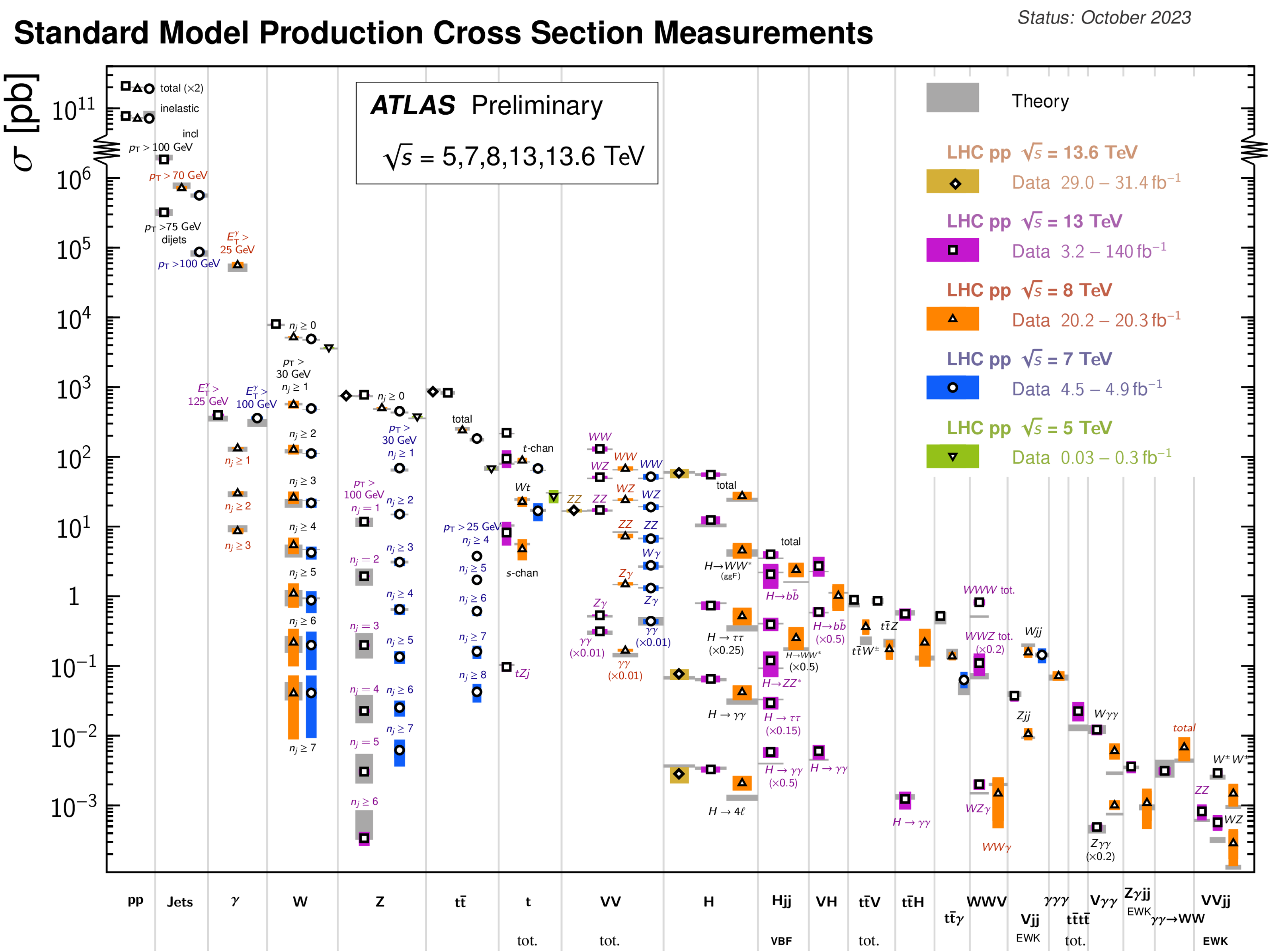
The precision frontier of QCD

Samuel Abreu
CERN & The University of Edinburgh

Higgs-Maxwell Meeting — Edinburgh, 2024

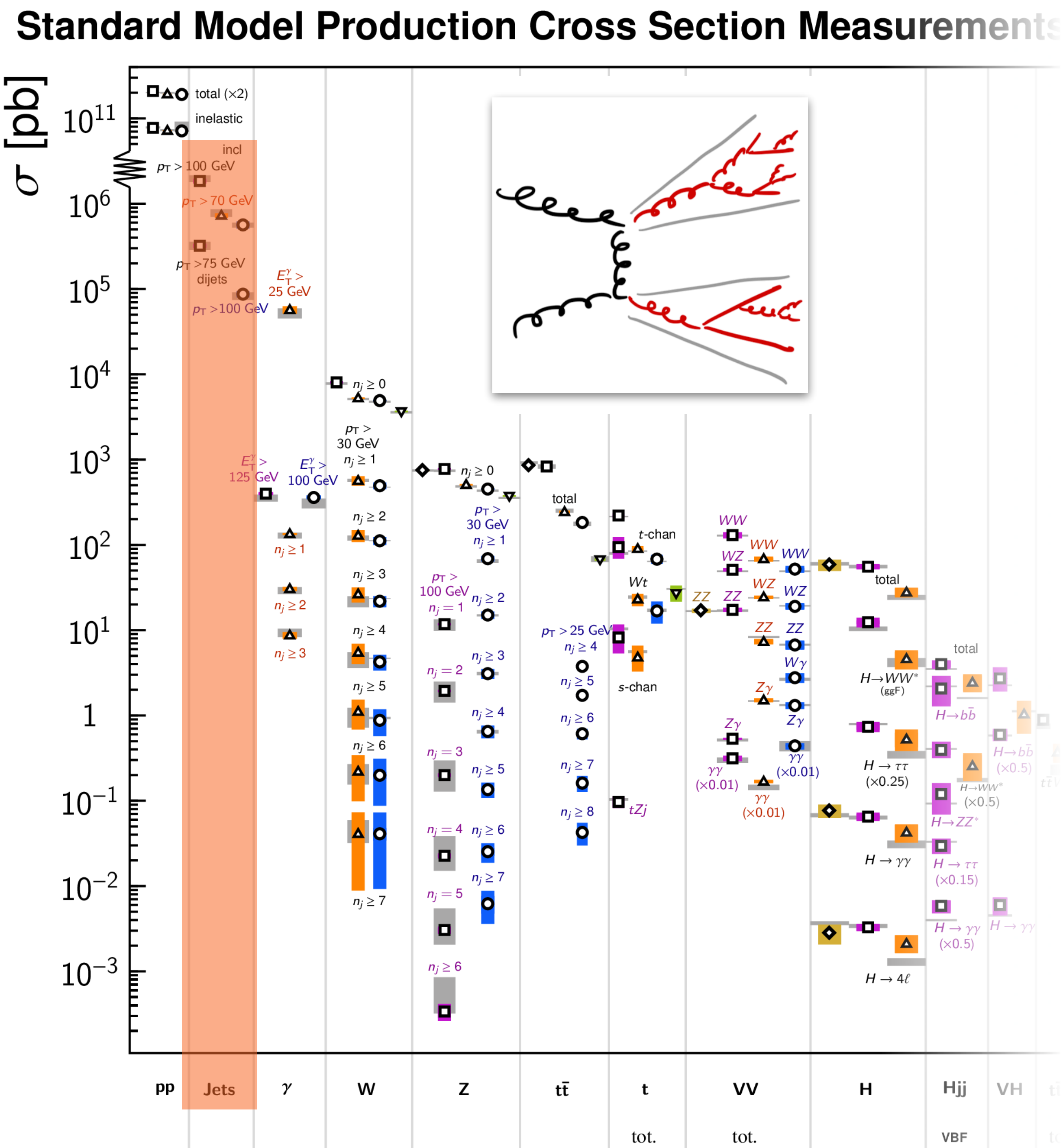
Very impressive results in collider physics



https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PUBNOTES/ATL-PHYS-PUB-2023-039/

