

Inflaton Fragmentation during Preheating :
Formation & Decay of Oscillons
NEHOP 2024 @ Edinburgh

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(With **Ed Copeland, R Mahbub, Md Shafi & S Basak**)

20th June 2024

Cosmic Inflation: Background dynamics, Quantum fluctuations and Reheating

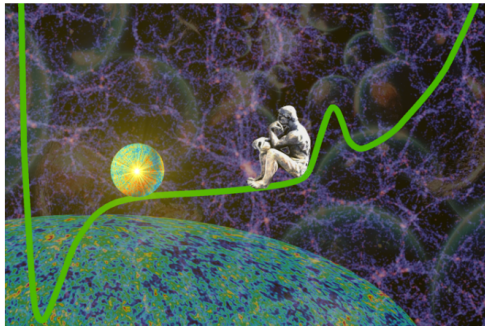
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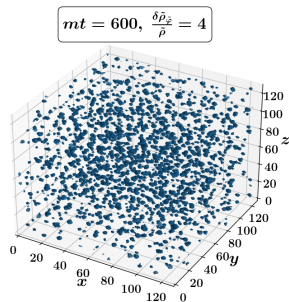
March 19, 2024

arXiv:2403.10606v1 [gr-qc] 15 Mar 2024



PBH Formation Mechanisms

- 1 Collapse of large (iso-)curvature fluctuations
- 2 Topological Defects
- 3 First-order phase transitions
- 4 **Scalar field fragmentation**



**Inflaton Fragmentation
during preheating**

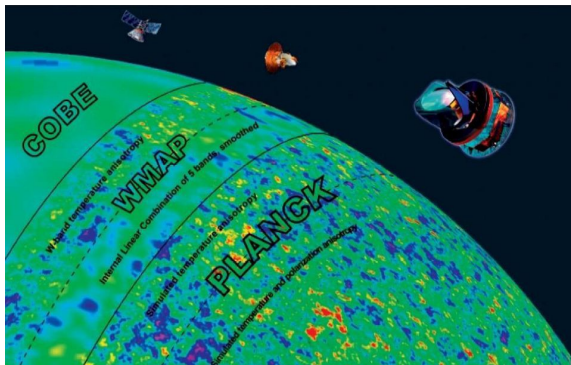
(CAVEAT: Not as Generic as often loosely claimed!)

Talks by **V. Takhistov** (Monday), **P. Serpico** (Tuesday)

Best Probes of the Early Universe

Cosmic Microwave Background (CMB) Radiation

+ **BBN** + **LSS** + **BAO**



- 1 What is the origin of the constituents of the plasma?
- 2 What is the origin of the fluctuations in the plasma?

Part-I: Primordial Fluctuations

(Quantum) Initial conditions for structure

Initial $\zeta(\vec{x}) \longrightarrow$ CMB \longrightarrow LSS (gravitational instability)

Properties of initial fluctuations:

① Adiabatic $\zeta(\vec{x})$

② Almost scale-invariant

$$\mathcal{P}_\zeta = A_S \left(\frac{k}{k_*} \right)^{n_S - 1} \quad k_* = 0.05 \text{ Mpc}^{-1}$$

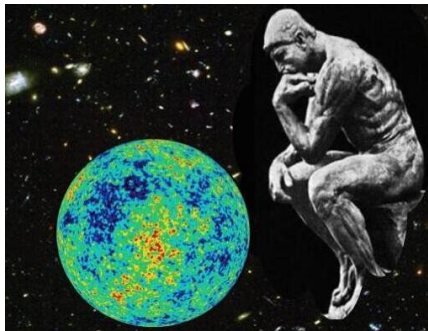
$$A_S \simeq 2 \times 10^{-9}, \quad n_S - 1 \simeq -0.035$$

③ Nearly **Gaussian** ($\sigma \simeq 10^{-4}$)

$$P[\zeta] = \mathcal{B} \exp \left[\frac{-\zeta^2}{2\sigma^2} (1 + f_{\text{NL}} \zeta + \dots) \right]$$

\rightarrow LSS, CMB \Rightarrow Large-scale primordial fluctuations

\rightarrow Origin \Rightarrow Quantum fluctuations during Inflation



Cosmic Inflation

System = Gravity ($g_{\mu\nu}$) + Scalar Field (φ)

$$S[g_{\mu\nu}, \varphi] = \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R - \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu} - V(\varphi) + \dots \right)$$

(1) **Background Evolution/Dynamics $\phi(t)$: qdS Expansion**

$$\ddot{a} > 0, ; \quad a_{\text{end}} = a_i e^{\Delta N}, \quad \Delta N > 60$$

(Makes the universe isotropic, uniform and flat)

(2) **Linear Perturbations: Two light fields ($m \ll H$)**

$$ds^2 = -\beta^2(t) dt^2 + a^2(t) \left[\left(e^{2\Psi(t, \vec{x})} \delta_{ij} + 2h_{ij}(t, \vec{x}) \right) dx^i dx^j \right]$$

→ **Comoving Curvature Perturbations:** $-\zeta(t, \vec{x}) = \Psi + \frac{H}{\dot{\phi}} \delta\varphi$

