# Fuzzy dark matter solitons in gravitational lensing time delays and the $H_0$ measurement

Mostly based on Blum, Teodori 2021 [2105.10873]

Teodori Luca

**Dark Matter Beyond the Weak Scale II**

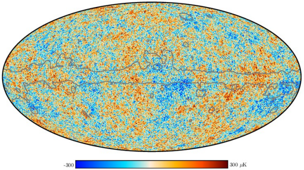
Durham University, March 2024

# Dark matter

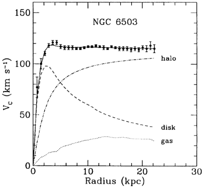
$m_{\rm dm}[\text{eV}]$ $\sim 10^{-9} M_\odot$

$10^{-20}$ $10^{-18}$ $10^{-16}$ ≀≀ $10^{6}$ $10^{8}$ $10^{10}$ $10^{12}$ $10^{14}$ ≀≀ $10^{53}$ $10^{55}$ $10^{57}$

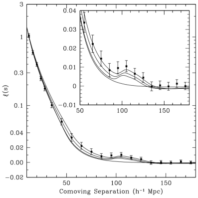
- Dark matter is an “easy” problem!
- What can gravity alone tell us?



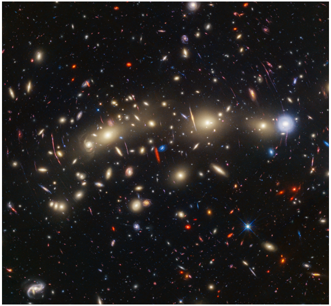

Planck 2018 [1807.06205]




K. Freese 2008 [0812.4005]

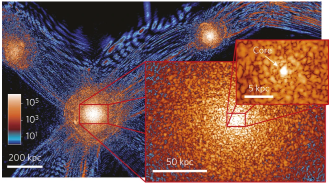



Eisenstein et al 2005 [astro-ph/0501171]

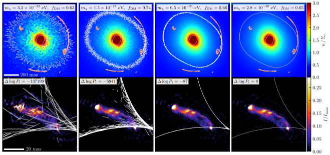


MACS 0416 (Nasa, Esa)

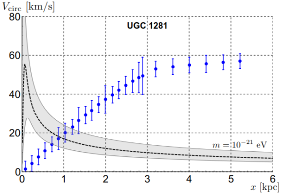
# Effects of FDM




H.-Y. Schive et al 2014 [1406.6586]



D.M. Powell et al 2023 [2302.10941]

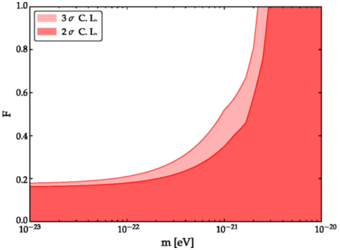



N. Bar et al 2018 [1805.00122]

- Main features: wave-interference phenomena and inner cored profile (solitons)
- Lyman-alpha forest
- Dynamical heating
- Galaxy rotation curves (Soliton-halo relation)
- Gravitational lensing anomalies

# Bounds on Fuzzy Dark Matter



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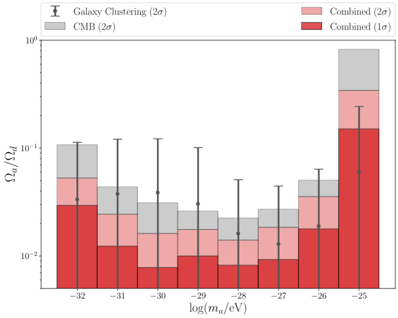
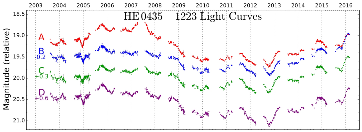



A. Laguë et al 2021 [2104.07802]

- Rotation curves, Dynamical heating $\implies m \simeq 1 \times 10^{-20}$ eV
- Lymann-$\alpha$ and CMB for fractions
- Gravitational lensing: time delays and $H_0$ with FDM cores




V. Bonvin et al 2016 [1607.01790]

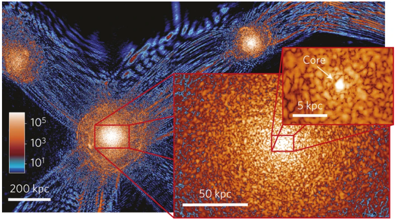
# Outline



- Solitons in FDM
- $H_0$ from time delays (?)
- Soliton profile affecting time delays inference
- Can we constrain the FDM paradigm with an $H_0$ prior?