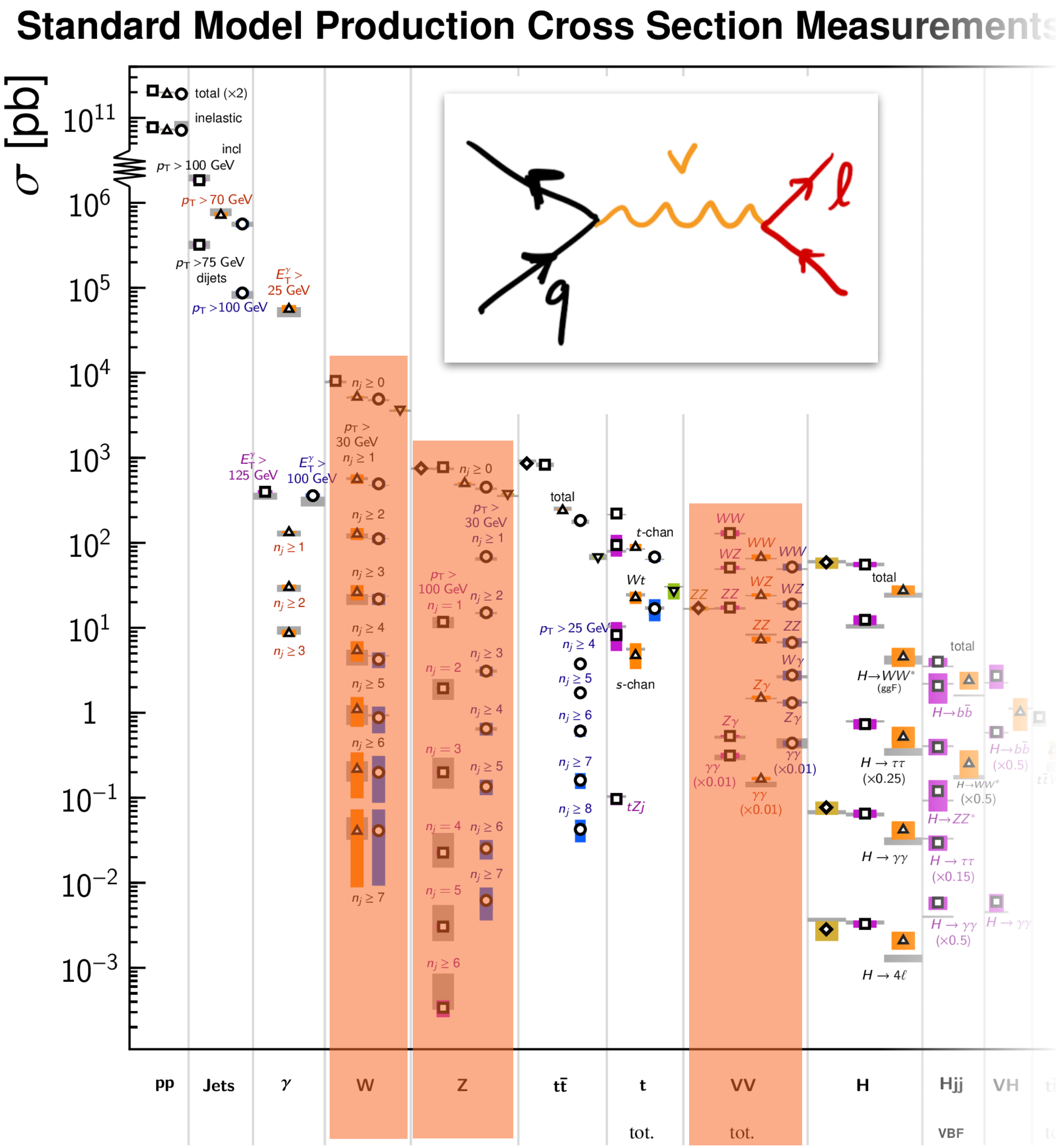
Jets

- Ubiquitous at hadron colliders
- Probe QCD dynamics over broad range of scales
- Used for α_s determination
- Background subtraction for BSM searches
- Recent new developments in flavoured jets
  - What is an IR-safe definition?
  - Higgs couplings ($H \to b\bar{b}$)
  - Top physics (PDFs, α_s, BSM)
  - Jet + V (PDFs, α_s)
  - Jet + missing E_T (BSM)



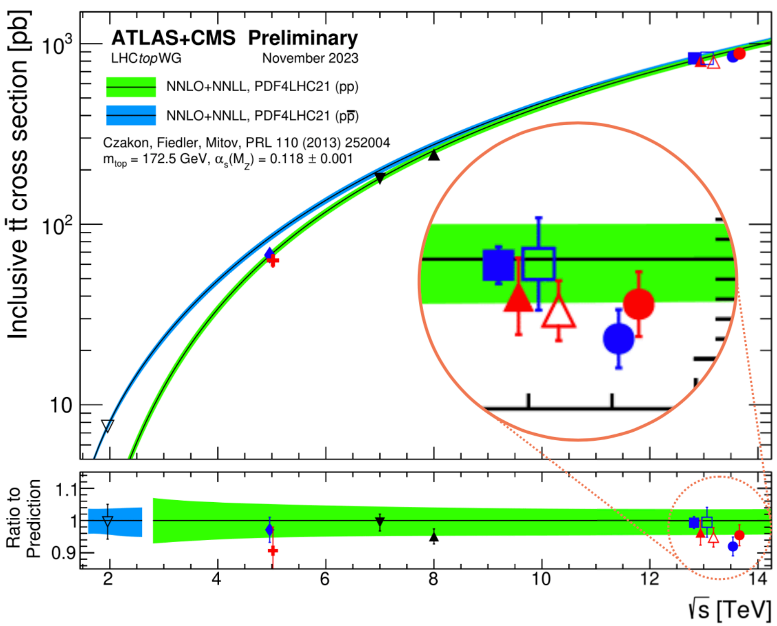
EW bosons

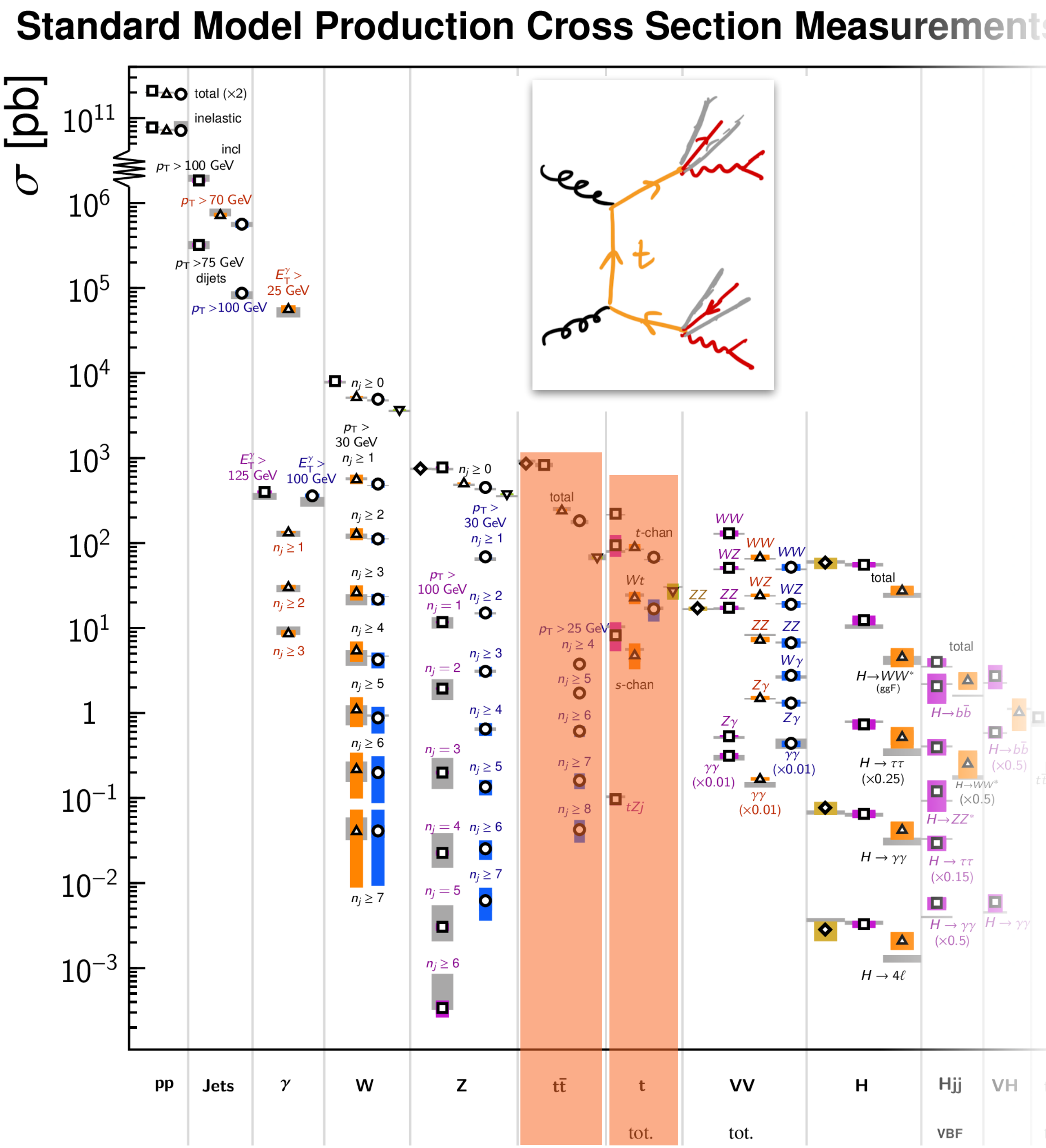
- ✓ Standard candles with very clear experimental signatures
- ✓ LHC from discovery machine to precision physics
  - Determination of EW parameters
  - Precise determination of M_W



Tops

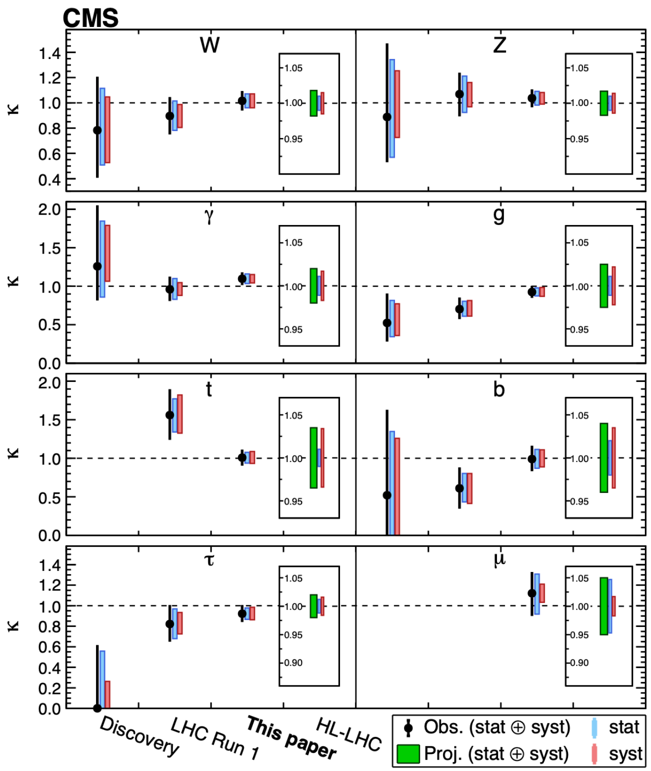
- Only quark that decays before it hadronises: precision test of the SM
- Heaviest SM particle: window into BSM physics
- Very precise experimental results, challenging theory precision



