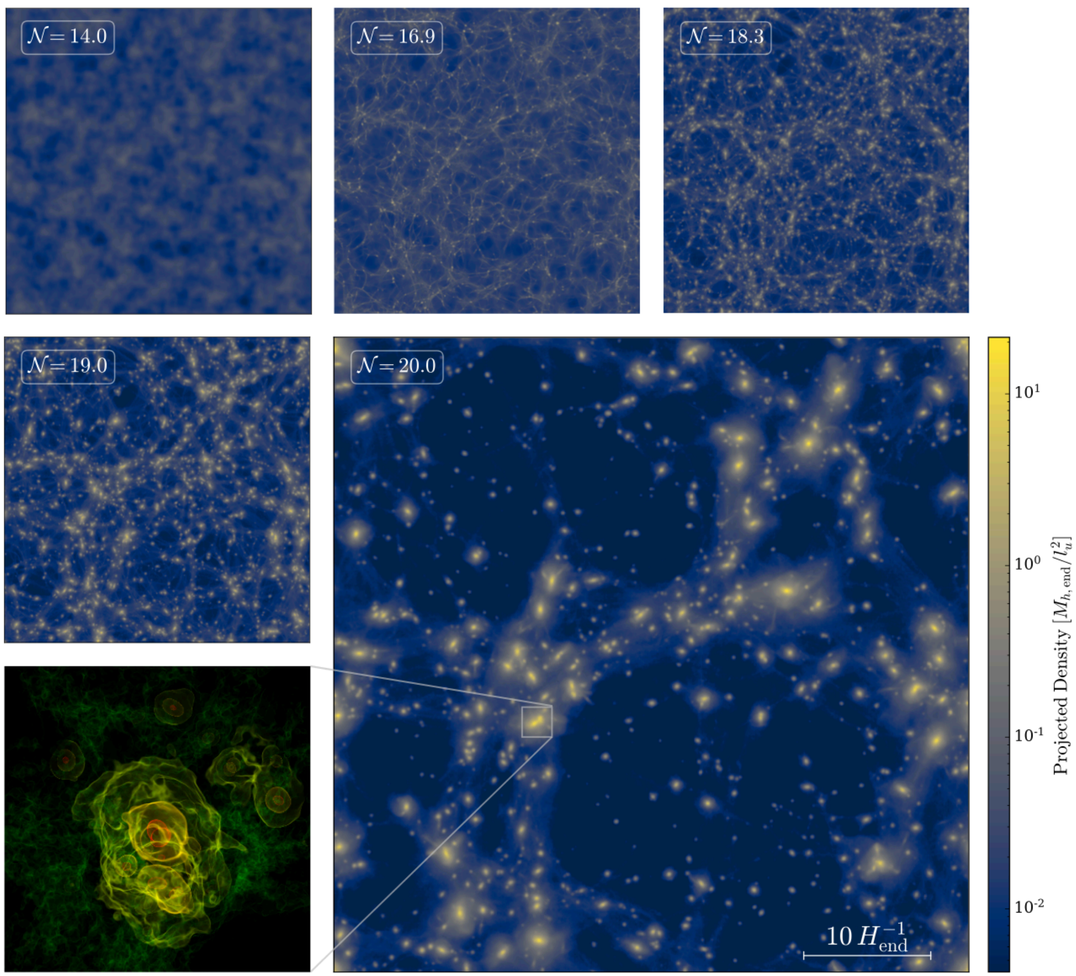
Very idealised conditions, can we trust the result?

→

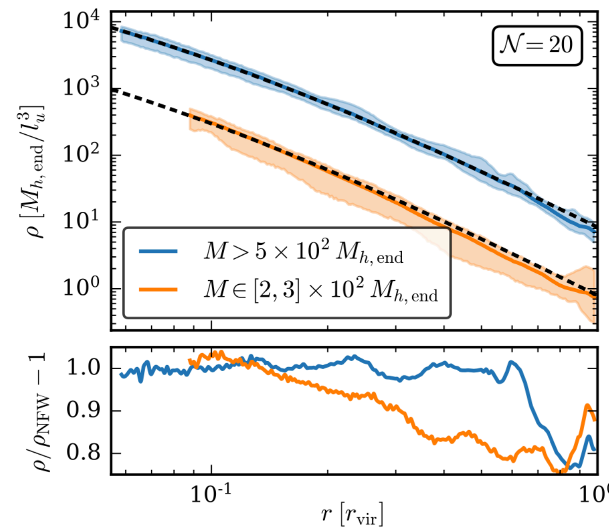
- Under same approximation, all DM fluctuations in MD would lead to BH formation
- We know that halo formation takes place, supported by velocity dispersion: Anything analogous for scalar fields?

