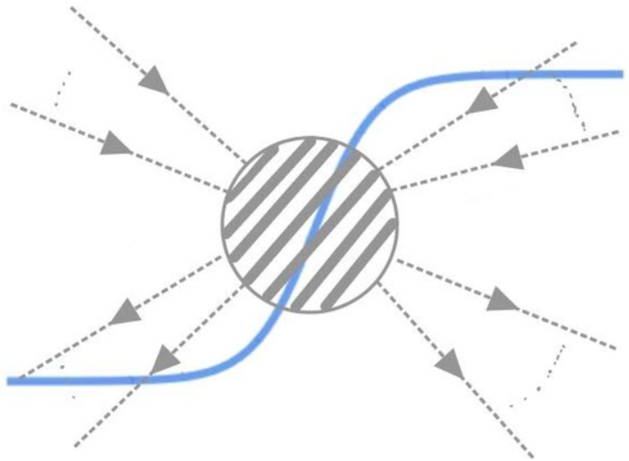

Dark Matter in & around Phase Transitions

Rudin Petrossian-Byrne (ICTP, Trieste)

Connected to work in collaboration with

Isabel Garcia Garcia (U. Washington),
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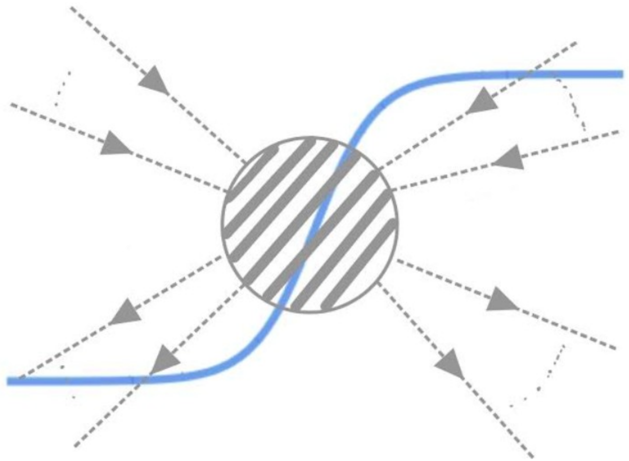

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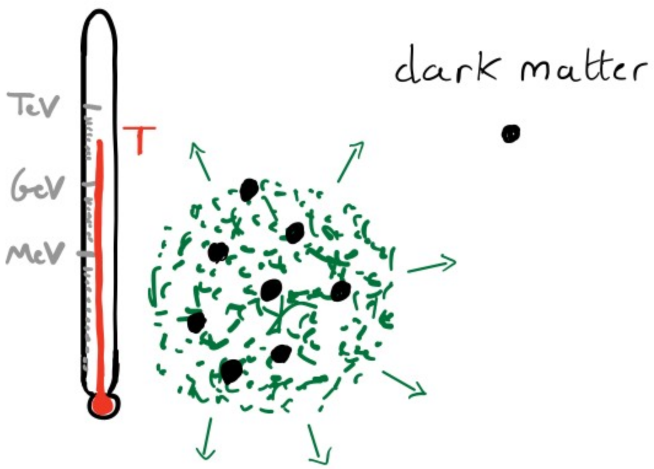
[Beyond WIMPs]

exemple of

Thermally connected



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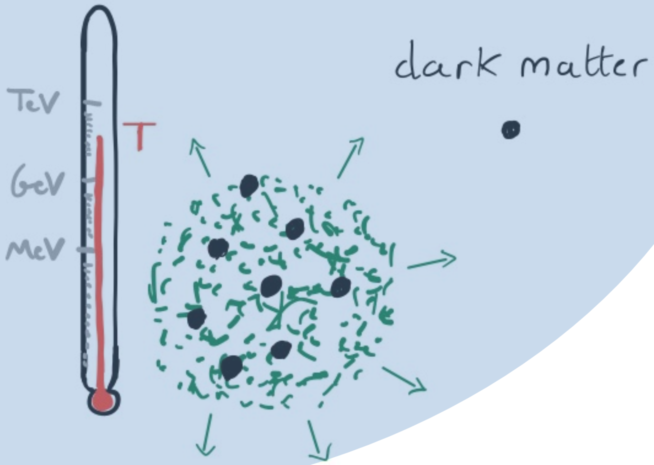

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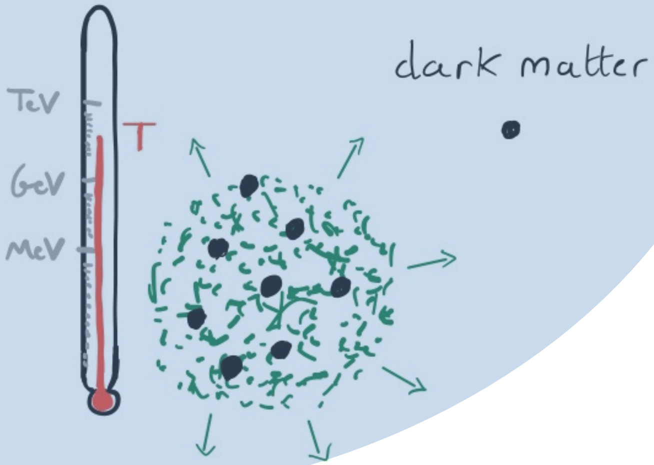

'I exist, therefore I am' (dark matter)

[Axion / ALPs , ~Vectors]

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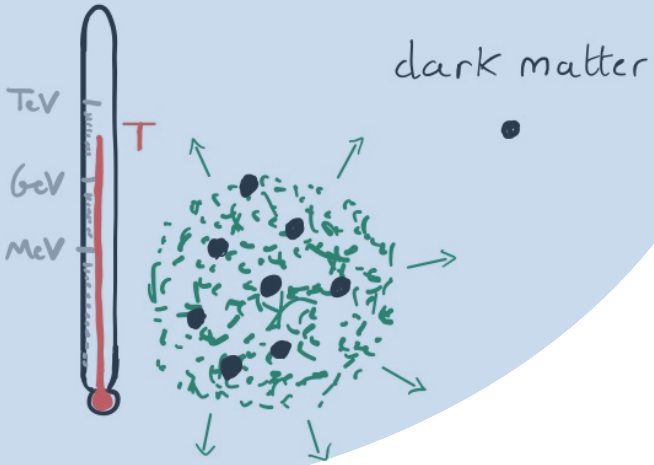

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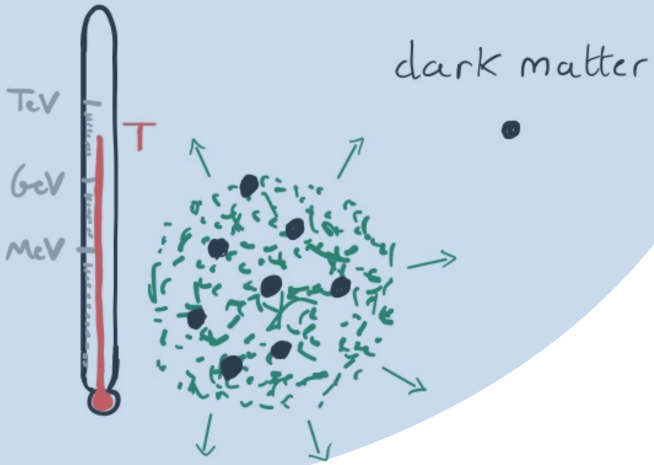

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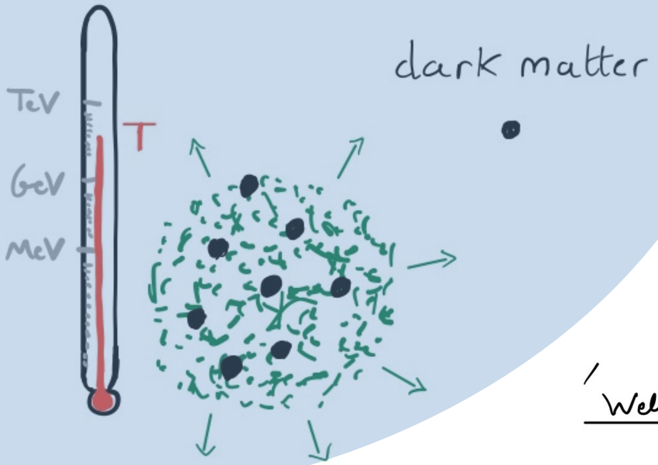

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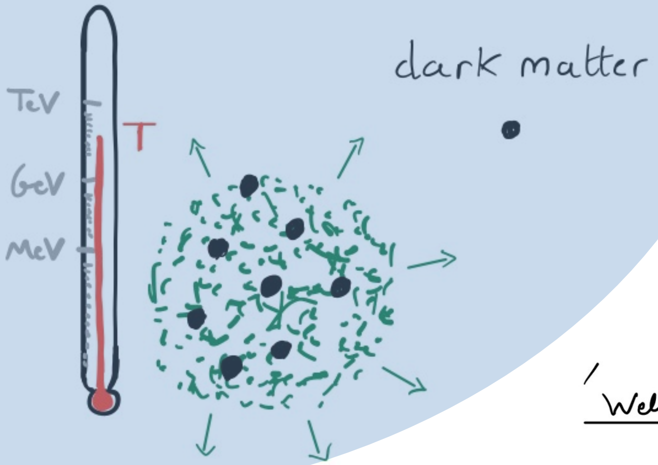

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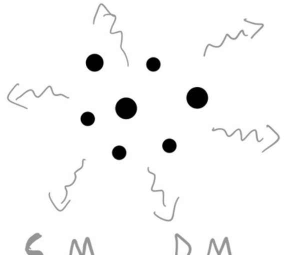

'Hawking Genesis'

[Lennon, March-Russell, RPB, Tillim (2018)]

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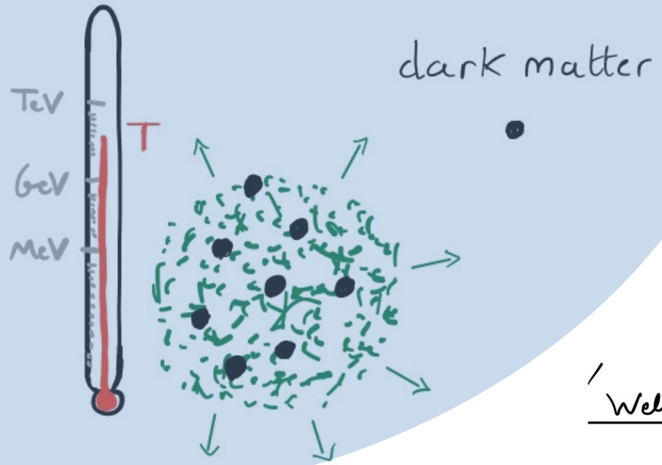

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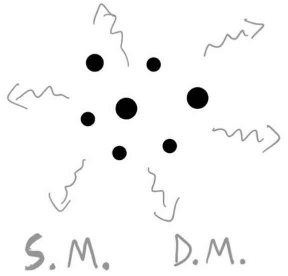


'Hawking Genesis'

Cosmological Phase Transitions

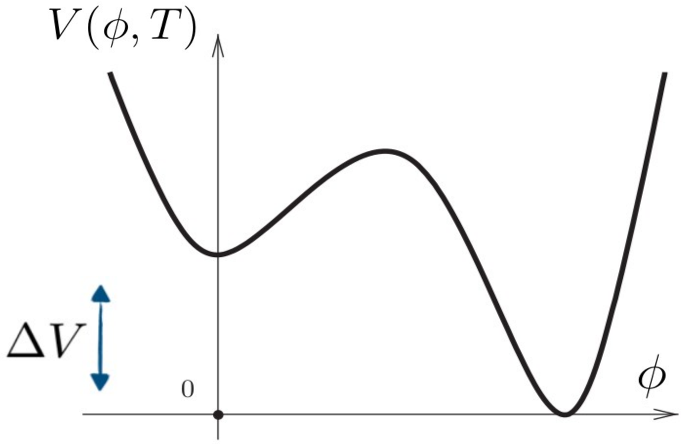
The universe 'boils' & departs from equilibrium

$\Longrightarrow$ Dark Matter produced or affected by Bubble dynamics

[Lennon, March-Russell, RPB, Tillim (2018)]

1^{st} Order Phase Transitions (FOPTs) in the Early Universe

Proposed long ago as a possibility for the big bang phase *Kirzhnits, Linde (1972), Weinberg (1974), Witten (1984)....*



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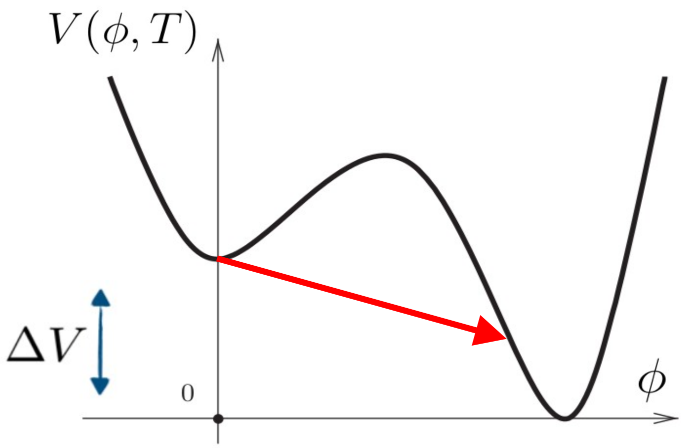

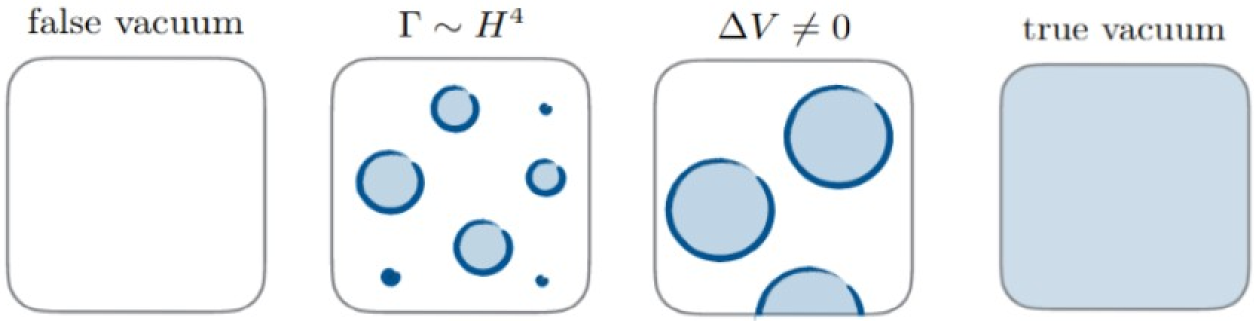

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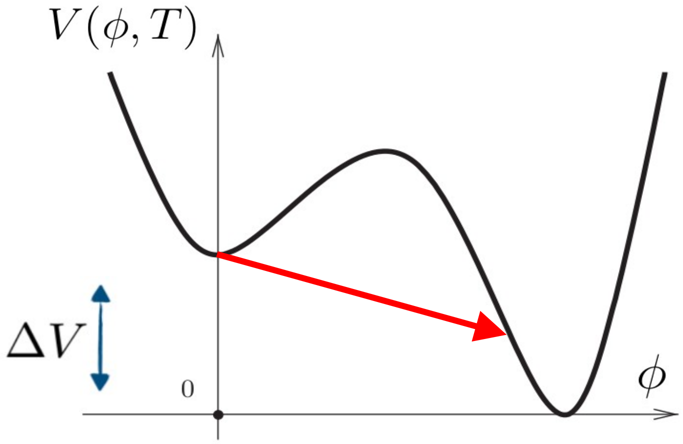

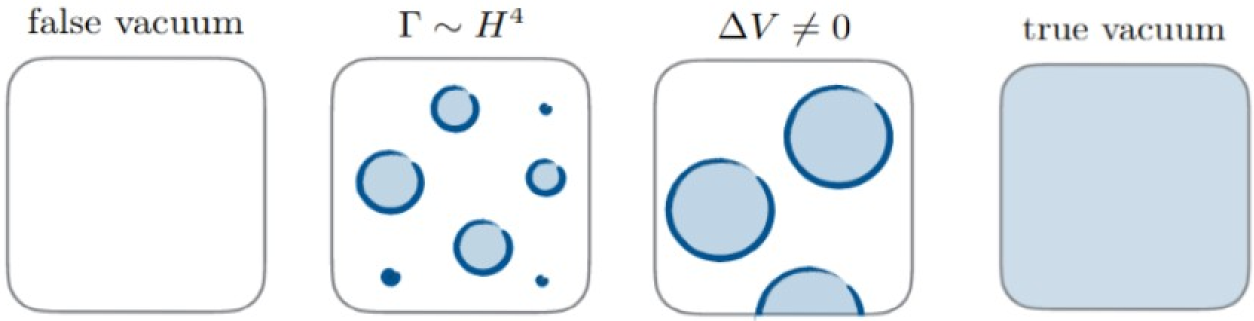

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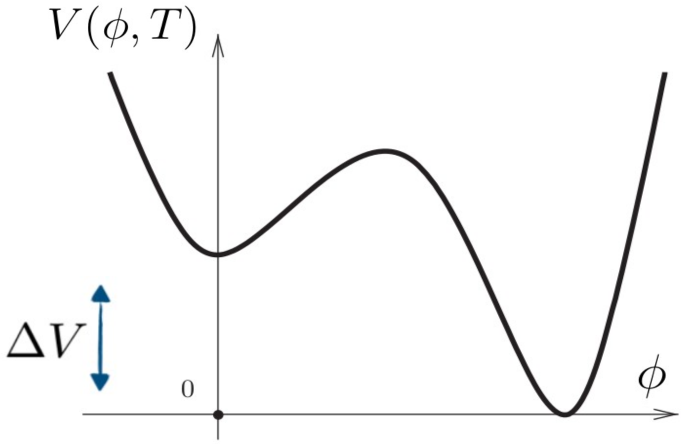
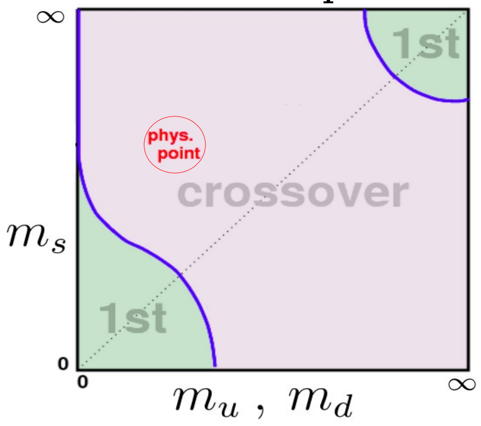


$T \sim 150$ MeV **QCD**

'Columbia plot'



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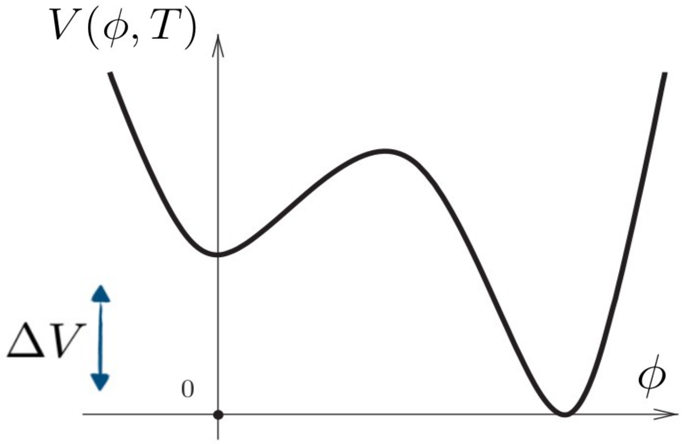