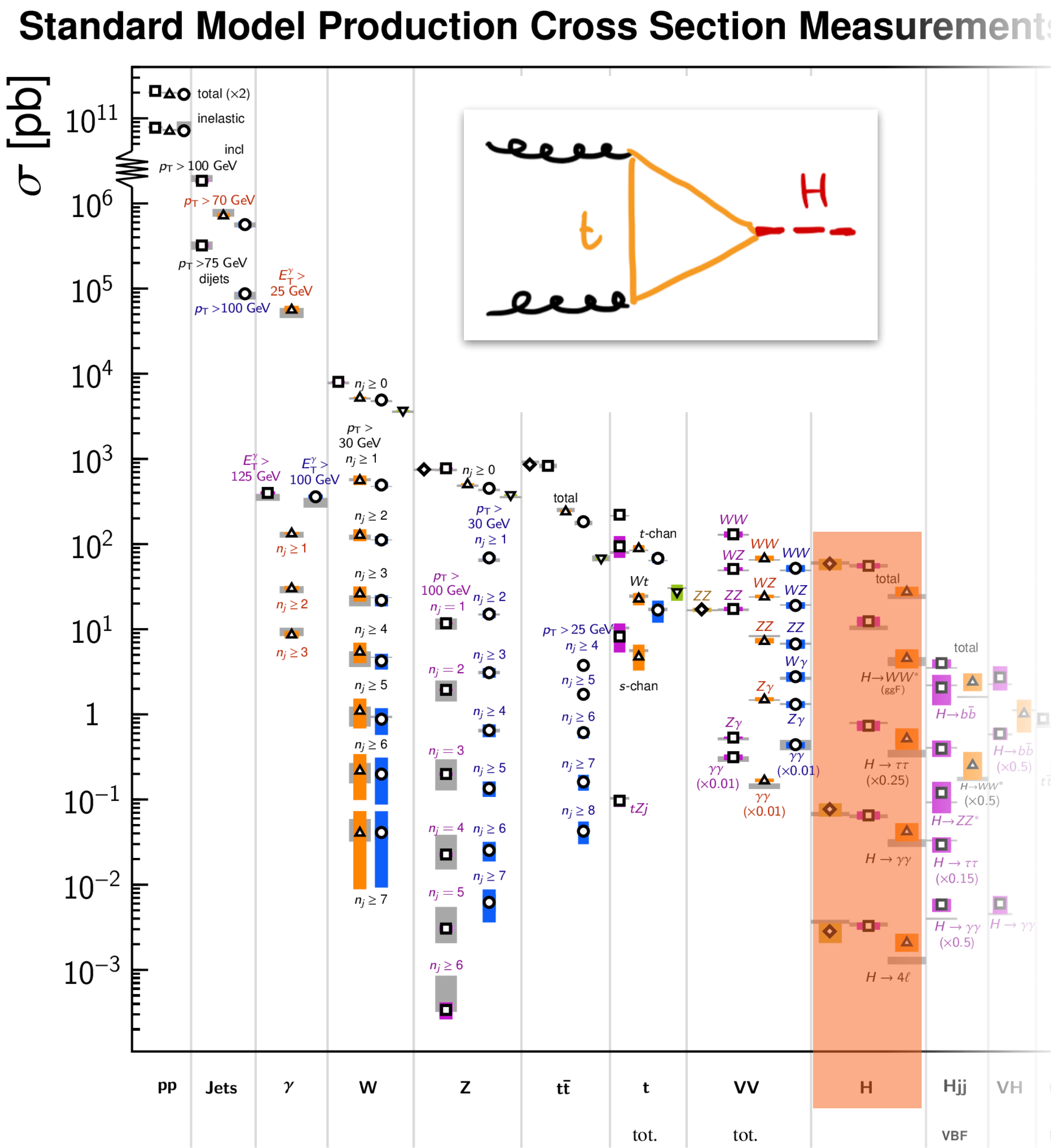


Higgs

- ✓ Main target of LHC program
- ✓ After discovery, measure properties of the Higgs boson
- ✓ Look for deviations from SM Higgs (no evidence so far)

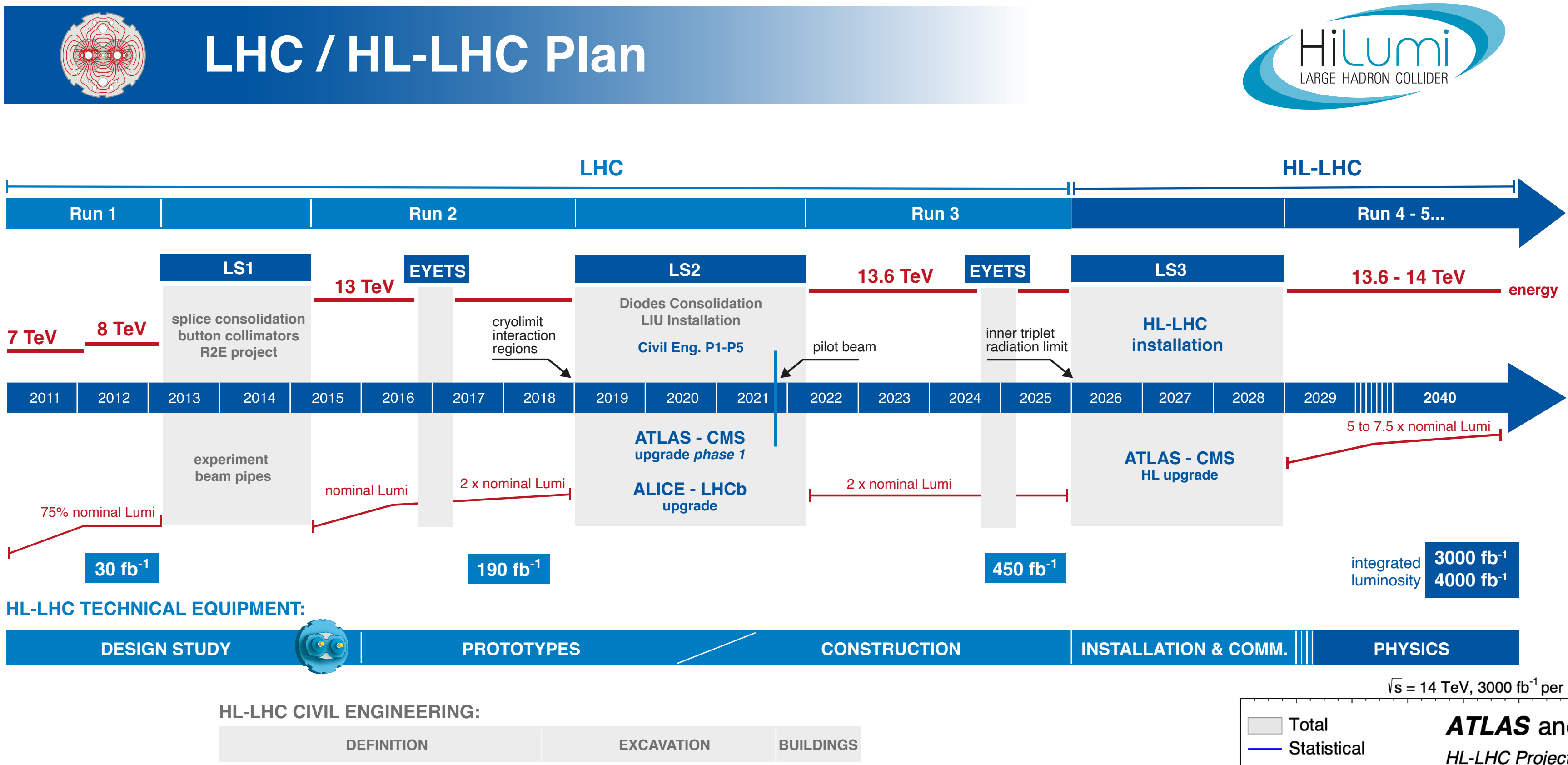


[Nature 607 (2022) 60]

