26 March 2024

Durham University

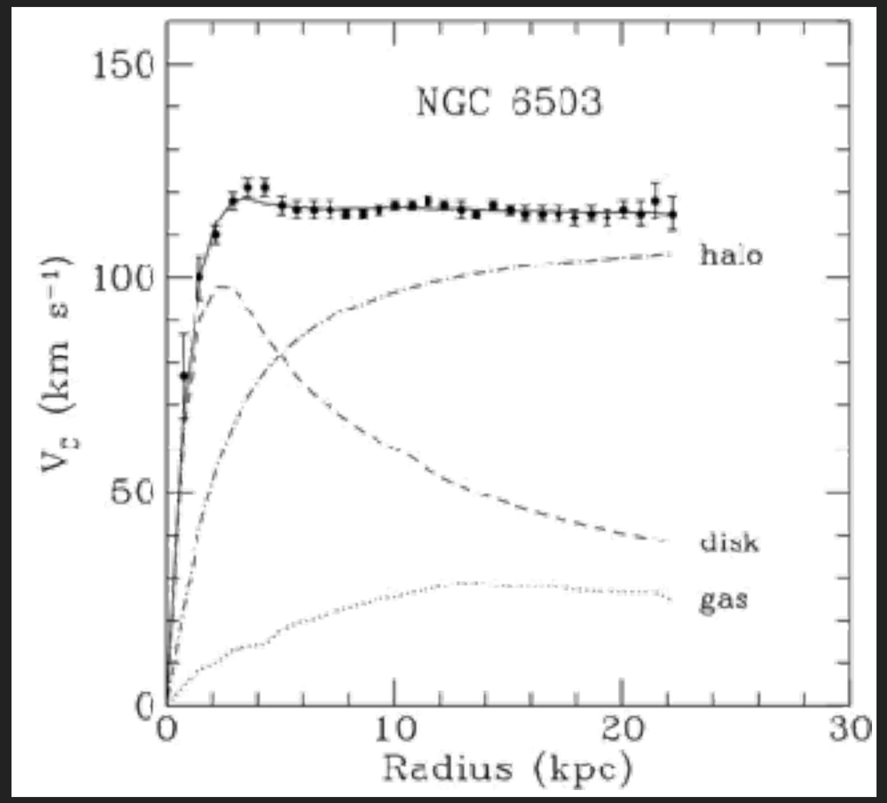
Dark Matter beyond the weak scale II

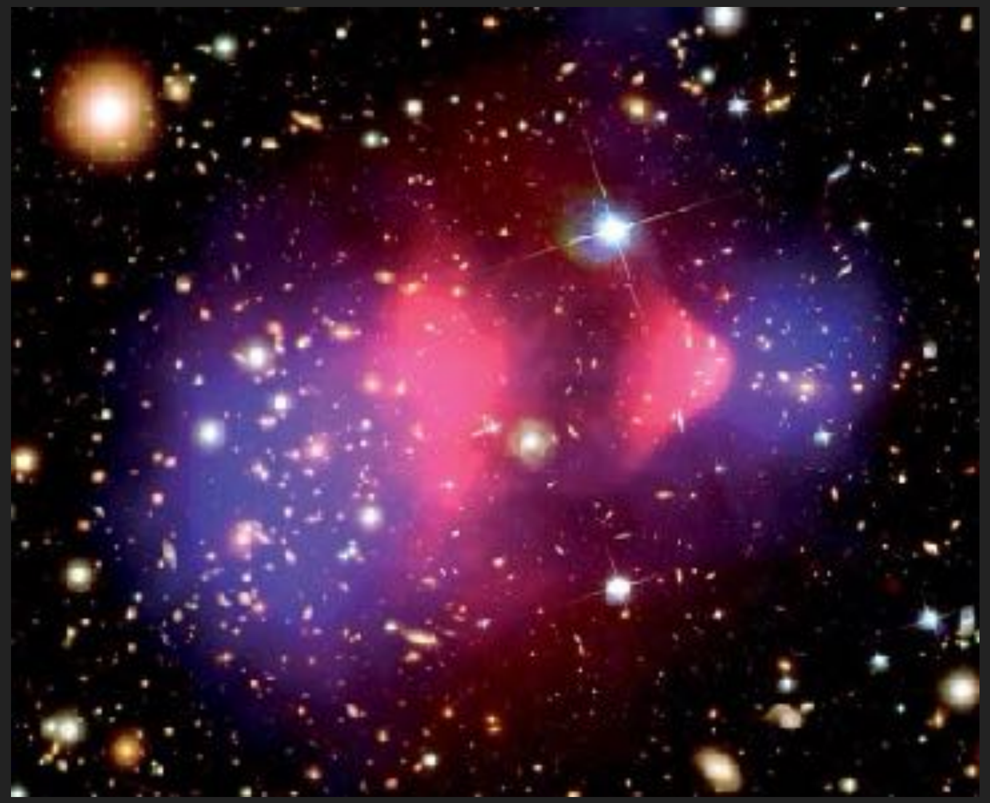
Production mechanisms for DM: from freeze-in to gravitational production

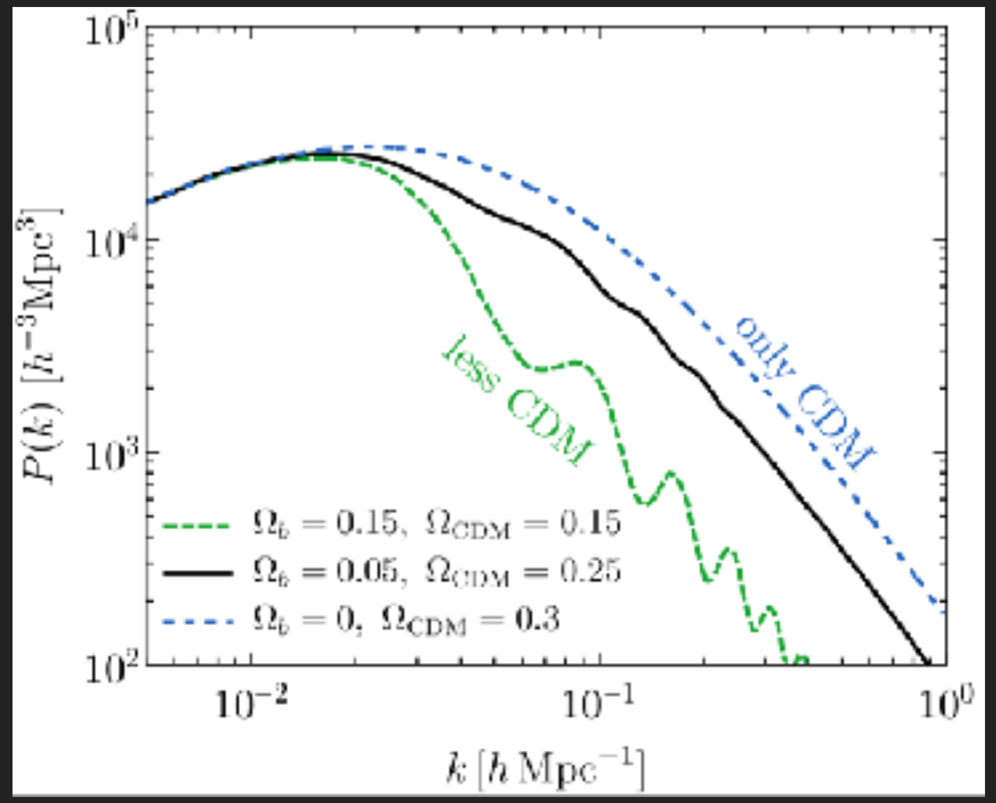
Davide Racco

ETH zürich

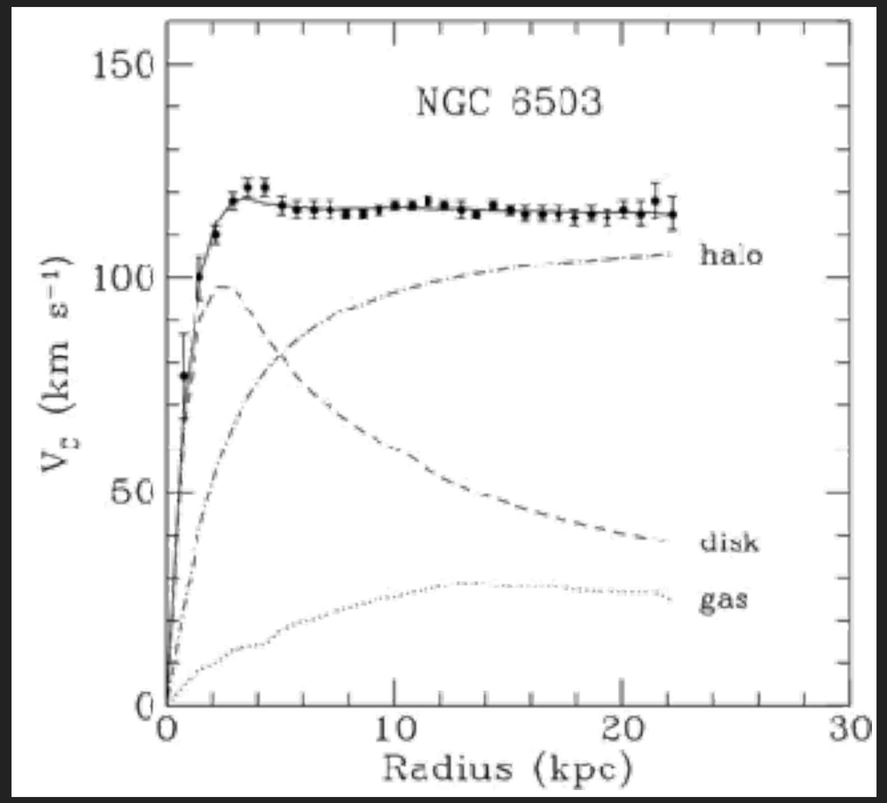
Universität Zürich UZH

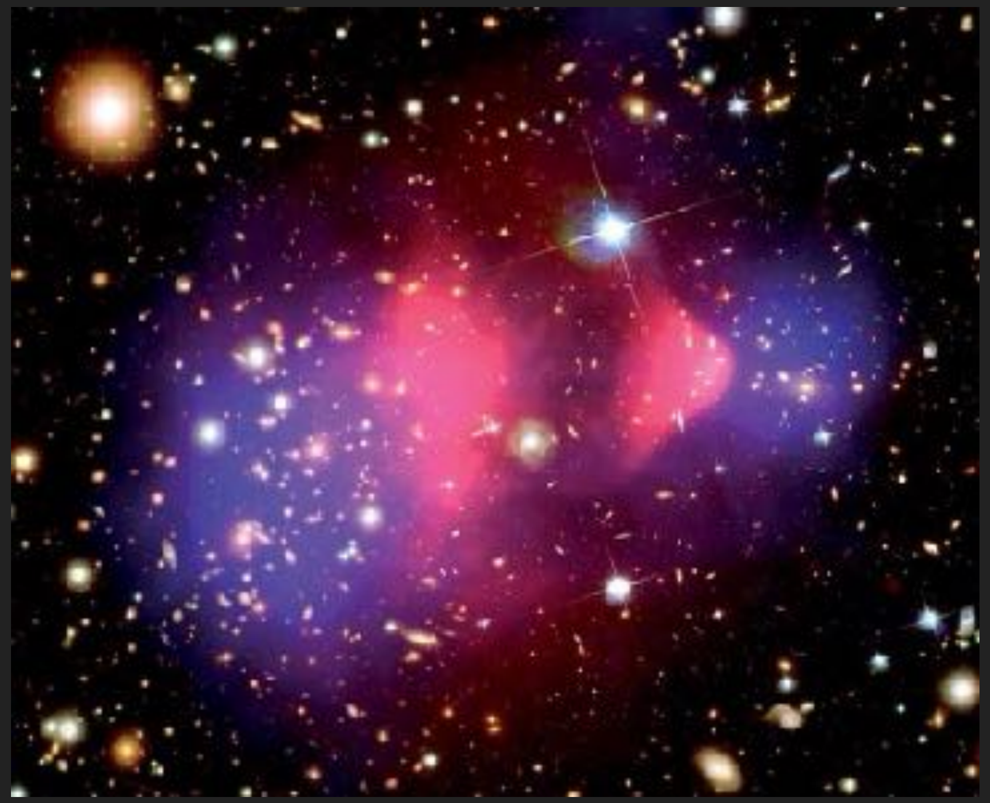


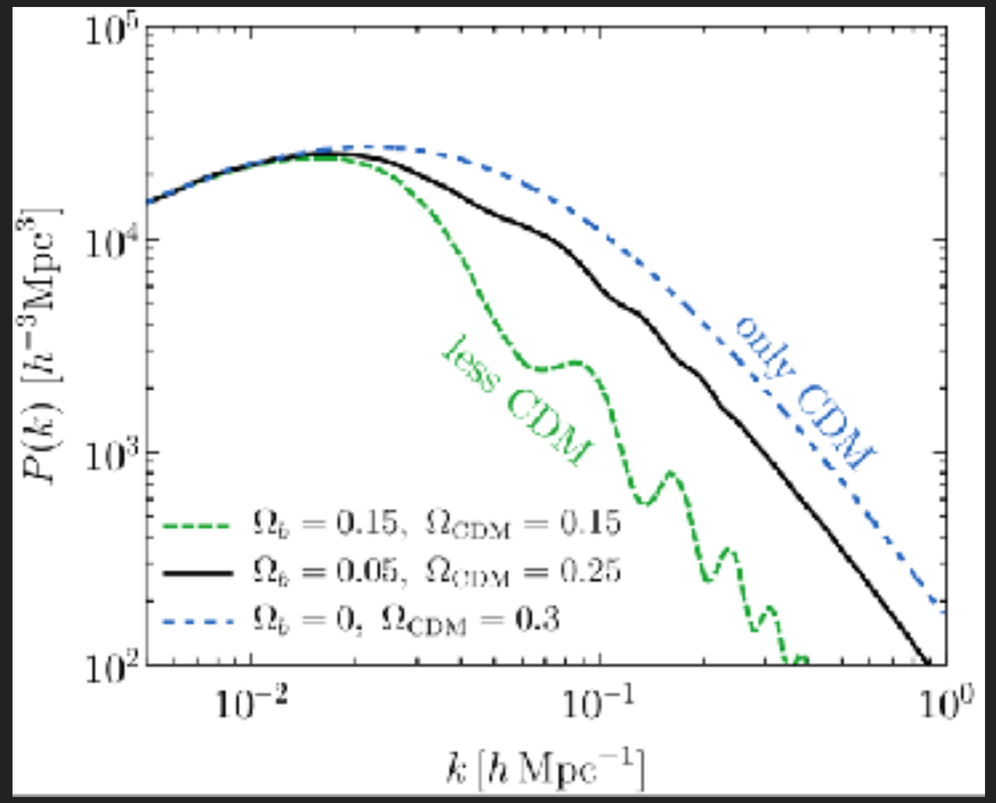




- All evidence from *gravitational* interactions
- Exp. searches look for other interactions with us







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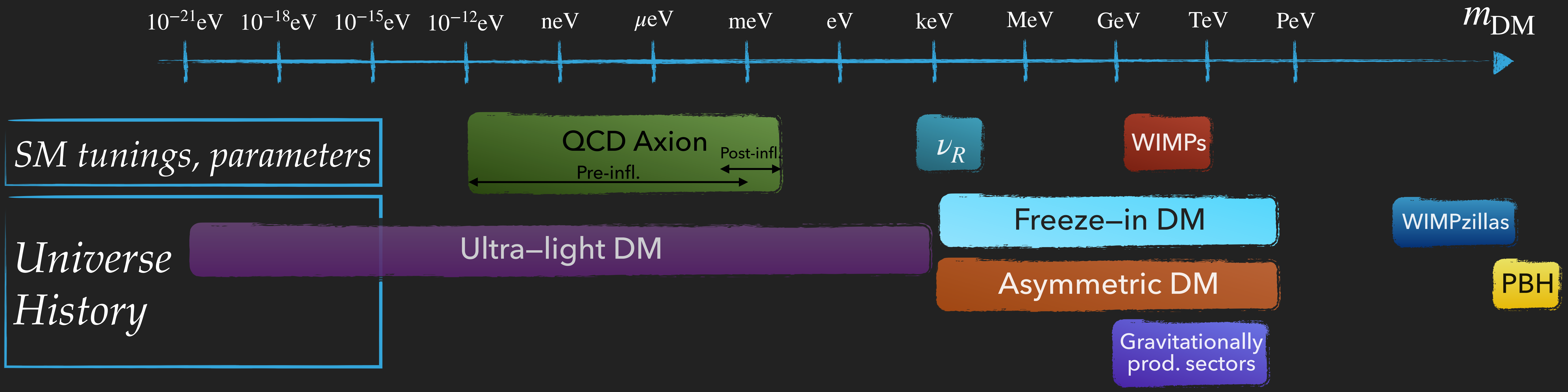
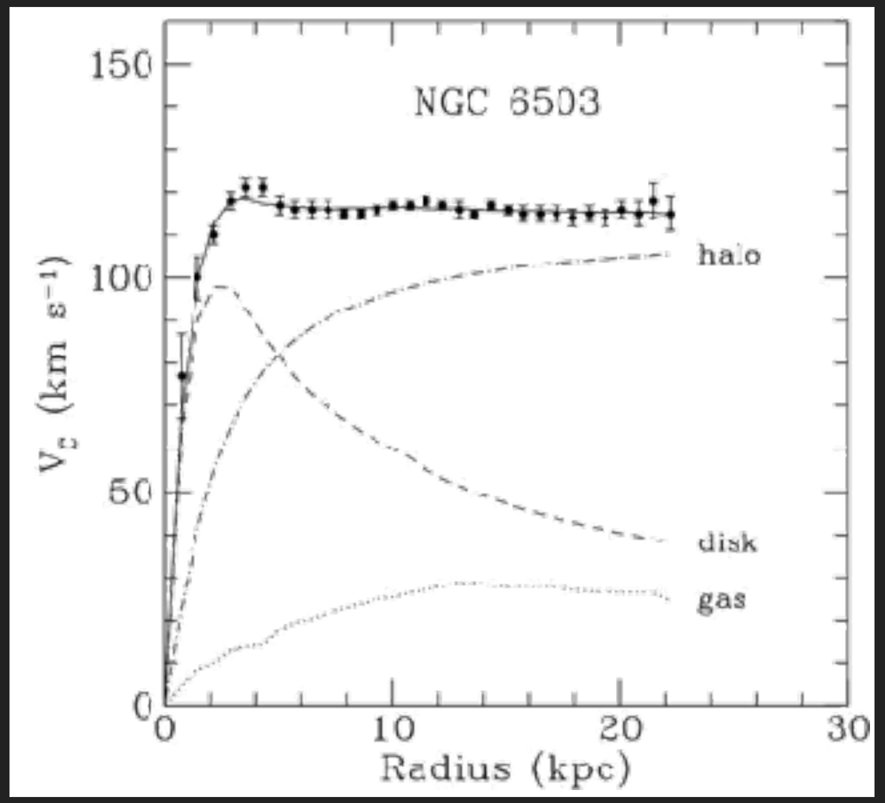
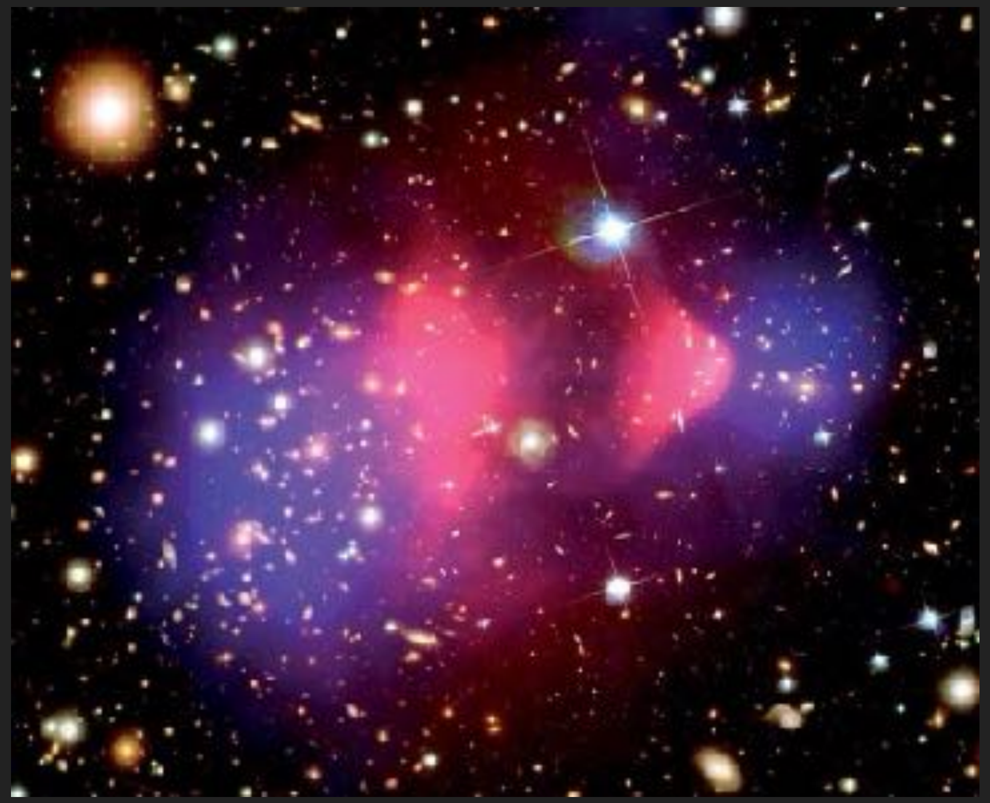
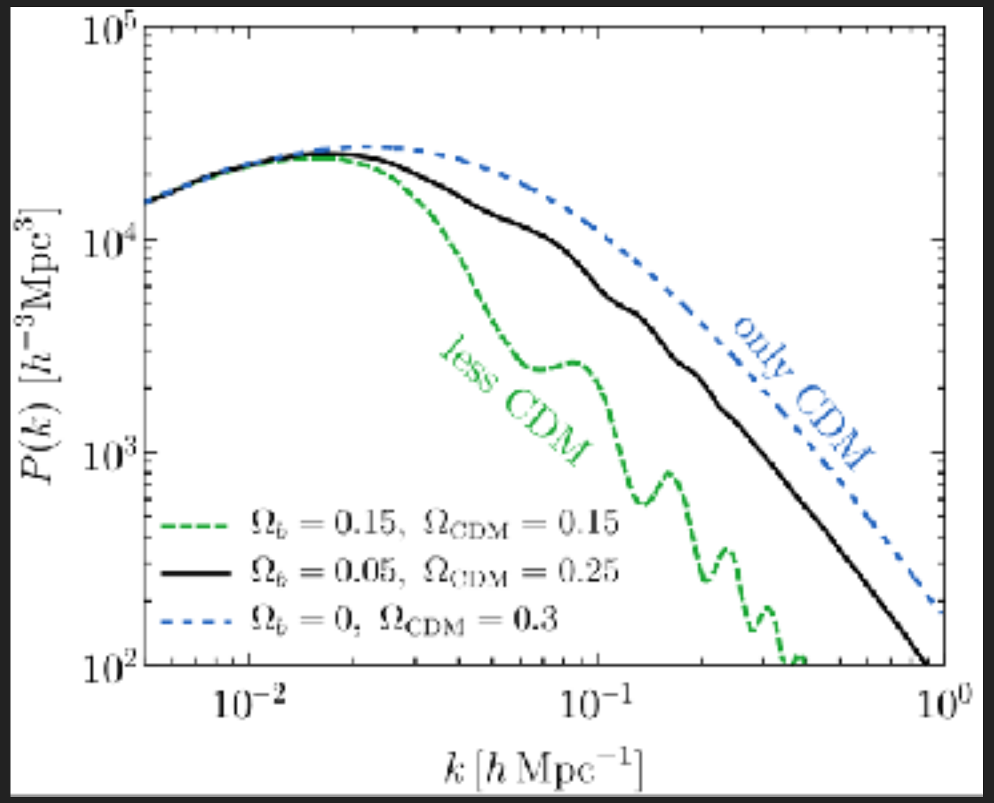


DARK MATTER PHENOMENOLOGY







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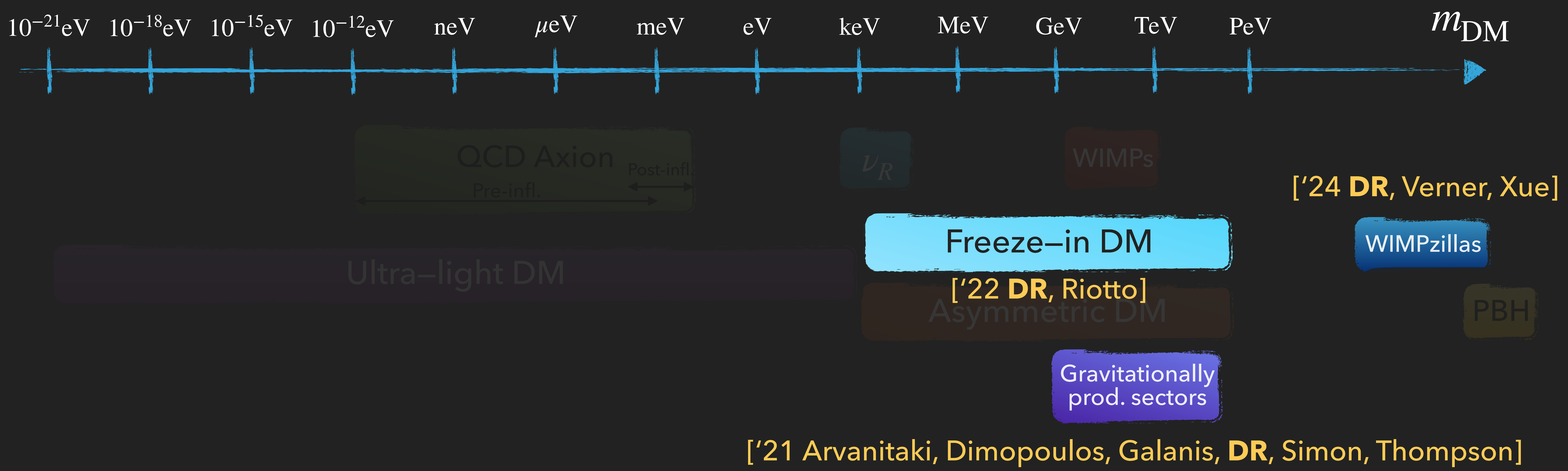
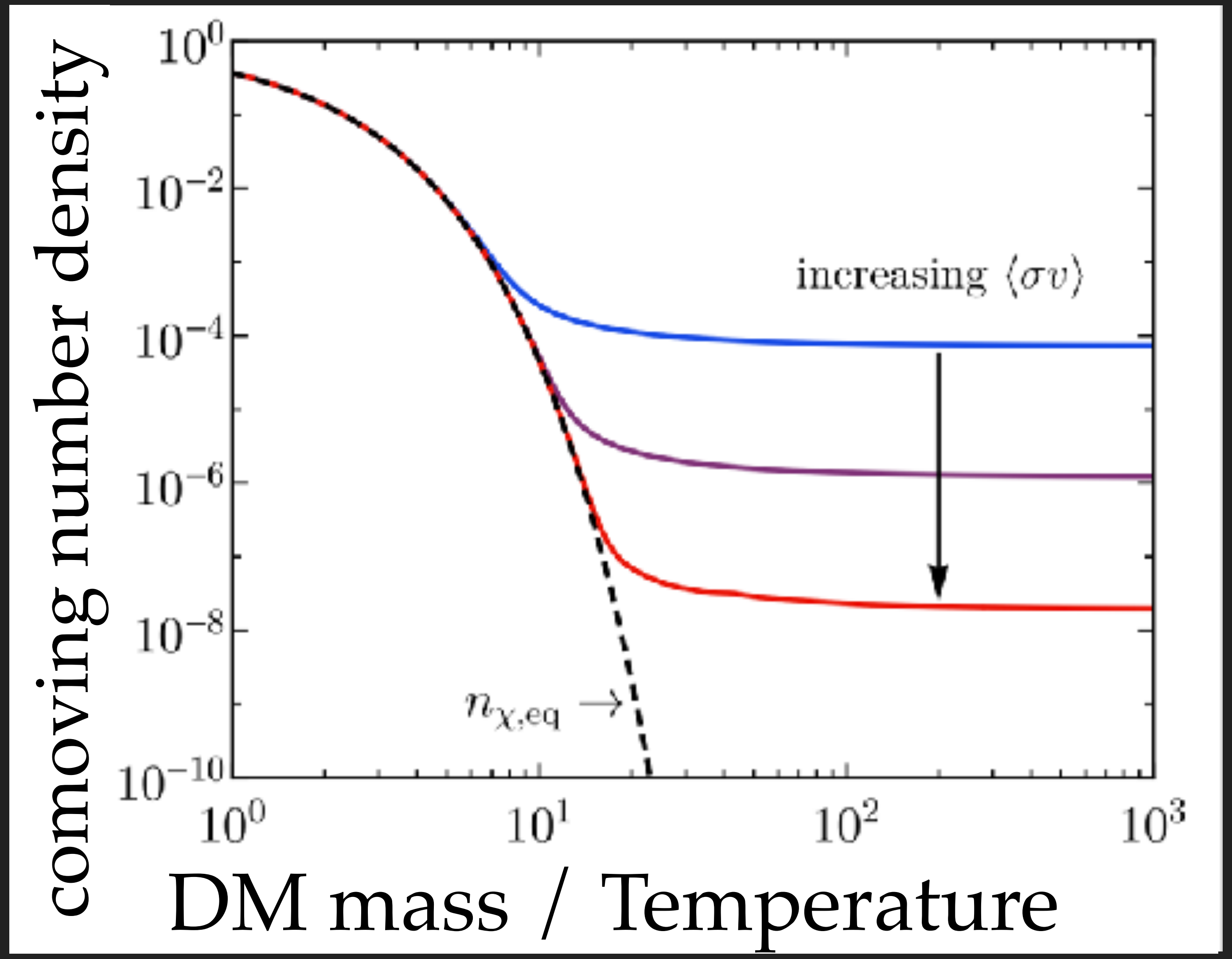
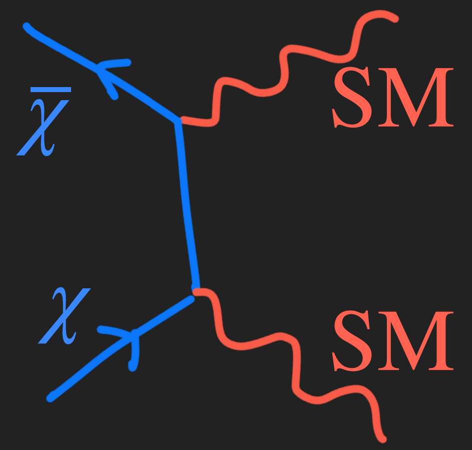
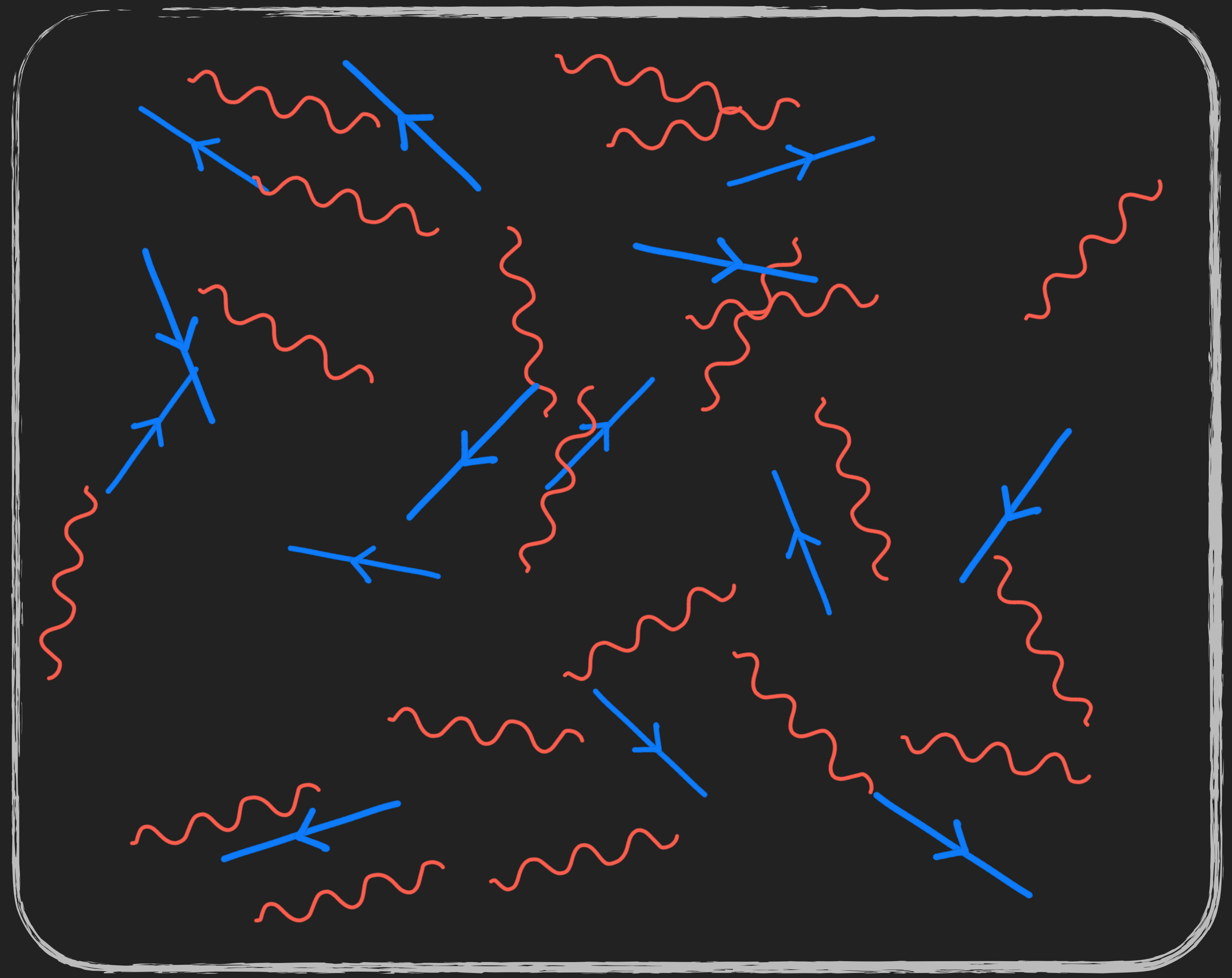


DARK MATTER PRODUCTION MECHANISMS: TARGET FOR SEARCHES



Freeze-Out





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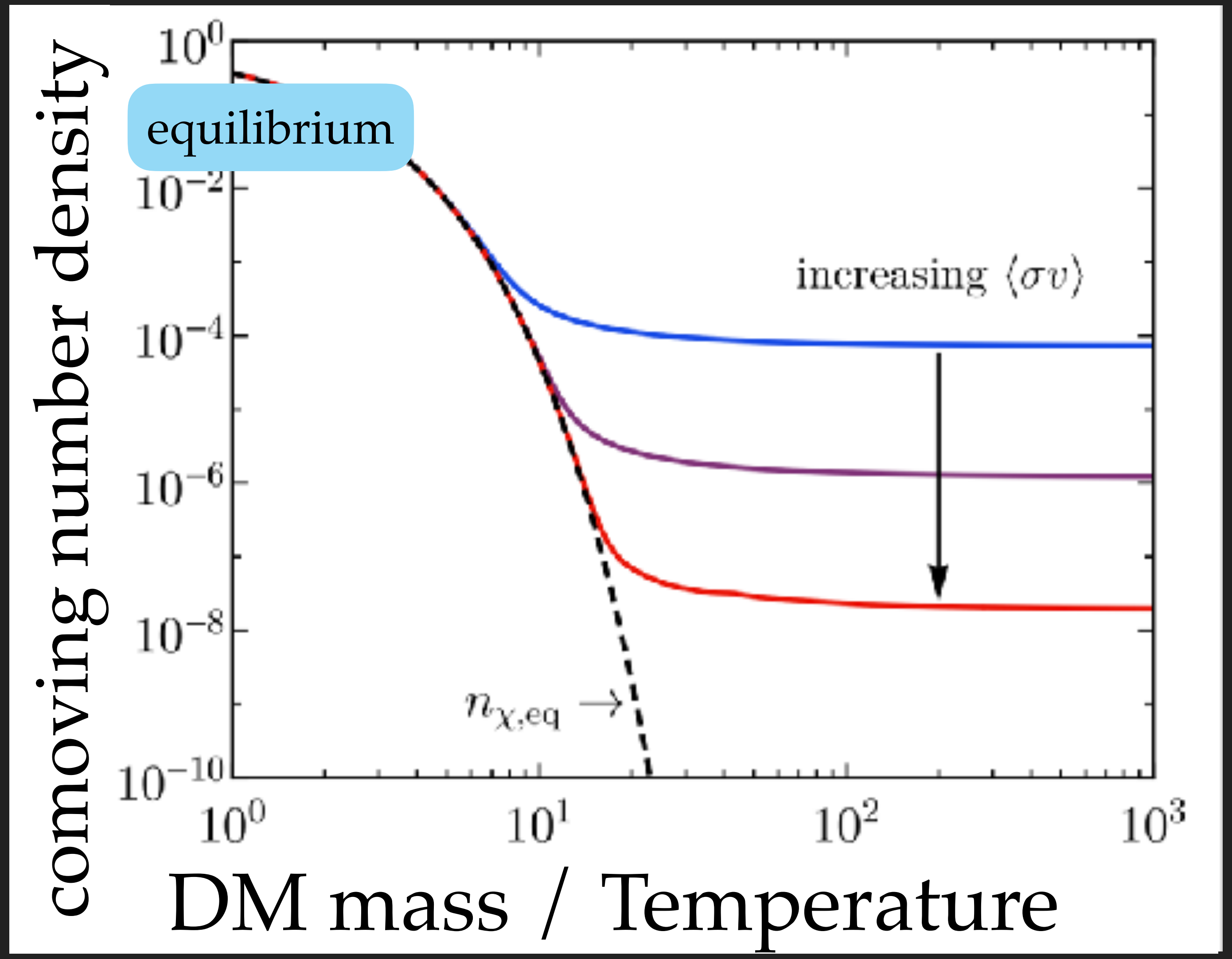
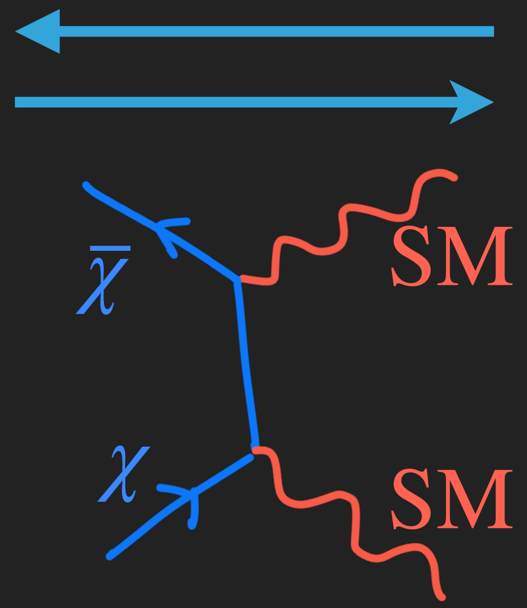
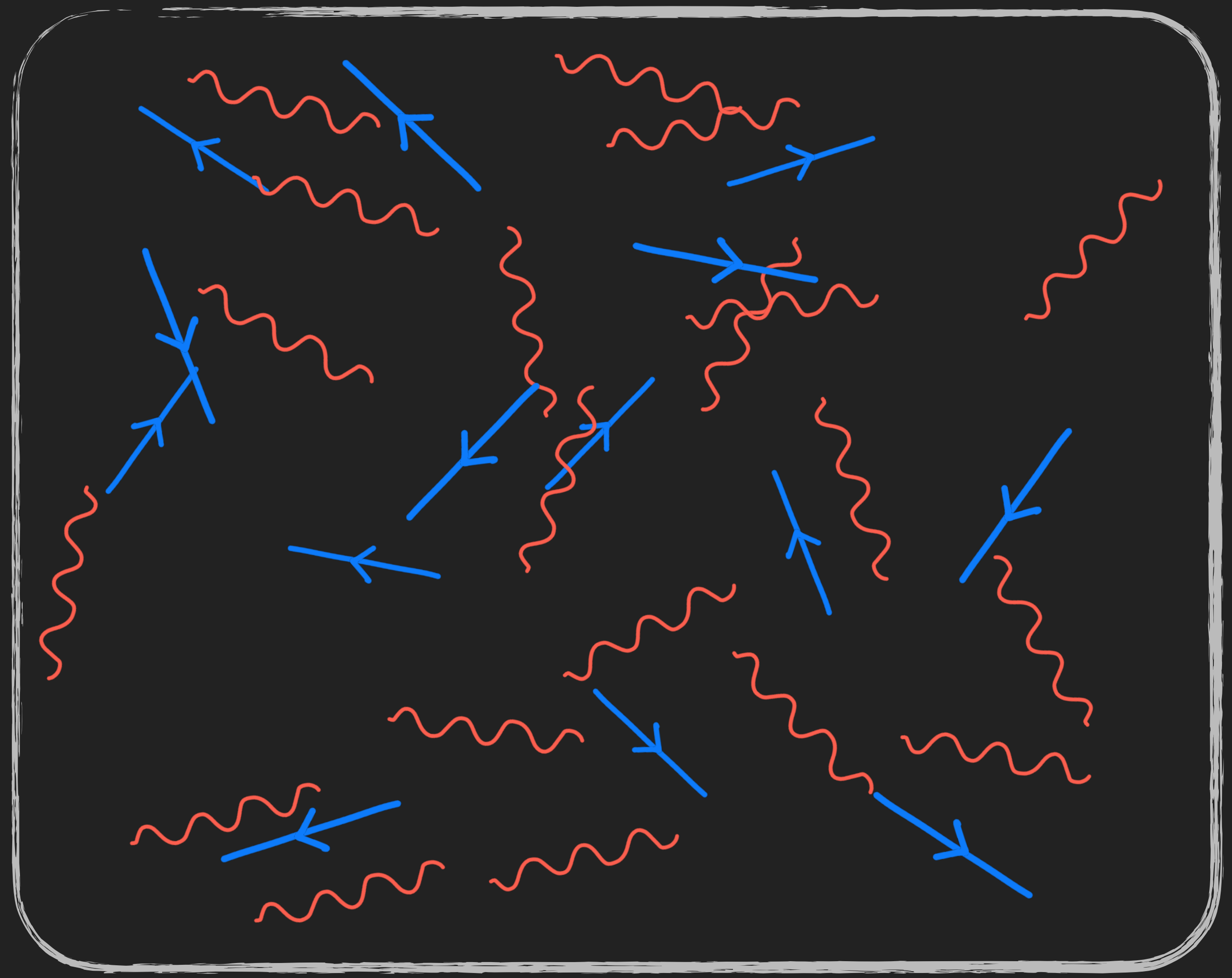
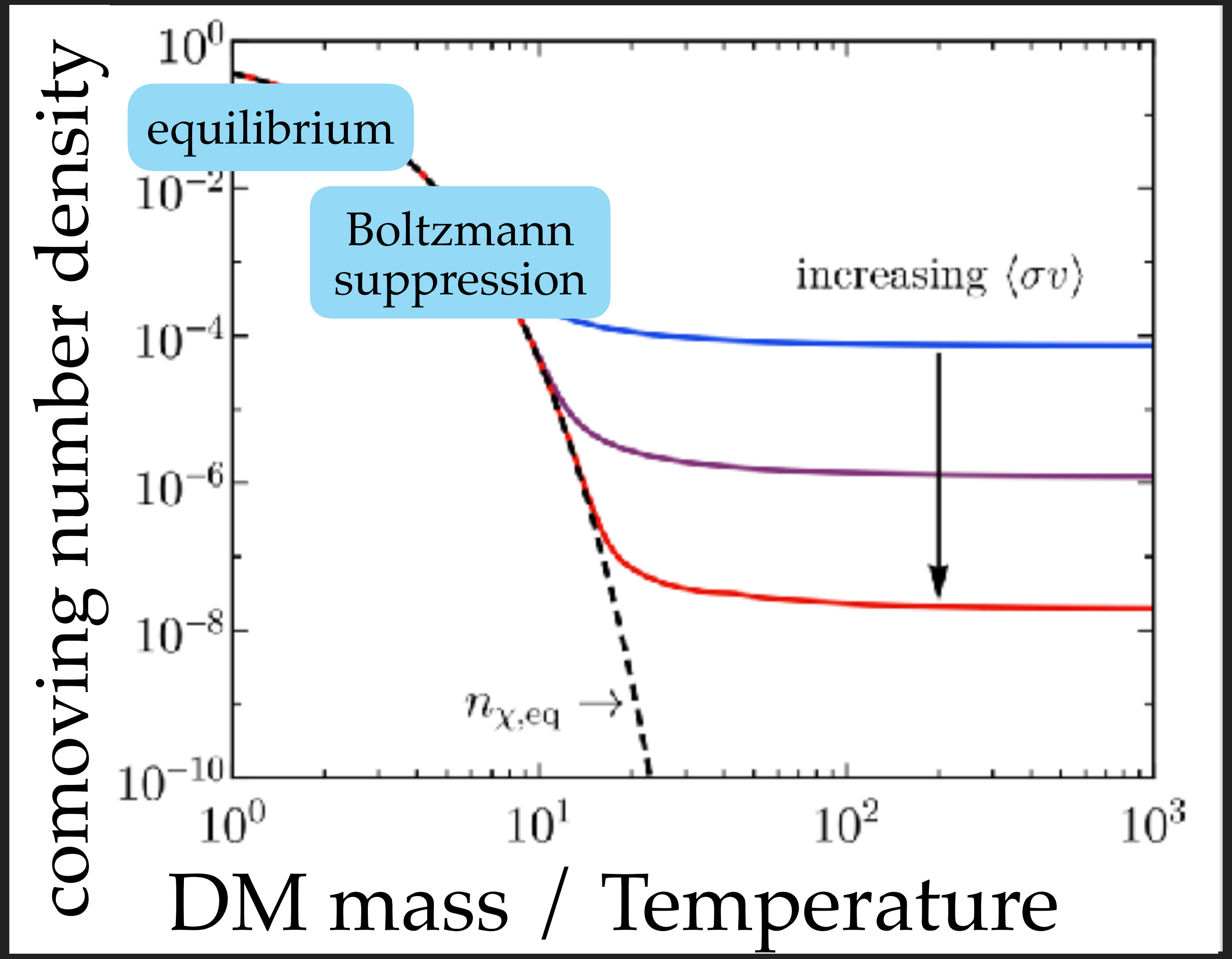