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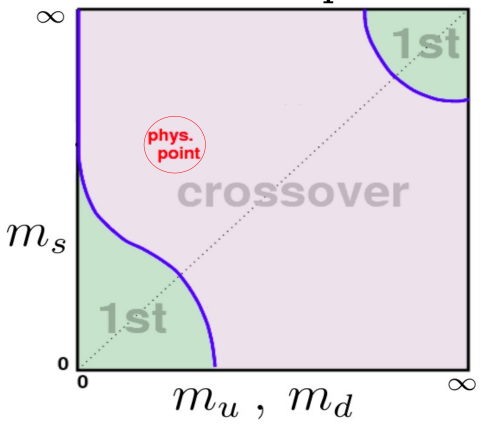

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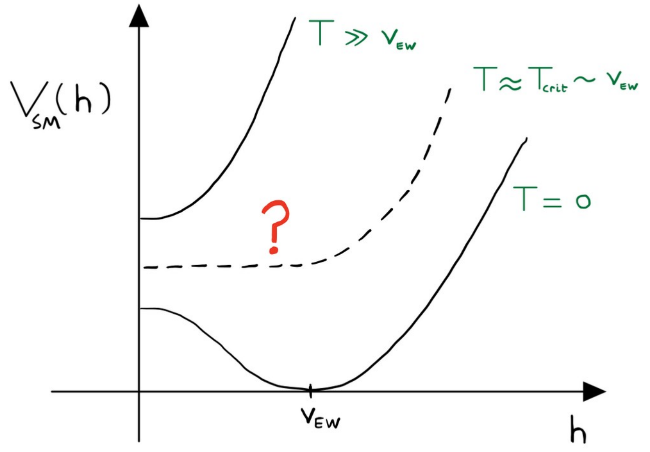

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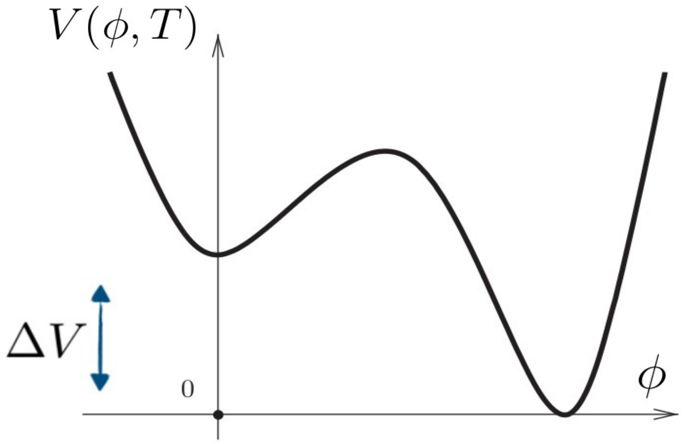

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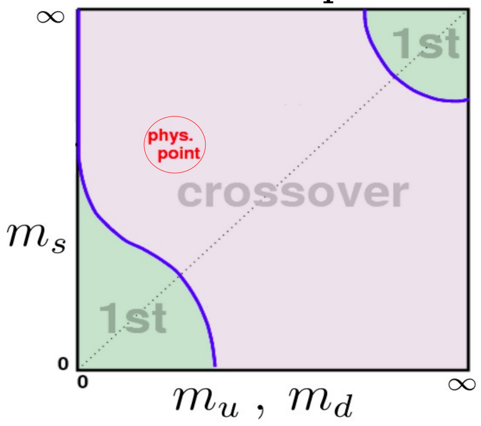

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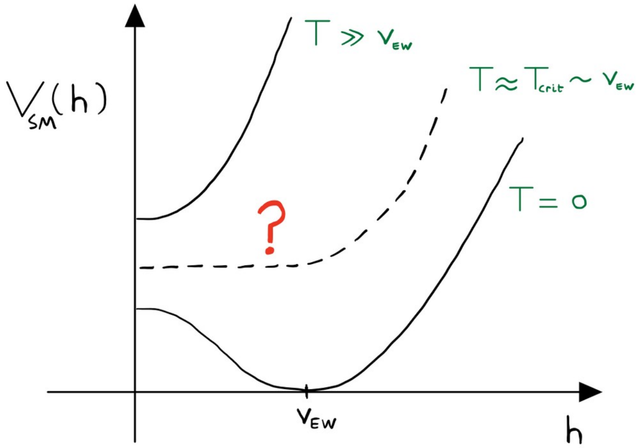

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...Both are now understood to be smooth crossovers

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In fact...

Today, there is no FOPT of the fundamental interactions in 4d at $\mu = 0$ **for any** T!

Higgs instability *[e.g. Degrassi et al. (2012)]* far from conclusively established

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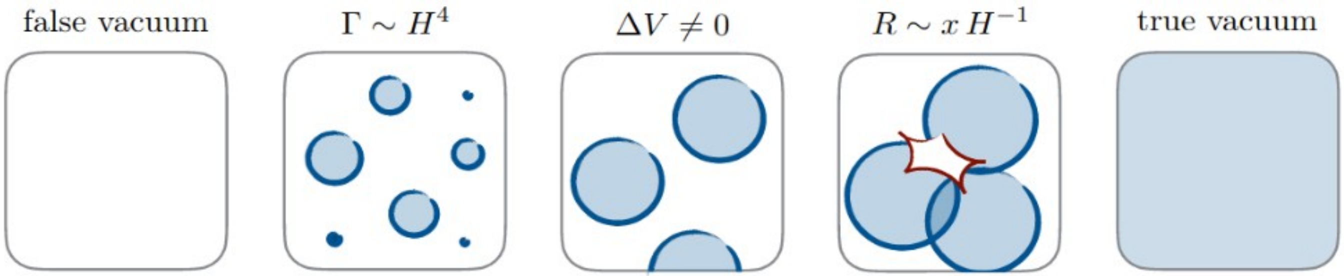
- **Gravitational waves**: at upcoming detectors (and even completely decoupled sectors become interesting)

Recent detection by PTOs - though most likely from super massive black holes – is a validation of the prospects

Gravitational Waves



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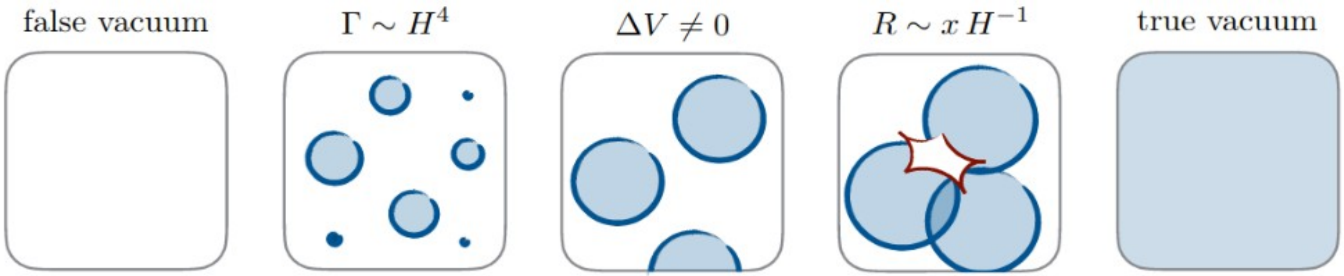

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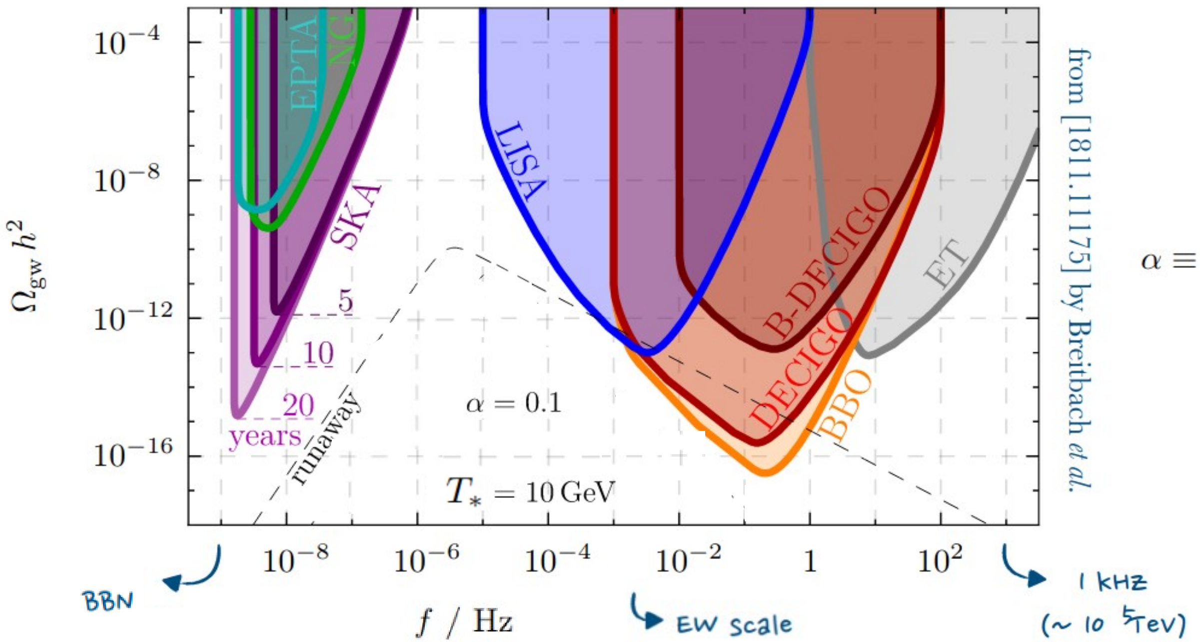
$\Rightarrow$ current/future observatories may be sensitive to the resulting stochastic background



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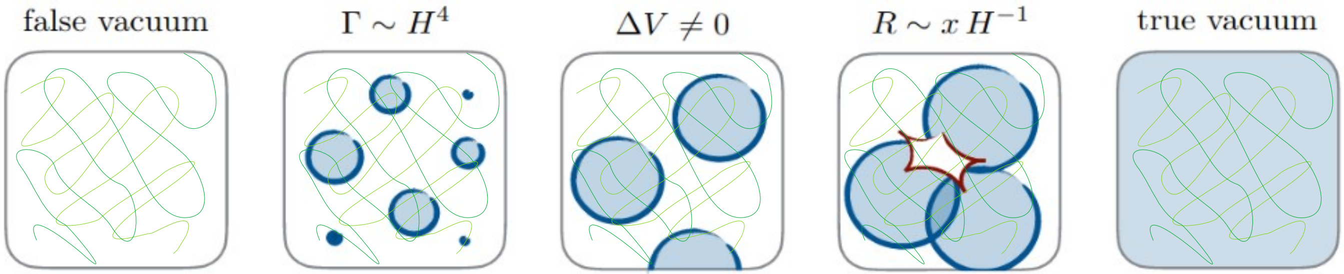
Gravitational Waves

Bubbles are surrounded by ‘stuff’…..



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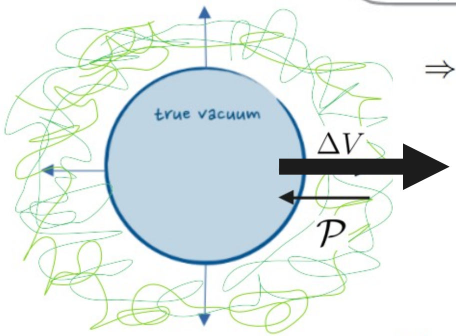

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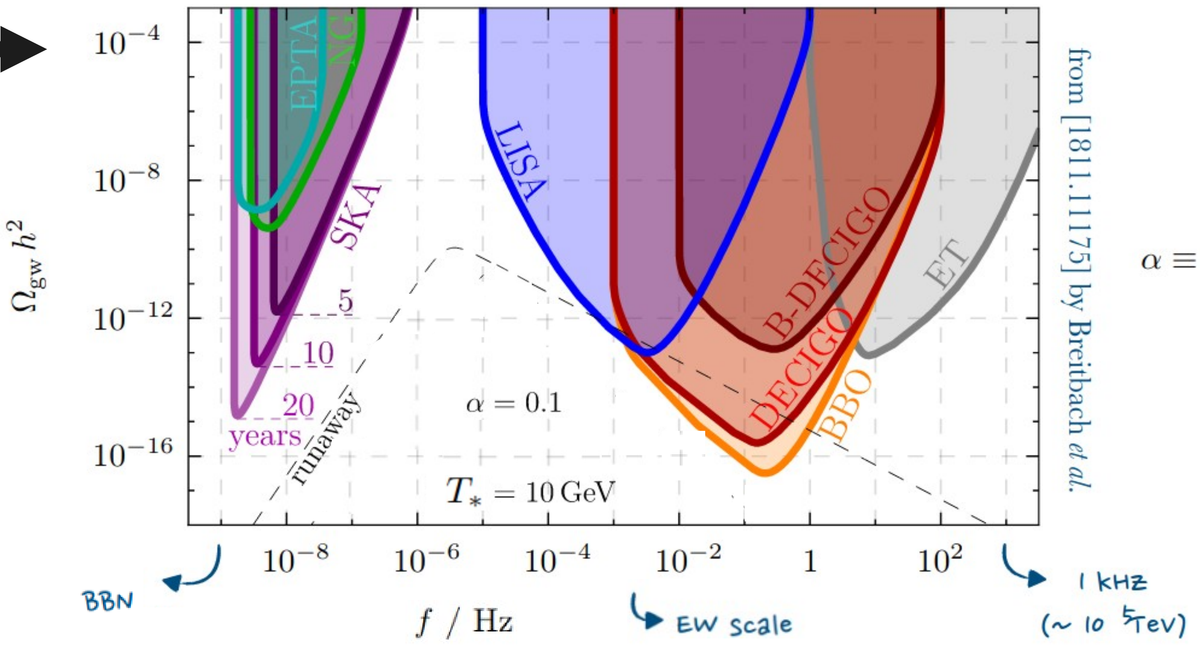

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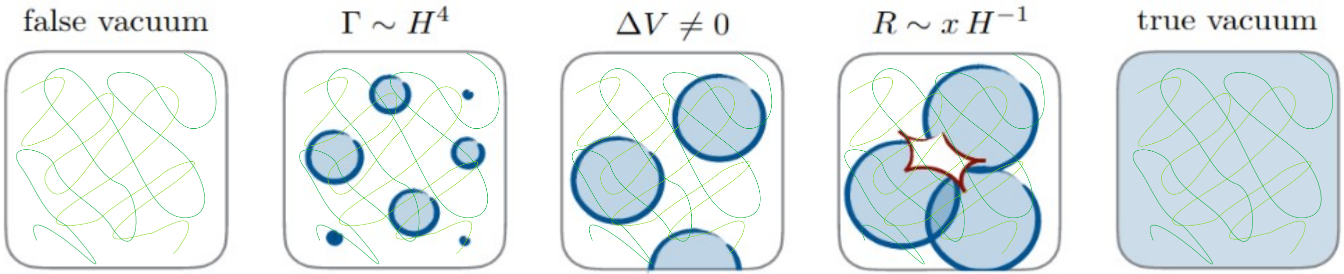

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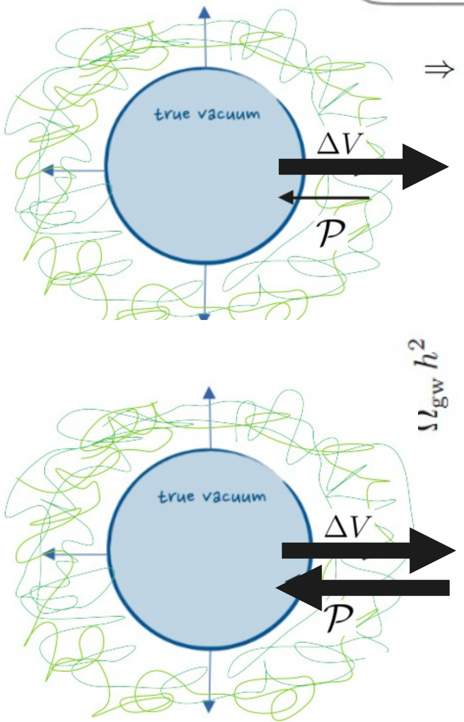

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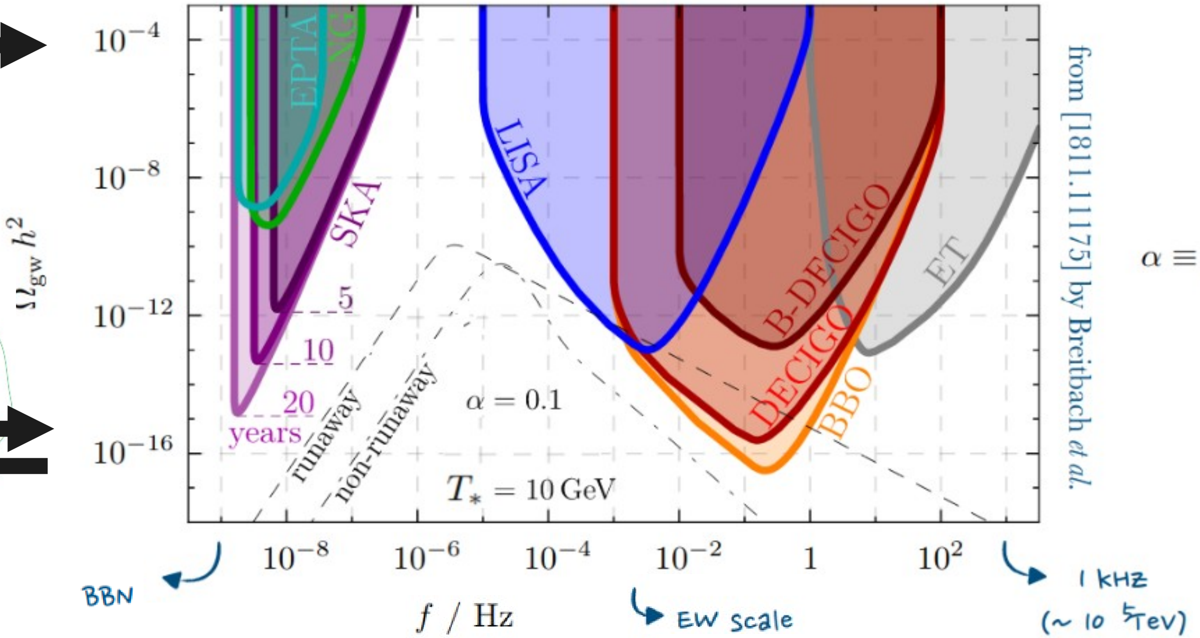

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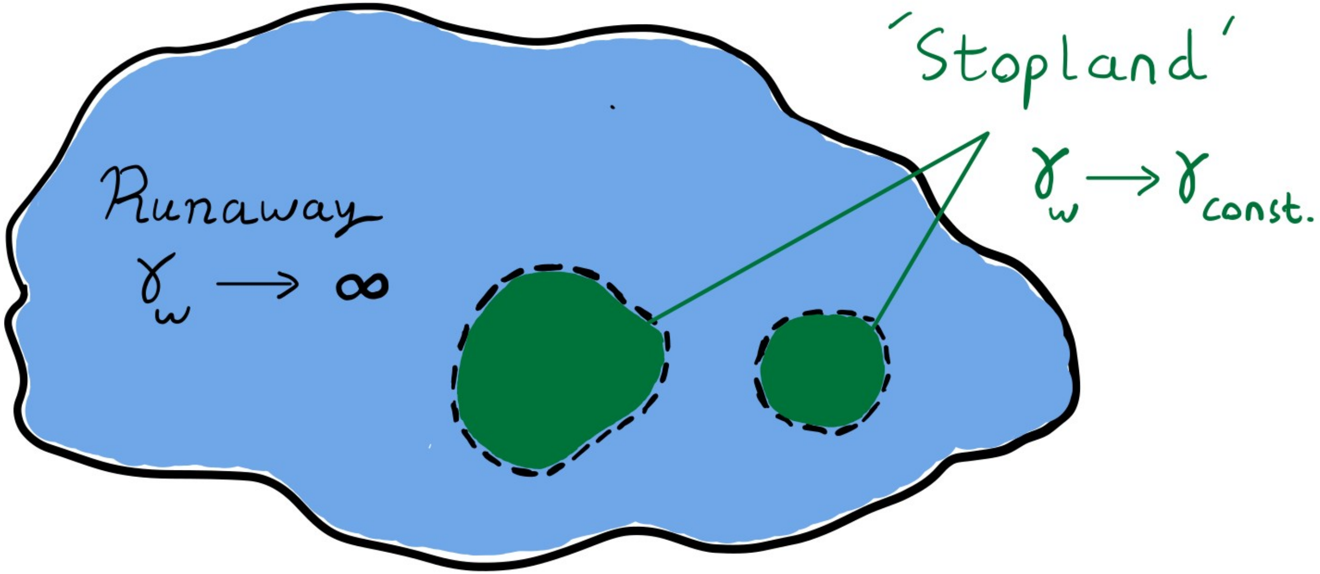
In the landscape of FOPTs.... which subset are runaway and which are obstructed ?