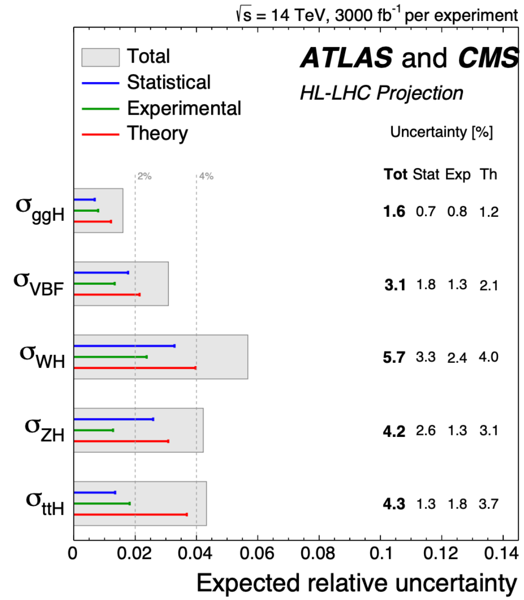


The future — High-Luminosity LHC

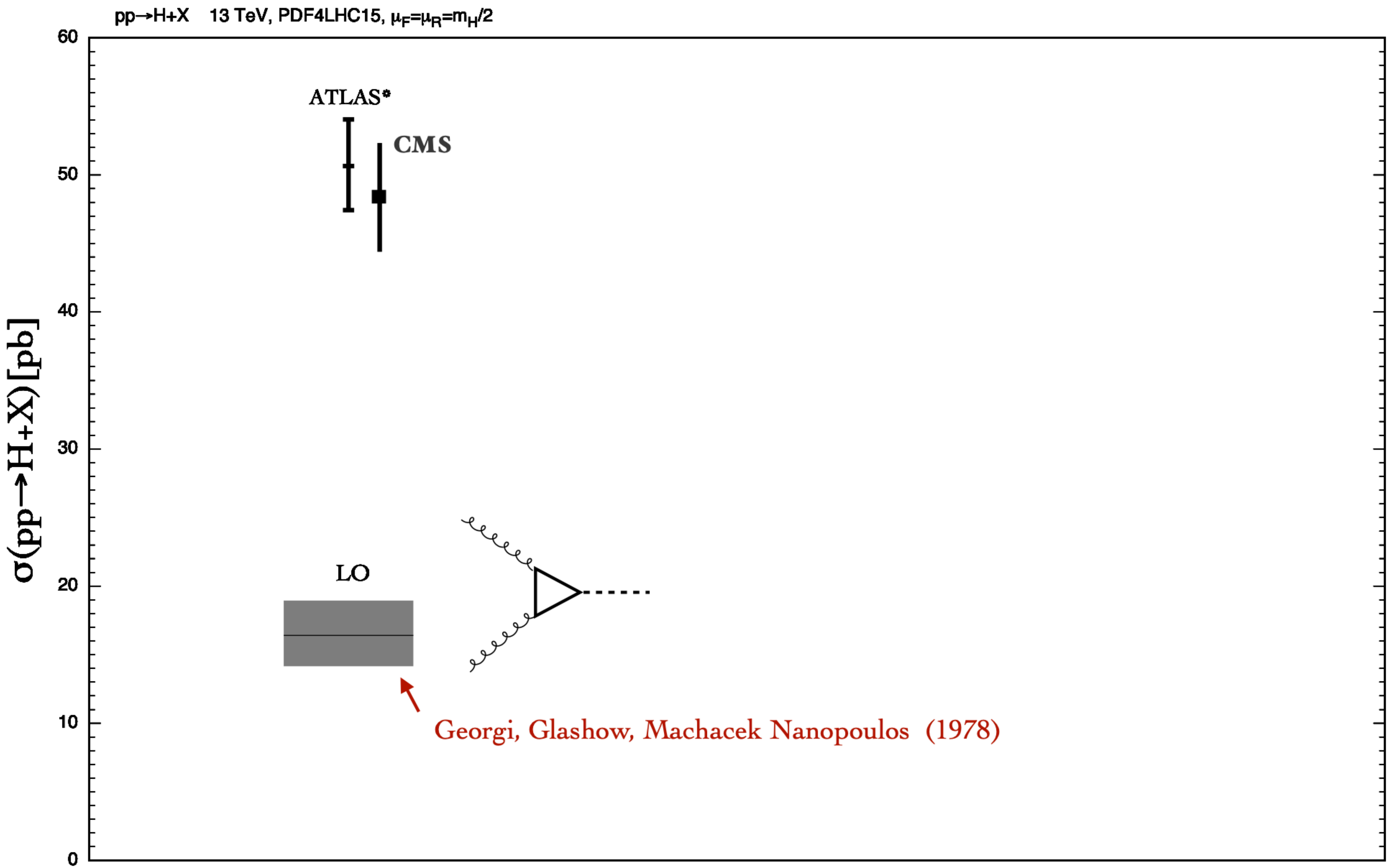
https://hilumilhc.web.cern.ch/content/hl-lhc-project



- ✓ 20 times more data
- ✓ Very high-precision measurements
- ✓ Access to new rare processes
- ✓ Bottleneck in theory predictions...

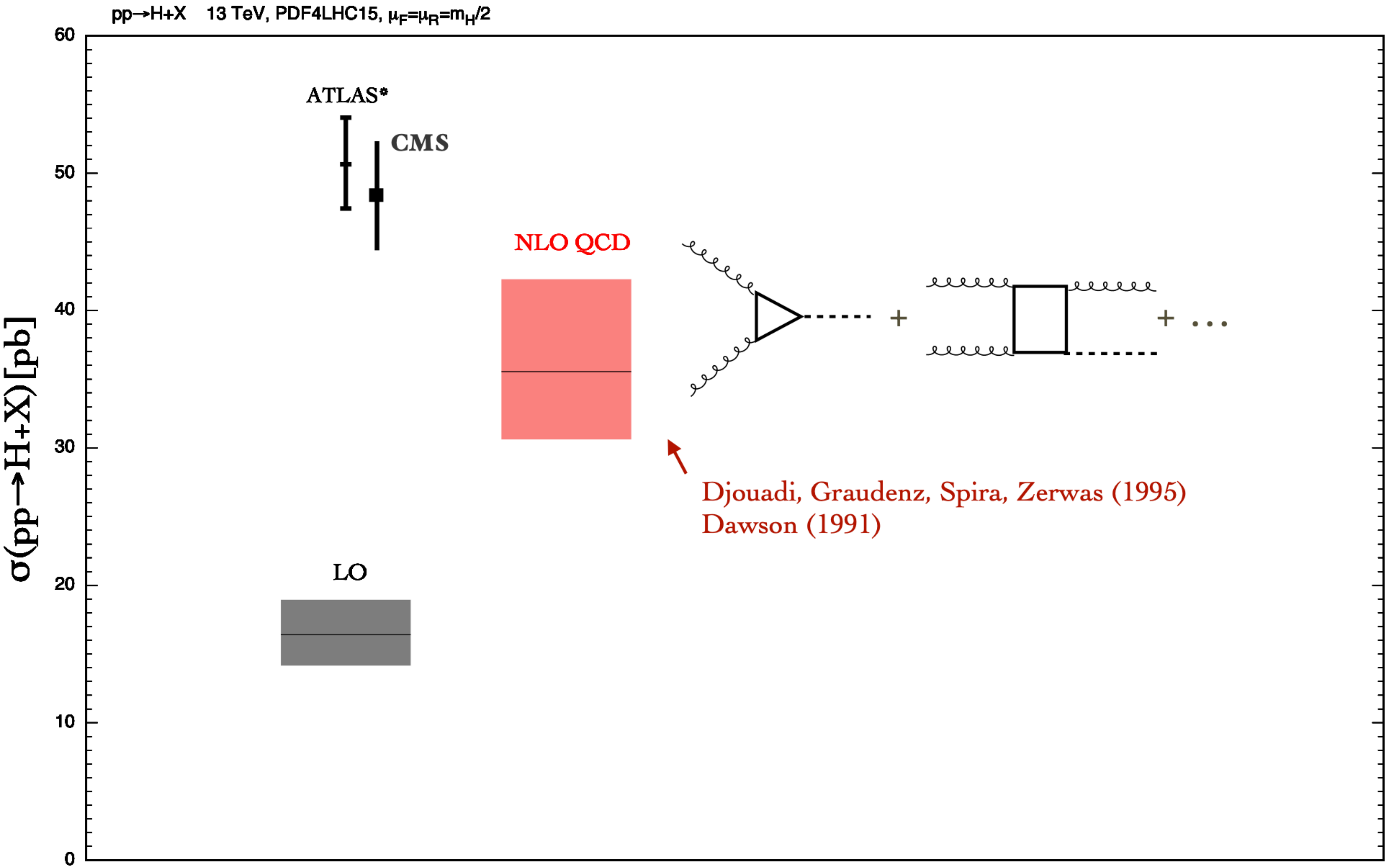


Why do we need precise theory predictions?

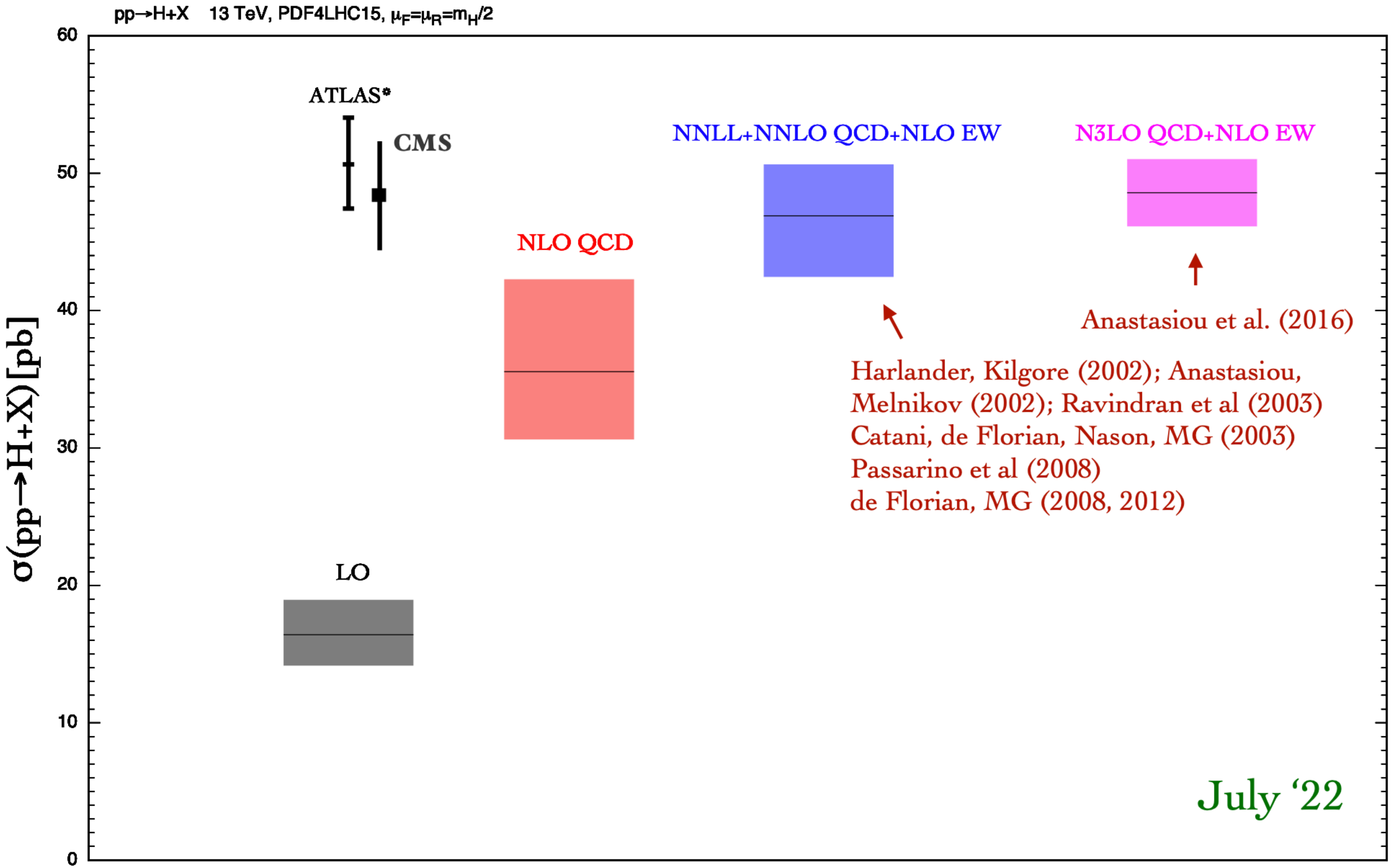


Slide by M.Grazzini

Why do we need precise theory predictions?