Freeze-Out

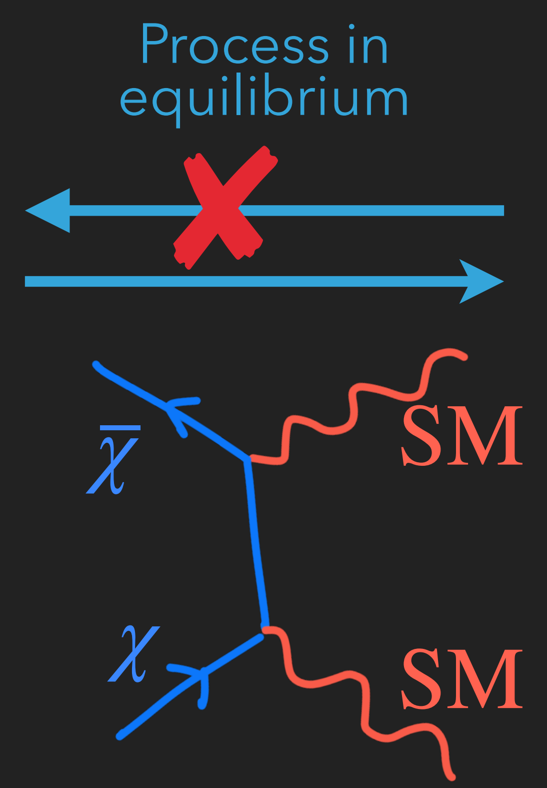
Process in equilibrium

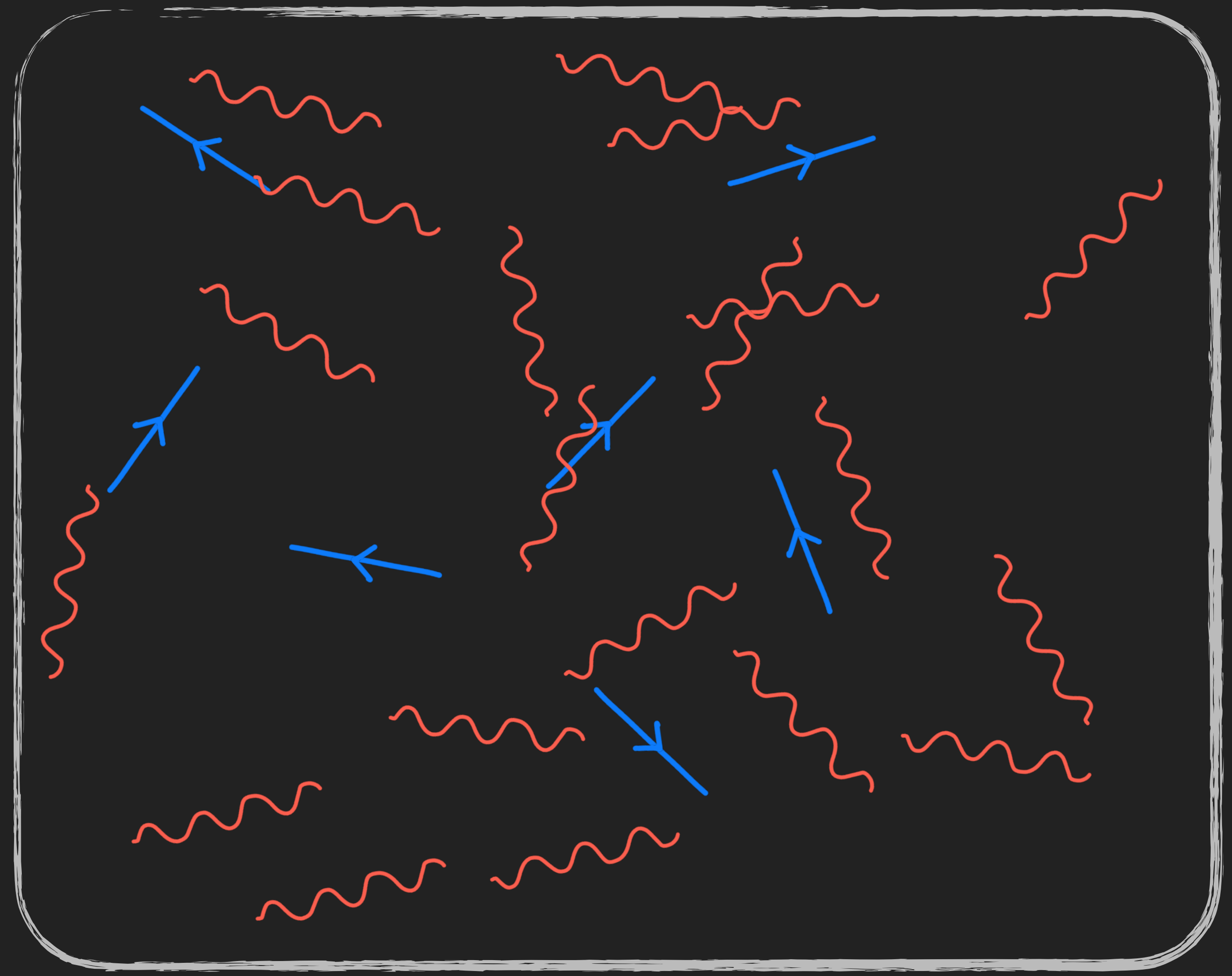


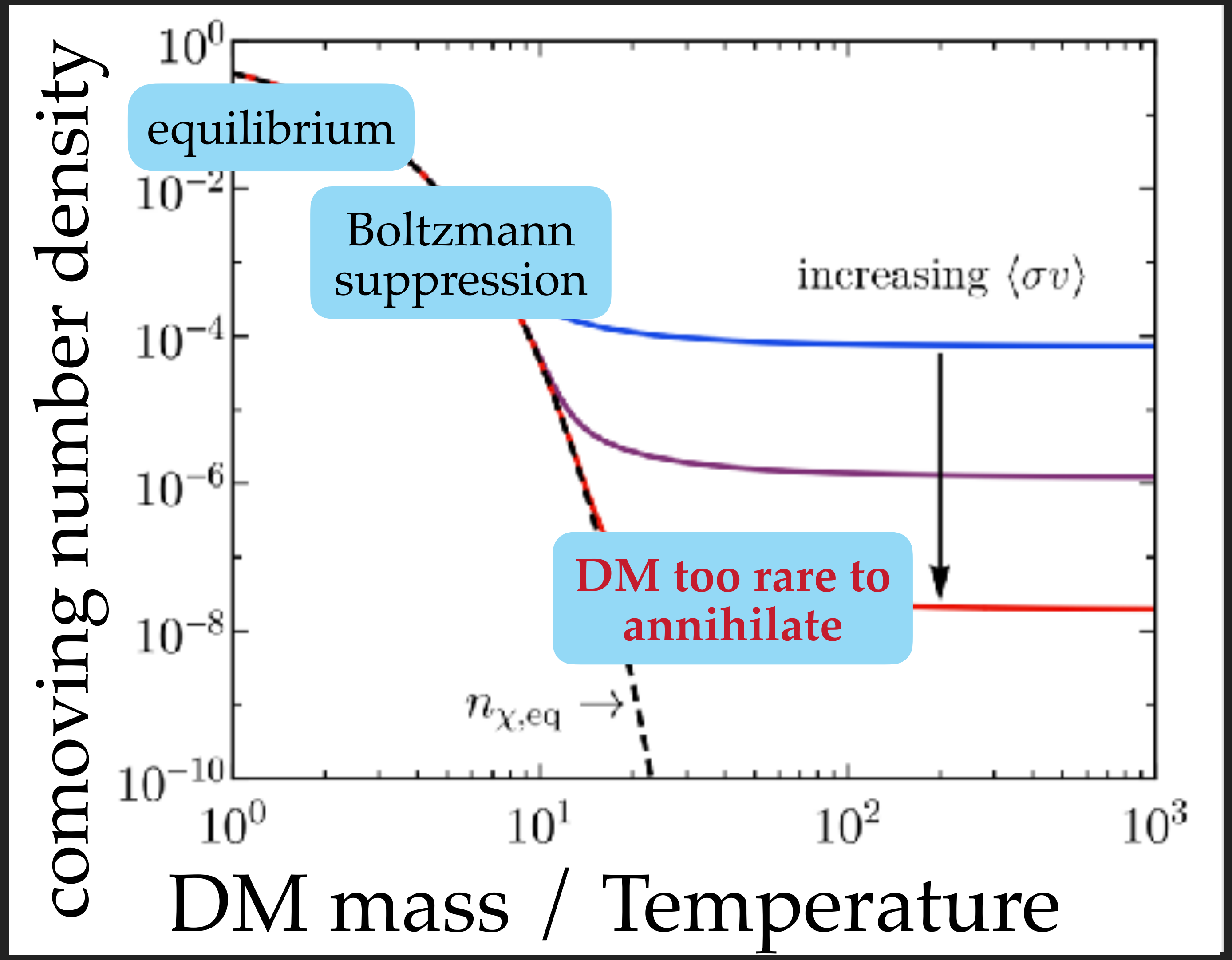



comoving number density
equilibrium
Boltzmann suppression
increasing ⟨σv⟩
$n_{\chi,\mathrm{eq}}$ →
DM mass / Temperature

Freeze-Out


Process in equilibrium
$\bar{\chi}$
χ
SM
SM





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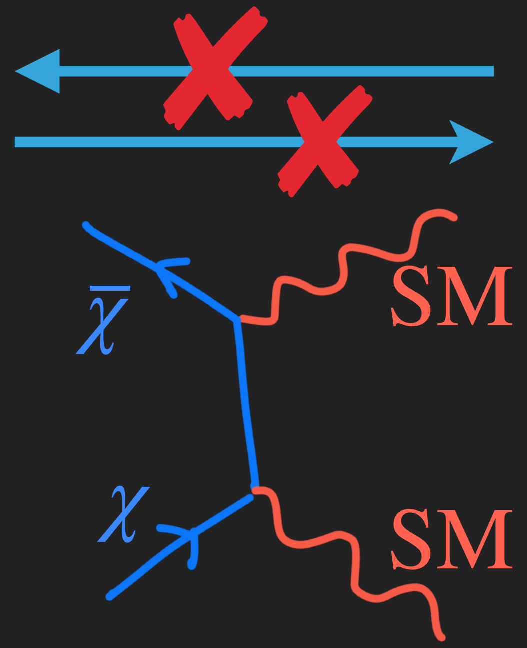

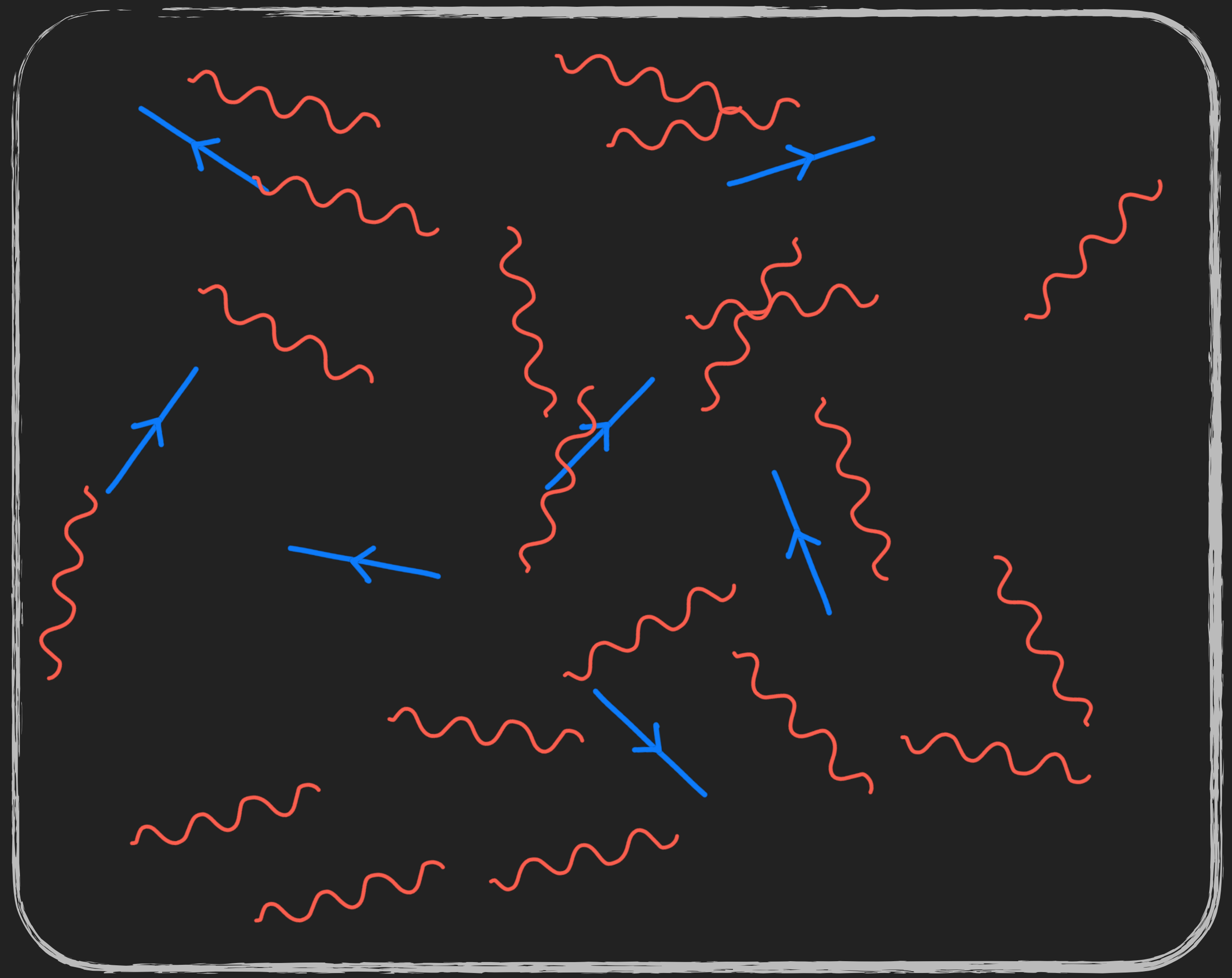

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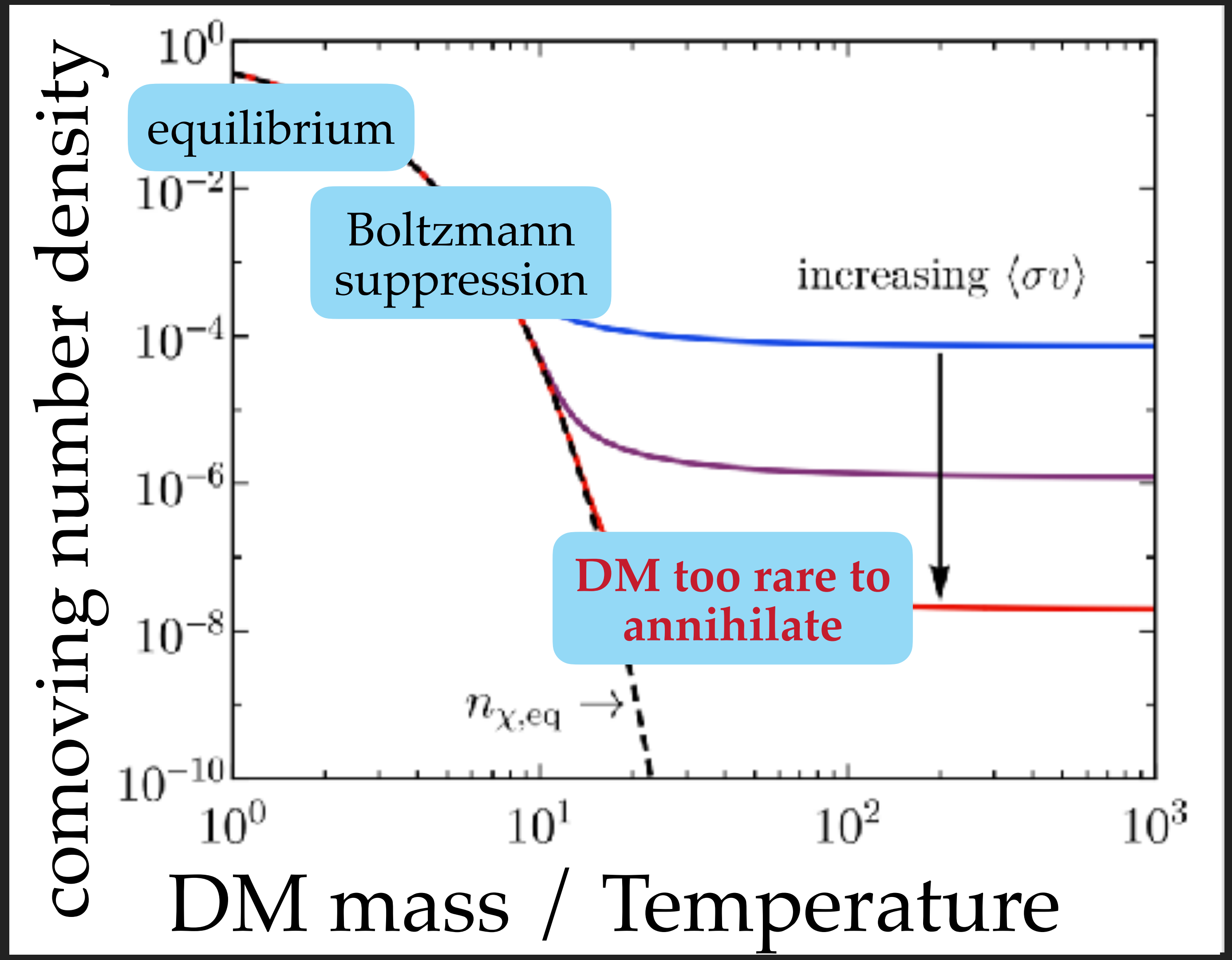

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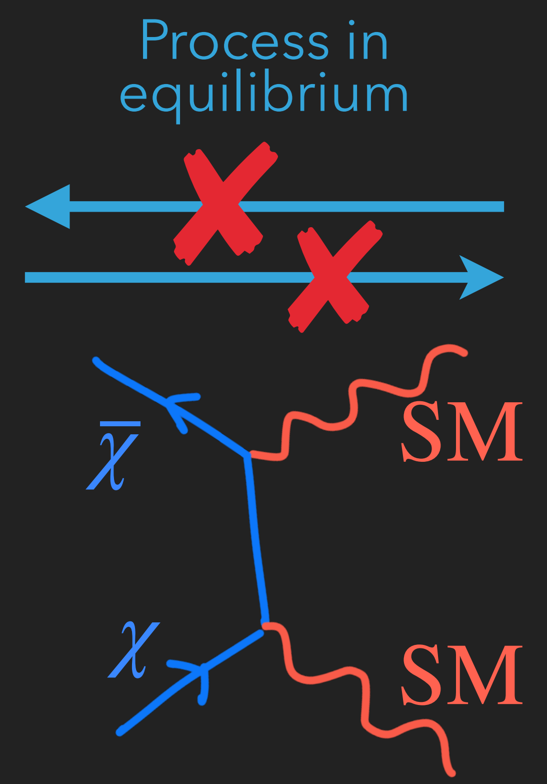
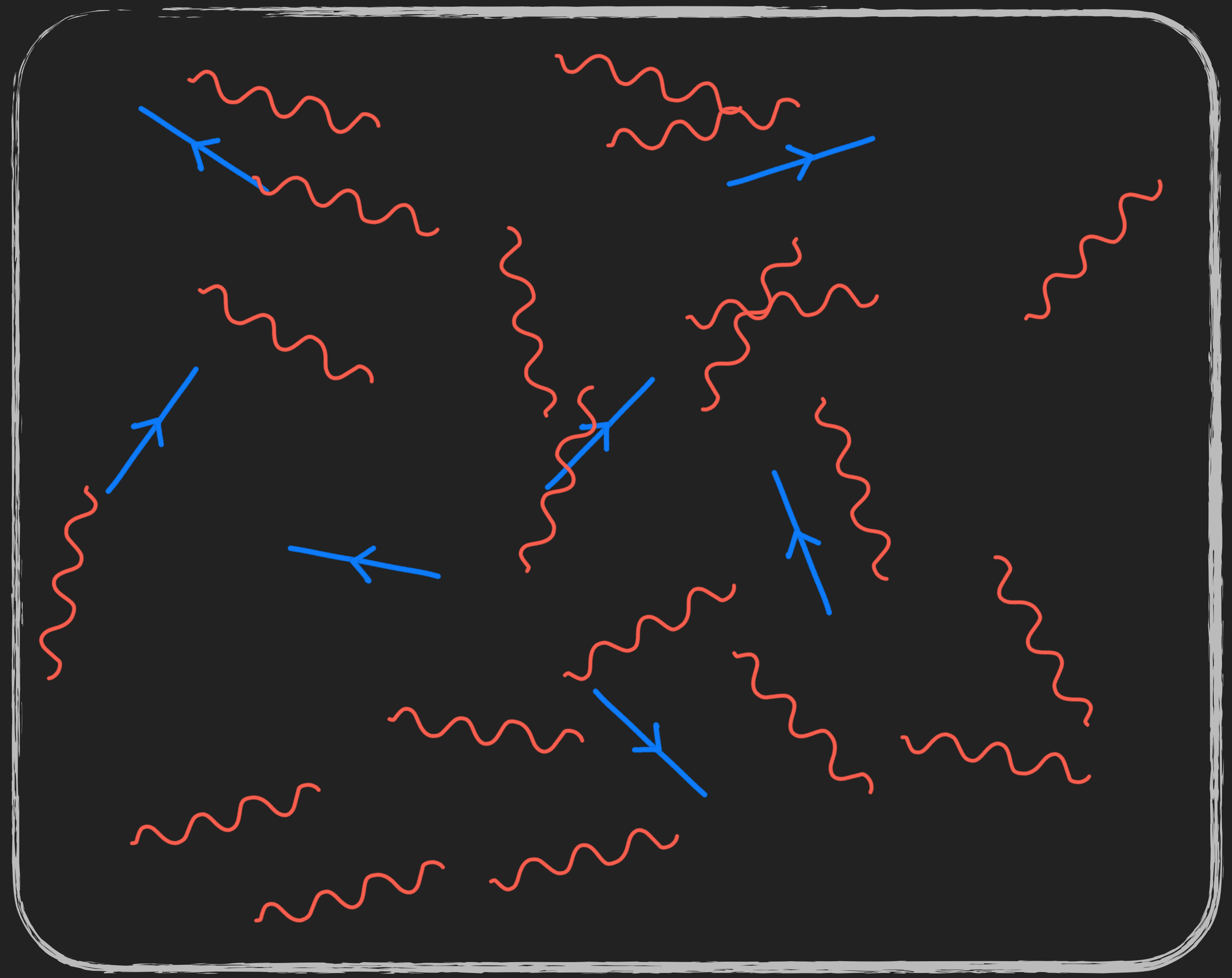
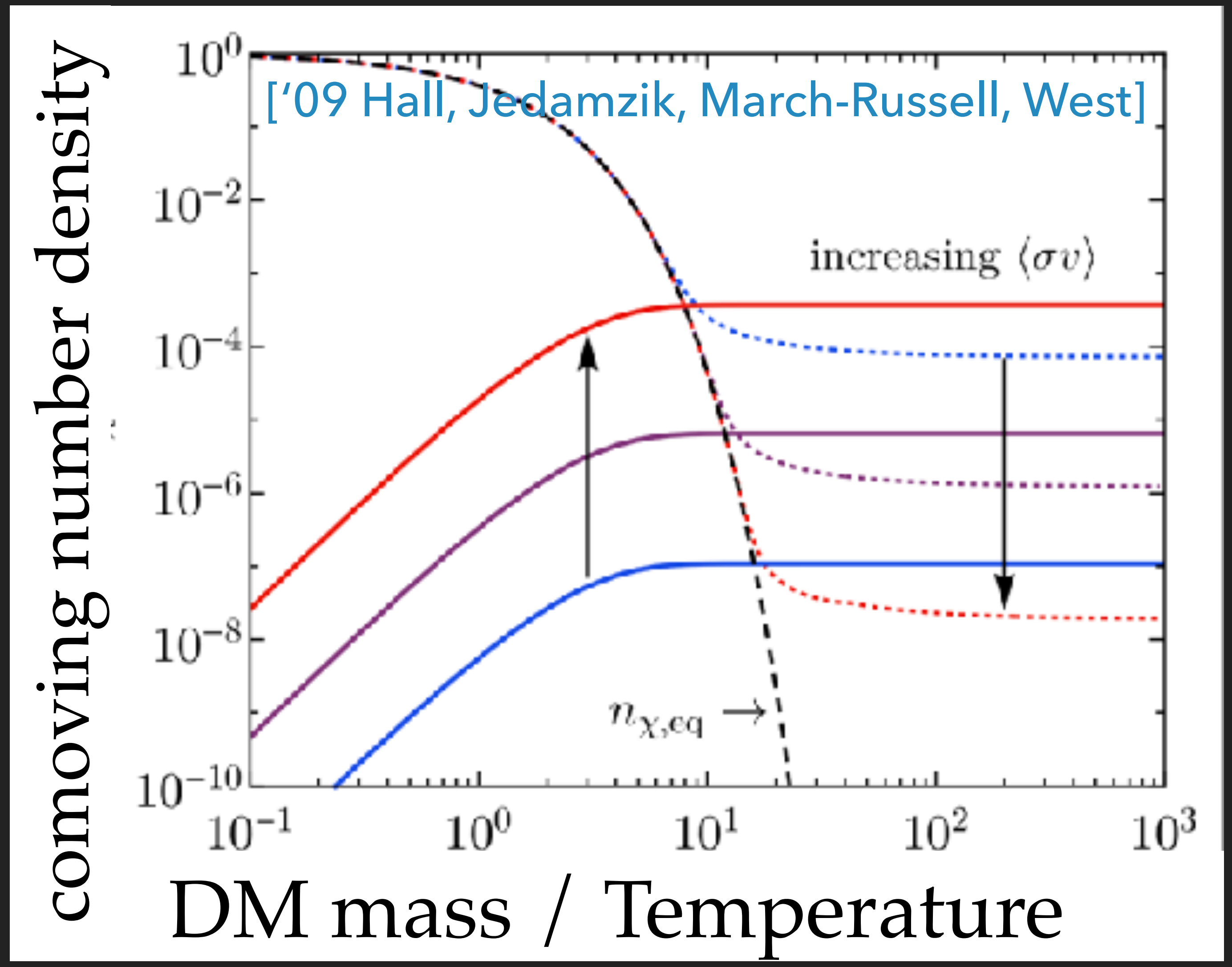
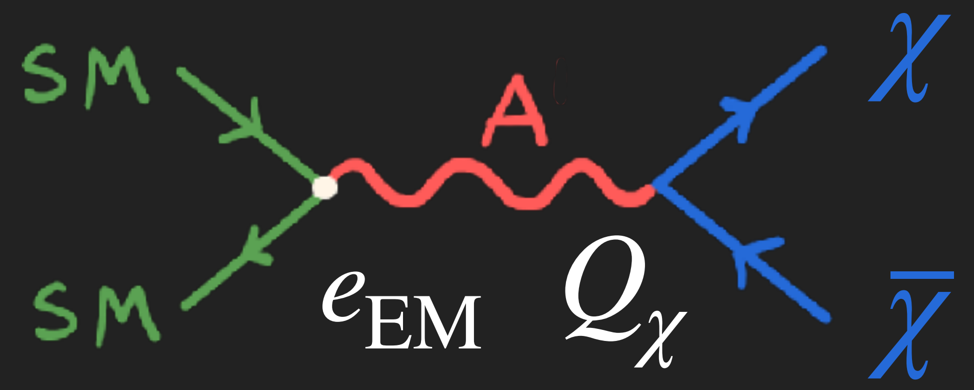






Freeze-In

- Particle with small coupling



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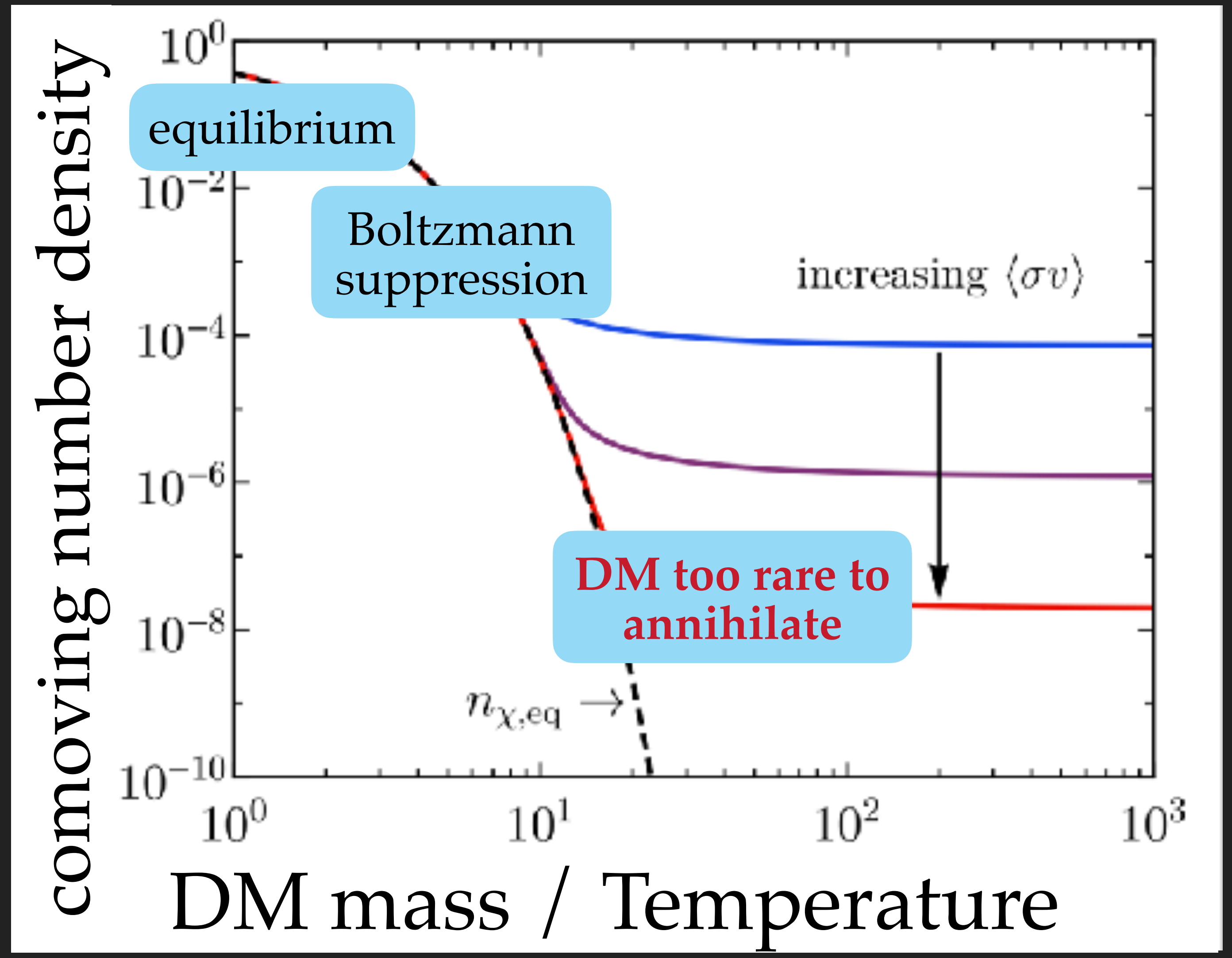

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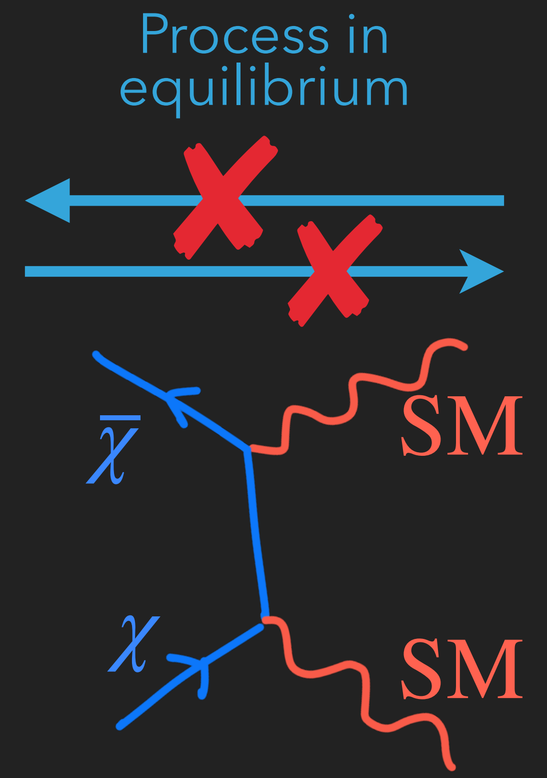

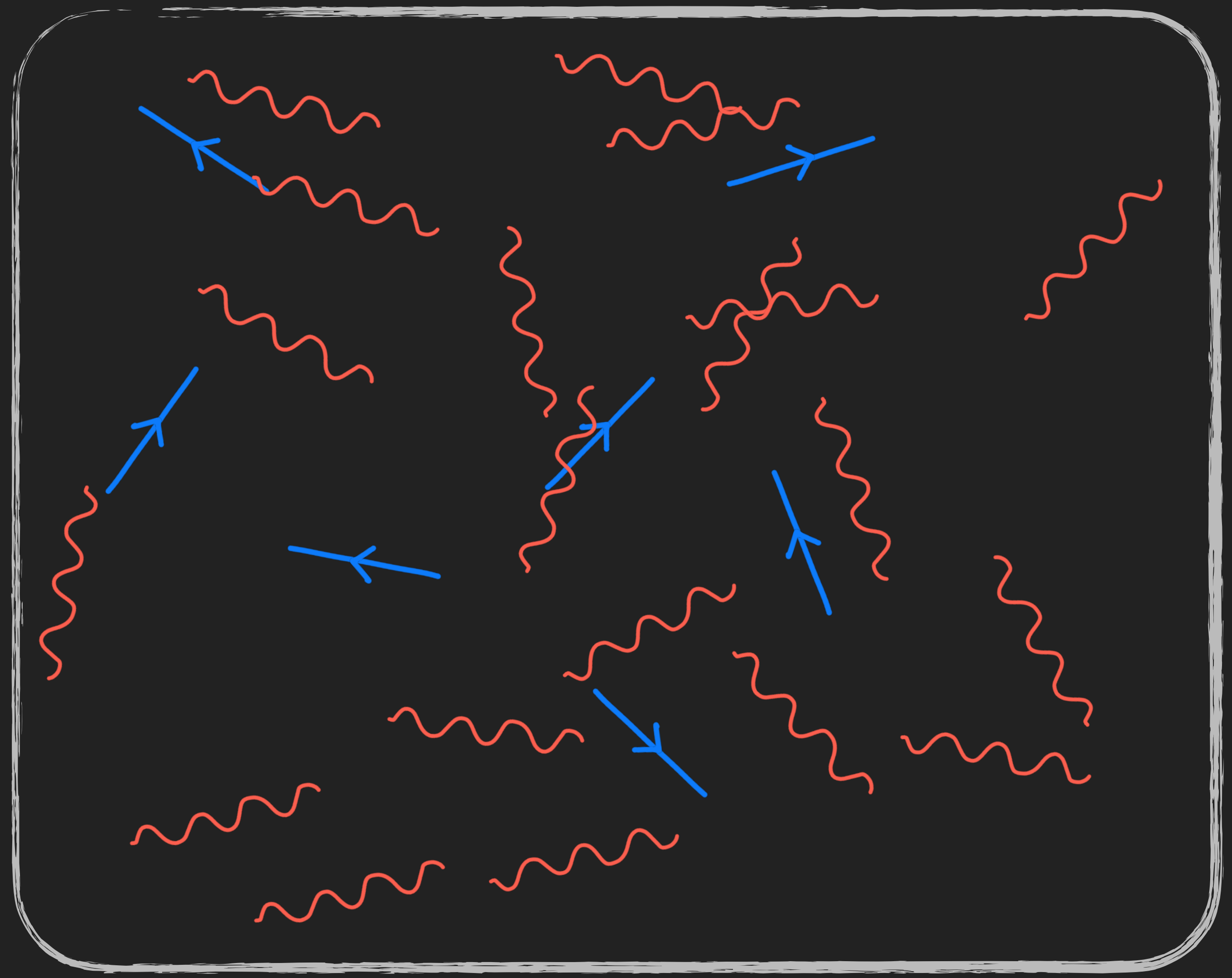

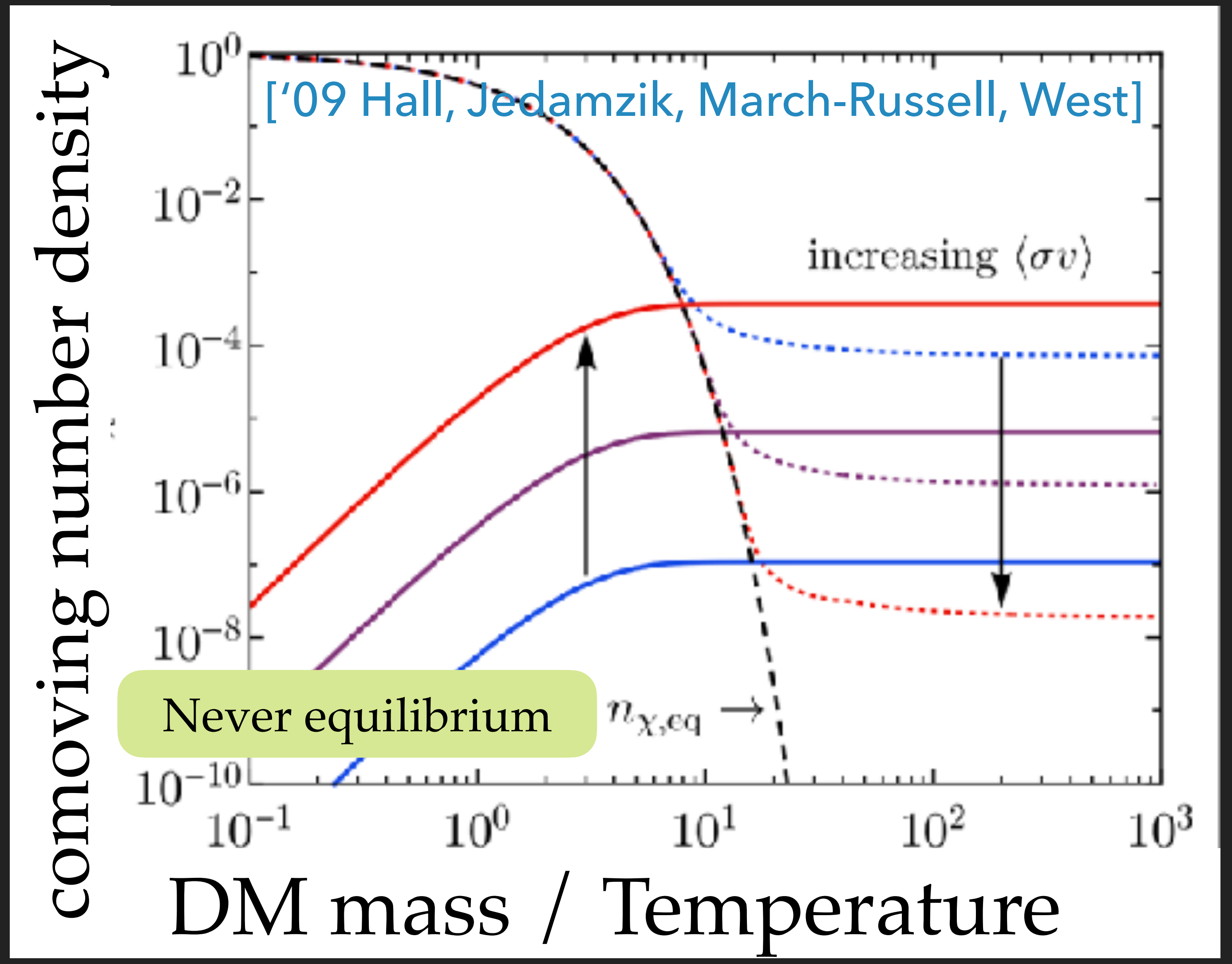

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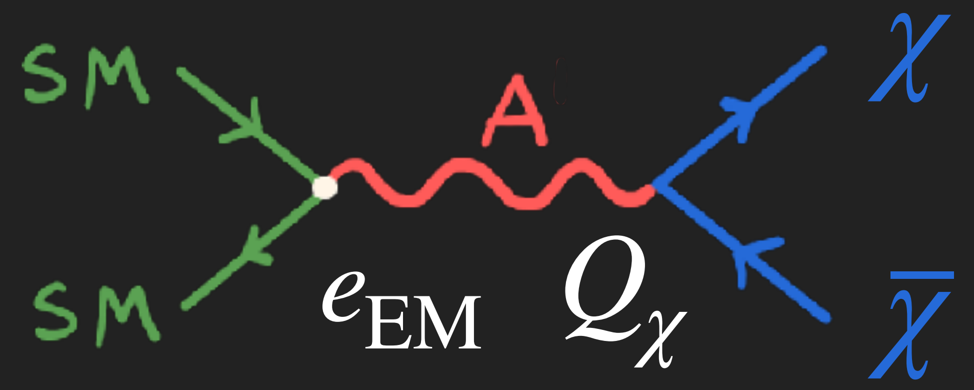

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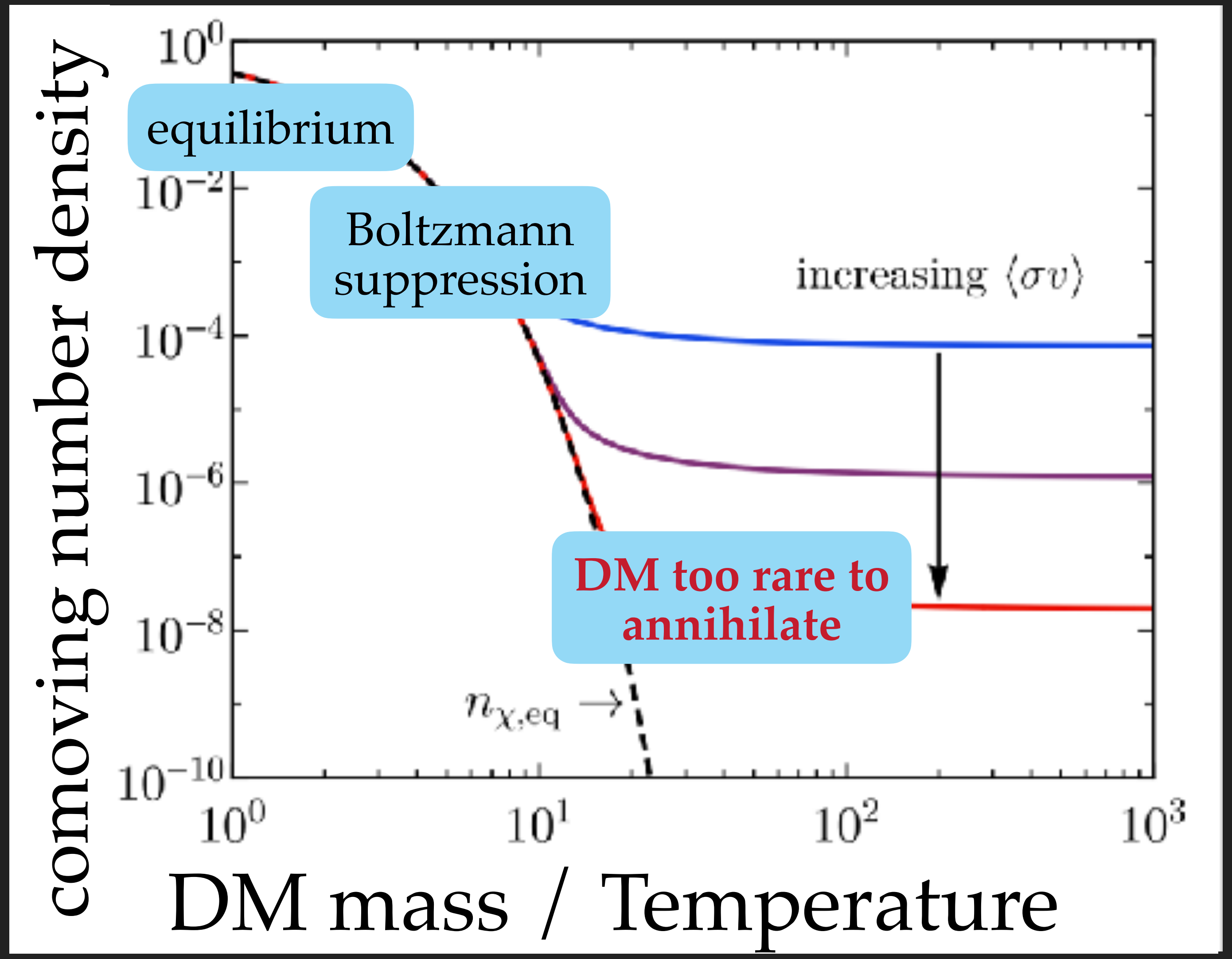
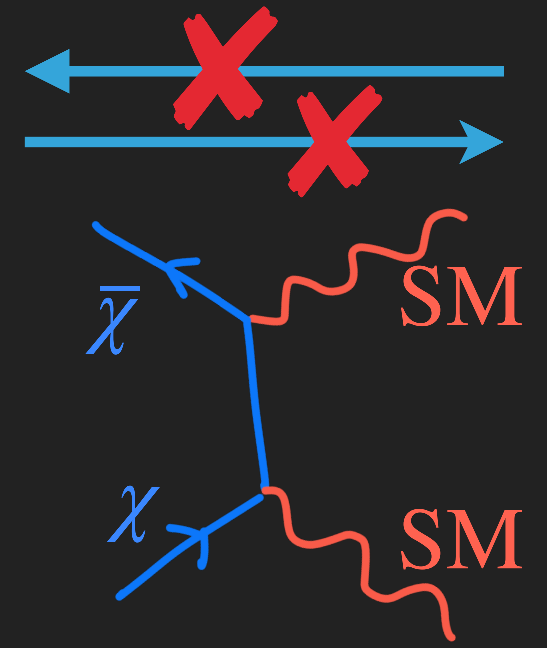
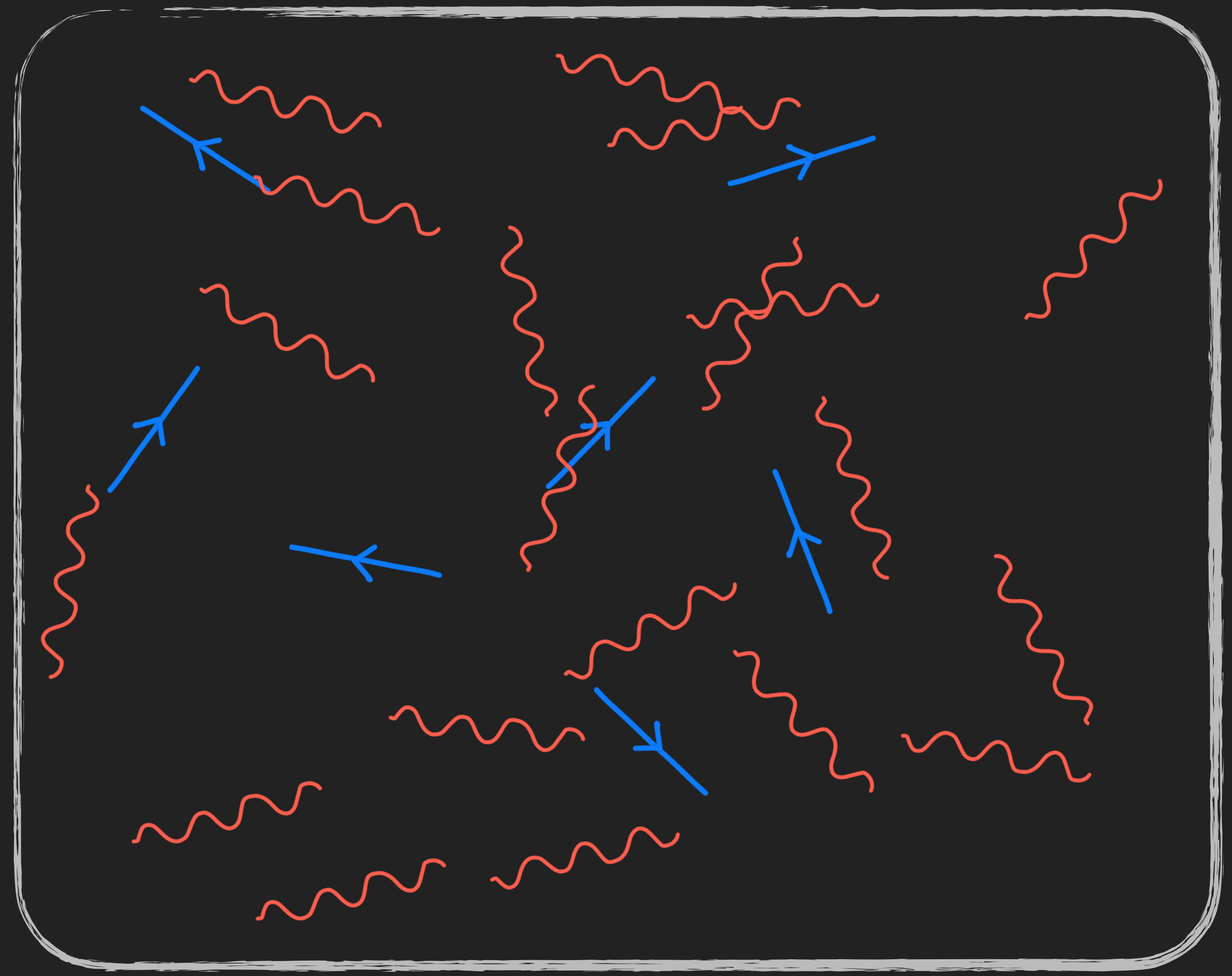
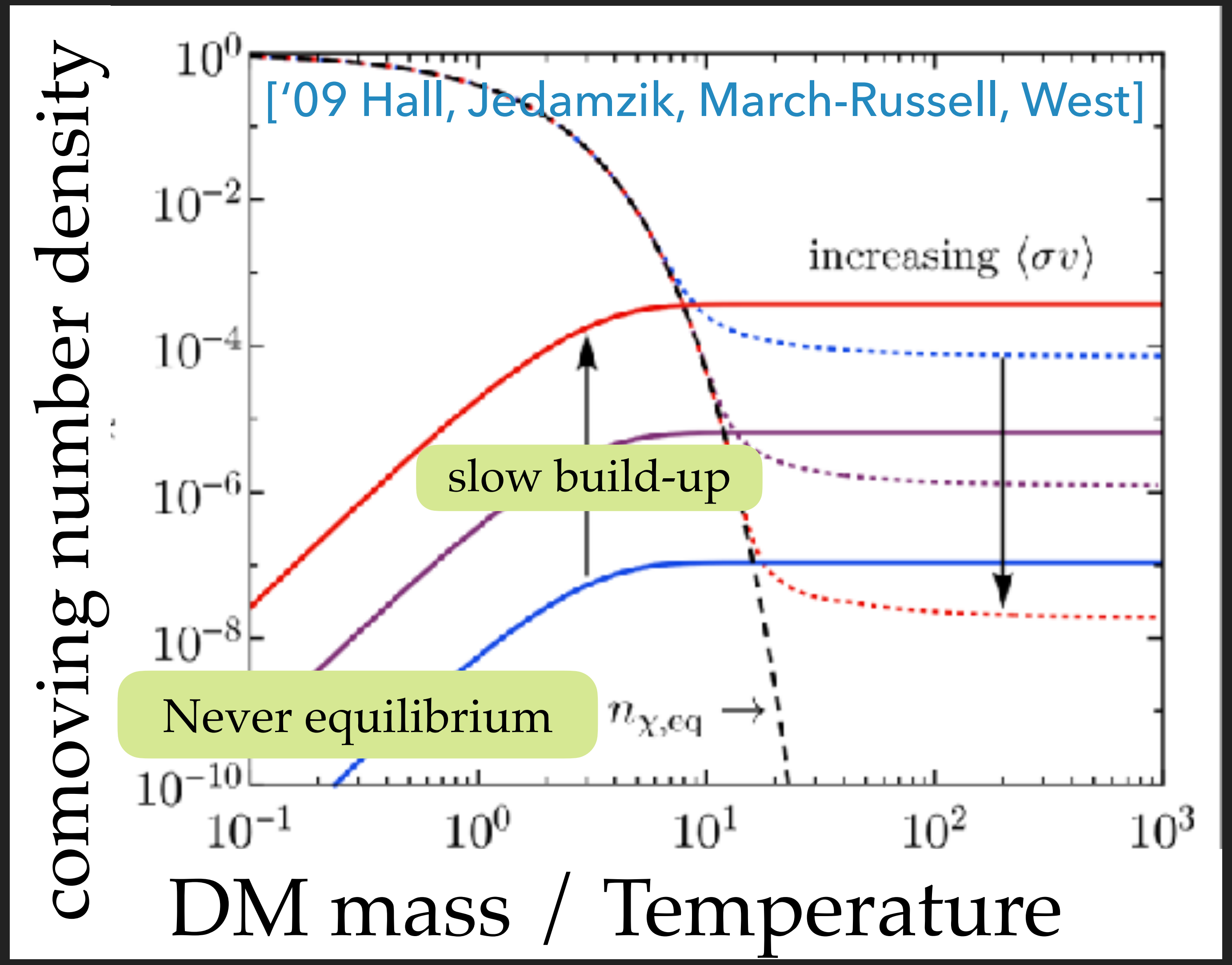


Freeze-Out

Process in equilibrium







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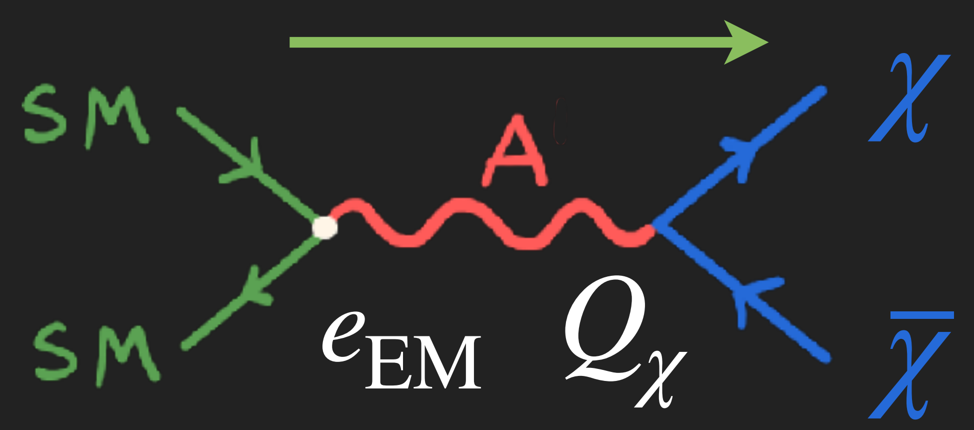