# Fuzzy Dark Matter essentials



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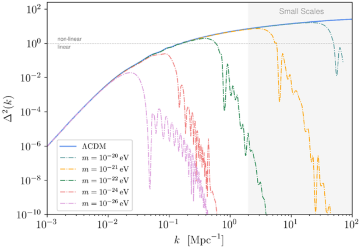



E.G.M. Ferreira 2021 [2005.03254]

- De-Broglie relevant in astrophysics scales

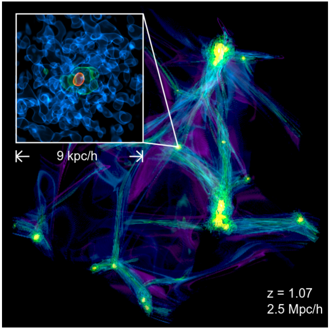
$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{mv} \simeq 3.8\,\mathrm{pc}\left(\frac{10^{-20}\,\mathrm{eV}}{m}\right)\left(\frac{10^2\,\mathrm{km\,s^{-1}}}{v}\right)$$

- Cannot squeeze too much mass in little volume (from uncertainty principle): small scales power spectrum suppression, plus formation of cored profiles
- Huge occupation number $\Longrightarrow$ classical field

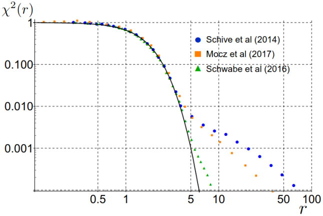
$$\mathcal{N} \simeq \frac{\rho_{\rm dm}}{m(mv)^3} \simeq 10^{84}\left(\frac{\rho_{\rm dm}}{0.4\,\mathrm{GeV\,cm^{-3}}}\right)\left(\frac{10^{-20}\,\mathrm{eV}}{m}\right)^4$$

- In NR limit, we have Schrödinger-Poisson equations

$$i\partial_t\psi = -\frac{\nabla^2\psi}{2m} + m\Phi\psi\,,$$
$$\nabla^2\Phi = 4\pi G|\psi|^2\,.$$




J. Veltmaat et al 2018 [1804.09647]




N. Bar et al 2018 [1805.00122]

- Ground state solution of Schrödinger-Poisson

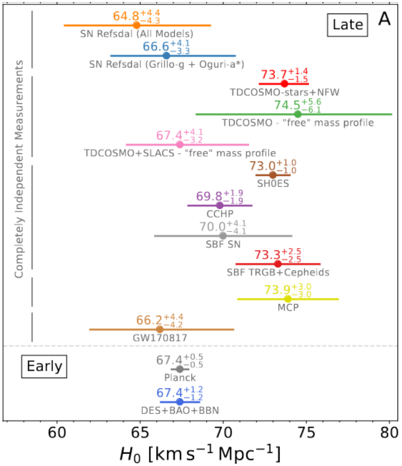
$$\psi(\vec{x},t) = \frac{mM_{\rm pl}}{\sqrt{4\pi}} e^{-i\gamma mt}\chi(\vec{x})\ ,\ x = rm$$

$$\partial_x^2\chi + \frac{2}{x}\partial_x\chi = 2(\Phi + \Phi_{\rm ext} - \gamma)\chi$$

$$\partial_x^2\Phi + \frac{2}{x}\partial_x\Phi = \chi^2$$

- Can cores affect time delays?

# The $H_0$ tension




P.L. Kelly et al 2023 [2305.06367]

- CMB and LSS (early);
- Distance ladder (late);
- Strong gravitational lensing from TDCOSMO (COSMOGRAIL, HoLiCOW, STRIDES, SHARP, COSMICLENS);
- Use SLACS lenses to put priors on lens population parameters.