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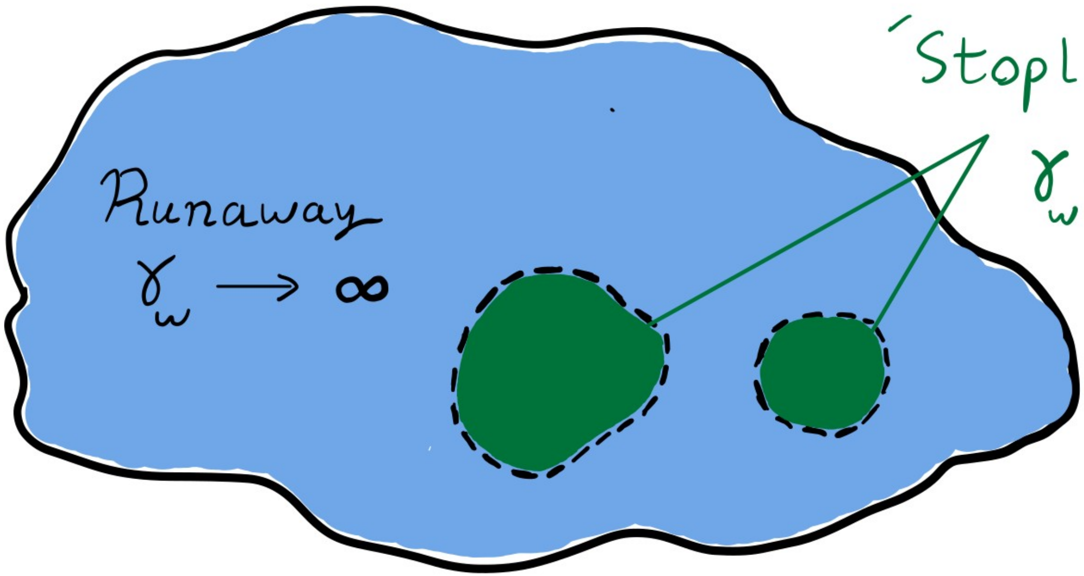

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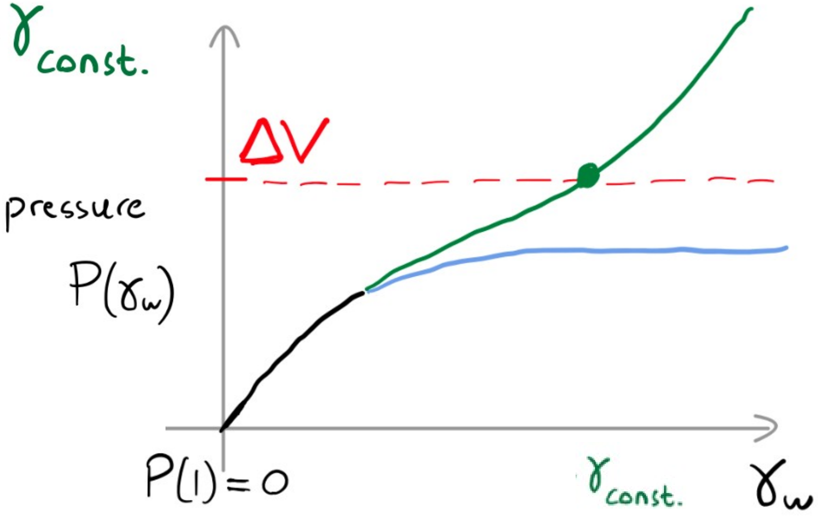

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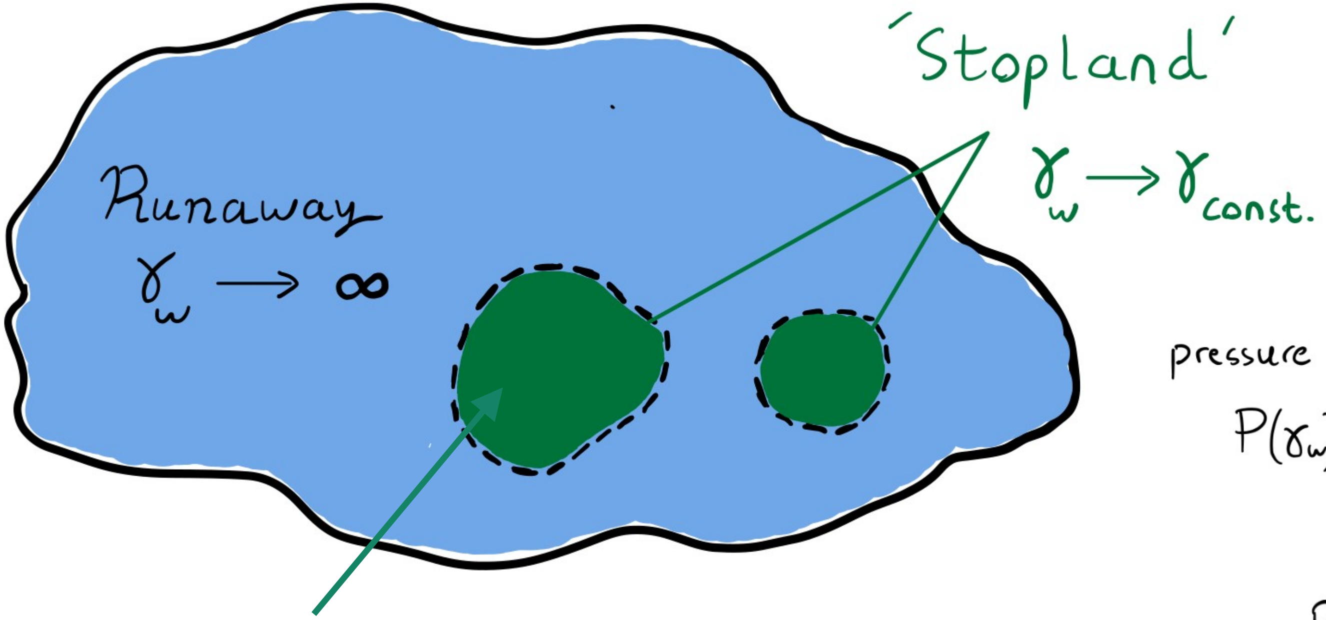

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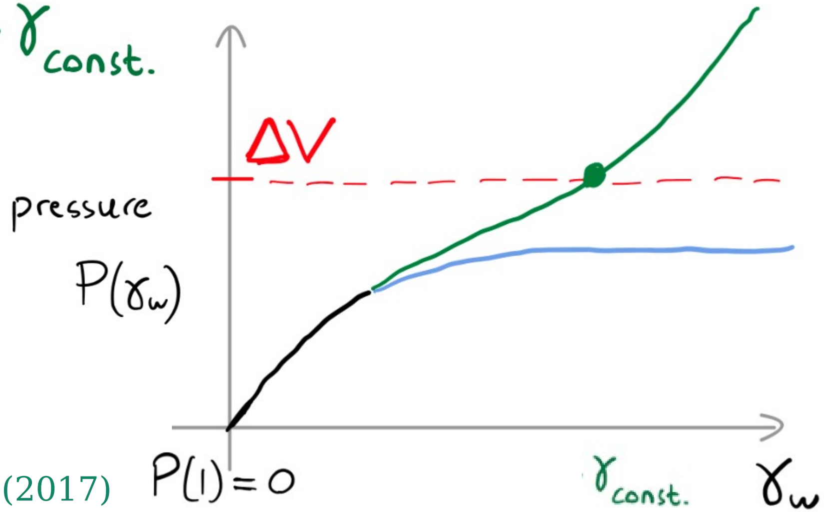

- Spontaneous breaking of gauge theories
- (and restoring)

  Azatov, Barni, **RPB** (2024 to appear)

Bodecker, Moore (2017)
Hoeche, Kozaczuk, Long, Turner, Wang (2020)
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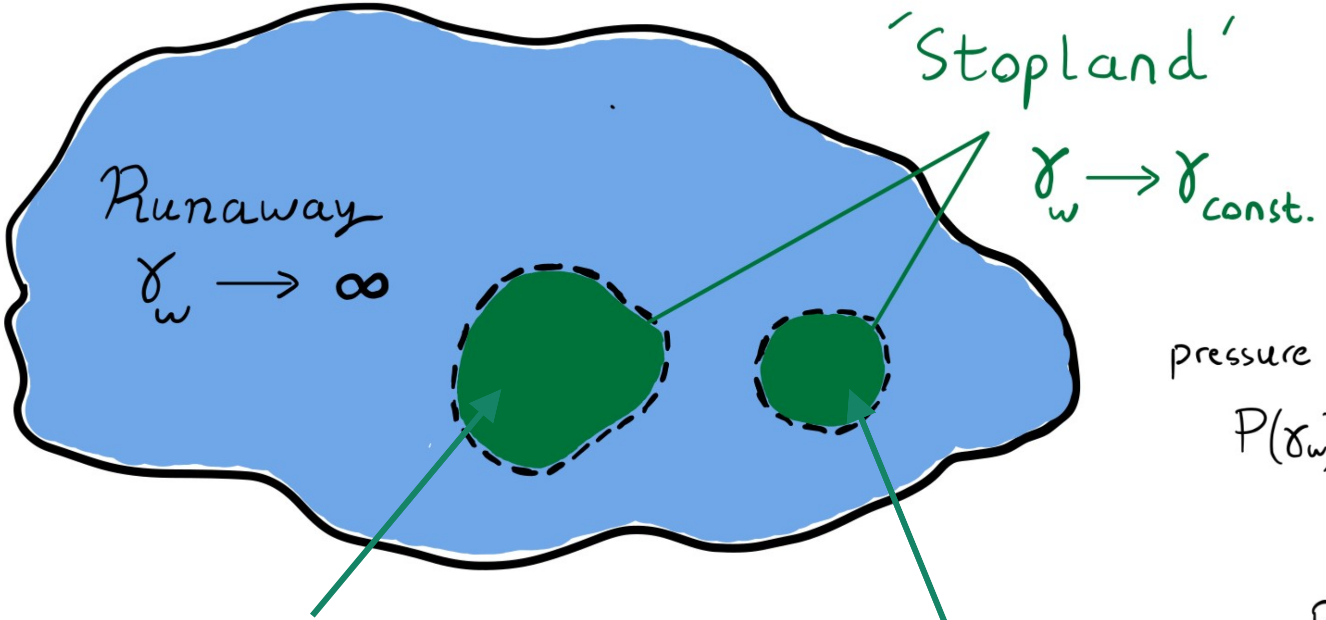

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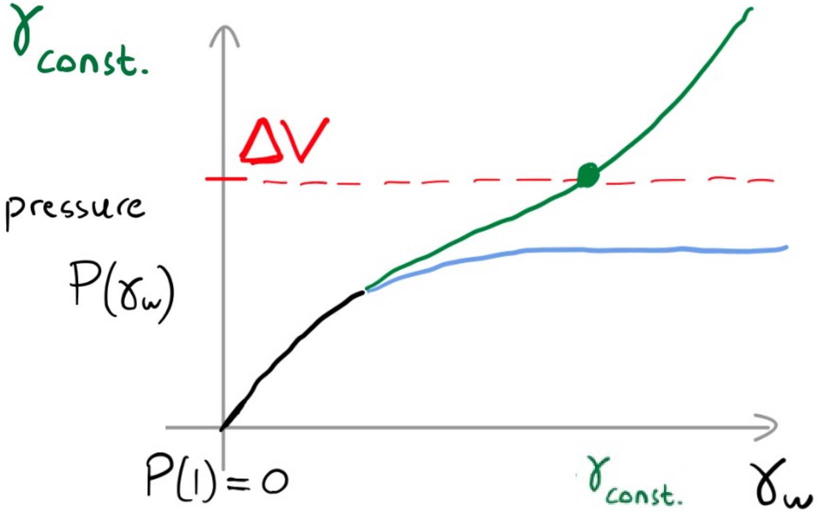

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Thin walls & light **dark matter**

Garcia Garcia, Kosegzi, **RPB** (2022) & (2024 to appear)

Medium–Bubble interactions, systematically

Consider the theory $\mathcal{L}_\Lambda[\phi, \psi]$ ϕ order parameter ψ any other field

In the domain wall background,

$$\mathcal{L}_\Lambda[\phi = v(z,t) + \tilde{\phi},\ \psi]$$

travelling planar wall

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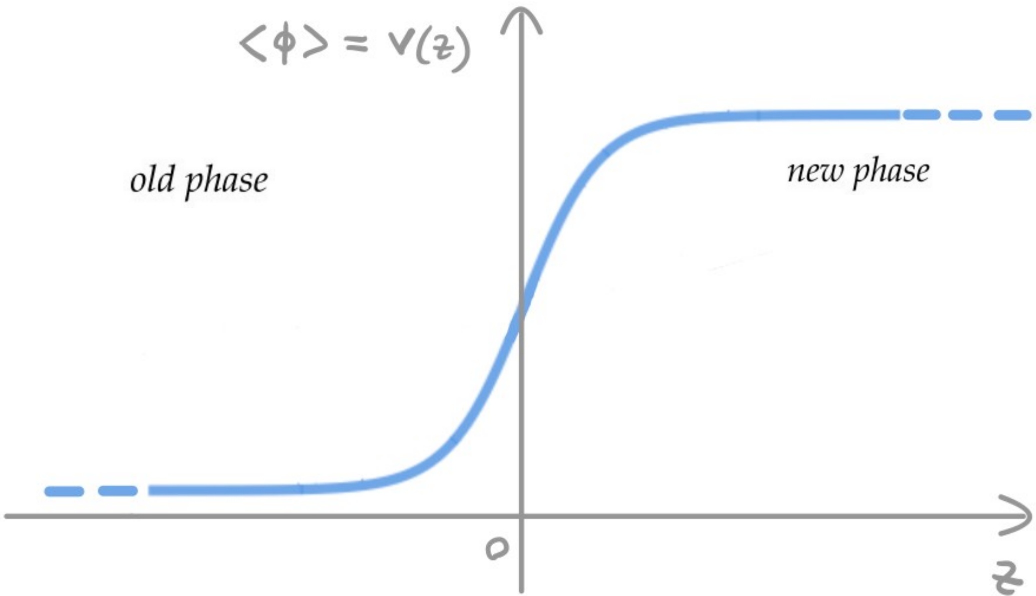
work in wall frame



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$$\mathcal{L}_\Lambda[\phi = v(z) + \tilde{\phi},\ \psi] = \left(1 + \Delta f^2(z)\right)(\partial\psi)^2 - \left(m^2 + \Delta m^2(z)\right)\psi^2 + \text{interactions}(z)$$

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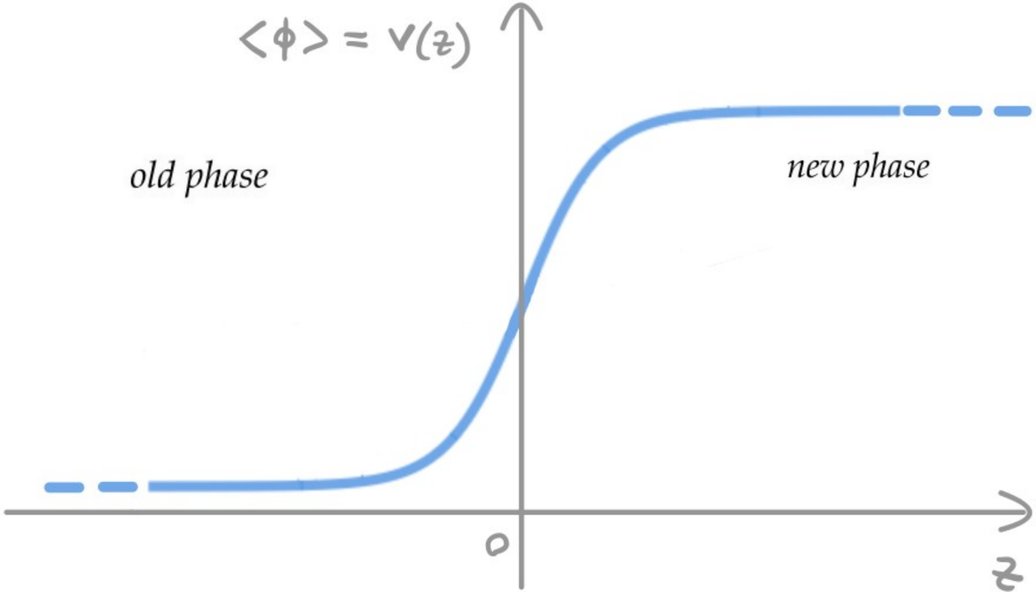

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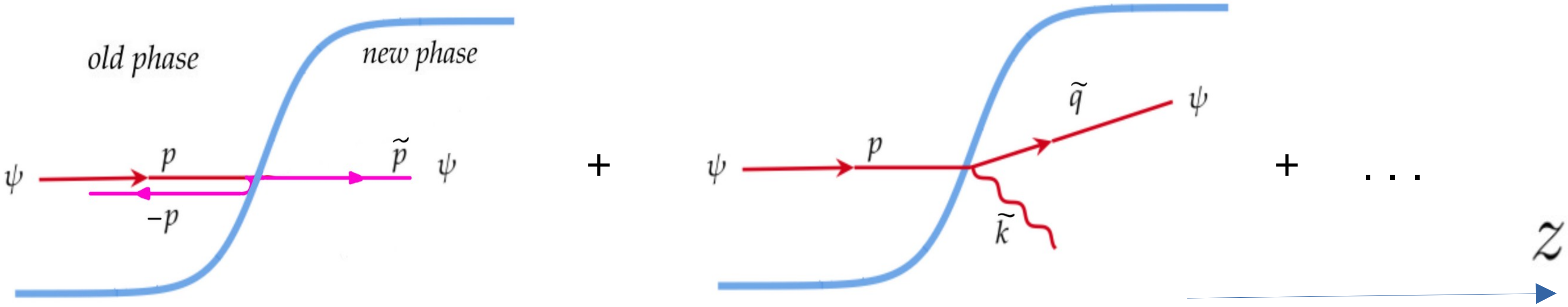
A particle hitting the wall can undergo several (z-momentum breaking) interactions



Leading order (LO): reflection / transmission

Next-to-LO (NLO):

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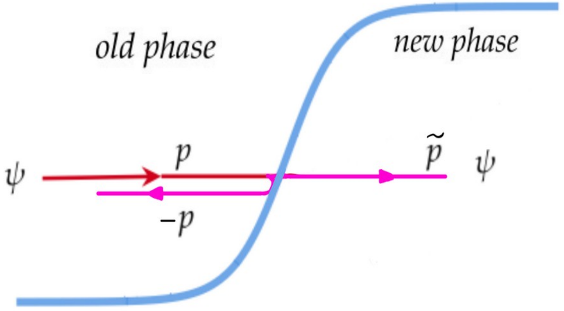
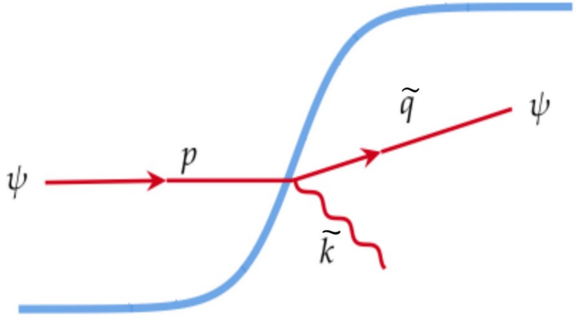