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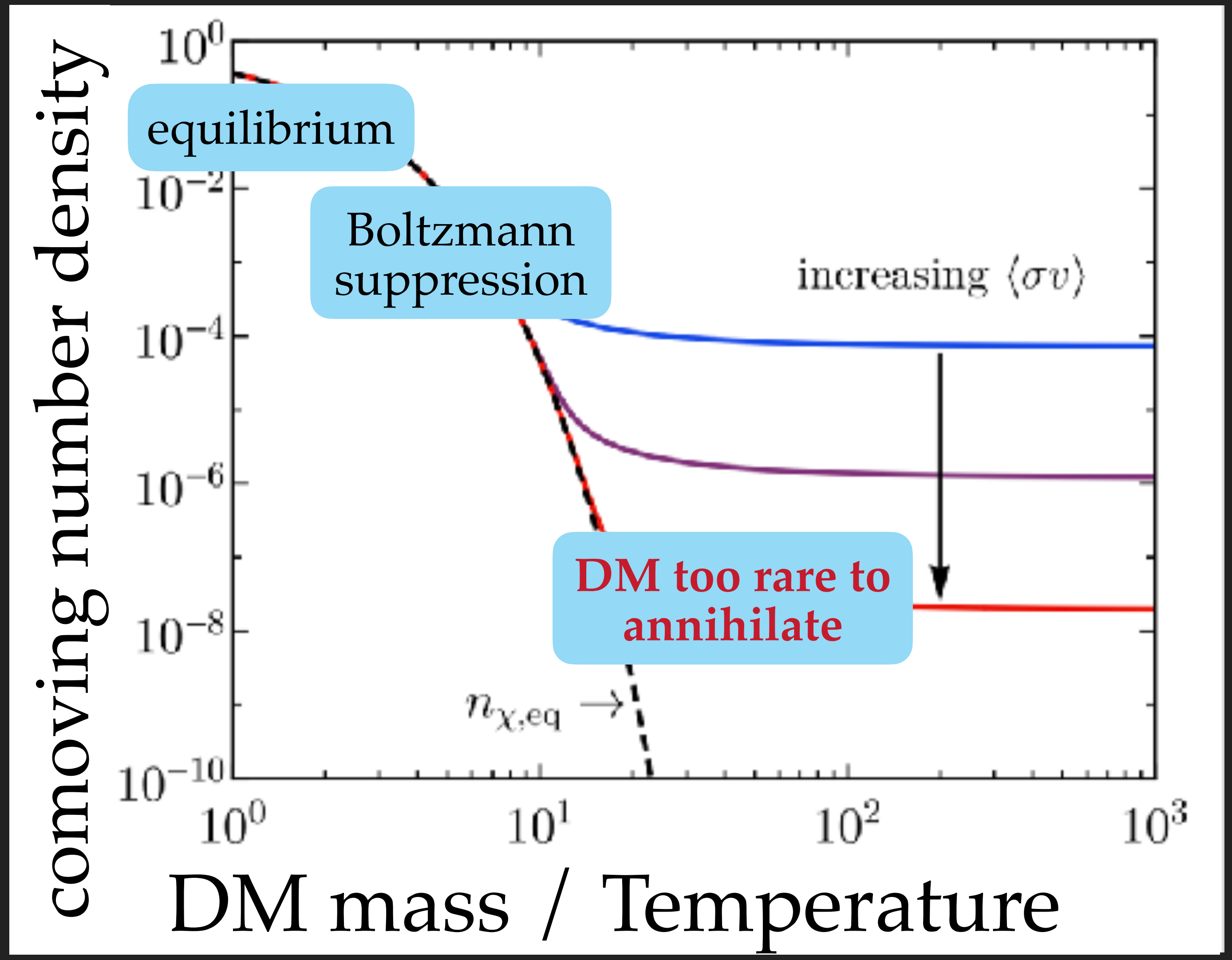

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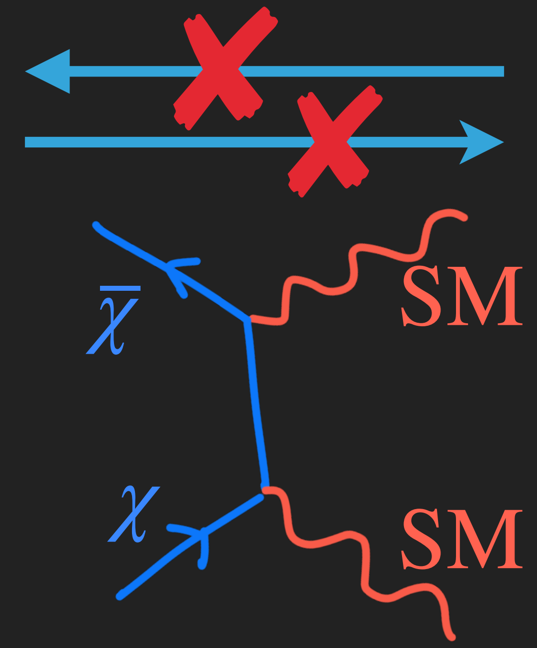

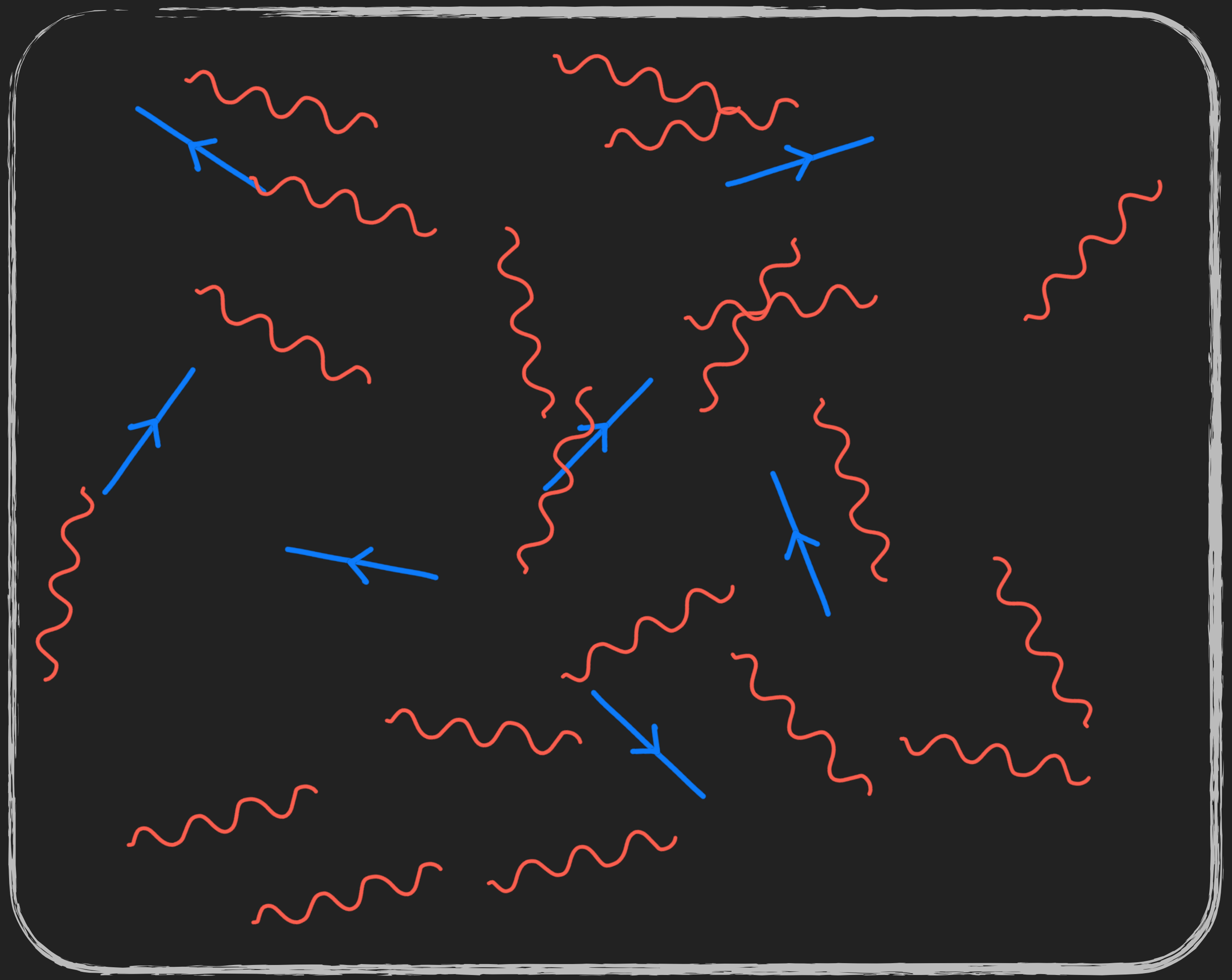

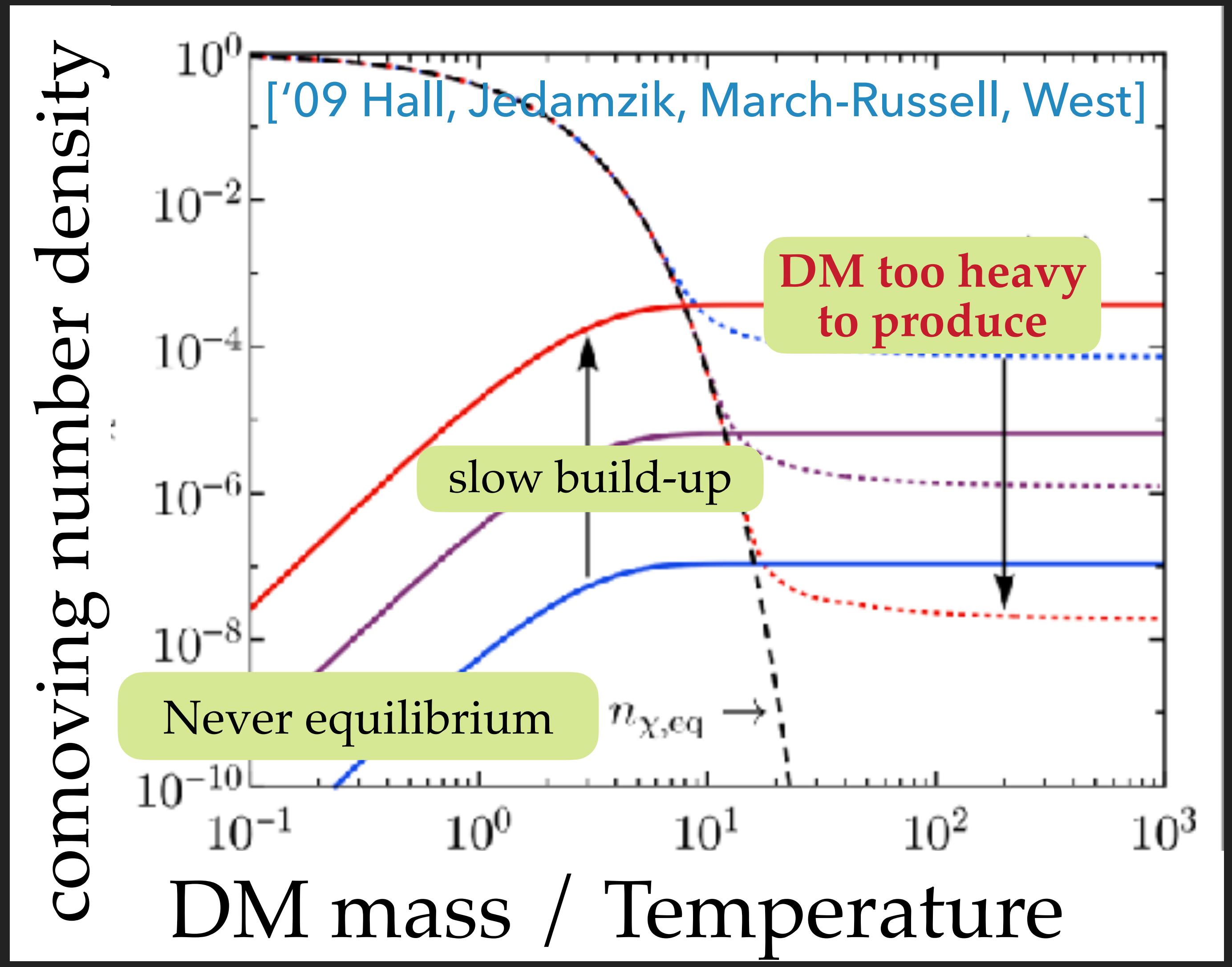

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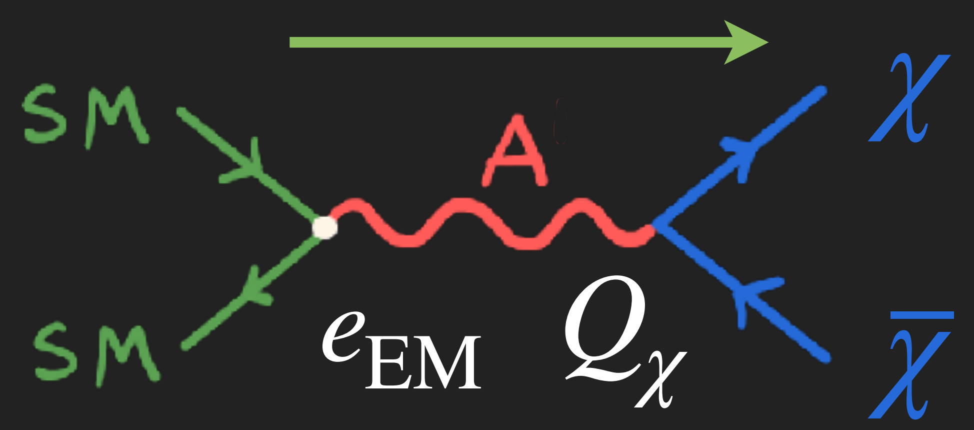

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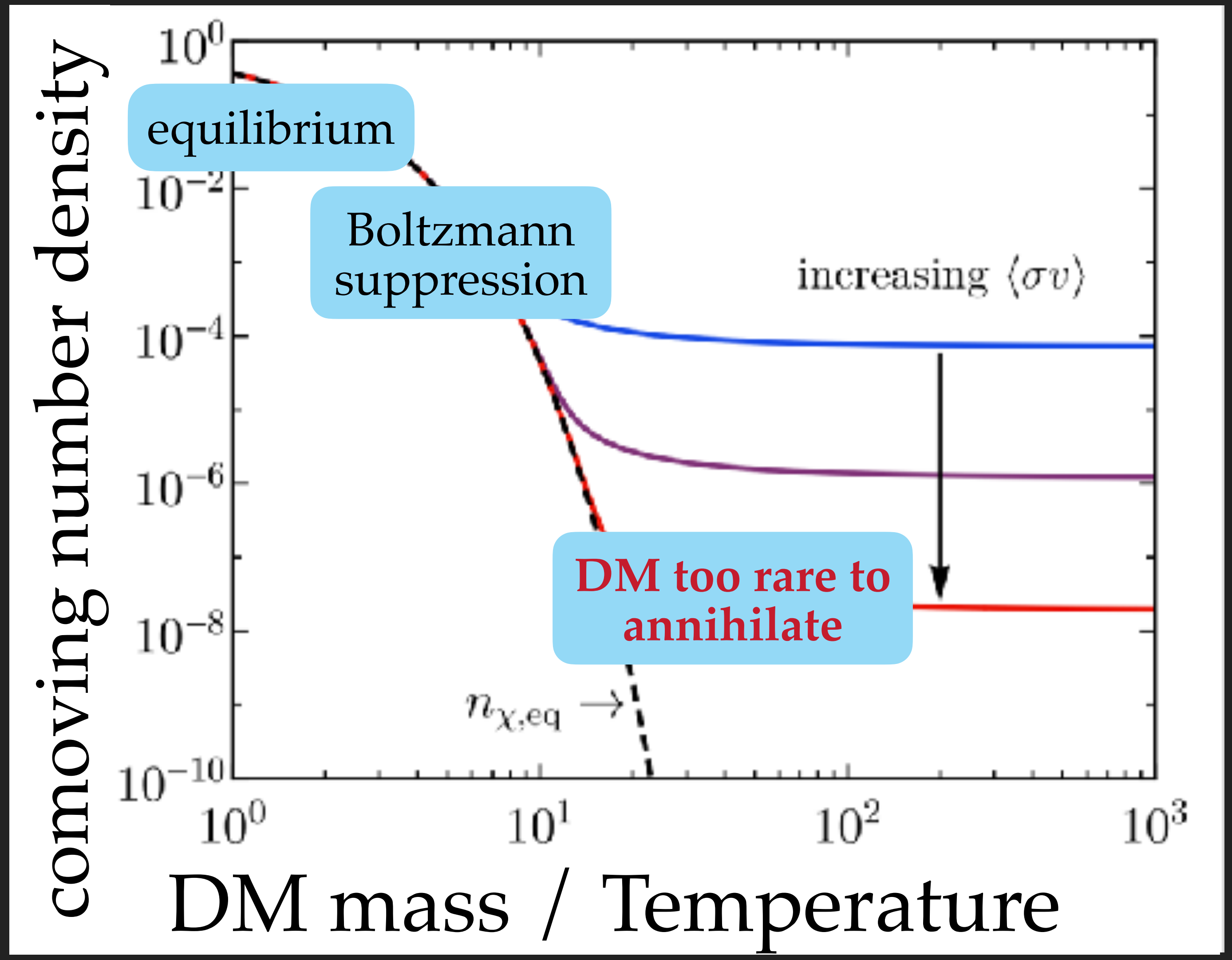

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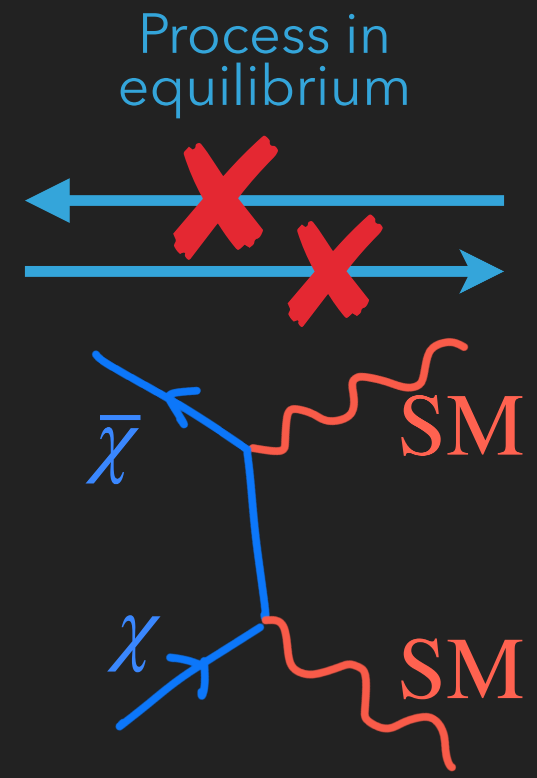

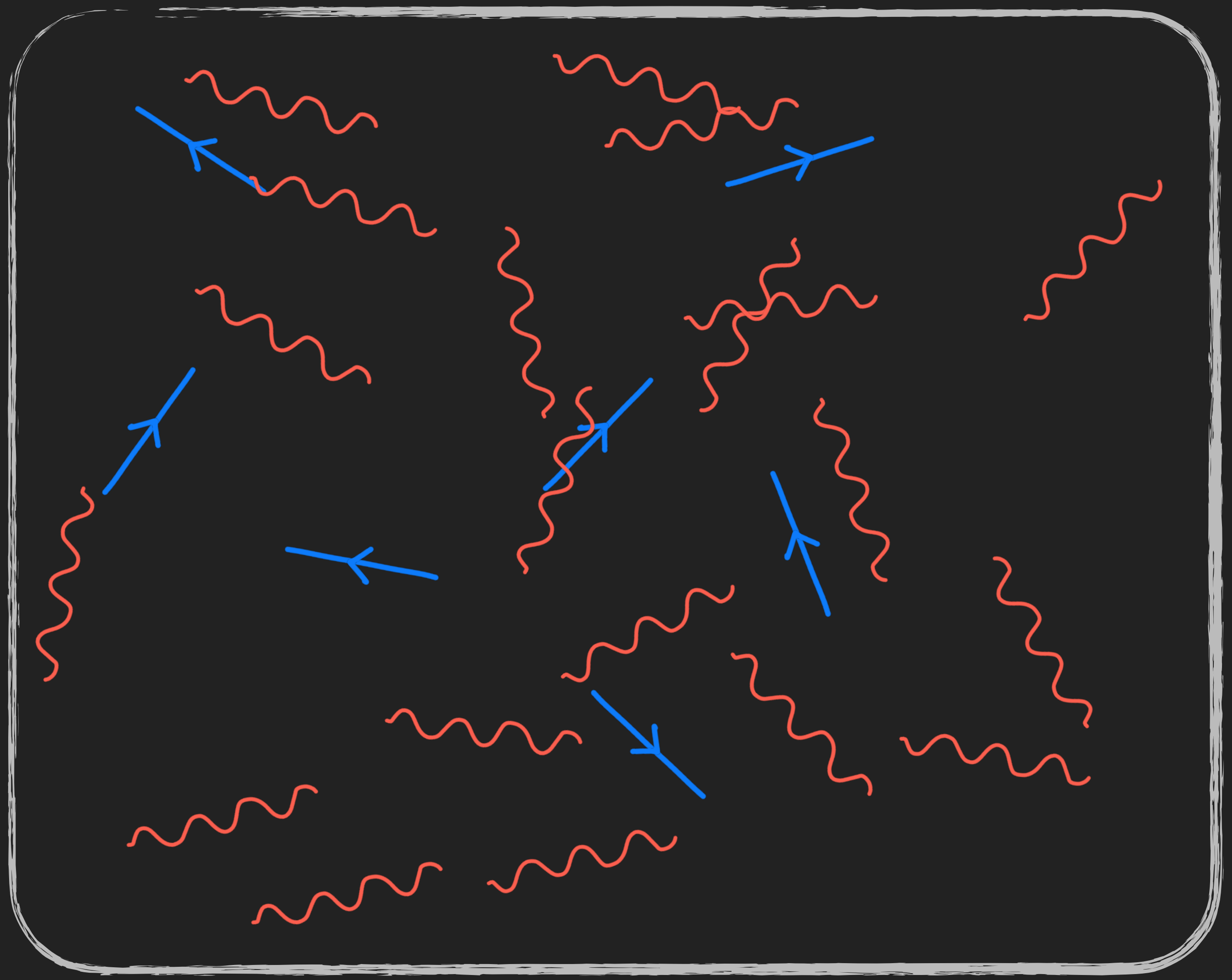

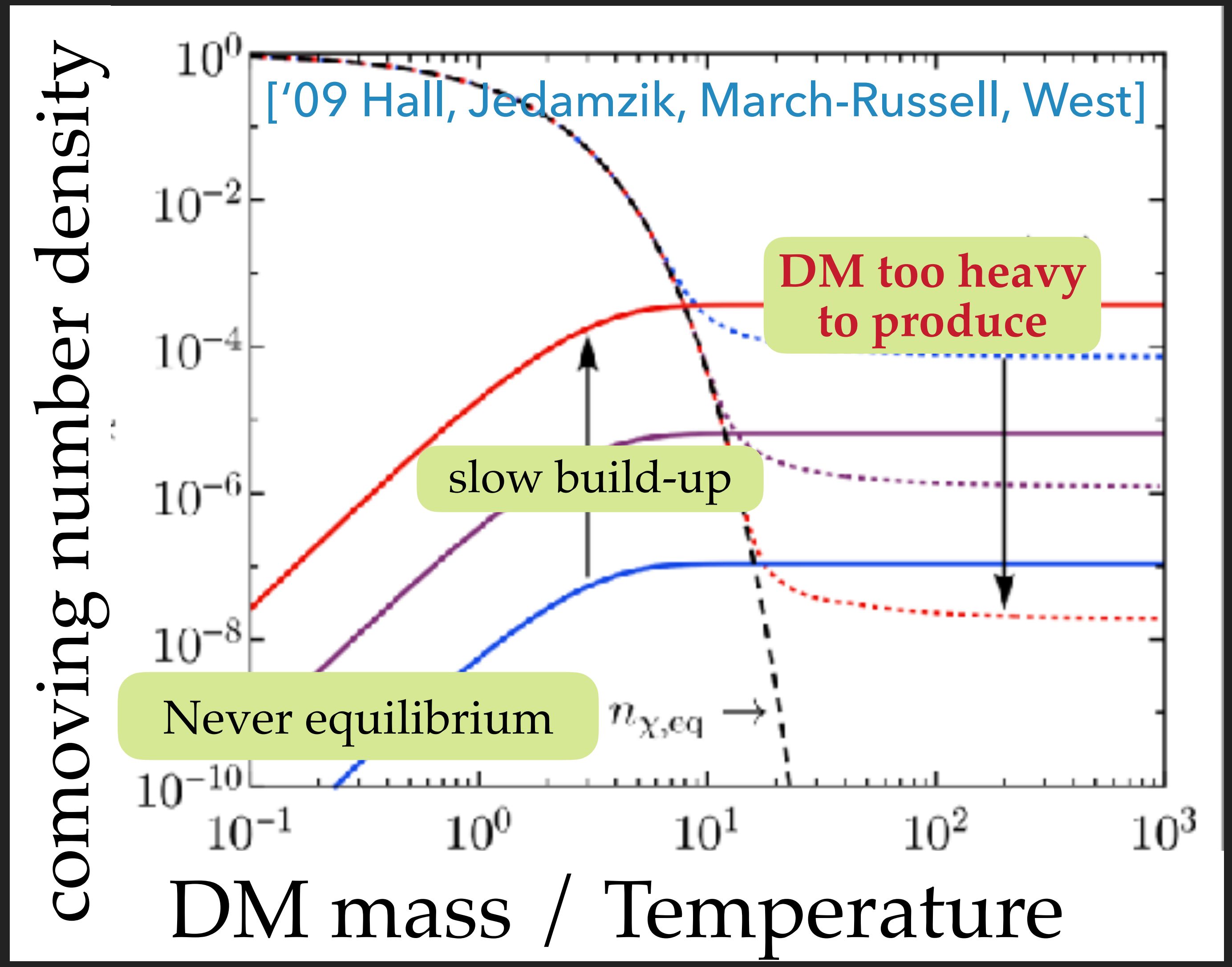

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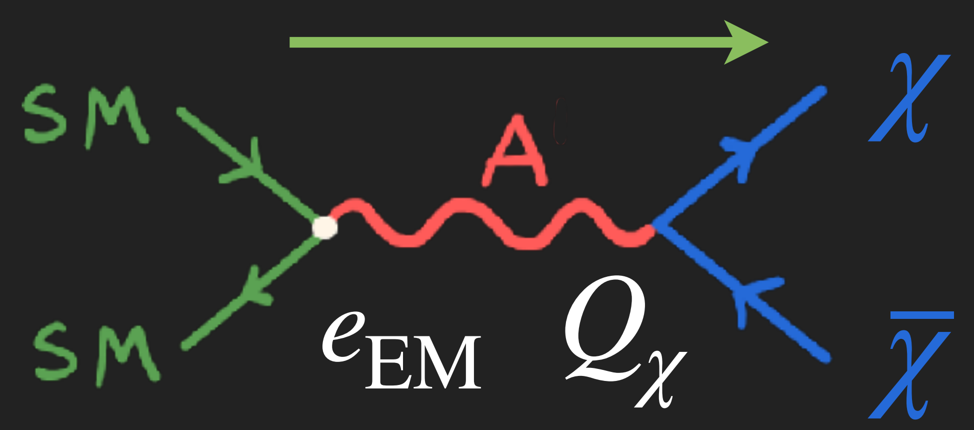
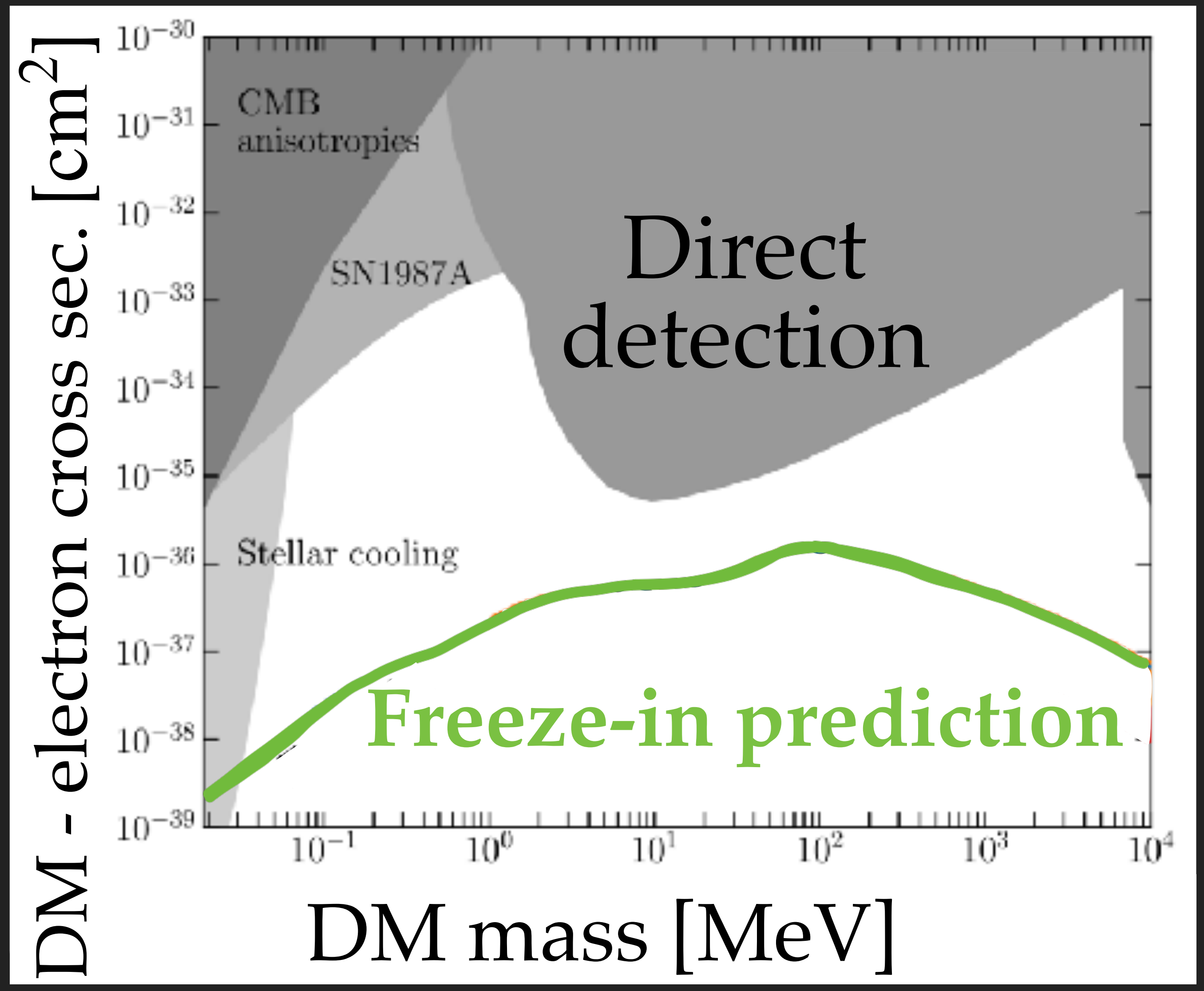
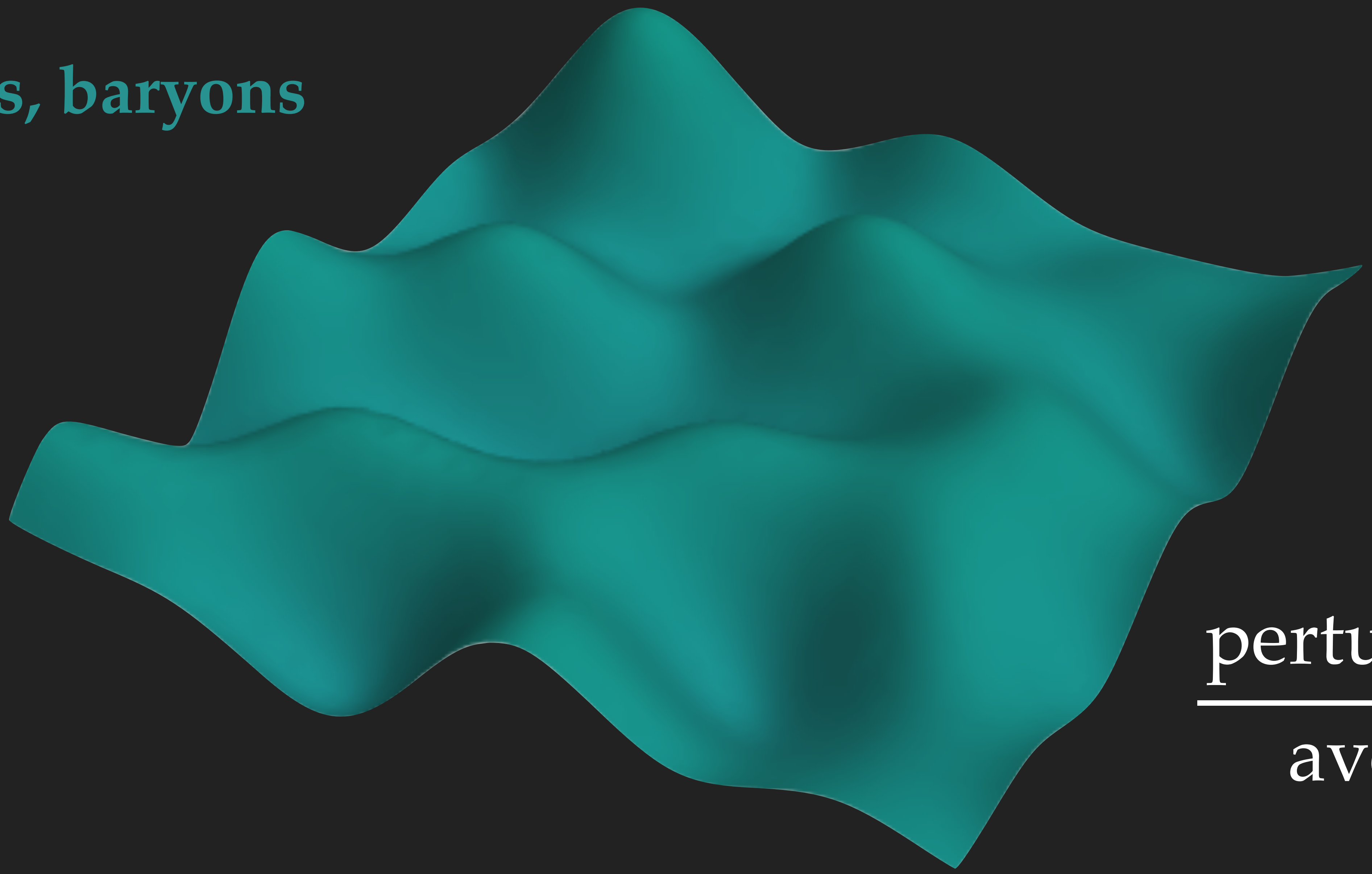




- DM and SM never in equilibrium. Do they share same perturbations? ['22 Bellomo, Berghaus, Boddy]

Photons, baryons



$$\frac{\text{perturbation}}{\text{average}} \to \frac{\delta\rho(t,\mathbf{x})}{\rho(t)}$$

FREEZE-IN AND COSMOLOGICAL PERTURBATIONS ON LARGE SCALES

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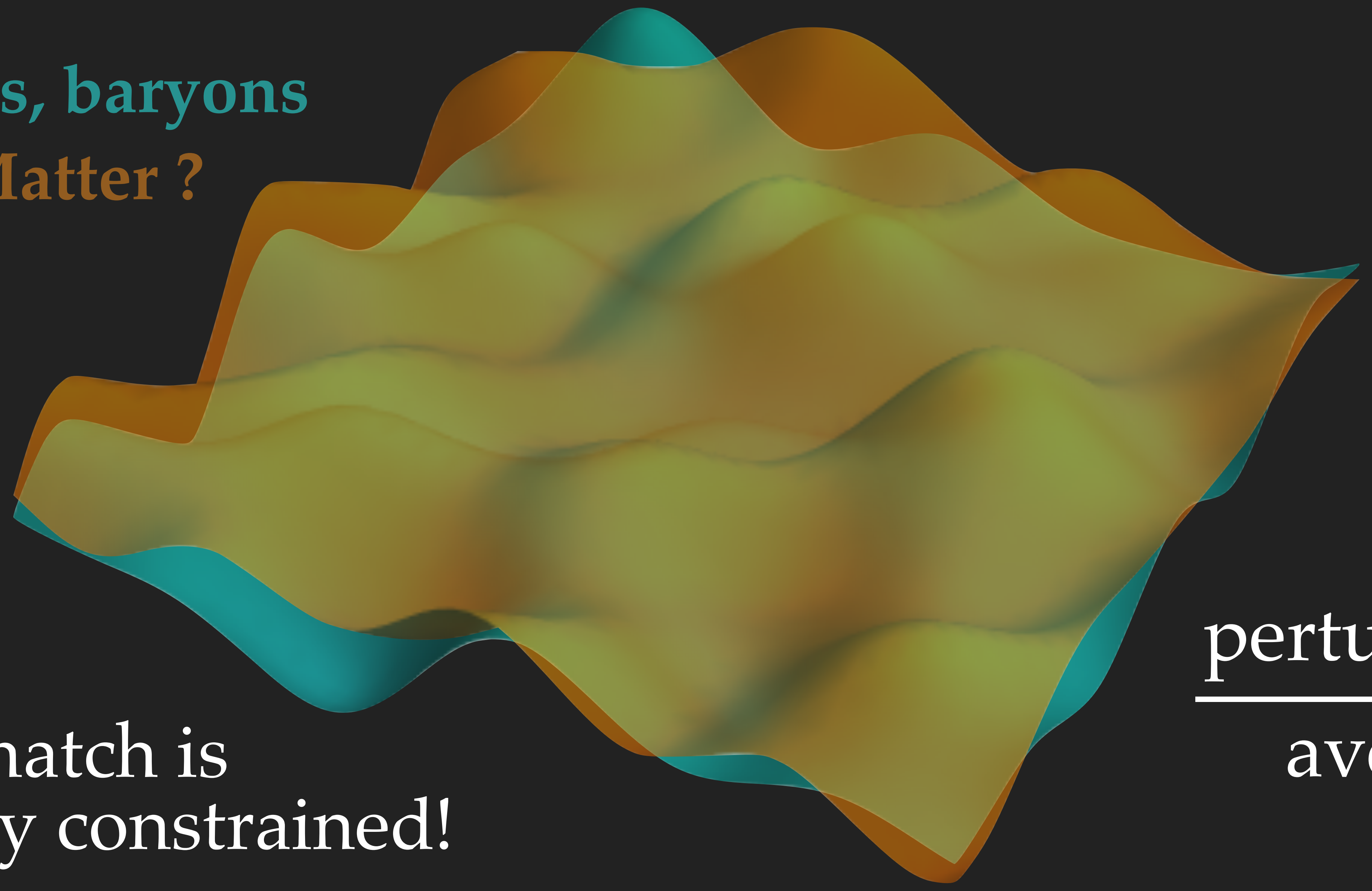

A mismatch is strongly constrained!

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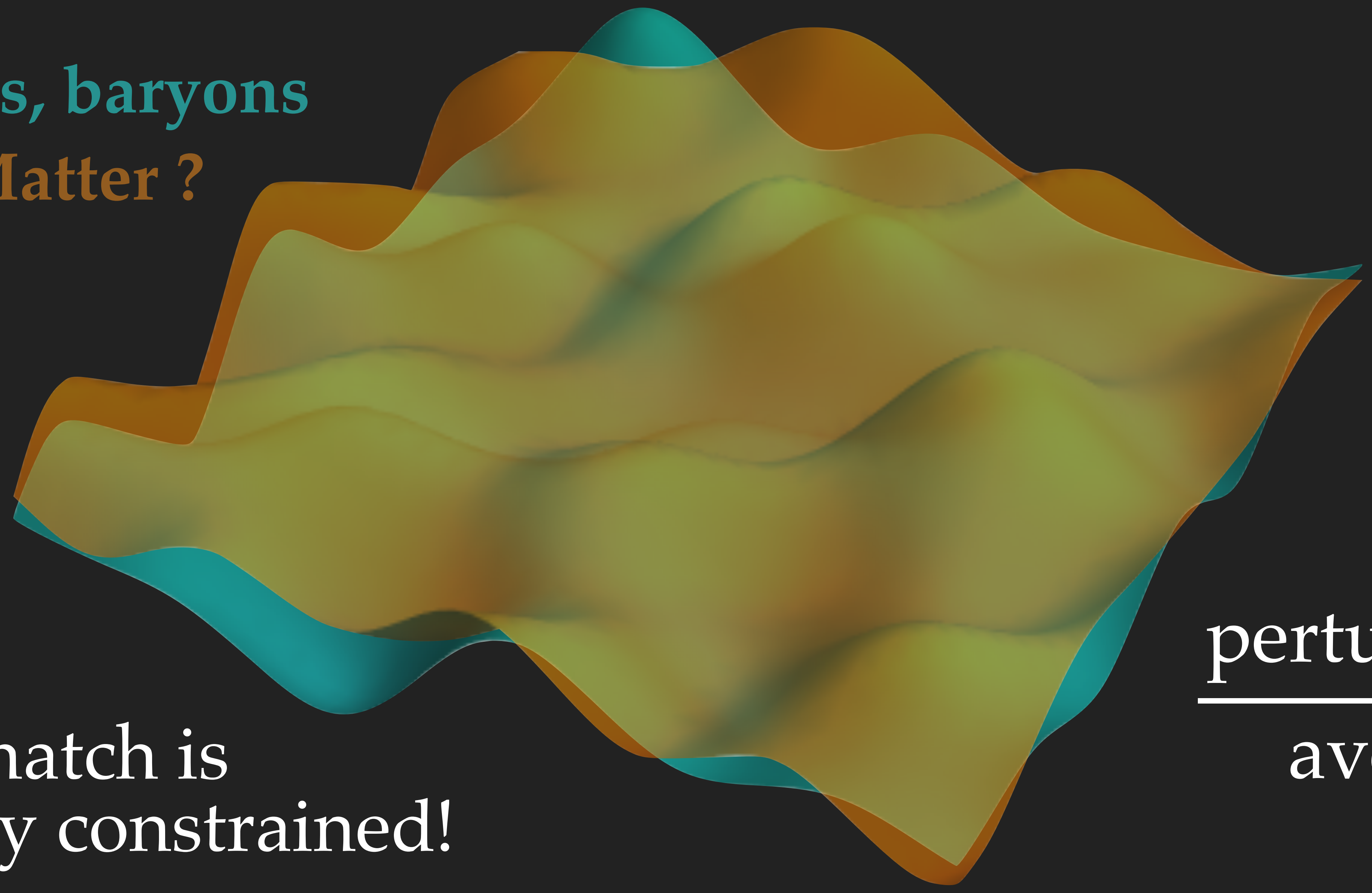

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$$\frac{\text{perturbation}}{\text{average}} \to \frac{\delta\rho(t, \mathbf{x})}{\rho(t)}$$

- different gauge: $\delta\rho(t, \mathbf{x}) \to \delta\rho(t, \mathbf{x}) + \dot{\rho}(t)\delta t$
- Gauge invariant curvature perturbation:

$$\zeta = -\psi - H\frac{\delta\rho_{\text{tot}}(t, \mathbf{x})}{\dot{\rho}_{\text{tot}}(t)}$$

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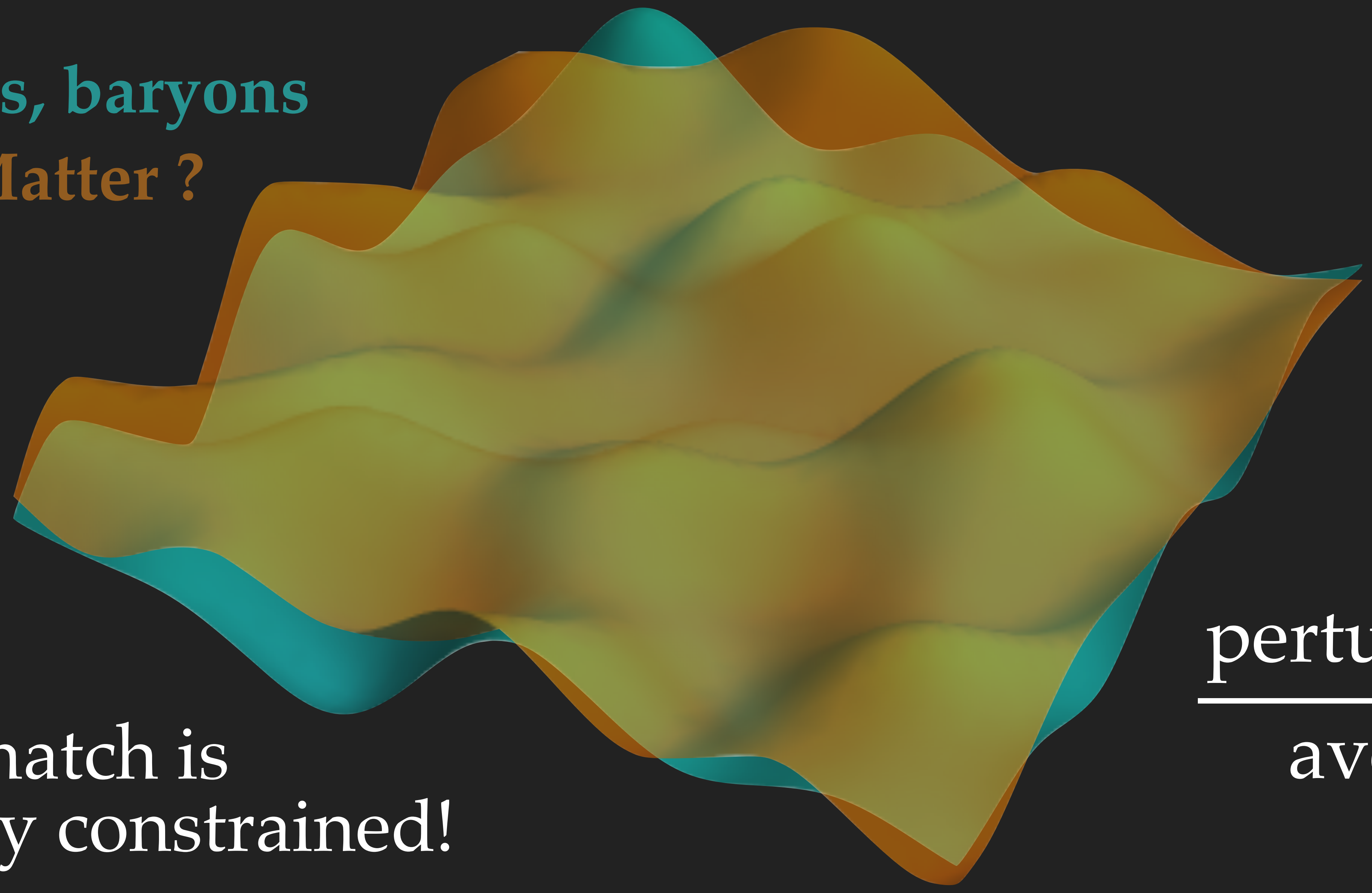
Dark Matter ?