(Induce CMB density and temperature fluctuations)

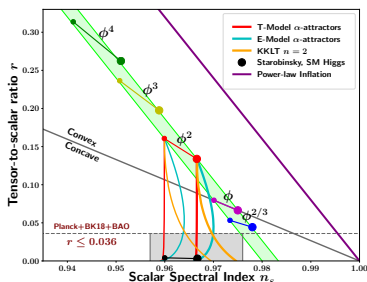
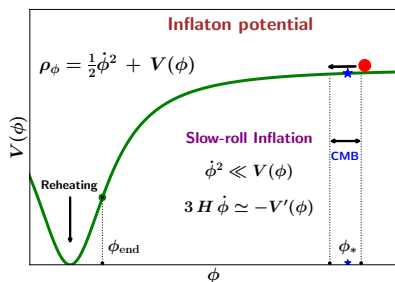
→ **Transverse & traceless Tensor Perturbations:** $h_{ij}(t, \vec{x})$

(Induce stochastic Gravitational Waves)

Observational Constraints: Single field slow-roll Inflation

Scalar tilt $n_s - 1 = -\epsilon_2 - 2\epsilon_1 \in [-0.043, -0.024]$ (red tilt)

Tensor-to-scalar ratio: $r = \frac{A_T}{A_S} = 16\epsilon_1 \leq 0.036$ (low)



\Rightarrow **Single-field slow-roll paradigm of Inflation & Asymptotically flat concave potentials!**

**PLANCK Inflation (2018); **BICEP/Keck (2021); **SSM & Sahni(2022)

What happened to other fields during inflation?

- Observations favour ‘**single-field slow-roll**’ inflation.
- ‘**Cold inflationary paradigm:**’

⇒ Negligible coupling to external fields $\frac{1}{2}g^2\varphi^2\chi^2, h\varphi\psi\bar{\psi}$

$$S[\varphi, \chi] = - \int d^4x \sqrt{-g} \left[\begin{aligned} & \frac{1}{2} \partial_\mu \varphi \partial^\nu \varphi + V(\varphi) + \frac{1}{2} \partial_\mu \chi \partial^\nu \chi + \frac{1}{2} m_{0\chi}^2 \chi^2 \\ & + \bar{\psi} (i\gamma^\mu \partial_\mu + m_{0\psi}) \psi \\ & + \frac{1}{2} g^2 \varphi^2 \chi^2 + h \psi \bar{\psi} \varphi + \dots \end{aligned} \right]$$

$$g^2, h \ll 1$$

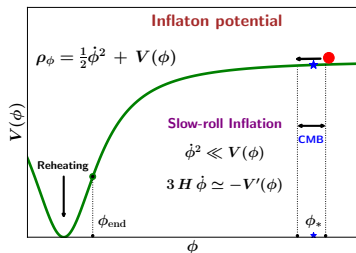
⇒ particle production during inflation can be neglected.

- **Effects of the small coupling?**

- 1 **Primordial Non-Gaussianity:** inflaton interactions.
- 2 Decay of the inflaton field: **Reheating the universe.**

Part-II : Post-inflationary Dynamics of the Inflaton condensate

Post-inflationary Oscillations



Potential near minimum $\phi < \phi_e$,

$$V(\phi) \simeq \frac{1}{2}m^2\phi^2 \pm \frac{\mu}{3}\phi^3 \pm \frac{\lambda}{4}\phi^4 + \dots$$

For a quadratic potential $V(\phi) \propto \phi^2$

$$\langle w_\phi \rangle = 0 \Rightarrow \rho_\phi \propto a^{-3}$$

→ SR Inflation: Friction dominated

$$\Rightarrow m^2 \equiv V_{,\phi\phi} \ll H^2$$

→ After inflation: $m^2 \gtrsim H^2$

→ When $m^2 \gg H^2$, inflaton exhibits

Coherent Oscillations for which

$$\Rightarrow \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \simeq \text{const.}$$

⇒ Inflaton behaves like Matter

Since $m > H \Rightarrow T_{\text{osc}} < H^{-1}$

Under **adiabatic approximation**

$$\phi(t) = \phi_0(t) \cos(mt);$$

with **amplitude** $\phi_0(t) \propto \frac{1}{t}$

However, we have ignored two important effects:

→ **External coupling:** (to other fields χ, ψ)

$$\mathcal{I}(\varphi, \chi) = \frac{1}{2} g^2 \varphi^2 \chi^2$$

→ **Asymptotically flat potentials:** (CMB observations)

$$V(\phi) = V_0 \left(\frac{\phi}{m_p} \right)^{2n} - |U(\lambda; \phi)|$$

have ‘attractive self-interaction’

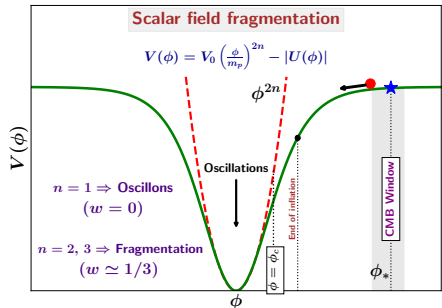
⇒ **Self – resonance** (due to λ)

⇒ Rapid Growth of $\delta\varphi$

⇒ **Scalar field fragmentation**

⇒ **Cosmological Solitons :**

‘**Oscillons**’



**Amin et. al & Lozanov et. al (2010-2020)

Existence of quasi-Solitons: Oscillons

→ Self-supported, localised, long-lived non-linear ‘solitary’ configurations.