Adapted from M. Millon et al 2019 (TDCOSMO I) [1912.08027]

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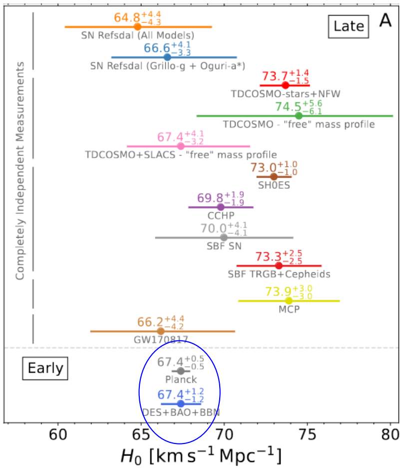


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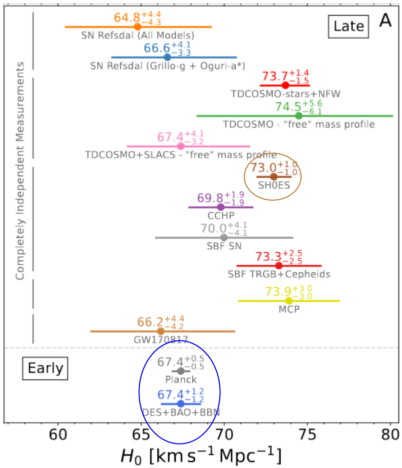


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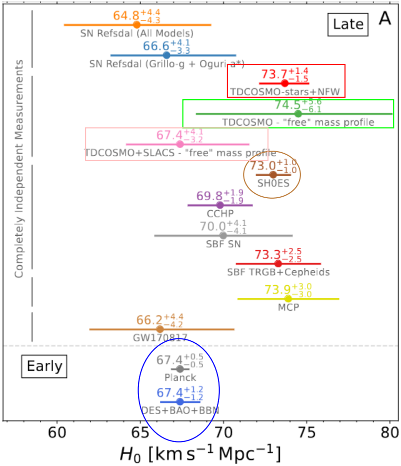


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| Galaxy | $H_0$ |
|---|---|
| PG1115 | $81.1^{+8.0}_{-7.0}$ |
| RXJ1131 | $78.2^{+3.4}_{-3.4}$ |
| WFI2033 | $71.6^{+3.8}_{-4.9}$ |
| HE0435 | $71.7^{+4.8}_{-4.5}$ |
| DESJ0408 | $74.2^{+2.7}_{-3.0}$ |
| B1608+656 | $71.0^{+2.9}_{-3.3}$ |
| J1206 | $68.9^{+5.4}_{-5.1}$ |

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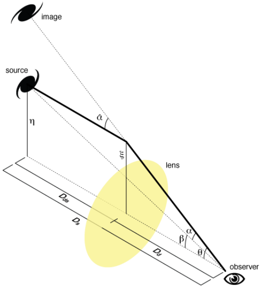
# Cores affecting time delays?




credit: Wikipedia

- $\hat{\alpha} = 2 \int \nabla_\perp \Phi \, \mathrm{d}\lambda \implies \vec{\beta} = \vec{\theta} - \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} \hat{\alpha}(\vec{\theta})$
- Convergence $\kappa \sim \int \mathrm{d}z \, \rho$.
- Lens model + time delay measurement

$$\Delta t_{ij} \propto \frac{1}{H_0} \, .$$

- Degeneracies (mass sheet degeneracy): source position and mass of galaxy unknown

$$\vec{\beta} \to \lambda \vec{\beta} \, , \; \kappa \to \lambda \kappa + (1 - \lambda) \implies H_0 \to \lambda H_0$$

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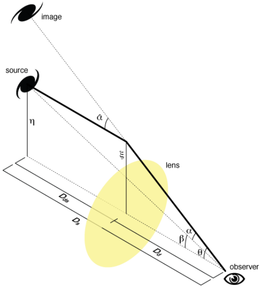


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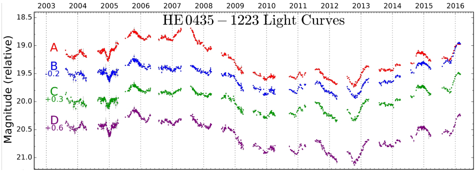


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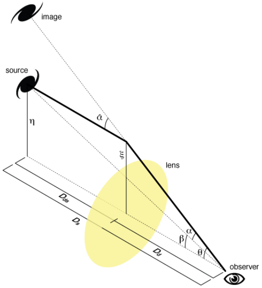


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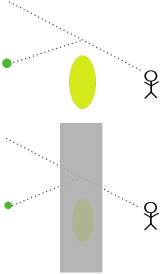

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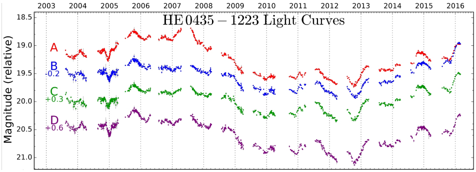