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$$\frac{\text{perturbation}}{\text{average}} \to \frac{\delta\rho(t,\mathbf{x})}{\rho(t)}$$

Single-clock argument ['04 Weinberg]

$$\rho_{\rm DM}(t,\mathbf{x}) \leftrightarrow T_{\rm SM}(t,\mathbf{x})$$

$$\frac{\delta\rho_{\rm DM}}{\dot\rho_{\rm DM}} = \frac{\delta T}{\dot T} = \frac{\delta\rho_\gamma}{\dot\rho_\gamma}$$

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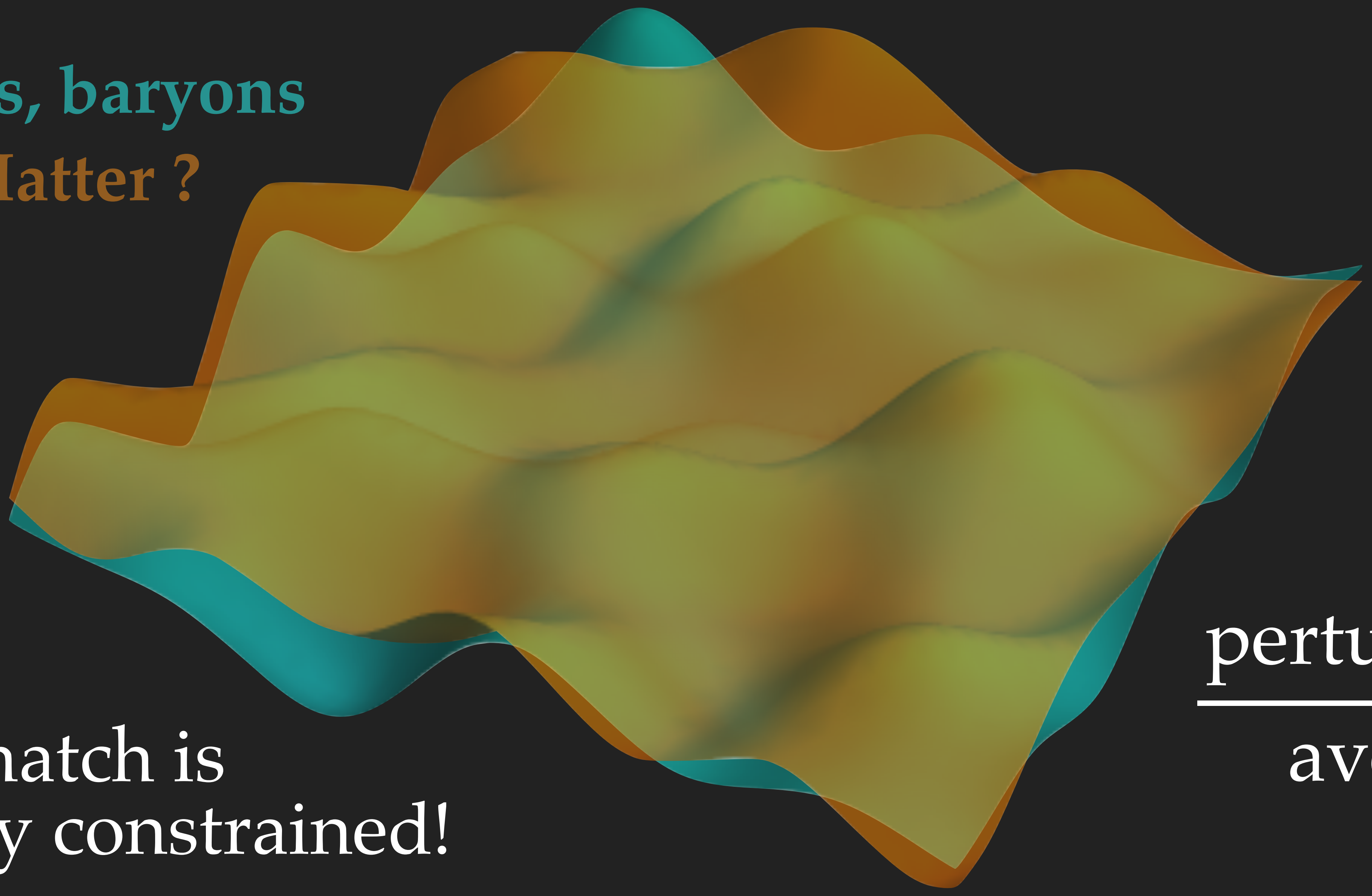

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['22 **DR**, Riotto] ['22 Strumia]

- DM energy density $\rho_{\rm DM}(t,\mathbf{x}) \leftrightarrow T_{\rm SM}(t,\mathbf{x})$ — regardless of thermalisation!
- DM and SM share origin of perturbations $\Rightarrow$ cannot differ later
- Tiny corrections suppressed by k^2 ['23 Holst, Hu, Jenks]

- different gauge: $\delta\rho(t,\mathbf{x}) \rightarrow \delta\rho(t,\mathbf{x}) + \dot\rho(t)\delta t$
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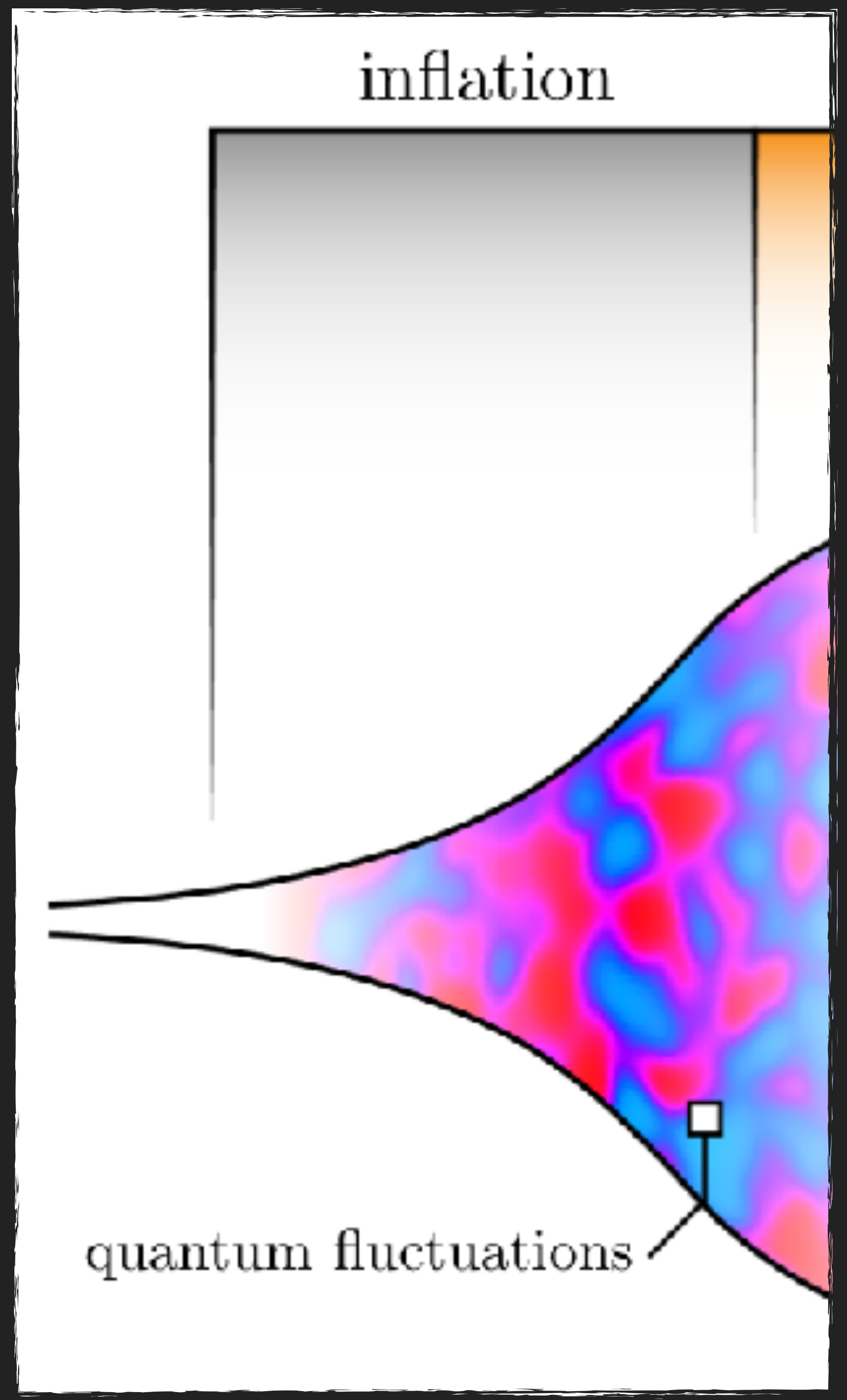
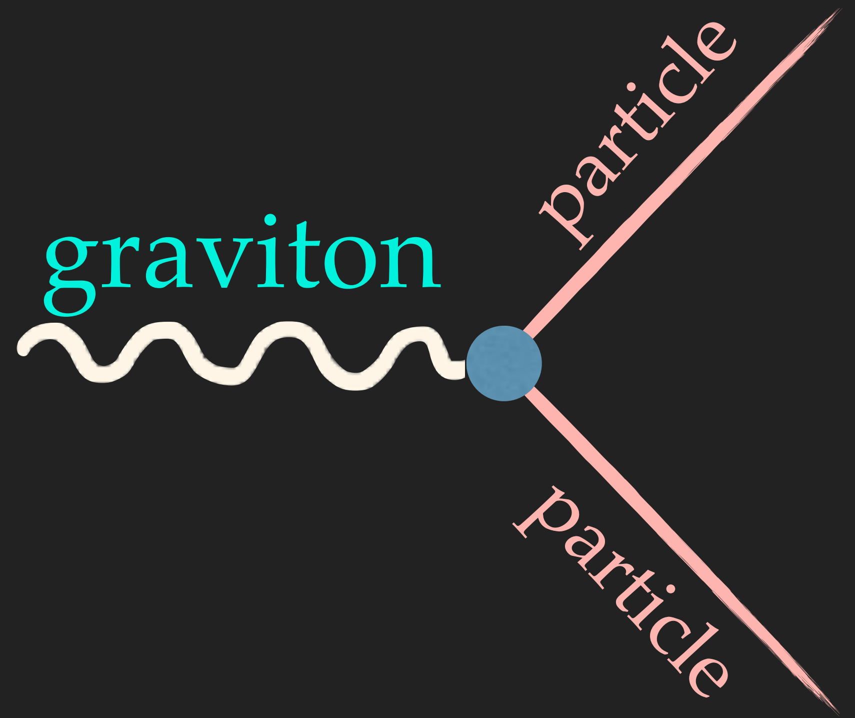
GRAVITATIONAL PRODUCTION (1)

['39 Schrödinger; '69 Parker; '77 Gibbons, Hawking; '79 Birrell, Davies; '87 Ford; …]





Oscillator with $\omega(t)$ → level crossing

time-dependent $\omega_k(t)$ in expanding Universe → from initial vacuum to non-vacuum later

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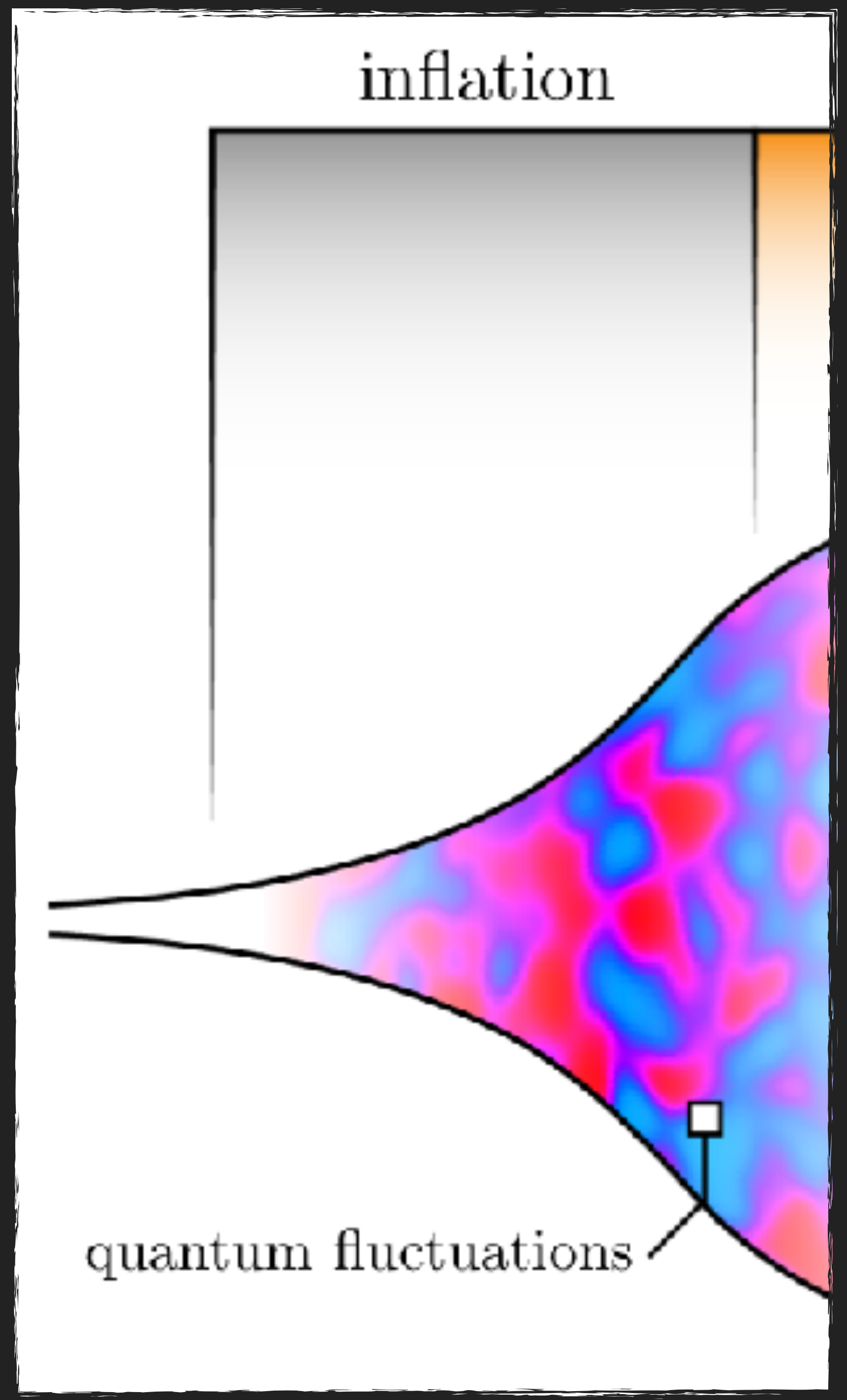


Oscillator with $\omega(t)$ → level crossing

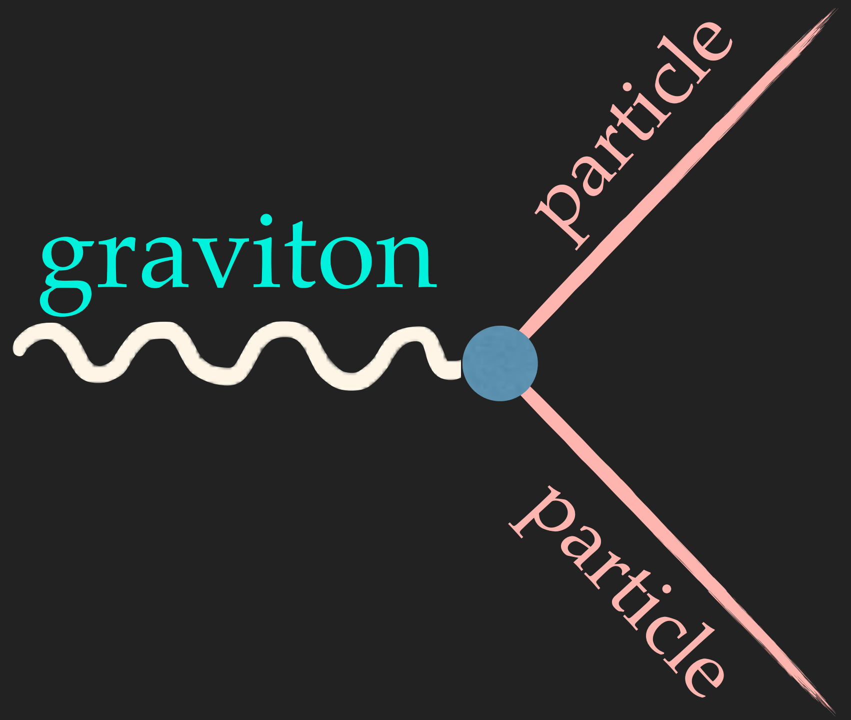
time-dependent $\omega_k(t)$ in expanding Universe → from initial vacuum to non-vacuum later

$$\ddot{u}_k(t) + \omega_k^2 u_k(t) = 0$$

$$u_k(t) = \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t}$$

$$\phi \sim \int \left(a_k u_k + a_k^\dagger u_k^* \right)$$

$$a_k |0\rangle = 0 \quad \forall k$$



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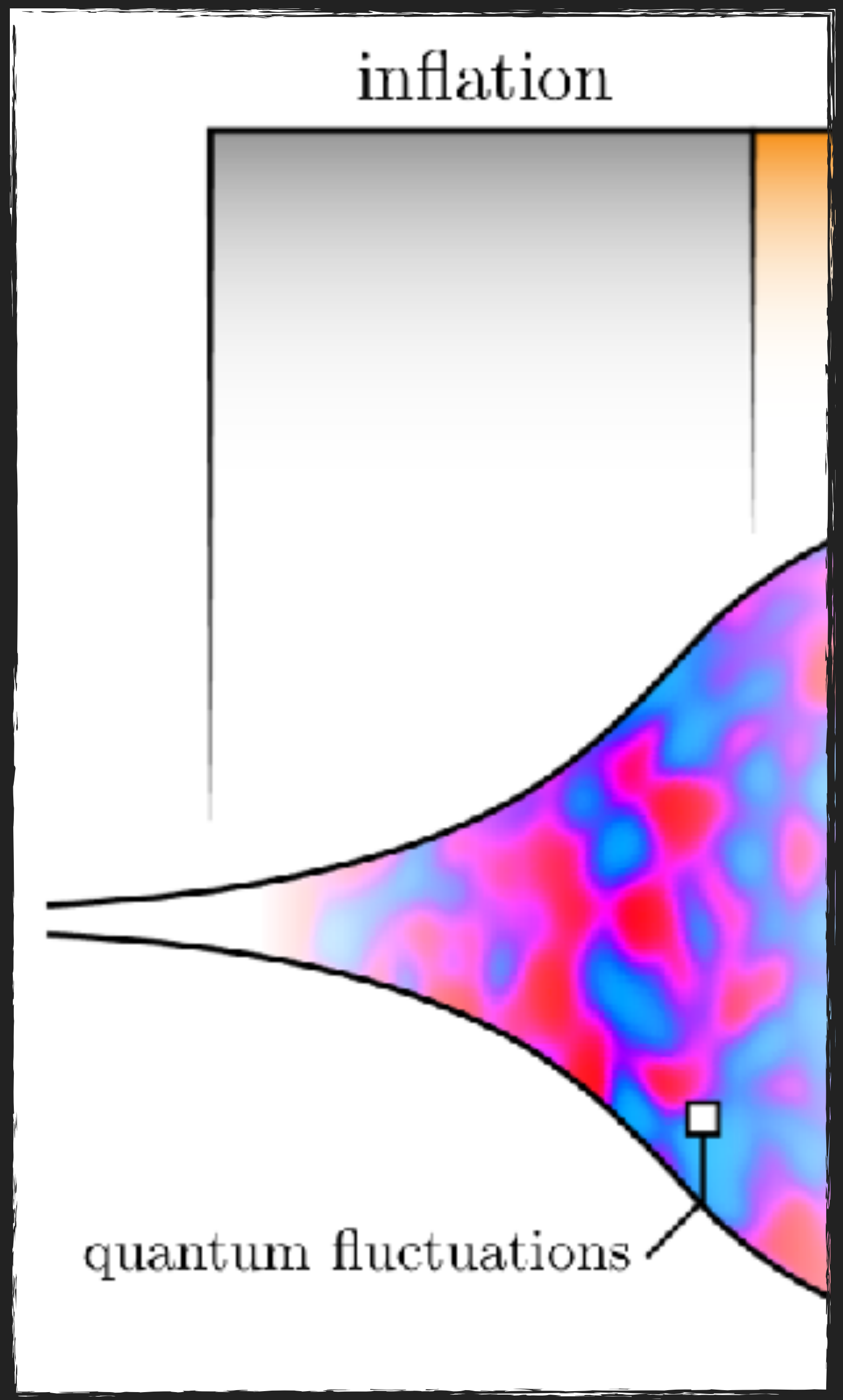
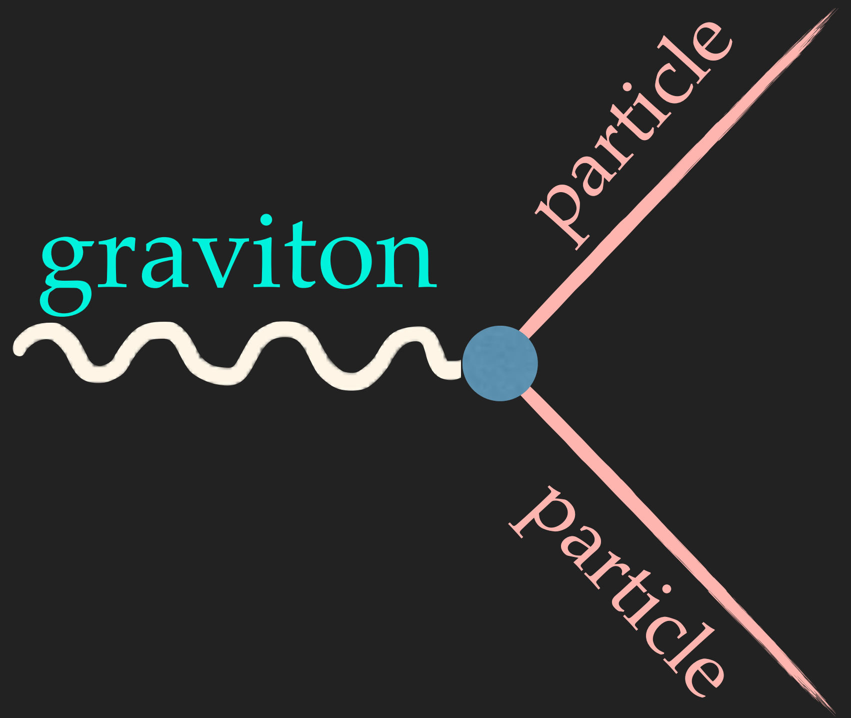




Oscillator with $\omega(t)$ → level crossing

time-dependent $\omega_k(t)$ in expanding Universe → from initial vacuum to non-vacuum later

$$\ddot{u}_k(t) + \omega_k^2\, u_k(t) = 0 \qquad \ddot{u}_k(t) + \omega_k^2(t)\, u_k(t) = 0$$

$$u_k(t) = \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t} \qquad u_k(t) \approx \frac{1}{\sqrt{\omega_k(t)}} e^{-i\int^t \omega_k(t')t'}$$

$$\phi \sim \int \left(a_k u_k + a_k^\dagger u_k^* \right) \qquad \phi \sim \int \left(a_k^{(\text{out})} u_k^{(\text{out})} + a_k^{\dagger(\text{out})} u_k^{*(\text{out})} \right)$$

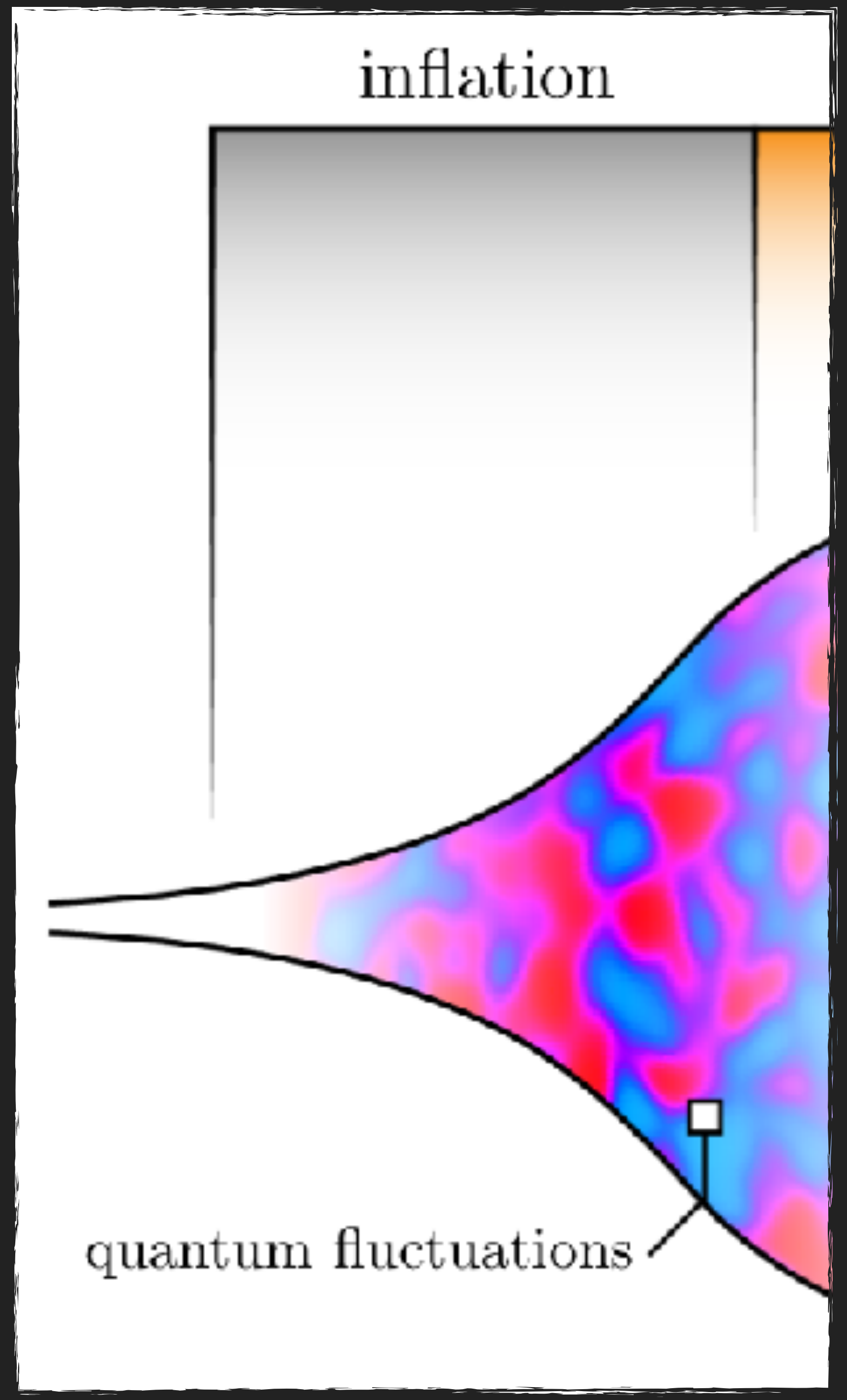
$$a_k |0\rangle = 0 \quad \forall k \qquad a_k^{(\text{in})} \neq a_k^{(\text{out})} \implies |0^{(\text{in})}\rangle \neq |0^{(\text{out})}\rangle$$

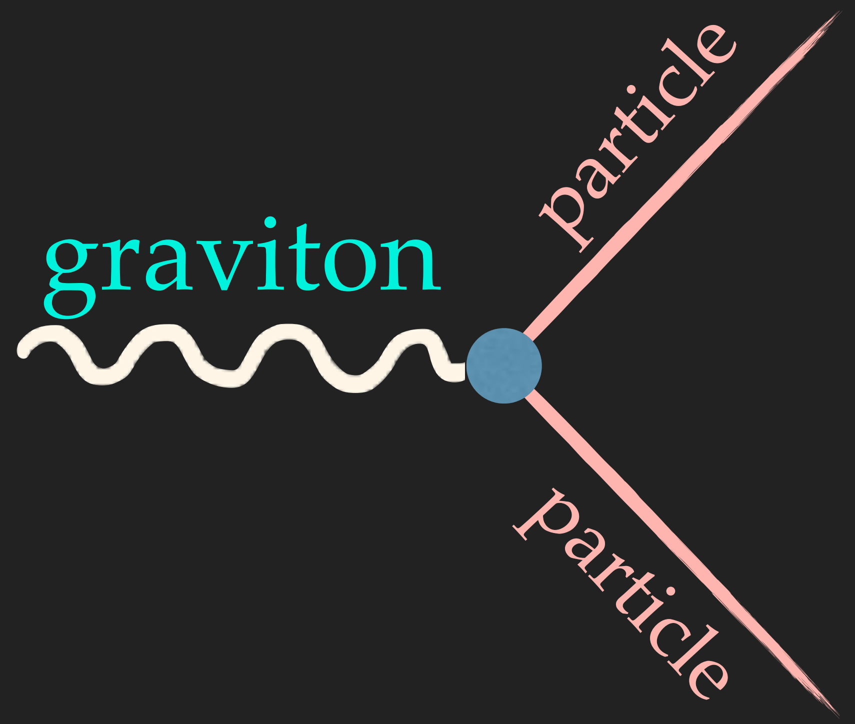
Particle production!

$\omega_k(t)$: mass term, …

GRAVITATIONAL PRODUCTION (2)

['39 Schrödinger; '69 Parker; '77 Gibbons, Hawking; '79 Birrell, Davies; '87 Ford; ...]





Time-varying bkg → $\omega_k(t)$ → particle production

Violation of scale invariance → time-dependent equations

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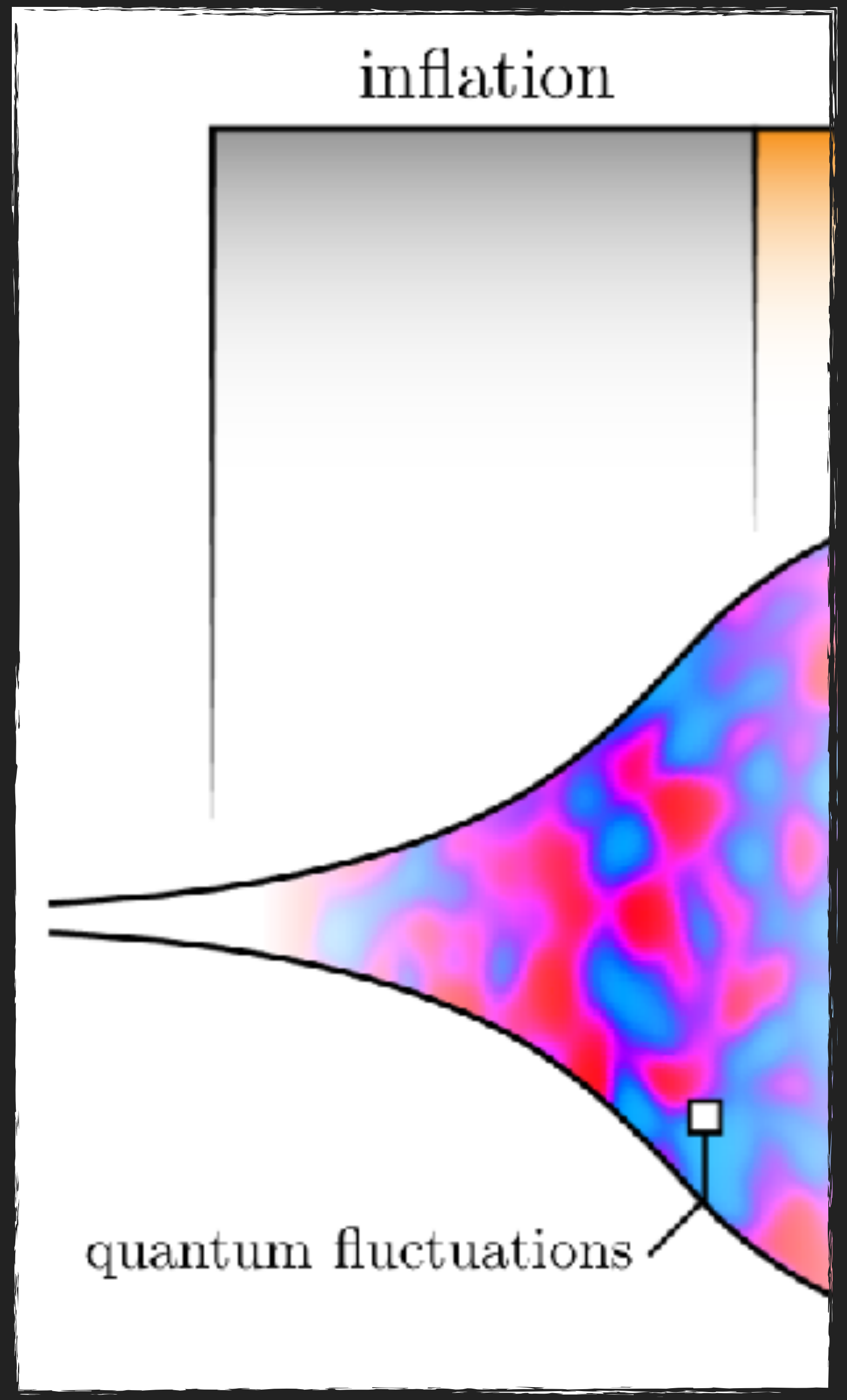
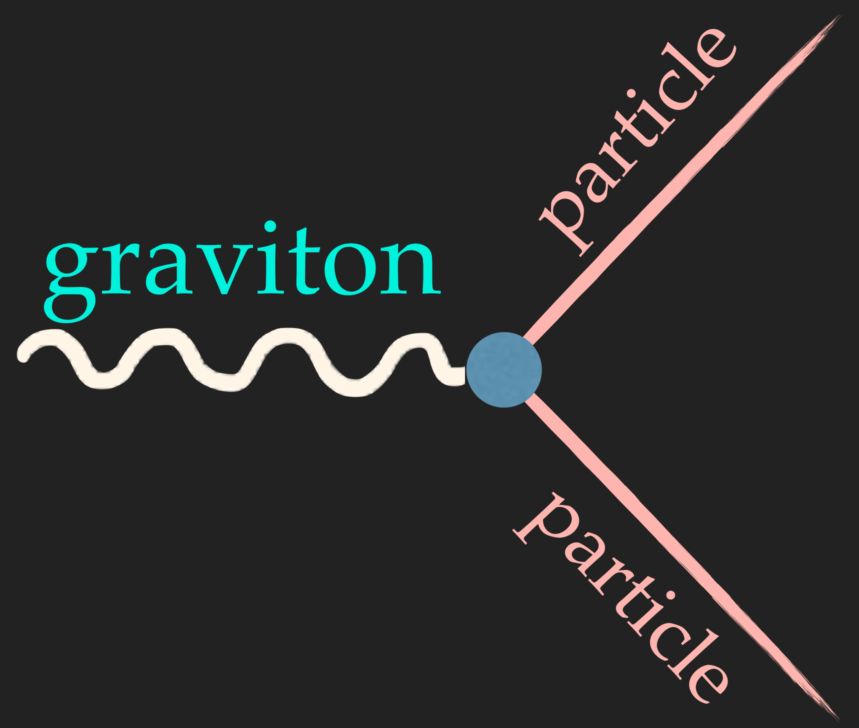




Time-varying bkg → $\omega_k(t)$ → particle production

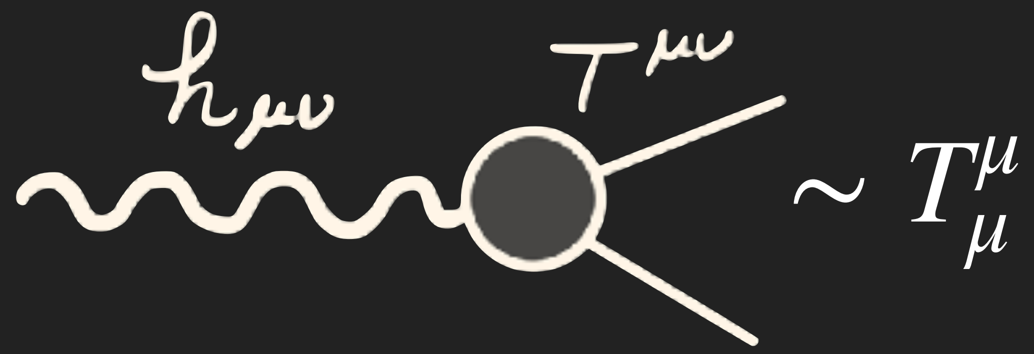
Violation of scale invariance → time-dependent equations

Scale transformation: $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}\,,\ \phi \to \lambda^{\#}\phi$

Irreducible time dependence in equations of motion

Scale invariance: $T^\mu_\mu = 0$, broken e.g. by mass

Coupling matter-gravity: $\mathscr{L} = h_{\mu\nu}T^{\mu\nu}$



GRAVITATIONAL PRODUCTION (3)

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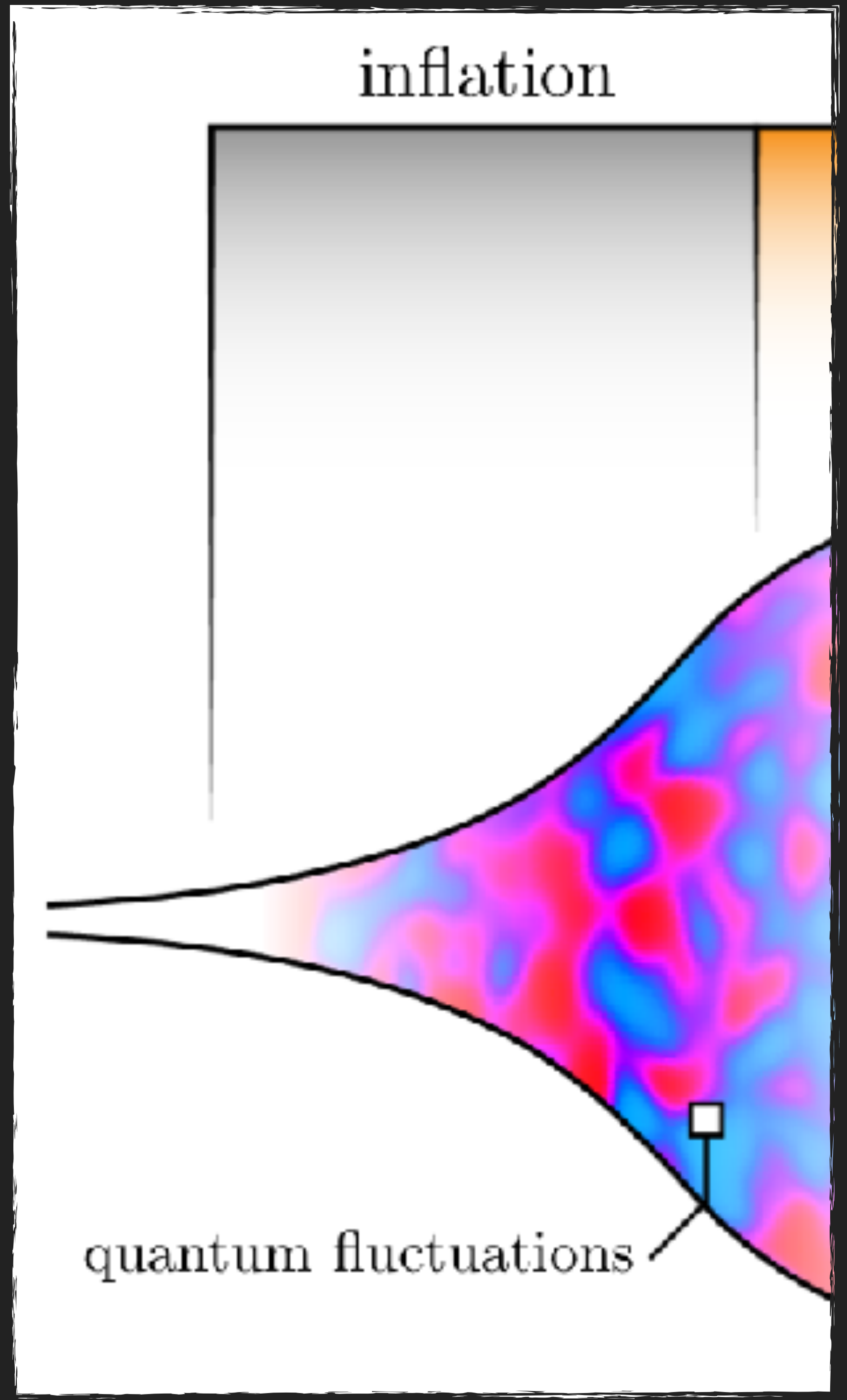

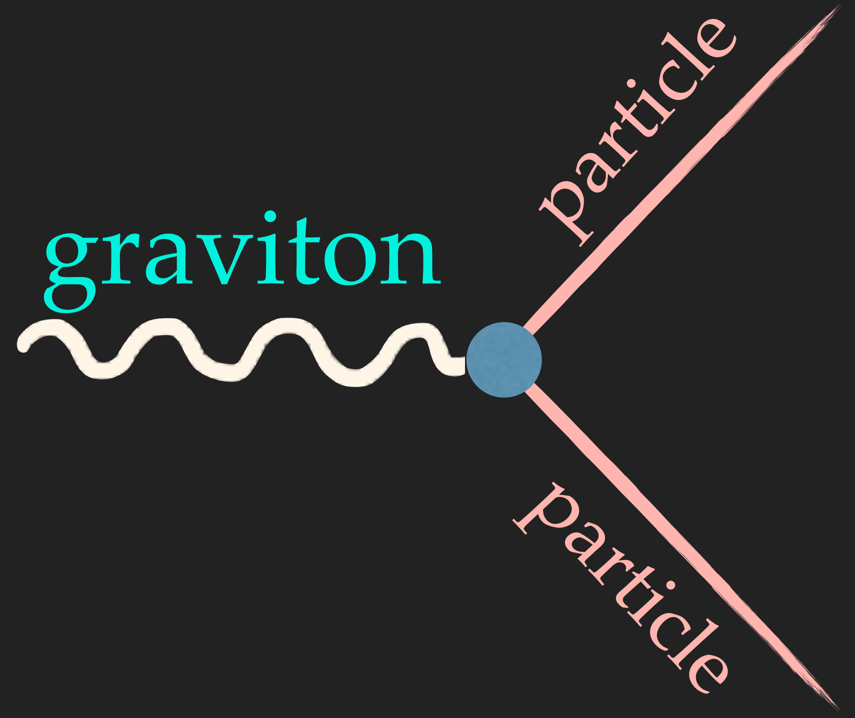

Time-varying bkg → $\omega_k(t)$ → particle production

Violation of scale invariance → time-dependent equations

Inflationary de-Sitter ≈ "bath" at "temperature" $T_{\rm dS} \sim \dfrac{H_I}{2\pi}$

$$\rho_{k,\rm exit}^{\rm (scalar)} \sim H_I^4$$

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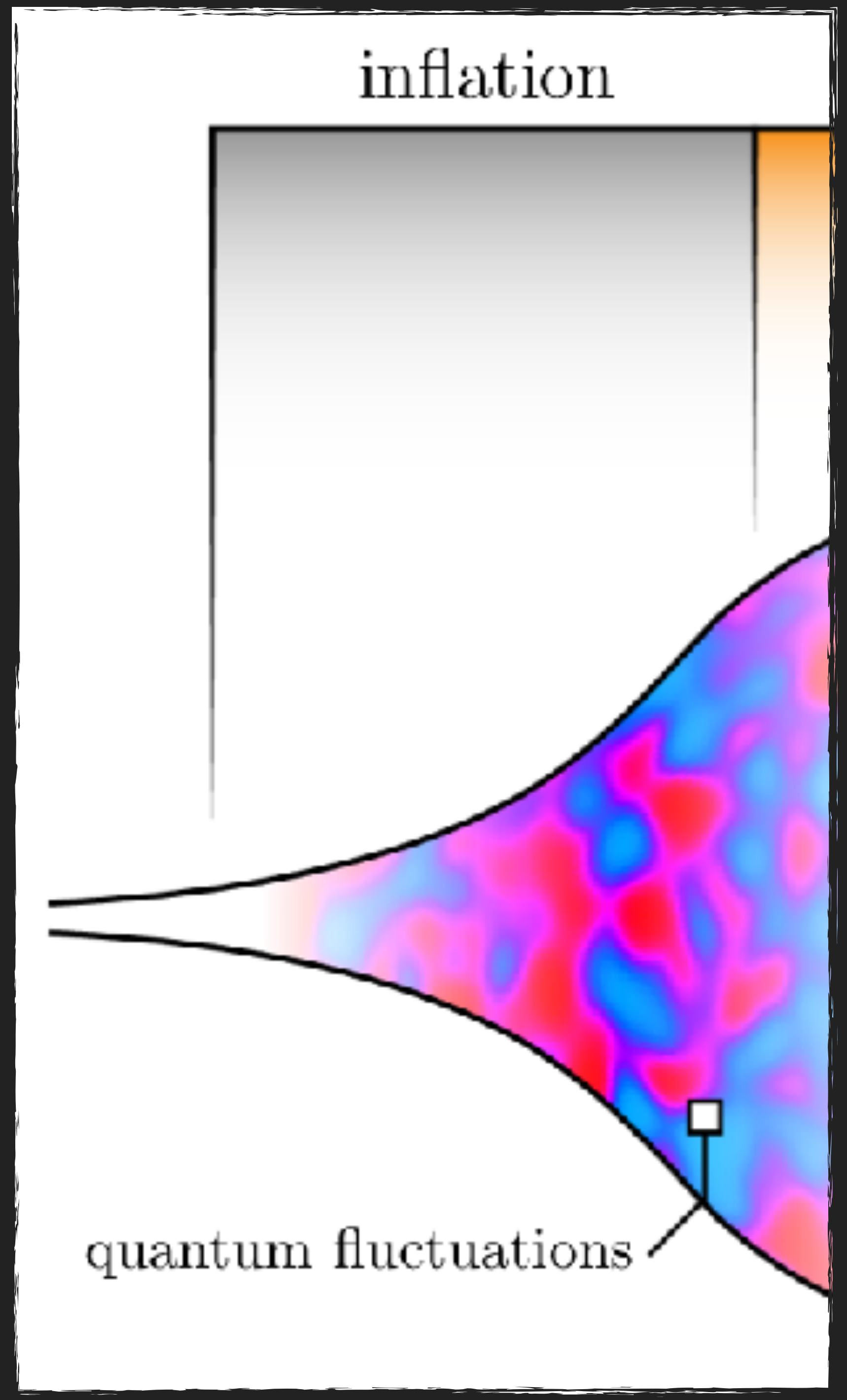

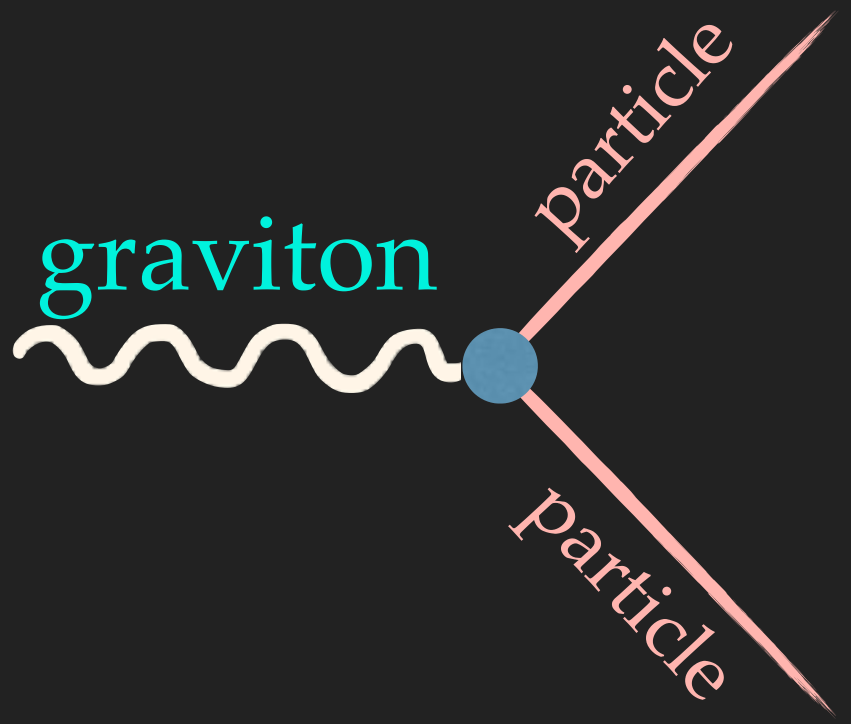

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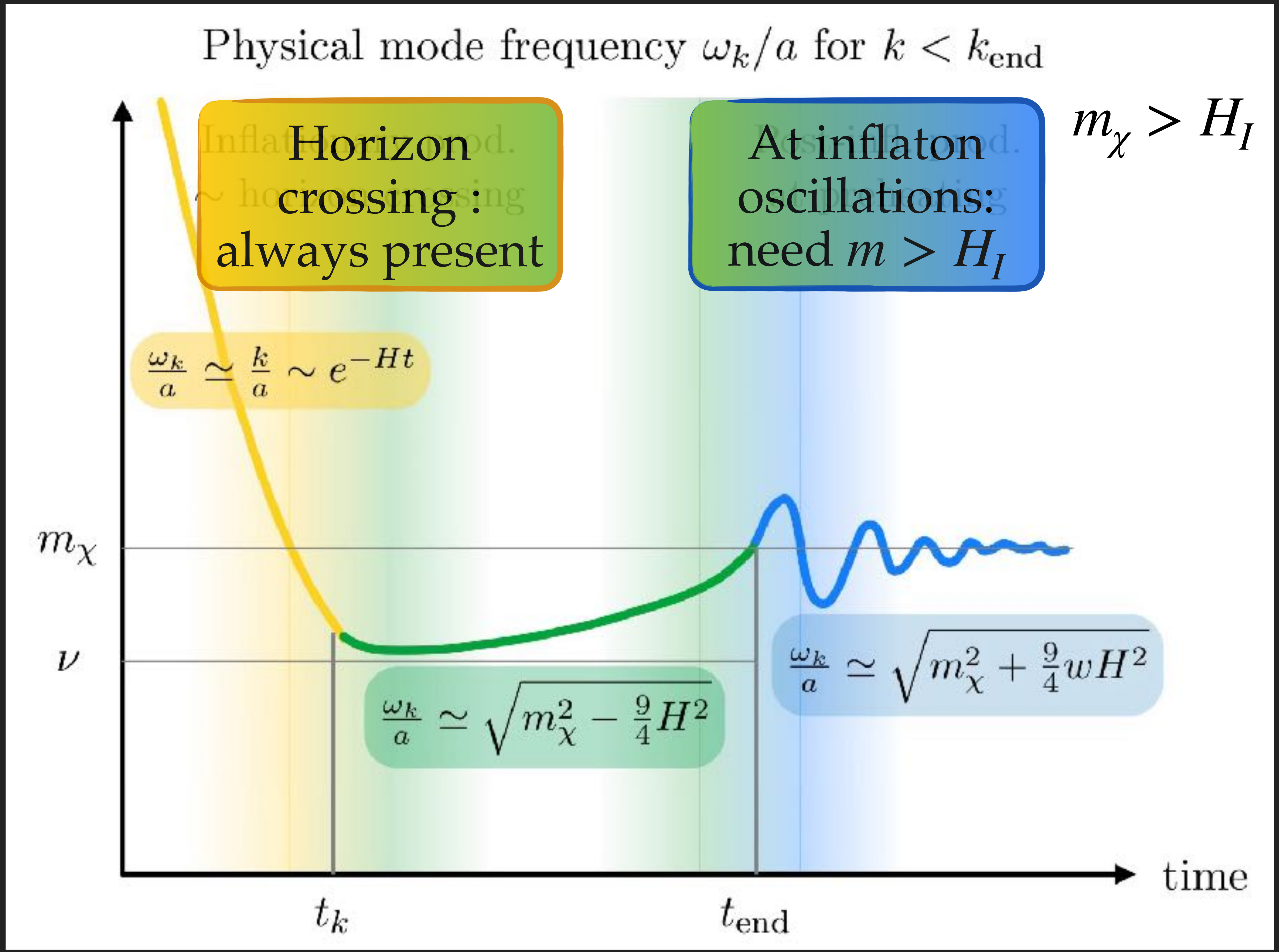
- Useful, but only analogy, not a strict equivalence

EPOCHS OF GRAVITATIONAL PRODUCTION

$$\frac{\omega_k(t)}{a(t)} = \sqrt{\frac{k^2}{a(t)^2} + m^2 + \frac{9}{4} w(t) H(t)^2}$$

[(in preparation, '24) **DR**, Verner, Xue]



['98 Chung, Kolb, Riotto; '99 Kofman, Linde, Starobinsky; '18 Chung, Kolb, Long; '19 Li, Nakama, Sou, Wang, Zhou; '21 Ling, Long; '23 Brandenberger, Kamali, Ramos; ...]

