

# Back to the phase space: Thermal Axions

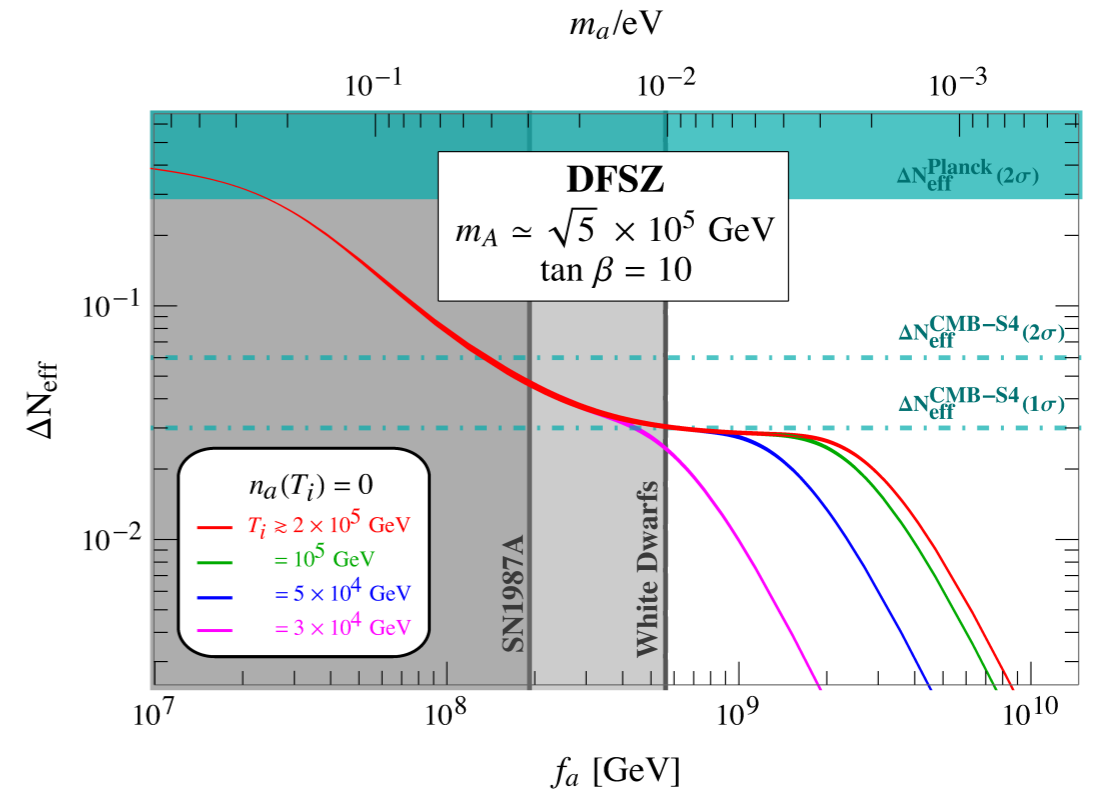
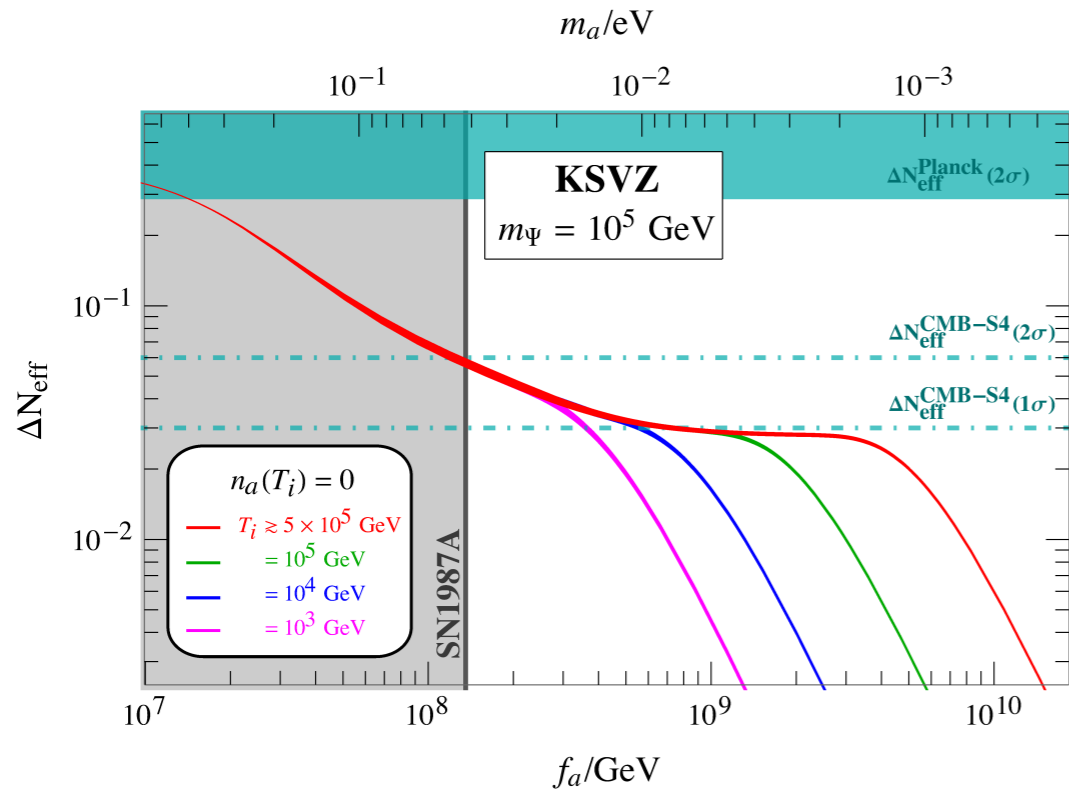
**Francesco  
D'Eramo**



UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA

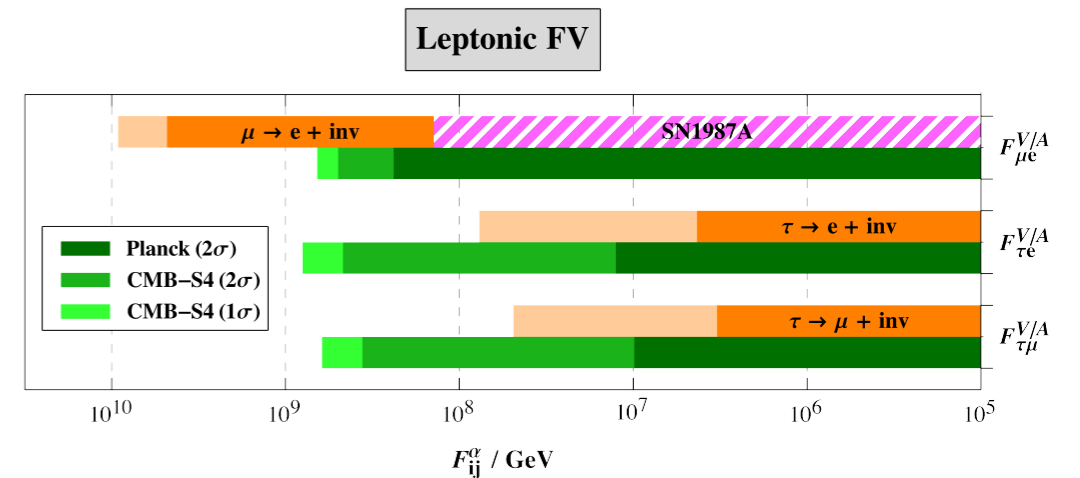
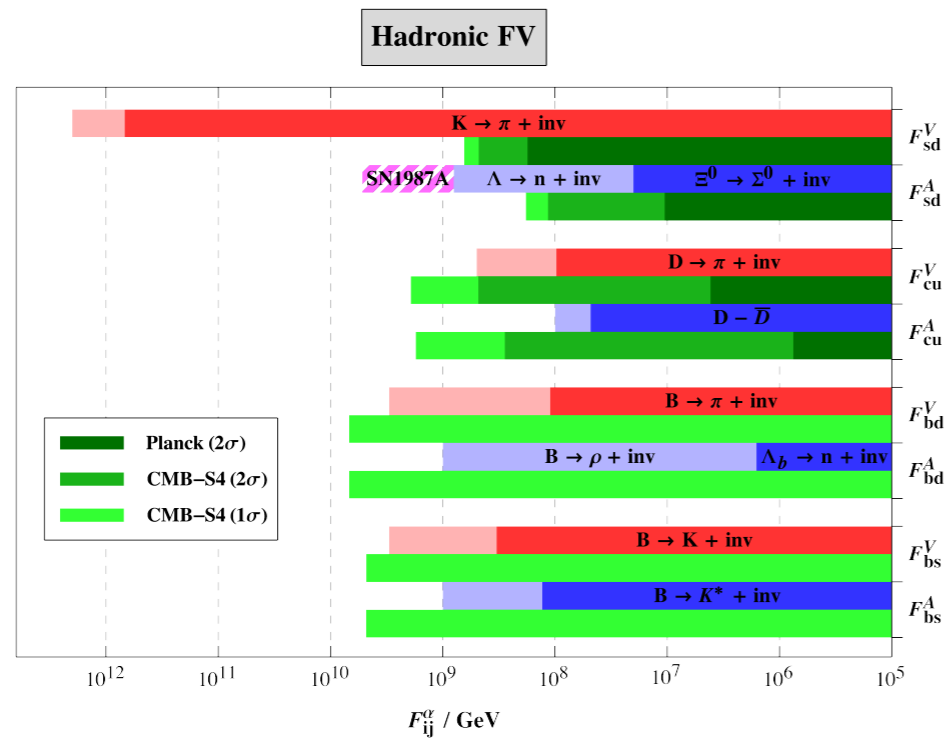


# The way back I: $\Delta N_{\text{eff}}$



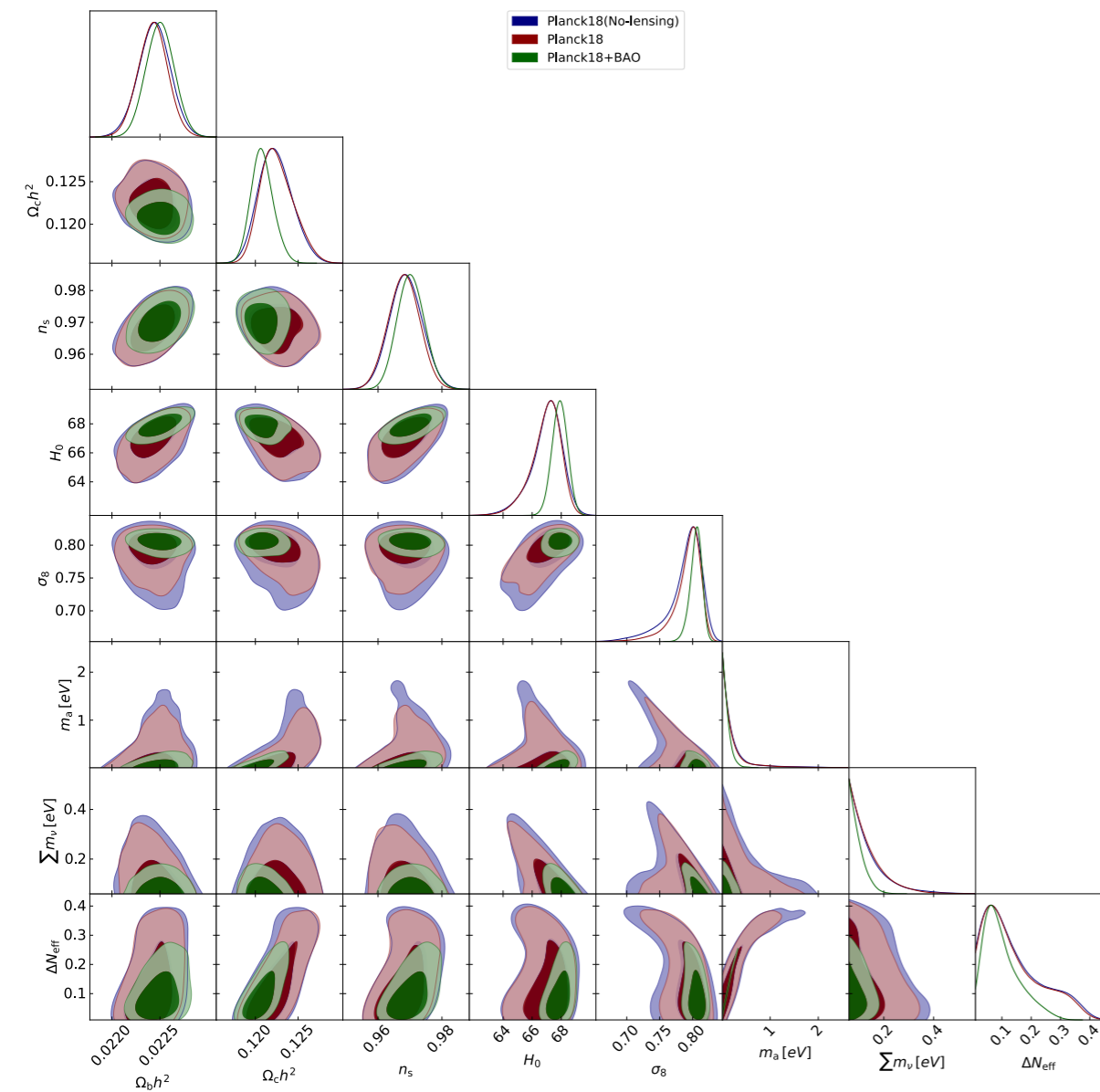
FD, Hajkarim, Yun, **Phys.Rev.Lett. 128 (2022)**