Leading order (LO): reflection / transmission

$+$



Next-to-LO (NLO):

$+ \ \ldots$

z

Medium–Bubble interactions, systematically

Consider the theory $\mathcal{L}_\Lambda[\phi, \psi]$ ϕ order parameter ψ any other field

In the domain wall background, the theory of ψ to quadratic order is schematically

$$\mathcal{L}_\Lambda[\phi = v(z) + \tilde{\phi},\ \psi] = \left(1 + \Delta f^2(z)\right)(\partial\psi)^2 - \left(m^2 + \Delta m^2(z)\right)\psi^2 + \text{interactions}(z)$$

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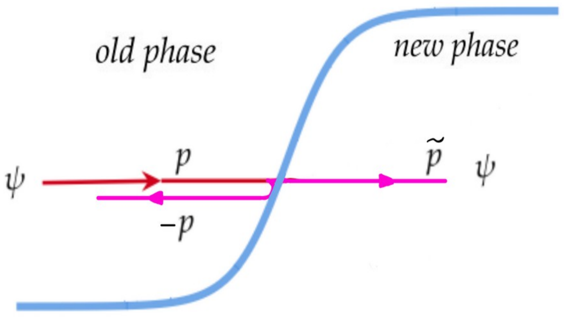

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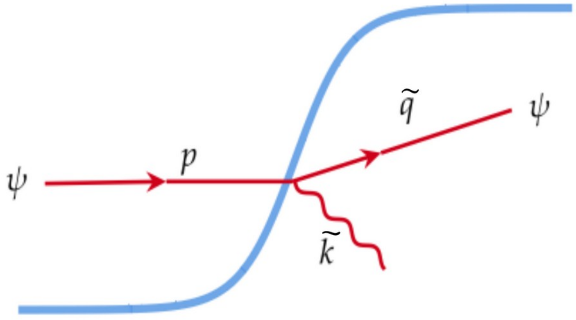

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In large γ limit, wall interacts with single particles: $\mathcal{P} = \gamma_w n v_w \langle \Delta p \rangle$

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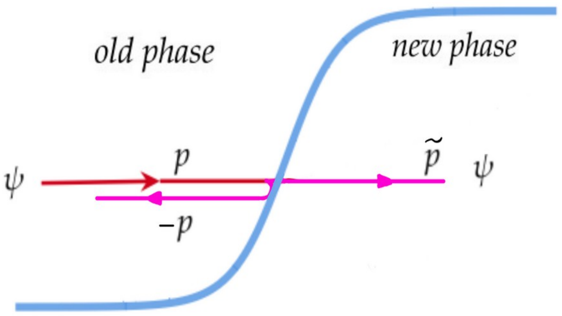

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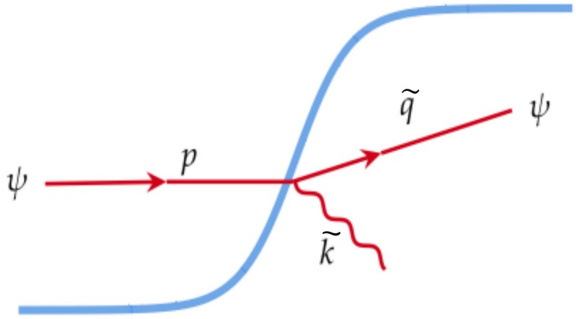

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work in wall frame $\quad$ $O\left(\frac{1}{\Lambda}\right)$ e.g. $\phi(\partial\psi)^2$ $\quad$ e.g. $\phi\psi^2$

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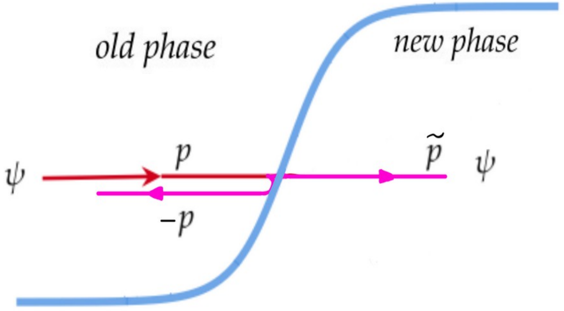

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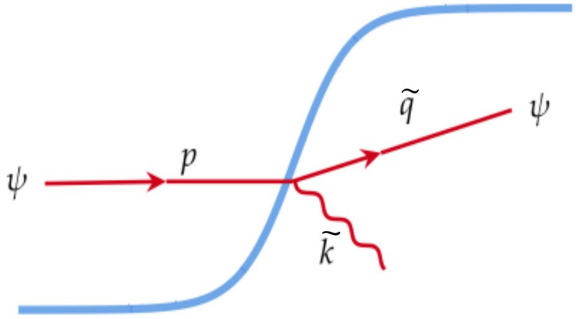

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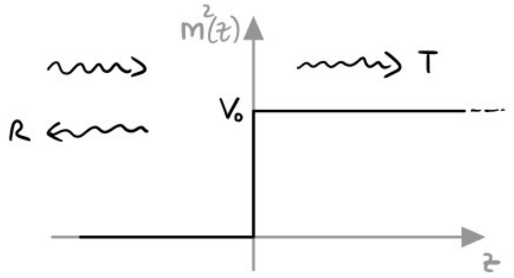
Change in mass: step function scattering

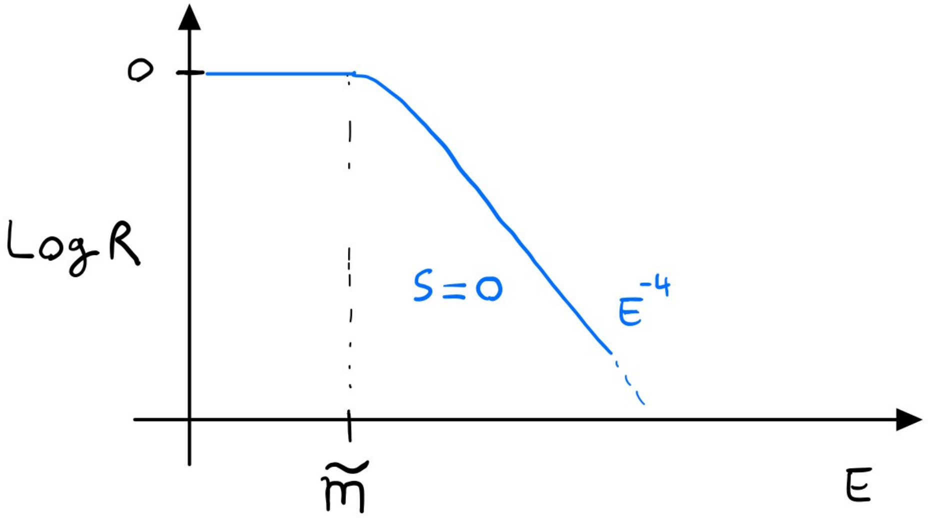
Klein-Gordon

$$\left(\partial^2 + m^2(z)\right)\phi = 0$$



Reflection Probability

$$R = \left(\frac{k^z - \tilde{k}^z}{k^z + \tilde{k}^z}\right)^2 \quad \text{where} \quad k^z = \sqrt{E^2 - m^2}\,, \quad \tilde{k}^z = \sqrt{E^2 - \tilde{m}^2}$$



Change in mass: step function scattering

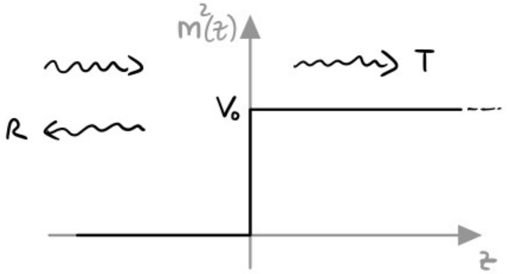
Dirac

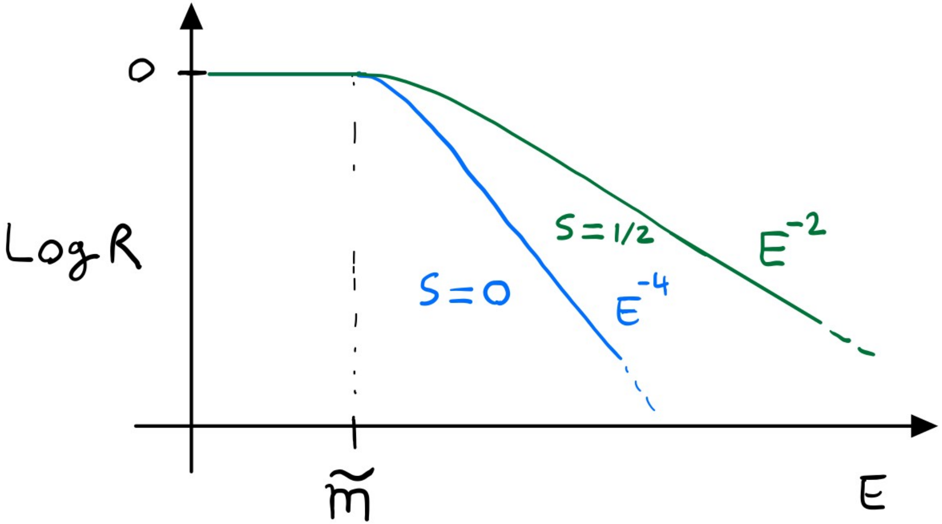
$$(i\not{\partial} - m)\Psi(x) = 0$$

Reflection Probability

$$R = \frac{k(E+\tilde{m}) - \tilde{k}(E+m)}{k(E+\tilde{m}) + \tilde{k}(E+m)}$$

where $k^z = \sqrt{E^2 - m^2}$, $\tilde{k}^z = \sqrt{E^2 - \tilde{m}^2}$

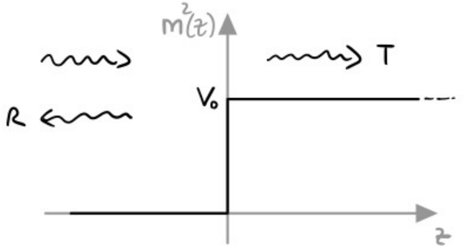




Change in mass: step function scattering

Proca

$$\partial_\mu F^{\mu\nu} - m^2(z) A^\nu = 0$$

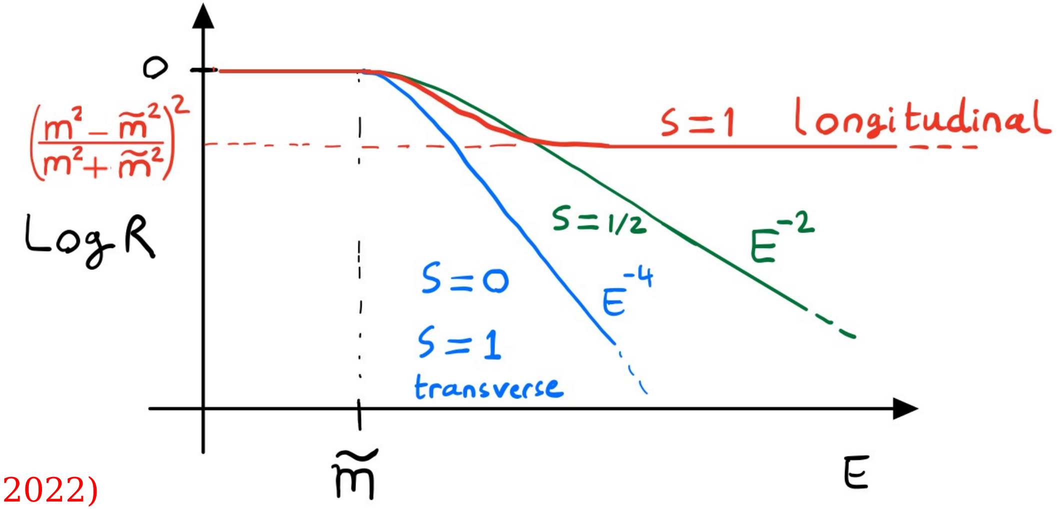


Reflection Probability

$$R = \left(\frac{k^z - \tilde{k}^z}{k^z + \tilde{k}^z}\right)^2$$ Transverse

$$R = \left(\frac{k^z \tilde{m}^2 - \tilde{k}^z m^2}{k^z \tilde{m}^2 + \tilde{k}^z m^2}\right)^2$$ Longitudinal

$k^z = \sqrt{E^2 - m^2}$, $\tilde{k}^z = \sqrt{E^2 - \tilde{m}^2}$



Garcia Garcia, Kosegzi, **RPB** (2022)

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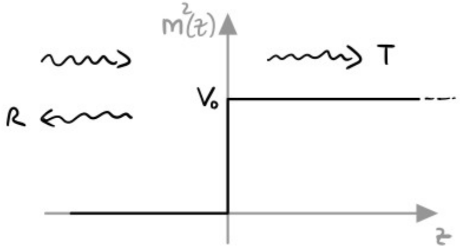

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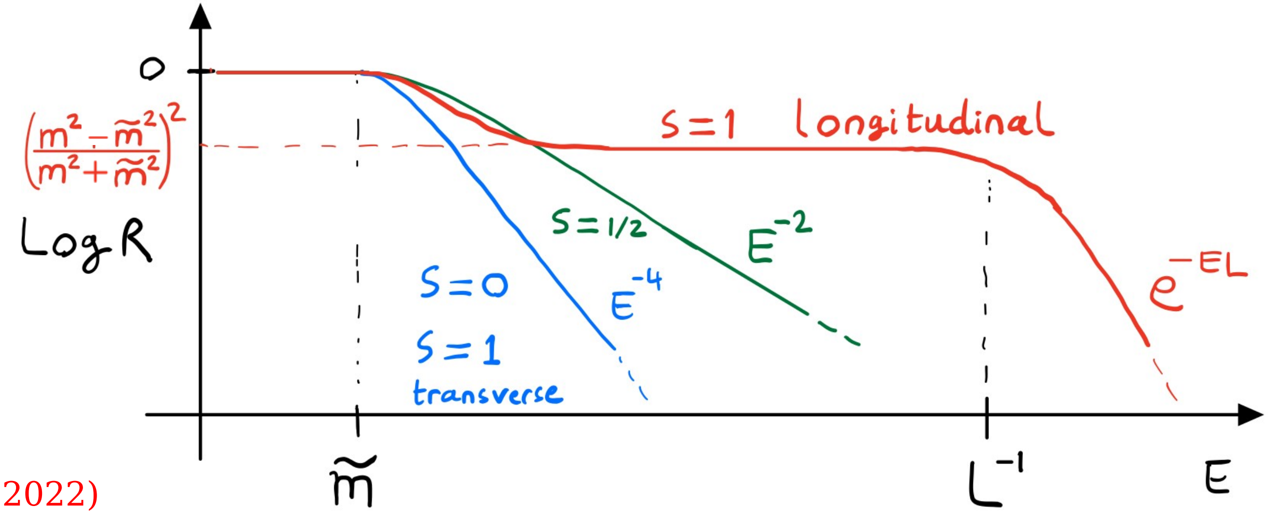

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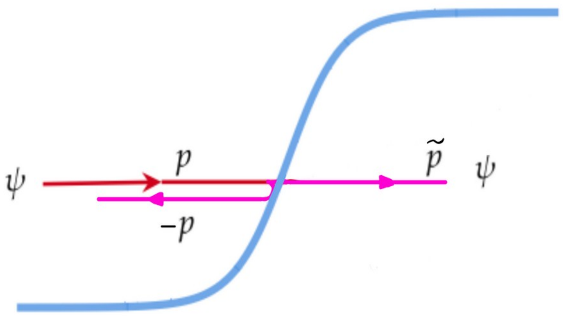
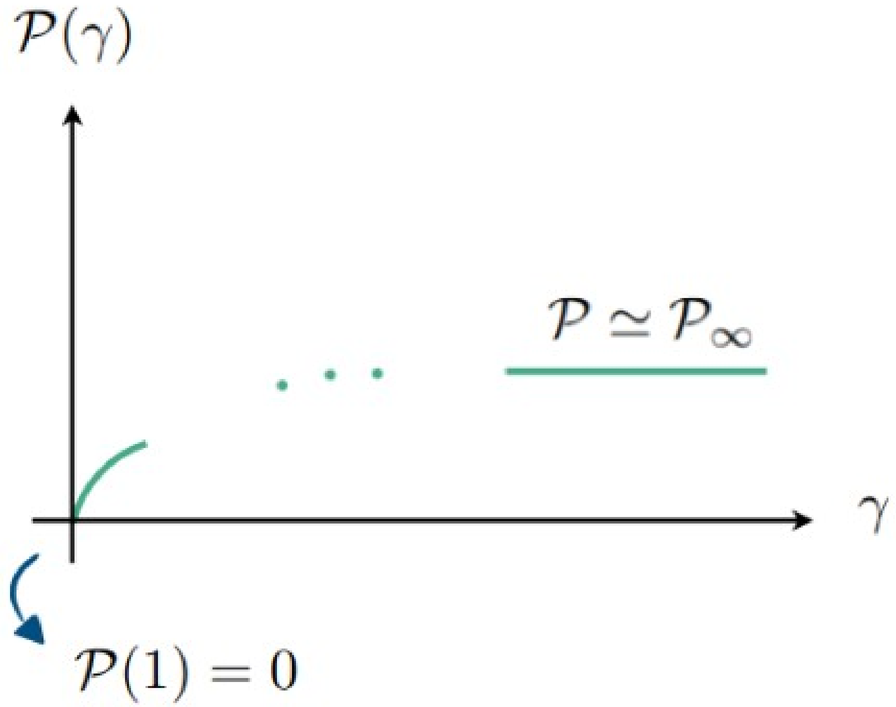


$$\mathcal{P} \approx \gamma n v \left[R\, 2k^z + (1-R)(\tilde{k}^z - k^z)\right]$$

Ignoring reflections, LO pressure is constant

Bodecker, Moore (2009)



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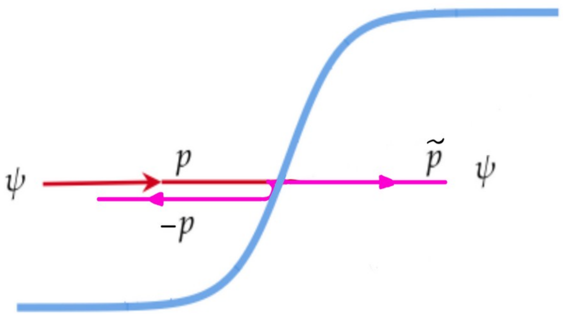

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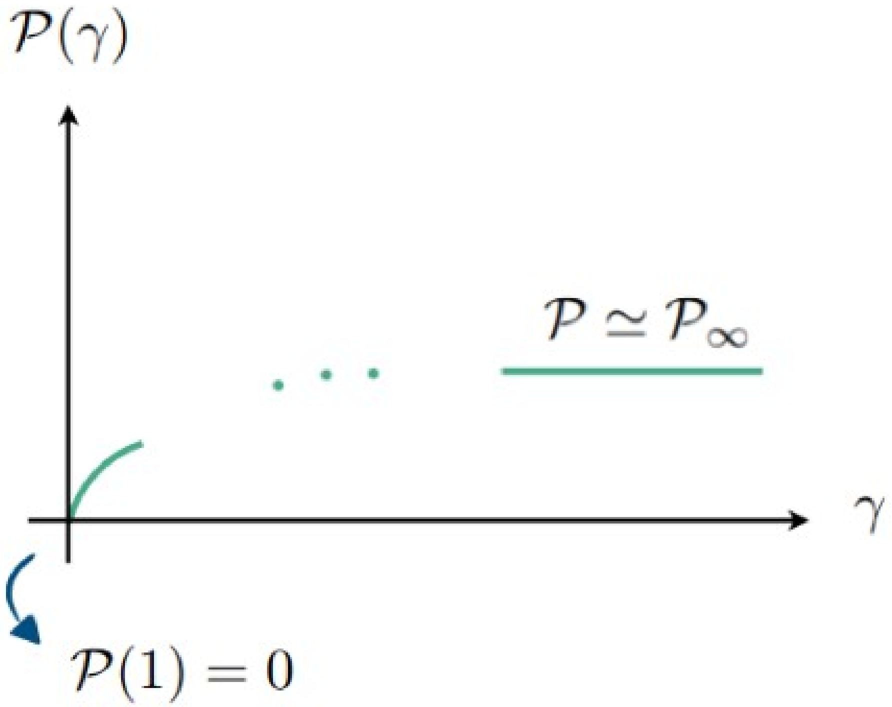
Reflection of cold longitudinal vectors by thin walls

$$\mathcal{P} \simeq \rho_{\mathrm{DM}} \gamma^2 R \quad \text{while} \quad \gamma \lesssim (mL)^{-1}$$