- Time-dependent $\omega_k(t)$:

$$n_{k,\text{(late times)}} \sim k^3 |\beta_k|^2 \qquad |0_k\rangle^{\text{(in)}} \neq |0_k\rangle^{\text{(out)}}$$

$$\beta_k = \int_{t_i}^{t} dt' \frac{1}{2} \frac{\dot\omega_k}{\omega_k} \exp\left(-2i \int_{t_i}^{t'} \frac{\omega_k}{a} dt'' \right)$$

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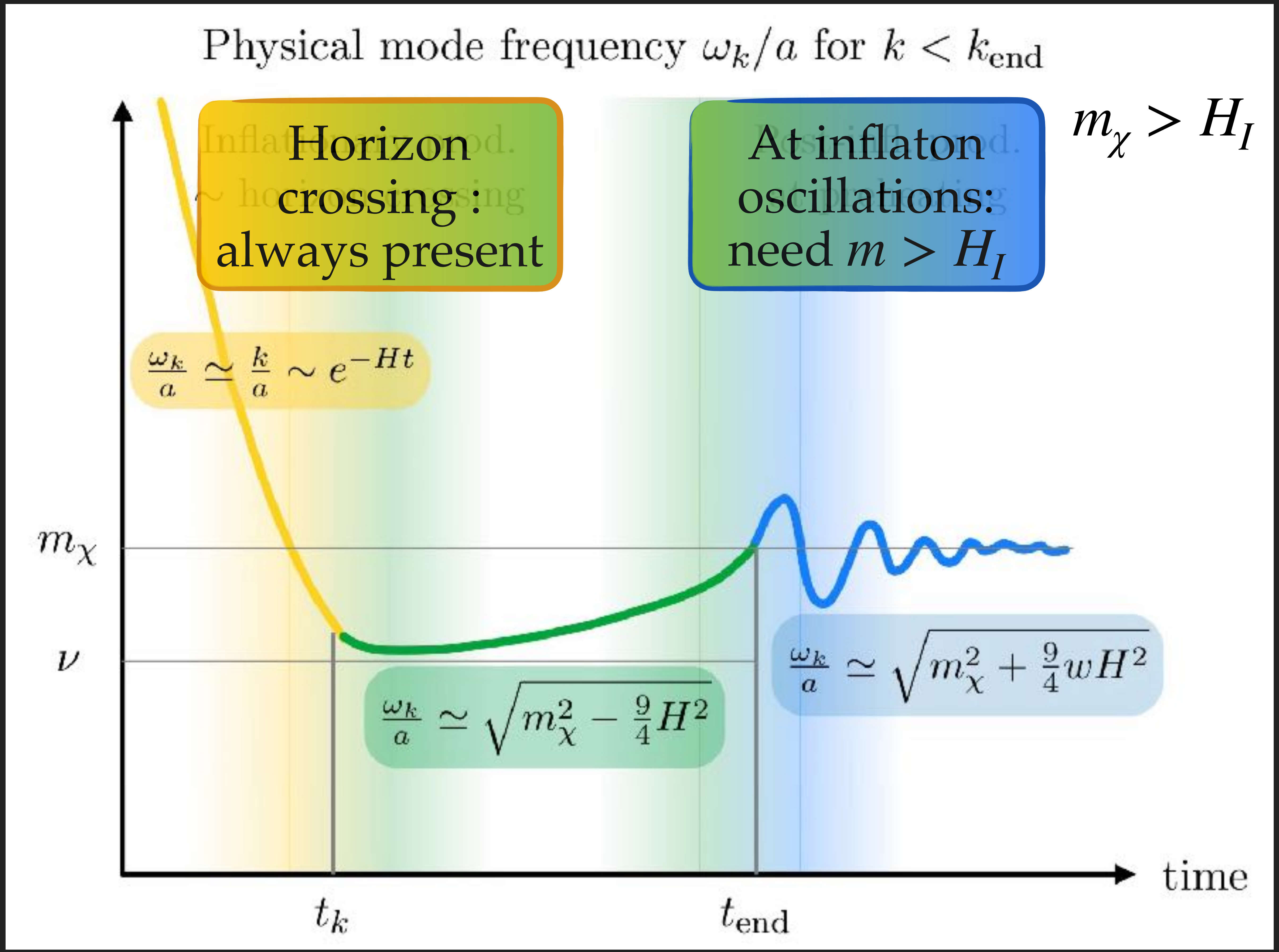

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  - heavy $m > H_I$: after end of inflation (*depends on preheating*)

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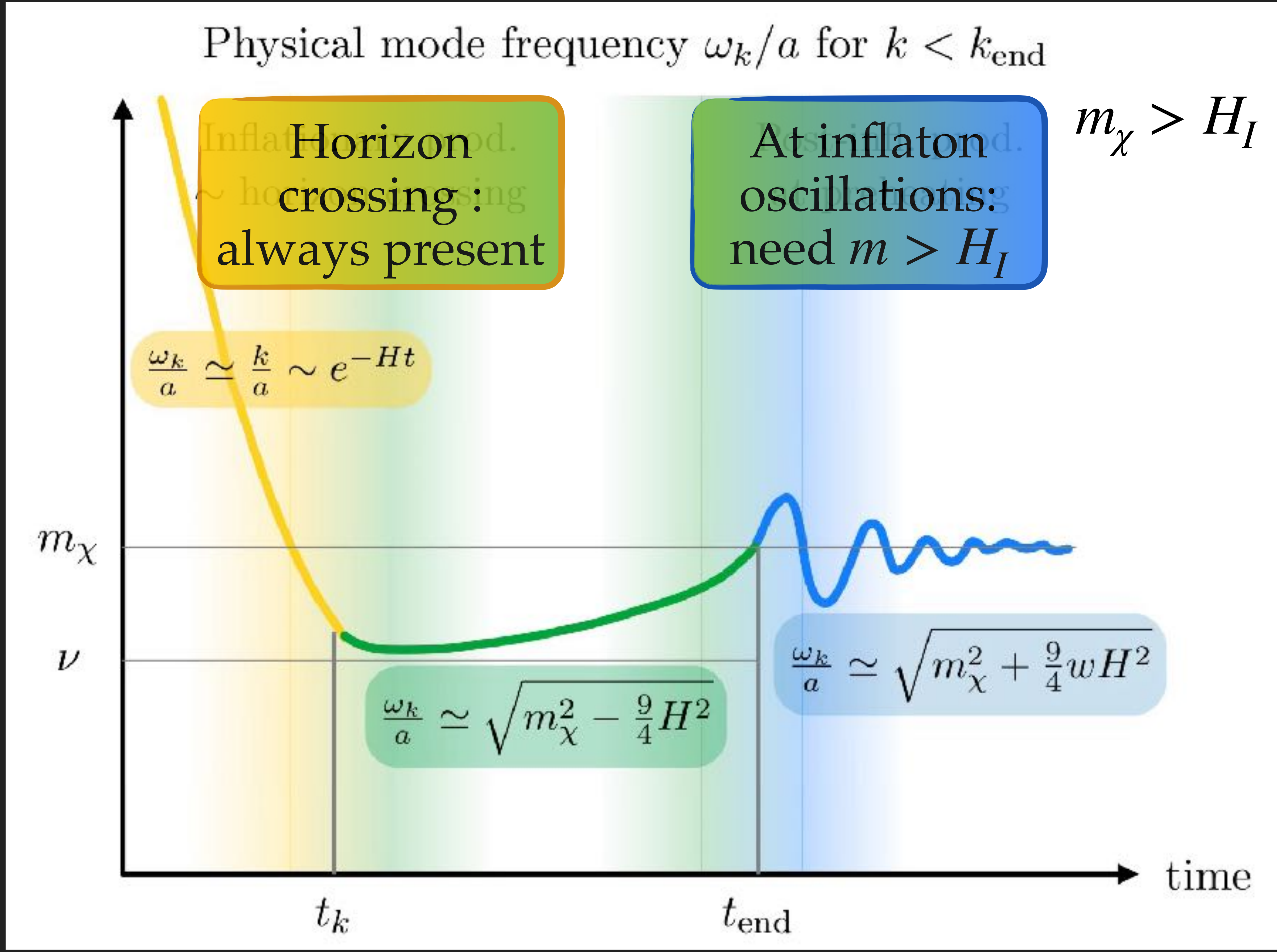

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  - Hubble crossing (*model independent*)
  - heavy $m > H_I$: after end of inflation (*depends on preheating*)
- Phase $\exp\left(i\int \omega dt\right)$
  - $e^{2ik\eta}$ at early times
  - heavy $m > H_I$: rapid phase → saddle appr. → $n_k \sim \exp(-\pi m/H_I)$

Gravitational production

TARGETS FOR DARK MATTER FROM GRAVITATIONAL PRODUCTION

Gravitational production

neV | μeV | meV | eV | keV | MeV | GeV | TeV | 10^6 GeV | 10^9 GeV | 10^{12} GeV | 10^{15} GeV | $m_{\rm DM}$

Single Species

Massive vector A'

['15 Graham, Mardon, Rajendran]

Fermion ψ

['99 Kuzmin, Tkachev]

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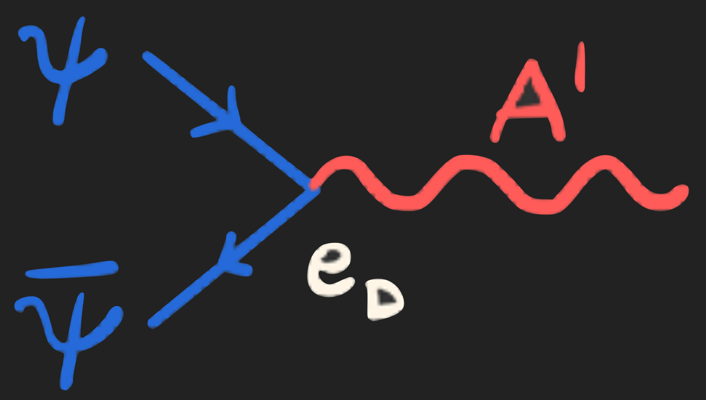
Dark Sector

['21 Arvanitaki, Dimopoulos, Galanis, **DR**, Simon, Thompson]

Massive Dark QED

ψ dark matter

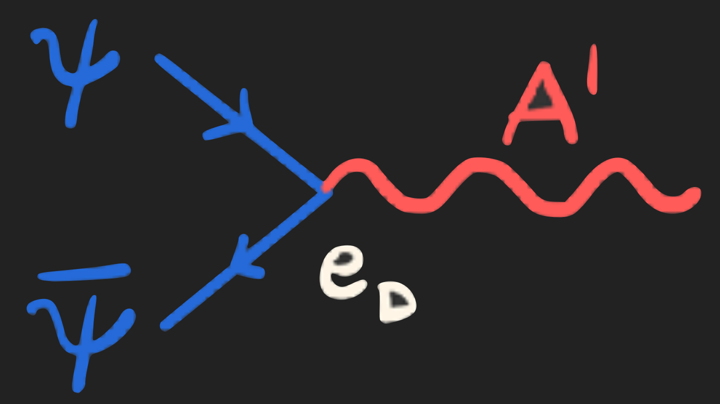
A' grav. prod.



- Complexity and thermalisation in the dark sector change the Dark Matter target

DYNAMICS IN A DARK SECTOR

['21 Arvanitaki, Dimopoulos, Galanis, **DR**, Simon, Thompson]



Gravitational production of A' during inflation → End of Inflation → Interactions: $A' \to A', \psi$ → Thermalisation → Thermalised Dark sector → Freeze-out → ψ Dark Matter

$m_\psi \in [\text{GeV}, 10\,\text{TeV}]$

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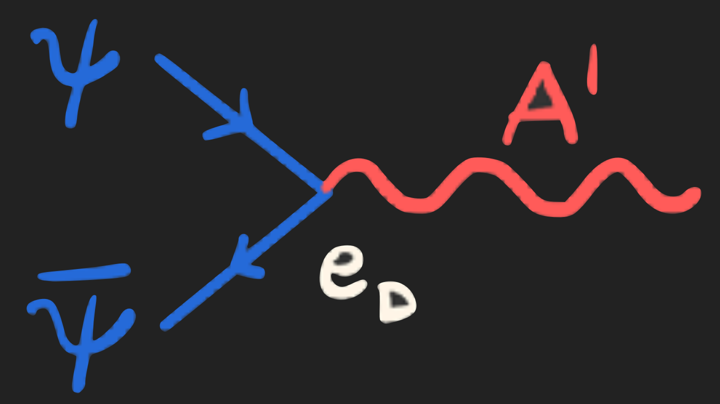

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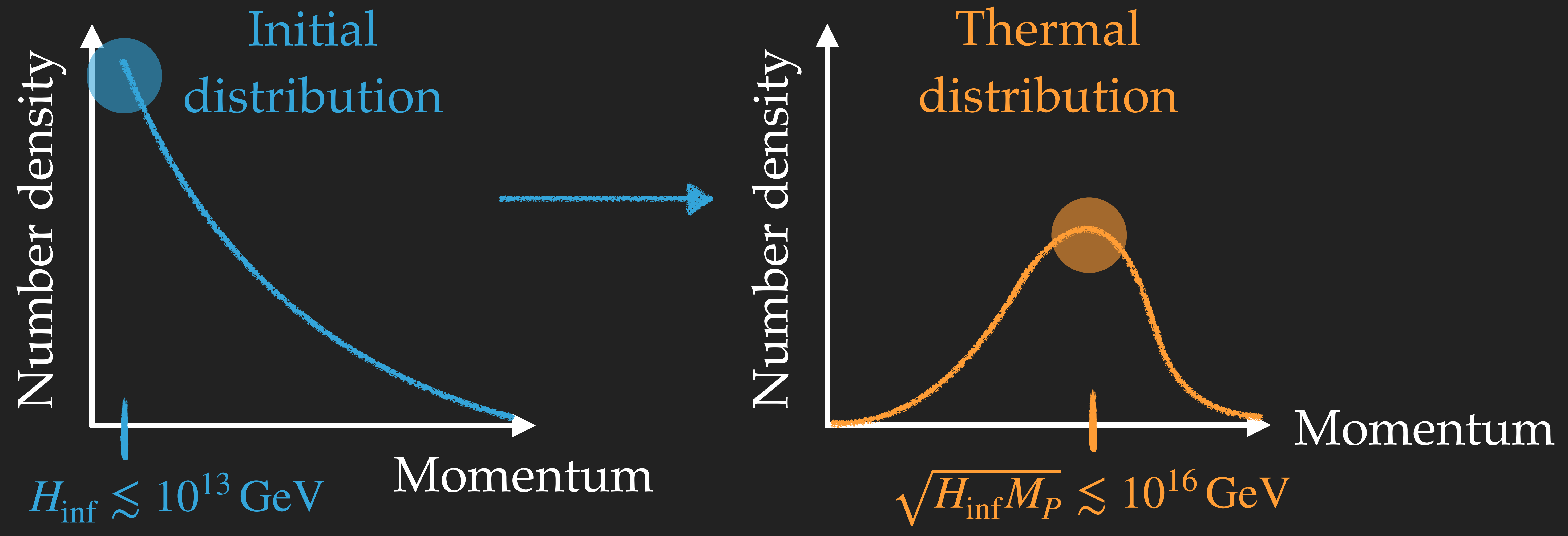
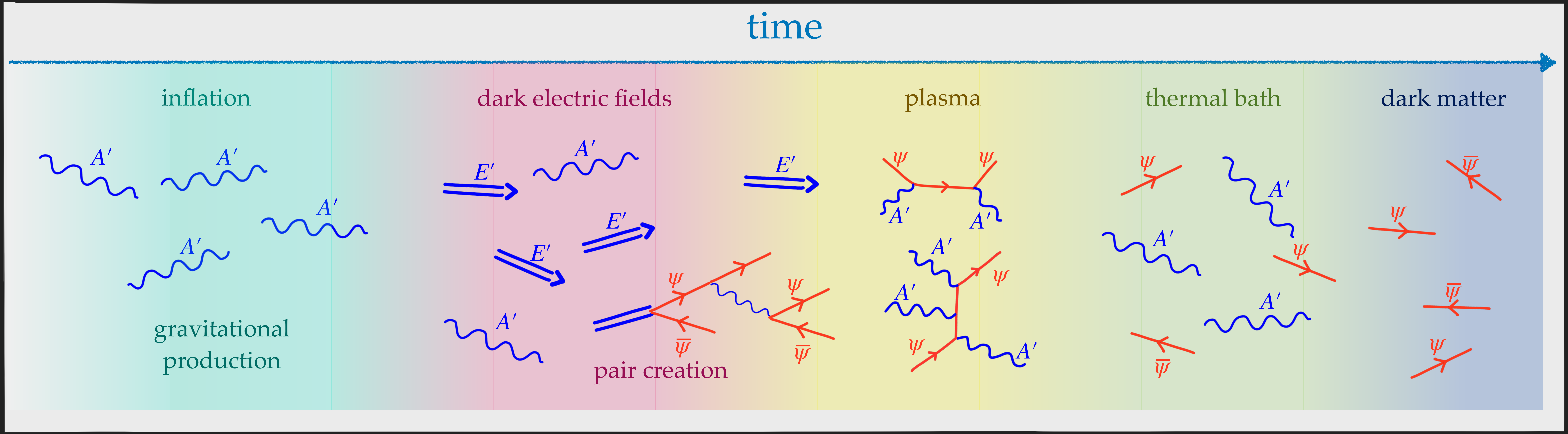


Universality:

$$T_{\rm DM} \sim \sqrt{\frac{H_{\rm inf}}{M_P}}\, T_{\rm SM}$$

 ['21 Arvanitaki, Dimopoulos, Galanis, **DR**, Simon, Thompson]



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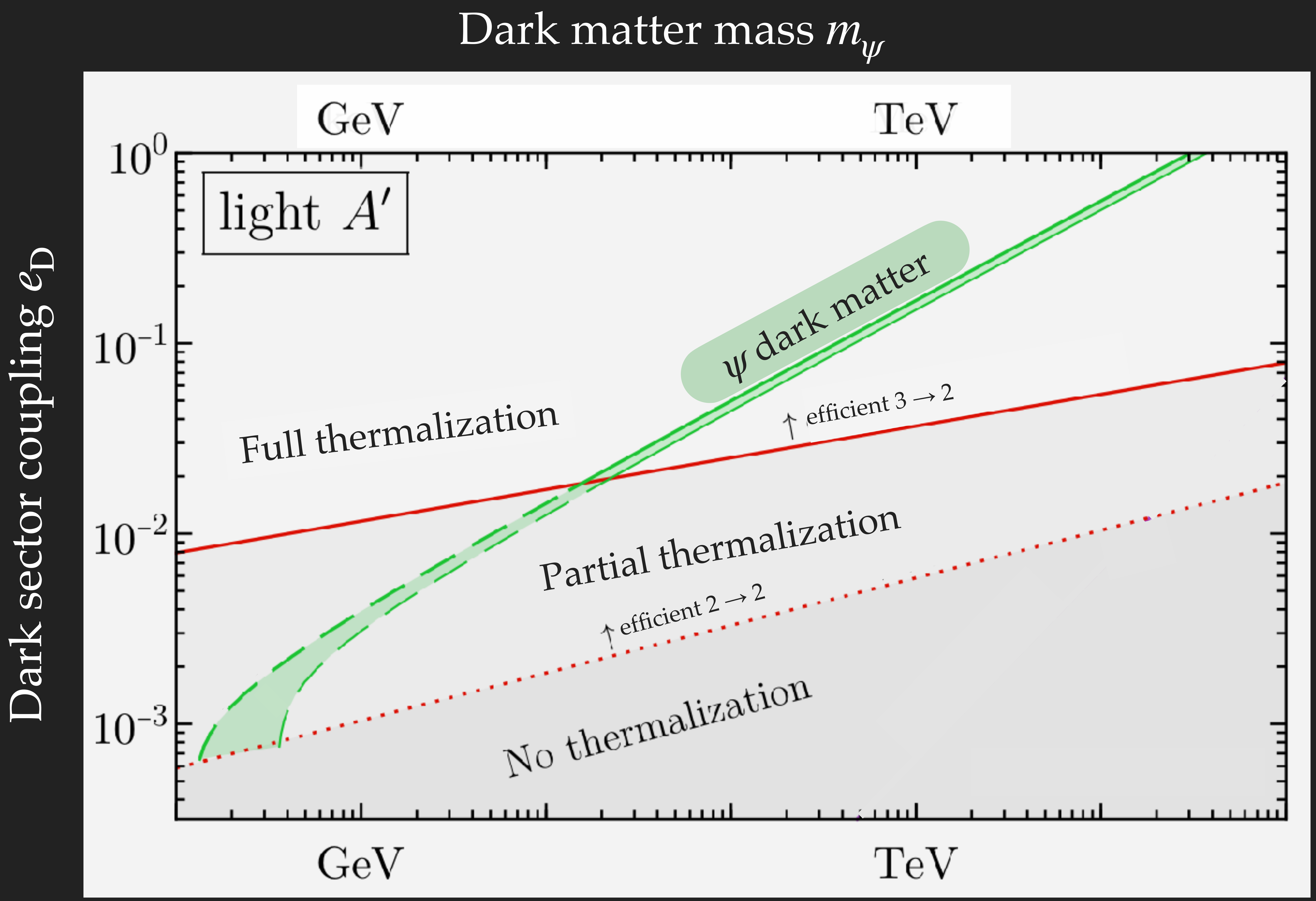
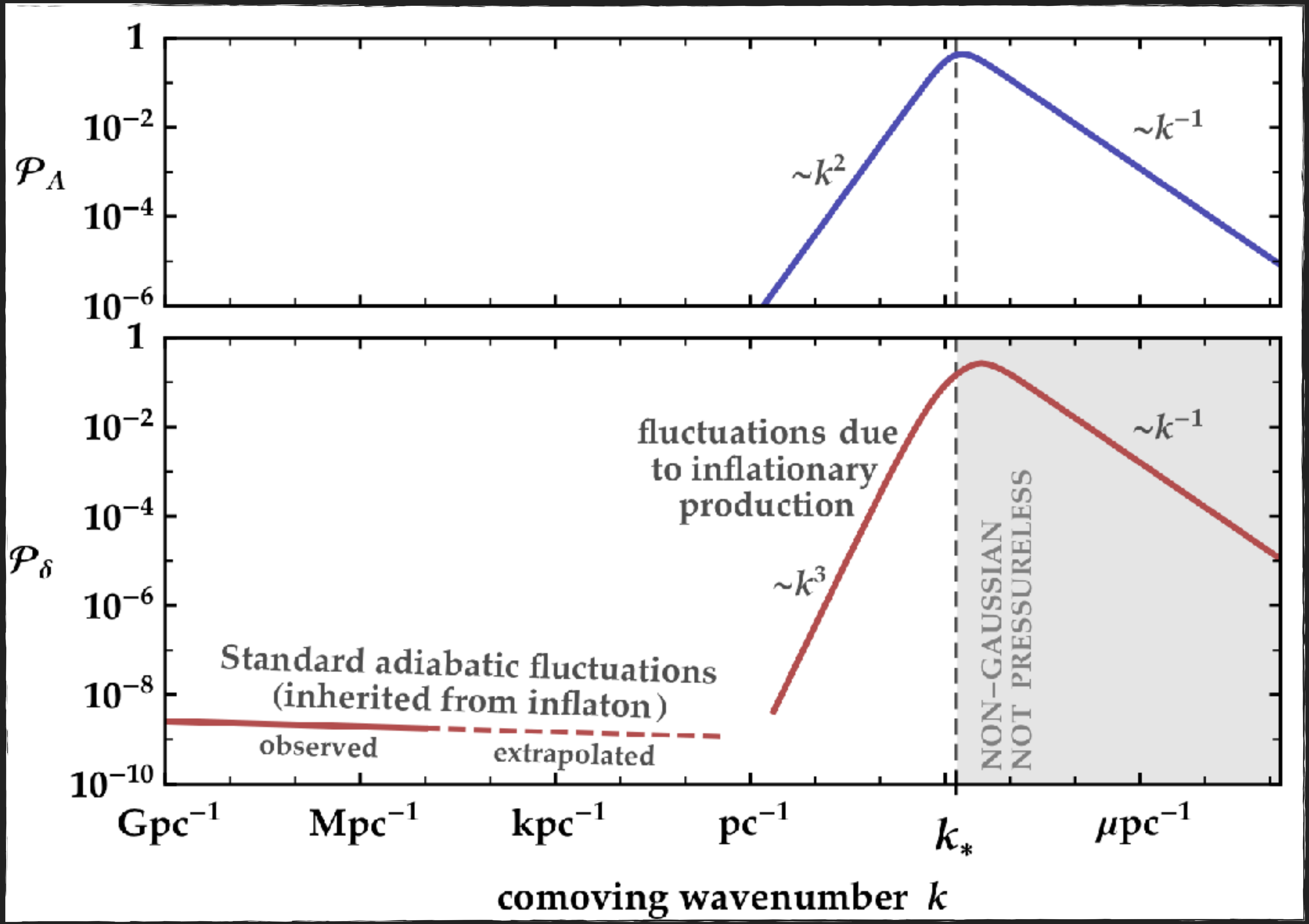
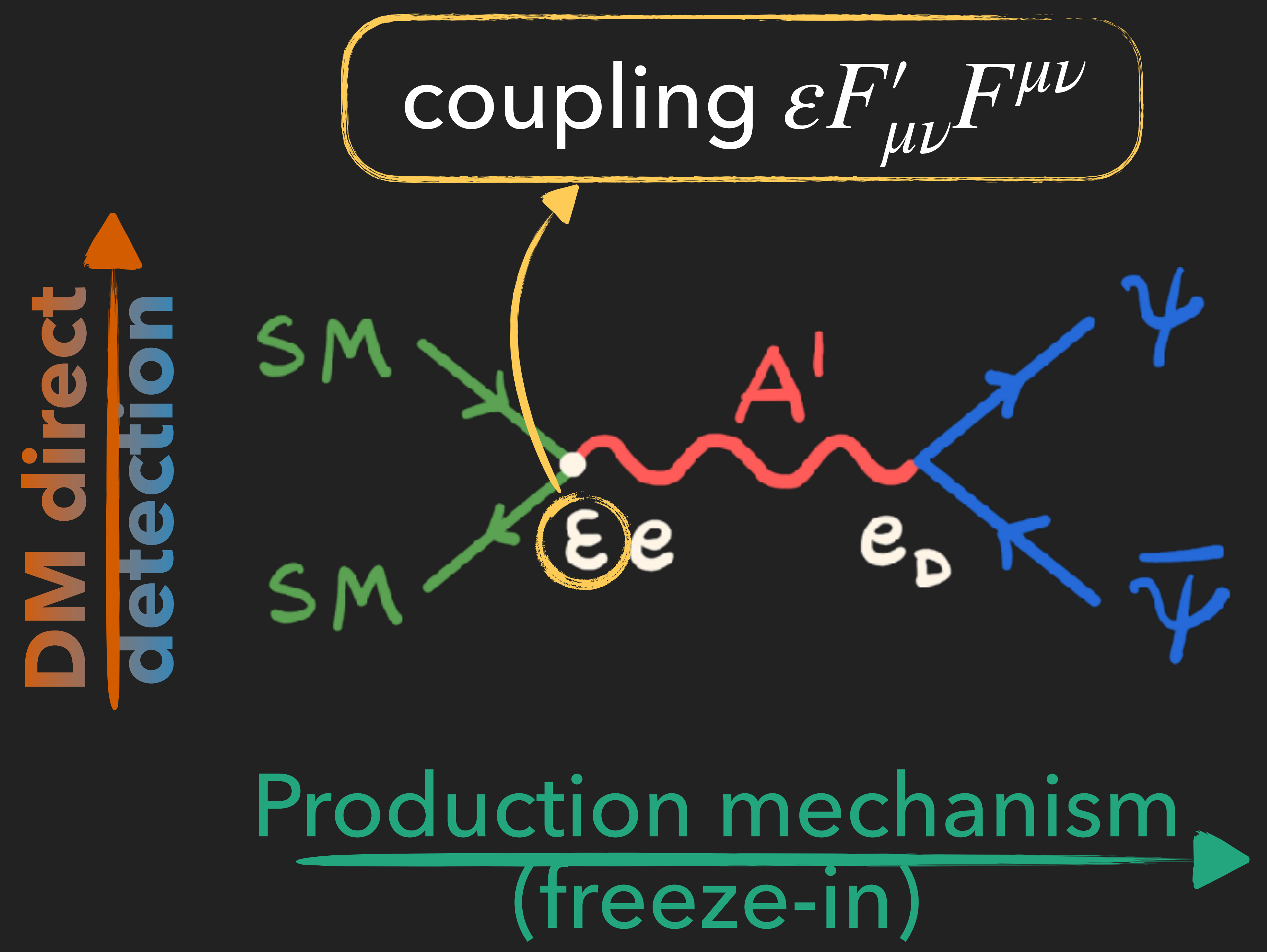


NO TROUBLE WITH ISOCURVATURE PERTURBATIONS

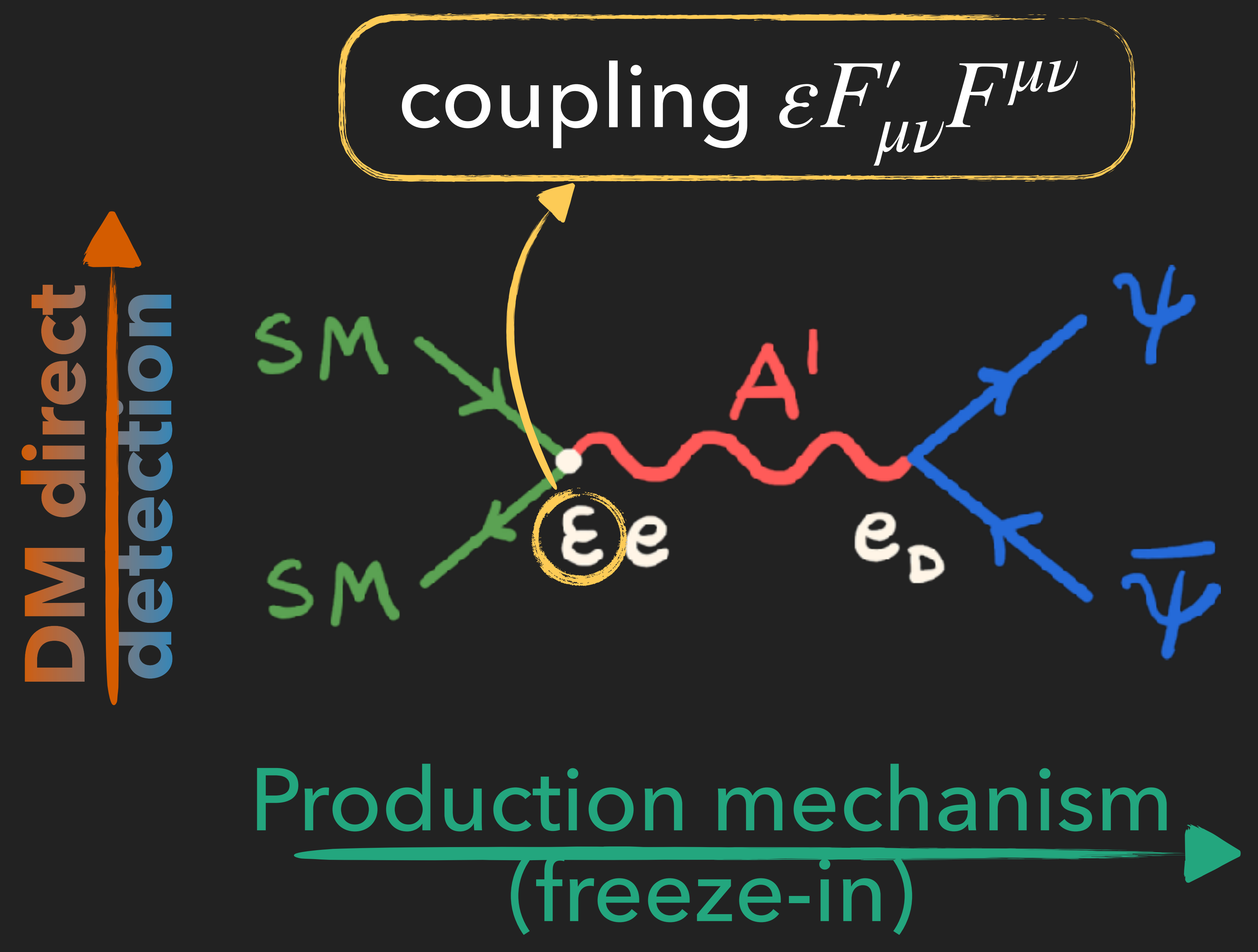
['15 Graham, Mardon, Rajendran]



$$k_*^{-1} \sim 10^{10}\,\text{km} \cdot \sqrt{\frac{10^{-5}\text{eV}}{m_{A'}}} \sim 0.3\,\text{mpc} \cdot \sqrt{\frac{10^{-5}\text{eV}}{m_{A'}}}$$



coupling $\varepsilon F'_{\mu\nu} F^{\mu\nu}$
DM direct detection
SM
SM
A'
εe
e_D
ψ
$\overline{\psi}$
Production mechanism (freeze-in)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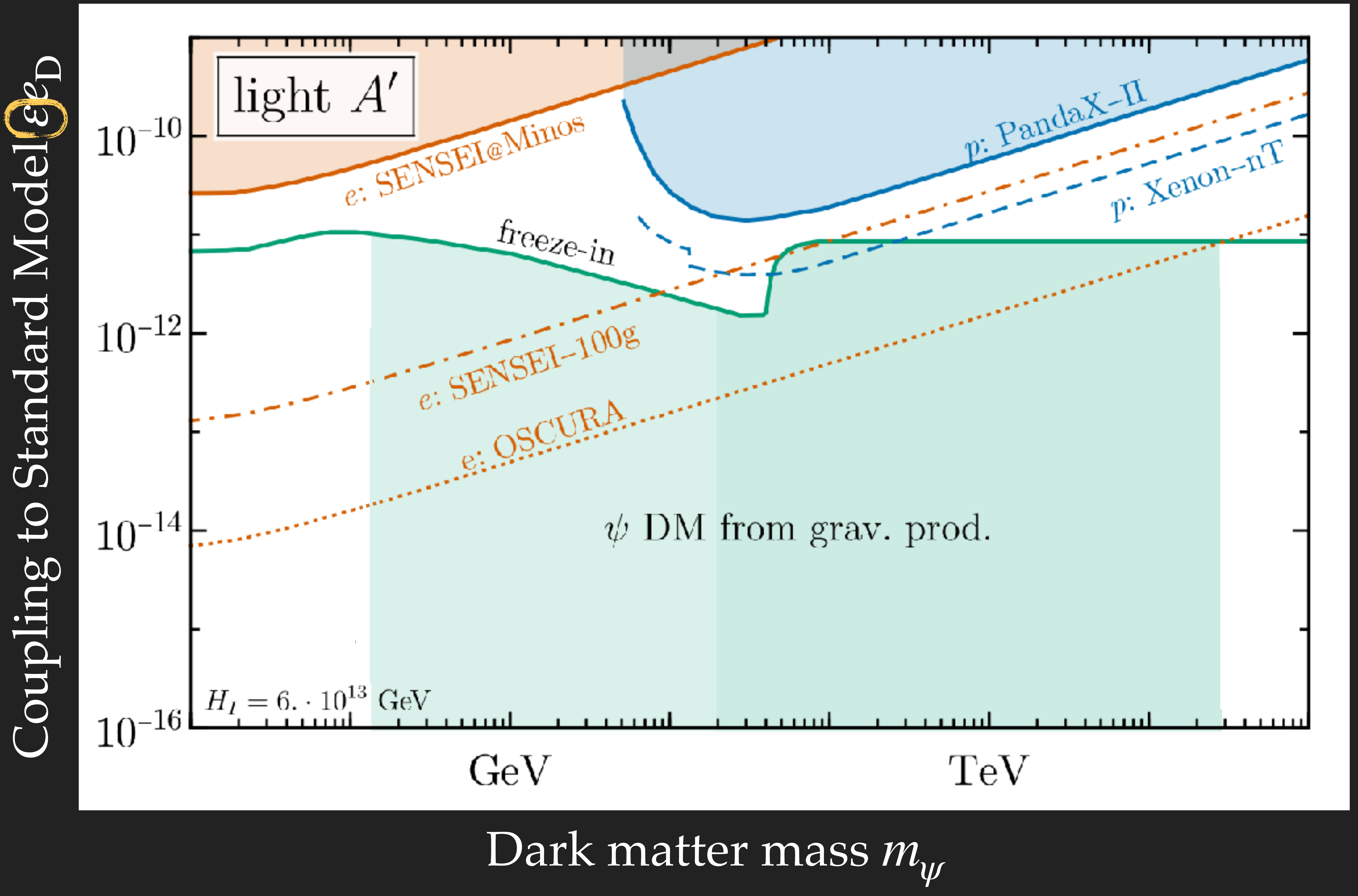
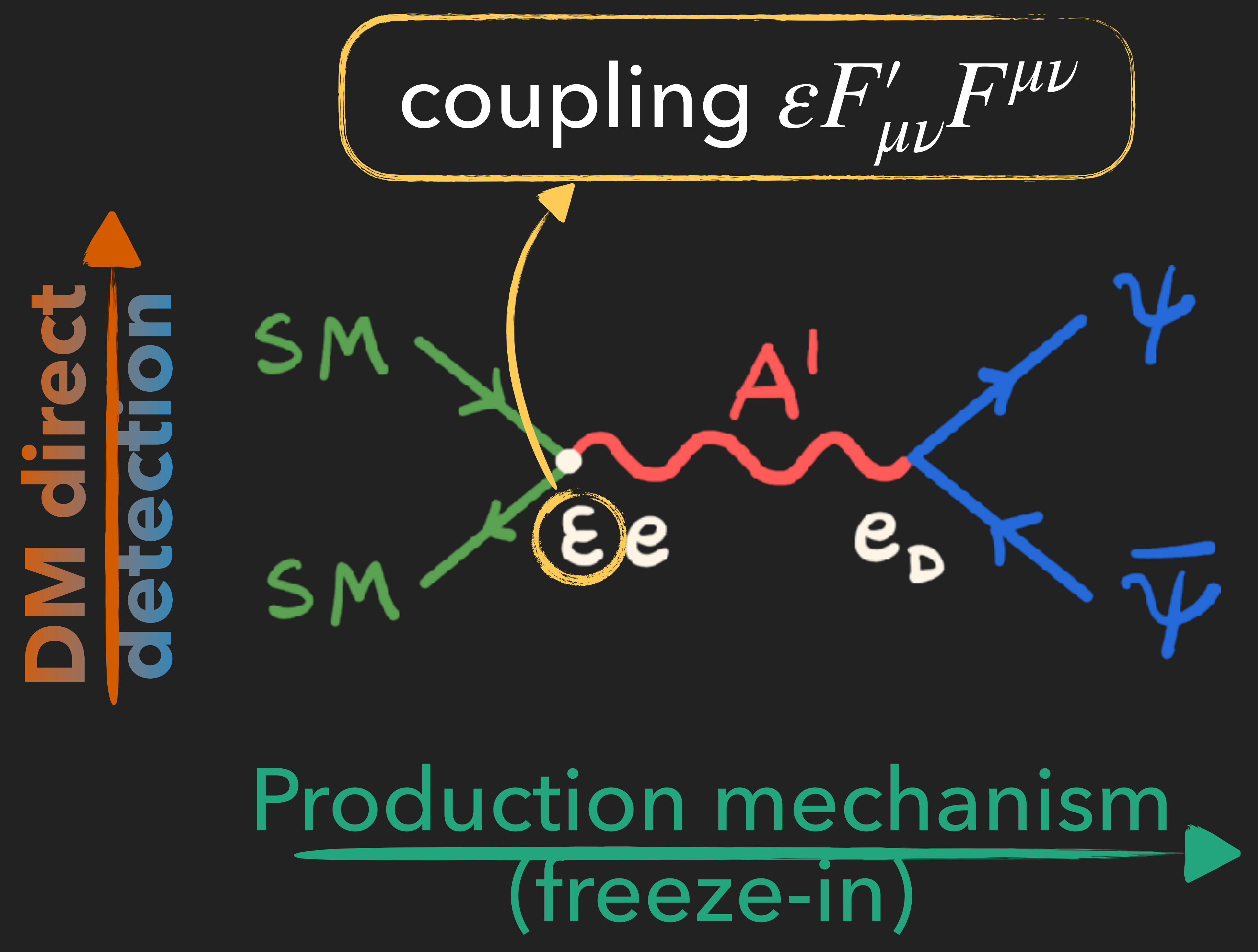
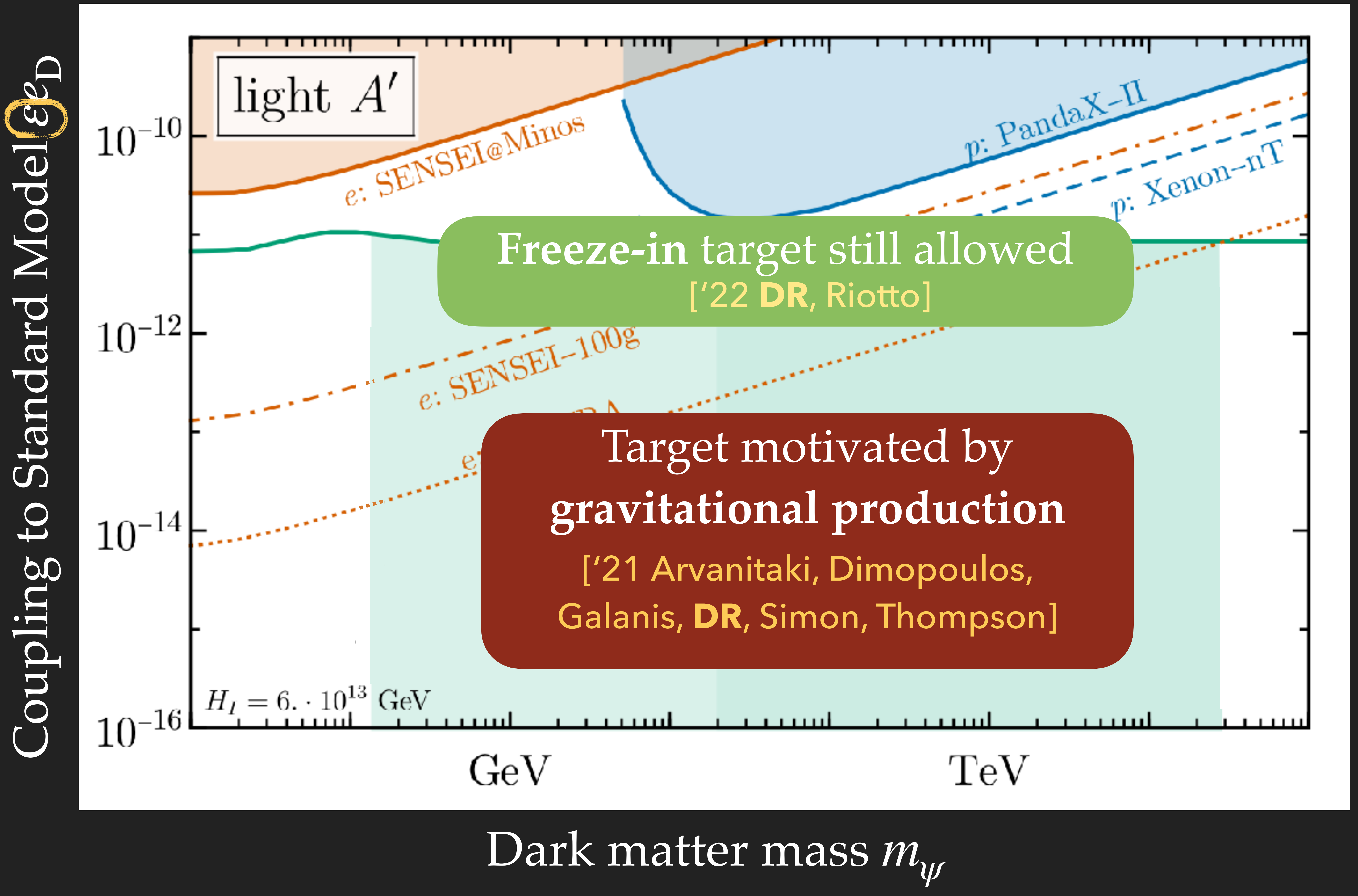

Coupling to Standard Model εe_D
light A'
10^{-10}
10^{-12}
10^{-14}
10^{-16}
e: SENSEI@Minos
p: PandaX-II
p: Xenon-nT
freeze-in
e: SENSEI-100g
e: OSCURA
ψ DM from grav. prod.
$H_I = 6. \cdot 10^{13}$ GeV
GeV
TeV
Dark matter mass m_ψ

TARGETS FOR MILLICHARGED PARTICLES






Thank you for your attention!

BACKUP SLIDES

- Choose spatial coordinates to reabsorb $\zeta_L(\mathbf{x})$ on long scales:

perturbations ζ_L on large scales

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 e^{2\zeta(\mathbf{x})}\mathrm{d}\mathbf{x}^2$$

- Choose spatial coordinates to reabsorb $\zeta_L(\mathbf{x})$ on long scales:

perturbations ζ_L on large scales → homogeneous on large scales

$$ds^2 = -\,dt^2 + a(t)^2 e^{2\zeta(\mathbf{x})} d\mathbf{x}^2$$

$$\mathbf{x}' = e^{\zeta_L(\mathbf{x})}\mathbf{x}$$

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- Super-horizon fluctuations seeded by inflation:

$$T(\mathbf{x}', t) = T_{\rm bkg}(t) e^{\zeta_L(\mathbf{x})/5}$$

- Compute any quantity in this frame:

$$\Gamma\Big(T(\mathbf{x}', t)\Big)$$

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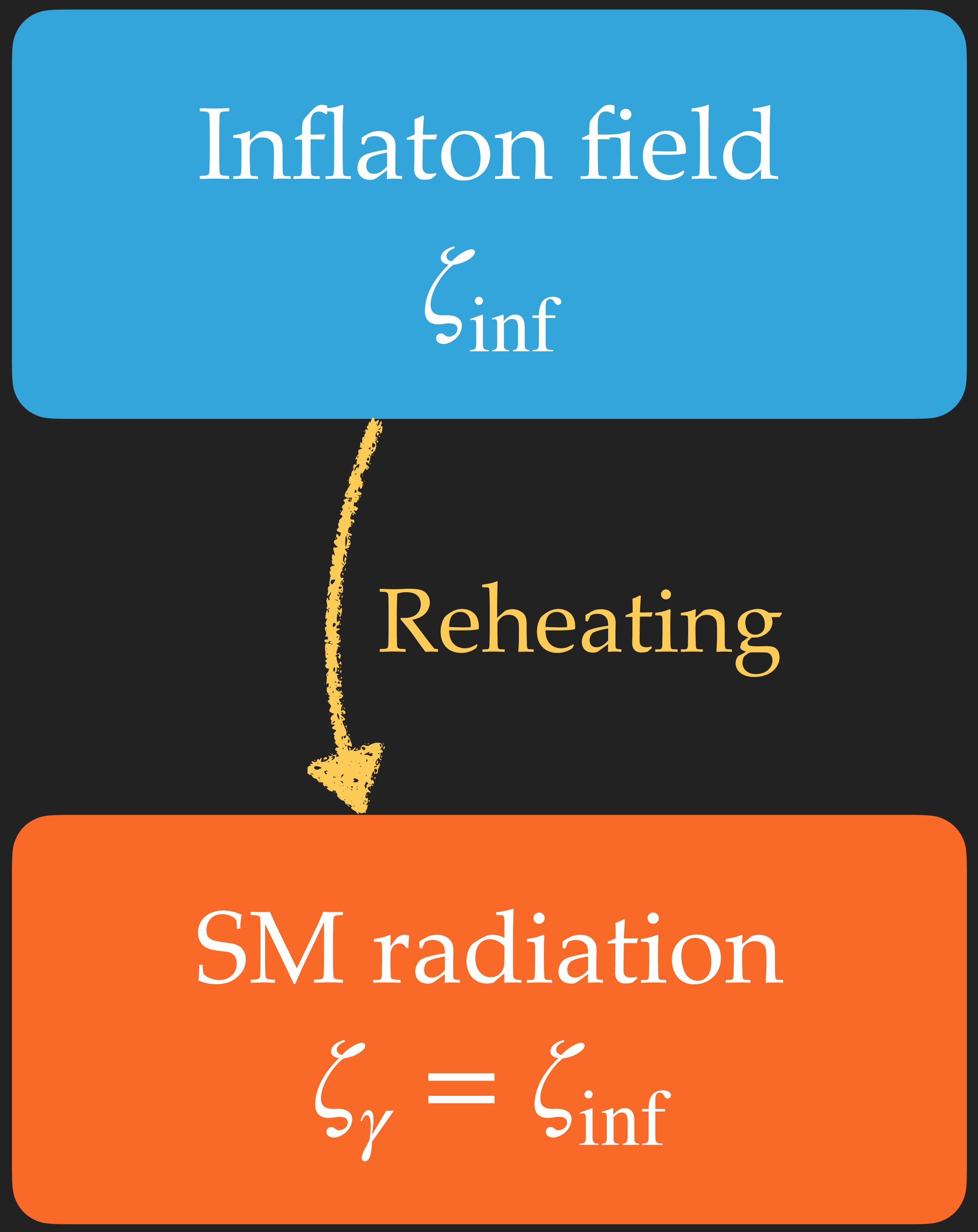
- Thermal bath:
  $\zeta_\gamma(t, \mathbf{x}) \leftrightarrow \delta T(t, \mathbf{x})$
- **Adiabatic** perturbations:
  any fluid component
  following these pert.