FD, Hajkarim, Yun, **JHEP 10 (2021)**



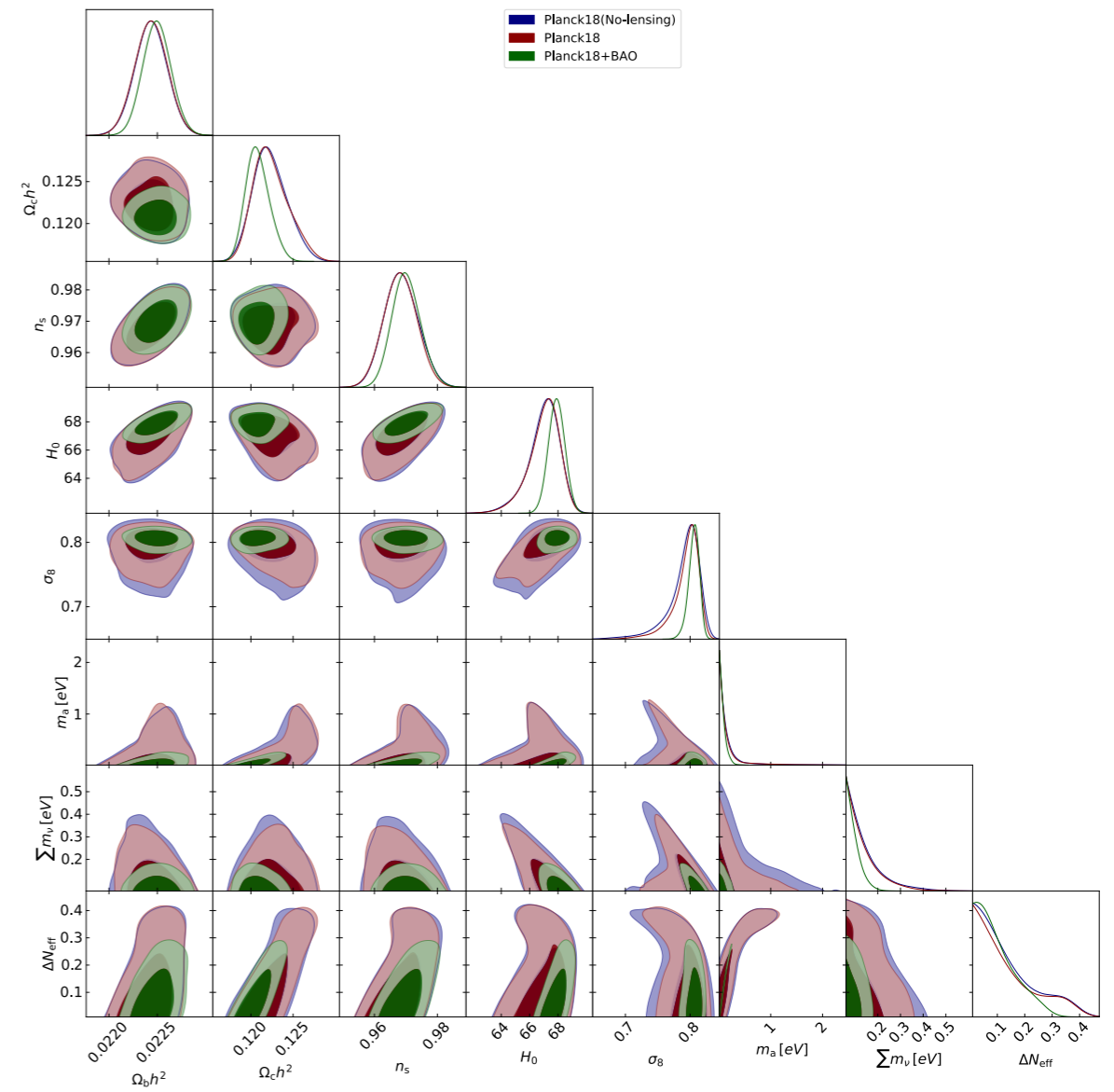
FD, Yun, **Phys.Rev.D 105 (2022)**

# The way back II: Mass Bound



**KSVZ**

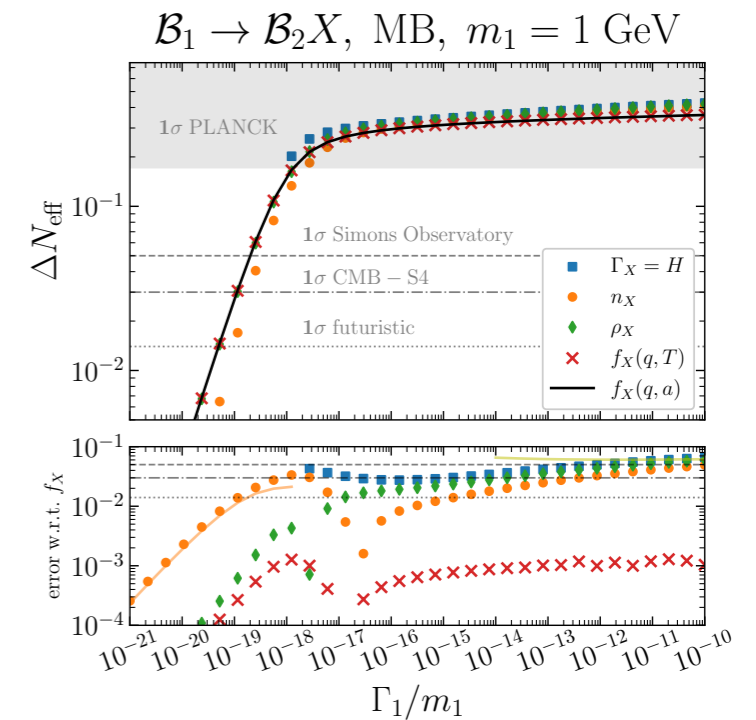
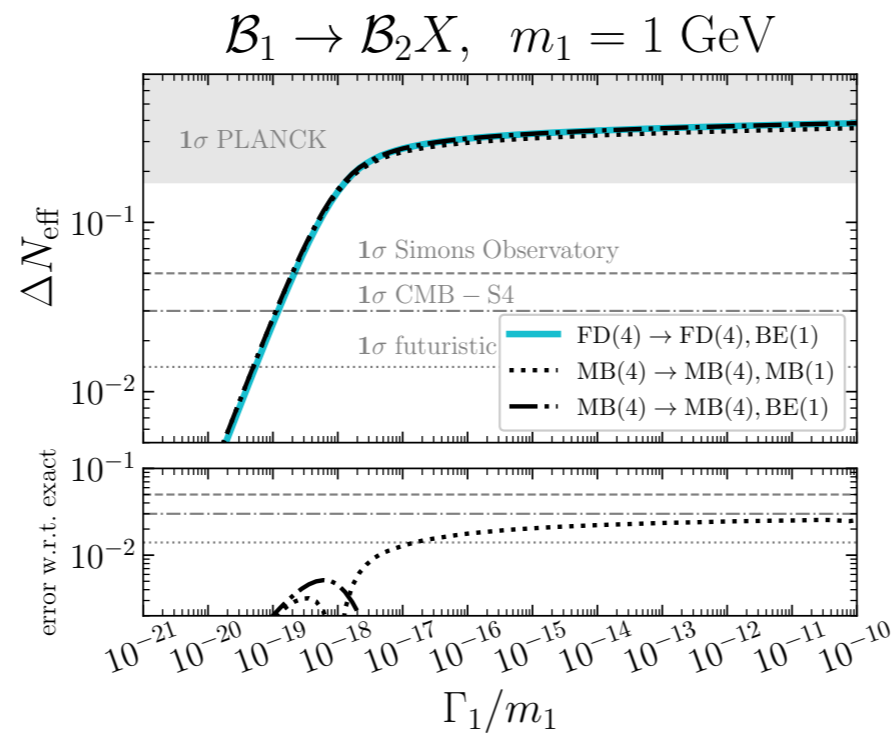
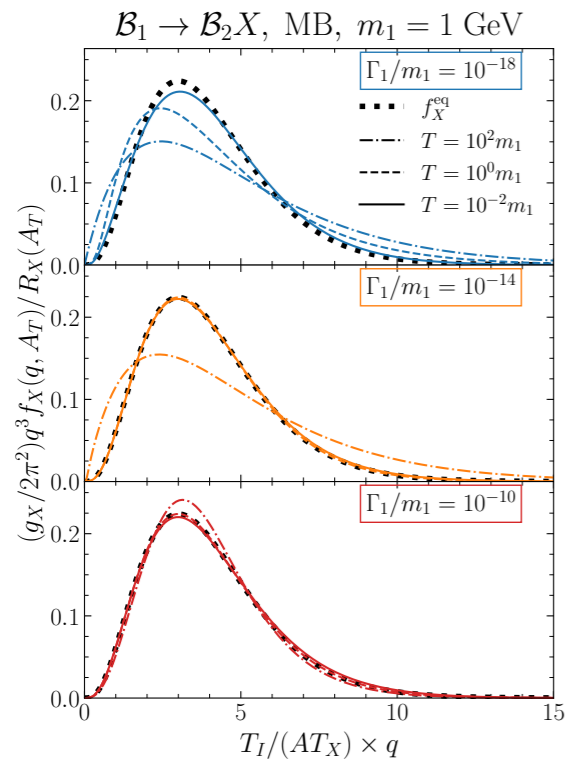
$$m_a \leq 0.282(0.420) \text{ eV}$$



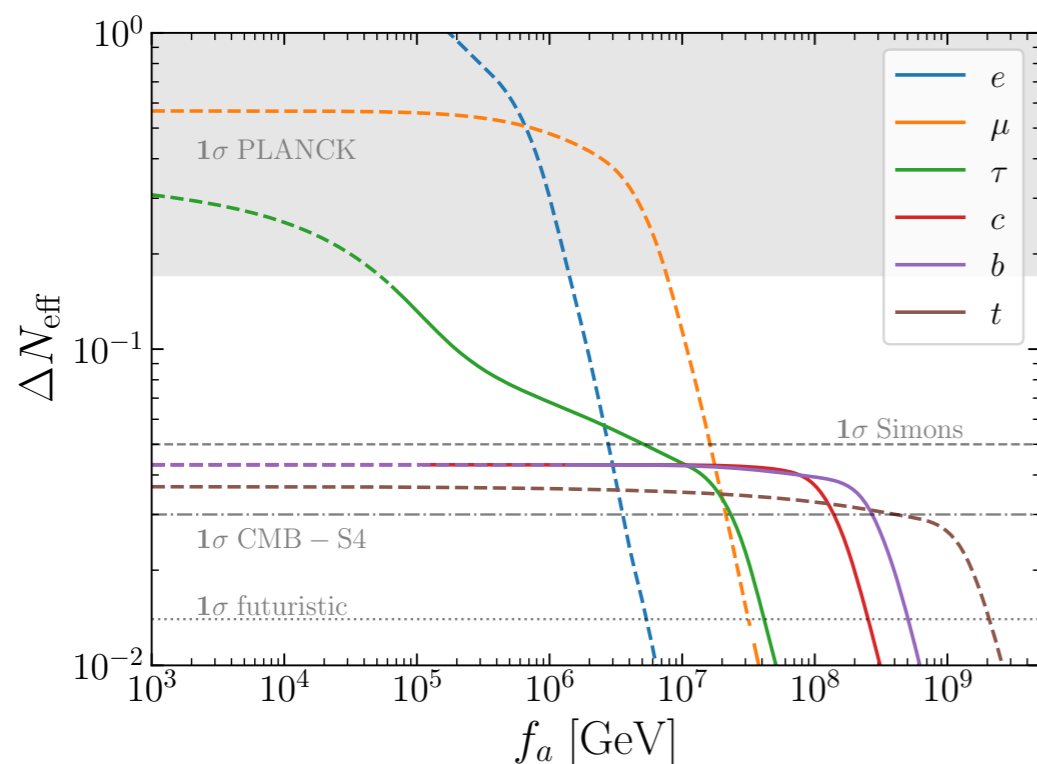
**DFSZ**

$$m_a \leq 0.209(0.293) \text{ eV}$$

# The way back III: Phase Space



FD, Hajkarim, Lenoci, **JCAP 03 (2024)**



Updated predictions  
via a rigorous phase space analysis

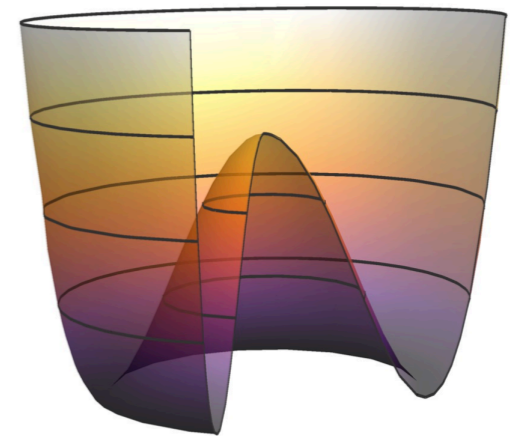
FD, Lenoci, **in preparation**

# The QCD Axion

## New global $U(1)_{PQ}$ symmetry

- spontaneously broken at the scale  $f_a$  (with  $f_a \gg$  weak scale)
- anomalous under strong interactions

Pseudo Nambu-Goldstone boson in the low-energy spectrum (“**QCD axion**”) with (“anomalous”) coupling to gluons

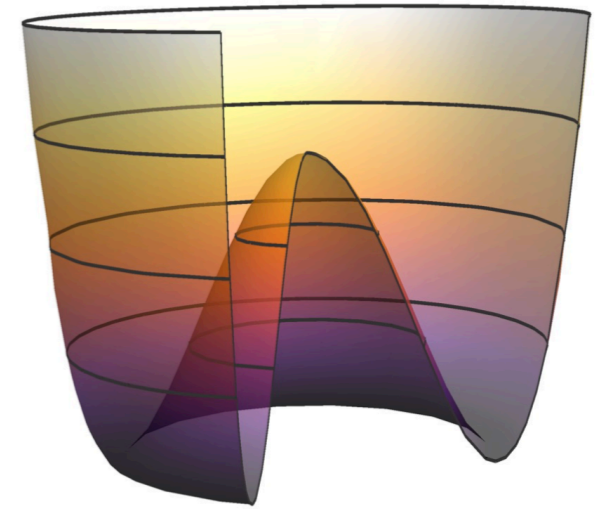


$$\frac{a}{f_a} \frac{\alpha_s}{8\pi} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

$$m_a \simeq 5.7 \left( \frac{10^{12} \text{ GeV}}{f_a} \right) \mu\text{eV}$$

# Axion-Like-Particles (ALPs)

Ubiquitous in motivated  
extension of the Standard Model



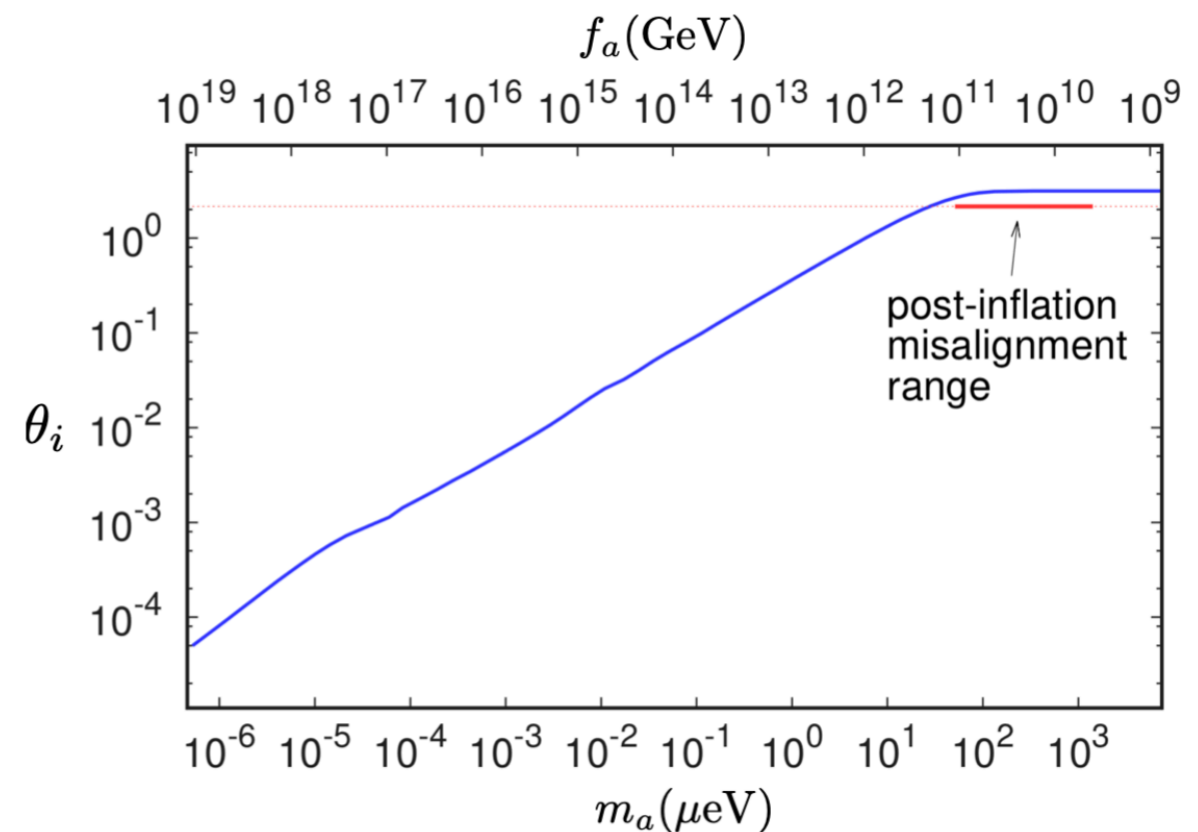
- Pseudo-Nambu-Goldstone-bosons in field theory
- Axions in string theory

$$\mathcal{L}_{\text{int}} = c_X \frac{a}{f_a} \frac{\alpha_X}{8\pi} X^{\mu\nu} \tilde{X}_{\mu\nu} + c_\psi \frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi$$
$$m_a \simeq \frac{\Lambda^2}{f_a}$$

Results in this talk mostly about the QCD axion, easily generalized  
(especially when the mass does not play any role)

# Cold Axions – Dark Matter

## Predicting $f_a$ (with caveats)



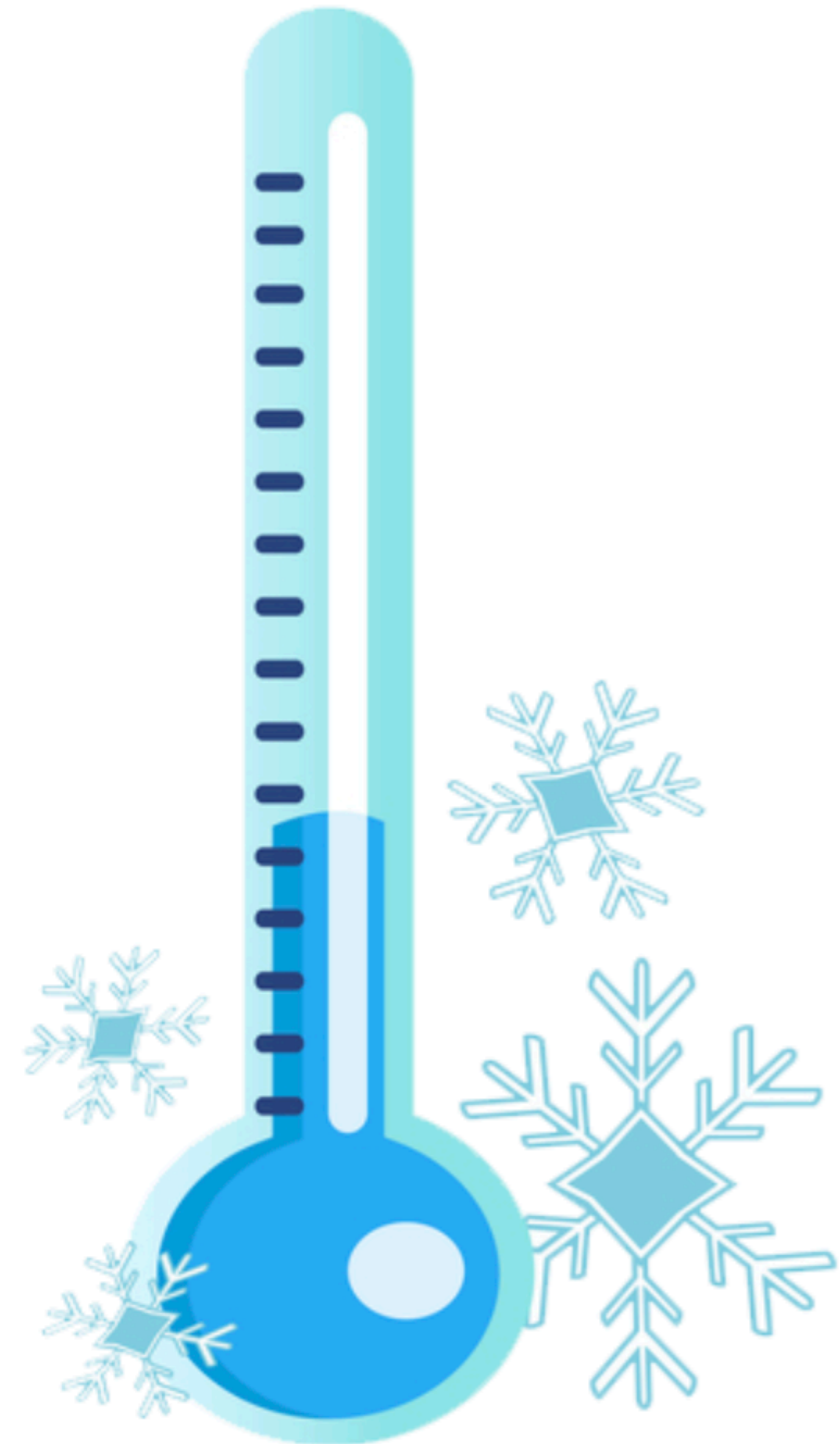
Borsanyi et al., Phys.Lett.B 752 (2016) and Nature 539 (2016)

## Axion string contribution?

Klaer and Moore, JCAP 10 and JCAP 11 (2017)

Gorghetto, Hardy, Villadoro  
JHEP 07 (2018) and SciPost Phys.10 (2021)

Buschmann, Foster, Safdi  
PRL124 (2020) and Nature Commun. 13 (2022)