→ **Solitons are ubiquitous in nature!**

(1834 J. S. Russell: solitary wave in a canal in ‘Edinburgh’!)

(Appearing in fluids, smoke rings, condensed matter physics, optics, HEP, topological defects and Cosmology.)

→ **Oscillons are oscillating non-topological solitons!**

→ Analytical results based on small-amplitude oscillations

$$V(\varphi) \approx \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4} \varphi^4 + \frac{g}{6} \varphi^6$$

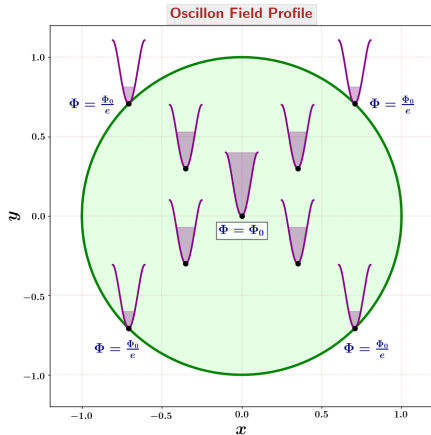
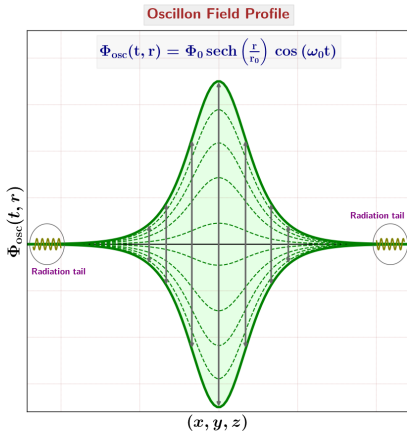
→ Supports **Oscillon-like solution** of the form

$$\varphi_{\text{osc}}(t, r) \approx \Phi(r) \cos(\omega_0 t) + \dots; \quad \Phi(r) \approx \Phi_0 \operatorname{sech}\left(\frac{r}{r_0}\right)$$

**Rajaraman(1987), **Gleiser *et. al*; **Amin *et. al* ; **Mahbub, SSM (2023)

Oscillon Profile

$$\Phi(t, r) \approx \phi_0 \operatorname{sech}\left(\frac{r}{r_0}\right) \cos(\omega_0 t)$$



Interesting compact (non-linear) field configurations !

Existence of Oscillons ?

1 Oscillons exit as analytic (stationary) solutions

(of post-inflationary oscillations around asymptotically flat potentials)

(a) For symmetric plateau potentials:

→ small-amplitude oscillations

$$V(\varphi) \approx \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4} \mu \varphi^4 + \frac{g}{6} \lambda \varphi^6$$

→ Supports **Oscillon-like solution** of the form

$$\Phi(t, r) = \phi_0 \operatorname{sech} \left(\frac{r}{r_0} \right) \cos(\omega_0 t)$$

(b) For asymmetric plateau potentials?

$$V(\varphi) \approx \frac{1}{2} m^2 \varphi^2 - \frac{1}{3} \mu \varphi^3 + \frac{1}{4} \lambda \varphi^4$$

2 Can they form dynamically?

(starting from **natural conditions at the end of inflation**)

*Copeland *et. al*(1995); *Amin *et. al*(2011); *Mahbub, SSM(2023); *Kim, McDonald

Self-resonance and inflaton fragmentation

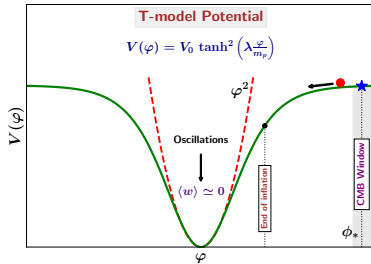
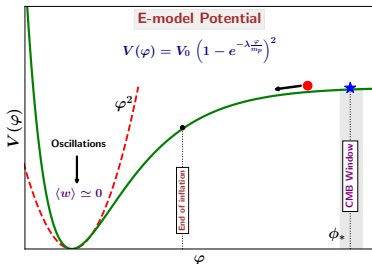
In the linear regime, Fourier mode functions satisfy

$$\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k + \left[\frac{k^2}{a^2} + V_{,\phi\phi}(\phi) \right] \delta\varphi_k = 0$$

(Equation of a **damped parametric oscillator**)

⇒ **Resonant growth** of inflaton fluctuations $\delta\varphi_k(t) \propto e^{\mu_k mt}$

$$V(\phi) = \frac{1}{2}m^2\phi^2 - |U(\lambda; \phi)| \quad (\mathbf{E}\text{-Model} \ \& \ \mathbf{T}\text{-Model})$$