CURVATURE PERTURBATIONS

- Thermal bath: $\zeta_\gamma(t, \mathbf{x}) \leftrightarrow \delta T(t, \mathbf{x})$
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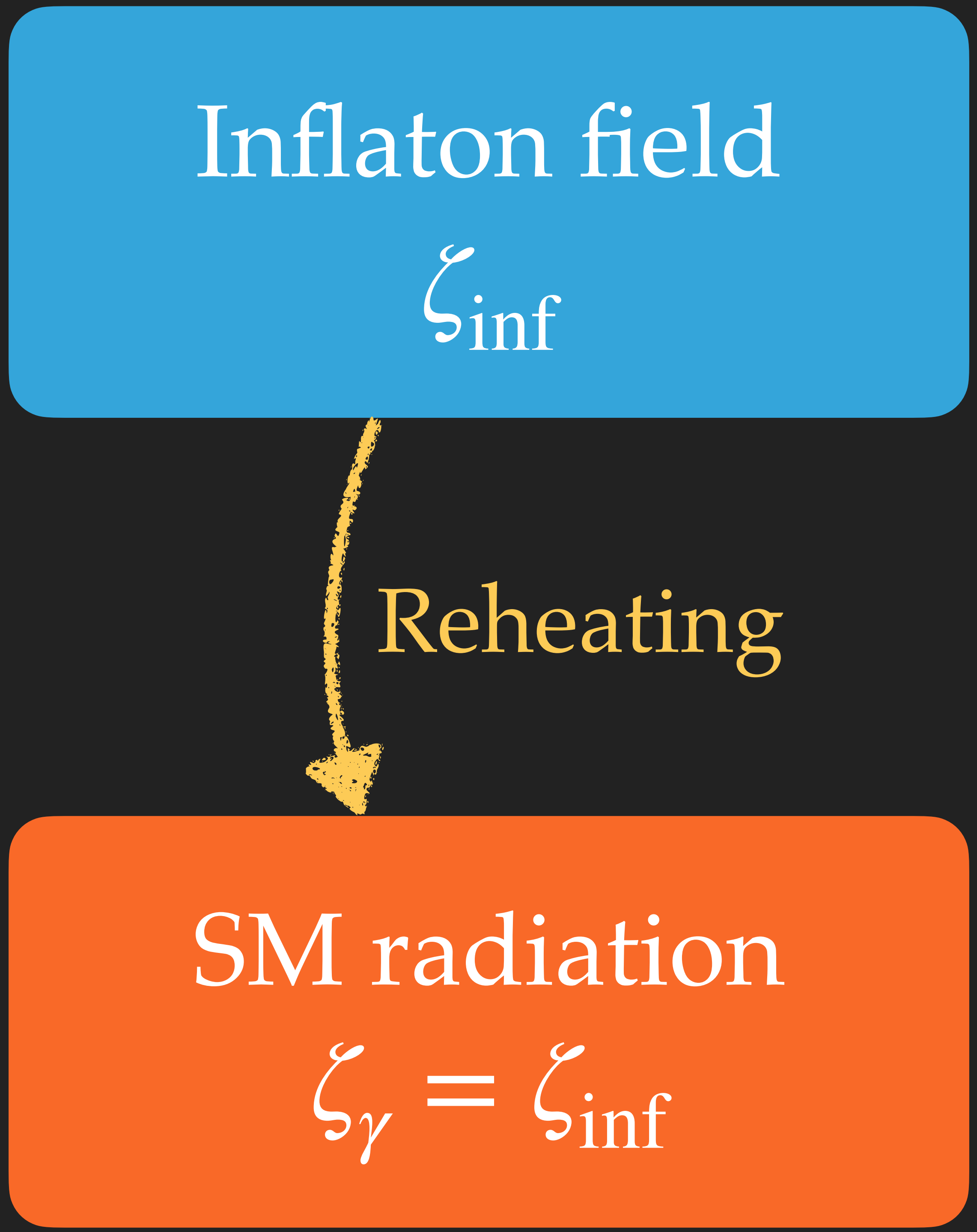


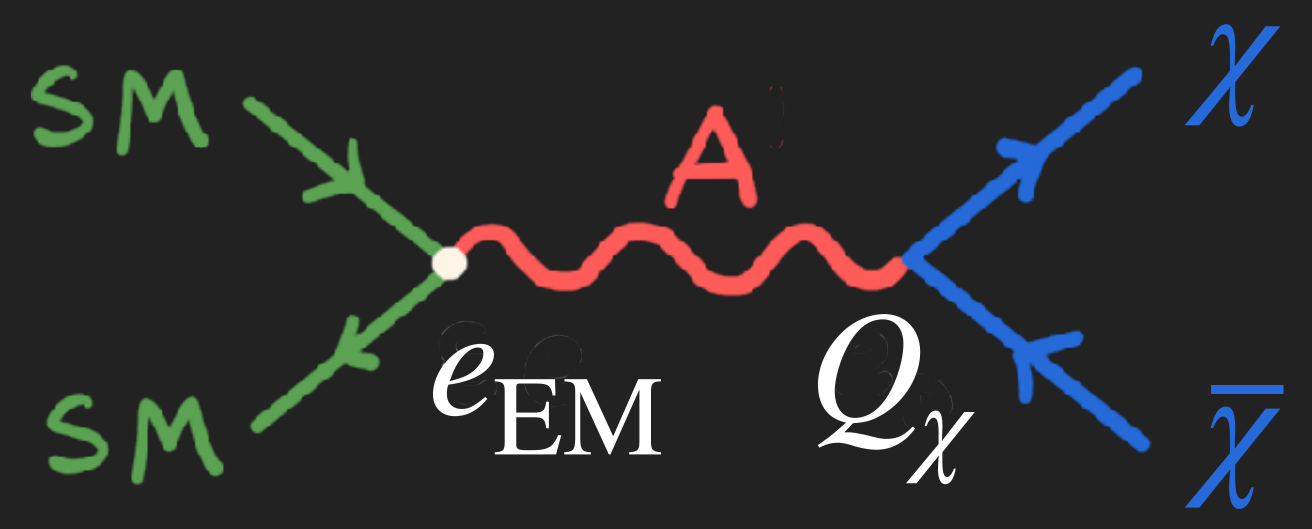
Different source of ζ?

$$\zeta_\alpha = -\psi - H \frac{\delta\rho_\alpha(t, \mathbf{x})}{\dot{\rho}_\alpha(t)}$$

$$\mathcal{S}_{\alpha,\beta} = 3(\zeta_\alpha - \zeta_\beta)$$

- **Isocurvature** perturbations: a fluid component with $\neq$ curv. pert.

CURVATURE PERTURBATIONS FOR FREEZE-IN DM



- Freeze-in DM is never in thermal equilibrium
- It originates from SM though:

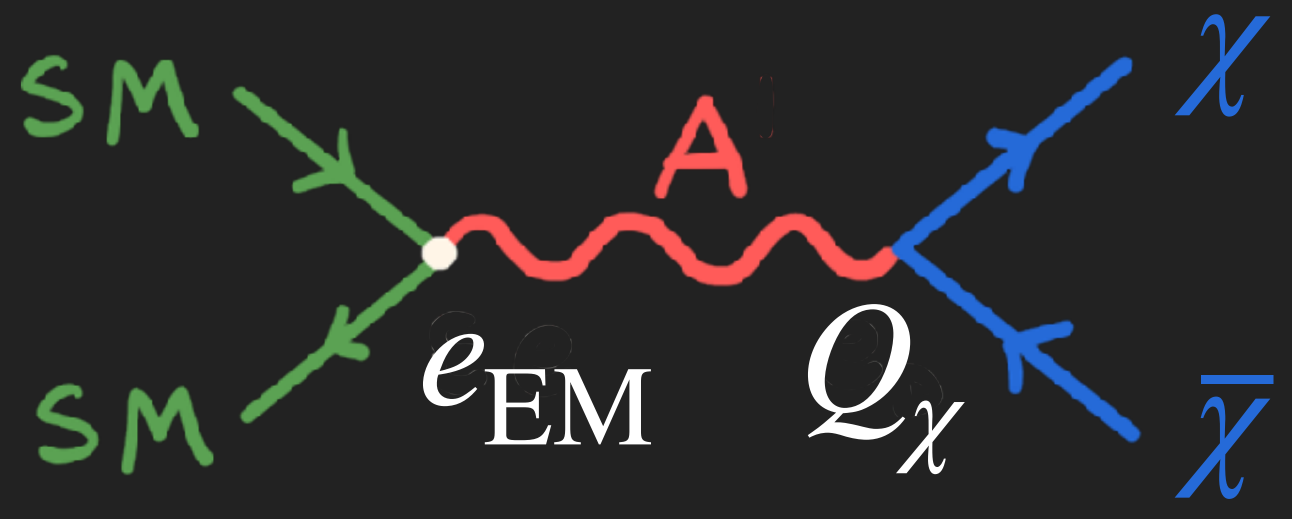
$$\dot{\rho}_{\text{DM}}(t,\mathbf{x}) = -3H\left(\rho_{\text{DM}}(t,\mathbf{x}) + P_{\text{DM}}(t,\mathbf{x})\right) + \Gamma(t,\mathbf{x})$$

must be included in $\dot{\rho} \to \zeta$

$$\Gamma = \langle T_{\text{SM}}\sigma_{\text{ann}}v_{\text{rel}}\rangle\, n_{\text{SM}}^2$$

Millicharge DM: $\Gamma = \left(\dfrac{9\alpha_{\text{EM}}Q_\chi^2\zeta(3)^2}{2\pi^4}\right) T_{\text{SM}}^5(t,\mathbf{x})$

CURVATURE PERTURBATIONS FOR FREEZE-IN DM



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['04 **Weinberg**]

SINGLE-CLOCK ARGUMENT

$$\rho_{\mathrm{DM}}(t,\mathbf{x}) \leftrightarrow T_{\mathrm{SM}}(t,\mathbf{x})$$

$$\frac{\delta\rho_{\mathrm{DM}}}{\dot{\rho}_{\mathrm{DM}}} = \frac{\delta T}{\dot{T}} = \frac{\delta\rho_\gamma}{\dot{\rho}_\gamma}$$

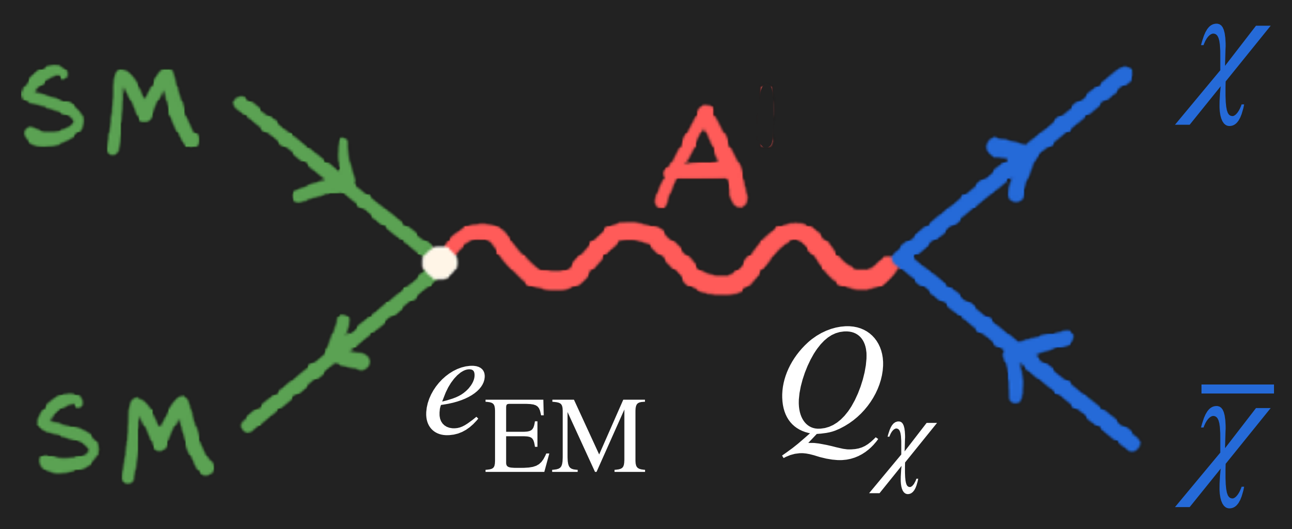
NB: regardless of thermalisation!

FREEZE-IN DM: EVOLUTION OF PERTURBATIONS

- Time evolution for energy density:

$$\dot{\rho}_{\rm DM} = -3H\left(\rho_{\rm DM} + P_{\rm DM}\right) + Q_{\rm DM}\,,$$
$$\dot{\rho}_{\gamma} = -3H\left(\rho_{\gamma} + P_{\gamma}\right) + Q_{\gamma}\,.$$

$$Q_{\rm DM} = \Gamma(\rho_\gamma)\,,$$
$$Q_\gamma = -\Gamma(\rho_\gamma)\,.$$



- Derive evolution equation for curvature perturbations on *large scales*
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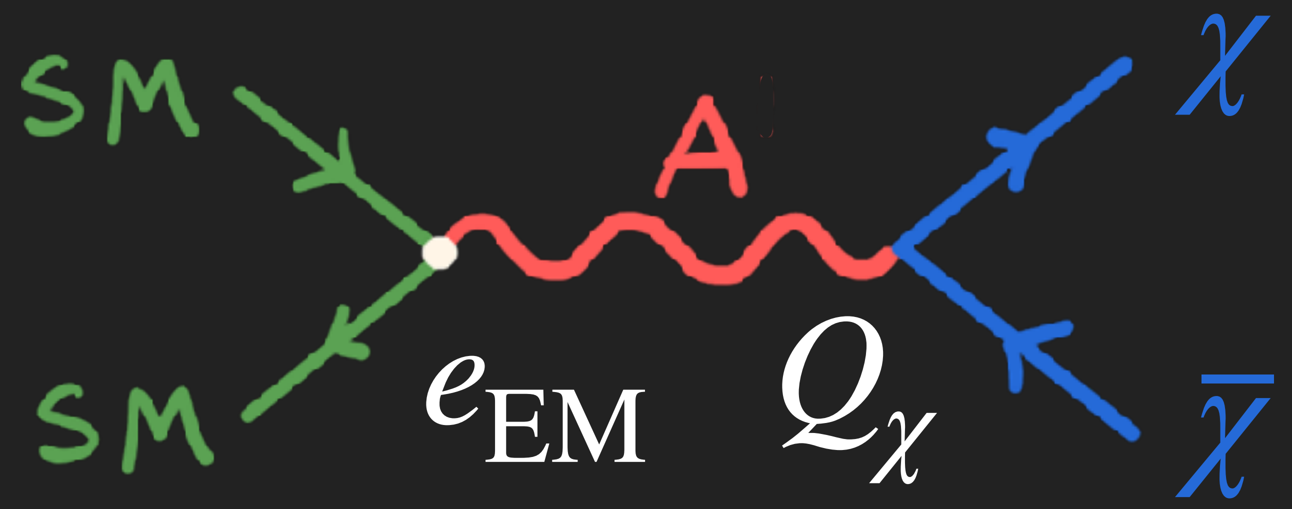

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$$P_{\mathrm{DM}} = P_{\mathrm{DM}}(\rho_{\mathrm{DM}}) \rightsquigarrow P_{\mathrm{DM}}(T_{\mathrm{SM}}) \qquad \text{DM pressure}$$

$$Q_{\mathrm{DM}} = Q_{\mathrm{DM}}(\rho_{\mathrm{SM}}) \rightsquigarrow Q_{\mathrm{DM}}(T_{\mathrm{SM}}) \qquad \text{Energy transfer rate}$$

- Single-clock argument: T_{SM} only source of perturbations here

['22 **DR**, Riotto]

- Gauge-invariant result:

$$\dot{\zeta}_{\rm DM} = \frac{3H^2}{\dot{\rho}_{\rm DM}} \delta P_{\rm intr,DM} - \frac{H}{\dot{\rho}_{\rm DM}} \left(\delta Q_{\rm intr,DM} + \delta Q_{\rm rel,DM} \right)$$

$$\dot{\zeta}_{\gamma} = \frac{3H^2}{\dot{\rho}_{\gamma}} \delta P_{\rm intr,\gamma} - \frac{H}{\dot{\rho}_{\gamma}} \left(\delta Q_{\rm intr,\gamma} + \delta Q_{\rm rel,\gamma} \right)$$

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$$\delta P_{\rm intr,DM} = \delta P_{\rm DM} - \frac{\dot{P}_{\rm DM}}{\dot{\rho}_{\rm DM}} \delta \rho_{\rm DM}$$

$$\delta P_{\rm intr,\gamma} = \delta P_{\gamma} - \frac{\dot{P}_{\gamma}}{\dot{\rho}_{\gamma}} \delta \rho_{\gamma}$$

- Intrinsic non-adiabatic pressure perturbation
- Vanish when $P_{\rm DM}$ is only a function of $\rho_{\rm DM}$

- Gauge-invariant result:

$$\dot{\zeta}_{\rm DM} = \frac{3H^2}{\dot{\rho}_{\rm DM}}\delta P_{\rm intr,DM} - \frac{H}{\dot{\rho}_{\rm DM}}\left(\delta Q_{\rm intr,DM} + \delta Q_{\rm rel,DM}\right)$$

$$\dot{\zeta}_\gamma = \frac{3H^2}{\dot{\rho}_\gamma}\delta P_{\rm intr,\gamma} - \frac{H}{\dot{\rho}_\gamma}\left(\delta Q_{\rm intr,\gamma} + \delta Q_{\rm rel,\gamma}\right)$$

$$\delta P_{\rm intr,DM} = \delta P_{\rm DM} - \frac{\dot{P}_{\rm DM}}{\dot{\rho}_{\rm DM}}\delta\rho_{\rm DM}$$

$$\delta P_{\rm intr,\gamma} = \delta P_\gamma - \frac{\dot{P}_\gamma}{\dot{\rho}_\gamma}\delta\rho_\gamma$$

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- Intrinsic non-adiabatic pressure perturbation
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- If Γ is only a function of $T_{\rm SM}$, it's $\propto \mathcal{S}_{{\rm DM}\gamma}$

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- If Γ is only a function of $T_{\rm SM}$, it's $\propto \mathcal{S}_{\rm DM\gamma}$

$$\delta Q_{\rm rel,DM} = \frac{Q_{\rm DM}\dot\rho}{2\rho}\left(\frac{\delta\rho_{\rm DM}}{\dot\rho_{\rm DM}} - \frac{\delta\rho}{\dot\rho}\right) = -\frac{Q_{\rm DM}}{6H\rho}\dot\rho_\gamma\mathcal{S}_{\rm DM,\gamma}$$

$$\delta Q_{\rm rel,\gamma} = \frac{Q_\gamma\dot\rho}{2\rho}\left(\frac{\delta\rho_\gamma}{\dot\rho_\gamma} - \frac{\delta\rho}{\dot\rho}\right) = \frac{Q_\gamma}{6H\rho}\dot\rho_{\rm DM}\mathcal{S}_{\rm DM,\gamma}$$

- Non-adiabatic perturbed energy transfer
- It is $\propto \mathcal{S}_{\rm DM\gamma}$

FREEZE-IN DM: NO ISOCURVATURE ON LARGE SCALES

- Final result:

$$\Gamma = \langle T_{\rm SM}\sigma_{\rm ann}v_{\rm rel}\rangle\, n_{\rm SM}^2$$

$$\dot{\mathcal{S}}_{\rm DM,\gamma} = \frac{3\dot\rho}{\dot\rho_{\rm DM}^2}\left(\frac{\dot\rho_\gamma^2 - \dot\rho_{\rm DM}^2}{\dot\rho_\gamma}\frac{\Gamma}{2\rho} - \dot\Gamma\right)(\zeta - \zeta_\gamma) \propto \mathcal{S}_{\rm DM,\gamma}$$

- Isocurvature can be only sourced by itself, **and** only if $\Gamma \neq 0$
- It is exactly zero on large scales, so it remains zero:

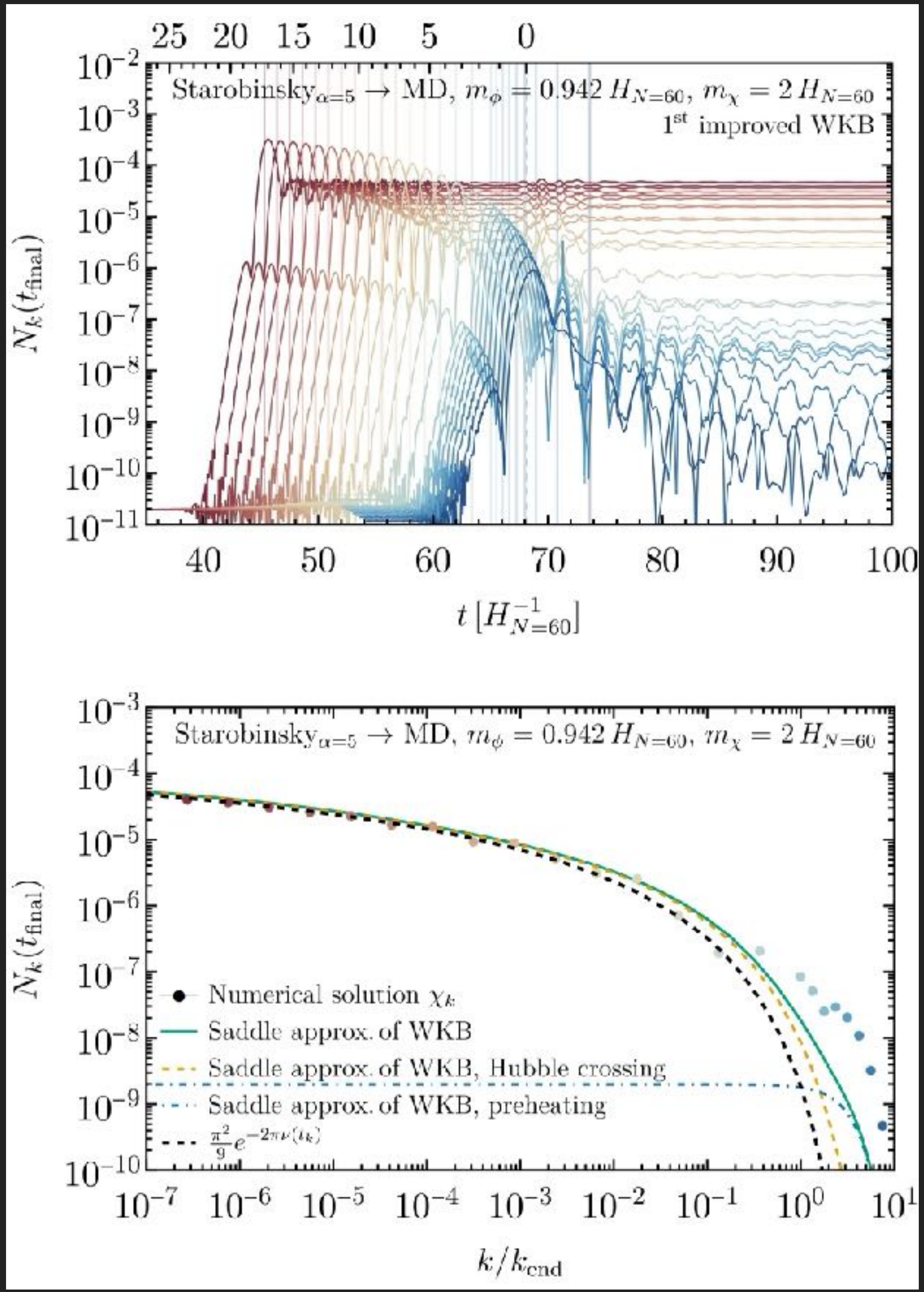
$$\zeta(\mathbf{x}, t \ll t_{\rm F-IN}) = \zeta_\gamma(\mathbf{x}, t \ll t_{\rm F-IN}) \qquad \mathcal{S}_{\rm DM,\gamma}(\mathbf{x}, t \gg t_{\rm F-IN}) = 0$$

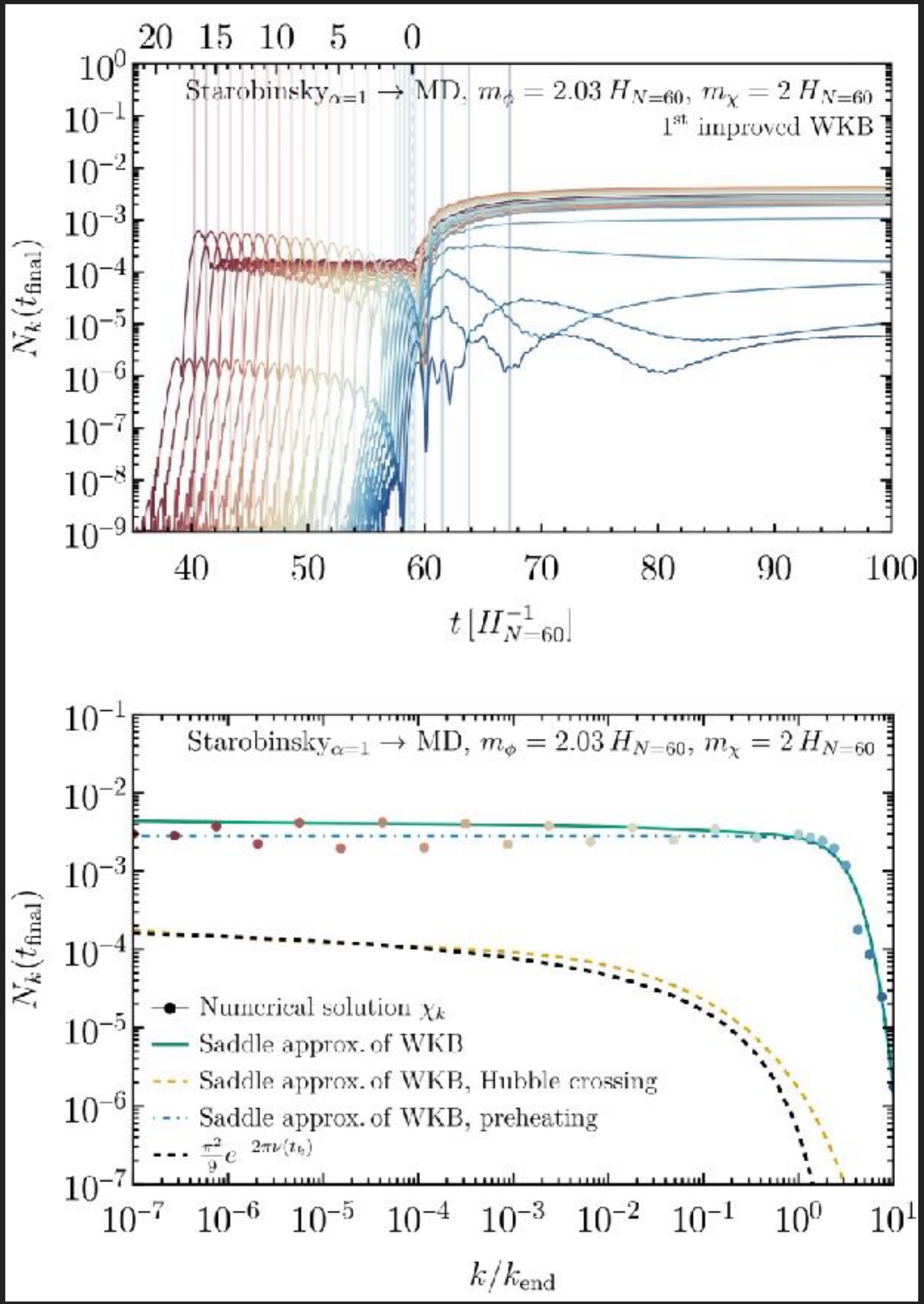
- Large gap of scales ($> 10^6$) between horizon and freeze-in and CMB scales

GRAVITATIONAL PRODUCTION OF SUPERHEAVY DM

['24 **DR**, Verner, Xue]

Production at Horizon crossing during inflation





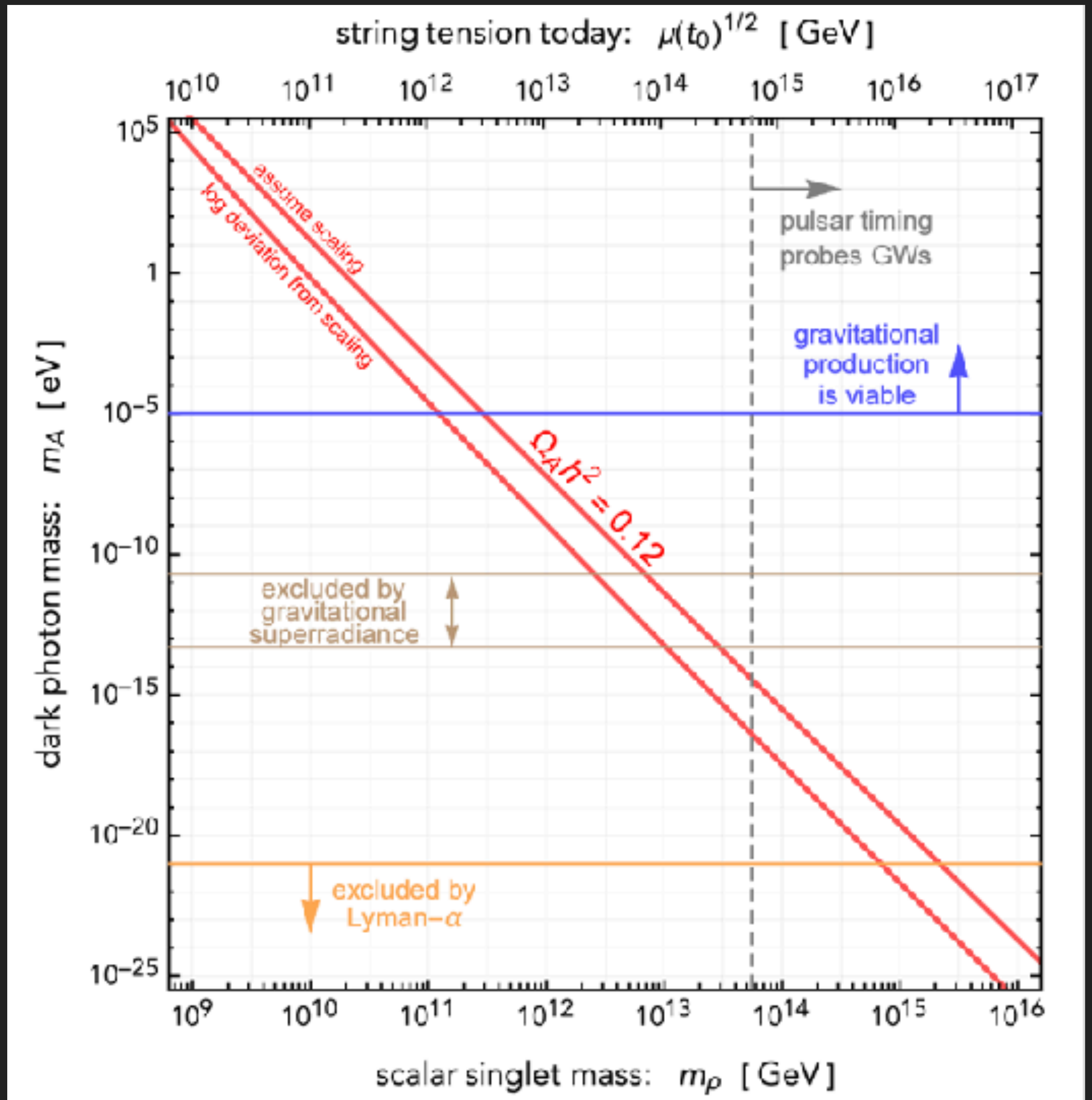
Production with inflaton oscillations

MASS OF DARK PHOTON FROM DARK HIGGS?

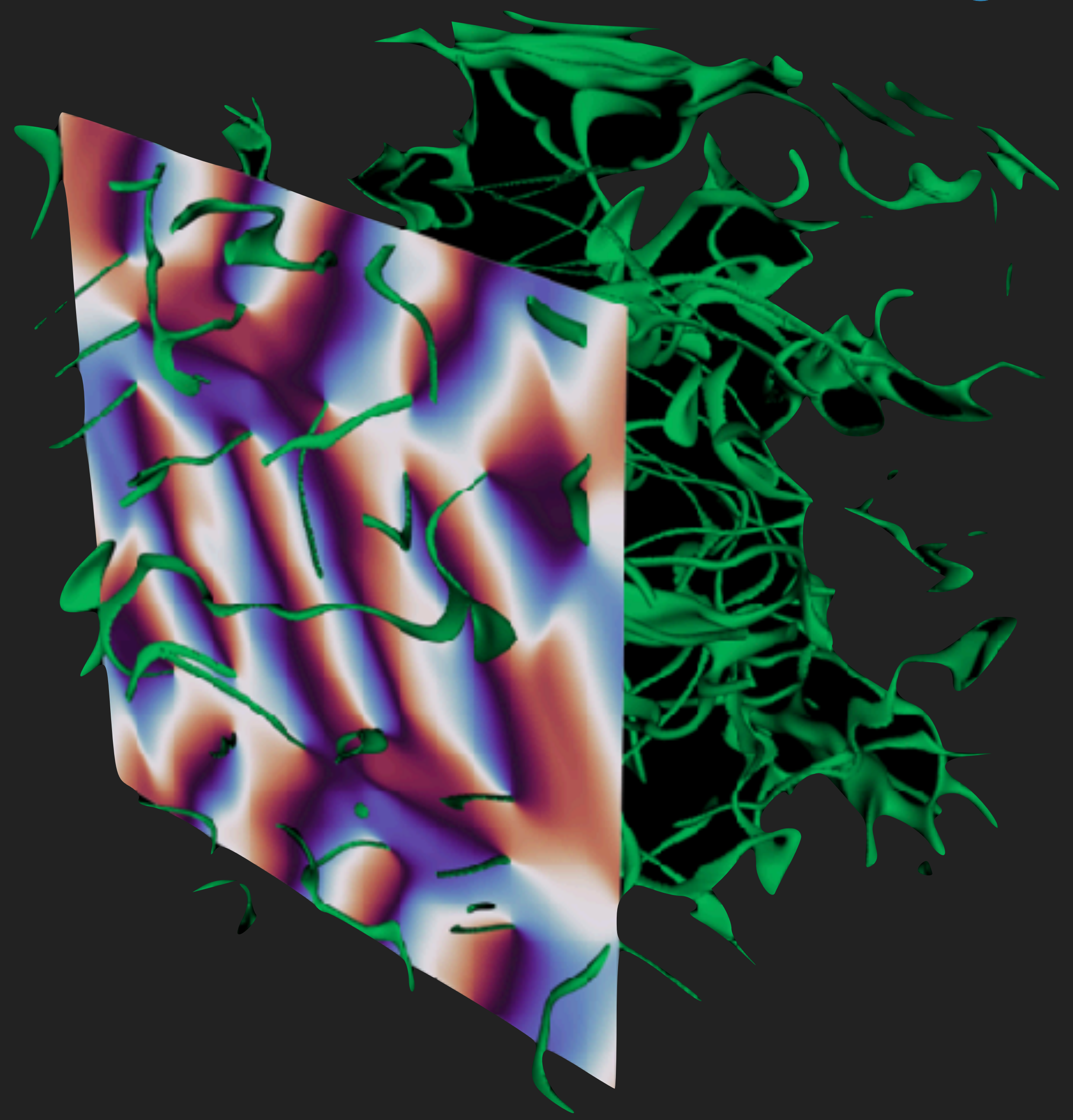
- We consider pure Stückelberg mass: Proca theory, perfectly valid QFT
- If assume Higgs mechanism, string network produced if

$$v < H_I \to \sim g > \frac{m_A}{H_I}$$

['19 Long, Wang]

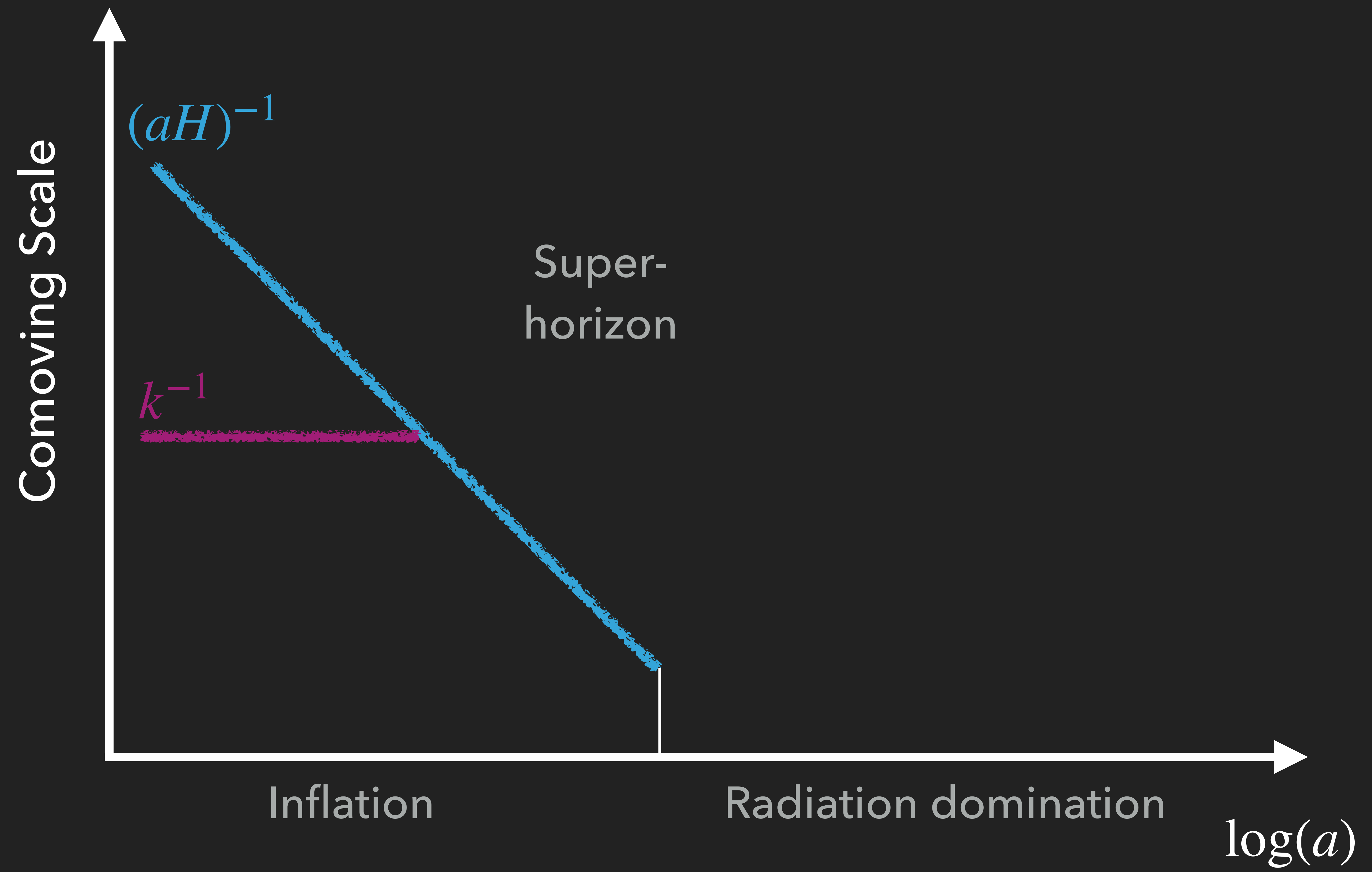


['22 Redi, Tesi]
['22 East, Huang]



- For dark QED, even assuming dark Higgs: ψ can be DM for $e_D \gg g$, or A' is DM with $e_D \gtrsim g$ ($H_I \lesssim \mathcal{O}(10)$ GeV, $m_{A'} \sim \mathcal{O}(1)$ GeV)
- This constraint is milder in dark QED than pure A': we can afford much larger $m_{A'}$

VECTOR DARK MATTER — LONGITUDINAL A'_L



- $\rho_{k,\mathrm{exit}} \sim H_I^4$, the mode $A'_{L,k}$ freezes

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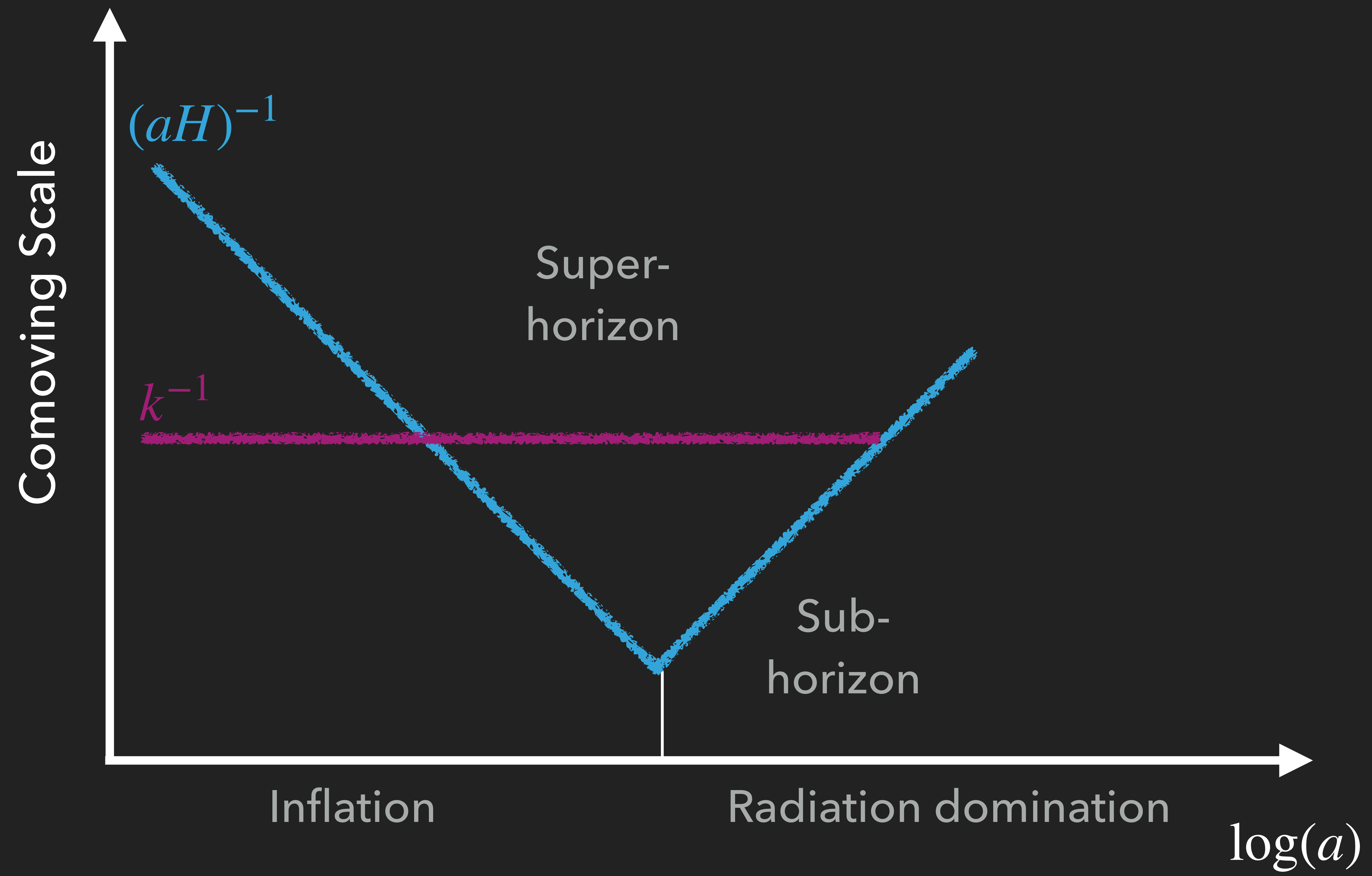
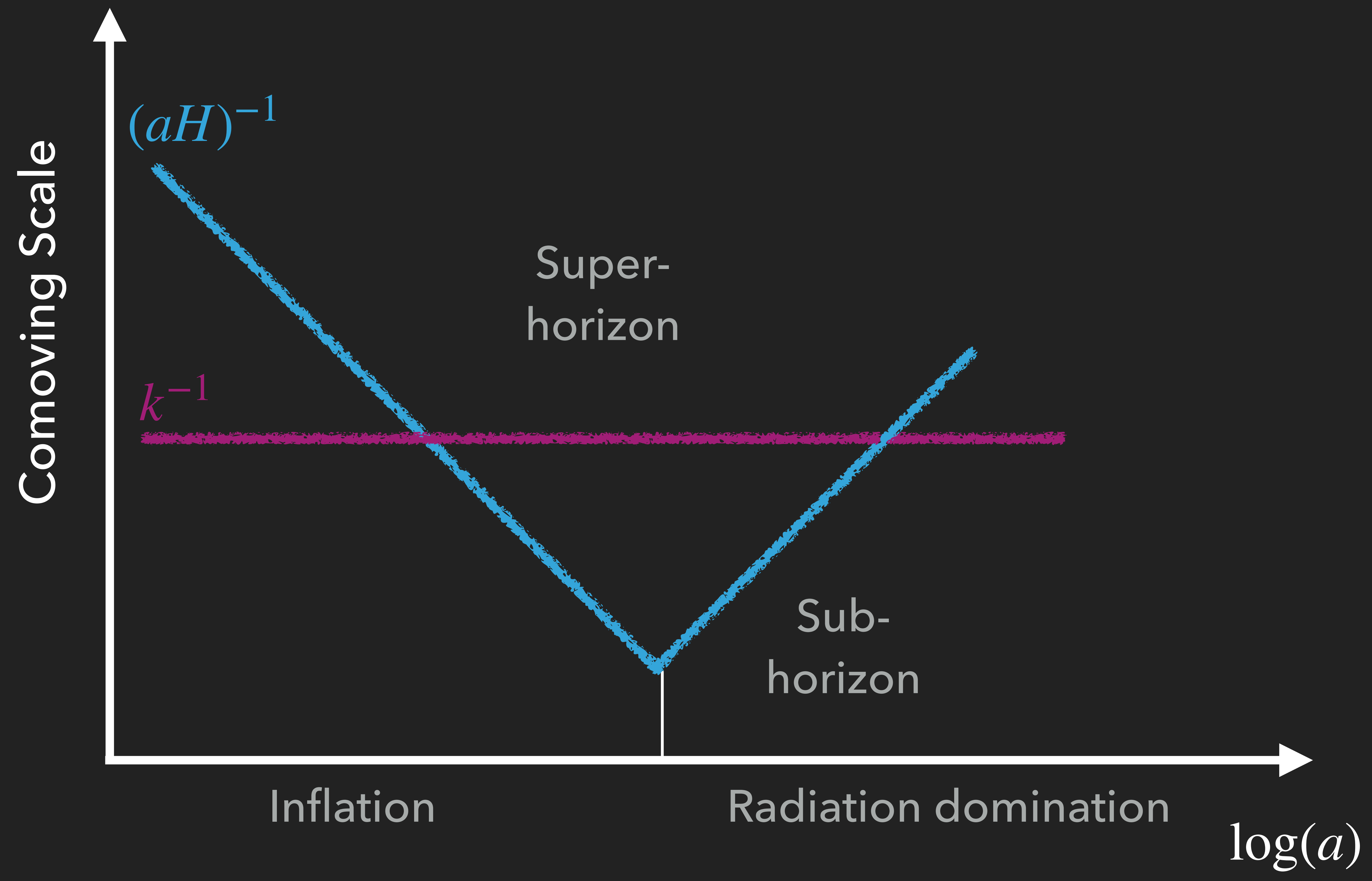


- $\rho_{k,\mathrm{exit}} \sim H_I^4$, the mode $A'_{L,k}$ freezes
- Super-horizon: $\rho_k \sim m_{A'}^2 A'^2 \sim a^{-2}$

['15 Graham, Mardon, Rajendran]



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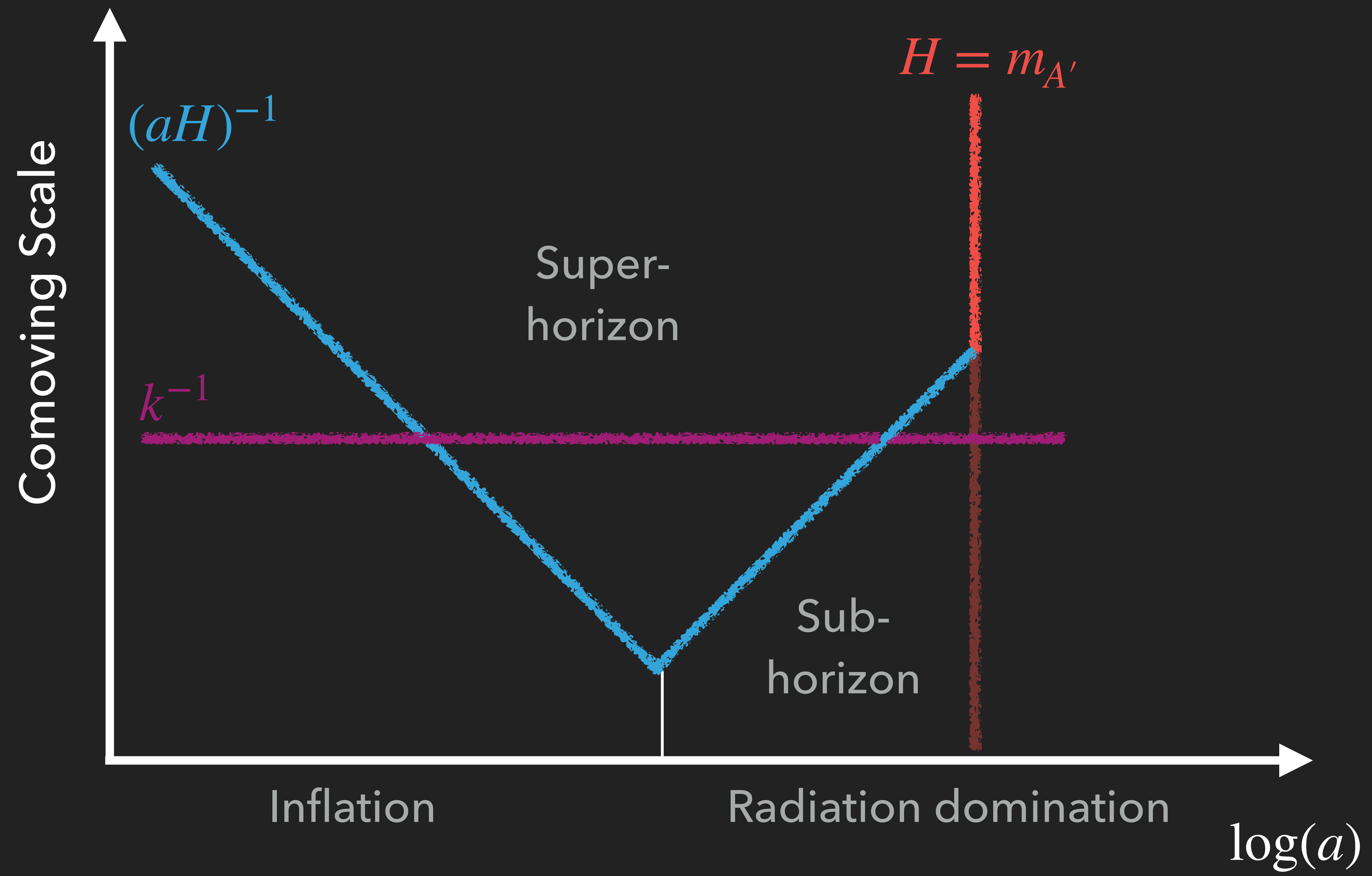