# Hot Axions — Dark Radiation

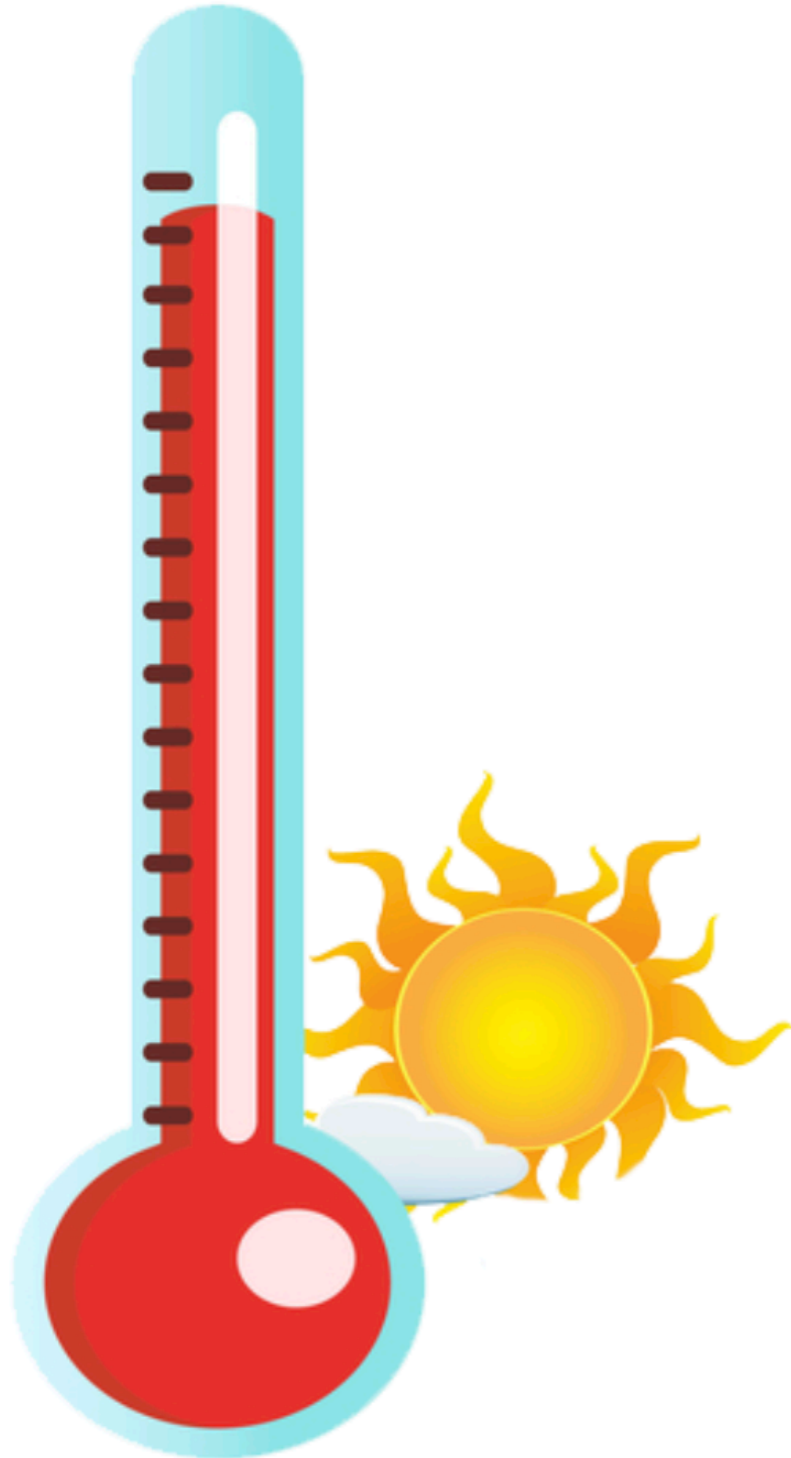
Axions produced with kinetic energy much larger than their mass (i.e. “hot”)

Additional radiation at:

- BBN ( $m_a \lesssim \text{MeV}$ )
- CMB formation ( $m_a \lesssim 0.3 \text{ eV}$ )

$$\rho_{\text{rad}} = \left[ 1 + \frac{7}{8} \left( \frac{T_\nu}{T_\gamma} \right)^4 N_{\text{eff}} \right] \rho_\gamma$$

$$\Delta N_{\text{eff}} = \frac{8}{7} \left( \frac{11}{4} \right)^{4/3} \frac{\rho_a}{\rho_\gamma}$$

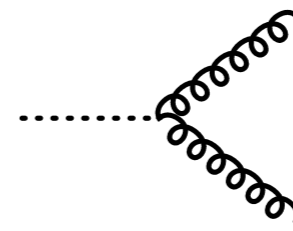




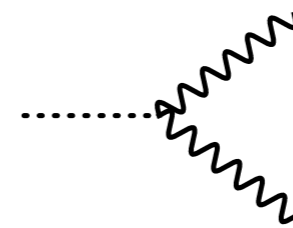
# Thermal Axions

Scatterings and/or decays involving  
primordial thermal bath particles  
(axion energy  $\gg m_a$ , i.e. “hot”)

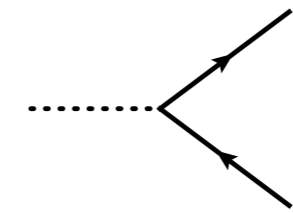
Unavoidable  
Production Source!



$$\frac{\alpha_s}{8\pi} \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu}$$



$$c_{\gamma\gamma} \frac{\alpha_{\text{em}}}{8\pi} \frac{a}{f_a} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

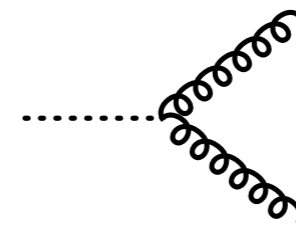


$$c_\psi \frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi$$

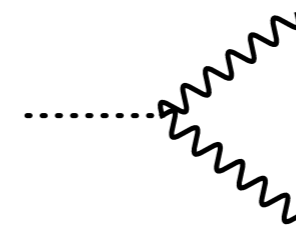
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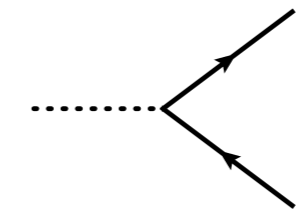
Unavoidable  
Production Source!



$$\frac{\alpha_s}{8\pi} \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu}$$



$$c_{\gamma\gamma} \frac{\alpha_{\text{em}}}{8\pi} \frac{a}{f_a} F^{\mu\nu} \tilde{F}_{\mu\nu}$$



$$c_\psi \frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi$$

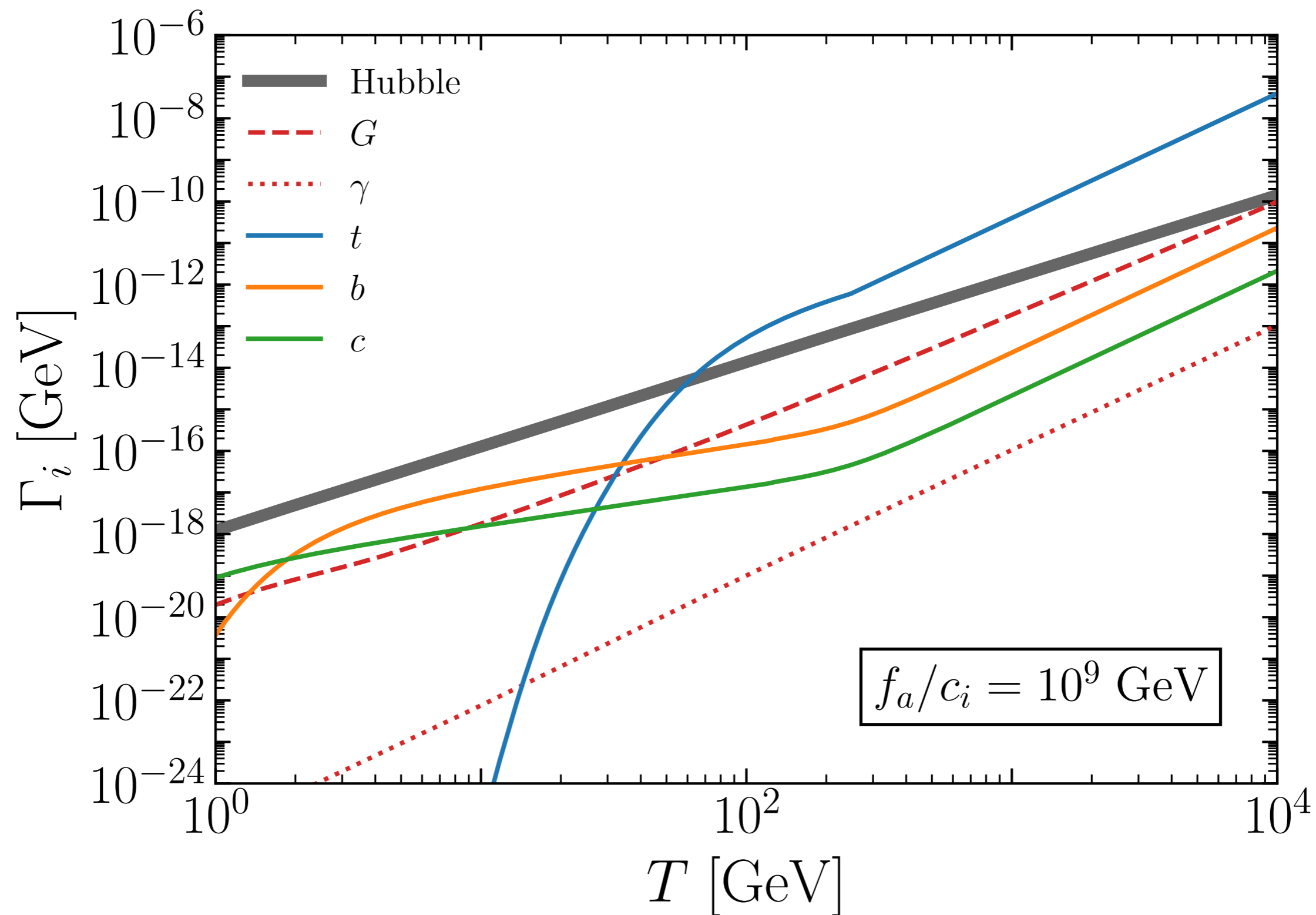
## ROUGH ESTIMATE

### (instantaneous decoupling)

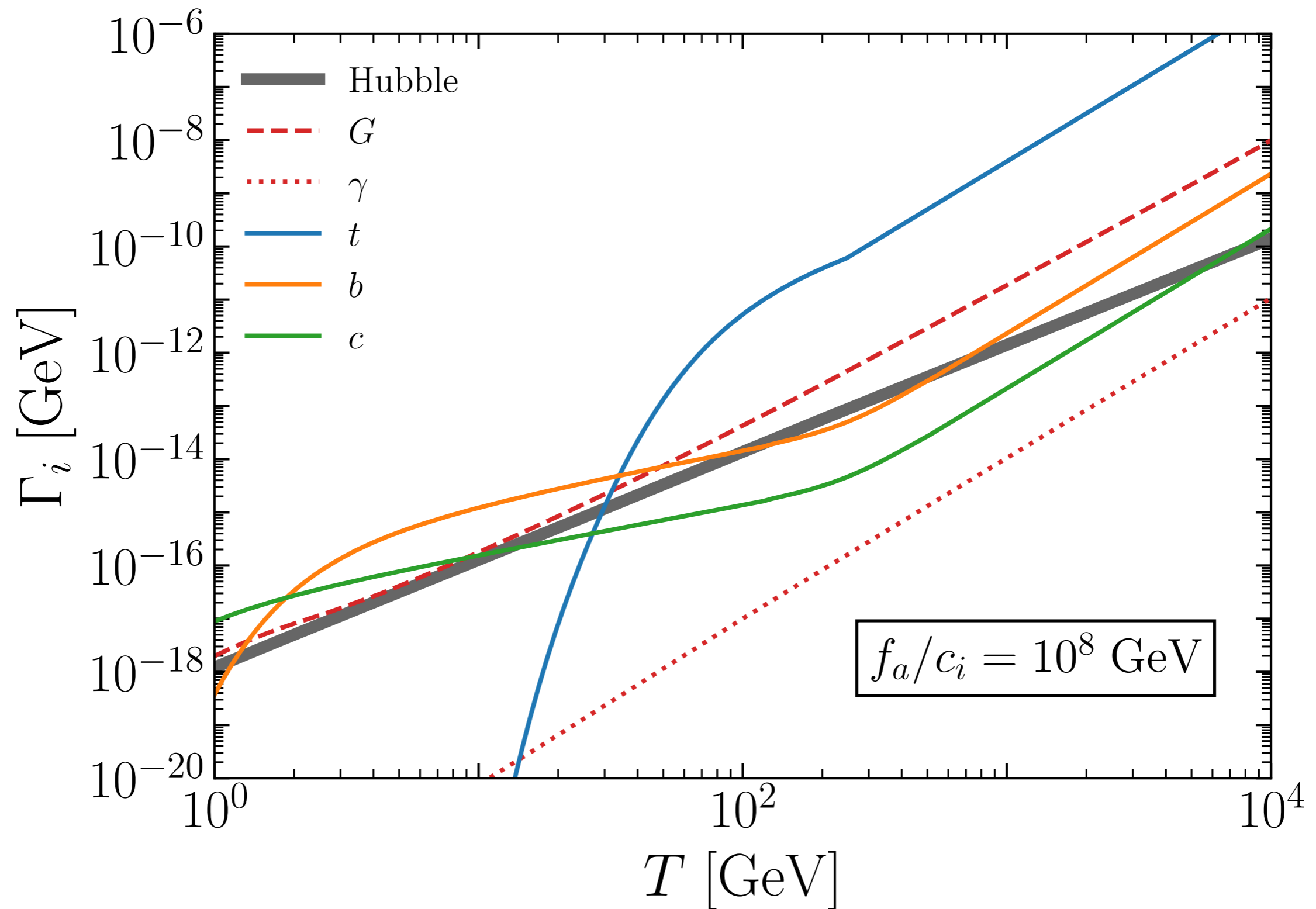
- Assume they thermalize at early times
- Estimate the decoupling temperature,  $\Gamma(T_D) = H(T_D)$ , and the resulting  $\Delta N_{\text{eff}}$

$$\Delta N_{\text{eff}} \simeq 0.027 \left( \frac{106.75}{g_{*s}(T_D)} \right)^{4/3}$$

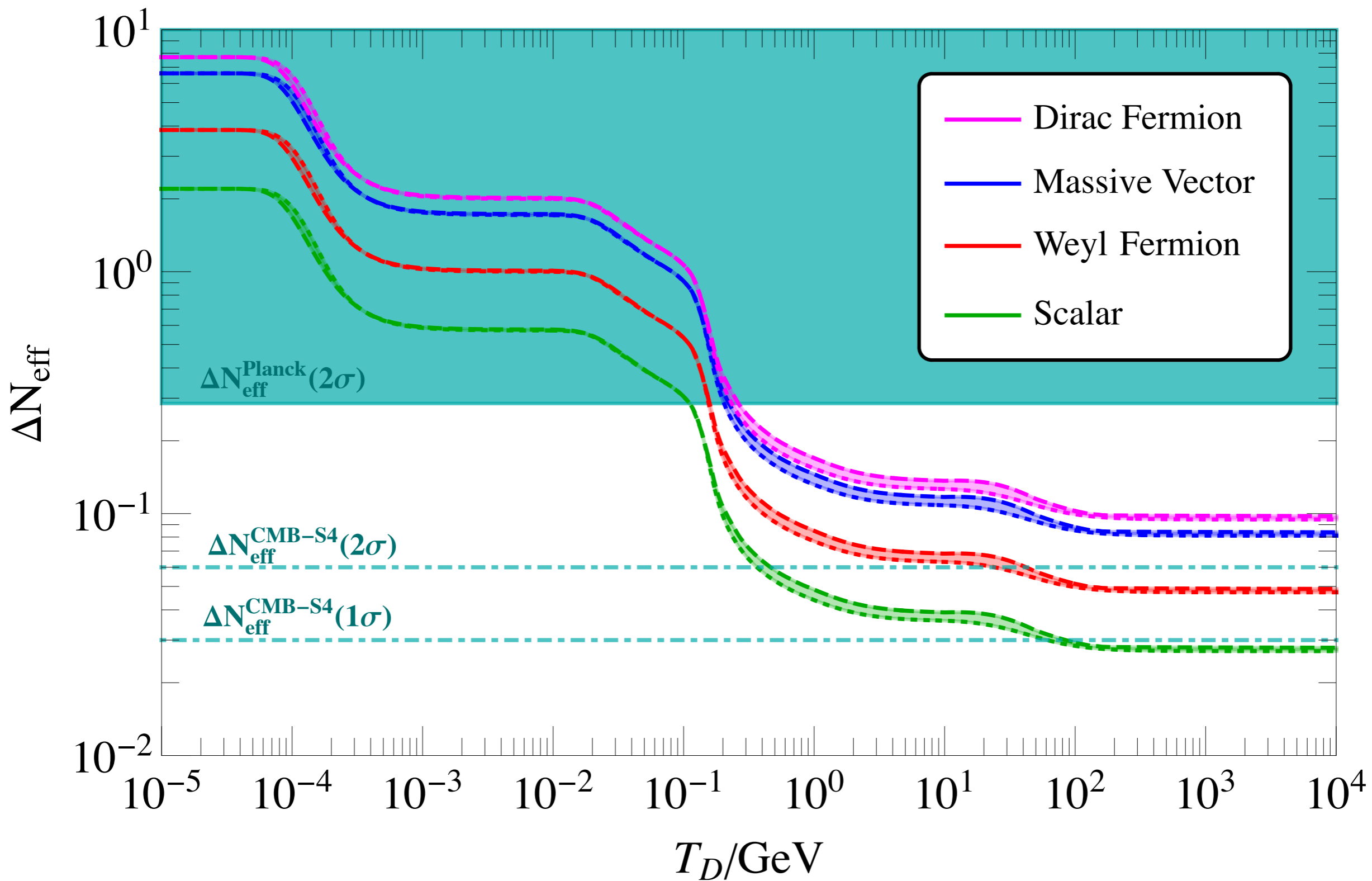
# Do they thermalize? And when?