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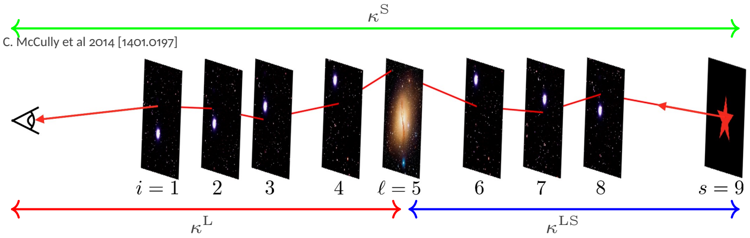
# Examples of mass sheets






- LSS on the line of sight;
- Host dark matter halo from group/cluster;
- Stellar and galaxy kinematics $\Longrightarrow$ proxy for real mass.
- All the previous was (sort of) accounted in TDCOSMO I, but what about a mass sheet on the dark matter halo of the lens galaxy itself?

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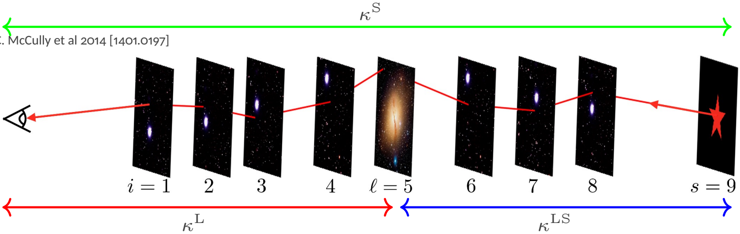


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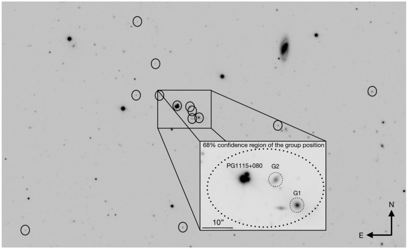


G.C.-F. Chen et al 2019 [1907.02533]

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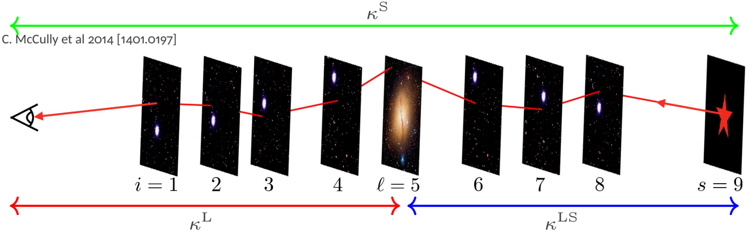


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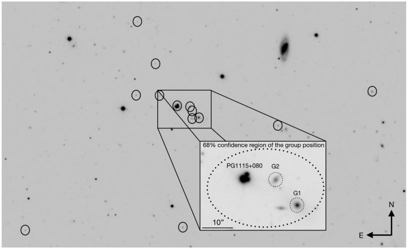


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# ULDM cores as approximate Mass Sheets?



- Soliton has core-like density profile

$$\rho = \frac{\beta^4 m^2}{4\pi G}\chi_1^2 \,, \quad \chi_1 \simeq \frac{1}{(1+a^2r^2)^b}$$

- To make the idea work, one needs:

$$\delta_{\rm E} := \frac{\alpha_{\rm c}(\theta_{\rm E})}{\theta_{\rm E}} - \kappa_{\rm c}(\theta_{\rm E}) \lesssim 0.01 \,,$$

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$$\theta_{\rm core} \sim \frac{1}{\beta a m D_{\rm l}} > \theta_{\rm E}$$

$$m \lesssim 10^{-24}\,\mathrm{eV}\left(\frac{1''}{\theta_{\rm E}}\right)^{3/2}\left(\frac{1\,\mathrm{Gpc}}{D_{\rm d}}\right)\left(\frac{\delta H_0/H_0}{0.1}\right)^{-1/2}$$

- ULDM whole DM: needs $m \sim 10^{-24}$ eV, tension with Lymann-$\alpha$ forest and CMB.
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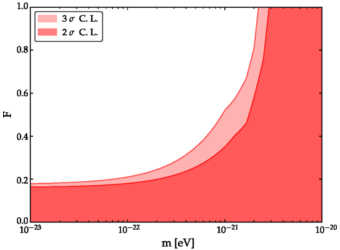