Why do we need precise theory predictions?



Slide by M.Grazzini

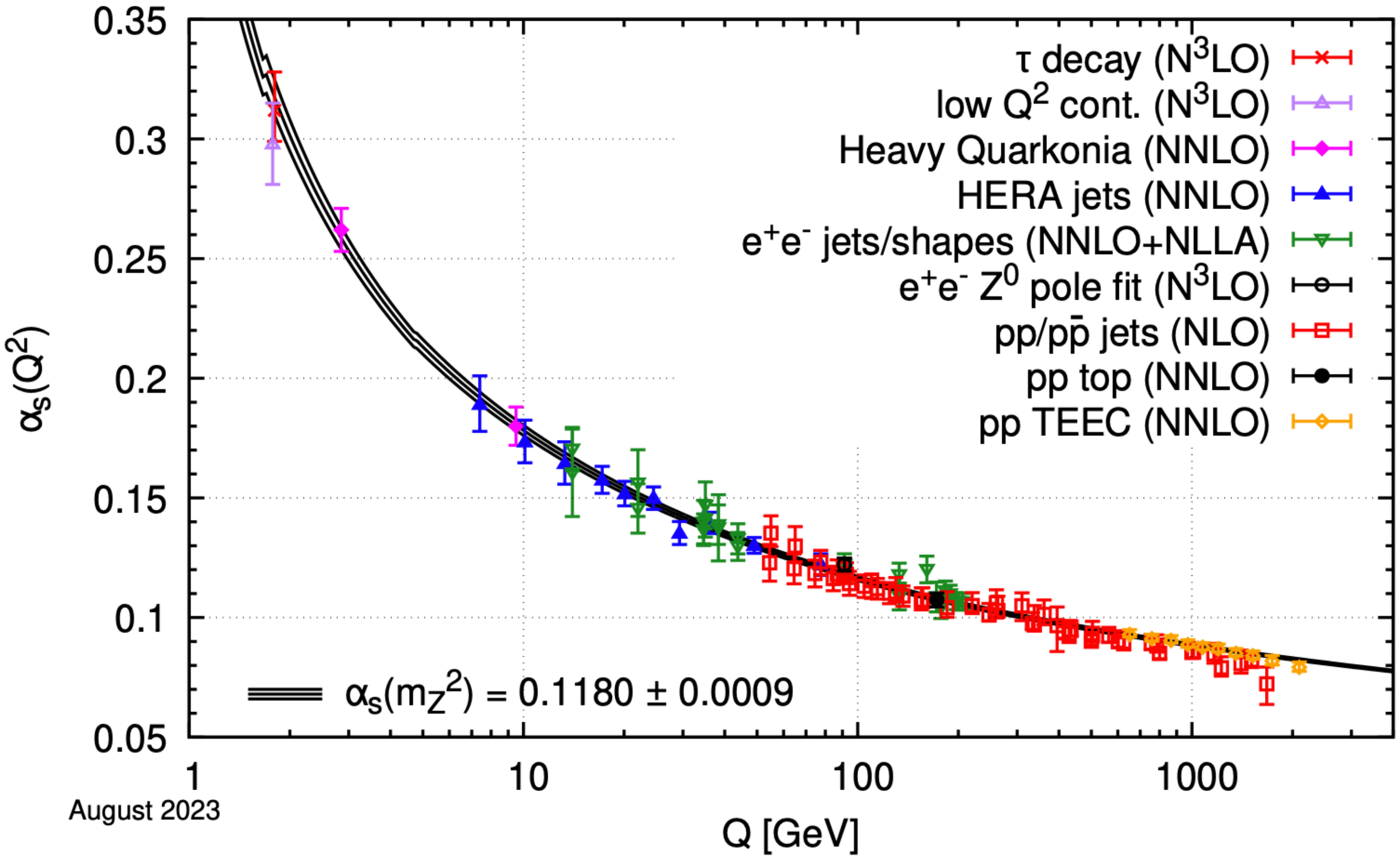
PERTURBATIVE QCD CALCULATIONS

> Quantum chromodynamics is conceptually simple. Its realisation in nature, however, is usually very complex. But not always.

Franck Wilczek [Phys.Today 53N8 (2000) 22-28]

Why is (perturbative) QCD a complex theory?

https://pdg.lbl.gov/2023/



- ✓ QCD looks very different at different energies
- ✓ Particles participating in high-energy interactions are not what detectors measure
    - How do we relate the two perspectives?

But why not always?

- If sufficiently inclusive over final state (i.e., don't ask too many questions about it)

$$\sigma_{AB\to X} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_{a|A}(x_a) f_{b|B}(x_b) \, \sigma_{ab\to X}(x_a, x_b) \left(1 + \mathcal{O}(\Lambda_{QCD}/Q)\right)$$

Parton Distribution Functions (PDFs): non perturbative, but universal

Hard scattering: perturbation theory

Non-perturbative effects: power suppressed

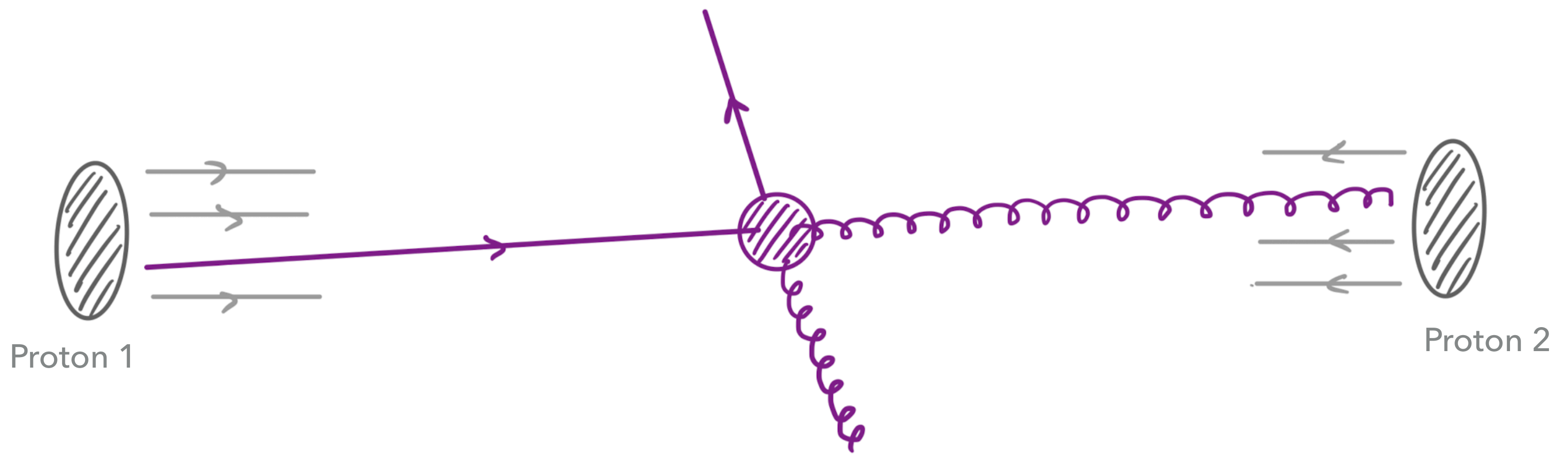
- Collinear factorisation
  - Can define a universal object (the proton) and measure its distribution of quarks and gluons

- Asymptotic freedom: at high-energies, the theory is perturbative
  - Can compute the hard scattering in perturbation theory

- Non-perturbative corrections to factorisation formula: largely unstudied…
  - Start to become an obstruction to increase of theory precision