Indeed: "Inflaton halos"!

N. Musoke, S. Hotchkiss, and R. Easther, 1909.11678
B. Eggemeier, J. C. Niemeyer, and R. Easther, 2011.13333



Solve Schrödinger-Poisson equations, eventually finding NFW-like halos.



Newtonian approximation always good ($\Phi \ll 1$), **no sign of possible PBH formation**

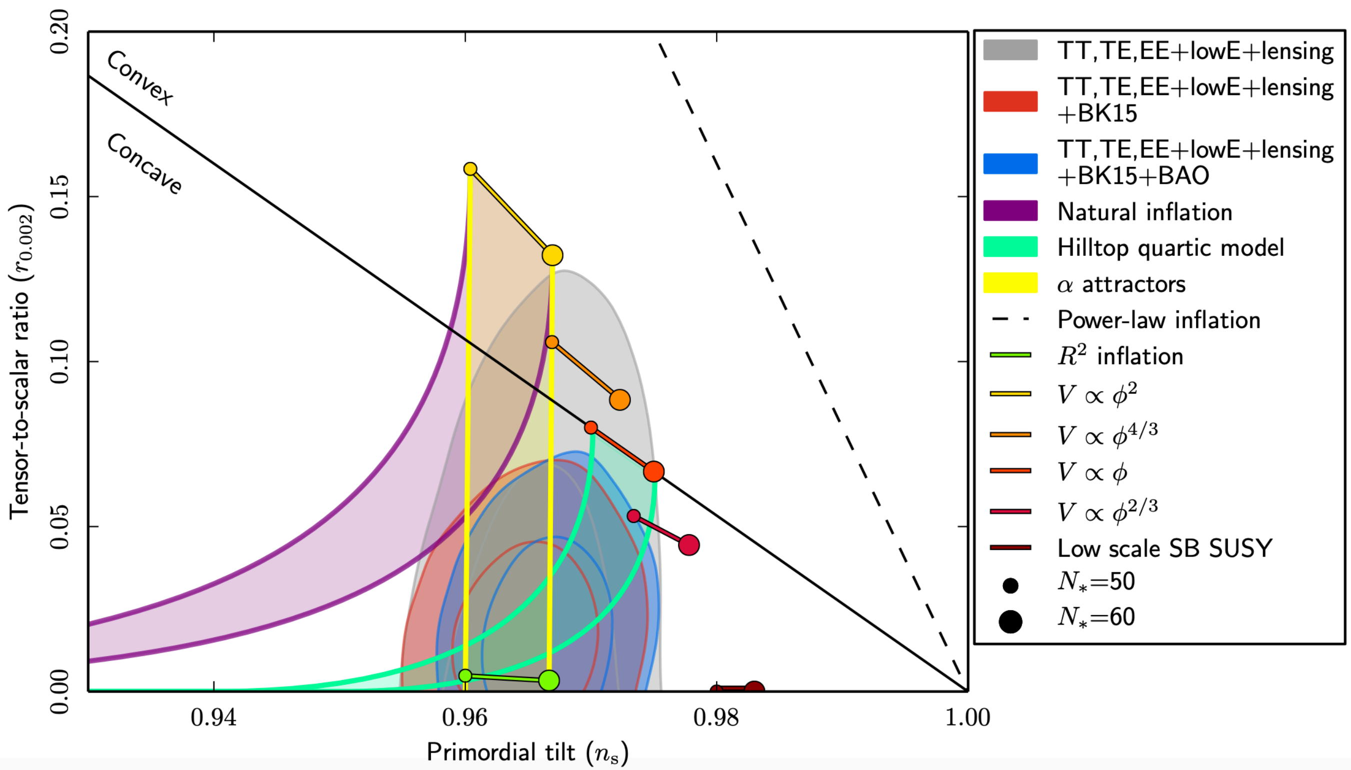
Unless solitonic core forms at N>30 e-folds (!!!) after end of inflation, different mechanism

III. Quadratic approximation and beyond

Questioning the quadratic potential approximation

Strictly speaking, quadratic potential is ruled out by CMB observations.