# Do they thermalize? And when?



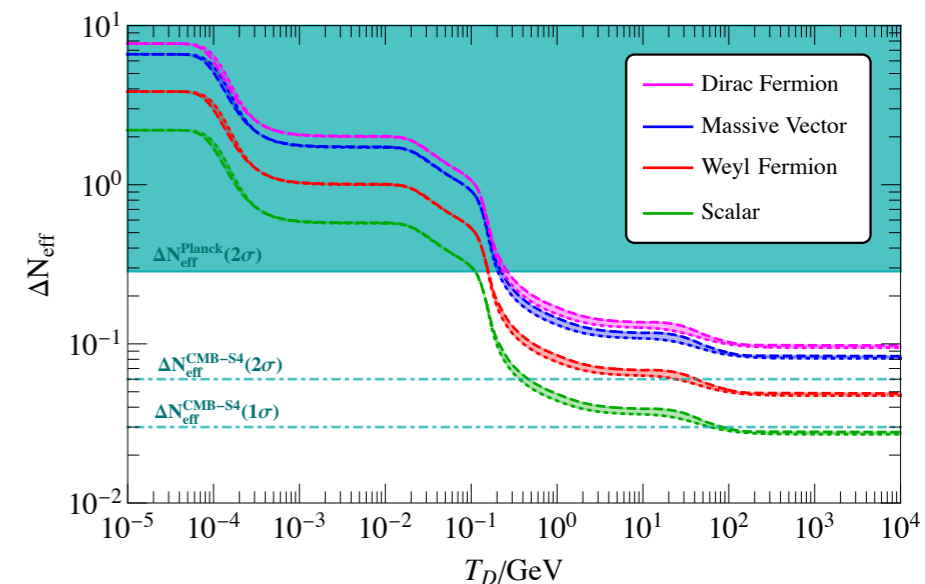
# Bounds and Prospects



# Predicting $\Delta N_{\text{eff}}$

Axions may never thermalize

If they do, decoupling detail relevant  
(effect larger the experimental error)



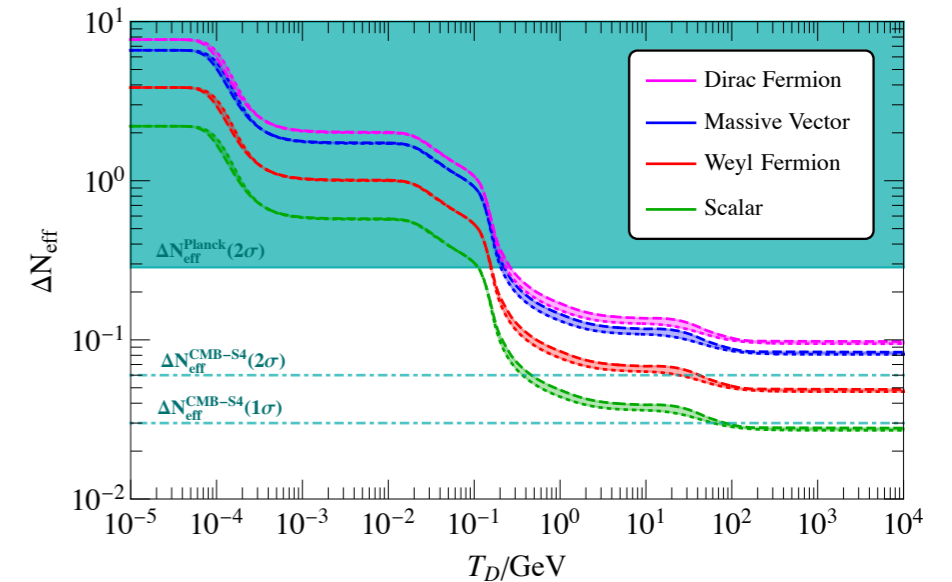
# Predicting $\Delta N_{\text{eff}}$

Axions may never thermalize

If they do, decoupling detail relevant  
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## An improved methodology

- Track the number density of axions
- Convert the asymptotic result to  $\Delta N_{\text{eff}}$  via the equilibrium distribution



$$\frac{dn_a}{dt} + 3Hn_a = \sum_{\alpha} \gamma_{\alpha}$$

$$\Delta N_{\text{eff}} \simeq 74.85 Y_a^{4/3}$$

# Predicting $\Delta N_{\text{eff}}$

Axions may never thermalize

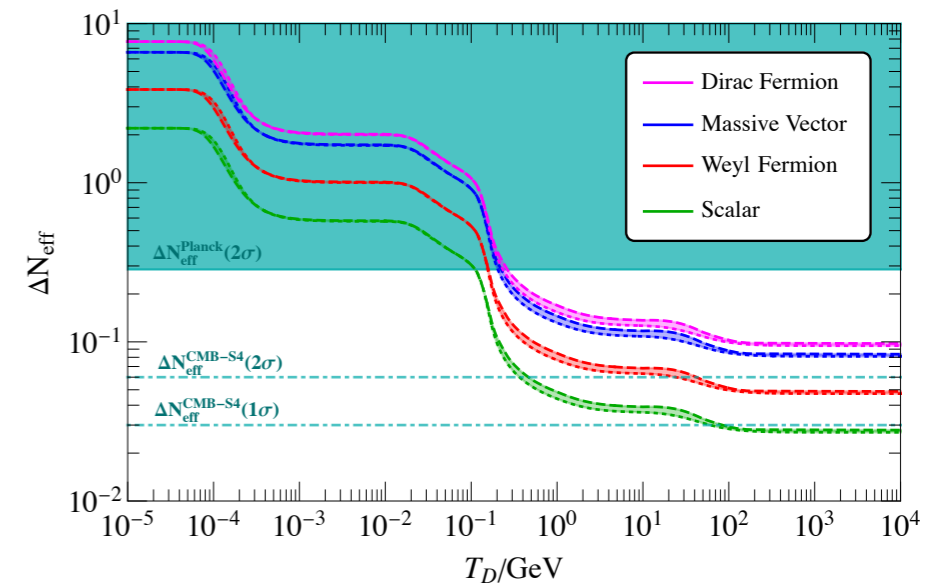
If they do, decoupling detail relevant  
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## An improved methodology

- Track the number density of axions
- Convert the asymptotic result to  $\Delta N_{\text{eff}}$  via the equilibrium distribution



Equilibrium thermodynamics, Spectral distortions neglected,  
Maxwell-Boltzmann statistics (i.e., no quantum effects),  
Static thermal bath (i.e., no energy exchanged)



$$\frac{dn_a}{dt} + 3Hn_a = \sum_{\alpha} \gamma_{\alpha}$$

$$\Delta N_{\text{eff}} \simeq 74.85 Y_a^{4/3}$$



# Irreducible Part for the QCD Axion

Strong CP Problem

$$\frac{a}{f_a} \frac{\alpha_s}{8\pi} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

**Above**

$$gg \rightarrow ga$$

$$\bar{q}q \rightarrow ga$$

$$q/\bar{q} g \rightarrow q/\bar{q} a$$

Long-range of gluon-mediated interactions  
give rise to unpleasant IR behavior

Masso, Rota, Zsembinski, PRD66 (2002)

Graf, Steffen, PRD 83 (2011)

Salvio, Strumia, Xue, JCAP 01 (2014)

**Below**

Pion scattering

$$\pi\pi \rightarrow \pi a$$

Chang, Choi, Phys.Lett.B 316 (1993)

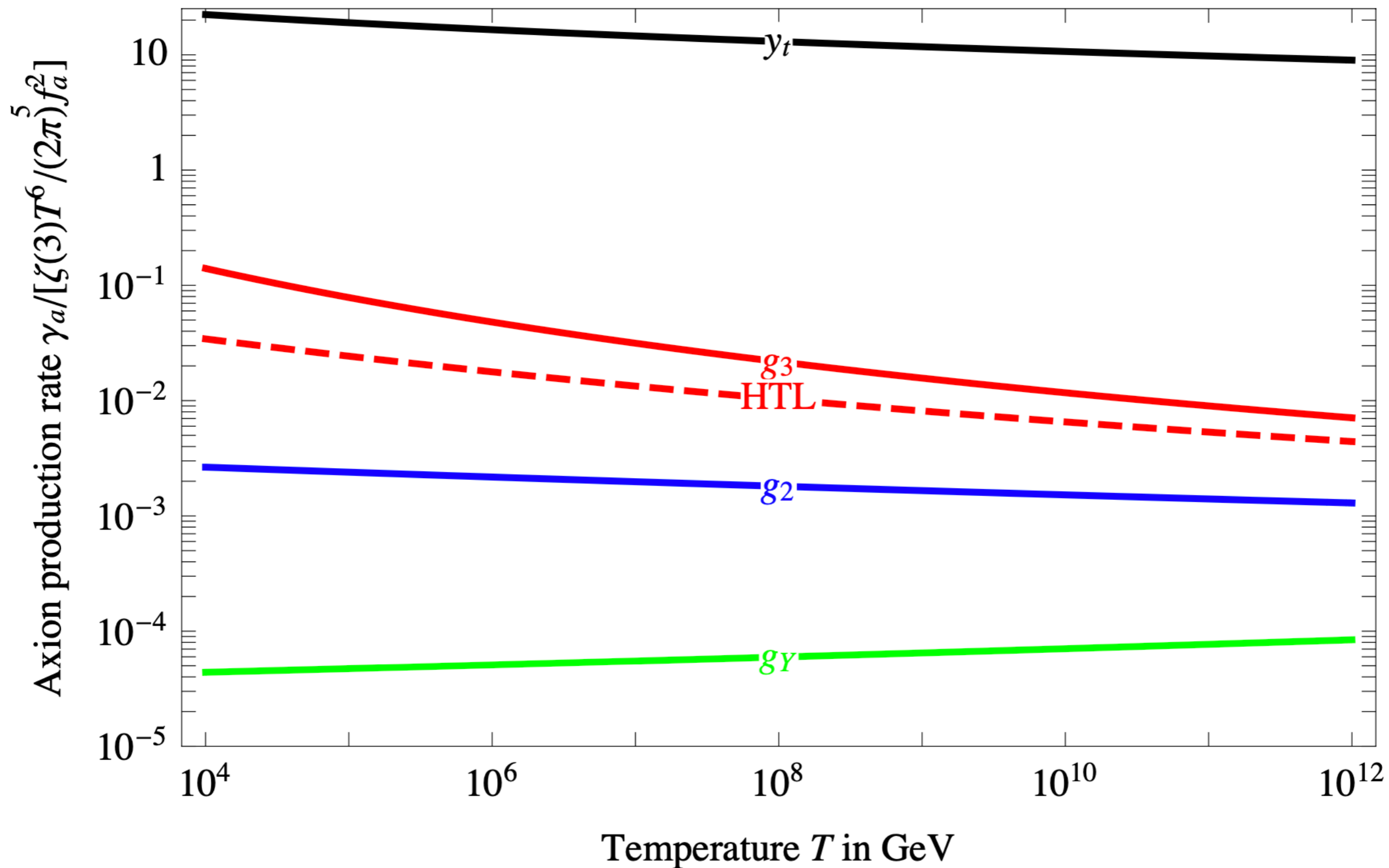
Recent studies:

Di Luzio, Martinelli, Piazza, Phys.Rev.Lett. 126 (2021)

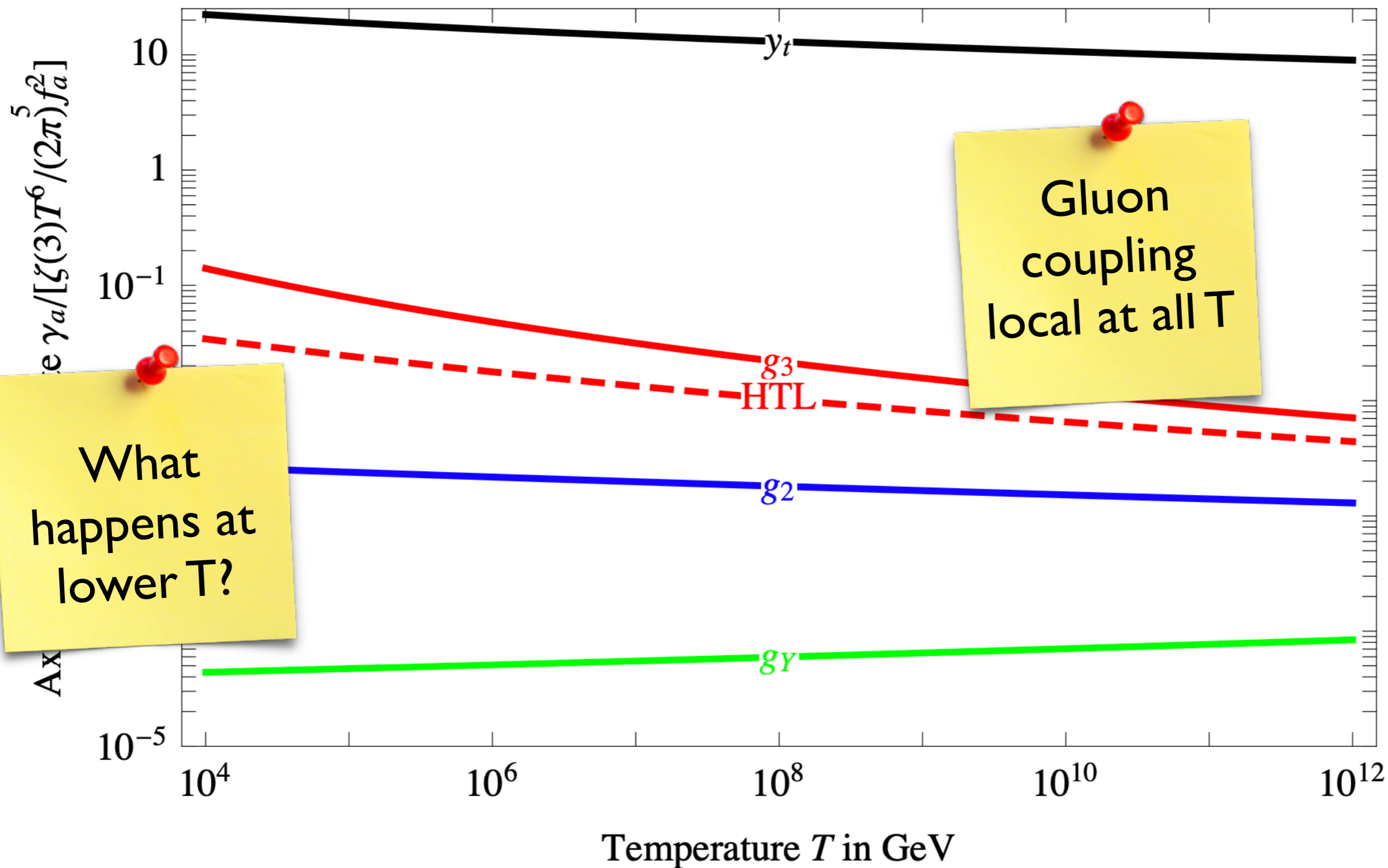
Notari, Rompineve, Villadoro, Phys.Rev.Lett. 131 (2023)

Di Luzio, Camalich, Martinelli, Piazza, Phys.Rev.D 108 (2023)