**Lozanov, Amin(2017); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

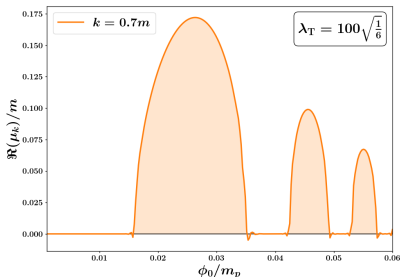
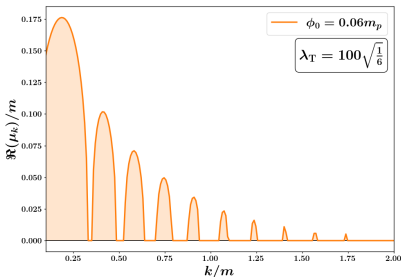
Linear Parametric Self-resonance

Equation of a **linear parametric oscillator**

$$\frac{d^2 \delta \varphi_k}{dT^2} + \Omega_\varphi^2(k, T) \delta \varphi_k = 0 ; \quad \Omega_\varphi^2 \text{ is oscillatory in } T$$

Floquet Theorem $\delta \varphi_k(T) = \mathcal{M}_k^{(+)}(T) e^{\mu_k T} + \mathcal{M}_k^{(-)}(T) e^{-\mu_k T}$

\Rightarrow **Exponentially growing** for $\text{Re}(\mu_k) \neq 0$ (**in resonance bands**)



**SSM Lecture Notes (2024)

Non-thermal particle production

Part-III : Non-linear Dynamics & Lattice Simulations

Non-linear dynamics: CosmoLattice

Fully non-linear (dimensionless version of) field equations

$$\ddot{\tilde{\varphi}} + 3\tilde{H}\dot{\tilde{\varphi}} - \frac{\tilde{\nabla}^2}{a^2}\tilde{\varphi} + \tilde{V}_{,\tilde{\varphi}} = 0$$

$$\tilde{H} \equiv \frac{\dot{a}}{a} = \frac{1}{3m_p^2} \left\langle \tilde{K}_{\tilde{\varphi}} + \tilde{G}_{\tilde{\varphi}} + \tilde{V}(\tilde{\varphi}) \right\rangle$$

Where $\tilde{t} = m t$; $\tilde{x} = m x$; $\tilde{\varphi}, \tilde{\chi} = \frac{1}{\beta} \frac{\varphi, \chi}{m_p}$; $\tilde{F} = \frac{F}{\beta^2 m^2 m_p^2}$

$$\tilde{K}_{\tilde{\varphi}} = \frac{1}{2} \left(\frac{\partial \tilde{\varphi}}{\partial \tilde{t}} \right)^2; \quad \tilde{G}_{\tilde{\varphi}} = \frac{1}{2a^2(\tilde{t})} \left[\left(\frac{\partial \tilde{\varphi}}{\partial \tilde{x}} \right)^2 + \left(\frac{\partial \tilde{\varphi}}{\partial \tilde{y}} \right)^2 + \left(\frac{\partial \tilde{\varphi}}{\partial \tilde{z}} \right)^2 \right]$$

$$\tilde{\rho}_{\tilde{\varphi}} = \tilde{K}_{\tilde{\varphi}} + \tilde{G}_{\tilde{\varphi}} + \tilde{V}(\tilde{\varphi}); \quad \tilde{p}_{\tilde{\varphi}} = \tilde{K}_{\tilde{\varphi}} - \frac{1}{3}\tilde{G}_{\tilde{\varphi}} - \tilde{V}(\tilde{\varphi})$$

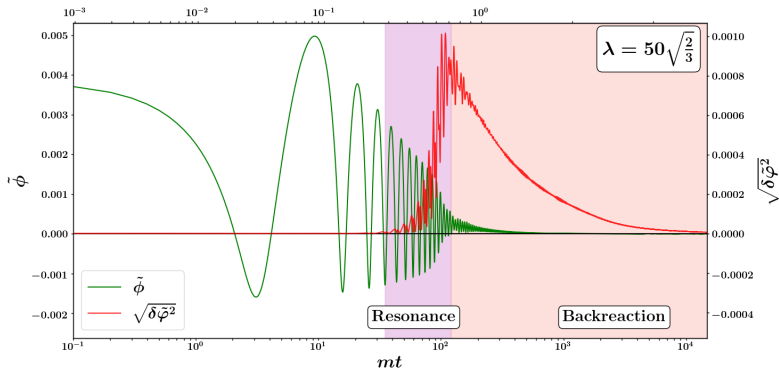
Lattice specifications:

$$N = 128^3; \quad 0.05 m^{-1} \leq k \leq 5 m^{-1}$$

**Figueroa *et. al* (2020, 2021); **Mahbub, SSM (2023)

Self-resonance and Inflaton Fragmentation

Strong self-resonance \Rightarrow Inflaton fragmentation
(Asymmetric E-Model potential)