Can we trust calculations based on it, once embedding it in a viable potential?



Planck collaboration, 1807.06211

Embedding in α-attractor classes of models

T-models

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{M}\right)$$

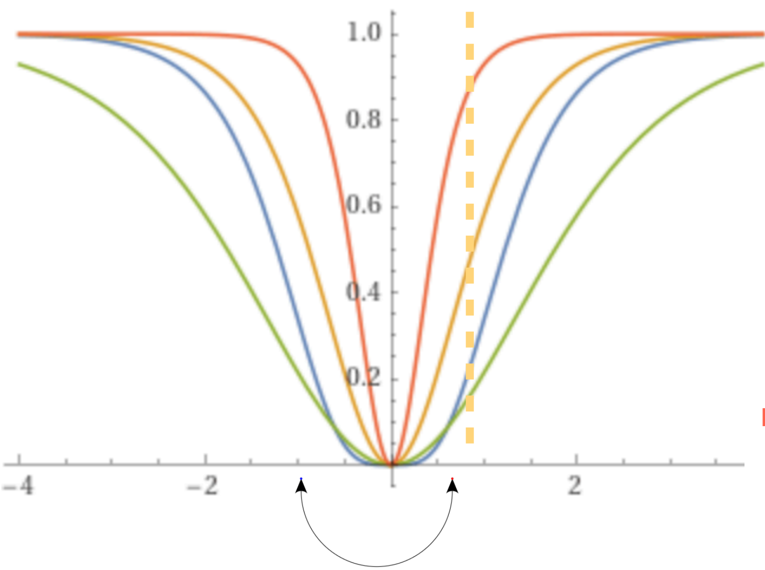
E-models

$$V(\phi) = \Lambda^4 \left|1 - \exp\left(-\frac{\phi}{M}\right)\right|^{2n}$$

Starobinsky for $M = \sqrt{3/2}M_{\rm P}$

n=1

$\phi \sim M$ Inflation (plateau)



Preheating

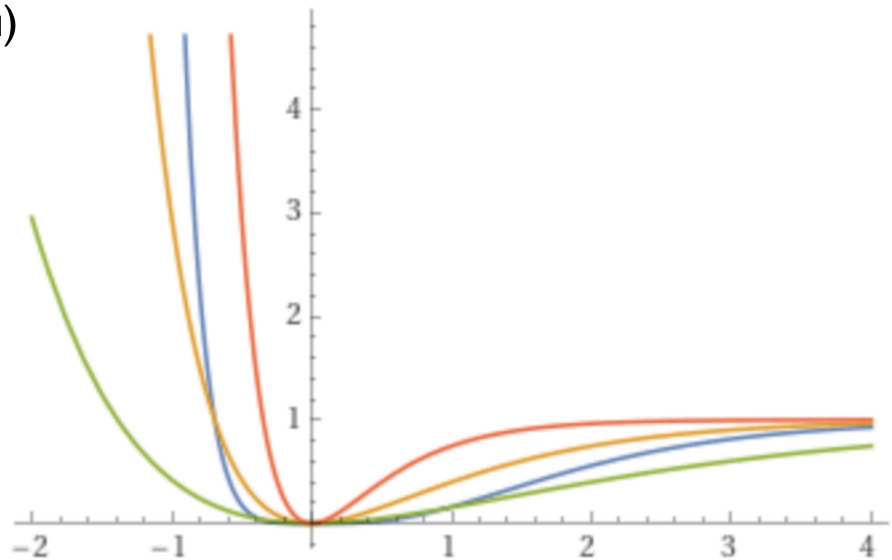
(Quadratic for n=1)