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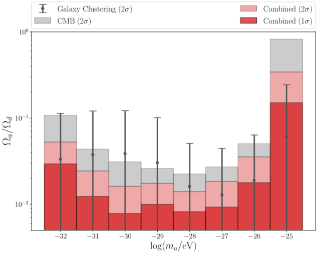


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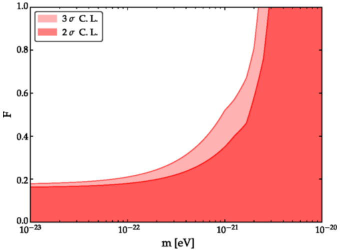


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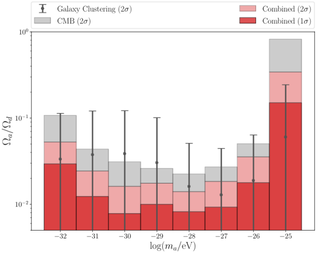


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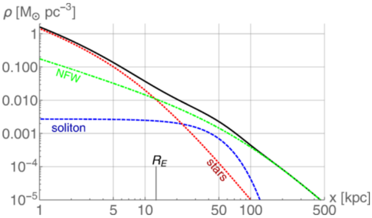
# An explicit example



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- An example: subdominant $m = 2 \times 10^{-25}$ component with $M_{\rm sol} \sim 10^{12} M_\odot$, $M_{200} = 2 \times 10^{13} M_\odot$ (resembles DESJ0408 system)
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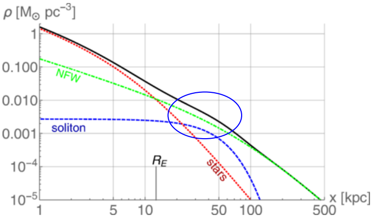


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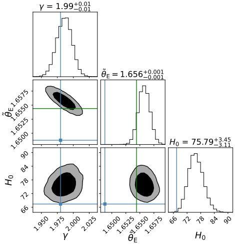
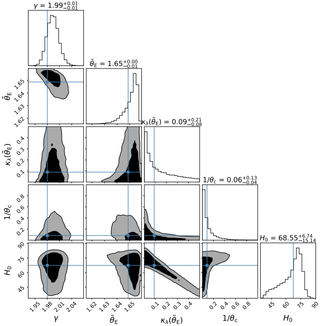
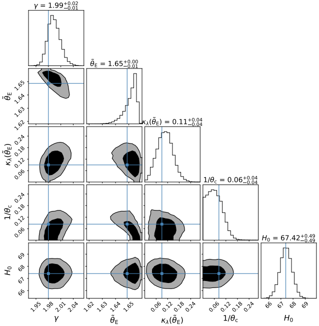
# Mock analysis



- Is our $\delta_{\mathrm{E}}$ criterium good enough?
  Yes, actually conservative
- Use `lenstronomy` utilities to see if the idea could work
- Mock: subdominant $m = 2 \times 10^{-25}$ component with $M_{\mathrm{sol}} \sim 10^{12} M_{\odot}$, $M_{200} = 2 \times 10^{13} M_{\odot}$ and $H_0 = 67.4$ km/s/Mpc
- Can we "relax" so much mass in the soliton?



# Can we relax such a mass?
## Comparison with TDCOSMO systems

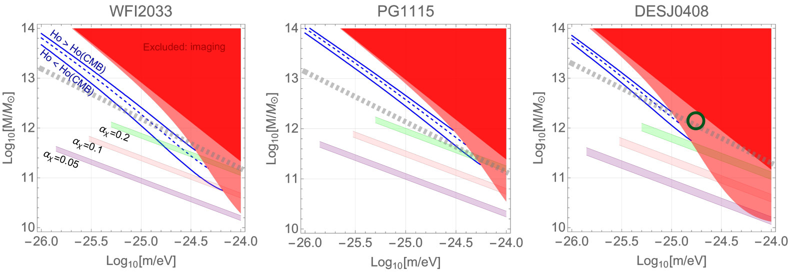


- Gravitational relaxation time scale from kinetic theory

$$\tau(R) = b\frac{\sqrt{2}}{12\pi^3}\frac{m^3\sigma^6(R)}{G^2\alpha_\chi^2\rho^2(R)\Lambda}$$