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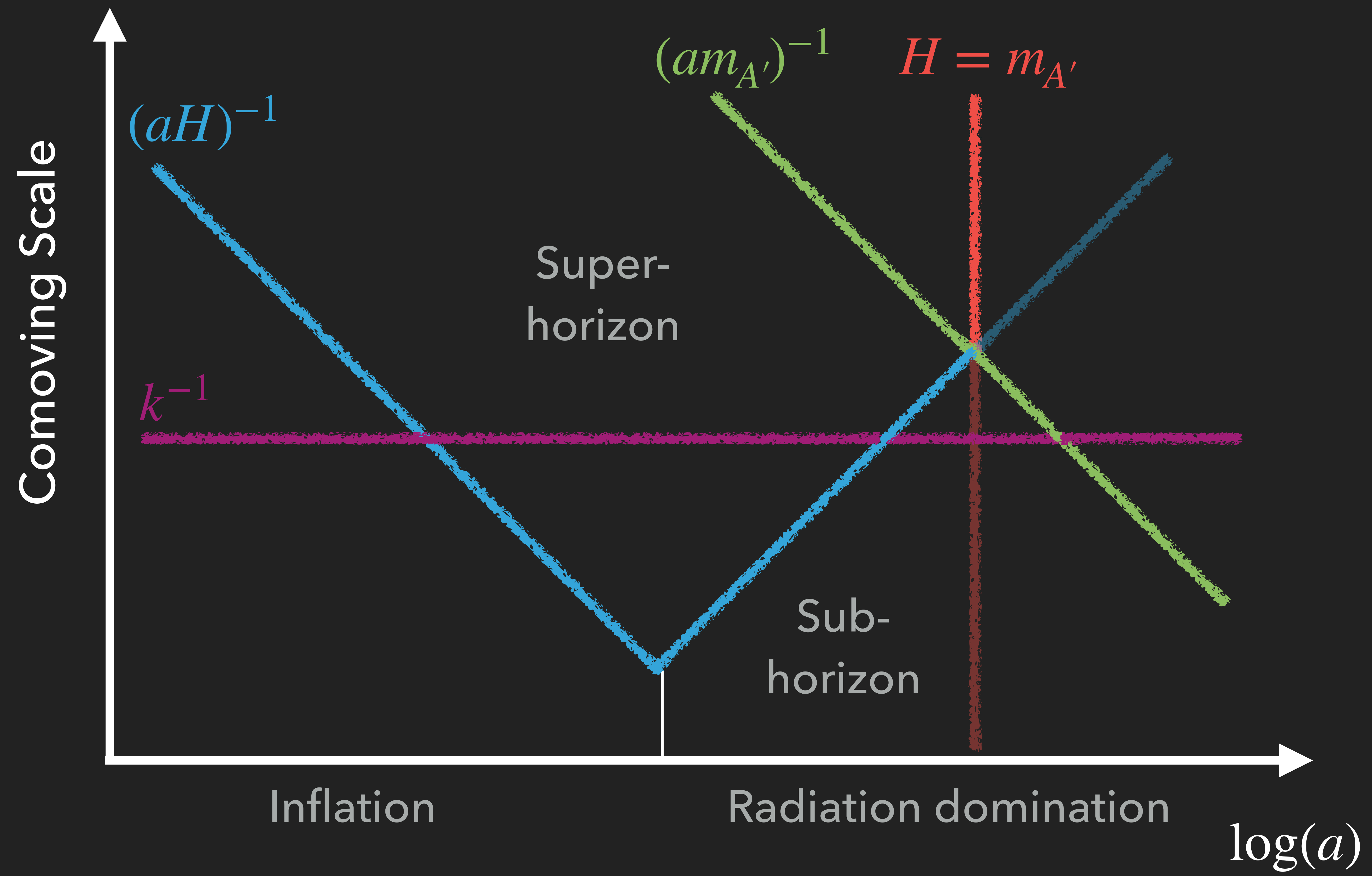

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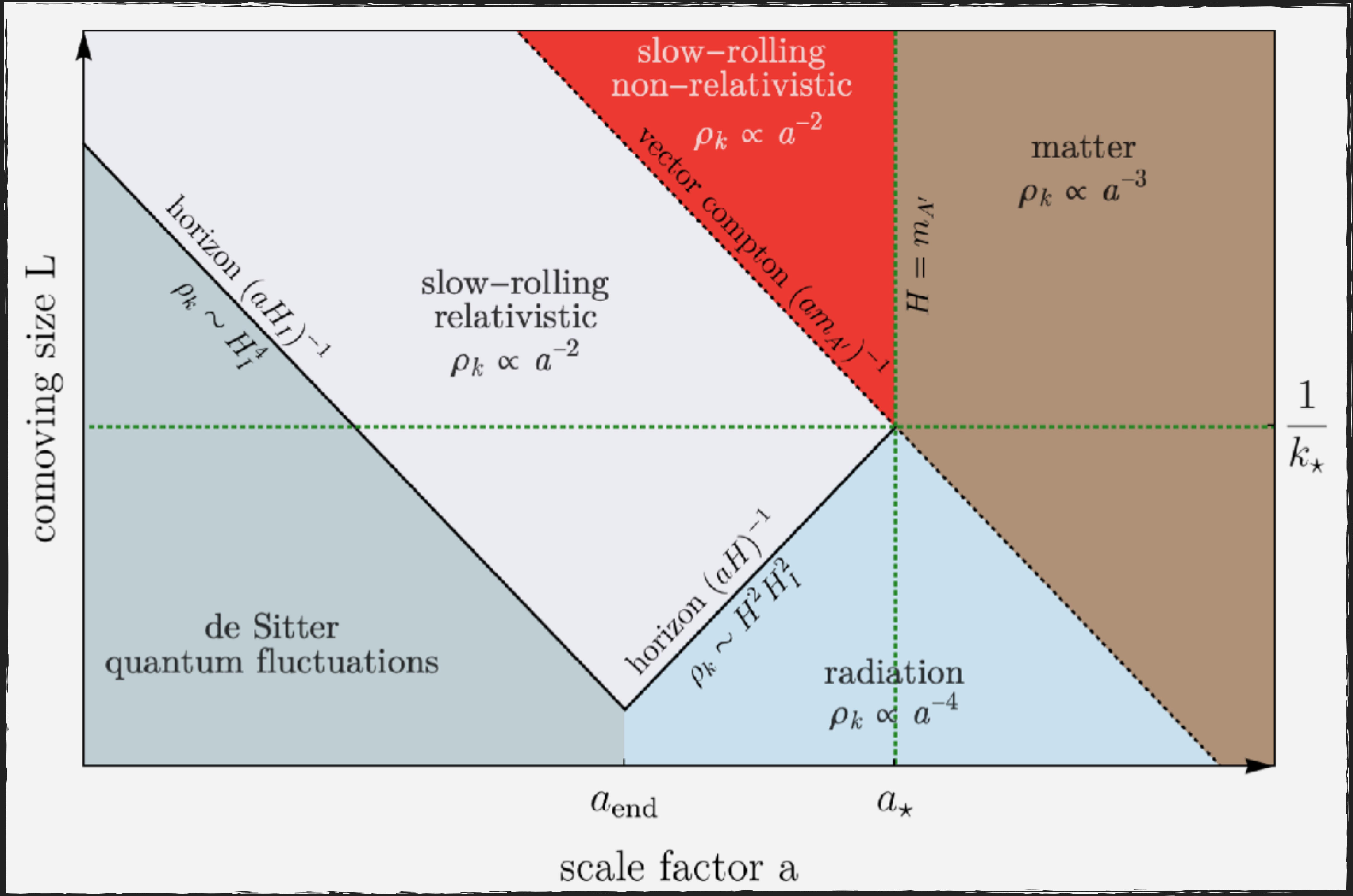
- Hor. entry: oscillation, $\rho_k \sim a^{-4}$
- Time $H = m_{A'}$: all modes oscillate
- Mode non-relativistic $\rho_k \sim a^{-3}$

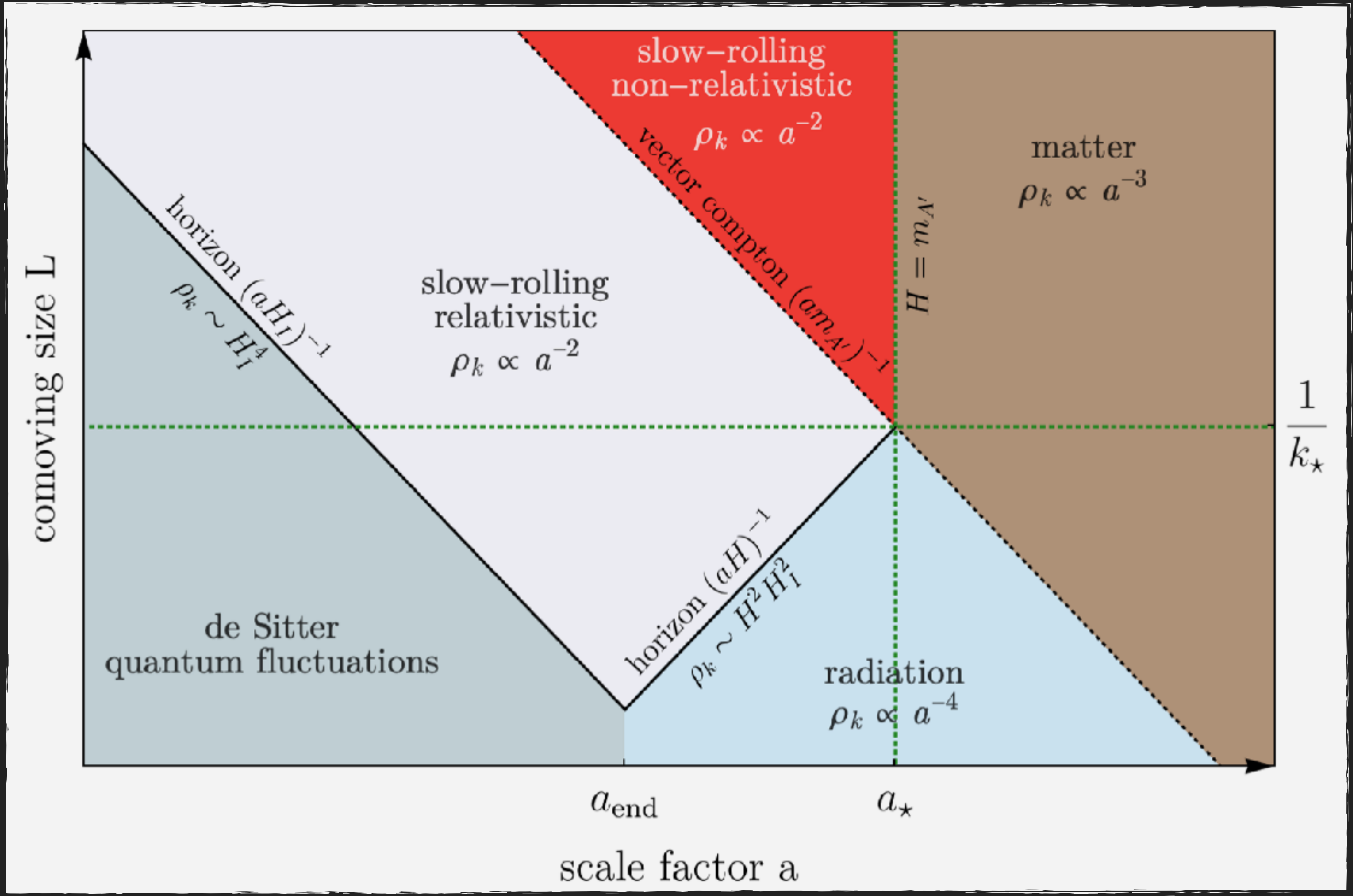
VECTOR DARK MATTER — LONGITUDINAL A'_L

['15 Graham, Mardon, Rajendran]



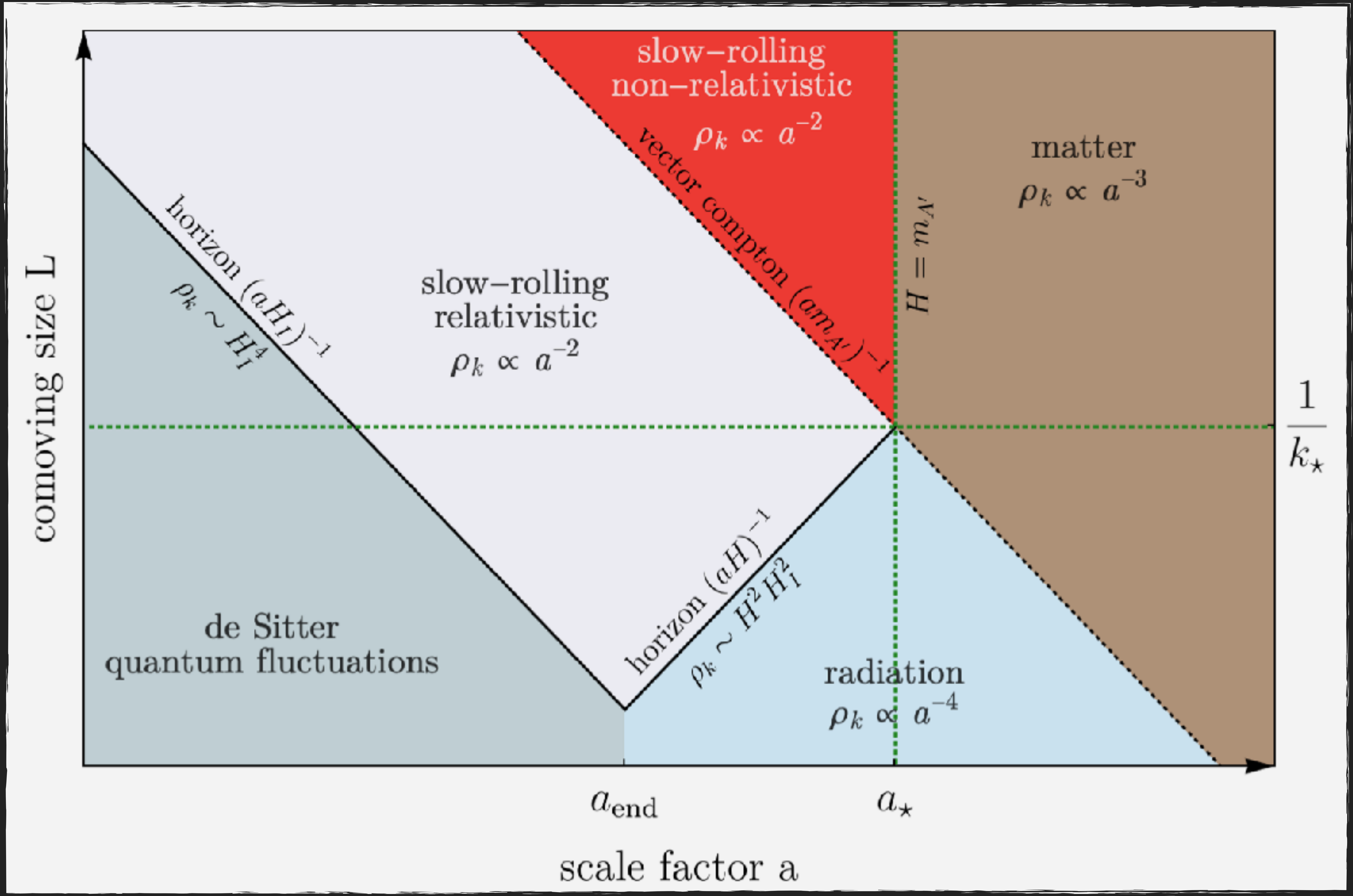
VECTOR DARK MATTER — LONGITUDINAL A'_L

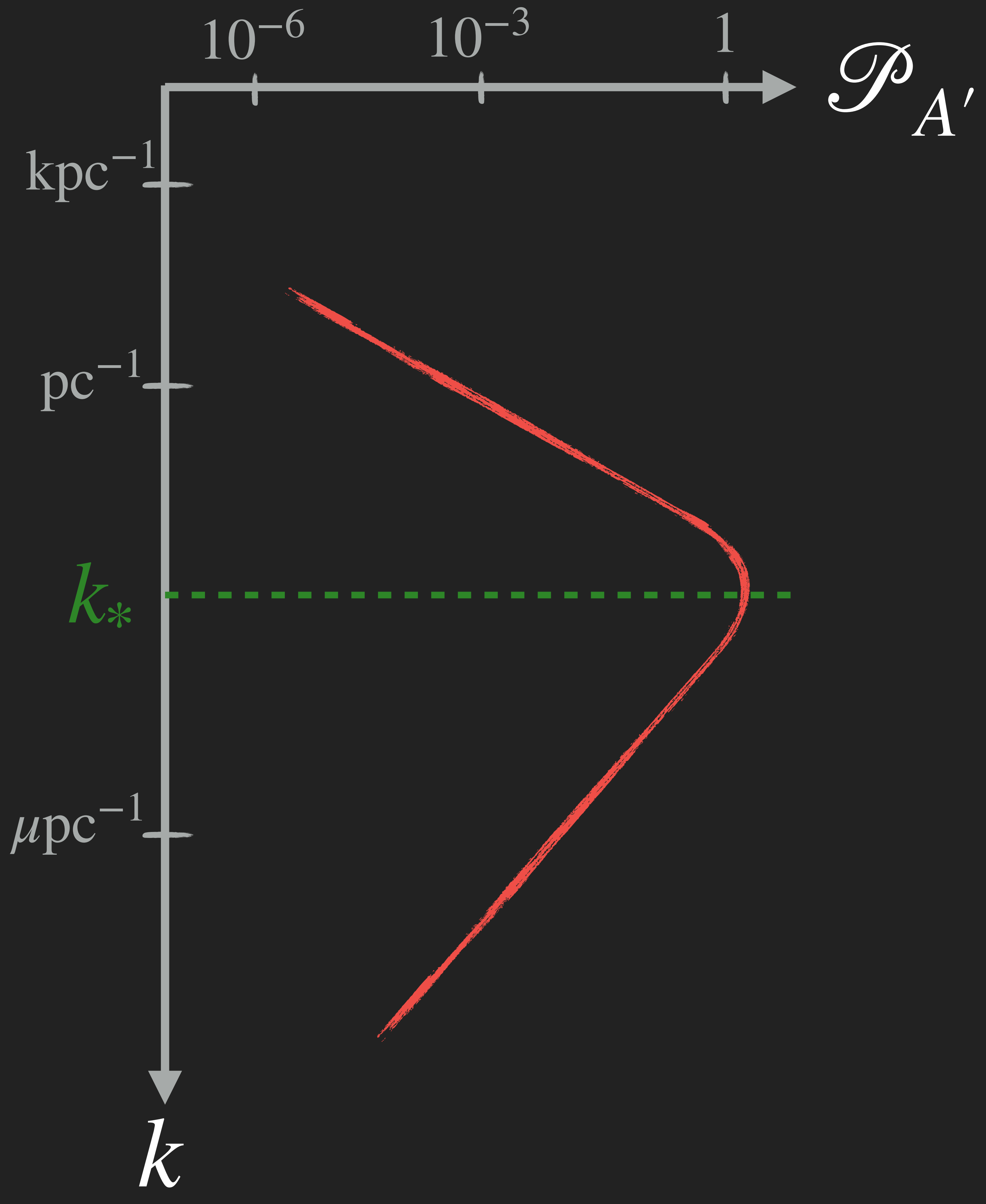
['15 Graham, Mardon, Rajendran]



$$\frac{\Omega_{A'}}{\Omega_{\rm DM}} \sim \sqrt{\frac{m_{A'}}{5\cdot 10^{-5}\,\text{eV}}}\left(\frac{H_I}{6\cdot 10^{13}\,\text{GeV}}\right)^2$$

['15 Graham, Mardon, Rajendran]





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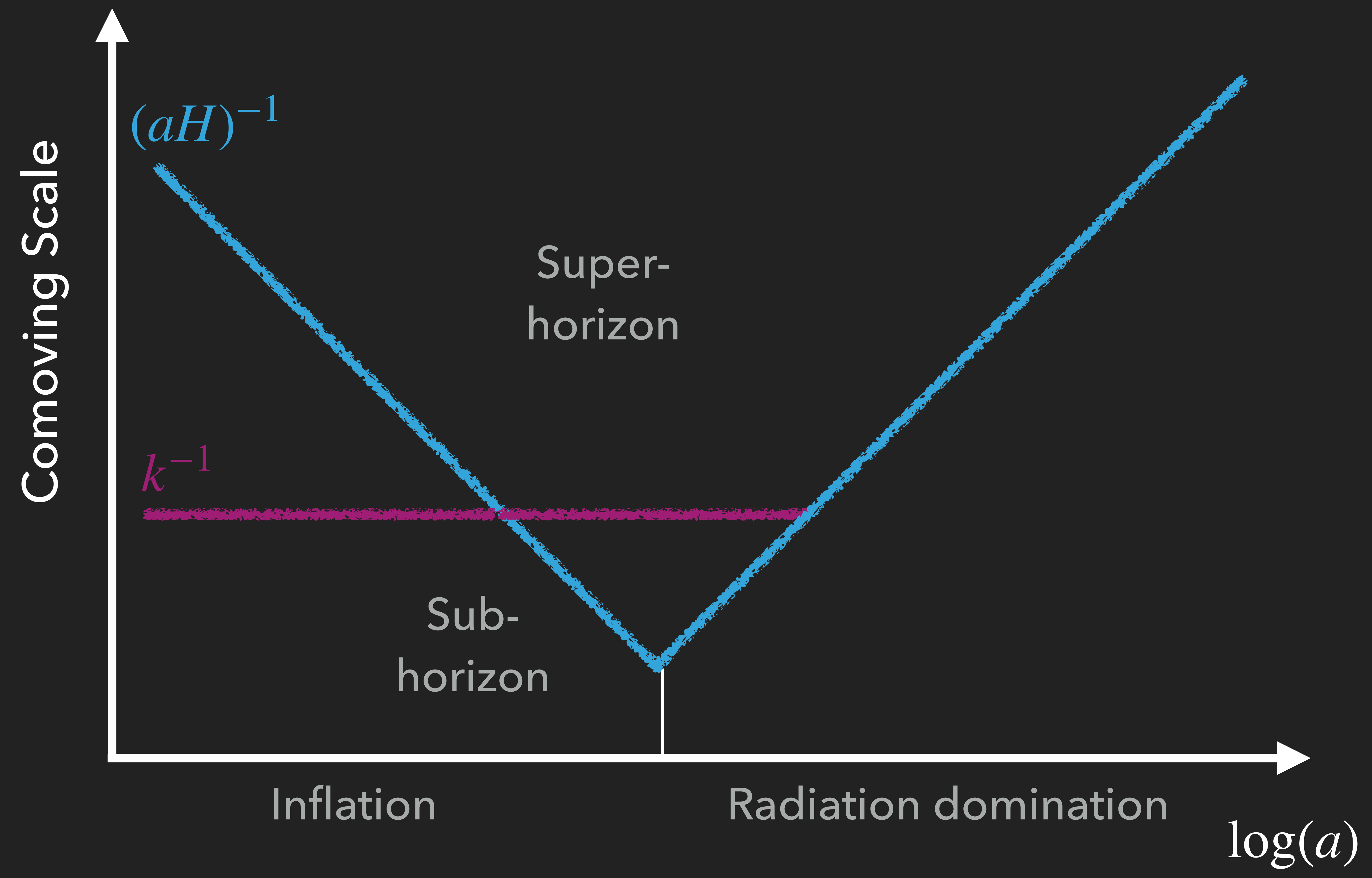
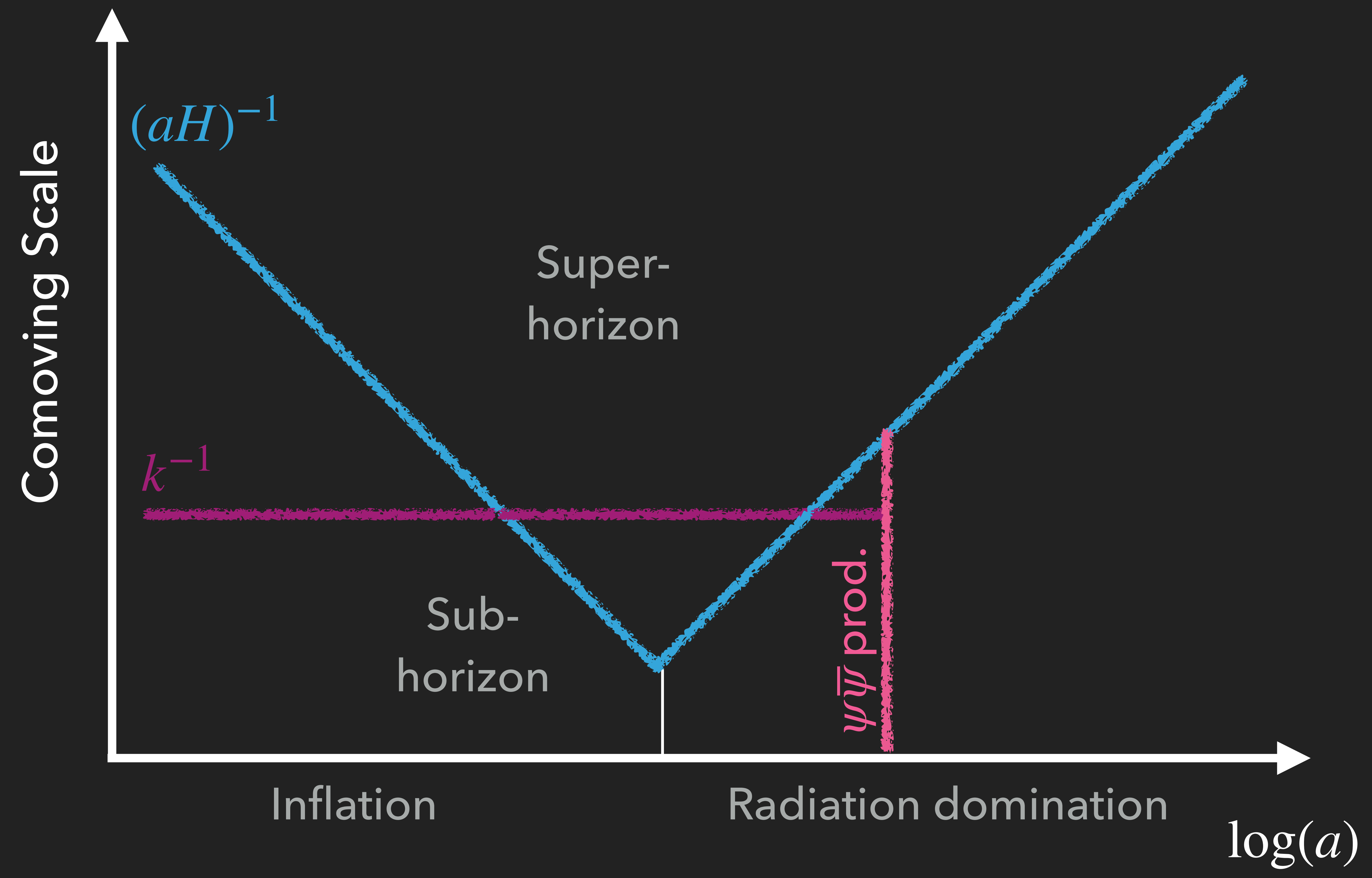
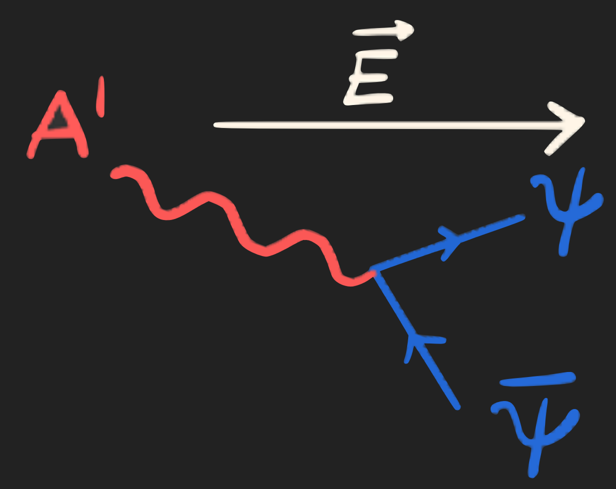
DARK MASSIVE QED: COSMOLOGICAL HISTORY



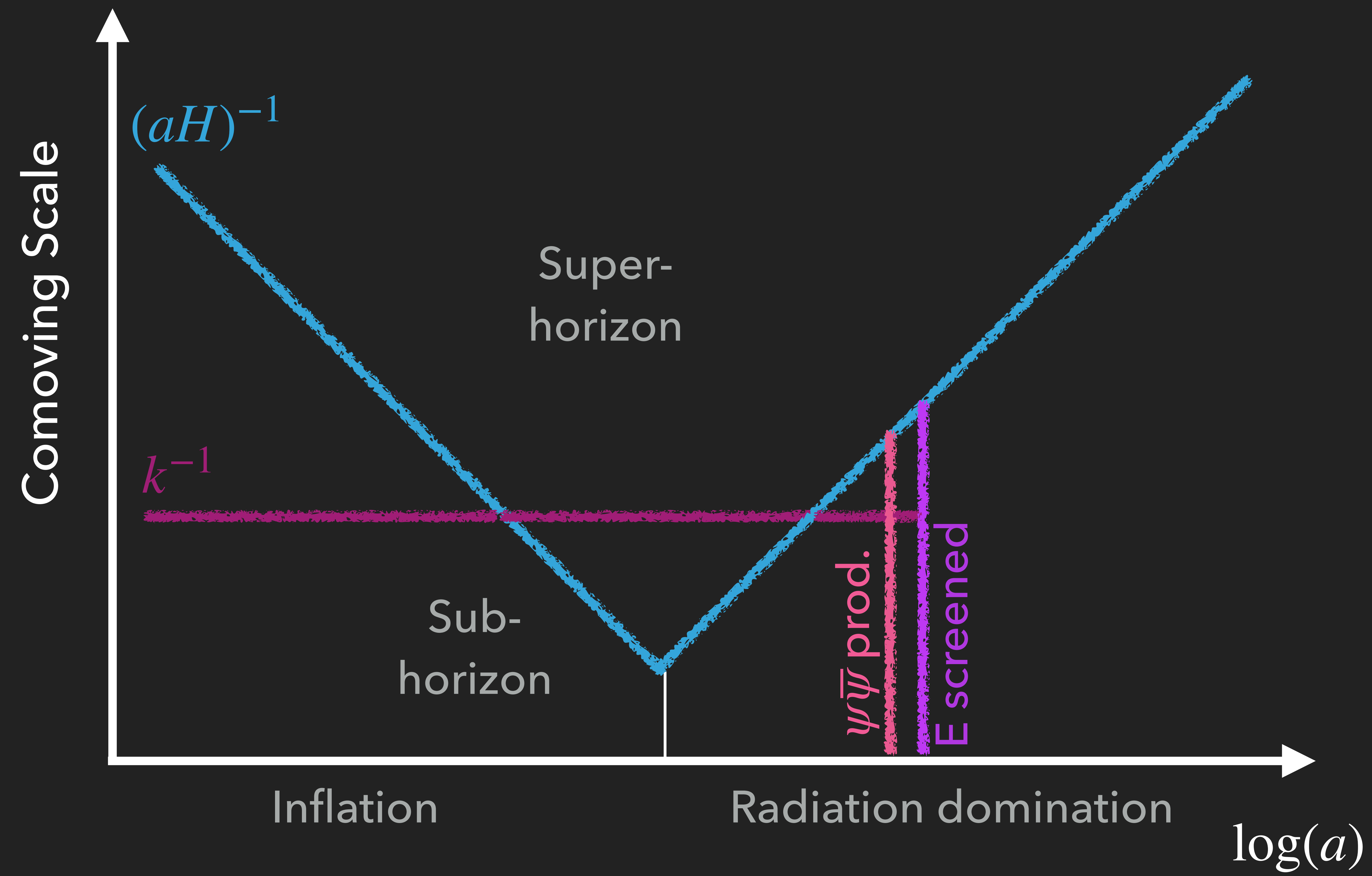
- $A'_{L,k}$ produced during inflation



- $A'_{L,k}$ produced during inflation
- Hor. crossing: strong electric fields



DARK MASSIVE QED: COSMOLOGICAL HISTORY



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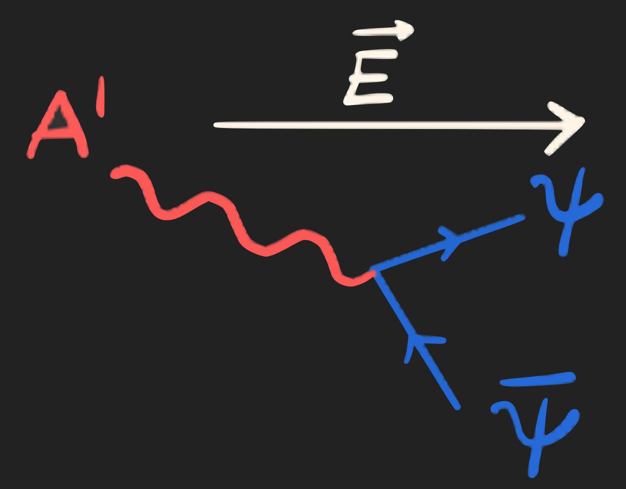

- ψ's partially screen the electric field

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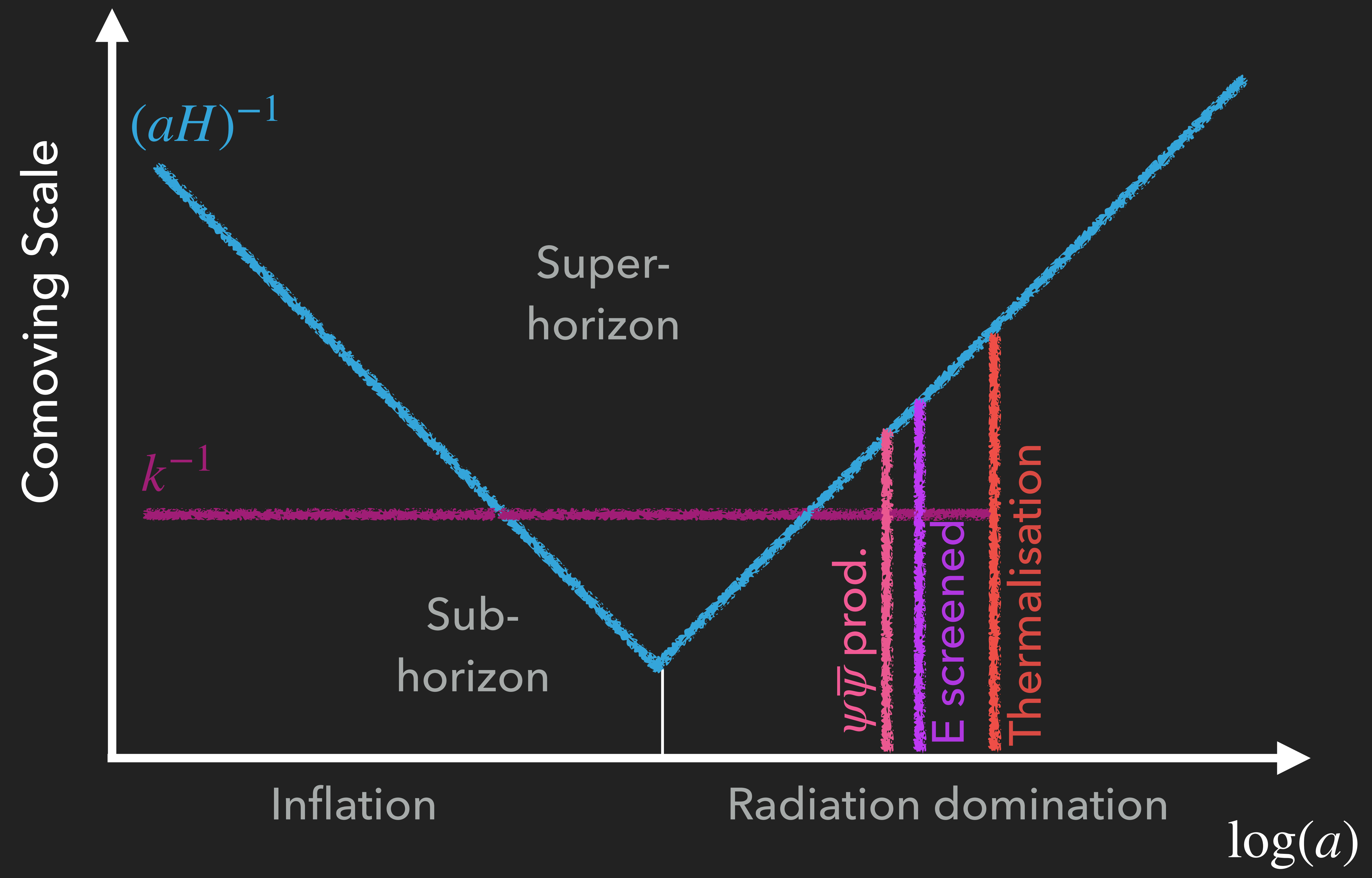

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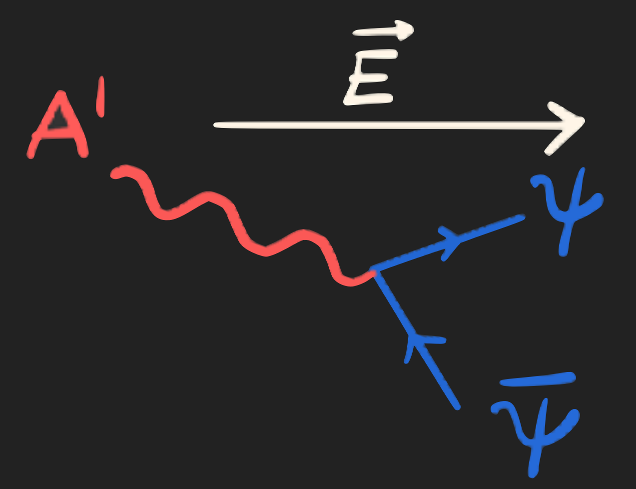
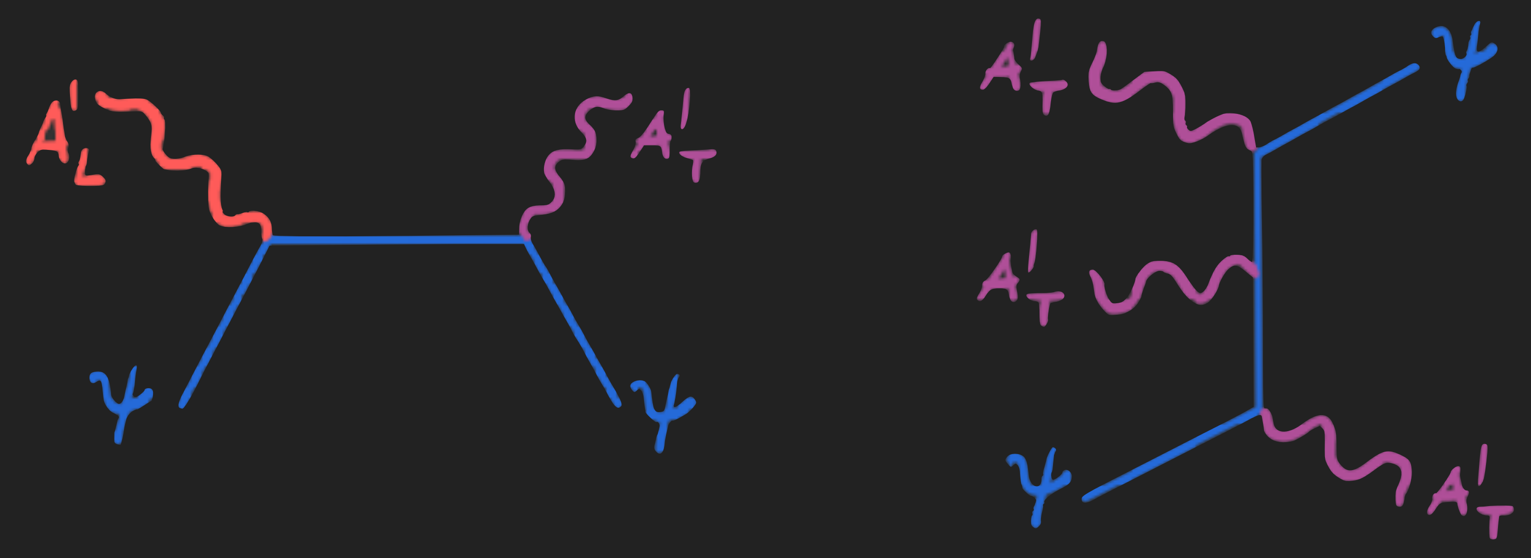


- ψ's partially screen the electric field
- Dark sector thermalises



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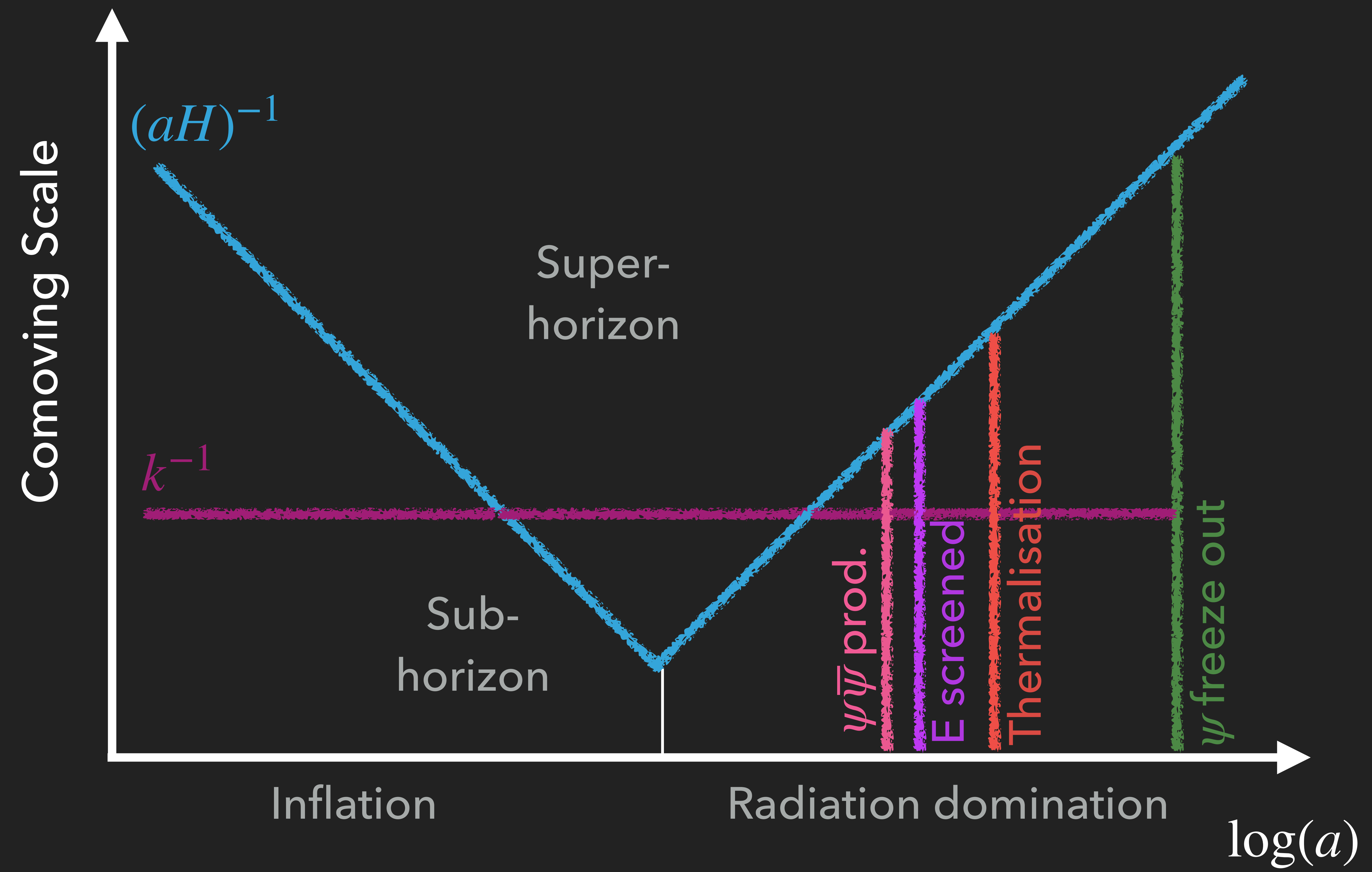

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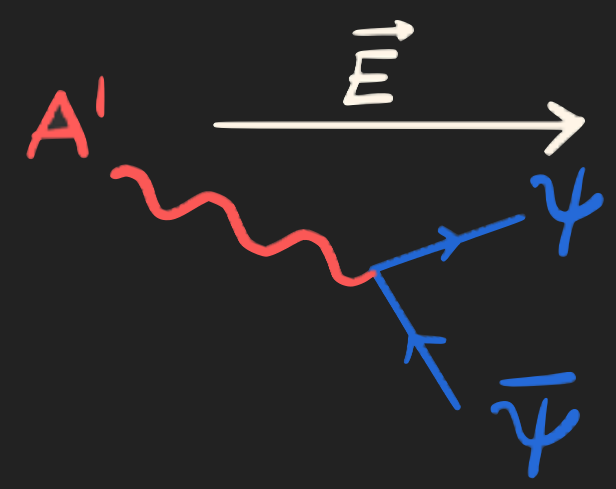

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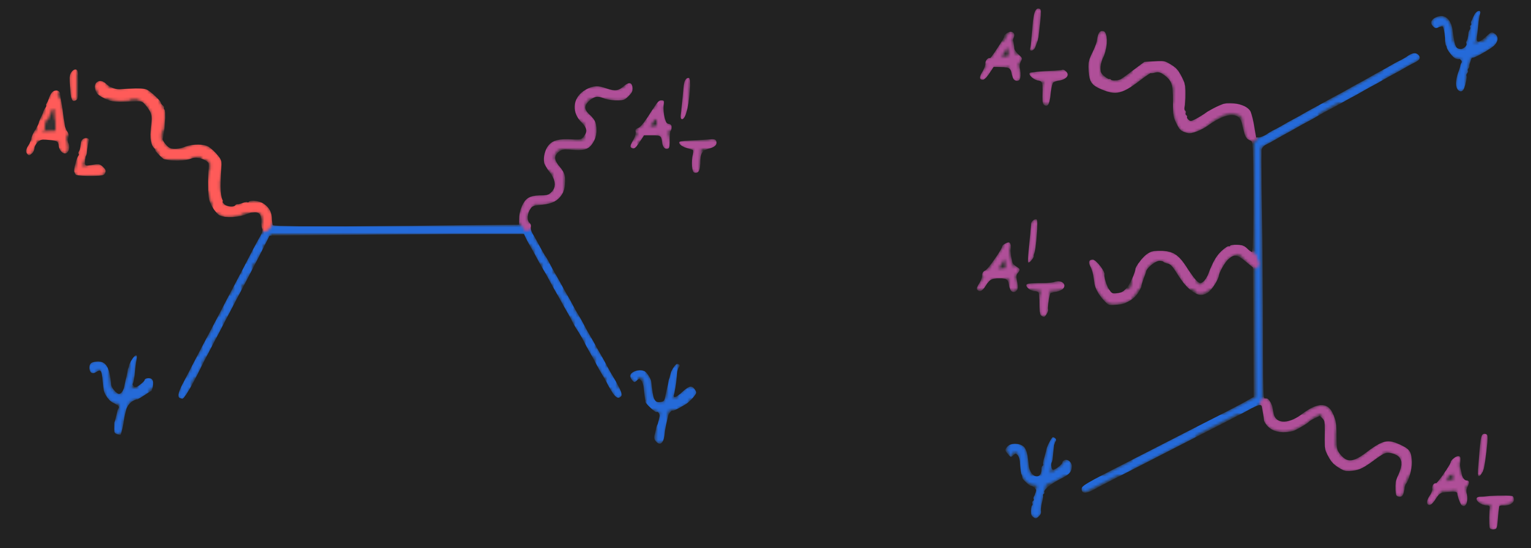
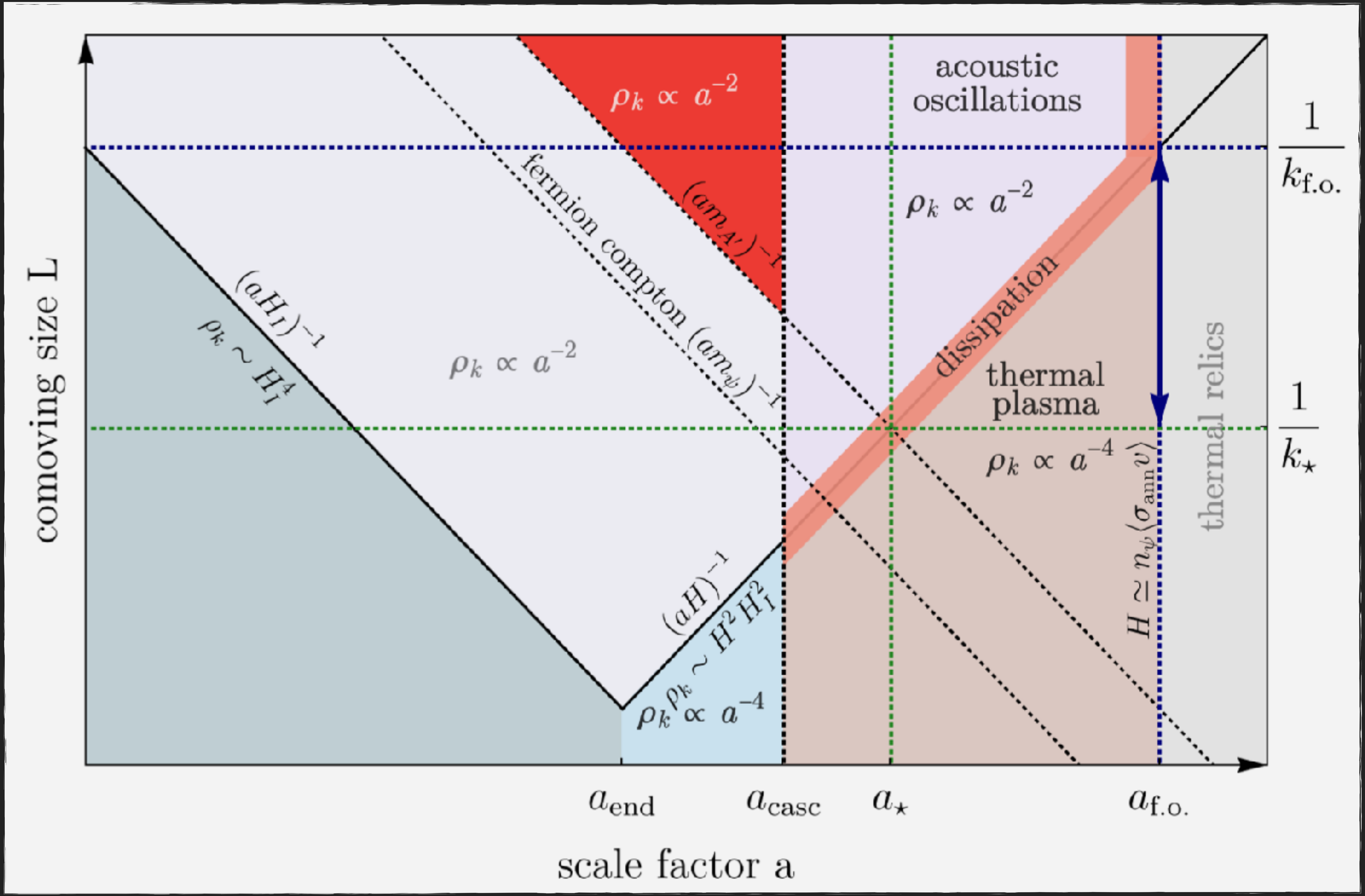
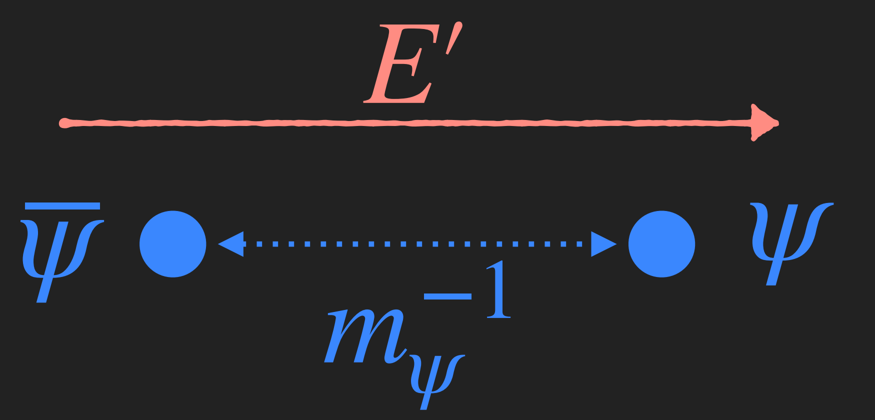