n=2, M=1

n=1, M=1

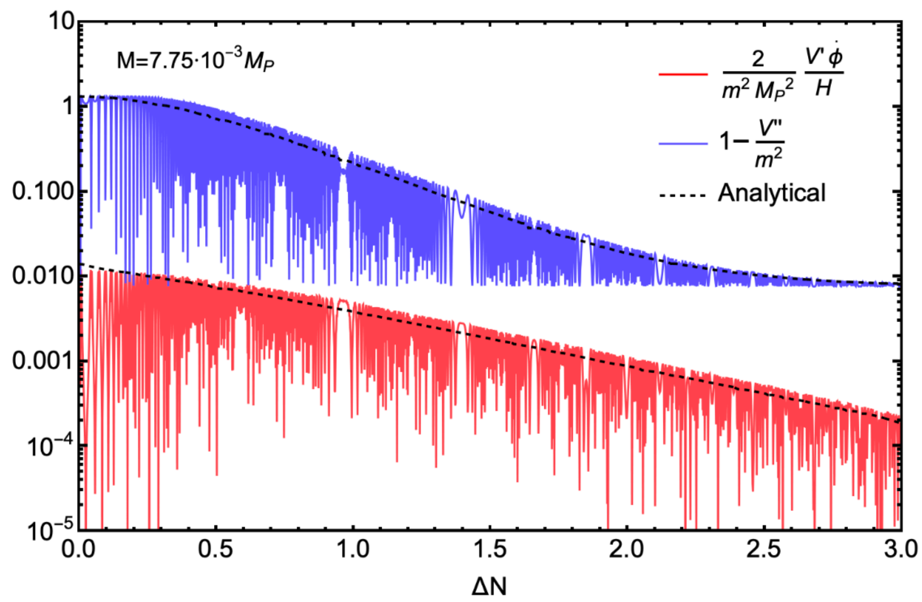
n=1, M=2

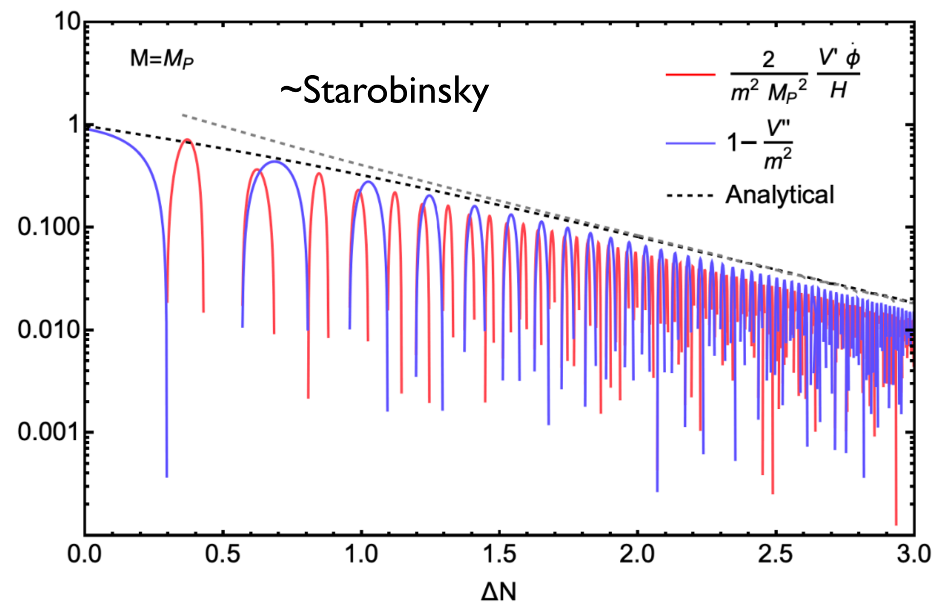
n=1, M=0.5



Data impose $M \lesssim 7\ M_{\rm P}$ (depending on details)

The importance of being anharmonic





Anharmonic terms are dominant (when $M \ll M_P$) or at least comparable (when $M \gtrsim M_P$) to metric terms, their ratio being of order $(M_P/\phi_{end})^2$