- $\tau(R) < t_{\rm gal} \implies$ find $R$ , $M_{\rm sol} < \alpha_\chi M_{\rm halo}(R)$
- An isothermal power-law with constant $\sigma$ yields the bound ($M_{\rm halo} \simeq \sigma^2 R/G$)

$$M_{\rm sol} \lesssim 10^{12} M_\odot \left(\frac{\alpha_\chi}{0.1}\right)^{3/2}\left(\frac{m}{5\times10^{-25}\,\text{eV}}\right)^{-3/4}\left(\frac{\sigma}{200\,\text{km s}^{-1}}\right)^{3/2}\left(\frac{t_{\rm gal}}{10\,\text{Gyrs}}\right)^{3/2}$$

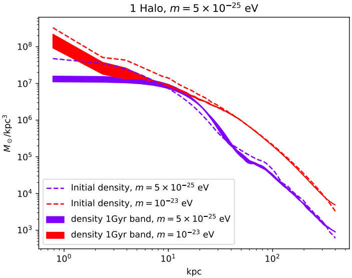


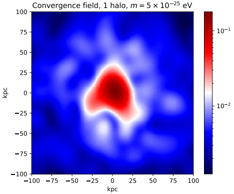

K. Blum, LT 2021 [2105.10873]

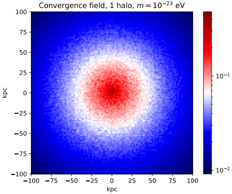
# Simulate the system? Preliminary

## Testing the relaxation bottleneck






- Constrain stringy axion fractions from time delays?
- 3D pseudo-spectral code for multiple field Schrödinger-Poisson equations
- Preliminary result: consistent with estimates, possibly motivating more cosmological simulations of such scenarios
- Work in progress...

# Summary



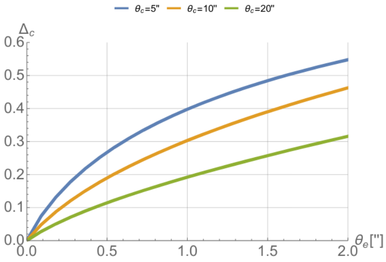
- We discuss the possibility that the $H_0$ tension in strong gravitational lensing measurements could come from an unaccounted approximate mass sheet degeneracy
- A possible physical dark matter model which can yield this is ULDM
- To make it work, one must consider ULDM as a subdominant part of the whole dark matter
- Scenario: $\mathcal{O}(0.1)$ fractions, $m \lesssim 1 \times 10^{-24}$ eV
- Refine estimates with simulations (work in progress)
- With an $H_0$ prior, time delays can be used to probe features of DM halos, difficult to spot otherwise, and possibility to put bounds on FDM scenario

# Using stellar kinematics

$$\sigma^2_{\rm los}(\theta) = \frac{2G}{I(\theta)}\int_1^\infty \frac{{\rm d}y}{y} K(y) I(yD_{\rm L}\theta) M(yD_{\rm L}\theta)$$

In internal MSD ($\lambda =: 1 - \kappa_{\rm c}$),

$$M = (1-\kappa_{\rm c})M^{\rm model} + M_{\rm core}\ ,\ M_{\rm core}(r) \propto \kappa_{\rm c}\frac{r^3}{r_c}\ ,\ \kappa_{\rm c} \sim \rho_0 r_c$$



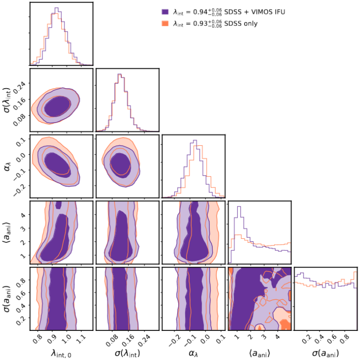

K. Blum, LT 2021 [2105.10873]

A perfect MSD limit is not conservative! Used in TDCOSMO IV

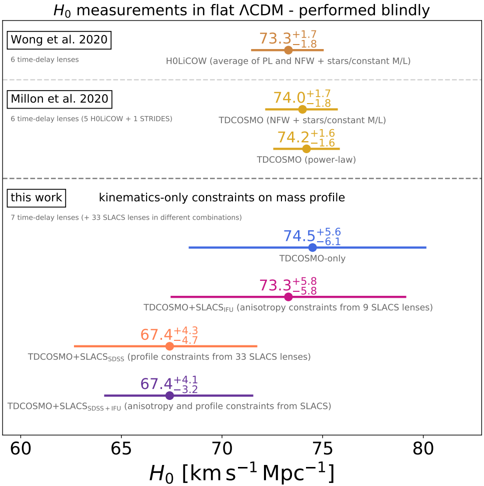
$$\left(\frac{\sigma_{\rm los}}{\sigma^{\rm model}_{\rm los}}\right)^2 = 1 - \kappa_{\rm c}(1-\Delta_{\rm c})$$

TDCOSMO XII: insert a parametrization of the core with finite (fixed!) core radius to catch this effect as well.