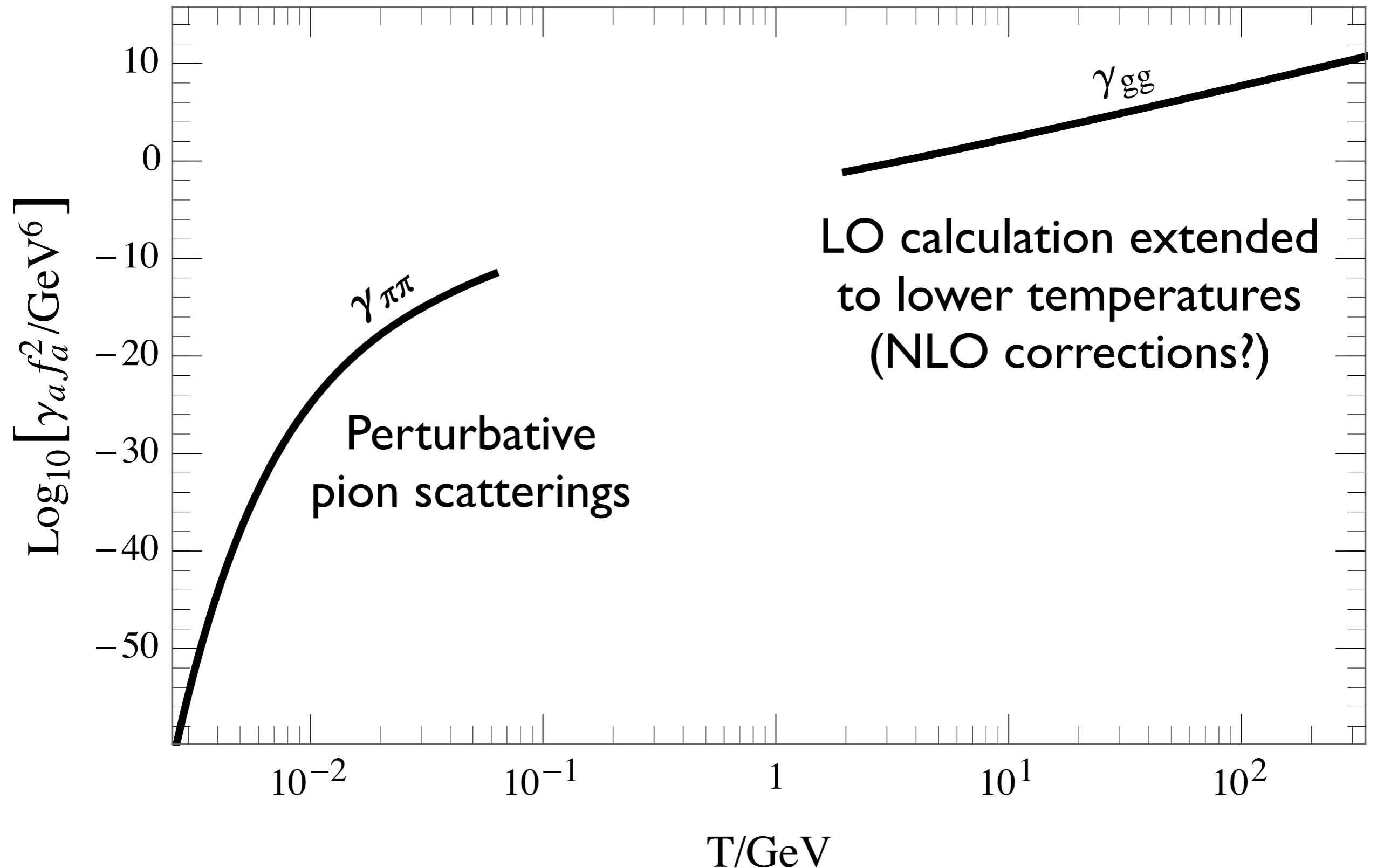
# Thermal Gluon Scattering



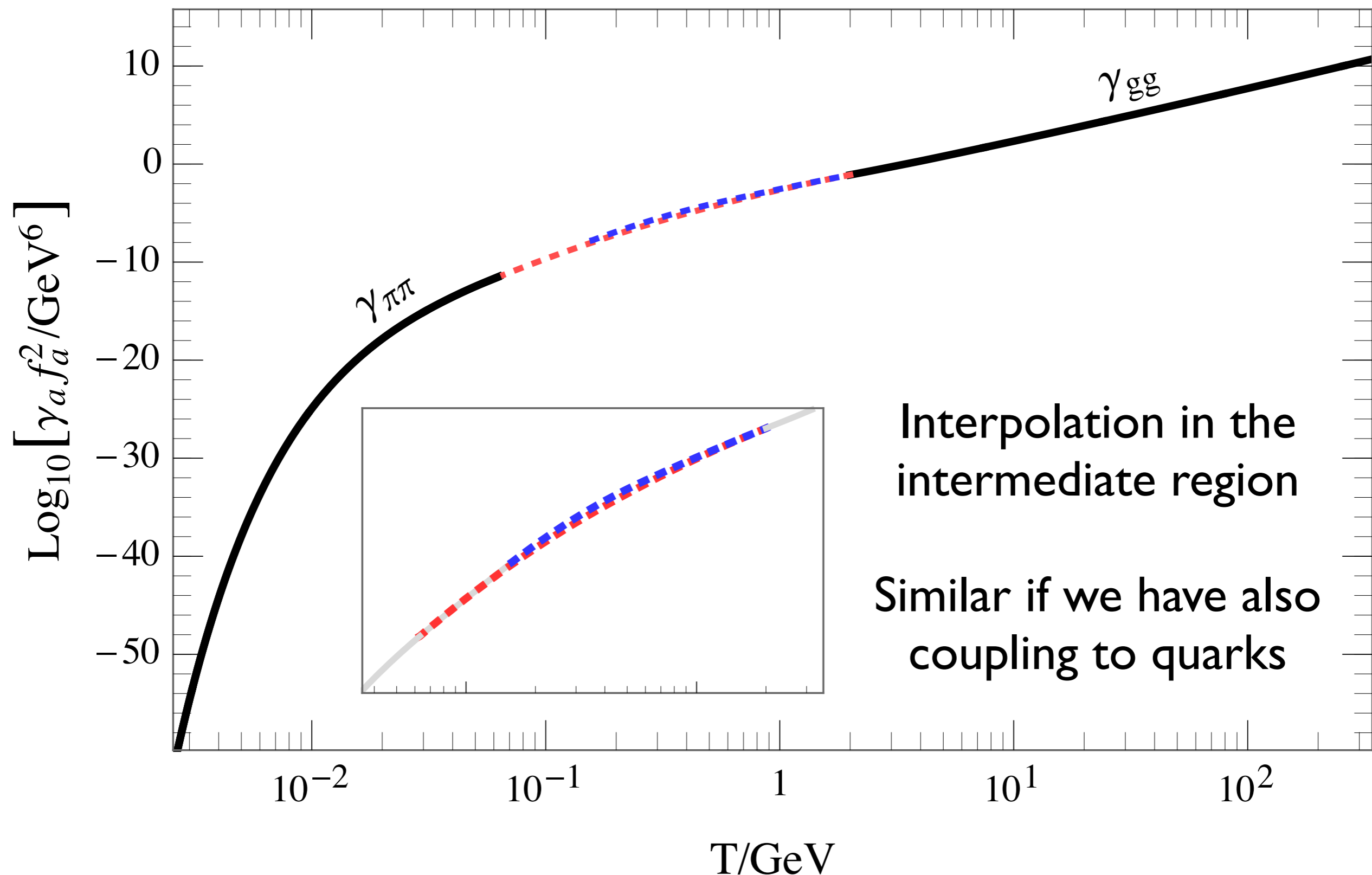
# Thermal Gluon Scattering



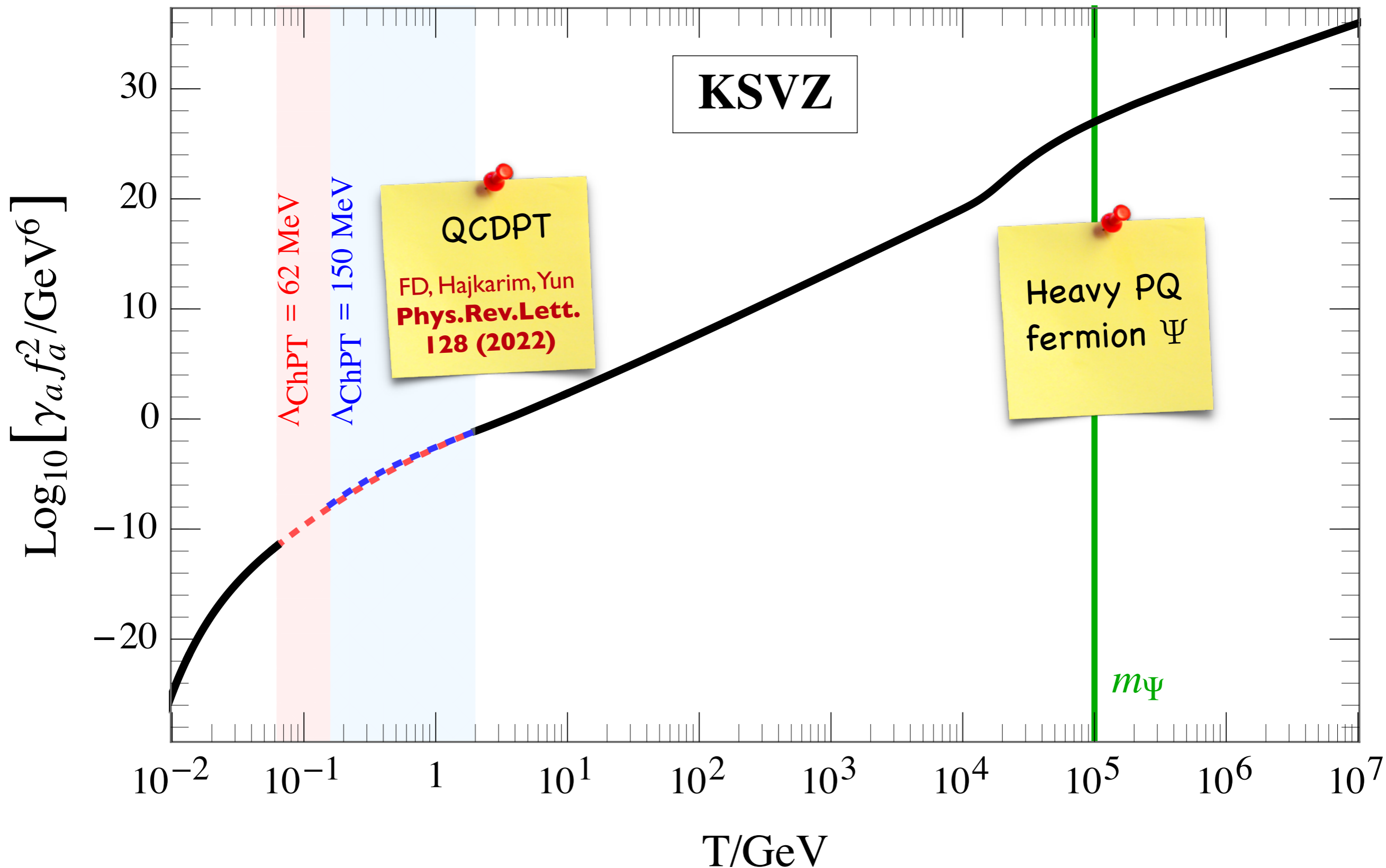
# Approaching the QCDPT



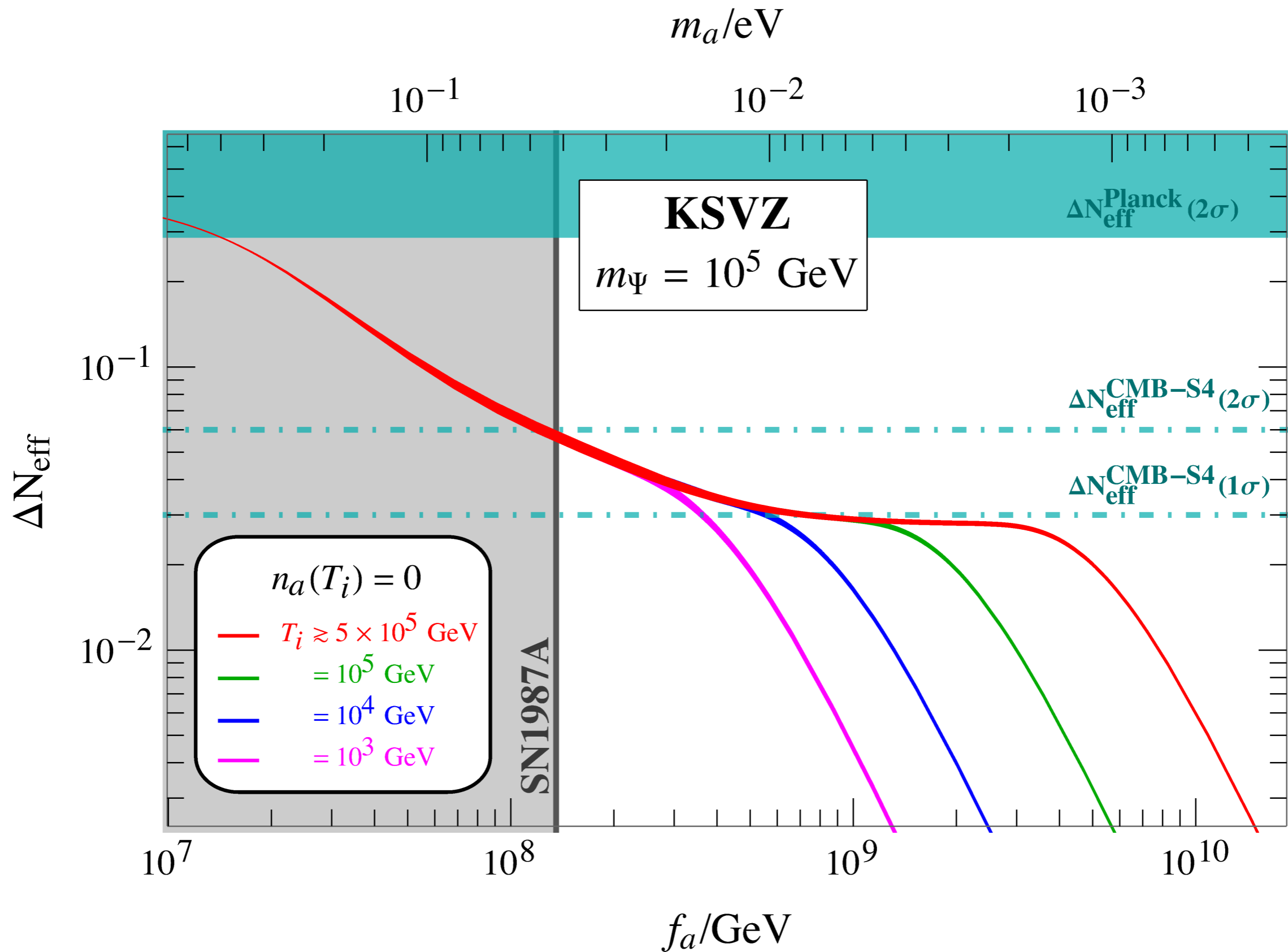
# Approaching the QCDPT



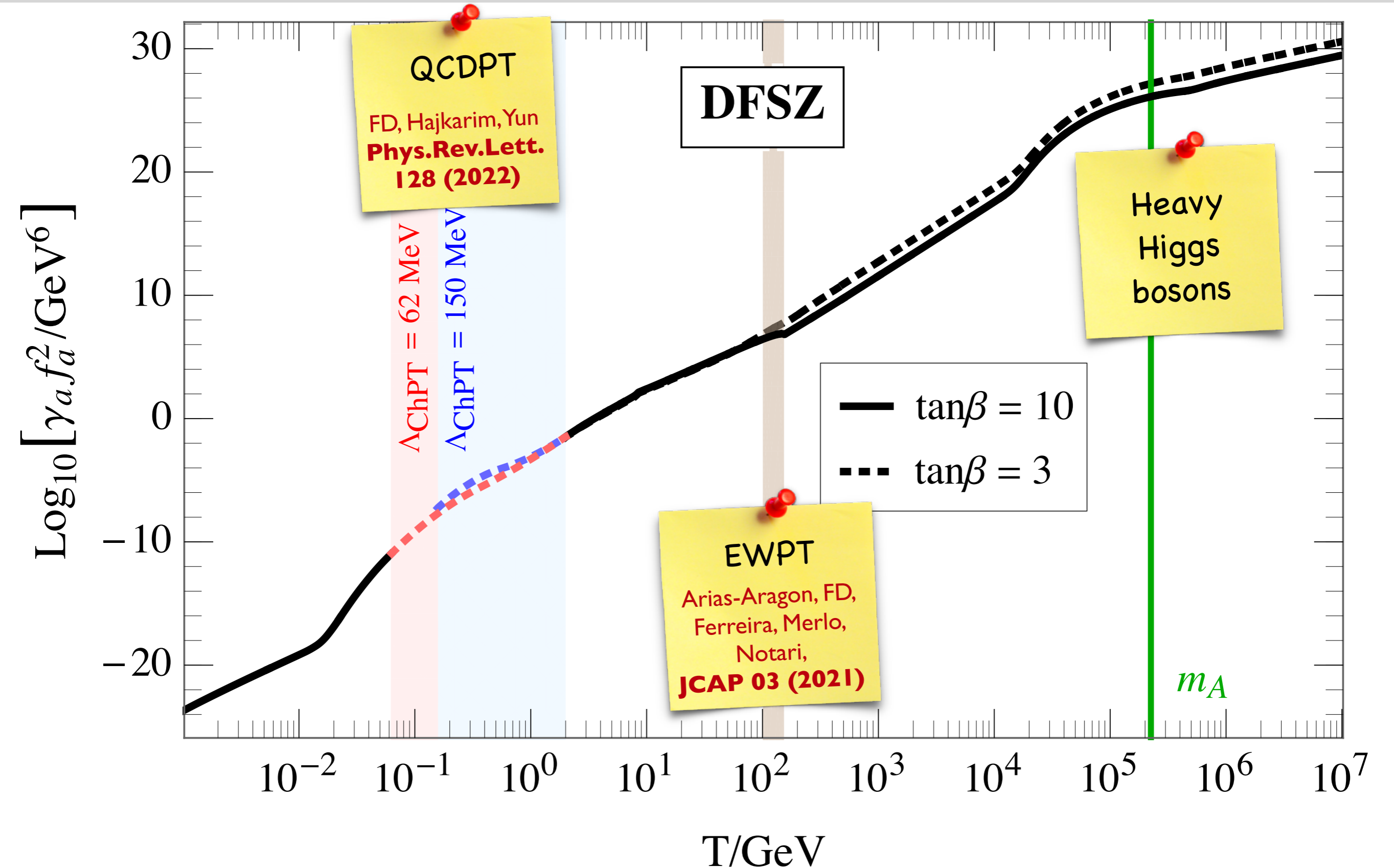
# KSVZ Axion — Production Rate



# KSVZ Axion — $\Delta N_{\text{eff}}$

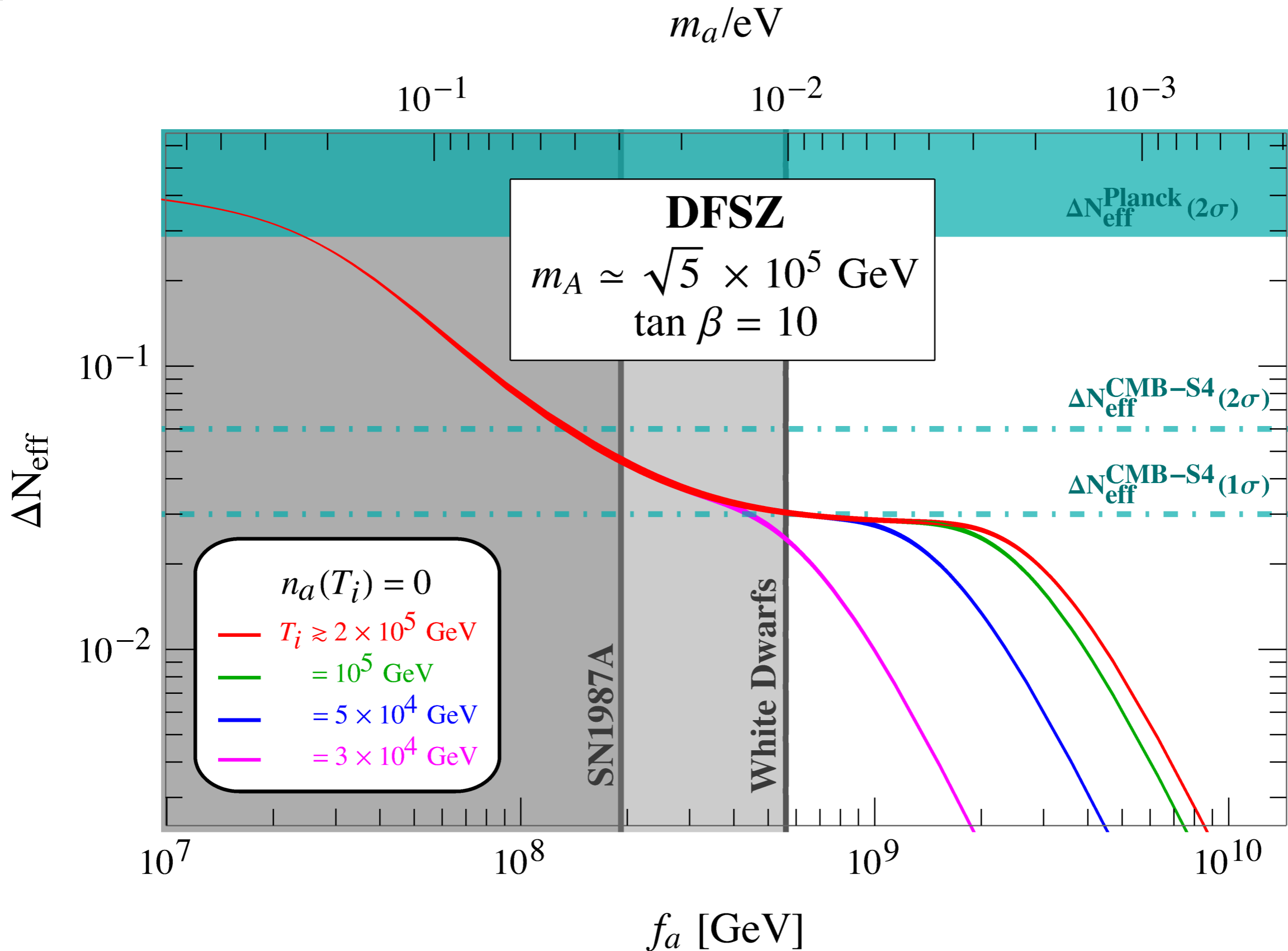


# DFSZ Axion — Production Rate

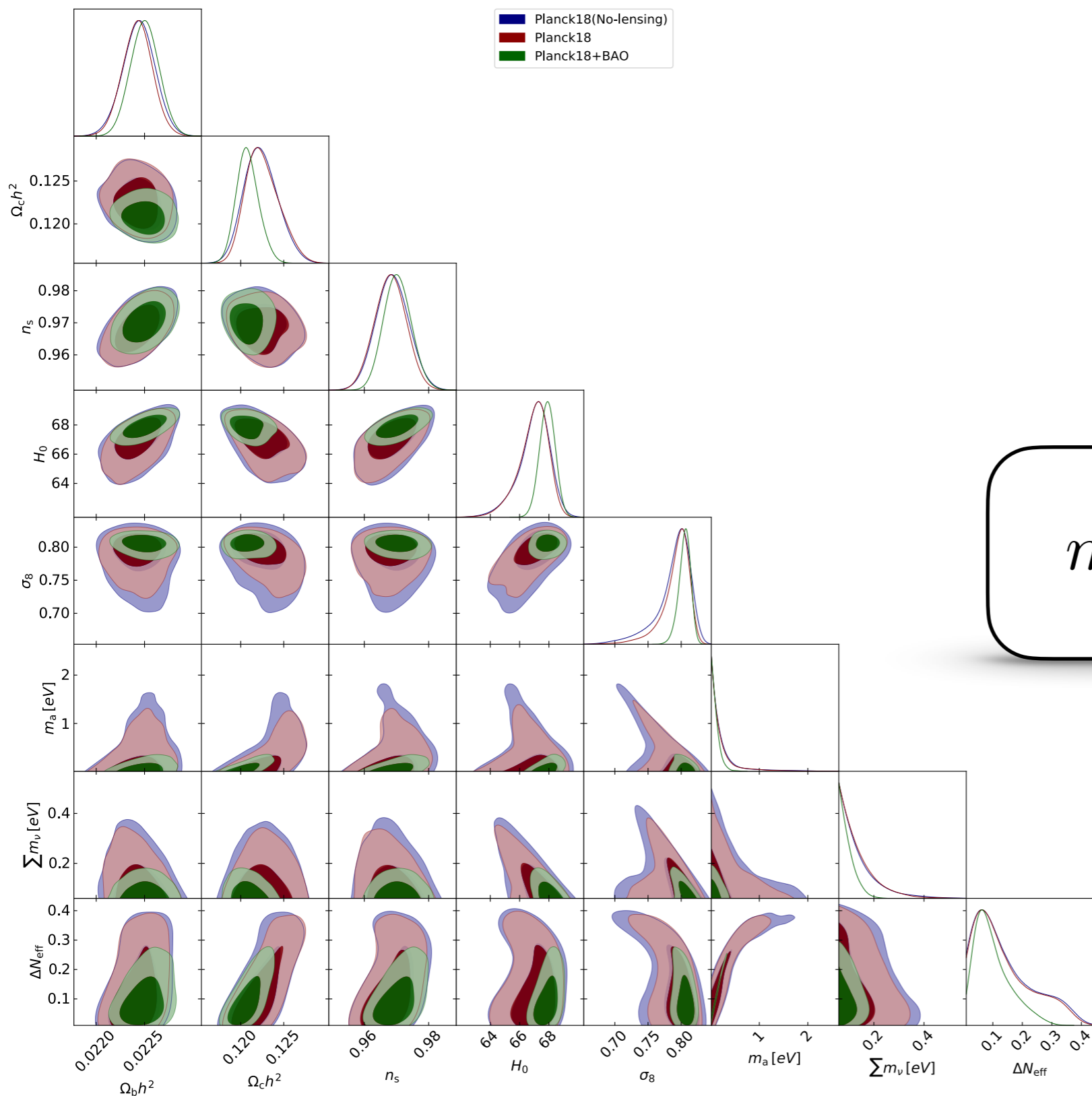




# DFSZ Axion — $\Delta N_{\text{eff}}$

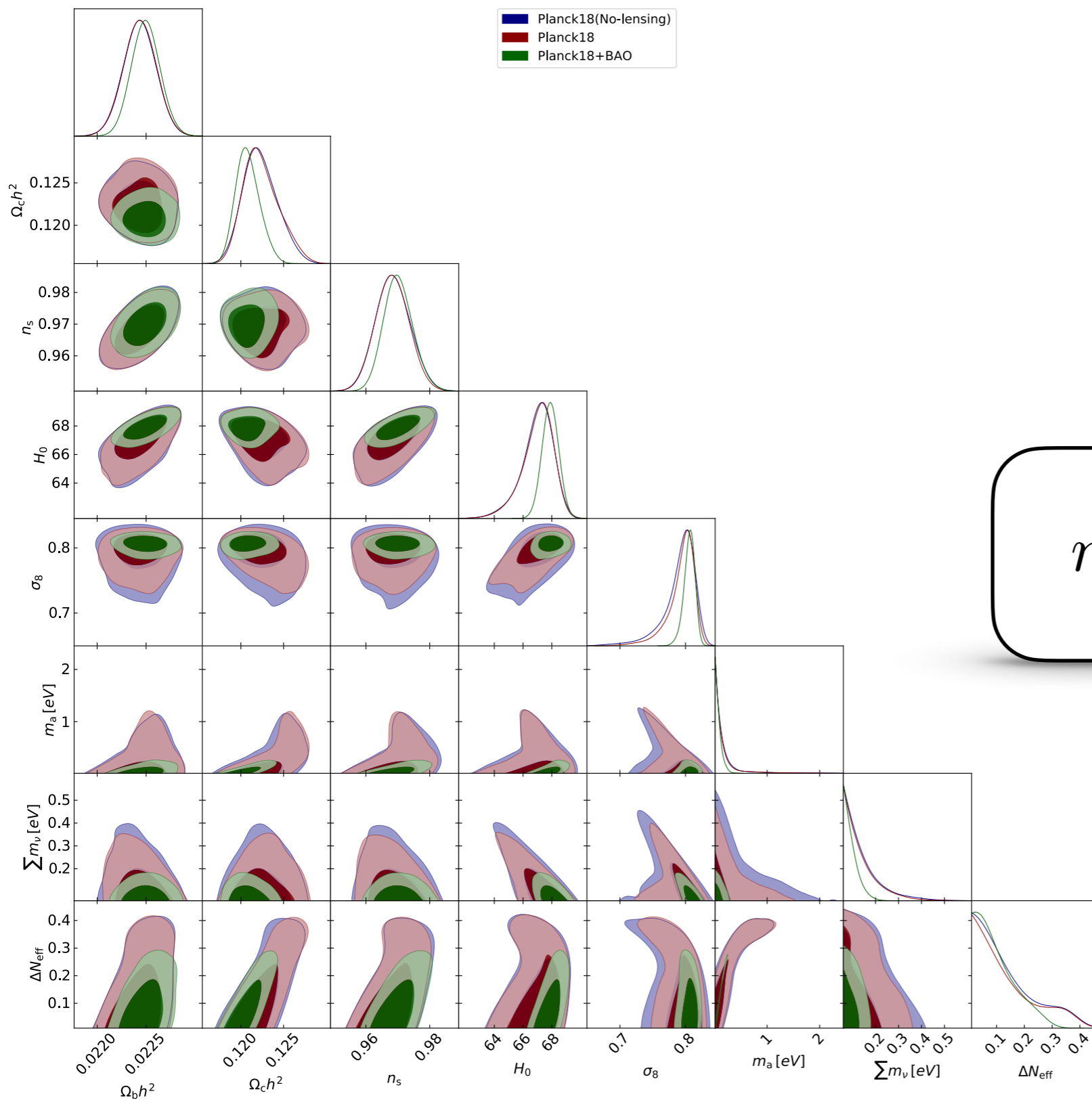


# KSVZ Axion Mass Bound



$$m_a \leq 0.282(0.420) \text{ eV}$$

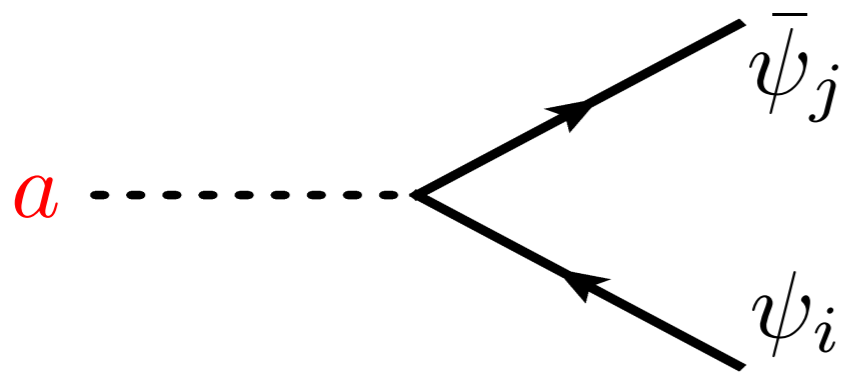
# DFSZ Axion Mass Bound



$$m_a \leq 0.209(0.293) \text{ eV}$$

# A Minor Variation: FV Axions

$$\mathcal{L}_{\text{FV}}^{(a)} = \frac{\partial_\mu a}{2f_a} \sum_{\psi_i \neq \psi_j} \bar{\psi}_i \gamma^\mu \left( c_{\psi_i \psi_j}^V + c_{\psi_i \psi_j}^A \gamma^5 \right) \psi_j$$