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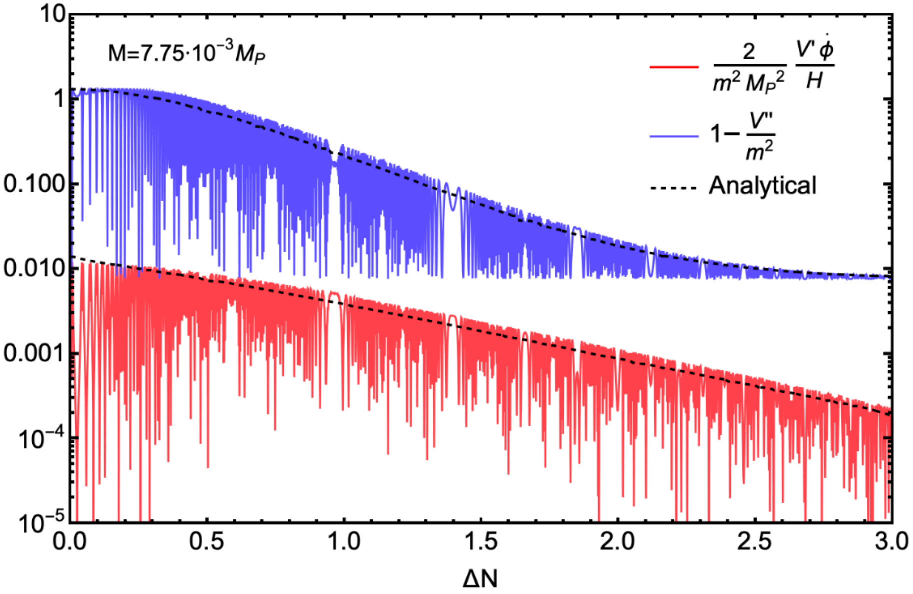

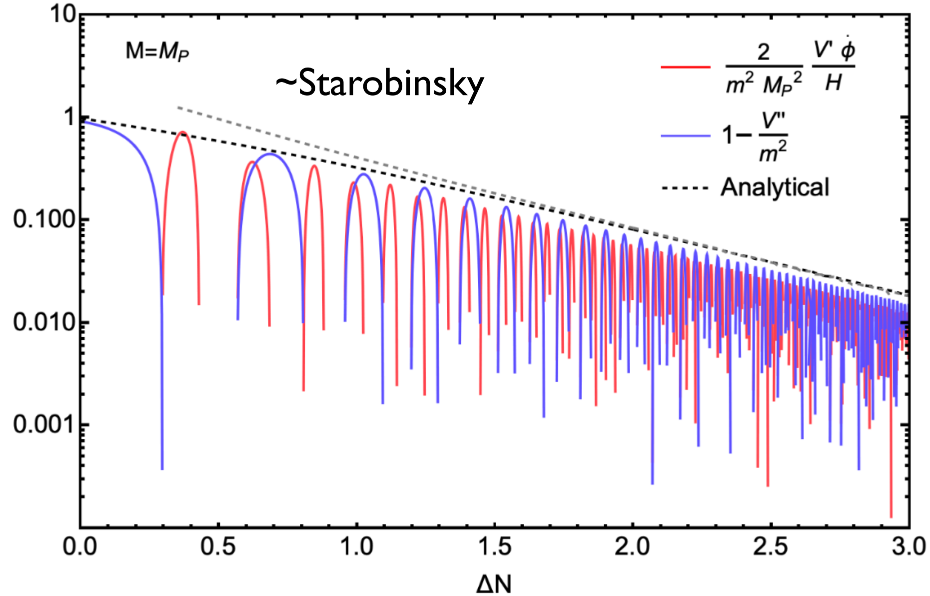

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Can we turn a failure of the approximation into a resource?

Can one rescue the mechanism thanks to anharmonic terms?

Since $\Re(\mu_k) m/H \propto M_P/M$ (M. Amin et al. 1106.3335...)

better to look at cases $M \ll M_P$ (metric terms can then be neglected w.r.t. the anharmonic ones)