Anatomy of pQCD calculation — Hard Interaction

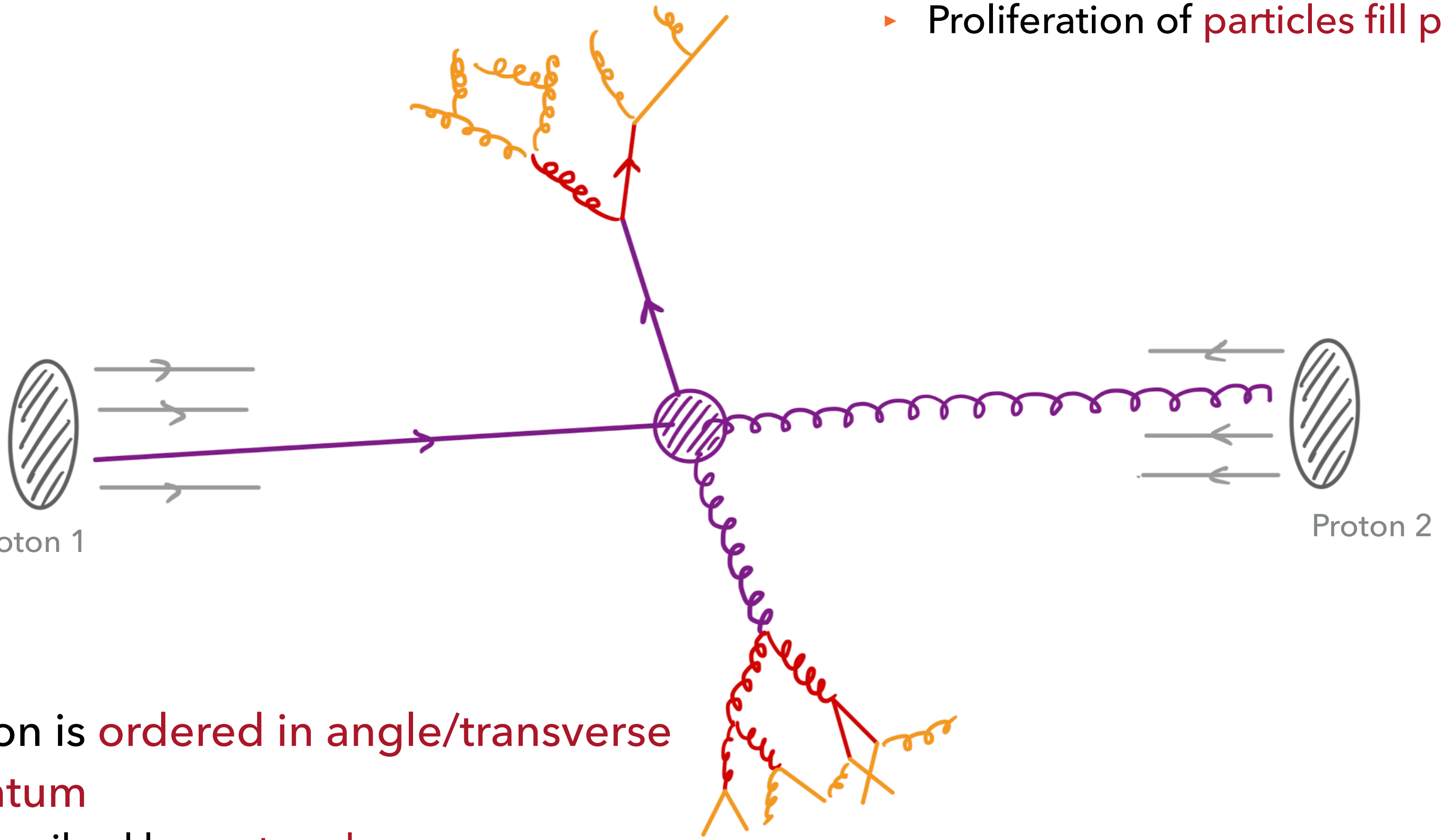
- A high-energy parton is extracted from each proton
  - Rely on non-perturbative PDFs to describe the proton



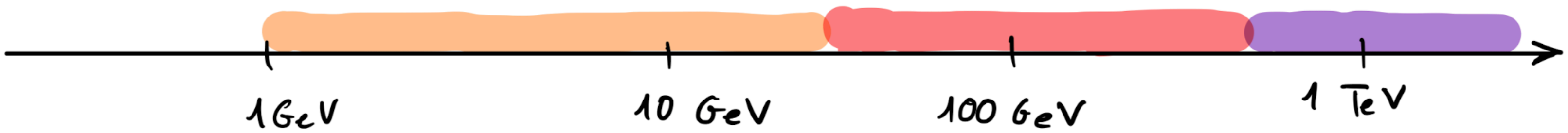
- High-energy interaction:
  - Computable in perturbative QCD
  - Produce high-energy particles

Anatomy of pQCD calculation — Parton Showers

- ✓ Particles produced in the final state radiate
  - Proliferation of particles fill phase-space

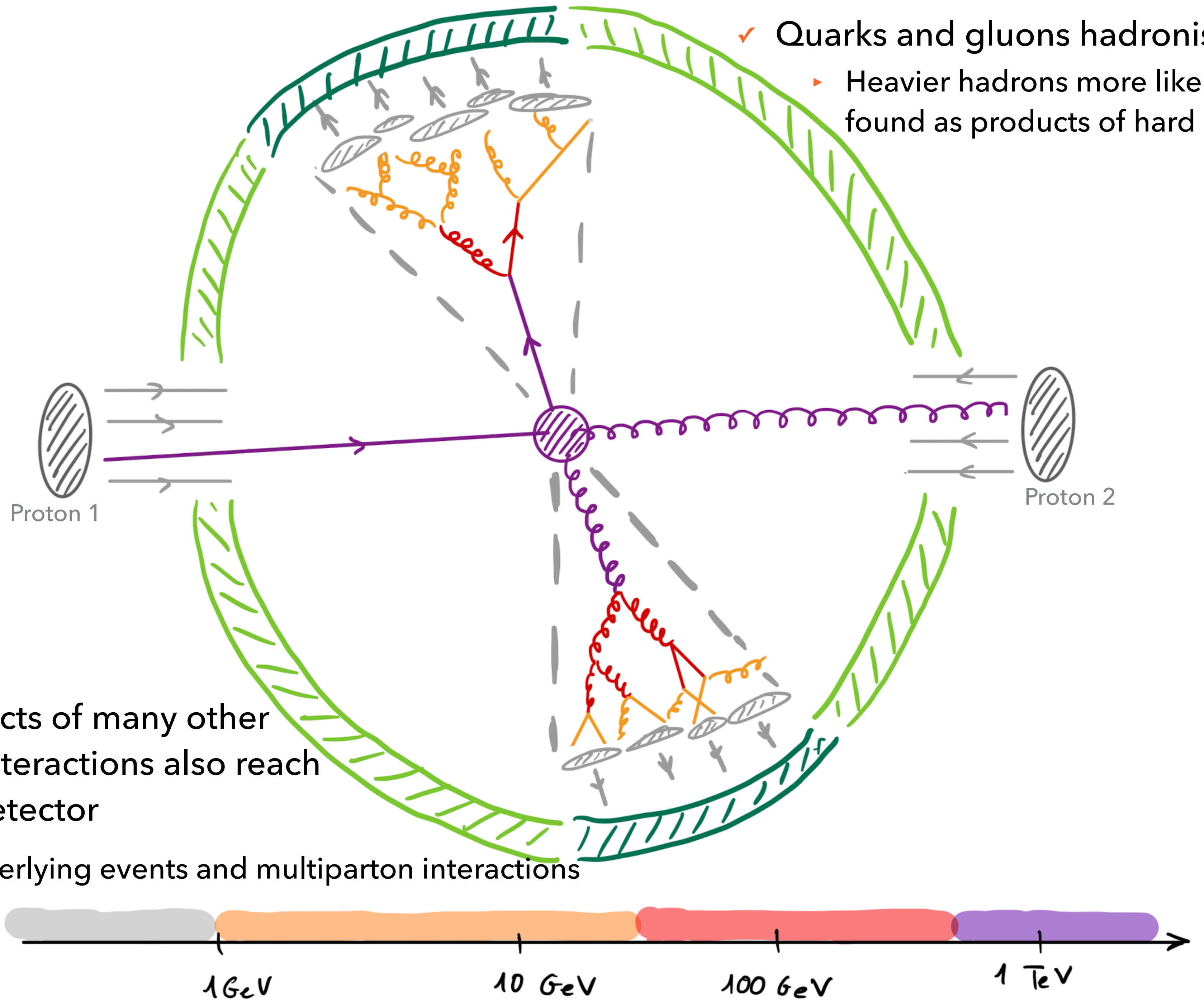


- ✓ Radiation is ordered in angle/transverse momentum
  - Described by parton showers
  - Form collimated jets of particles



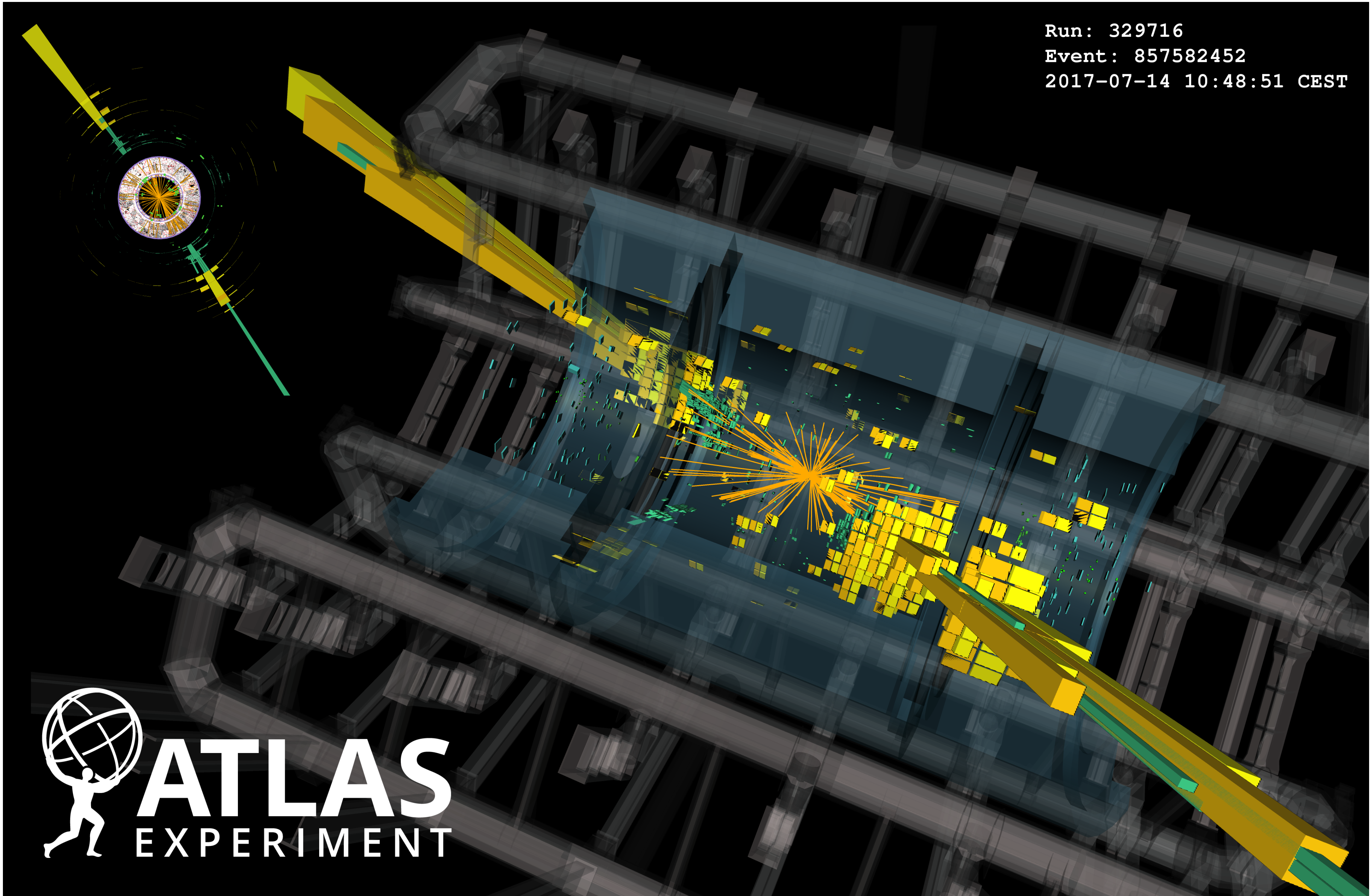
Anatomy of pQCD calculation — Hadronisation and UE&MPIs