# TDCOSMO IV



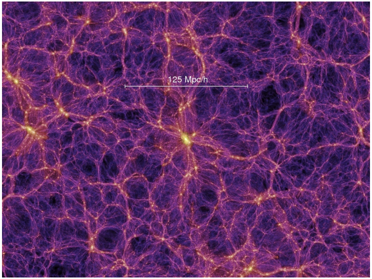

S. Birrer et al 2020 (TDCOSMO IV) [2007.02941]



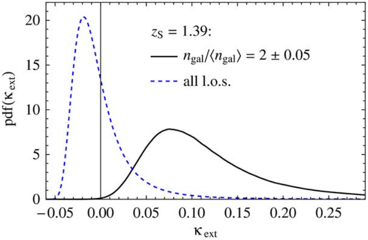
# Interpreting the Mass Sheet Degeneracy



Springer et al 2005




Suyu et al 2010




- Change $\lambda_{\mathrm{S}}$: changing one's mind about the true $\kappa^{\mathrm{S}}$
- 

$$\kappa^{\mathrm{R}} \longmapsto \lambda_{\mathrm{R}}\kappa^{\mathrm{R}} + (1 - \lambda_{\mathrm{R}})$$
$$\Gamma^{\mathrm{R}} \longmapsto \lambda_{\mathrm{R}}\Gamma^{\mathrm{R}}$$

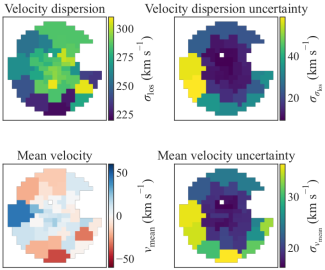
- Time delays do change!

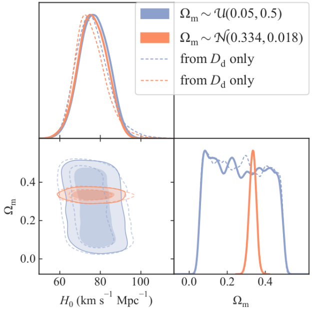
$$\Delta\tau \to \lambda_{\mathrm{S}}\lambda_{\mathrm{LS}}^{-1}\lambda_{\mathrm{L}}\Delta\tau$$
$$H_0 \to \lambda_{\mathrm{S}}\lambda_{\mathrm{LS}}^{-1}\lambda_{\mathrm{L}}H_0$$

- Estimate $\kappa^{\mathrm{S}}$ via ray-tracing through Millennium Simulation and characterization of the lens field
- Degeneracy is limited by priors on weak lensing quantities and constraints on mass of lens galaxy (stellar kinematics)

# TDCOSMO XII



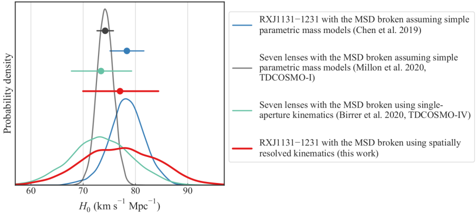



- Resolved kinematics with Keck Cosmic Web Imager (KCWI), to mitigate systematic effects
- New result for a single system:

$$H_0 = 77.1^{+7.3}_{-7.1}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- Goal for the future: Spatially resolved velocity dispersion measurements for around 40 time-delay lens galaxies will yield an independent 2% $H_0$ measurement without any mass profile assumption