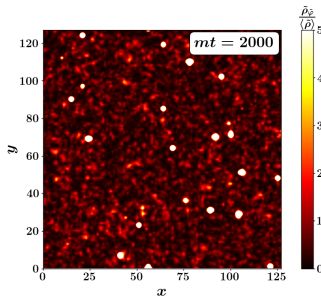
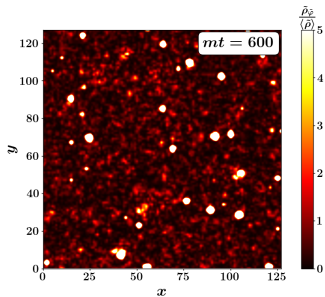
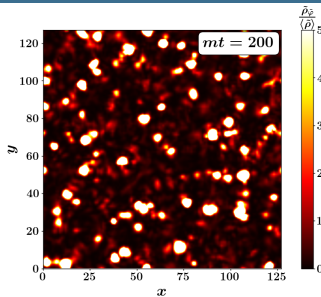
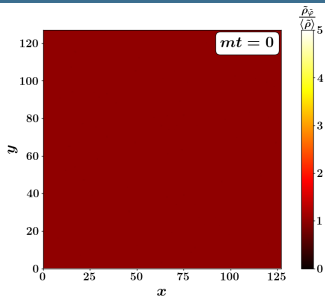


Inflation condensate

Inflation fluctuations

**Mahbub, SSM (2023); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

Oscillon formation in real time (Asymmetric)

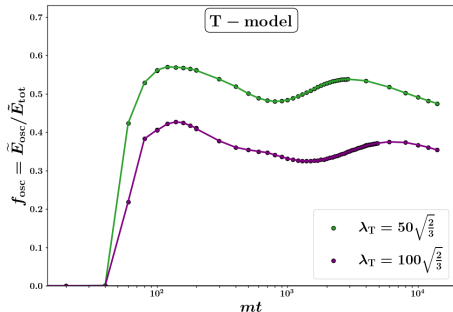
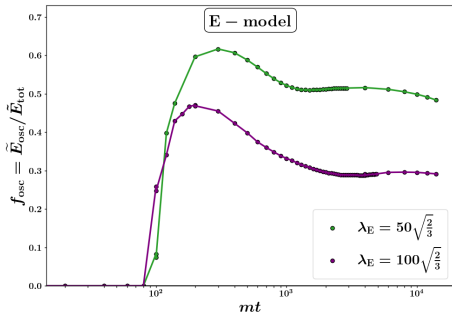


Fractional Energy Density of Oscillons

Energy/Mass fraction

$$f_{\text{osc}} \equiv \frac{E_{\text{osc}}}{E_{\text{tot}}} = \frac{\int_{\delta\rho_\varphi \gtrsim 4\bar{\rho}_\varphi} d^3\mathbf{x} \rho_\varphi(\mathbf{x}, t)}{\int d^3\mathbf{x} \rho_\varphi(\mathbf{x}, t)}$$

(Fractional energy density of oscillons)



$\gtrsim 40\%$ of the total density \Rightarrow Significant!

**Mahbub, SSM (2023)

Conclusions (so far)

- 1 Oscillons form **for both Symmetric and Asymmetric plateau potentials**.
- 2 **Oscillons do form** after inflation in absence of external coupling starting from **generic initial conditions**.

Important Questions

- 1 What is the **lifetime of oscillons**? How do they **decay**?
- 2 We have **ignored external coupling**; $g \rightarrow 0$

What happens if $g \neq 0$? **Do oscillons form?**

Our latest work!

****Hertzberg (2010); **Zhang, Amin, Copeland, Saffin, Lozanov (2020)**

PREPARED FOR SUBMISSION TO JCAP

Formation and decay of oscillons after inflation in the presence of an external coupling, Part-I: Lattice simulations

Mohammed Shafi ^a, Edmund J. Copeland ^b, Rafid Mahbub ^c,
Swagat S. Mishra ^b, Soumen Basak^a

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(To appear on arXiv 2406.00108)

Part-IV : Oscillon Formation in presence of an External Coupling

Dynamics of Inflaton Decay (Preheating)

System : Inflaton $\varphi \rightarrow$ massless offspring χ ; $m \gg m_{0\chi}$

Described by the **action**

$$S[\varphi, \chi] = - \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + V(\varphi) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \mathcal{I}(\varphi, \chi) \right]$$

With interaction $\mathcal{I}(\varphi, \chi) = \frac{1}{2} g^2 \varphi^2 \chi^2$

The corresponding **field equations** are

$$\ddot{\varphi} - \frac{\nabla^2}{a^2} \varphi + 3H\dot{\varphi} + V_{,\varphi} + \mathcal{I}_{,\varphi} = 0$$

$$\ddot{\chi} - \frac{\nabla^2}{a^2} \chi + 3H\dot{\chi} + \mathcal{I}_{,\chi} = 0$$

with **Hubble parameter**

$$H^2 = \frac{1}{3m_p^2} \left[\frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} \frac{\vec{\nabla}\varphi}{a} \cdot \frac{\vec{\nabla}\varphi}{a} + V(\varphi) + \frac{1}{2} \dot{\chi}^2 + \frac{1}{2} \frac{\vec{\nabla}\chi}{a} \cdot \frac{\vec{\nabla}\chi}{a} + \mathcal{I}(\varphi, \chi) \right].$$