Target of several terrestrial experiments

Camalich et al., Phys.Rev.D 102 (2020)

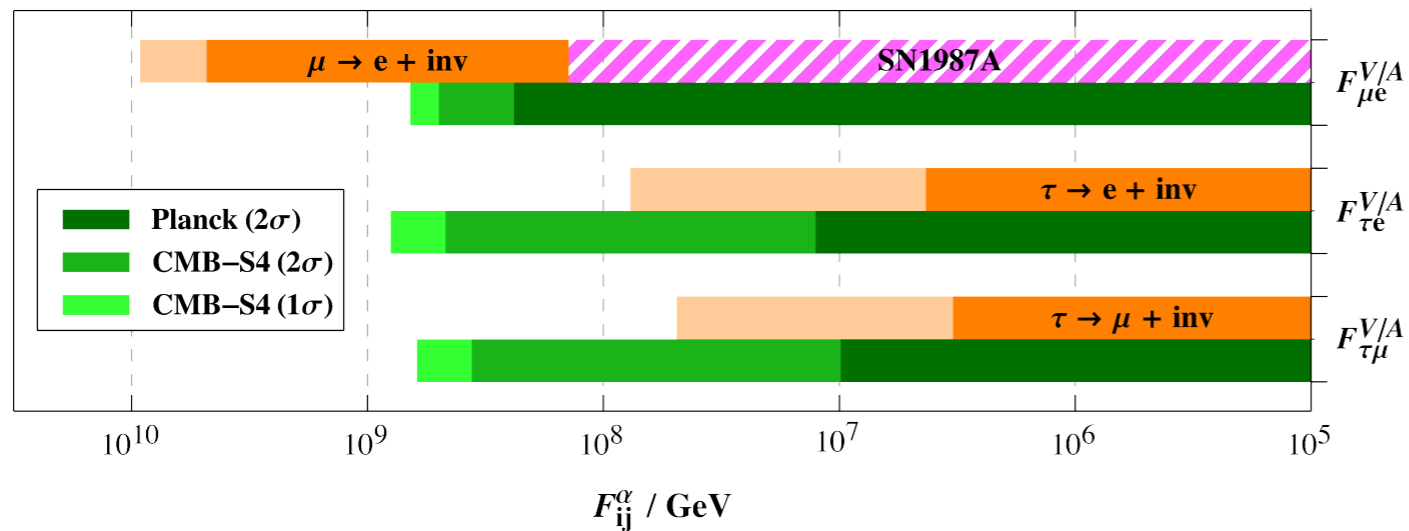
Calibbi et al., JHEP 09 (2021)

What about their role in the early universe?

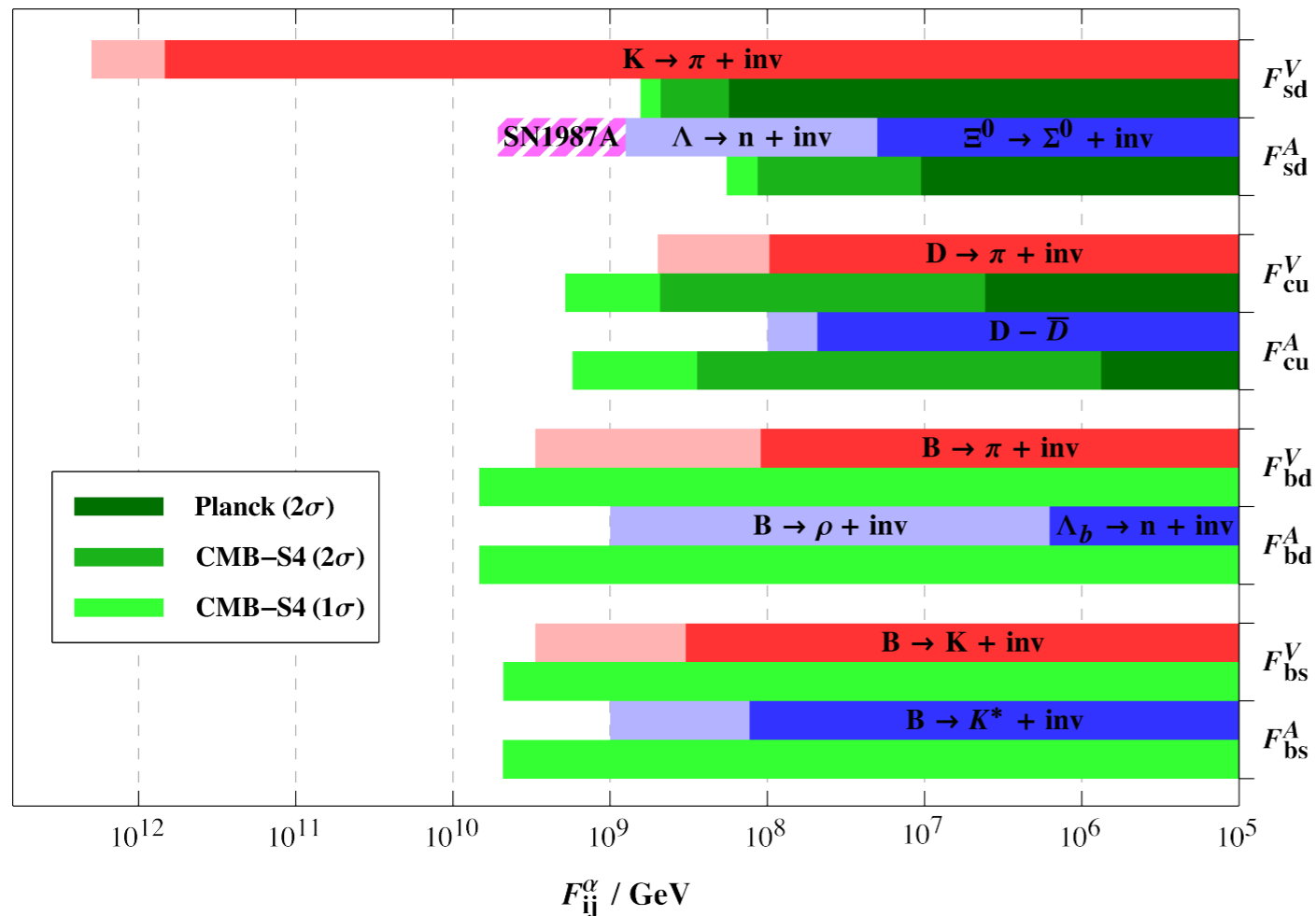
They mediate hot axion production!

# A Minor Variation: FV Axions

Leptonic FV

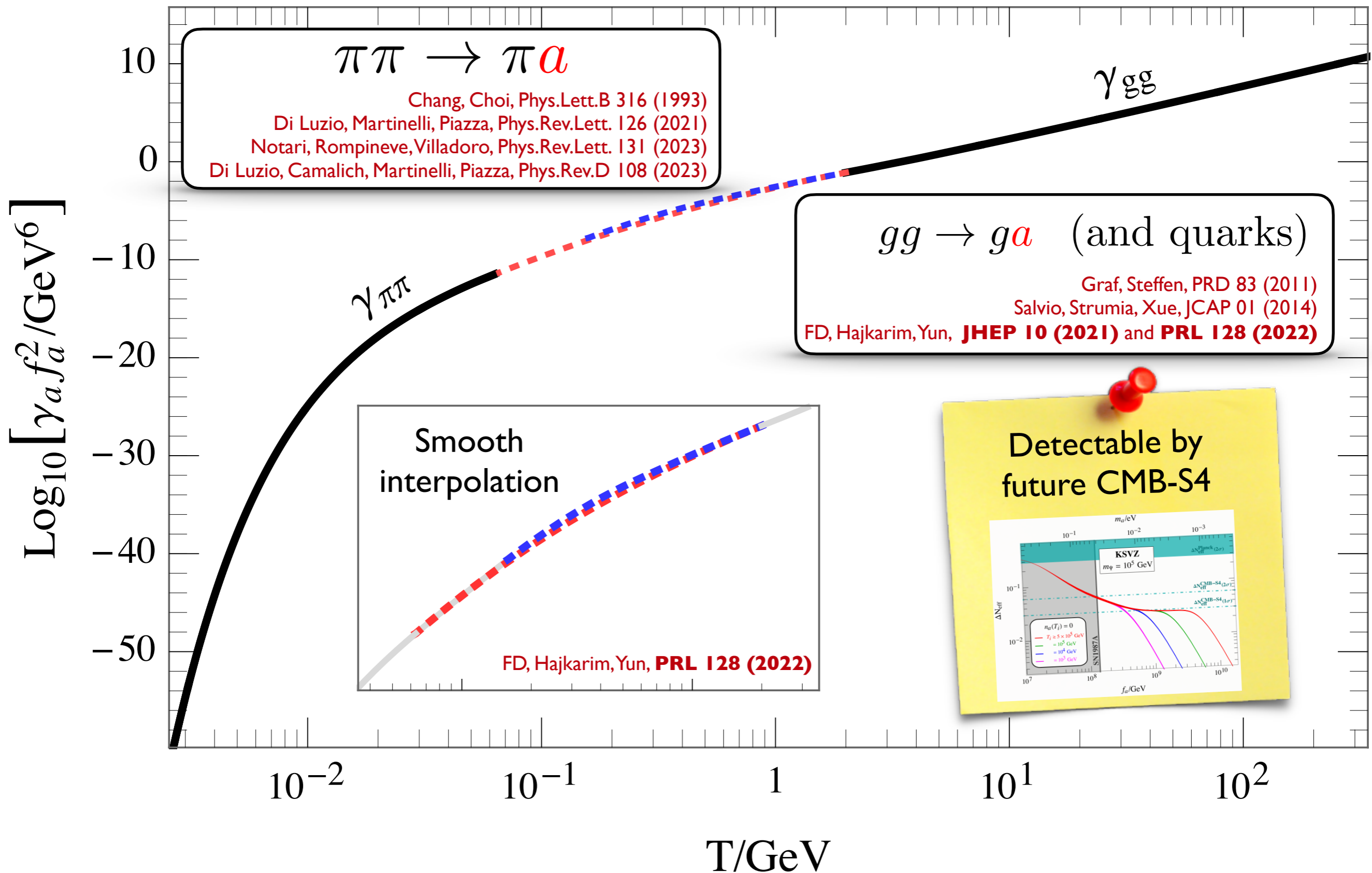


Hadronic FV



$$F_{\psi_i \psi_j}^\alpha \equiv \frac{2f_a}{c_{\psi_i \psi_j}^\alpha}$$

# Where Do We Stand?



# What's Next?

Axion production rate  
across the confinement scale still unknown

$$\gamma_a = n_i n_j \times \langle \sigma_{ij \rightarrow ja} v_{\text{rel}} \rangle$$

Thermal bath

Particle Physics

1. Cross sections with other hadrons?
2. Thermal bath description between 150 MeV and fews GeV?
3. Boltzmann equation evolution and cosmological observables?