- ✓ Quarks and gluons hadronise
  - ‣ Heavier hadrons more likely to be found as products of hard interaction

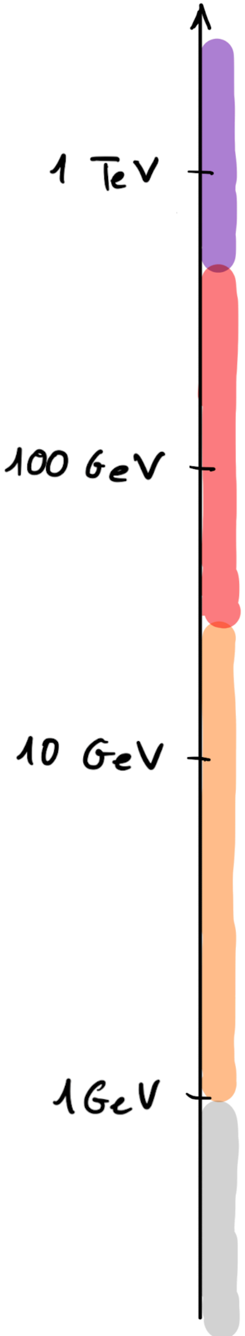


- ✓ Products of many other soft interactions also reach the detector
  - ‣ Underlying events and multiparton interactions

Anatomy of pQCD calculation —— Real Life



Multiple scales



- Hard interaction: perturbative QCD calculation

$$\sigma(Q) = \sigma_0 \left(1 + \alpha_S(Q)\sigma_1 + \alpha_S^2(Q)\sigma_2 + \dots\right) \qquad Q \sim 1\text{TeV}$$

LO NLO NNLO

- Relate processes between disparate scales $Q \sim 1\text{TeV}$ and $\mu_{\text{had}} \sim 1$ GeV
  - Large logarithms appear $L = \ln Q/\mu_{\text{had}} \sim 7$
  - Spoils convergence of perturbative series: $\alpha_s L^2$, $\alpha_s L$

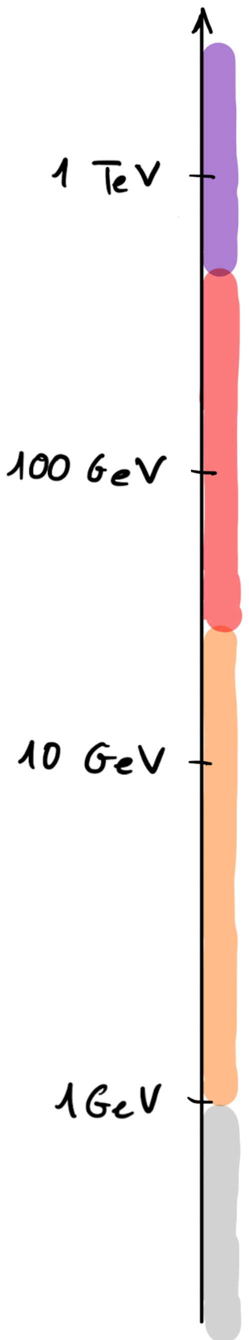
- Large logarithms must be resummed

$$\sigma = \sigma_0 \exp\left(\alpha_s^n L^{n+1} a_1 + \alpha_s^n L^n a_0 + \alpha_s^n L^{n-1} a_{-1} + \dots\right)$$

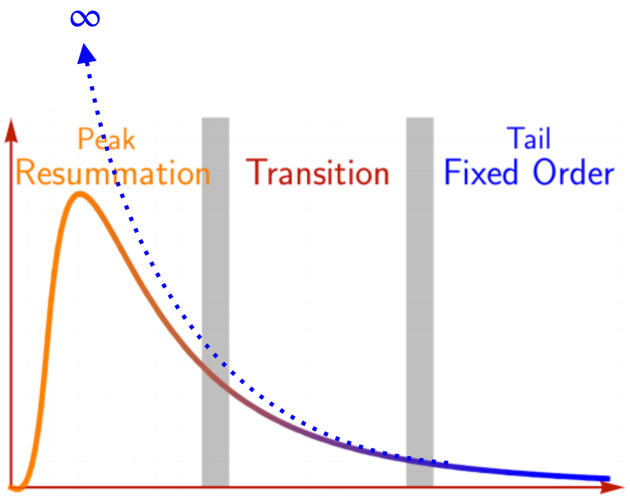
LL NLL NNLL

- Hadronisation, underlying events and multiparton interactions
  - Non-perturbative, model dependent
  - Different general purpose Monte Carlo codes implement different models
  - Differences included in theoretical error estimates

Parton showers and Resummation



$$\sigma = \sigma_0 \exp\left(\alpha_s^n L^{n+1} a_1 + \alpha_s^n L^n a_0 + \alpha_s^n L^{n-1} a_{-1} + \ldots\right)$$

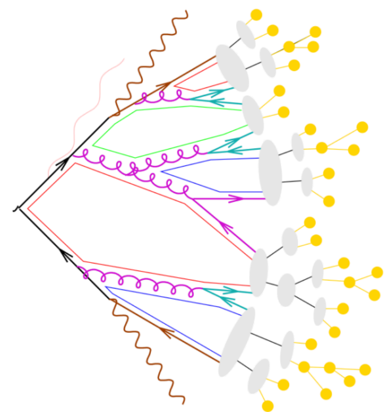


RESUMMATION

Inclusive processes, tailored to specific observables. Can reach very high logarithmic accuracy

- State of the art: $N^{2,3}LL$
- E.g.: For Drell Yan, N^3LO+N^3LL, giving theory predictions with few % error

DYTurbo [Camarda, Cieri], **Cute-MCFM** [Neumann, Campbell], **NNLOJet+RadISH** [Chen, Gehrmann, et al] 2022



PARTON SHOWERS

More exclusive processes, based on MC algorithms. Interfaced with hadronisation models in general purpose Monte-Carlo codes

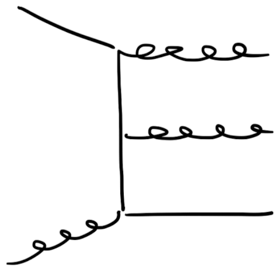
- Need to match fixed order and PS
- New generation of PS with controlled and systematically improvable accuracy
- State of the art: NLL, some NNLL

HARD INTERACTION: REAL RADIATION, FEYNMAN INTEGRALS AND AMPLITUDES

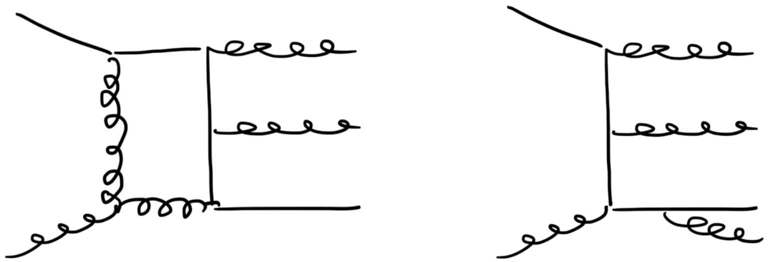
Loops and Legs

$$\sigma \sim \int d\Phi \, \left| \mathscr{A} \right|^2$$

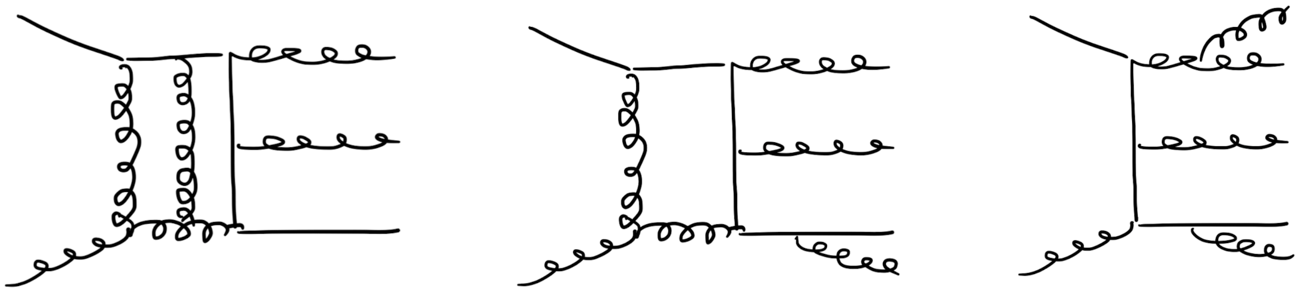
Leading Order



NLO



NNLO



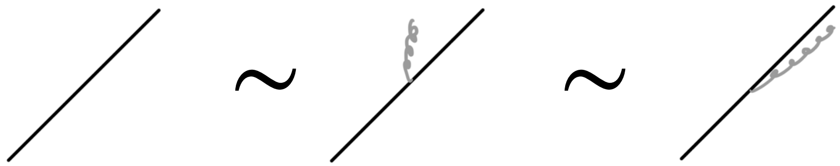
✓ The higher the order, the more loops and external legs we have