Preheating via Self & External Resonance

→ For inflaton decay, $|\phi(t)| \gg \delta\varphi(t, \vec{x})$, $\chi(t, \vec{x})$

$$\varphi(t, \vec{x}) = \phi(t) + \delta\varphi(t, \vec{x})$$

$$\chi(t, \vec{x}) = \bar{\chi}(t) + \delta\chi(t, \vec{x}) \quad (\chi \text{ field is in vacuum state})$$

→ At the end of inflation, $\rho_\phi \gg \rho_\chi$, $\rho_{\delta\varphi}$ (**Condensate dominated**)

→ Resulting **equations of dynamics in the linear regime**

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k + \left[\frac{k^2}{a^2} + V_{,\phi\phi}(\phi) \right] \delta\varphi_k = 0 \quad \text{Self - resonance}$$

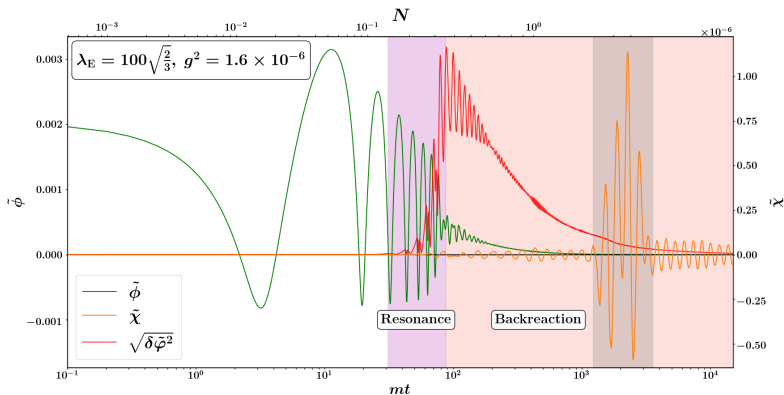
$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + g^2 \phi^2 \right] \chi_k = 0 \quad \text{External - resonance}$$

and the **Hubble parameter** $H^2 \simeq \frac{1}{3m_p^2} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right]$

**SSM Lecture Notes (2024)

Self-resonance and Oscillon decay for $g \neq 0$

Self-resonance \rightarrow Oscillons \rightarrow χ production



Asymmetric E-model potential

**Mahbub, SSM (2023); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

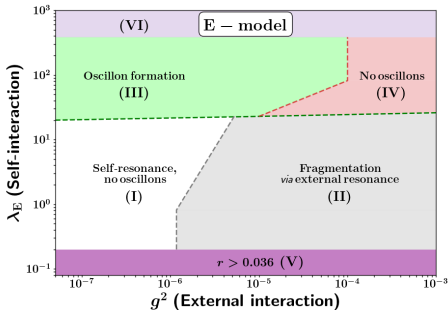
Parameter space of Oscillon formation $\{\lambda, g^2\}$

External Coupling

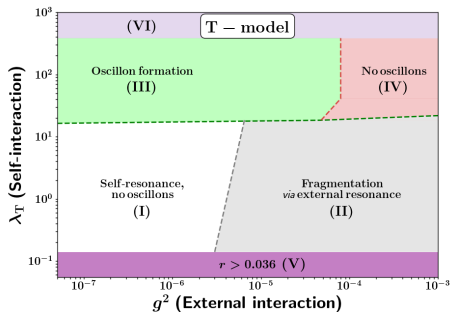
$$\mathcal{I}(\varphi, \chi) = \frac{1}{2} g^2 \varphi^2 \chi^2$$

$$V_E(\varphi) = V_{0E} \left(1 - e^{-\lambda_E \frac{\varphi}{m_p}}\right)^2$$

$$V_T(\varphi) = V_{0T} \tanh^2\left(\lambda_T \frac{\varphi}{m_p}\right)$$



(E-Model λ_E vs g^2)

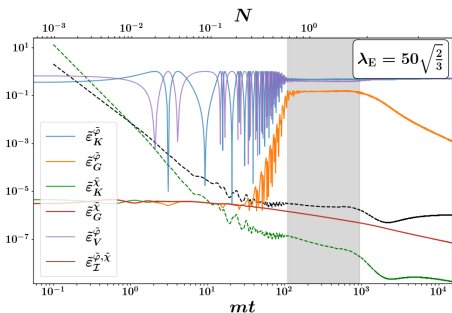


(T-Model λ_T vs g^2)