# Back to the Phase-Space

Model-independent analysis:  
generic thermal bath production of a  
single dark radiation particle  $X$

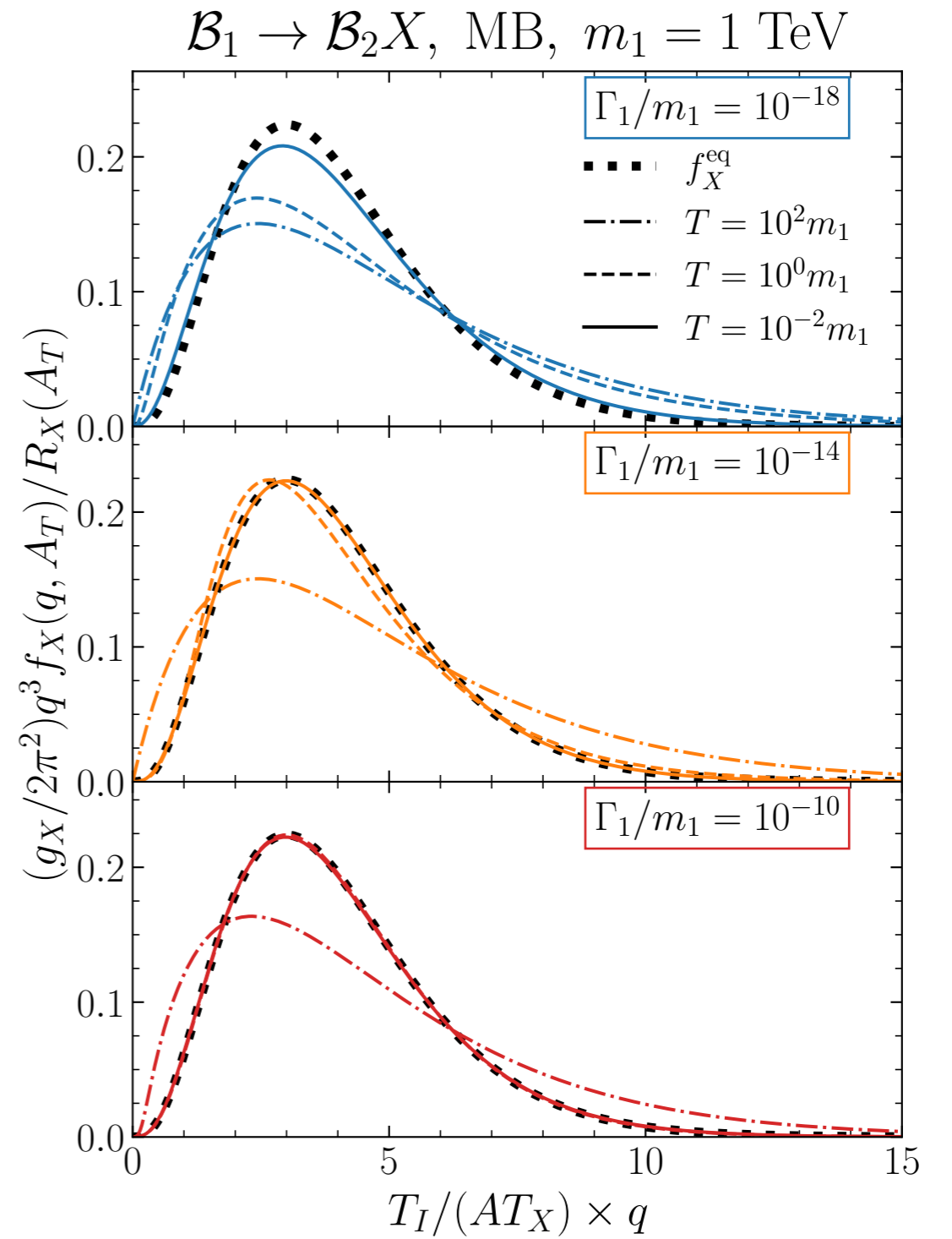
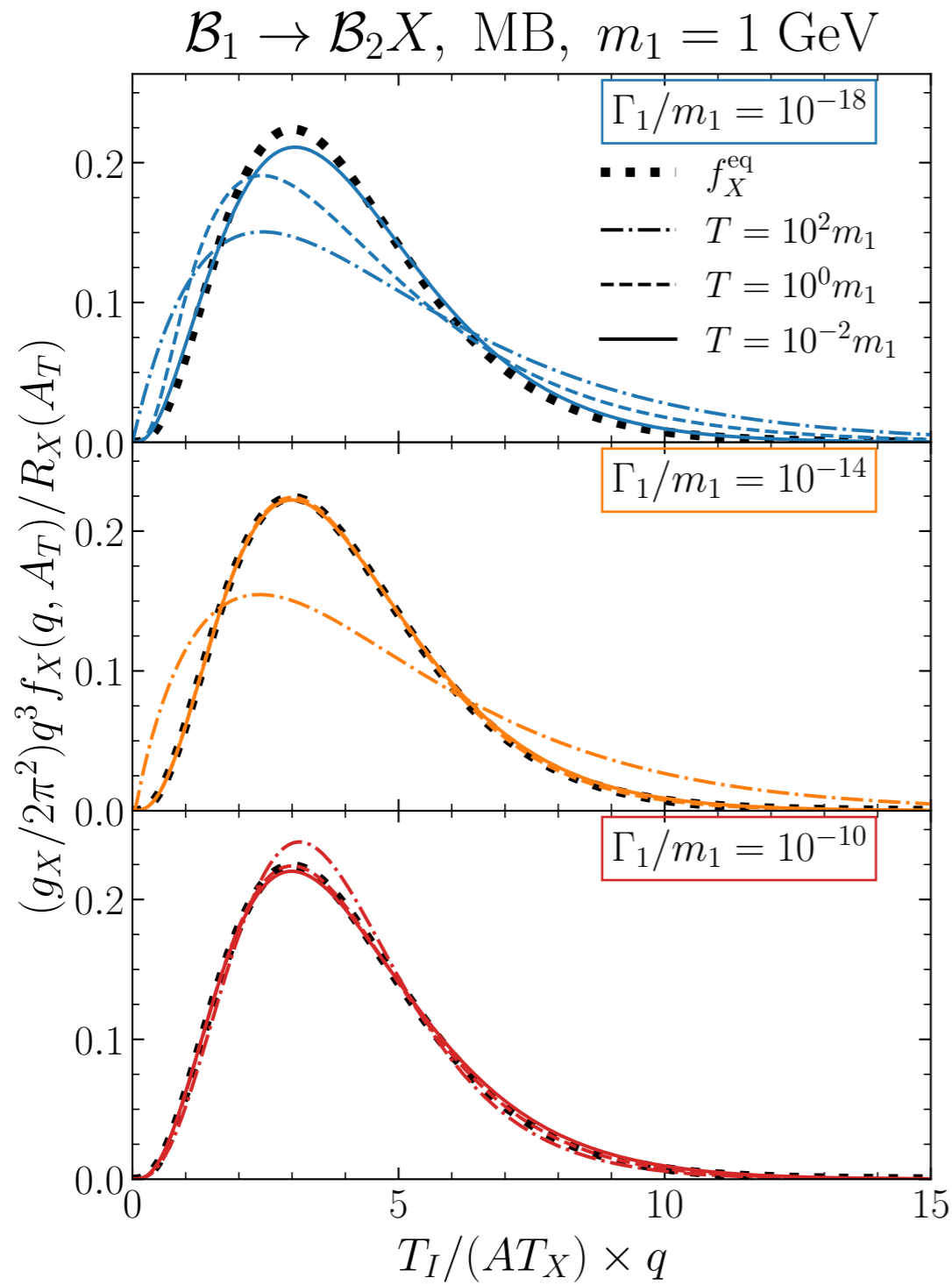
$$\mathcal{B}_1 \dots \mathcal{B}_n \rightarrow \mathcal{B}_{n+1} \dots \mathcal{B}_m X$$

$$\frac{df_X(k, t)}{dt} = \left( 1 - \frac{f_X(k, t)}{f_X^{\text{eq}}(k, t)} \right) \mathcal{C}_{n \rightarrow mX}(k, t)$$

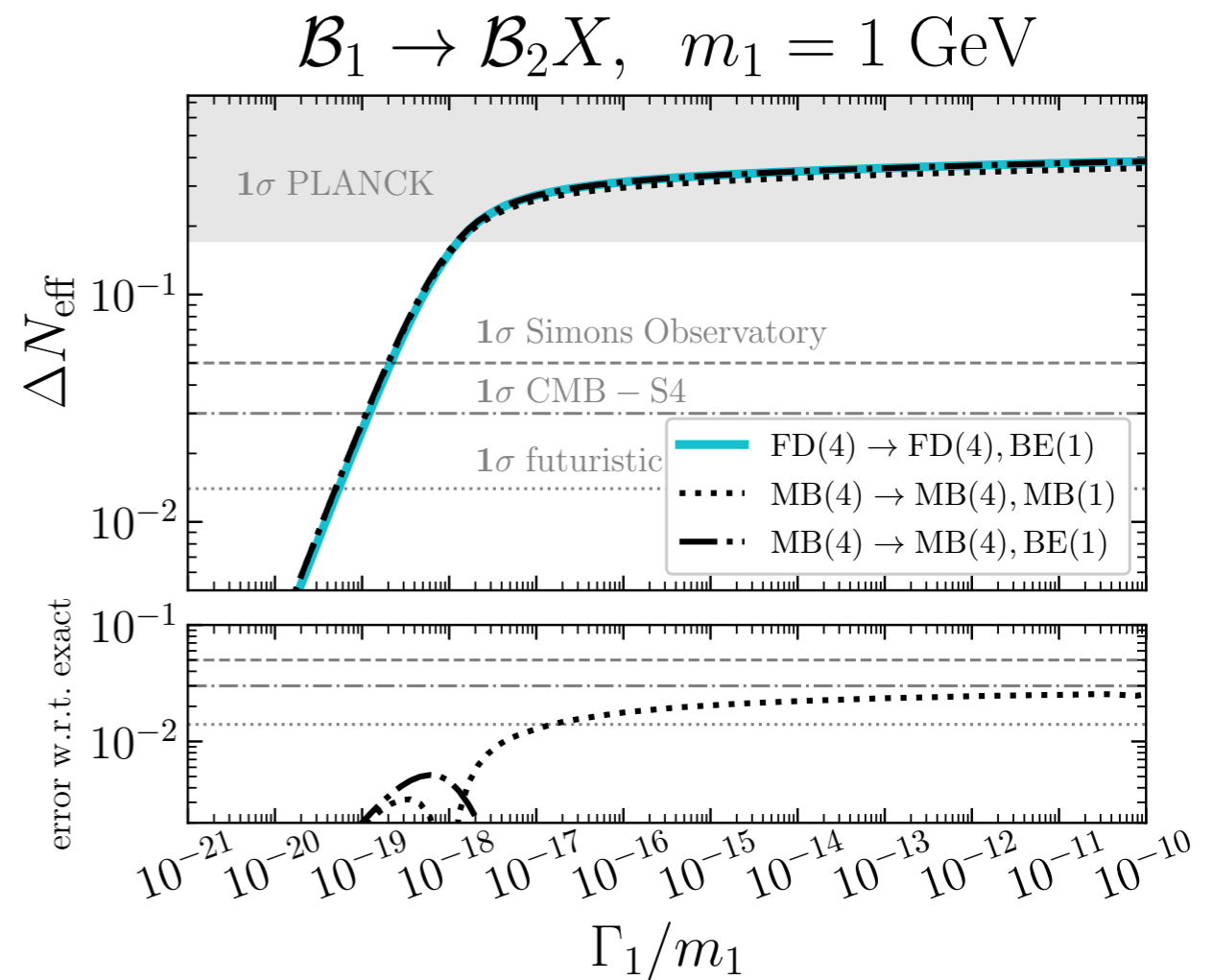
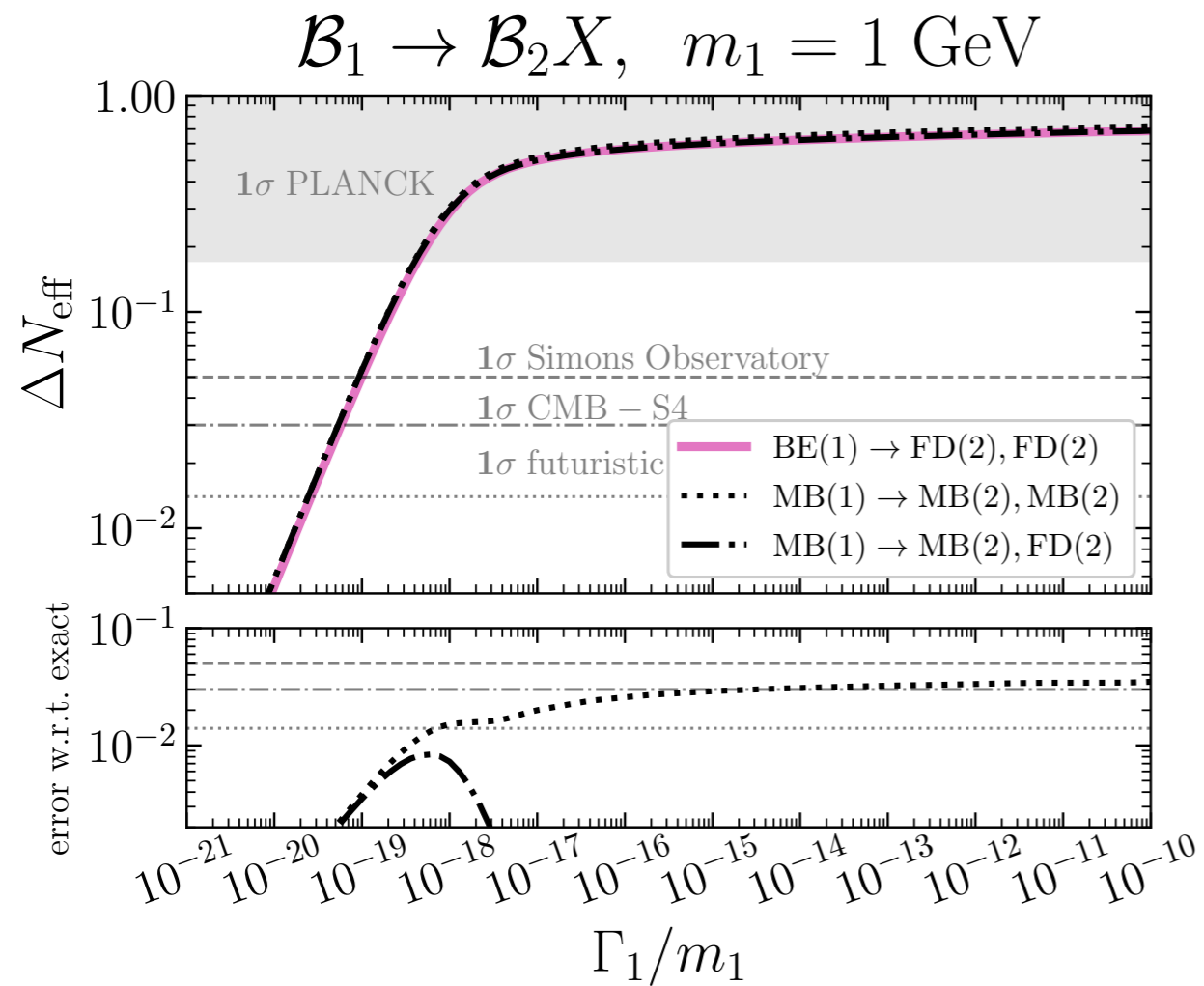
1. Keep track of phase-space and compute the energy density
2. Quantum statistical effects take into account
3. Energy exchanged with the thermal bath accounted for