Phase-space integration and singularities

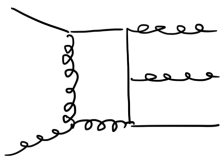
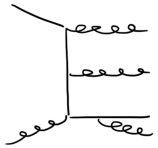
$$\sigma \sim \int d\Phi \, |\mathscr{A}|^2$$

- ✓ Loop amplitudes have IR singularities (after UV renormalisation)

- ✓ Phase-space integration has IR singularities



$E_g \to 0$ $\theta_g \to 0$

- ✓ Sum is finite: $\int d\Phi_3$  $+ \int d\Phi_4$ 

- ✓ Two approaches in phase-space integration:
  - Subtraction: build counter terms ⇒ process specific, very efficient
  - Slicing: introduce cut-off in integration ⇒ process independent, less efficient

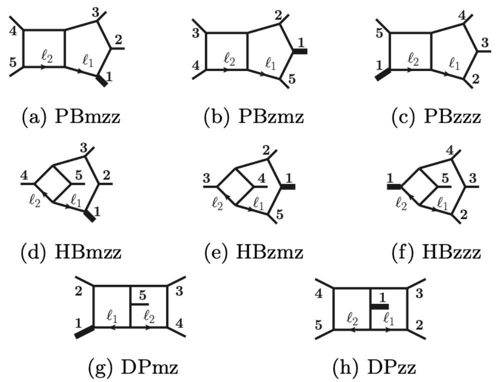
- ✓ State of the art: $2 \to 1$ at N^{3}LO, $2 \to 3$ at NNLO

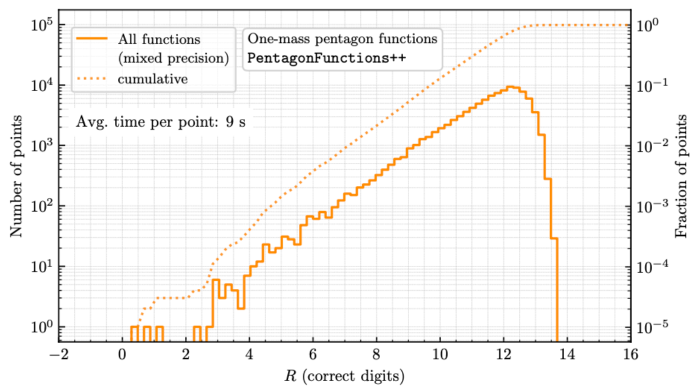
(Loop) Amplitudes — Master integrals

$$\mathscr{A} = \sum c_i(\vec{p};\epsilon)\ m_i(\vec{p};\epsilon)$$

Master coefficients **Master integrals**

- ✓ Feynman integrals form vector spaces: basis is theory independent ⇒ fundamental information about all QFTs
- ✓ Complicated multivalued functions ⇒ large overlap with pure mathematics
- ✓ Intricate analytic structure with interesting underlying geometry (elliptic, Calabi-Yau, ...)
- ✓ Goal: control analytic structure & fast and stable numerical evaluation
- ✓ Very advanced numerical approaches (differential equations, sector decomposition, ...)
- ✓ Example: master integrals for production of Higgs + 2 jets [Abreu et al, 2306.15431]
  - 6 variables
  - Hundreds of integrals
  - Hundreds of log singularities





(Loop) Amplitudes — Master coefficients

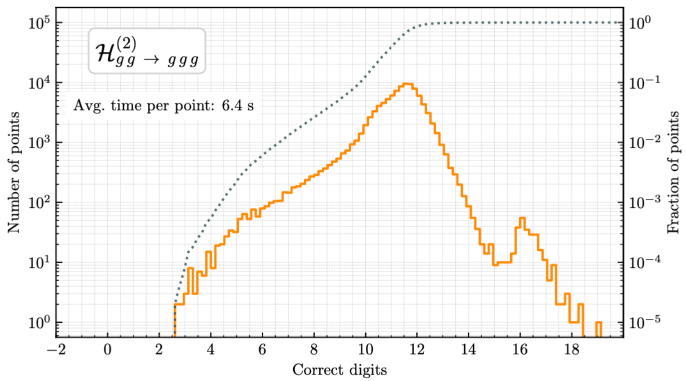
$$\mathscr{A} = \sum c_i(\vec{p};\epsilon)\; m_i(\vec{p};\epsilon)$$

Master coefficients **Master integrals**

- ✓ Theory specific ⇒ e.g., much more complicated in QCD than in $\mathcal{N} = 4$ SYM
  - Develop new techniques (unitarity, …) in simpler theories
- ✓ Main bottleneck: solving linear systems of Integration-By-Parts relations
- ✓ Complicated rational functions: use finite fields, tools from algebraic geometry, …
- ✓ State of the art: $2 \to 1$ at 4 loops, $2 \to 2$ at 3 loops, $2 \to 3$ at 2 loops
- ✓ E.g.: amplitudes for three-jet production at the LHC

[Abreu et al, 2102.13609]
[Agarwal et al, 2311.09870]
[de Laurentis et al, 2311.10086, 2311.18752]

  - Simple coefficients (after a lot of work)
  - Numerically stable, ready for pheno
  - Can we understand their analytic structure better?



A lot of progress in the past few years

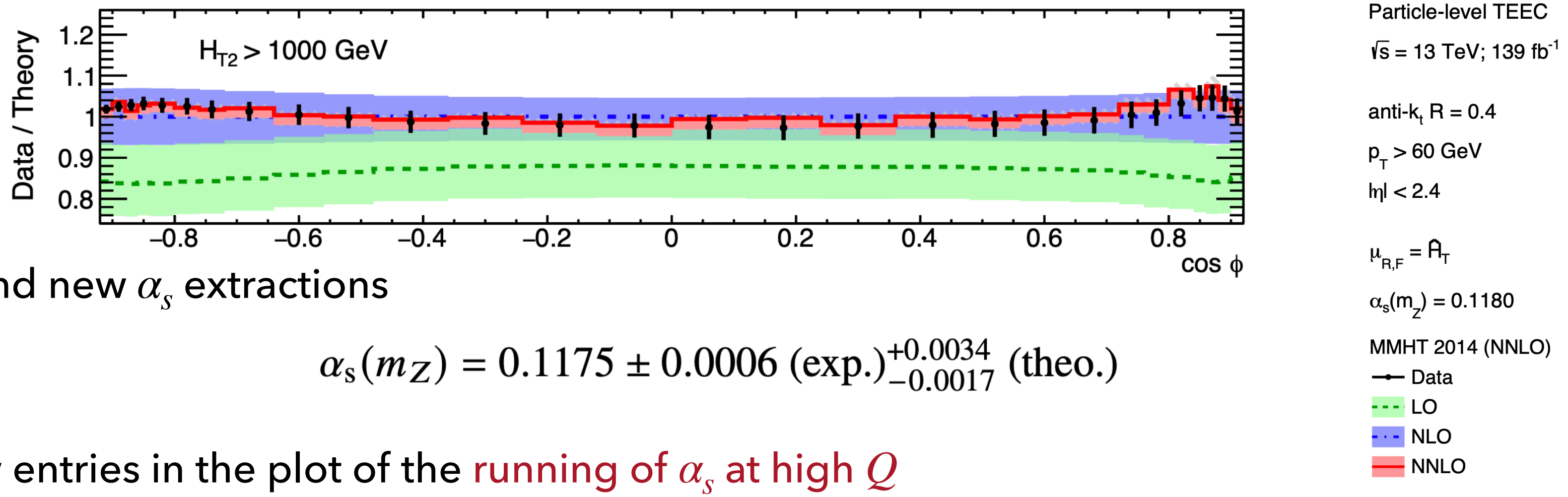
- More and more phenomenology studies at NNLO for 2 to 3 processes
  - $pp \to \gamma\gamma\gamma, pp \to \gamma\gamma + j, pp \to \gamma + jj, pp \to jjj$
  - $pp \to Wb\bar{b}, pp \to Ht\bar{t}, pp \to Wt\bar{t}$

- NNLO corrections to 3-jet production at the LHC

[Czakon, Mitov, Poncelet '21]

[ATLAS, JHEP 07 (2023) 85]

  - Energy-energy correlators…



  - … and new α_s extractions

$$\alpha_s(m_Z) = 0.1175 \pm 0.0006\,(\text{exp.})^{+0.0034}_{-0.0017}\,(\text{theo.})$$

  - New entries in the plot of the running of α_s at high Q

  - Among most complex NNLO calculations: 100M CPU hours ⇒ big problem we need to address for the future!

SUMMARY AND OUTLOOK

Summary and Outlook

- Precise theory predictions are crucial to exploit the full potential of the LHC
    - Wealth of new data will be available in the coming years
    - Great potential to test the SM and find new physics beyond it

- A lot of progress in QCD corrections
    - Extensive progress on parton showers and resummation
    - A lot of progress on amplitudes/Feynman integrals

- NNLO corrections: 2 to 2 processes largely done, 2 to 3 becoming a reality
    - Can we go to N^3LO? Ok for 2 to 1, partial results for 2 to 2
    - How do we make this useful for experimentalists?

- Next challenges towards percent-level phenomenology
    - Include EW corrections? More challenging because of masses
    - How to handle processes with more scales? Technically challenging
    - How to handle non-perturbative corrections? Conceptually challenging

THANK YOU!