Evolution of energy density components

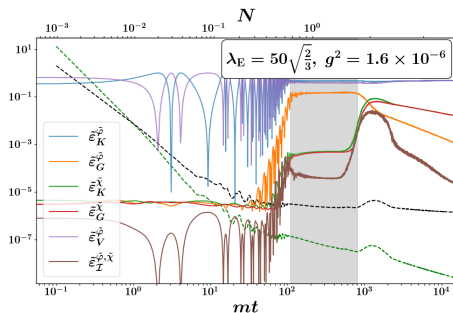
Absence of external interaction

Long-lived Oscillons



Presence of external interaction

oscillon decay into χ

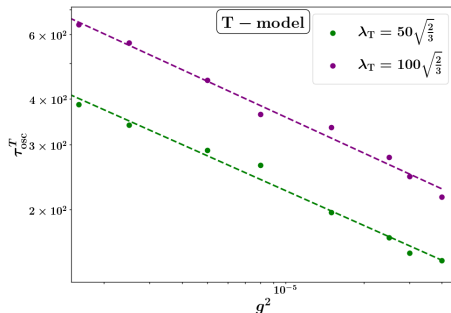
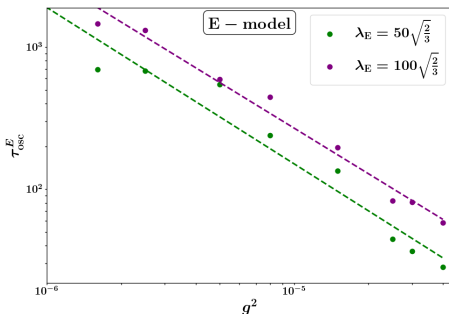


(Production of χ -particles due to oscillon decay!)

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Lifetime of (a population of) Oscillons

Robust oscillons for Gradient term $G_\varphi \propto a^{-3}$

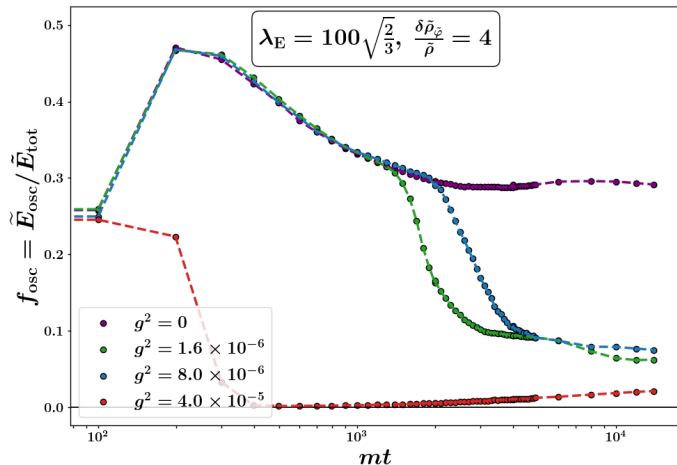


Empirically :

$$\tau_{\text{osc}}^E \propto (g^2)^{-1.1} ; \quad \tau_{\text{osc}}^T \propto (g^2)^{-0.3}$$

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Energy(Mass) Fraction of Oscillons



Reduction in f_{osc} due to χ -production

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Summary

① What happens if $g \neq 0$? Do oscillons form?

YES! (Preheating via Oscillon decay into χ)

② Lifetime of oscillons? How does an oscillon decay?

Our upcoming work (analytical)!

Phenomenological implications of oscillons?

- Universal (high frequency) GWs
- Late-time Gravitational clustering and GWs
- **Primordial Black Holes (NEHOP)**
- Oscillons in scalar field (fuzzy) dark matter
- Gravitational solitons (**Oscillatons**)

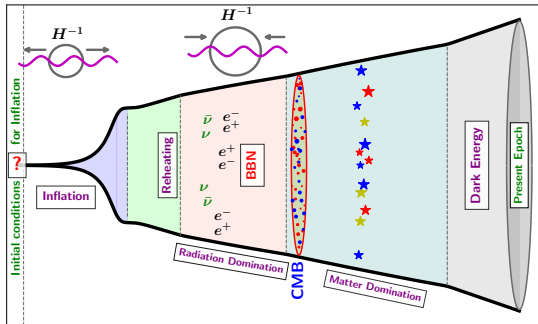
Arthur Conan Doyle conceived the idea of Sherlock Holmes in Edinburgh



Extra Slides

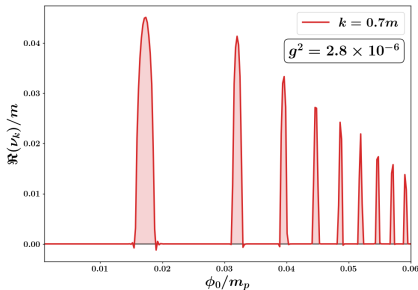
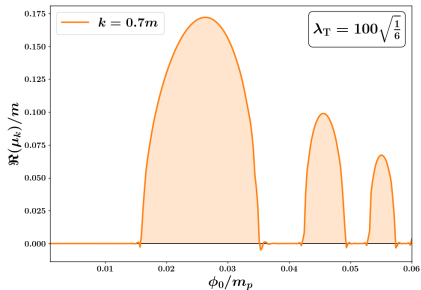
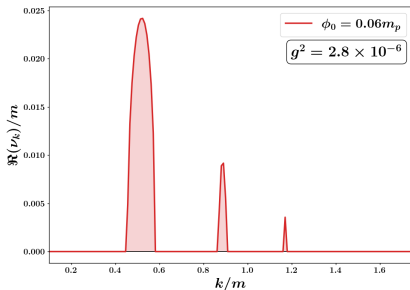
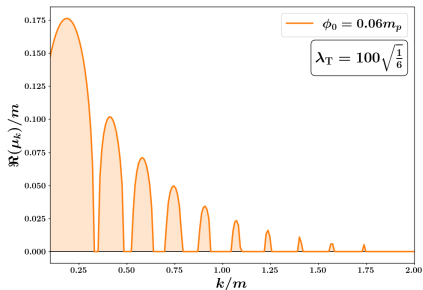
The hot Big Bang phase:

- **Beginning of the Universe** (×)
- **End of an earlier epoch of accelerated expansion** (✓)



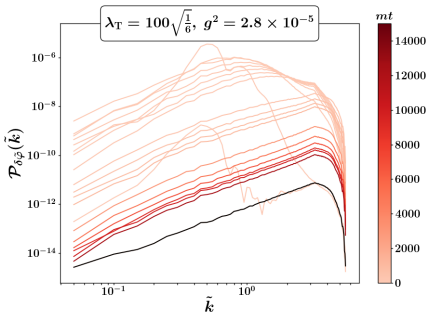
REHEATING: Origin of all primordial matter!

Structure of the resonance with $\{\lambda, g^2\}$

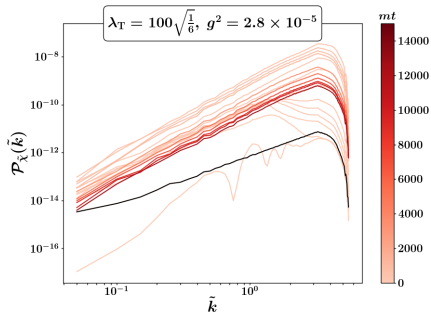


Power spectra of fluctuations: Resonance

Inflaton $\delta\varphi$ -fluctuations



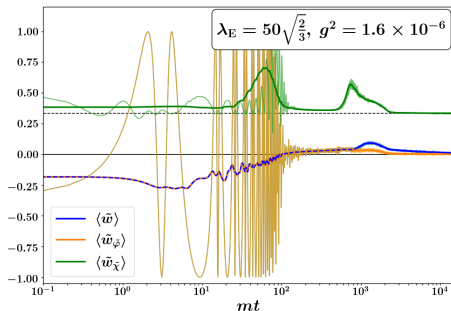
Offspring χ -fluctuations



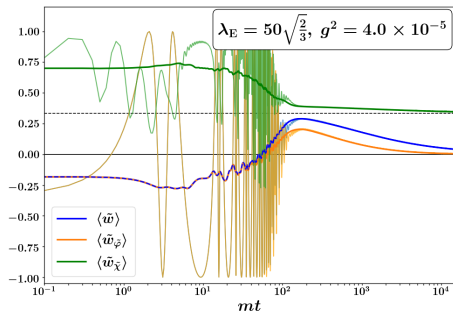
**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Evolution of Equation of State

Low external interaction



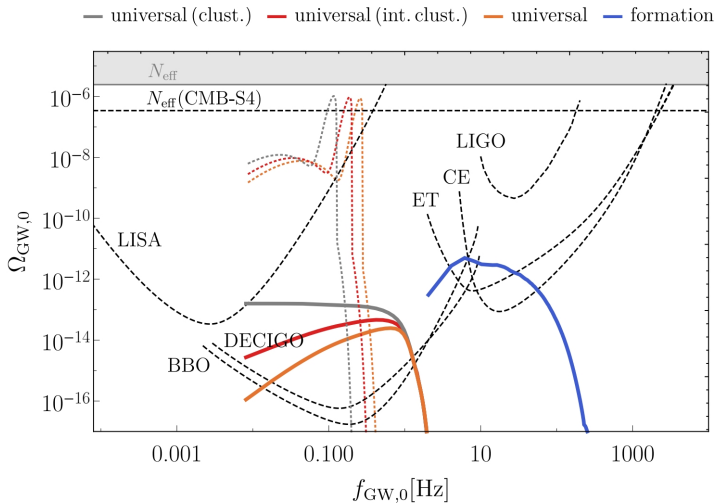
Large external interaction



$\langle w_{\varphi} \rangle \rightarrow 0$ asymptotically

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Universal Gravitational Waves



**Lozanov, Sasaki, Takhistov (2023)

Power-spectra: Linear Perturbation Theory

Slow-roll regime, $\epsilon_1, \epsilon_2 \ll 1$ (slow terminal speed)

$$\text{with } \epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{3m_p^2 H^2}; \quad \epsilon_2 = \frac{d \ln \epsilon_1}{dN}$$

Primordial power-spectrum on large scales –

$$\mathcal{P}_\zeta(k) = \frac{1}{8\pi^2} \left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_1} = A_S \left(\frac{k}{k_*}\right)^{n_S-1} \quad \text{Scalar spectral index}$$
$$n_S - 1 = -\epsilon_2 - 2\epsilon_1 \ll 1$$

$$\mathcal{P}_T(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^2 = A_T \left(\frac{k}{k_*}\right)^{n_T} \quad \text{Tensor spectral index}$$
$$n_T = -2\epsilon_1 \ll 1$$

CMB pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$

\Rightarrow Tiny fluctuations that are nearly scale-invariant