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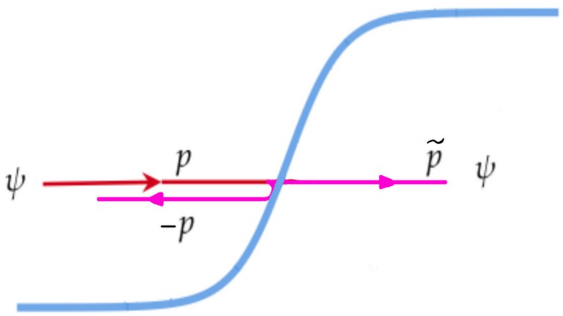

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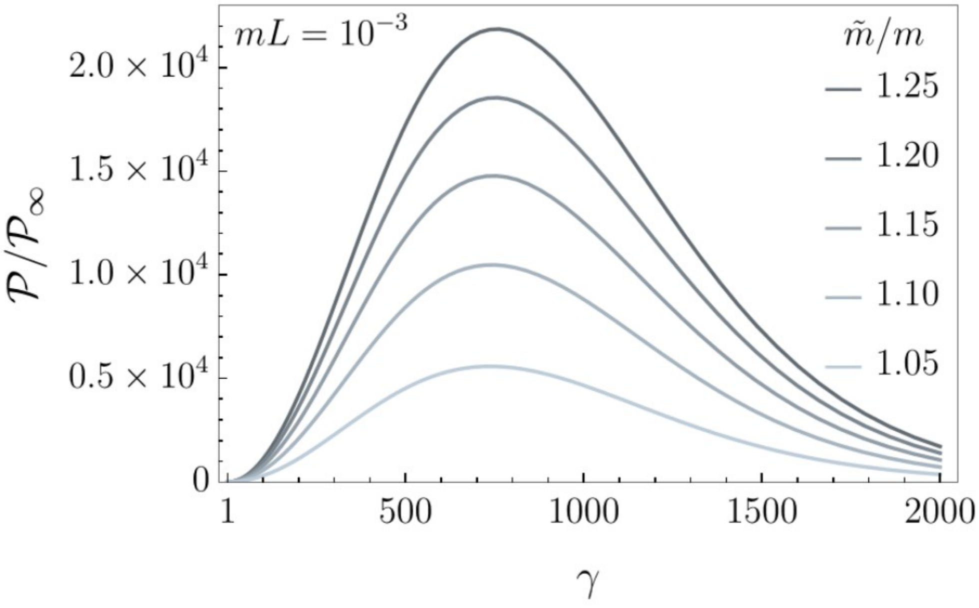

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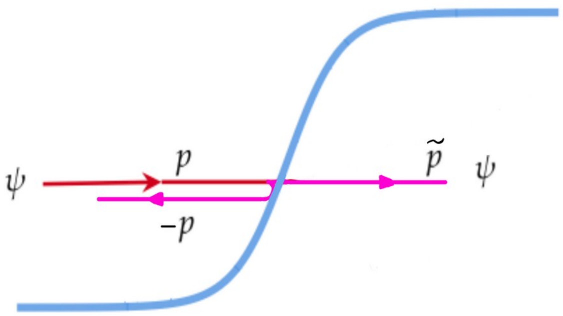

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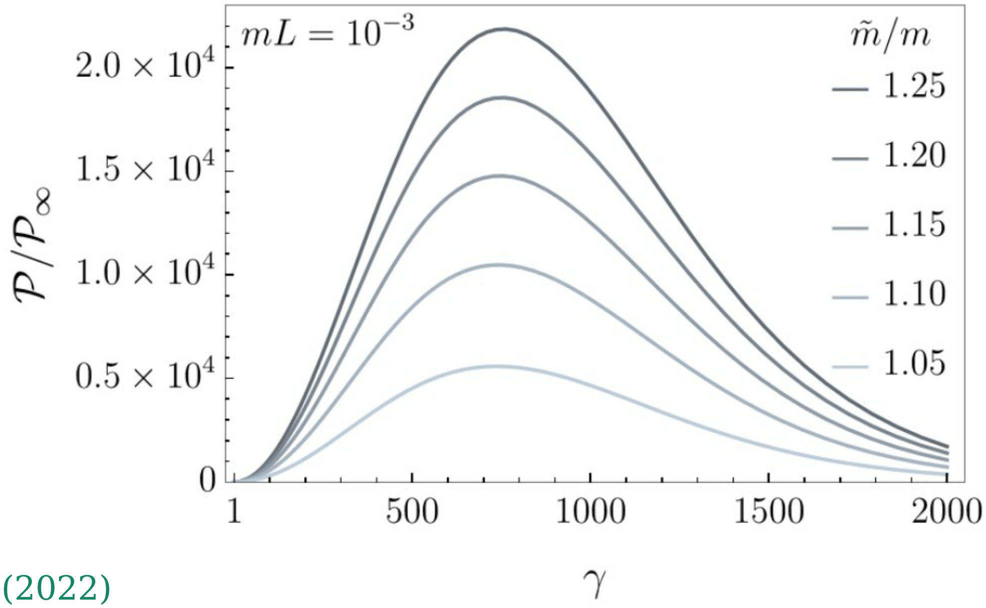
light vector dark matter!



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EFT Model

Massive vectors require some caution. Think in terms of a naive EFT...

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)}_{\text{scalar sector}} - \underbrace{\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2 V_\mu V^\mu}_{\text{massive dark photon}} + \boxed{\frac{\kappa}{2}\phi^2 V_\mu V^\mu} + \cdots$$

cannot be forbidden by symmetries

$$\Rightarrow \quad m_V^2 = m^2 + \underbrace{\kappa\langle\phi\rangle^2}_{\Delta m^2}$$

false vacuum

$\langle\phi\rangle = 0$

true vacuum

$\langle\phi\rangle \equiv v \neq 0$

this operator introduces a cut-off

$$\Lambda \lesssim \frac{4\pi m}{\sqrt{\kappa}} = \frac{4\pi v}{\sqrt{\Delta m^2/m^2}}$$

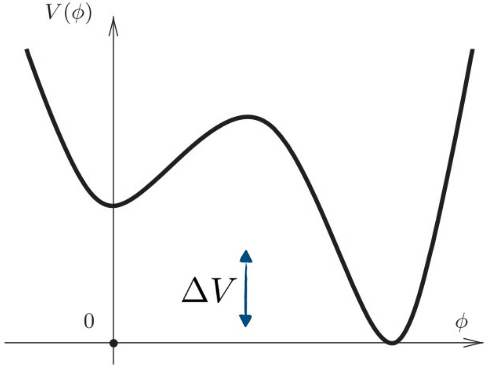
Here: focus on $\Delta m^2/m^2 \ll 1$ so that $\Lambda \gg 4\pi v$

Consequences

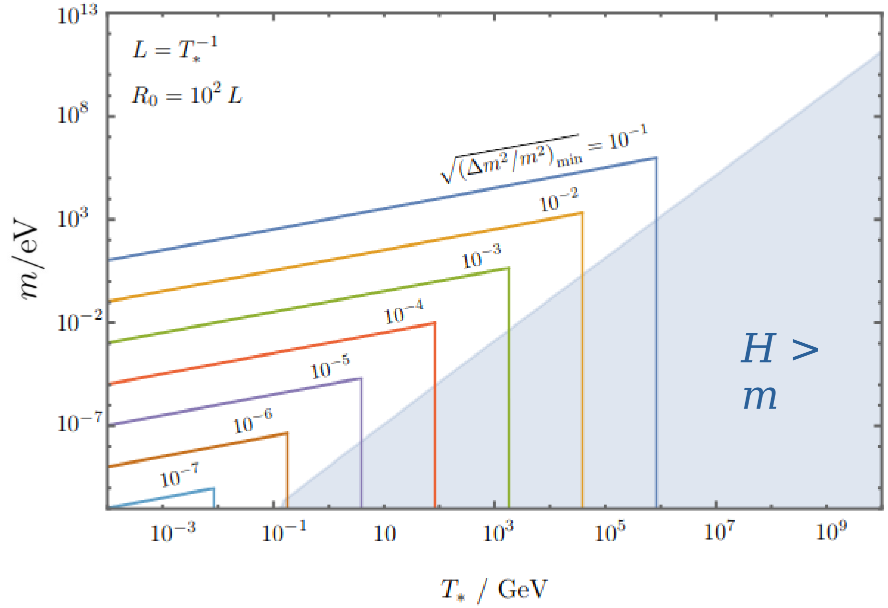
Given a phase transition:

$$T_* \,, \quad \alpha \equiv \frac{\Delta V}{\rho_{\rm SM}(T_*)} \,, L \,, R_0$$

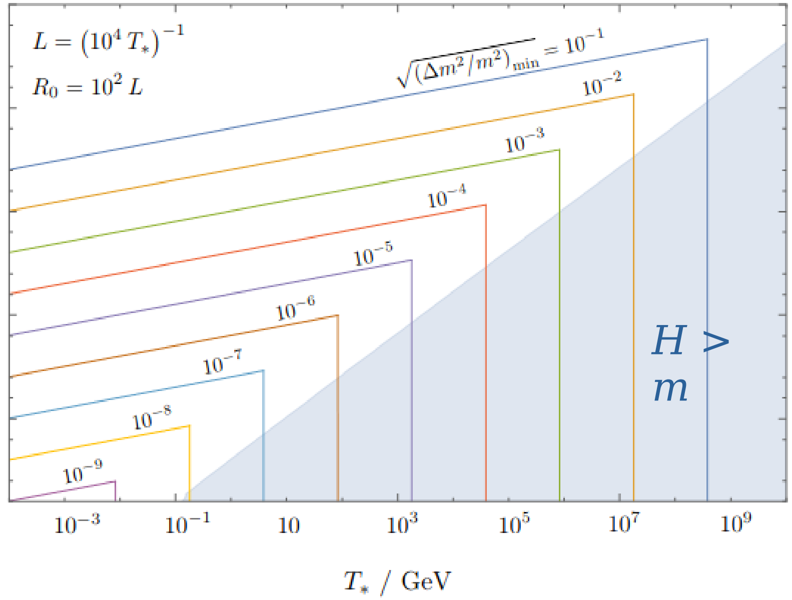
0.01



'thermal'



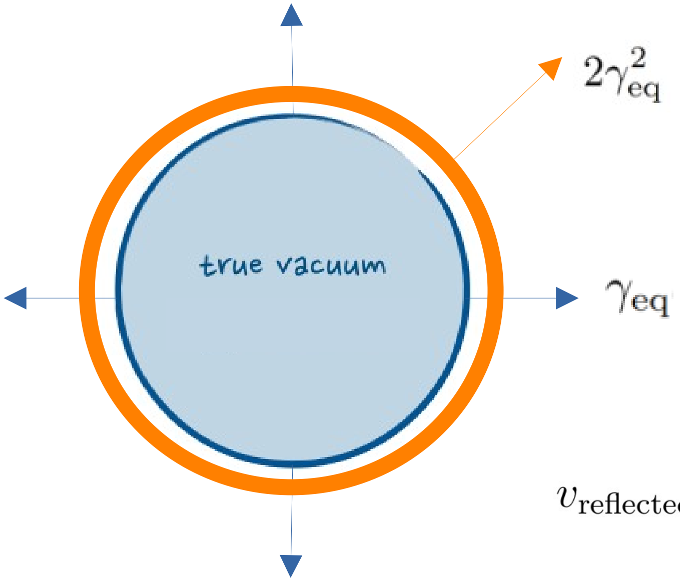
' some super-cooling'



Consequences

Equilibrium: Energy goes into reflected (now relativistic) longitudinal dark photons.

simple relativistic kinematics give:



$$v_{\rm reflected} - v_{\rm eq} \simeq \frac{|\vec{v}_{\rm eq}|}{2\gamma_{\rm eq}^2} \ll 1$$

Reflected longitudinals concentrated in a thin shell

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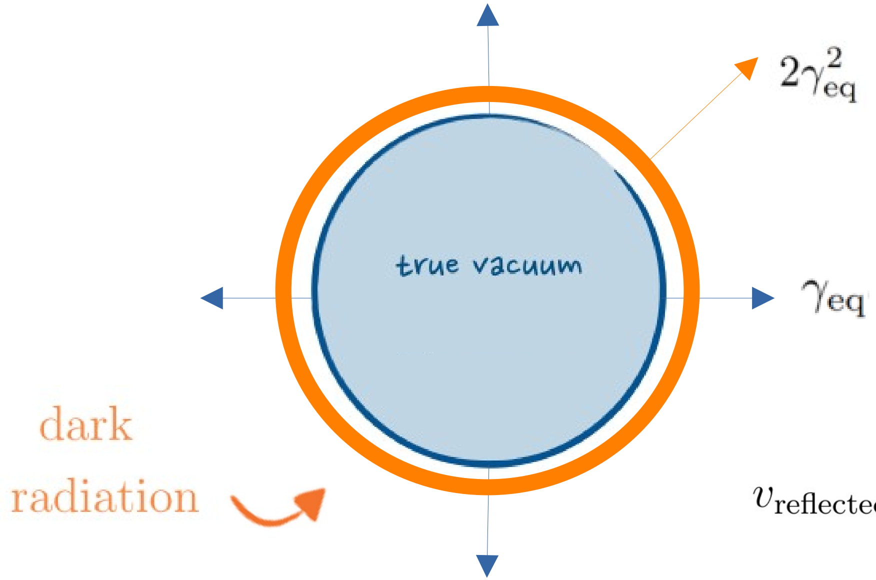

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Consequences

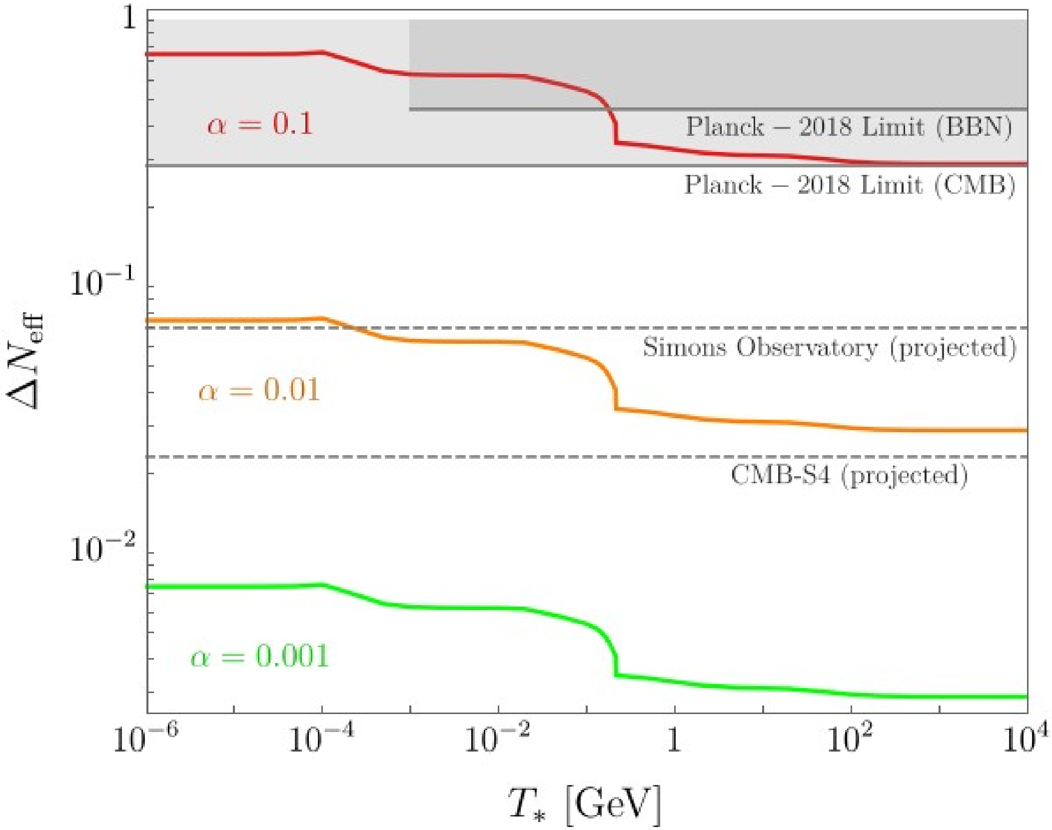
Dark radiation

BBN

$$\gamma_{\rm dr}(T \leq T_*) \simeq 2\gamma_{\rm eq}^2 \frac{a(T_*)}{a(T)} \approx 2\times 10^{13}\left(\frac{10^{-4}}{\Delta m^2/m^2}\right)^2\left(\frac{\alpha}{10^{-2}}\right)\left(\frac{\rho_{\rm dm}}{\rho_V}\right)\left(\frac{T}{1\ \rm MeV}\right) \overset{!}{\gtrsim} 1$$

$$10^{-6}\left(\frac{T}{1\ \rm eV}\right)$$

Recombination (CMB)



$$\Delta N_{\rm eff} = 0.3\left(\frac{\alpha}{0.1}\right)\left(\frac{g_*(100\ \rm GeV)}{g_*(T_*)}\right)^{1/3}$$

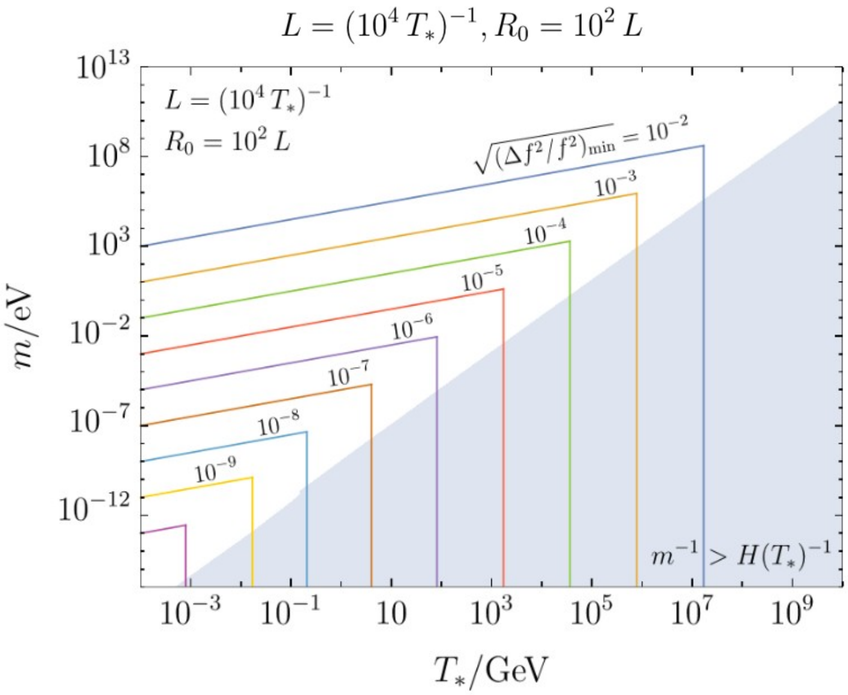
Generalisation to Axion /ALPs

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2 V_\mu V^\mu + \boxed{\frac{\kappa}{2}\phi^2 V_\mu V^\mu} + \cdots$$

Mass terms for vector are like kinetic terms for uneaten NGB

$$\Rightarrow \quad \mathcal{L} = \frac{1}{2}\left(f^2 + \Delta f(z)\right)\partial_\mu\theta\partial^\mu\theta + \ldots$$