IV. Some lattice (& numerical GR) results

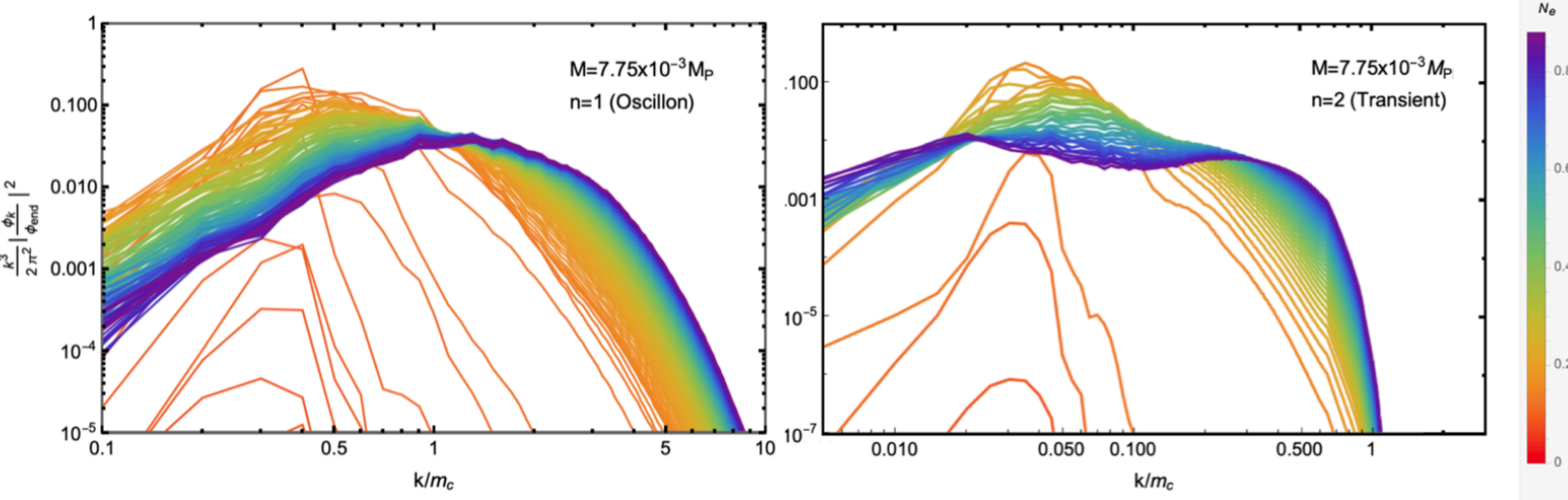
Lattice study for T model

https://cosmolattice.net/
See 2006.15122, 2102.01031

Backreaction of inflaton perturbations onto their own growth shuts off the resonant amplification well before BH-forming conditions are reached.

For potential matching a quadratic well, ***oscillons*** form (fixed physical size).

'Transient' metastable structures possible otherwise



Too fluffy for PBH: $|\mathcal{C}| \lesssim 10^{-3}$

In agreement with
Lozanov & Amin 1710.06851, 1902.06736

(motivated by oscillon physics, rather than PBH formation)

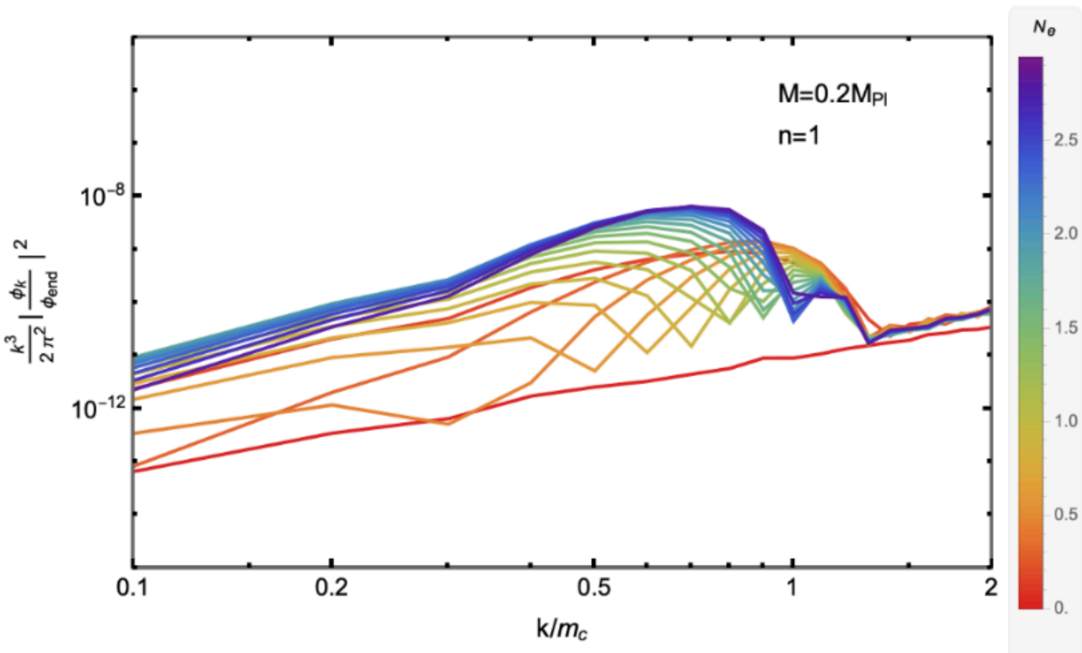
Can GR rescue PBH formation?