# Steps for the kinematic measurement



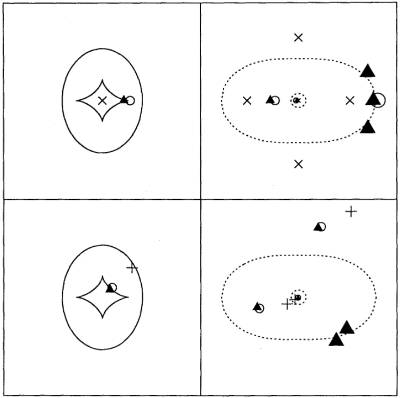
- For each pixel, compute the spectroscopic $S/N$ ratio, to perform Voronoi binning (have the best spatial resolution given a $S/N$ treshold).
- Fit the spectra in each Voronoi bin using pPXF and the X-shooter Spectral Library (XSL)
- Change template setup to estimate variance-covariance matrix (hopefully catching possible systematics)
- Solve the axisymmetric Jeans equation given an anisotropy profile and a gravitational potential. The gravitional potential is obtained from deprojecting $\kappa_{\rm gal}$ (they consider oblate and prolate case), which includes the finite core term.
- With this, infer a constraint on the possible mass of the lens $\implies$ constraint on the mass sheet degeneracy, hence on the error bars on the inferred $H_0$.

# Things to note



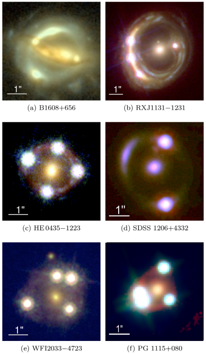
- $\kappa^{\mathrm{L}}$ is effectively set to zero (as far as I can tell);
- In deprojecting the 3D spheroid for the dinamic modelling, they discuss the purely prolate or purely oblate case, not considering triaxiality;
- For the core profile $\kappa_{\mathrm{s}}$, different parametrizations are not explored, and $\theta_{\mathrm{s}}$ is fixed to the lowest value which would not affect imaging;
- Anisotropy profiles explored are 2, and they are spatially constant ones; different anisotropy profiles (non constant ones like Osipkov-Merritt) are not explored;
- The prior they use for $\kappa^{\mathrm{ext}}$ is strictly speaking the prior for $\kappa^{\mathrm{S}}$ only; from the triangle plot, $\kappa_{\mathrm{s}}$ seems just to follow his prior;
- Also the velocity anisotropy parameters seems to be limited by just the prior. Same story for the inclination angle *i* (for it, the prior is not directly on *i*, but on $q_{\mathrm{int}}$ );
- There is no discussion of external shear (but it was included in previous analysis) and no discussion of host cluster effects
- $\lambda$ seems to be constrained from the lower end, but not from the upper end.

Caustic: critical line mapped on source plane



R. Narayan and S. Wallington

- $R_{\text{cusp}} = \frac{|\mu_A + \mu_B + \mu_C|}{|\mu_A| + |\mu_B| + |\mu_C|} \to 0$
- $R_{\text{fold}} = \frac{|\mu_{\text{min}}| - |\mu_{\text{sad}}|}{|\mu_{\text{min}}| + |\mu_{\text{sad}}|} \to 0$
- They hold only in lenses with a smooth potential and small angles between two bright lensed images




(a) B1608+656 (b) RXJ1131−1231
(c) HE0435−1223 (d) SDSS 1206+4332
(e) WFI2033−4723 (f) PG 1115+080

K.C. Wong et al 2019 [1907.04869]

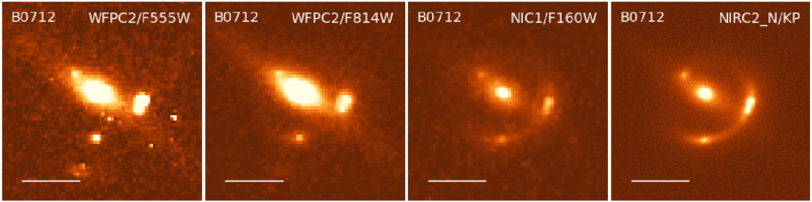
# Dark matter substructure or other?



- Substructures can cause magnification changes; in CDM, the halo and subhalo mass function are respectively

$$\frac{\mathrm{d}N}{\mathrm{d}M} \propto M^{-\alpha}\,,\, \alpha \sim 1.9\,,\; N(>\mu := m_{\mathrm{sub}}/M_{\mathrm{h}}) = \left(\frac{\mu}{\mu_1}\right)^{1+\alpha} \exp\left(-\left(\frac{\mu}{\mu_{\mathrm{cut}}}\right)^{b}\right)$$

- Numerical studies seem not to reproduce current flux anomaly strength with substructure alone
- Baryons effects: undetected disks, (microlensing of stars)
- Illustris simulation (Jen-Wei Hsueh et al 2018): baryonic components increase the probability of finding high flux-ratio anomalies in the early-type lenses by about 8% and by about 10-20% in the disc lenses, plus astrometric anomalies




J.-W. Hsueh et al 2017