Reflection probability $R = \left(\frac{\Delta f^2}{2f^2}\right)^2$



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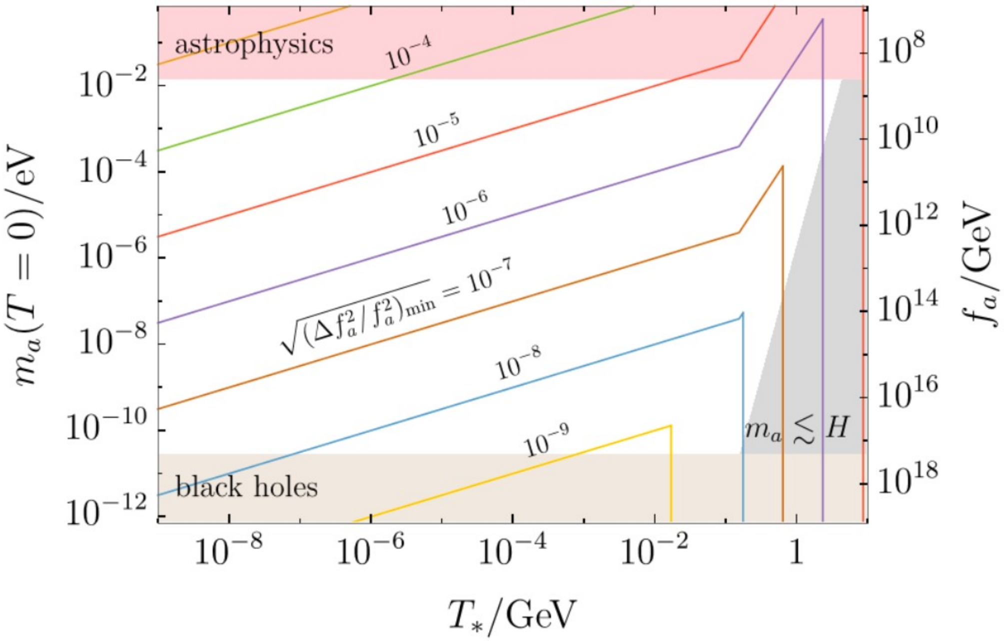
QCD Axion

$$m_a \approx 5.7\times 10^{-6}\,\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)$$

$$m(T) \approx \begin{cases} m_a\ , & T \lesssim T_c \\ m_a\,(T_c/T)^4\ , & T \gtrsim T_c \end{cases}$$

$L = (10^4\,T_*)^{-1},\ R_0 = 10^2\,L$



EWPT & QCD Axion

Perhaps a bit too general.... Let’s focus on the most constrained case possible

- QCD axion dark matter (missaligned mechanism)
- Electroweak Phase Transition

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FOPT at $100\,\mathrm{GeV} \lesssim T_* \lesssim 100\,\mathrm{MeV}$

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$$\Rightarrow \quad T_{\rm osc} \sim 150\,\mathrm{MeV}\left(\frac{(100\ \mathrm{GeV})^4}{\Delta V}\right)^{1/8}\left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right)^{1/4}$$

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…but are we sure 'frozen' fields don't give friction?

Friction from ‘frozen’ fields

Coupled action for the wall and the axion:

$$S[z_w(t),\theta] = \int dt \left\{-\sigma\sqrt{1-\dot{z}_w(t)^2} + z_w(t)\Delta V\right\} + \int d^2x \, \frac{1}{2} \left\{\left(f^2 + \Delta f^2(x,z_w)\right)(\partial_\mu\theta)^2 - f^2 m_a^2\theta^2\right\}$$

$$\Delta f^2 \Theta(\gamma(t)(z - z_w(t)))\, \Theta(t - t_n)$$

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Leads to coupled equations of motion

$$\frac{\ddot{z}_w(t)}{(1-\dot{z}_w(t)^2)^{3/2}} = \frac{\Delta V - \mathcal{P}(t)}{\sigma}\,, \qquad \mathcal{P} = \frac{\Delta f^2}{2}(\partial\theta)^2\Big|_{z=z_w(t)}$$

$$\partial_\mu \left(f^2 + \Delta f^2(t,z_w)\right)\partial^\mu\theta + f^2 m_a^2 \theta = 0$$

Friction from ‘frozen’ fields

Coupled action for the wall and the axion:

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Energy goes into sharp **'axion cliff '** features

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EWPT & QCD Axion

Perhaps a bit too general…. Let’s focus on the most constrained case possible

- QCD axion dark matter (missaligned mechanism)
- Electroweak Phase Transition

$$\mathcal{L}_{UV} \supset \eta|\Phi_{\rm PQ}|^2 h^2 \xrightarrow{\text{integrating out radial mode}} \mathcal{L}_{\rm EFT} \supset \left(f^2 + \eta\frac{h^2}{m_\rho^2}\right)(\partial_\mu\theta)^2 - m_a^2 f^2\theta^2$$
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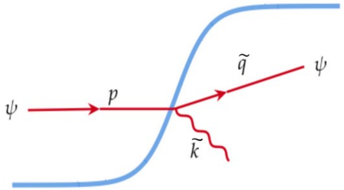

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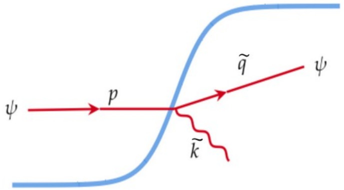

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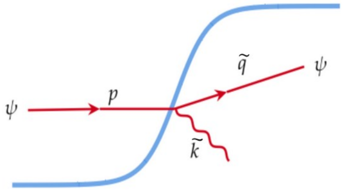

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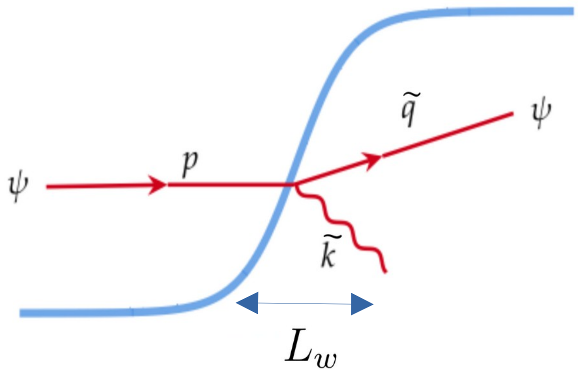
If < 1 then universe reborn form the collision of axion cliffs

NLO from Frist Principles



Azatov, Barni, **RPB**, Vanvlasselaer (2023)

We quantise field theories in the translation breaking background of a wall

Which wall? For a theorist the dream is a step wall (also phenomenologically relevant)

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Very schematically we compute:

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6 final state integrals – 3 conservation laws
(energy and perpendicular momenta)

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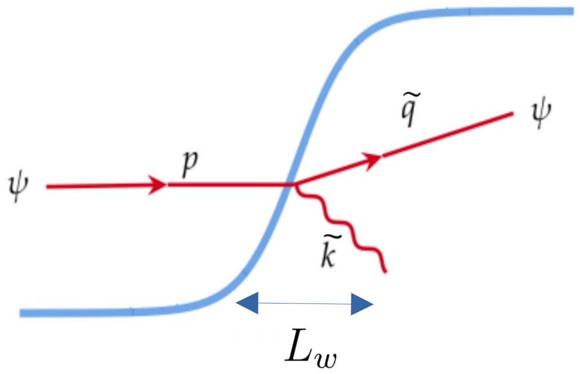

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Warm up: Simple Scalars

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Far from the wall the plane wave solution

$$\phi_{R,k} = e^{-ik_n x^n} \chi_{R,k}(z) = e^{-ik_0 t + ik_\perp x_\perp} \begin{cases} e^{ik_z z} + r_k e^{-ik_z z} & z \rightarrow -\infty \\ t_k e^{i\tilde{k}_z z} & z \rightarrow +\infty \end{cases}$$

$$n = t, x, y$$

where $k_0 = \sqrt{k_z^2 - m^2 - k_\perp^2}$ and $\tilde{k}^z = \mathrm{sign}(k^z)\sqrt{k^2 + m^2 - \tilde{m}^2}$

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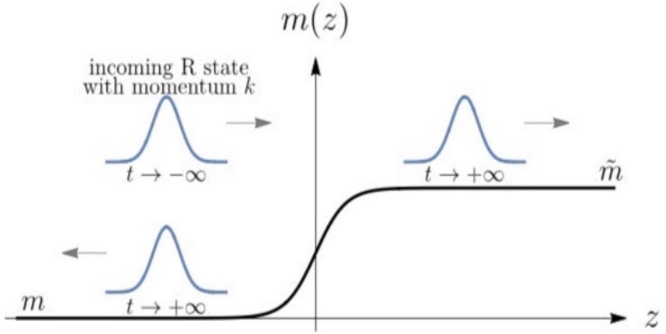

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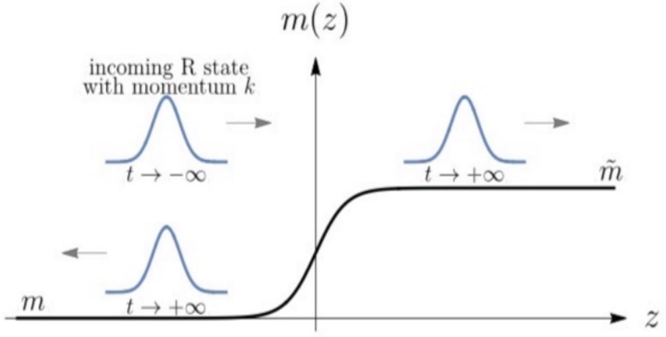

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Problem: NOT AN ORTHOGONAL BASIS!

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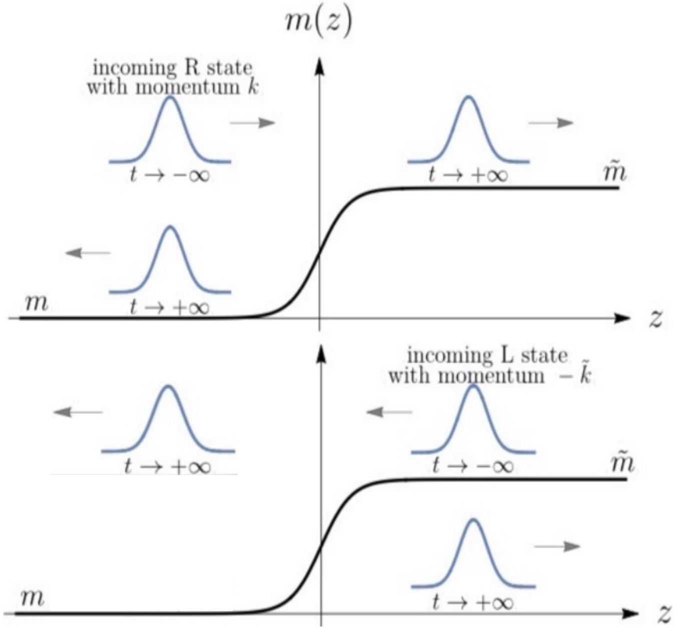

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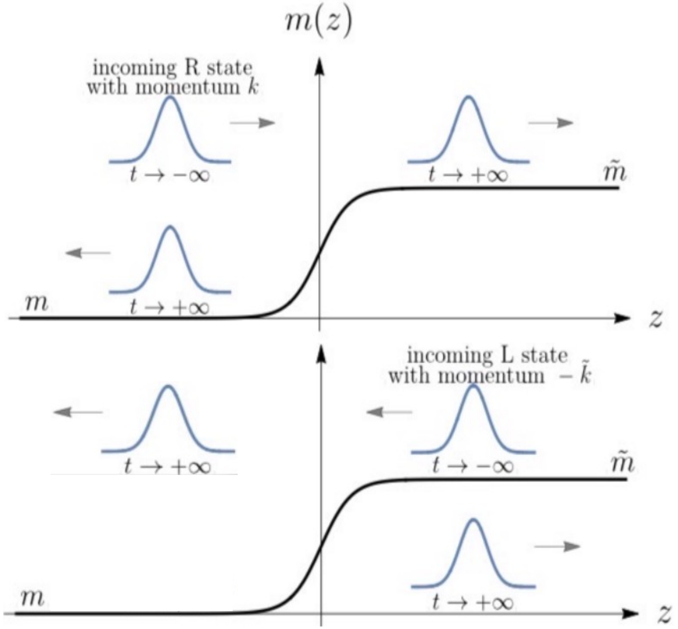

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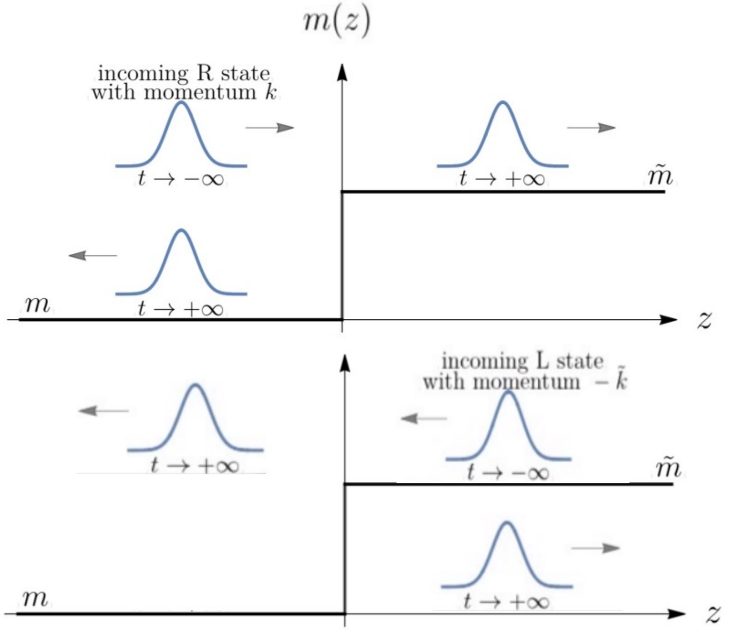

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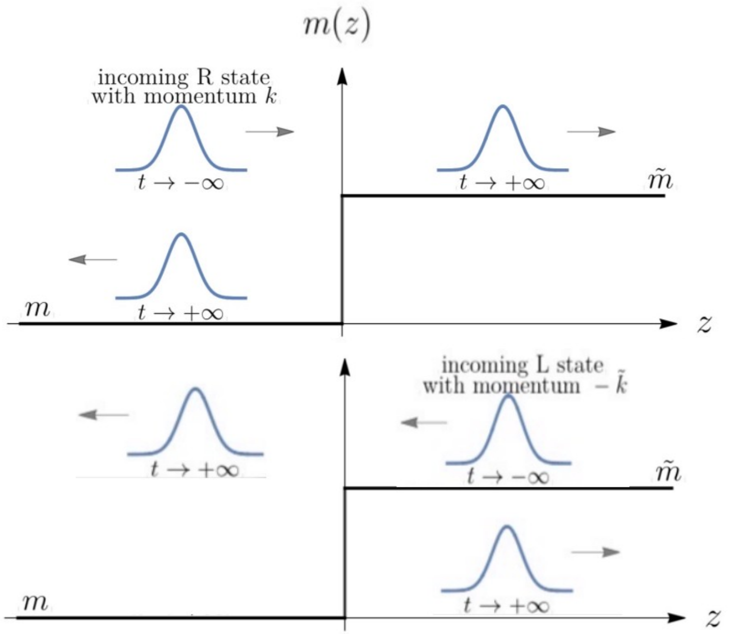

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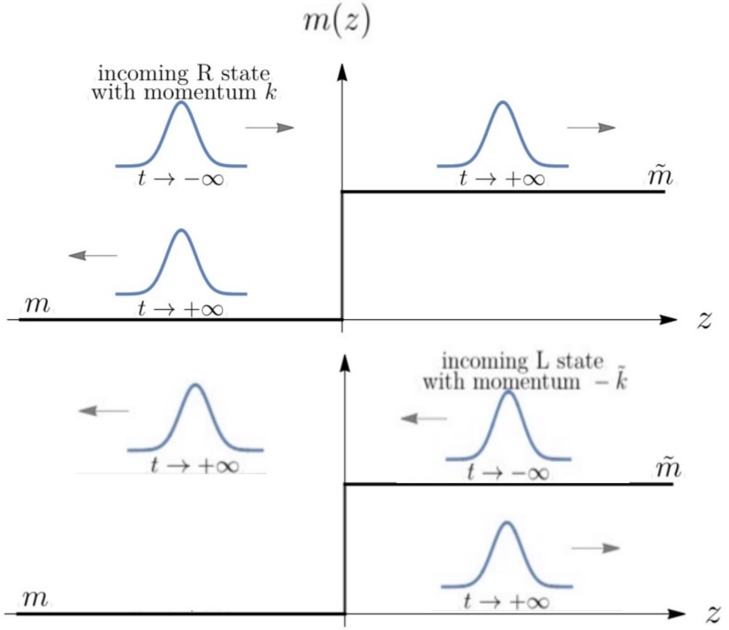
$$\phi_{L,k} = e^{-ik_n x^n}\chi_{L,k}(z) = e^{-ik_0 t + ik_\perp x_\perp}\sqrt{\frac{k_z}{\tilde{k}_z}}\begin{cases} \frac{\tilde{k}_z}{k_z} t_k e^{ik_z z} & z \to -\infty \\ -r_k e^{i\tilde{k}_z z} + e^{-i\tilde{k}_z z} & z \to +\infty \end{cases}$$

where $r_k = \dfrac{k^z - \tilde{k}^z}{k^z + \tilde{k}^z}$, $\quad t_k = \dfrac{2k^z}{k^z + \tilde{k}^z}$



$$\int_{-\infty}^{\infty} dz\ \chi_{I,k_z}\chi^*_{J,q_z} = 2\pi\delta_{IJ}\delta(k_z - q_z), \quad I, J \in L, R$$

using $\displaystyle\int_{-\infty}^{0} e^{i\beta z}dz = \mathrm{PV}\left(\frac{1}{i\beta}\right) + \pi\delta(\beta)$

Simple Scalars – quantisation

Having a complete set of states $\{\phi_{R,k_z}, \phi_{L,k_z}\}$ we can expand the field

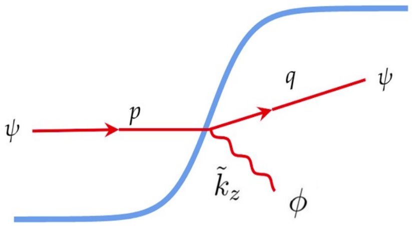
$$\phi(x,t) = \sum_{I=R,L} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} \left(a_{I,k}\phi_{I,k} + a^\dagger_{I,k}\phi^*_{I,k}\right)$$

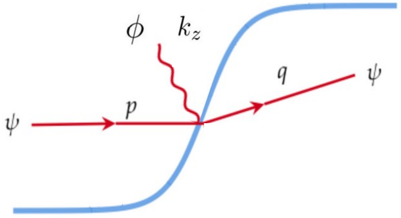
Promoting fields to operators and Poisson Brackets to commutators, gives:

$$[a_{I,k}, a^\dagger_{J,q}] = (2\pi)^3\delta(\vec{k}-\vec{q})\delta_{IJ} \;,$$
$$[a_{I,k}, a_{J,q}] = [a^\dagger_{I,k}, a^\dagger_{J,q}] = 0 \;, \qquad I, J \in \{R, L\}$$

We can define two types of states

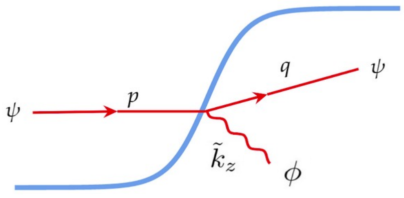
$$|k_z^R\rangle = \sqrt{2k_0}a^\dagger_{R,k_z}|0\rangle,$$
$$|k_z^L\rangle = \sqrt{2k_0}a^\dagger_{L,k_z}|0\rangle,$$





which should be thought as independent states in any process.