Relevant GR role only possible if M not too small compared to M_P (but want big enough to enhance resonance)

PBH formation found to be possible, starting from special initial conditions:

the objects we consider in this work have a compactness comparable to that of the corresponding black hole. The formation of such objects needs to be checked for each specific model via dedicated lattice simulations. In the simplest and most model independent scenarios, self-interactions of a single field are sufficient to make the quantum fluctuations grow and enter the non-linear regime.

F. Muia et al. arXiv:1906.09346

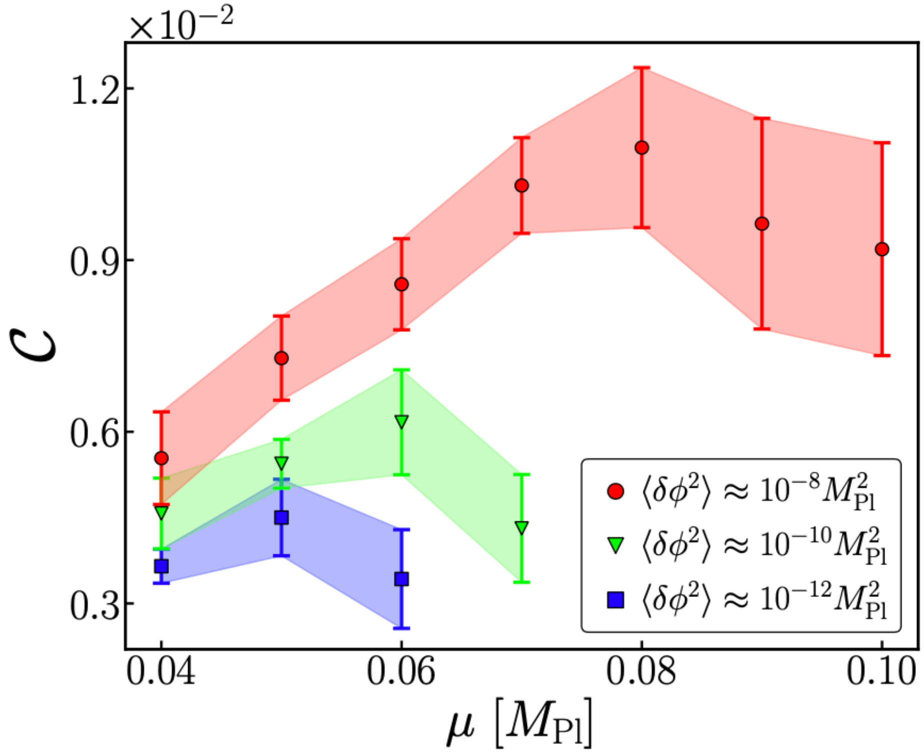


Our study suggests that the initial conditions needed to form PBH are very special, not automatically set by single-field inflaton dynamics

Numerical GR study

$$V(\phi) = \frac{m^2\mu^2}{2}\left(1 - e^{\frac{\phi}{M}}\right)^2$$

J.C. Aurrekoetxea, K. Clough, and F. Muia, 2304.01673



Maximum compactnesses $\mathcal{C}$ found: ~0.001- 0.01

increases with the scale μ [our *M*], *until it reaches a maximum value* [@ μ~0.05-0.1 M_P] *due to a balance between the overdensity growth rate and dilution due to the universe's expansion rate*