- ψ's freeze out $\longrightarrow$ Dark Matter



comoving size L
scale factor a
$a_{\rm end}$
$a_{\rm casc}$
$a_\star$
$a_{\rm f.o.}$
$\frac{1}{k_{\rm f.o.}}$
$\frac{1}{k_\star}$
$\rho_k \propto a^{-2}$
acoustic oscillations
$\rho_k \propto a^{-2}$
$(am_{A'})^{-1}$
fermion compton $(am_\psi)^{-1}$
$(aH_I)^{-1}$
$\rho_k \sim H_I^4$
$\rho_k \propto a^{-2}$
dissipation
thermal plasma
$\rho_k \propto a^{-4}$
$H \simeq n_\psi \langle \sigma_{\rm ann} v \rangle$
thermal relics
$(aH)^{-1}$
$\rho_k \sim H^2 H_I^2$
$\rho_k \propto a^{-4}$

Schwinger rate



$$e_{\rm D}E' \cdot m_\psi^{-1} > m_\psi$$

Cascade rate

$$\mathscr{W}_{\rm Schwinger} = \left(e_{\rm D}E'\right)^2 \exp\left(-\frac{\pi m_\psi^2}{e_{\rm D}E'}\right)$$

$$E' \sim \partial_t A'_L \sim H \frac{m_{A'}^2}{H^2} \frac{HH_I}{m_{A'}} \sim m_{A'}H_I$$

$$\chi \approx \frac{e_{\rm D}E'\omega_{A'}}{m_\psi^3}$$

$$\mathcal{W}_{\rm casc} = \frac{dN_{\psi\bar\psi}}{dt\,dV} \sim n_{A'} \frac{m_{A'}^2}{\omega_{A'}^2} \cdot \begin{cases} e_{\rm D}^3 \frac{E'}{m_\psi} e^{-8/(3\chi)} & \chi \lesssim 1 \\ e_{\rm D}^{8/3} \frac{E'^{2/3}}{\omega_{A'}^{1/3}} & \chi \gg 1 \end{cases}$$

Longitudinal suppression

DISPERSION RELATION OF PROCA FIELD IN PLASMA

Maxwell eq.

$$(\omega^2 - k^2 - m_{A'}^2)\vec{A} = -\vec{J}$$

$$\vec{J} = e_{\rm D} n_\psi \vec{u}$$

Lorentz eq.

$$m_\psi \partial_t \vec{u} = -e_{\rm D}\vec{E} - 2m_\psi \nu \vec{u}$$

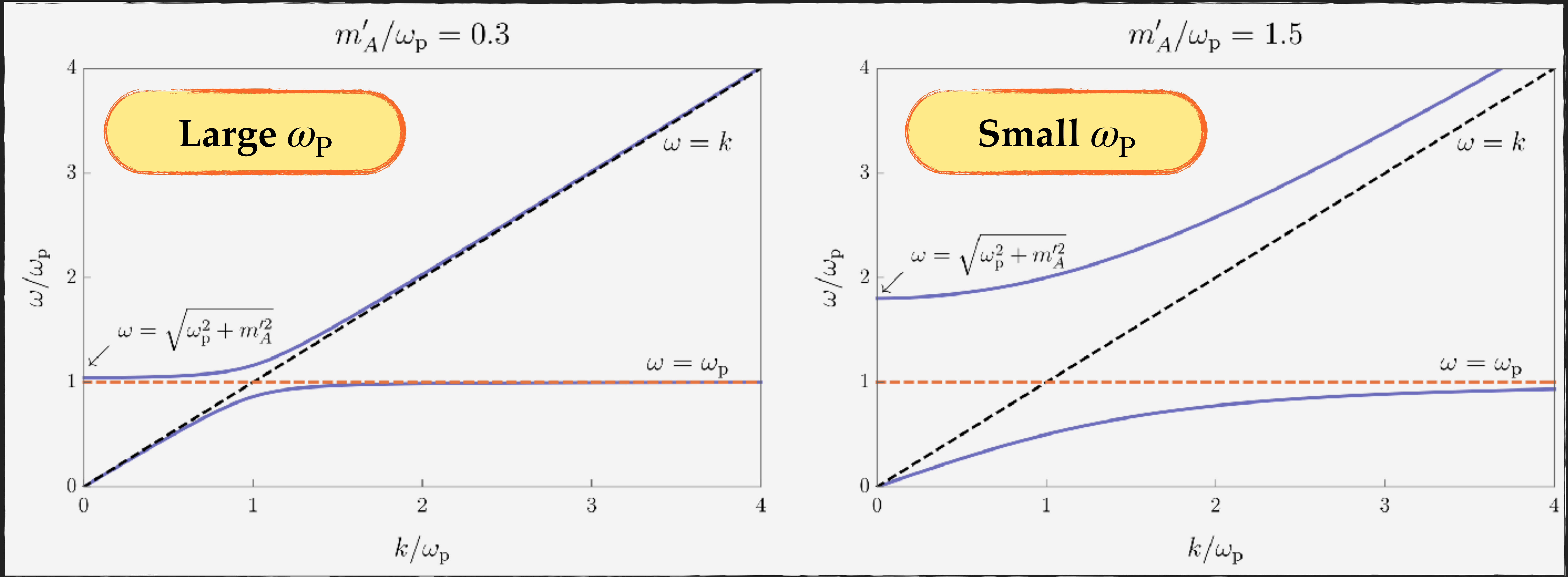
ν : collision rate of ψ

Dispersion relation for A'_L

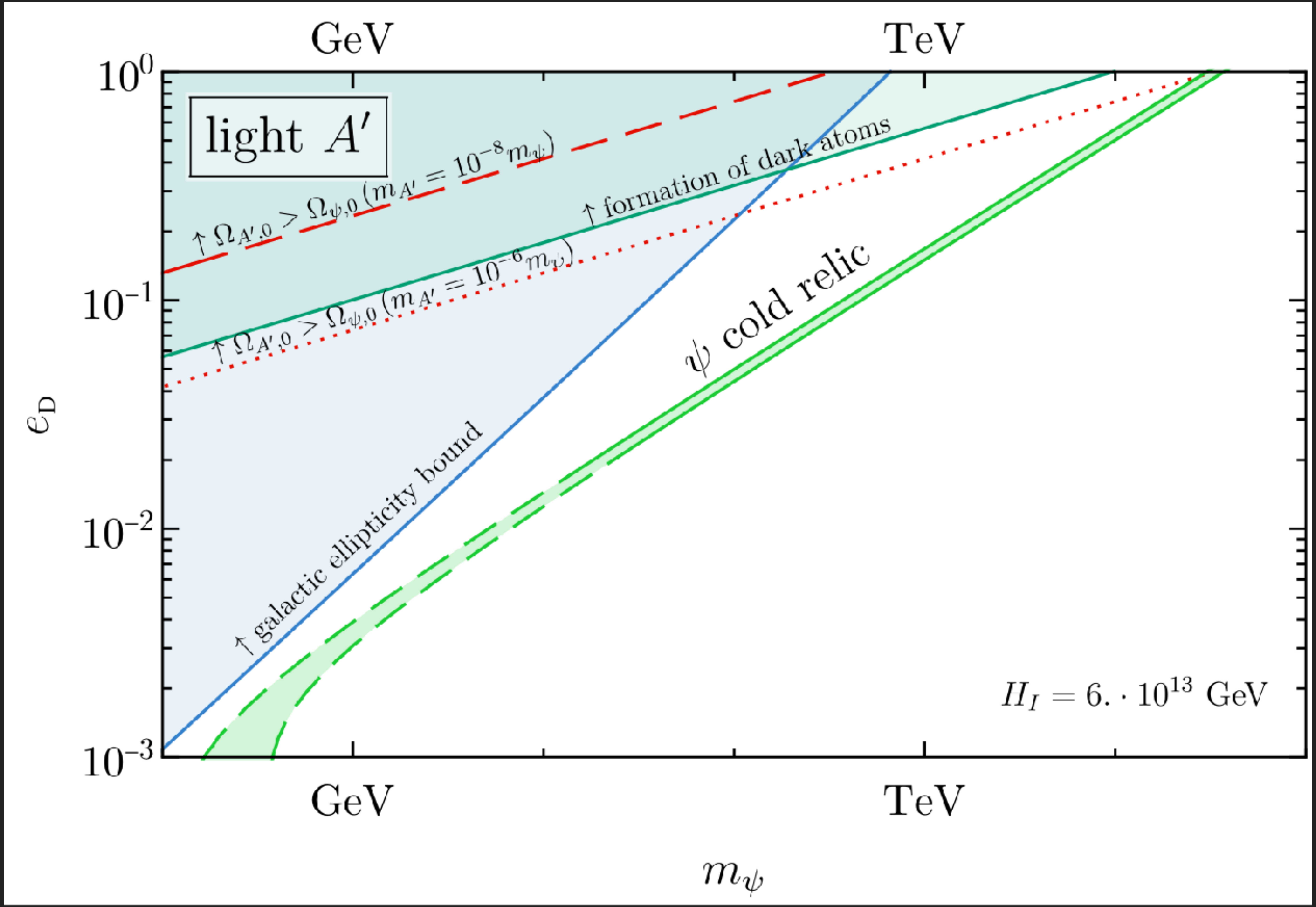
$$\omega^2 - k^2 - m_{A'}^2 = \omega_{\rm P}^2 \frac{\omega}{\omega + 2i\nu}\left(1 - \frac{k^2}{\omega^2}\right)$$

Plasma mass

$$\omega_{\rm P}^2 = \frac{e_{\rm D}^2 n_\psi}{E_\psi}$$

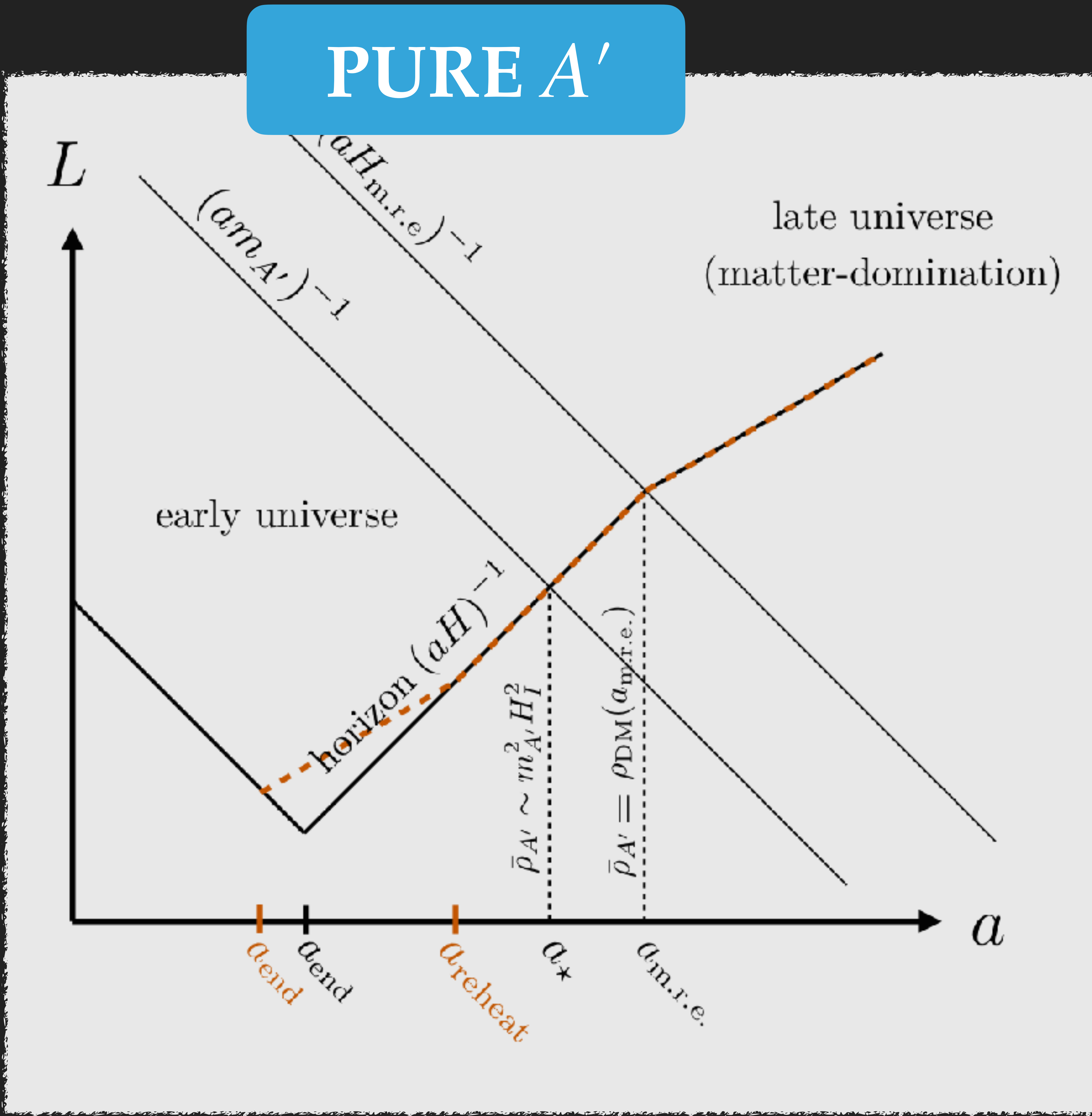


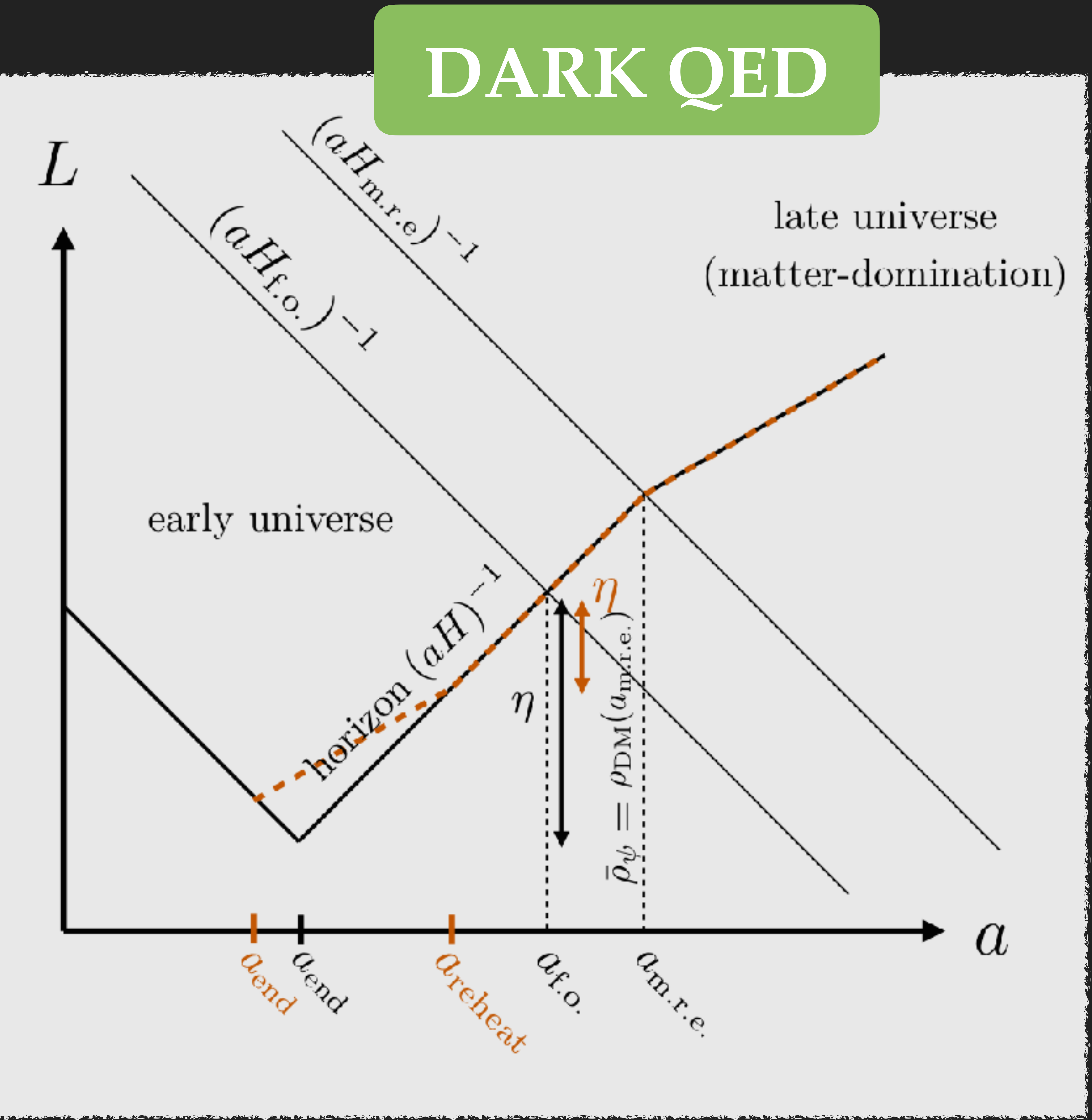
CONSTRAINTS FROM SELF-INTERACTIONS, DARK ATOMS



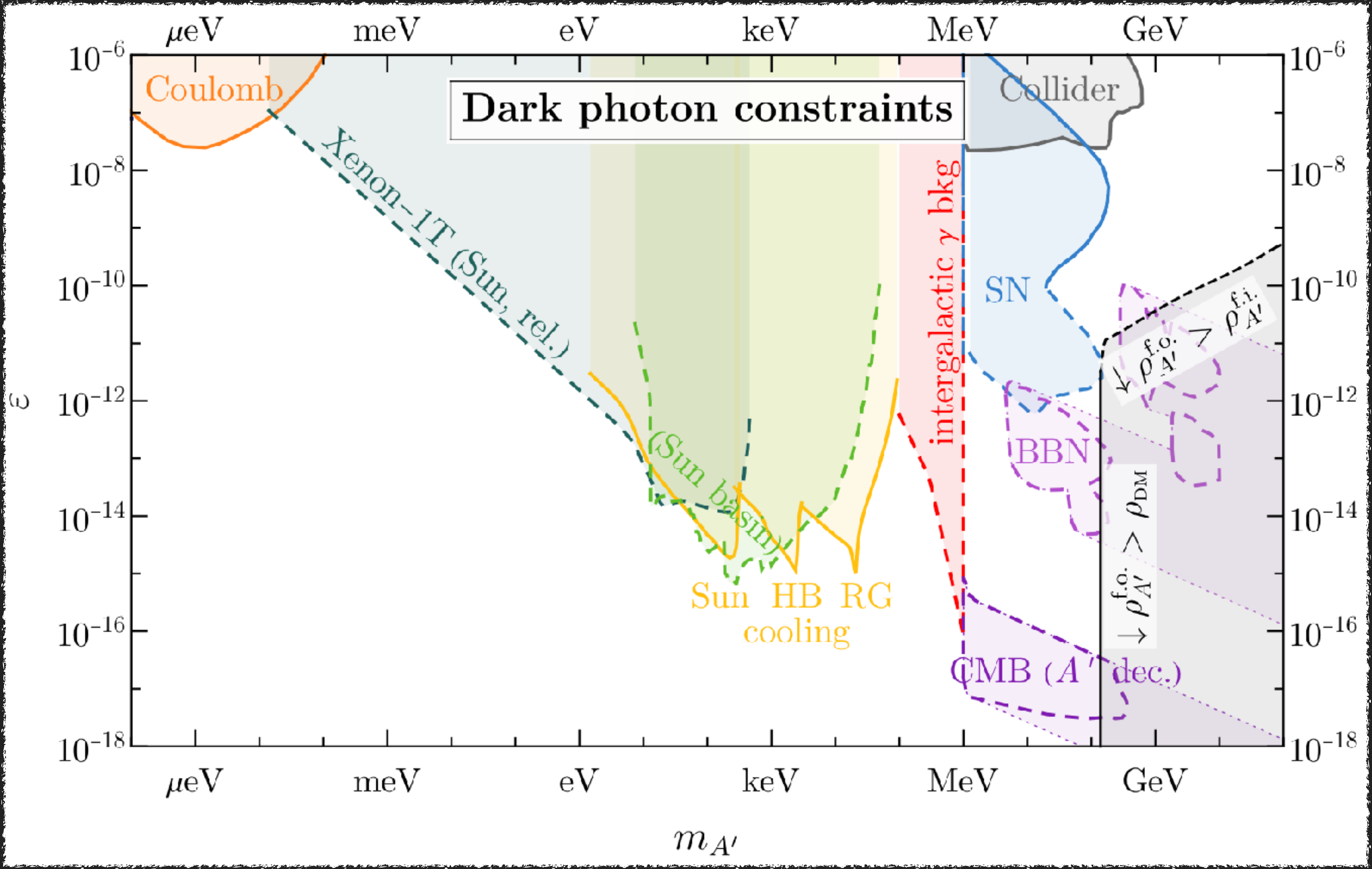
COMBINED PARAMETER SPACE OF A′ AND MILLICHARGED DM

(NEGLIGIBLE) IMPACT OF REHEATING





CONSTRAINTS ON DARK PHOTONS



- Pre-inflationary scenario: depends on

$$\begin{cases} m_{\text{axion}} & \to \text{experimental target} \\ \theta_{\text{misalignment}} & \to \text{astro / cosmo implications?} \end{cases}$$

QCD AXION AS DARK MATTER

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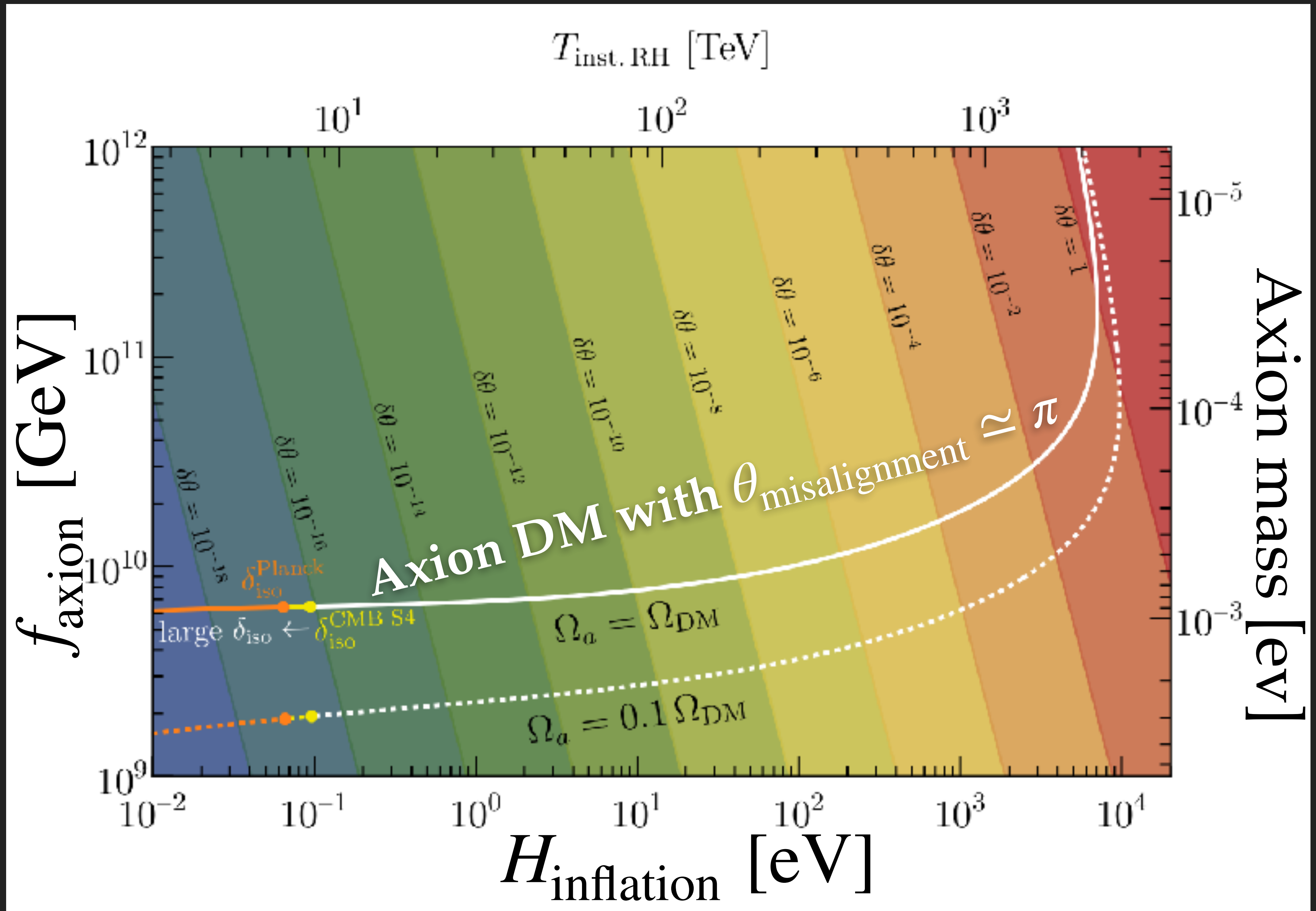
- $\theta_{\text{misalignment}} \simeq \pi \Rightarrow$ dense substructures ['19 Arvanitaki+]
- Can be realised in a minimal model

['20 Huang, Madden, **DR**, Reig]



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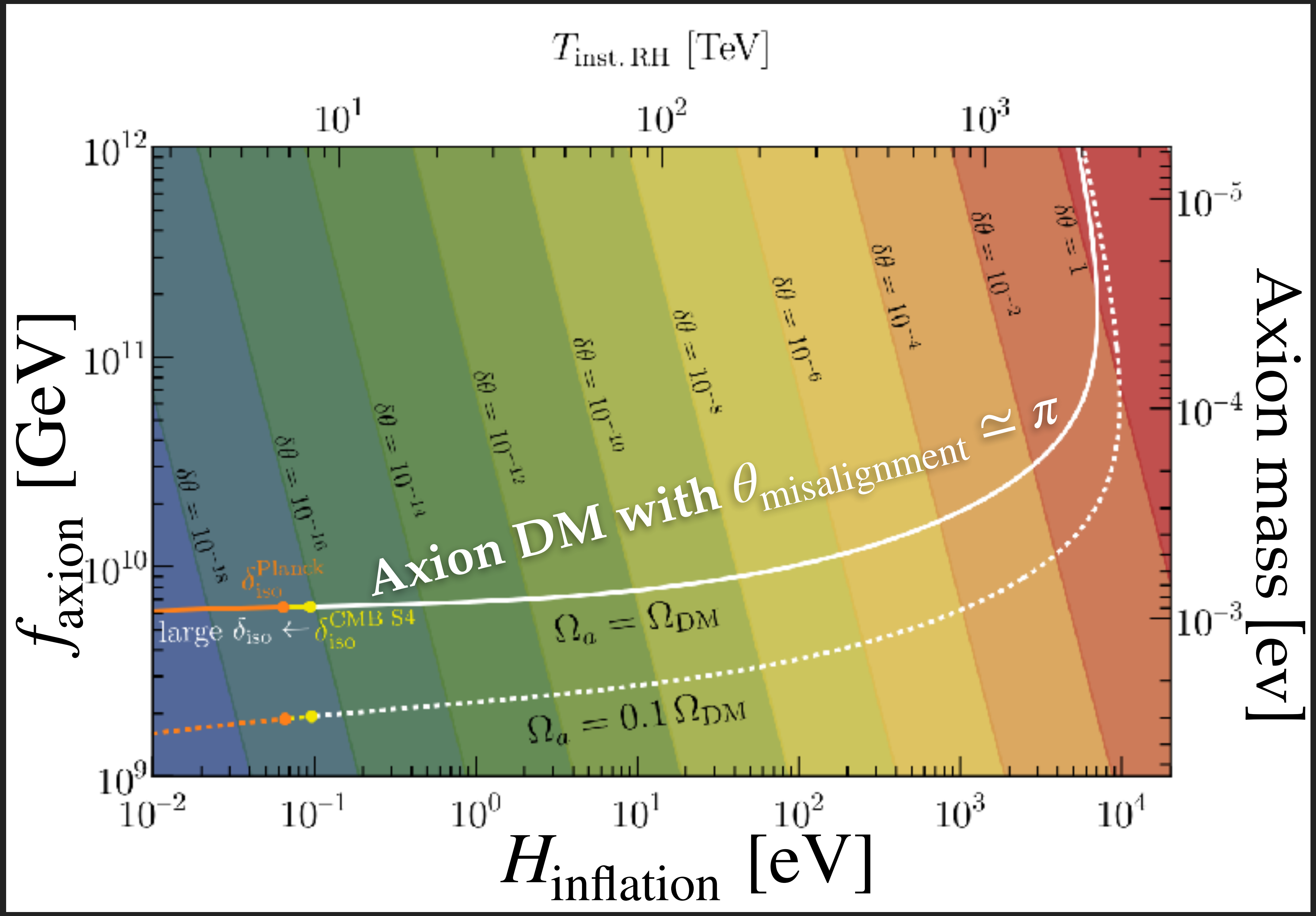

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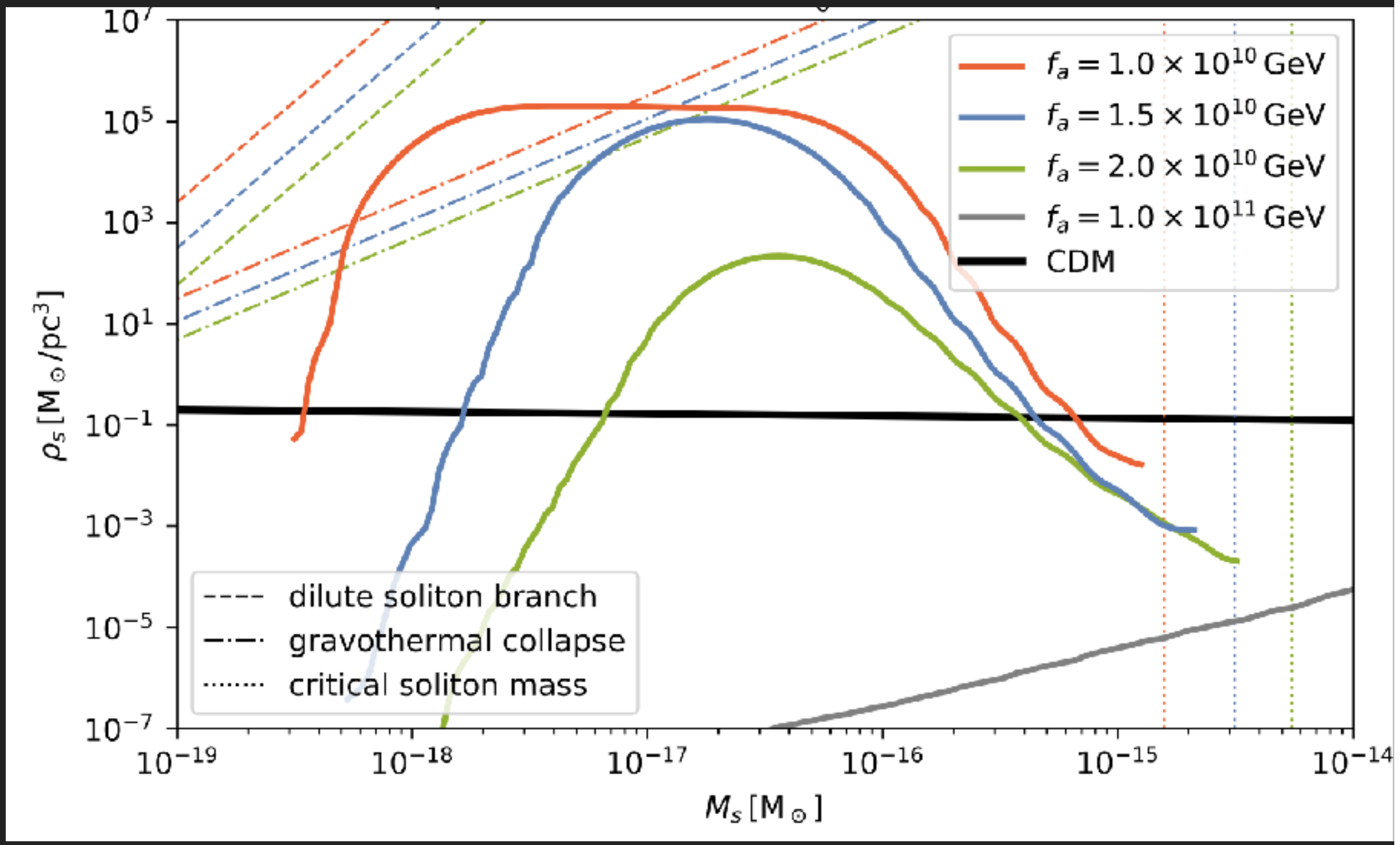
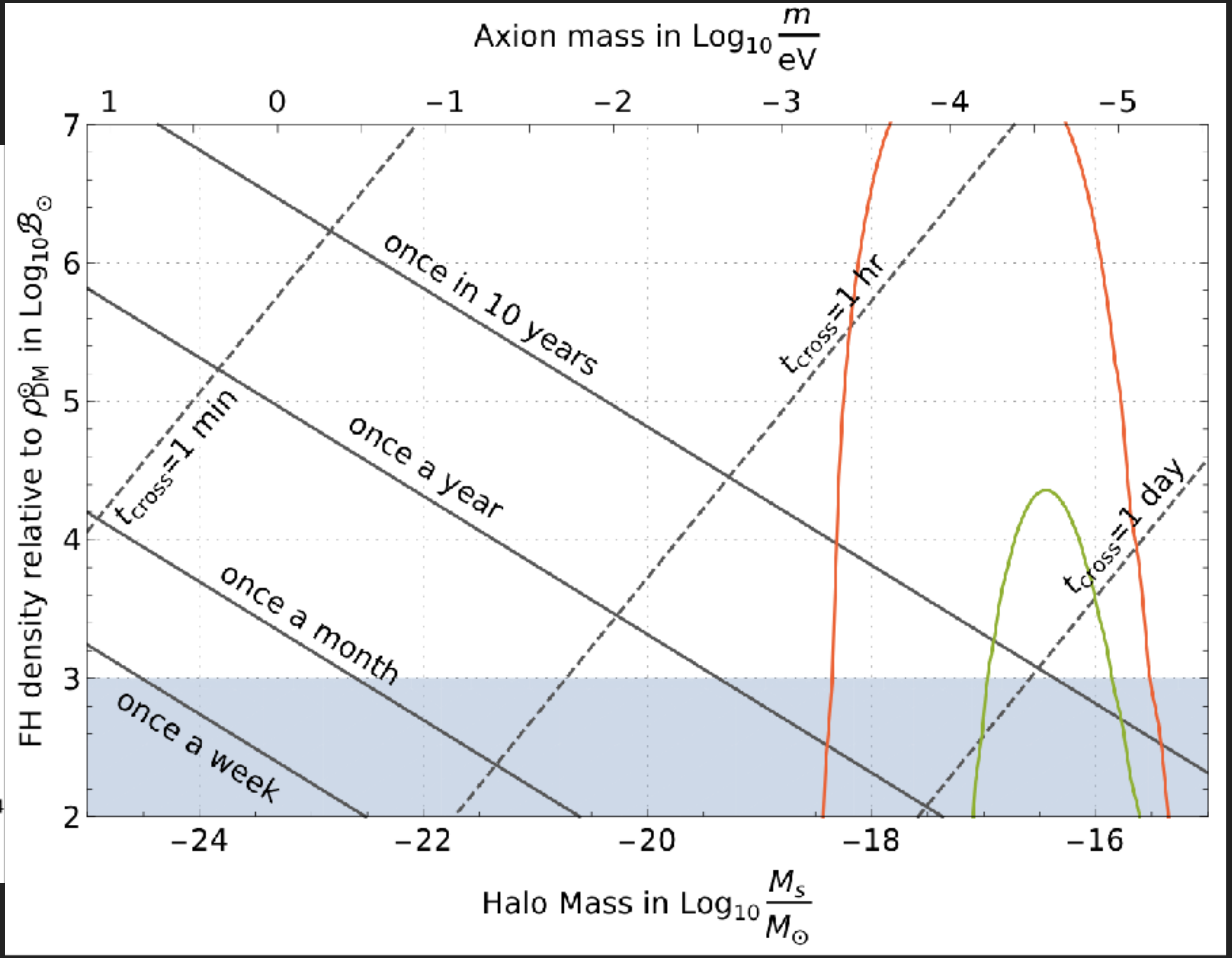
- Large $H_{\text{inflation}} \Rightarrow$ fluctuations in $\theta_{\text{misalignment}} \Rightarrow$ excluded by isocurvature
- How general is this conclusion? [(*work in progress*) Graham, **DR**]

LARGE MISALIGNMENT

['19 Arvanitaki, Dimopoulos, Galanis, Lehner, Van Tilburg, Thompson]





MAXIMAL MISALIGNMENT IN A MINIMAL MODEL

- Large initial misalignment: affect QCD axion mass, and clump DM substructures

$$\theta = \theta_{\rm SM} + \frac{a}{f_{\rm axion}} + \arg[M_{\rm quarks}] \qquad y\,\phi\,\bar{q}q$$

Colored fermion $\lesssim 10 - 100$ TeV

Scalar: flip sign → $m_{\rm axion\,DM} \sim$ meV

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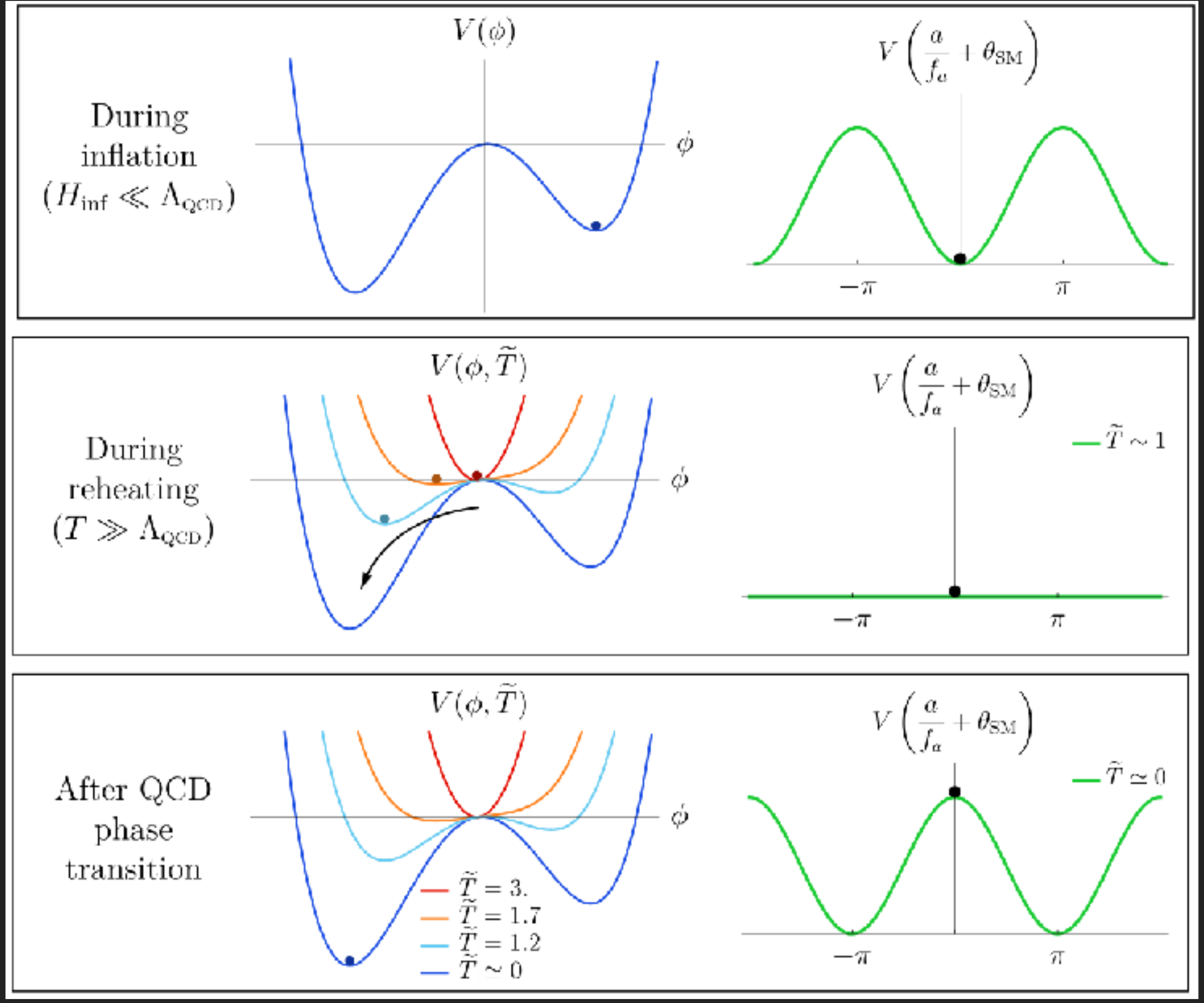
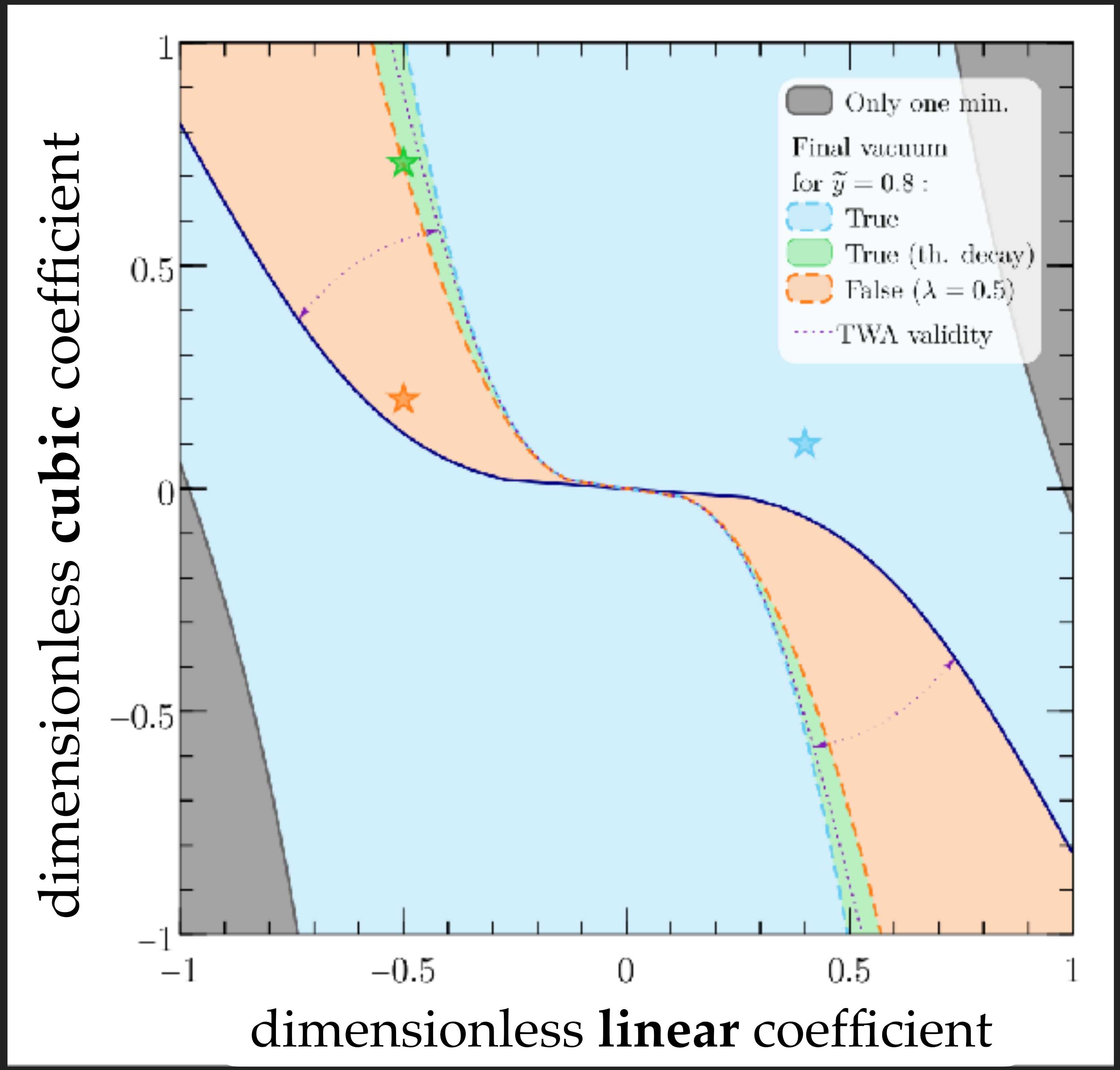
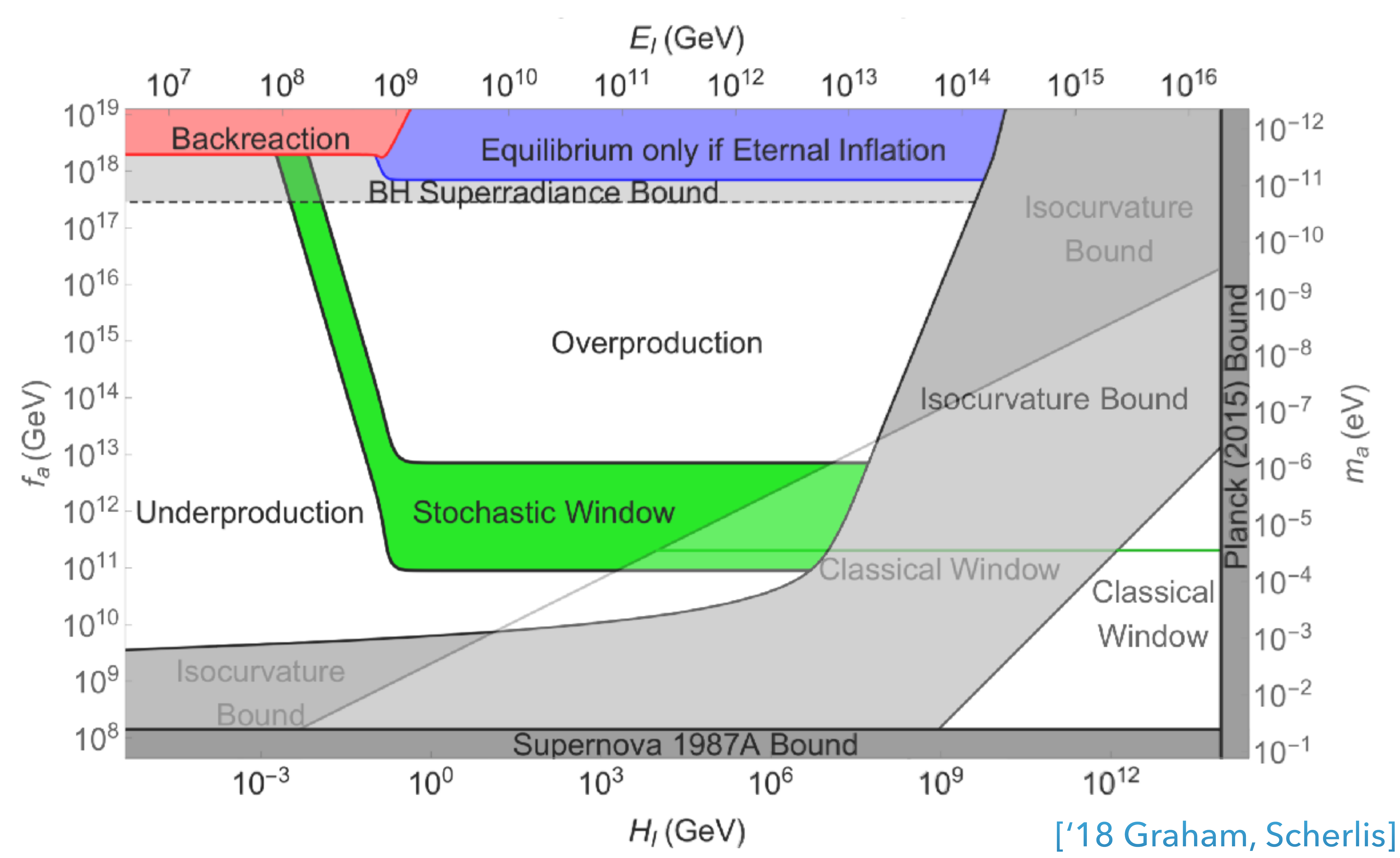




ISOCURVATURE BOUND FOR THE QCD AXION



['18 Graham, Scherlis]