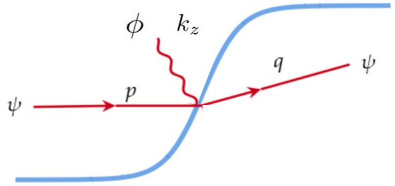
Simple Scalars – emission



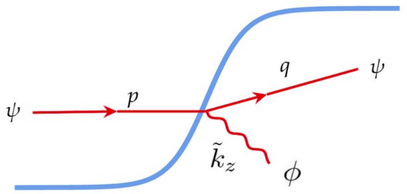
$$\mathcal{S} = \mathrm{T}\exp\left(-i\int d^4x\mathcal{H}_{\mathrm{Int}}\right) \qquad \mathcal{H}_{\mathrm{Int}} = -iy\psi^2(x)\phi(x)$$

$$\langle k_I^{\mathrm{out}}\, q|\, \mathcal{S}\, |p\rangle \equiv (2\pi)^3\delta^{(3)}(p^n - k^n - q^n)i\mathcal{M}_I \overset{\mathrm{tree}}{=} -i\int d^4x\langle k_I^{\mathrm{out}}\, q|\, \mathcal{H}_{\mathrm{Int}}\, |p\rangle$$

$$\text{with} \quad I = L, R\ ,$$



Simple Scalars – emission



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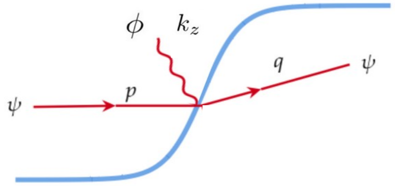

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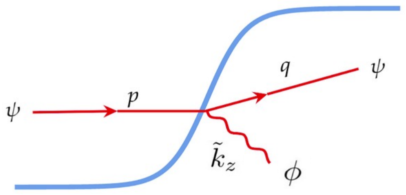
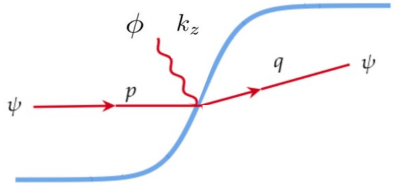
$$\text{with} \quad I = L, R\ ,$$

To compute $\langle \Delta p\rangle$ we need states with definite final momentum!

Simple Scalars – emission



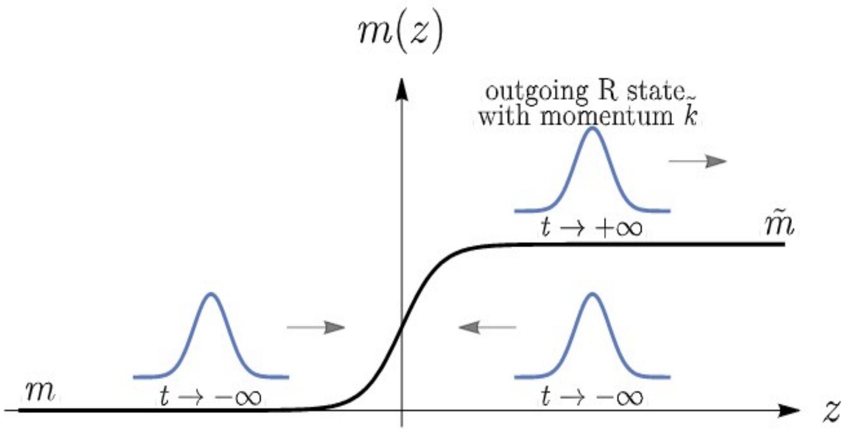
$$\mathcal{S} = \mathrm{T}\exp\left(-i\int d^4x\,\mathcal{H}_{\rm Int}\right) \qquad\qquad \mathcal{H}_{\rm Int} = -iy\psi^2(x)\phi(x)$$

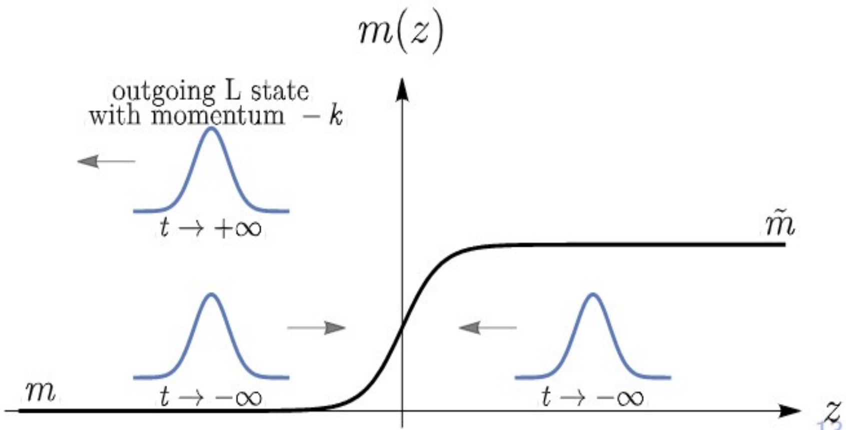


$$\langle k_I^{\rm out}\, q|\,\mathcal{S}\,|p\rangle \equiv (2\pi)^3\delta^{(3)}(p^n - k^n - q^n)\, i\mathcal{M}_I \overset{\rm tree}{=} -i\int d^4x\,\langle k_I^{\rm out}\, q|\,\mathcal{H}_{\rm Int}\,|p\rangle$$

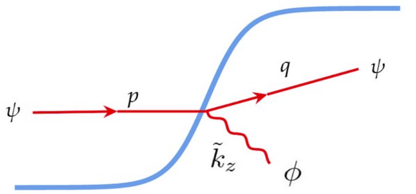
$$\text{with} \quad I = L, R\ ,$$

To compute $\langle \Delta p\rangle$ we need states with definite final momentum!
Then we define basis for outgoing states





Simple Scalars – emission



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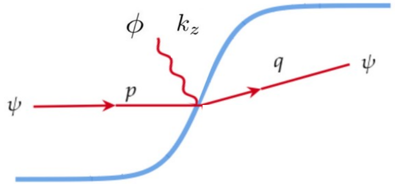


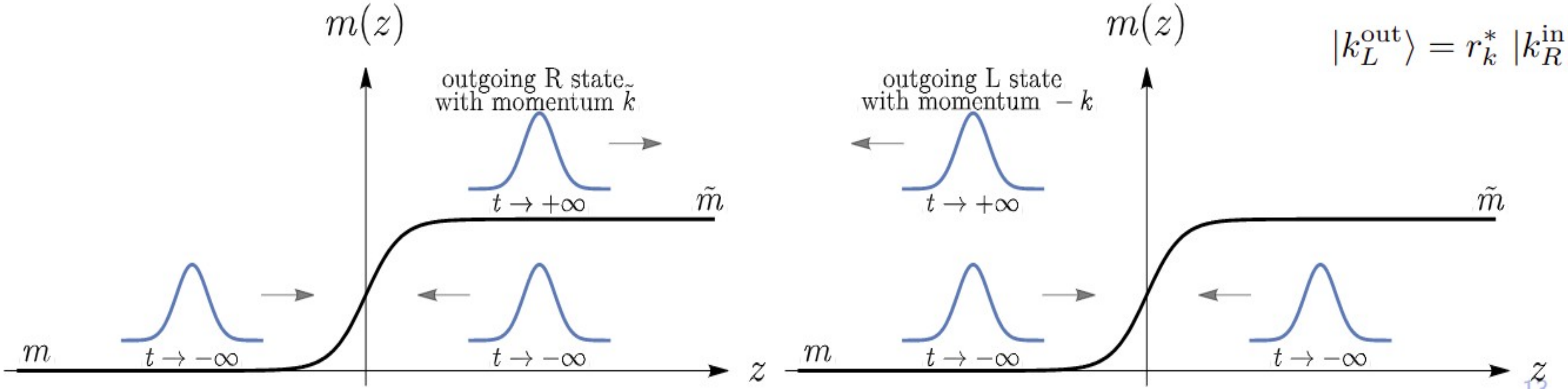
$$\langle k_I^{\text{out}}\ q|\ \mathcal{S}\ |p\rangle \equiv (2\pi)^3\delta^{(3)}(p^n - k^n - q^n)i\mathcal{M}_I \overset{\text{tree}}{=} -i\int d^4x\langle k_I^{\text{out}}\ q|\ \mathcal{H}_{\text{Int}}\ |p\rangle$$

$$\text{with} \quad I = L, R\ ,$$

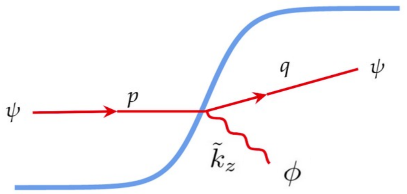
To compute $\langle\Delta p\rangle$ we need states with definite final momentum!
Then we define basis for outgoing states

$$|k_R^{\text{out}}\rangle = t_k^*\sqrt{\frac{\tilde{k}_z}{k_z}}\ |k_R^{\text{in}}\rangle - r_k^*\ |k_L^{\text{in}}\rangle,$$

$$|k_L^{\text{out}}\rangle = r_k^*\ |k_R^{\text{in}}\rangle + t_k^*\sqrt{\frac{\tilde{k}_z}{k_z}}\ |k_L^{\text{in}}\rangle\ \theta(\tilde{k})$$



Simple Scalars – emission



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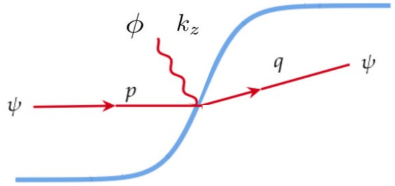

exact amplitudes

$$|\mathcal{M}_R|^2 = y^2\frac{4k^z\tilde{k}^z(k^z-\tilde{k}^z)^2}{(\tilde{k}_z^2-(p^z-q^z)^2)^2(k^z-p^z+q^z)^2}\ ,$$

$$|\mathcal{M}_L|^2 = y^2\frac{4k_z^2}{(k_z^2-(p^z-q^z)^2)^2}\begin{cases}\frac{(k^z-\tilde{k}^z)^2}{(\tilde{k}^z+p^z-q^z)^2}\ , & k^z > \Delta m\ ,\\ \frac{\Delta m^2}{\Delta m^2-k_z^2+(p^z-q^z)^2}\ , & k^z < \Delta m\ ,\end{cases}$$

Integrate over appropriate phase space $\langle\Delta p\rangle = \langle\Delta p_R\rangle + \langle\Delta p_L\rangle$

$$\equiv \int d\mathbb{P}_{\psi\to\psi\phi_R}\underbrace{(p^z-q^z-\tilde{k}^z)}_{\Delta p_R^z} + \int d\mathbb{P}_{\psi\to\psi\phi_L}\underbrace{(p^z-q^z+k^z)}_{\Delta p_L^z}$$

$$\int d\mathbb{P}_{\psi\to\psi\phi_I}\Delta p_I^z = \int_{k_{\mathrm{min}}^{z,I}}^{k_{\mathrm{max}}^z}\frac{dk_z}{2\pi}\frac{1}{2k_0}\int_0^{k_{\perp,\mathrm{max}}^2}\frac{dk_\perp^2}{4\pi}\cdot\frac{1}{2p^z}\left[\frac{1}{2|q^z|}|\mathcal{M}_I|^2\Delta p_I^z\right]_{q^z=\pm q_k^z}$$

Beyond the step wall: WKB

$k_z \lesssim L_w^{-1}$

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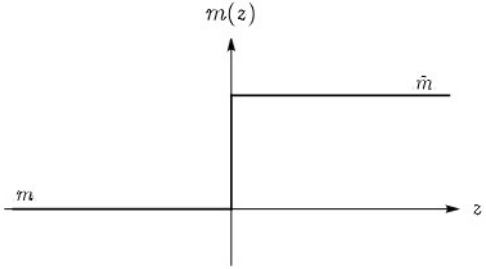
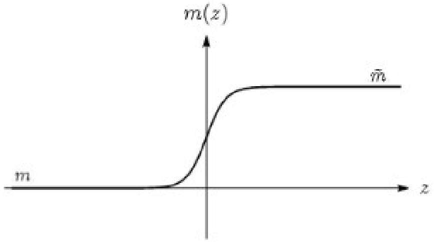




$$\mathcal{M}^{\text{wkb}} = \int_{-\infty}^{\infty} dz\; e^{i(p^z - q^z)} \chi_{R,k}(z) V(z)$$

Step wall: $\phi_{\zeta_{R,L}}$

WKB: $\chi_R(z) = \sqrt{\dfrac{k_z}{k_z(z)}}\, e^{-i\int_0^z dz' k_z(z') z'}$

On a case by case basis

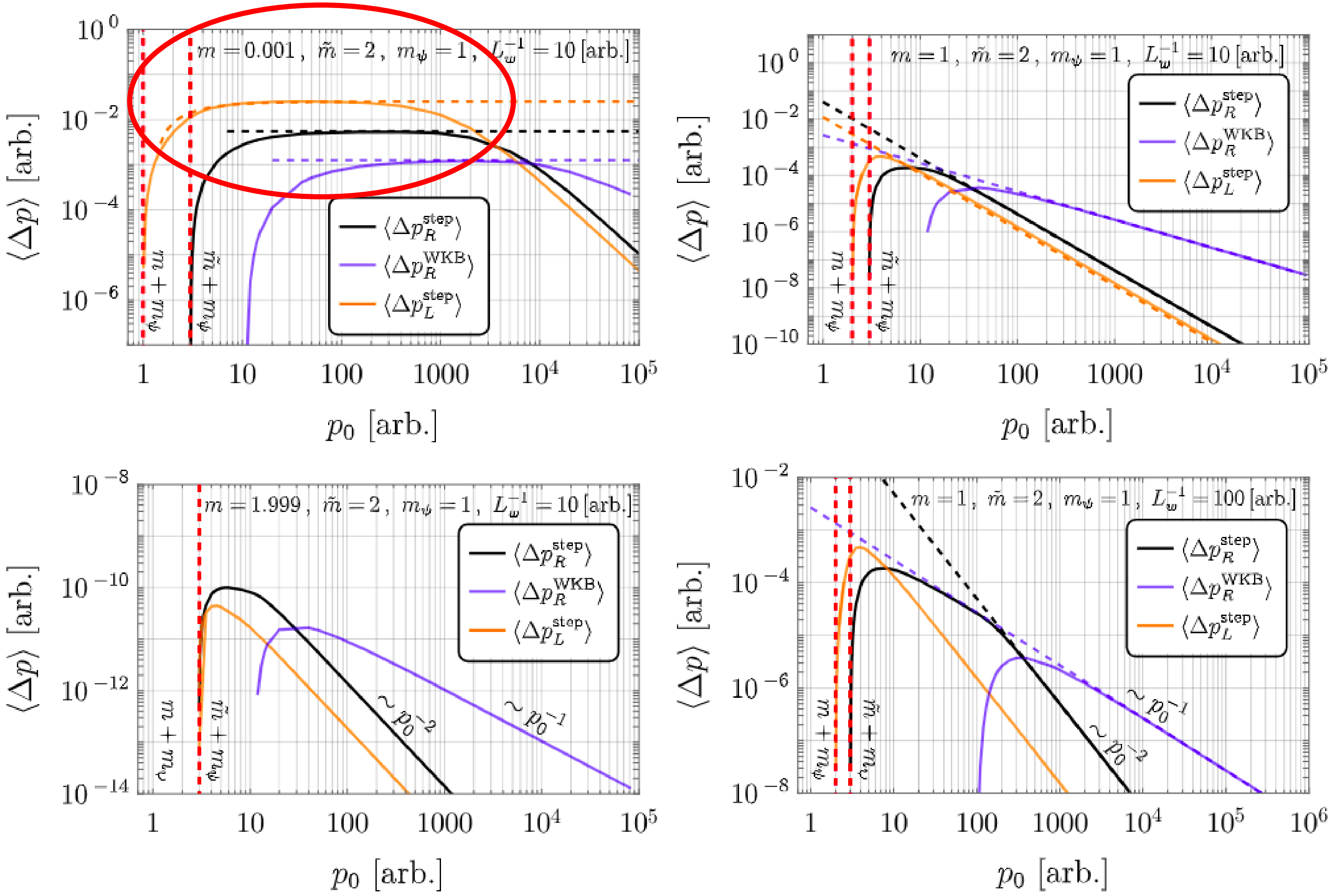
$$\mathcal{M}^{\text{wkb}} = \underbrace{\int_{-\infty}^{0} dz V(-\infty) e^{i\Delta p_z(-\infty)z} + e^{i\int_0^{L_w} dz' \Delta p(z')} \int_0^{\infty} dz V(+\infty) e^{i\Delta p_z(+\infty)z}}_{\mathcal{M}_{\text{outside}}} + \underbrace{\int_0^{L_w} dz V(z) e^{i\int_0^z dz' \Delta p(z')}}_{\mathcal{M}_{\text{inside}}}$$

$\mathcal{M} \to 0, \;\text{if}\; \Delta p_z^{\max} L_w \gg 1$ (z-momentum conservation is restored) using $\left(\int_{-\infty}^{0} e^{i\beta z} dz = \text{PV}\left(\frac{1}{i\beta}\right) + \pi\delta(\beta) \right)$

$$\mathcal{M}^{\text{wkb red.}} = \frac{V(-\infty)}{i\Delta p_z(-\infty)} - \frac{V(+\infty)}{i\Delta p_z(+\infty)} = \frac{-iy}{p^z - q^z - k^z} - \frac{-iy}{p^z - q^z - \tilde{k}^z}$$

For scalars example

Scalar Results



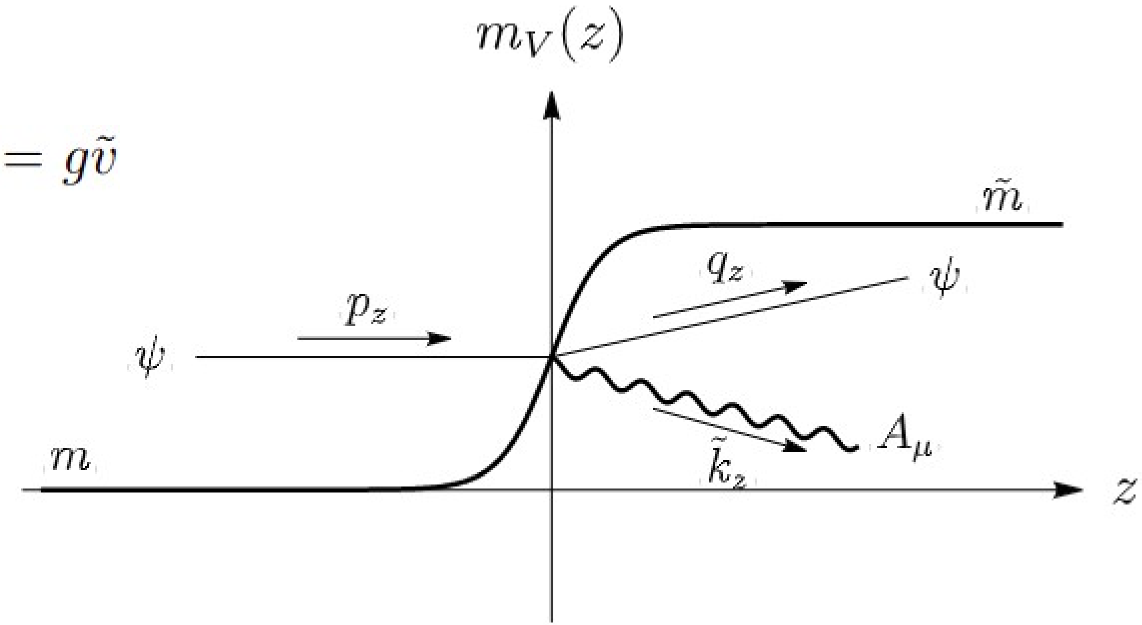
Gauge Symmetry Breaking

Gauge Symmetry Breaking

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(|\phi|) + |D_\mu\psi|^2 - \frac{1}{2}m_\psi^2\psi^2 + \text{gauge fixing}, \qquad D_\mu = \partial_\mu + igA_\mu$$

two minima at $\sqrt{2}|H| = v, \tilde{v}$

$m = gv$ and $\tilde{m} = g\tilde{v}$



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two minima at $\sqrt{2}|H| = v, \tilde{v}$

Extra complications: Gauge fixing

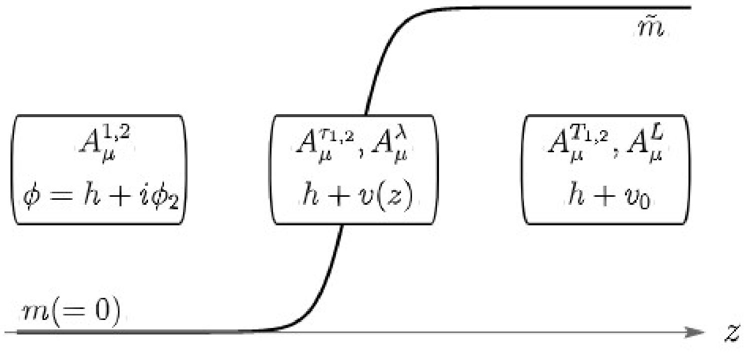
EOM: R_ξ gauge

$$\partial_z^2 v(z) = V'(v)$$

$$\Box h = -V''(v)h$$

$$\Box\phi_2 = -\xi g^2 v^2 \phi_2 - V'(v)\frac{\phi_2}{v} - 2g\partial_\mu v A^\mu$$

$$\partial_\nu F^{\mu\nu} = \frac{1}{\xi}\partial^\mu(\partial_\nu A^\nu) + g^2 v^2 A^\mu - 2g\phi_2\partial^\mu v$$



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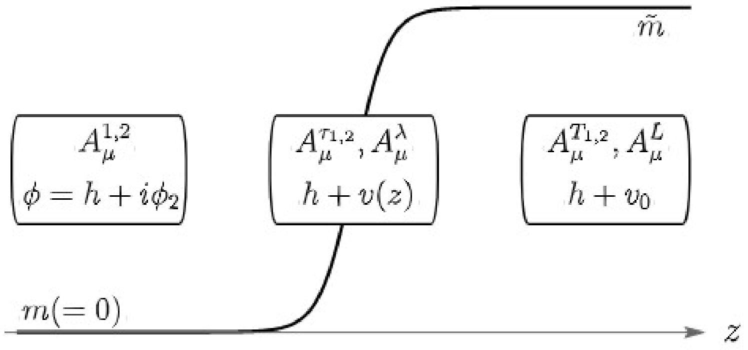
two minima at $\sqrt{2}|H| = v, \tilde{v}$

Extra complications: Gauge fixing $\rightarrow$ Unitary gauge works!!!