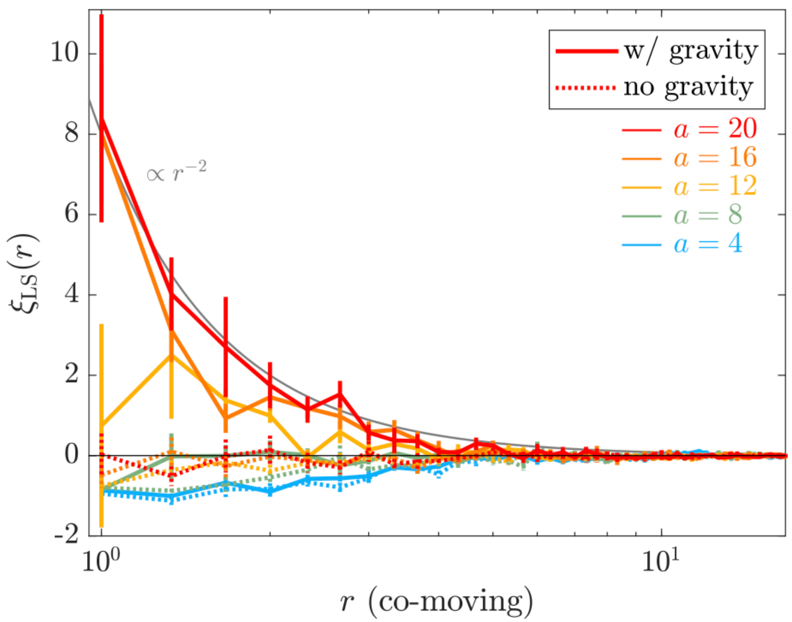
i.e. same 'ingredients' at play as in $\Re(\mu_k)m/H$ of previous slides

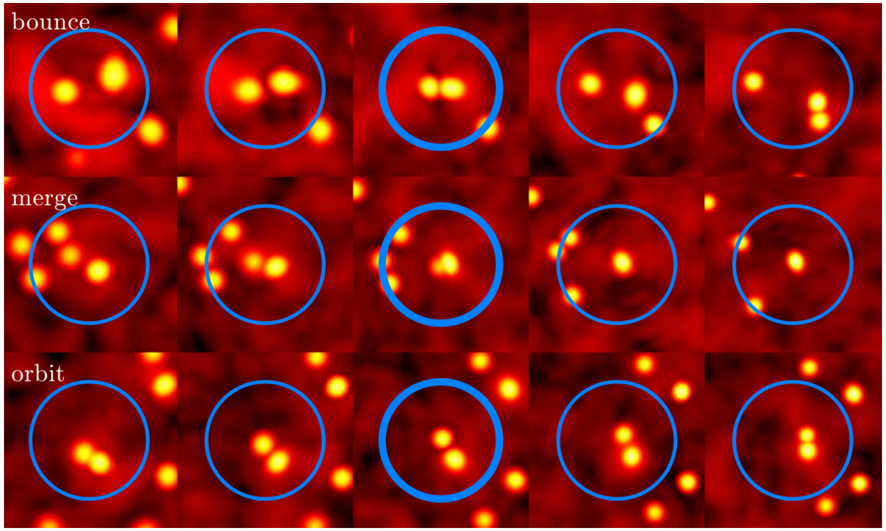
Bottom line

Can't form PBH unless fluctuations ≫ wrt what set by "standard" inflation

Could further oscillon dynamics lead to PBH formation?

Existing numerical studies on gravitational scattering of oscillons/solitons after preheating show some interesting dynamical clustering, but do not manifest any indication of PBH formation, suggesting that if PBH do form, it must be a rare process!





M. A. Amin and P. Mocz, 1902.07261

For an analytical theory encompassing different BSM scenarios, and more optimistic conclusions, see e.g. E. Cotner, A. Kusenko, M. Sasaki and V. Takhistov 1907.10613

V. Conclusions

- We revisited the interesting claim that PBH may be **generically** produced in the post-inflationary, preheating phase via self-resonant amplification of fluctuations. Metric preheating found sufficient, with quadratic potential proxy

- We found that the results is not robust when lifting some simplifying hypotheses, notably perturbation growth in absence of feedback onto the background and lack of gradients

- We found that anharmonic terms are leading (or comparable with) the metric terms, notably for strong resonance. Could have rescued the mechanism…

- … but lattice studies show that this is not the case. When self-gravitating objects form, they remain too 'fluffy' to lead to PBHs.

- Similarly, numerical relativity studies suggest that PBHs can only form when starting from conditions much more inhomogeneous than expected from standard inflation.

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PBH formation is not generic: Perhaps that is why they are fascinating !!!