EOM: Unitary gauge $\xi \to \infty$

$$\begin{aligned} \partial_z^2 v(z) &= V'(v) \\ \Box h &= -V''(v)h \\ \partial_\nu F^{\mu\nu} &= g^2 v^2(z) A^\mu \end{aligned}$$

(new!) Transversality condition $\begin{cases} \partial_\mu(m^2(z)A^\mu) = 0 \\ 3 \text{ propagating dofs} \end{cases}$



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two minima at $\sqrt{2}|H| = v, \tilde{v}$

Extra complications: Complete basis not convenient in terms of conventional transverse and longitudinal polarisations..

$$\partial_\nu F^{\mu\nu} = g^2 v^2(z) A^\mu \,.$$

Generalized Lorentz condition: $\boxed{\partial_\mu(m^2(z)A^\mu) = 0}$

The field is best expanded in terms of 'wall' polarisations τ and λ which correspond to whether $A^z = 0$ or not

$$A^\mu_{k_0,k_\perp} = e^{-ik_n x^n} \sum a_l \, \chi^\mu_{l,k^n}(z) \qquad \ell = \tau_1, \tau_2, \lambda$$

Gauge Symmetry Breaking

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(|\phi|) + |D_\mu\psi|^2 - \frac{1}{2}m_\psi^2\psi^2 + \text{gauge fixing}, \qquad D_\mu = \partial_\mu + igA_\mu$$

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$\boxed{\tau}$

$$\chi^\mu_\tau = \epsilon^\mu_{\tau_{1,2}} \, \chi_{\tau_{1,2}}(z) \,, \qquad \left[E^2 - \partial_z^2 + g^2 v^2(z)\right] \chi_{\tau_{1,2}}(z) = 0$$

$$\epsilon^\mu_{\tau_1} = (0,0,1,0), \qquad \epsilon^\mu_{\tau_2} = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2}$$

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$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(|\phi|) + |D_\mu\psi|^2 - \frac{1}{2}m_\psi^2\psi^2 + \text{gauge fixing}, \qquad D_\mu = \partial_\mu + igA_\mu$$

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$\boxed{\lambda}$

$$\chi^z_\lambda = \frac{E}{gv(z)}\lambda(z) \qquad \left(-E^2 - \partial_z^2 + U_\lambda(z)\right)\lambda = 0.$$

$$U_\lambda(z) = g^2 v^2(z) - v\,\partial_z\left(\frac{\partial_z v}{v^2}\right)$$

Quantisation

$$A^\mu = \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \left(a^{\text{in}}_{\ell,I,k}\, \chi^\mu_{\ell,I,k}(z) + h.c.\right)$$
$$= \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \left(a^{\text{out}}_{\ell,I,k}\, \zeta^\mu_{\ell,I,k}(z) + h.c.\right) \;,$$

where $I = R, L$ denote right and left movers, $\ell = \tau_1, \tau_2, \lambda$ sums over different polarisations.

Quantisation

$$A^\mu = \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \left(a^{\text{in}}_{\ell,I,k}\, \chi^\mu_{\ell,I,k}(z) + h.c.\right)$$

$$= \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \left(a^{\text{out}}_{\ell,I,k}\, \zeta^\mu_{\ell,I,k}(z) + h.c.\right) ,$$

where $I = R, L$ denote right and left movers, $\ell = \tau_1, \tau_2, \lambda$ sums over different polarisations. The wave modes $\chi^\mu(z)$ are in general constructed via

$$\chi^\mu_{\tau_i,I,k} = \epsilon^\mu_{\tau_i}\, \chi_{\tau_i,I,k}(z) ,$$

where $\epsilon^\mu_{\tau_1} = (0,0,1,0)$, $\epsilon^\mu_{\tau_2} = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2}$

$$\chi^\mu_{\lambda,I,k} = \left(\frac{-ik^n \partial_z(v\lambda_{I,k})}{gEv^2}, \frac{E}{gv}\lambda_{I,k}\right) \overset{\text{on shell}}{=} \bar{\partial}^\mu \left(\frac{\partial_z\left(v\lambda^z_{I,k}\right)}{Eg\, v^2}\right) + \frac{gv(z)}{E}\lambda_{I,k}\, \delta^\mu_z ,$$

Ingoing state wavefunctions

Quantisation

$$A^\mu = \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \left(a^{\text{in}}_{\ell,I,k}\, \chi^\mu_{\ell,I,k}(z) + h.c.\right)$$
$$= \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \left(a^{\text{out}}_{\ell,I,k}\, \zeta^\mu_{\ell,I,k}(z) + h.c.\right) \ ,$$

where $I = R, L$ denote right and left movers, $\ell = \tau_1, \tau_2, \lambda$ sums over different polarisations. The wave modes $\chi^\mu(z)$ are in general constructed via

$$\chi^\mu_{\tau_i,I,k} = \epsilon^\mu_{\tau_i}\, \chi_{\tau_i,I,k}(z) \ , \qquad \text{where} \qquad \epsilon^\mu_{\tau_1} = (0,0,1,0), \qquad \epsilon^\mu_{\tau_2} = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2}$$

$$\chi^\mu_{\lambda,I,k} = \left(\frac{-ik^n \partial_z(v\lambda_{I,k})}{gEv^2}, \frac{E}{gv}\lambda_{I,k}\right) \overset{\text{on shell}}{=} \bar{\partial}^\mu \left(\frac{\partial_z\left(v\lambda^z_{I,k}\right)}{Eg\, v^2}\right) + \frac{gv(z)}{E}\lambda_{I,k}\, \delta^\mu_z \ ,$$

Ingoing state wavefunctions

$$\zeta^\mu_{\tau_i,\{L,R\}} = \epsilon^\mu_{\tau_i}\, \chi^*_{\tau_i,\{R,L\}}(z),$$

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Outgoing state wavefunctions

Quantisation

$$A^\mu = \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3\sqrt{2k_0}} e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \left(a^{\rm in}_{\ell,I,k}\ \chi^\mu_{\ell,I,k}(z) + h.c.\right)$$
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where $I = R, L$ denote right and left movers, $\ell = \tau_1, \tau_2, \lambda$ sums over different polarisations. The wave modes $\chi^\mu(z)$ are in general constructed via

$$\chi^\mu_{\tau_i,I,k} = \epsilon^\mu_{\tau_i}\ \chi_{\tau_i,I,k}(z)\ ,$$ where $\epsilon^\mu_{\tau_1} = (0,0,1,0)$, $\epsilon^\mu_{\tau_2} = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2}$

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Outgoing state wavefunctions

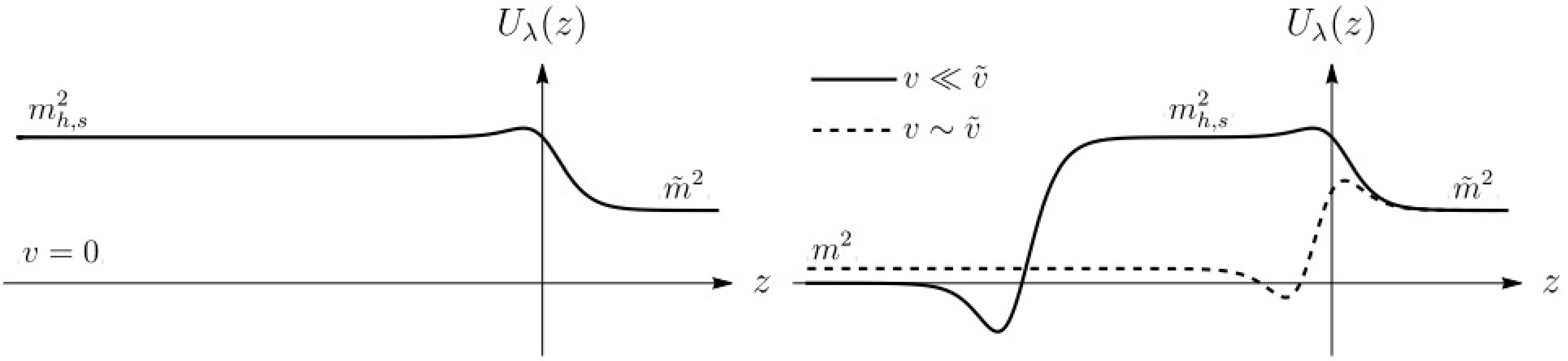
Constructed in terms of respective Schrodinger - like equations

$$\left[E^2 - \partial_z^2 + g^2v^2(z)\right]\chi_{\tau_{1,2}}(z) = 0 \qquad \left(-E^2 - \partial_z^2 + U_\lambda(z)\right)\lambda = 0$$

$$U_\lambda(z) = g^2v^2(z) - v\ \partial_z\left(\frac{\partial_z v}{v^2}\right)$$

The Lambda Potential

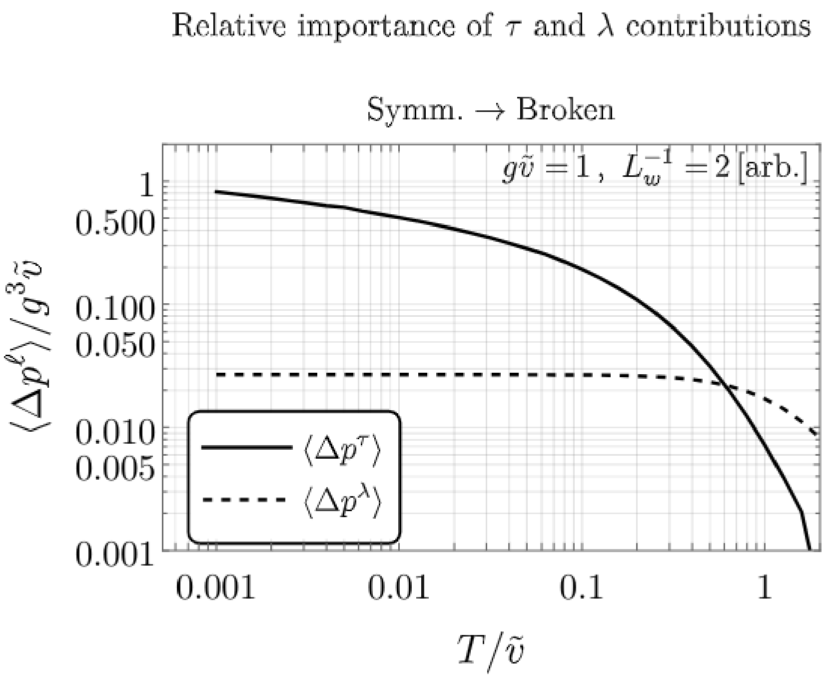
$$U_\lambda(z) = g^2 v^2(z) - v\,\partial_z\left(\frac{\partial_z v}{v^2}\right)$$

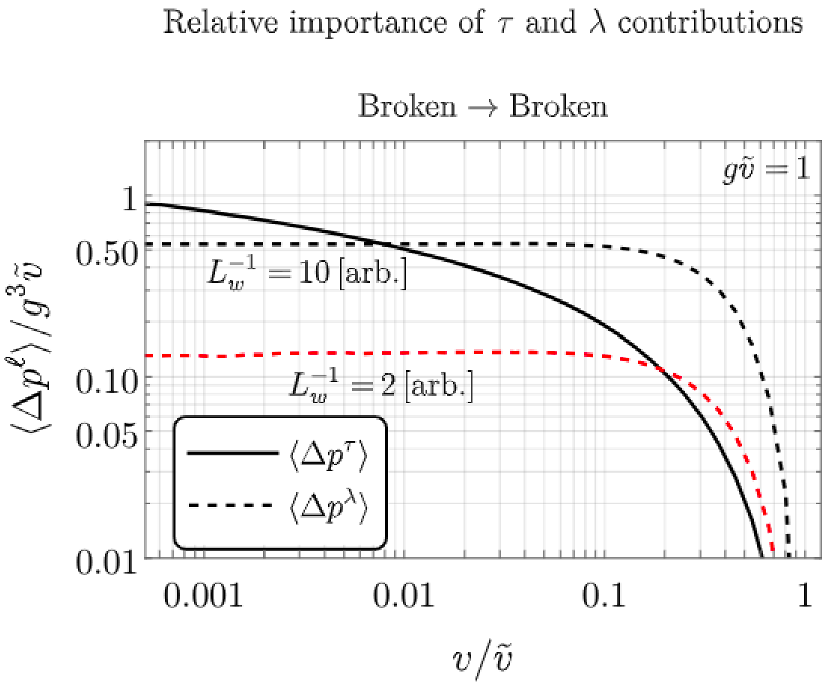


$m = gv$ and $\tilde{m} = g\tilde{v}$

Jump to Results

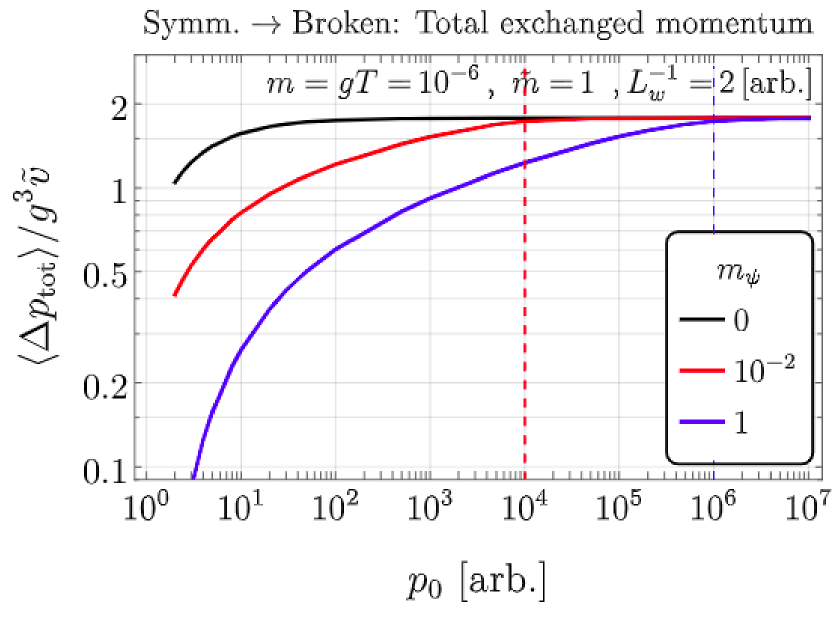
$\langle \Delta p \rangle$ In the asymptotic limit $\gamma_w \to \infty$

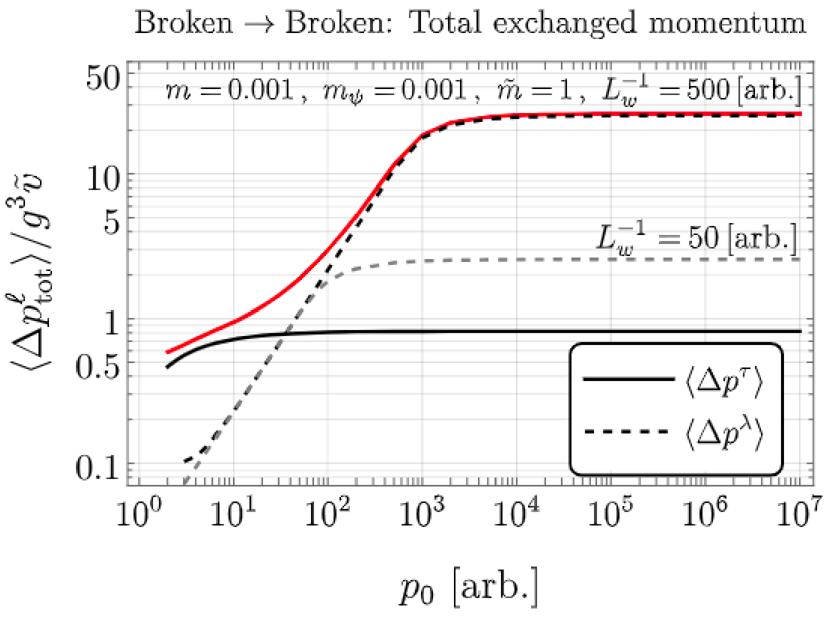




Results

We are also able to capture transient regimes
→ ultimately matters to determine equilibrium velocity





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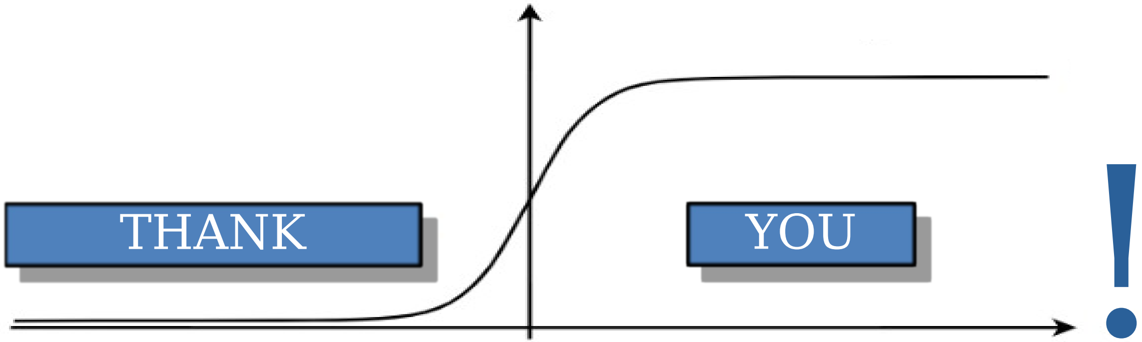
- Understanding medium – bubble interactions is a necessary step in calculating anything….


THANK
YOU
!