# Phase Space Distributions

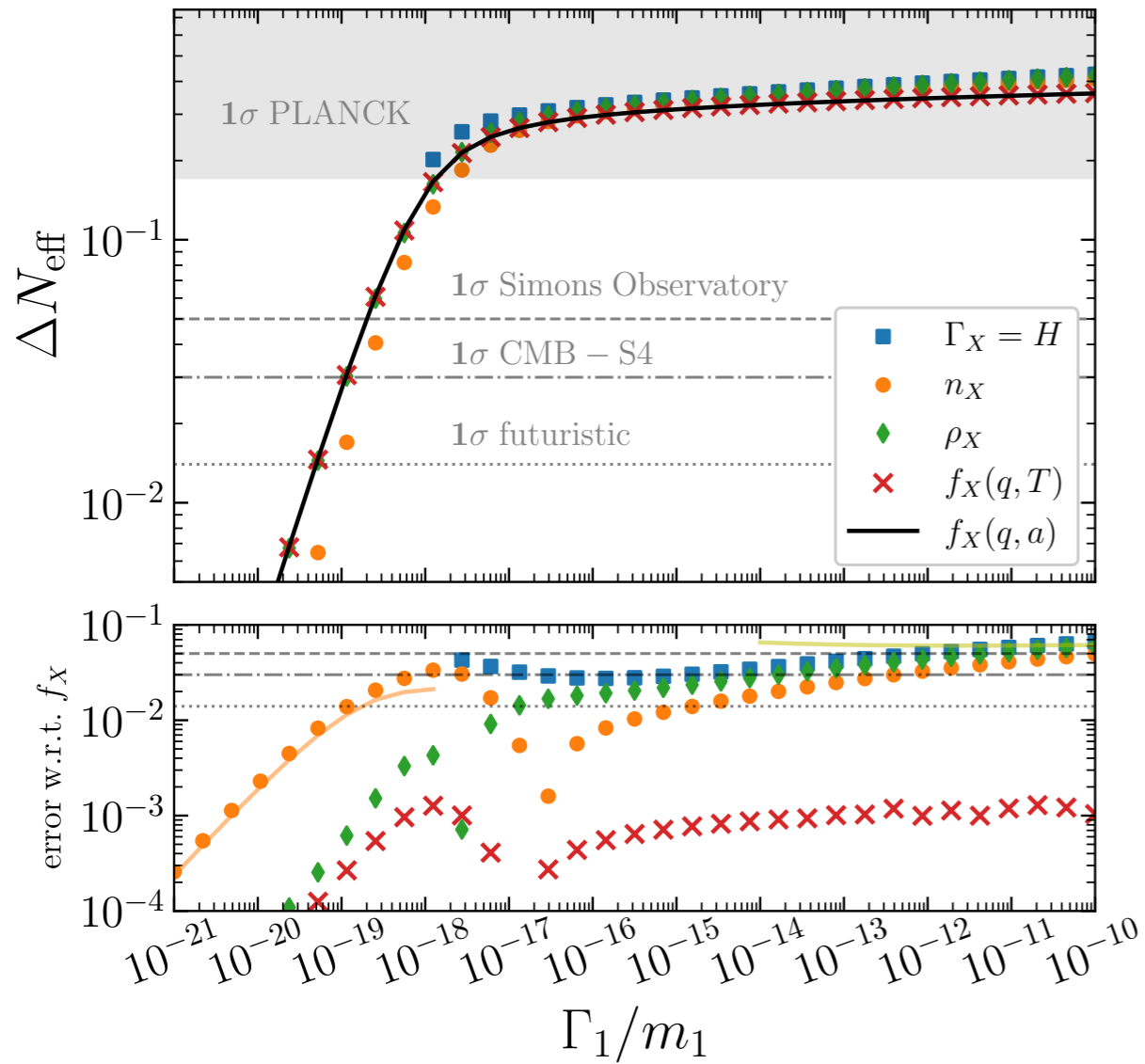


# Quantum Statistical Effects

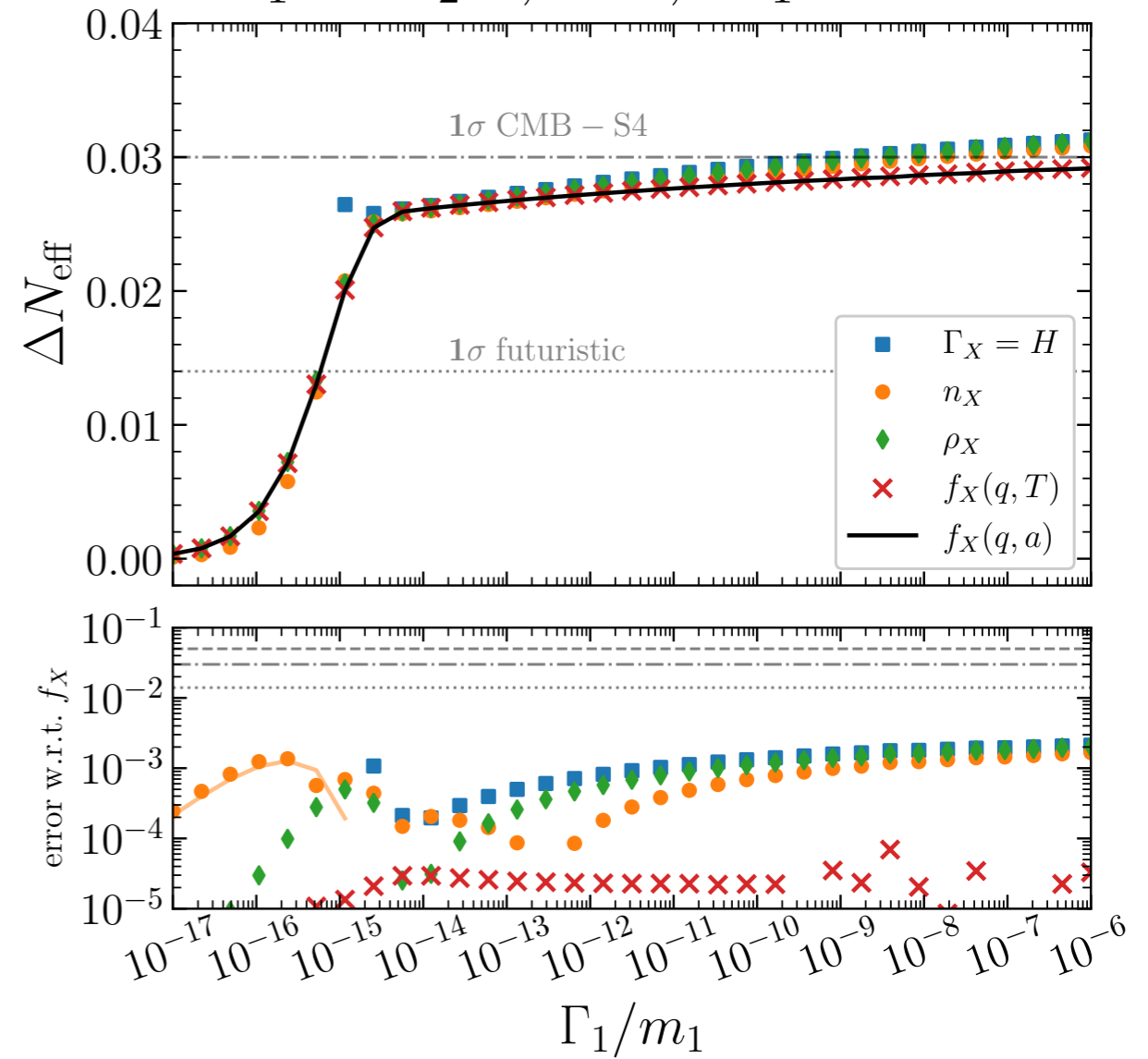


# Error in predicting $\Delta N_{\text{eff}}$

$\mathcal{B}_1 \rightarrow \mathcal{B}_2 X$ , MB,  $m_1 = 1 \text{ GeV}$



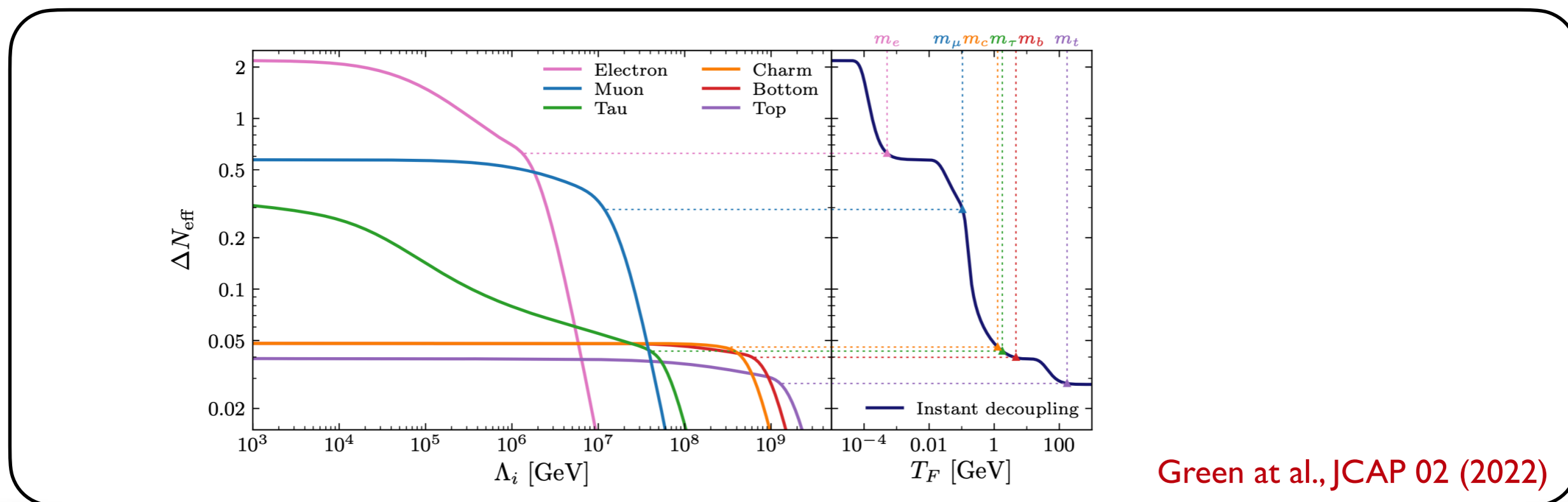
$\mathcal{B}_1 \rightarrow \mathcal{B}_2 X$ , MB,  $m_1 = 1 \text{ TeV}$



# Axion-Fermion Interactions

$$\mathcal{L}_{\text{int}} = \frac{\partial_\mu a}{2f_a} \sum_\psi c_\psi \bar{\psi} \gamma^\mu \gamma_5 \psi$$

Baumann et al, Phys.Rev.Lett. 117 (2016)  
 Ferreira, Notari, Phys.Rev.Lett. 120 (2018)  
 FD et al, **JCAP 11 (2018)**  
 Arias-Aragón et al., **JCAP 11 (2020)**  
 and **JCAP 03 (2021)**

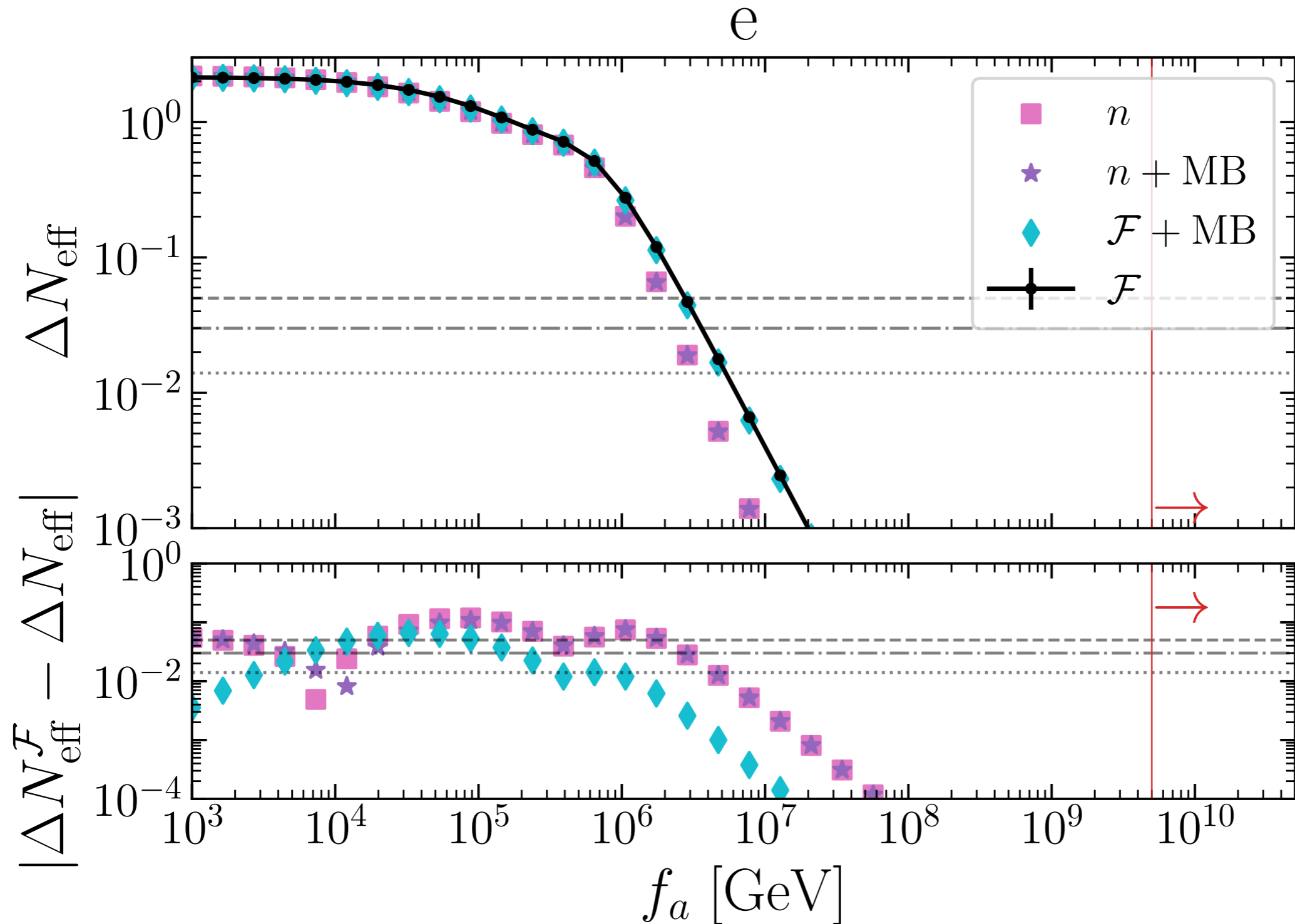


Related work:

KSVZ axion mass bound in the phase space (pion scattering only)

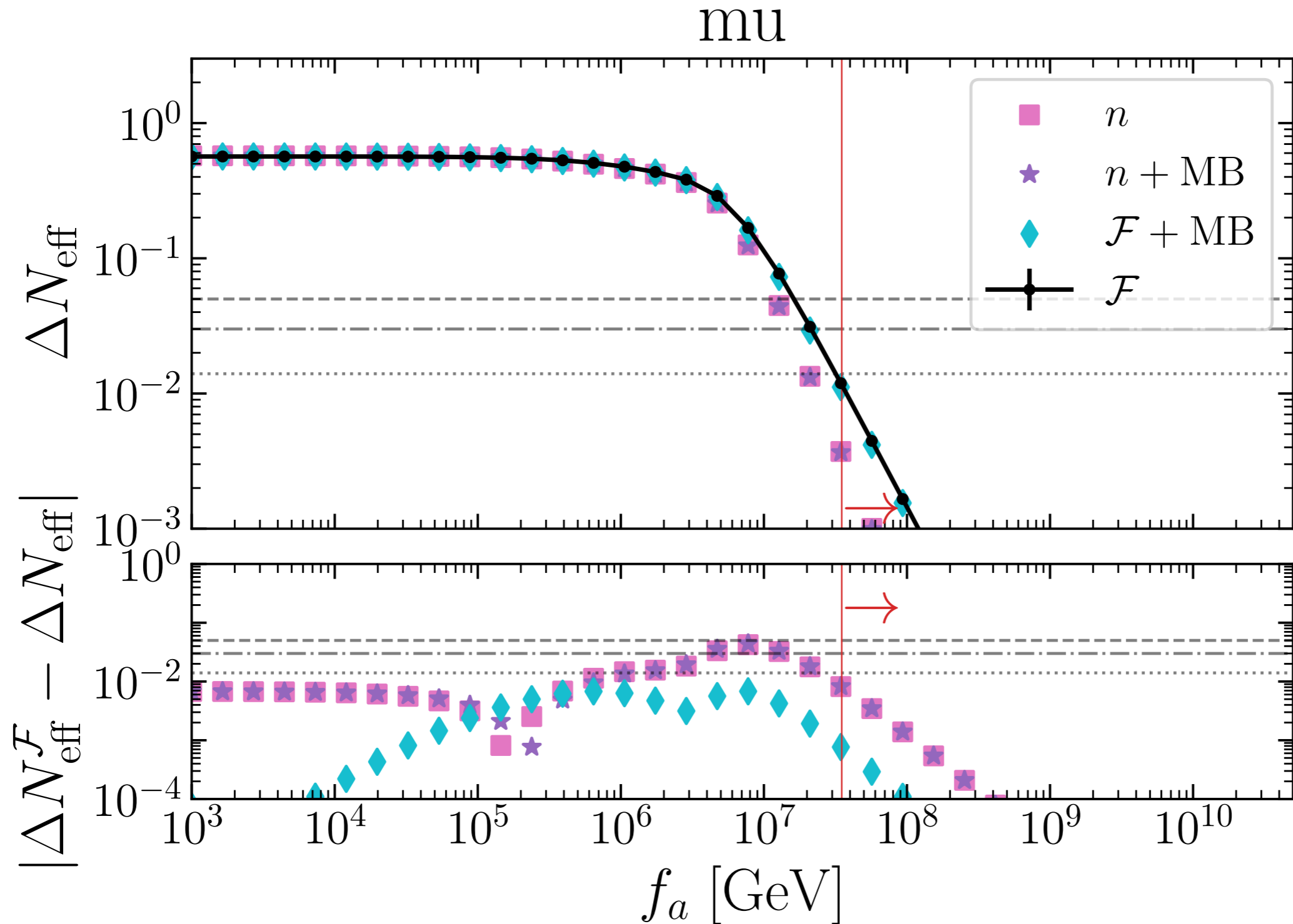
Notari, Rompineve, Villadoro, Phys.Rev.Lett. 131 (2023)  
 Bianchini, Grilli di Cortona, Valli, 2310.08169 [hep-ph]

# Axion-Fermion Interactions



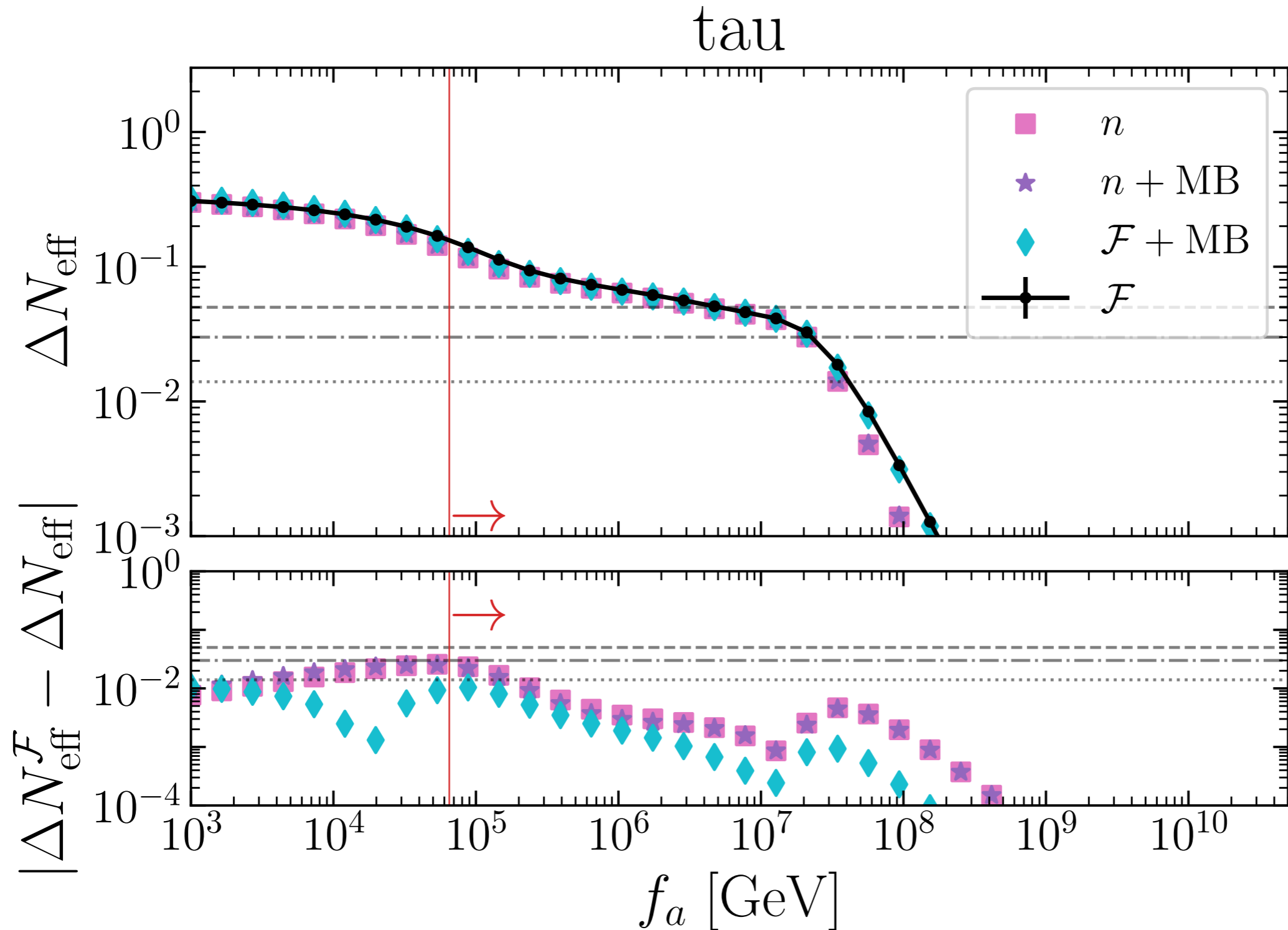
PRELIMINARY

# Axion-Fermion Interactions



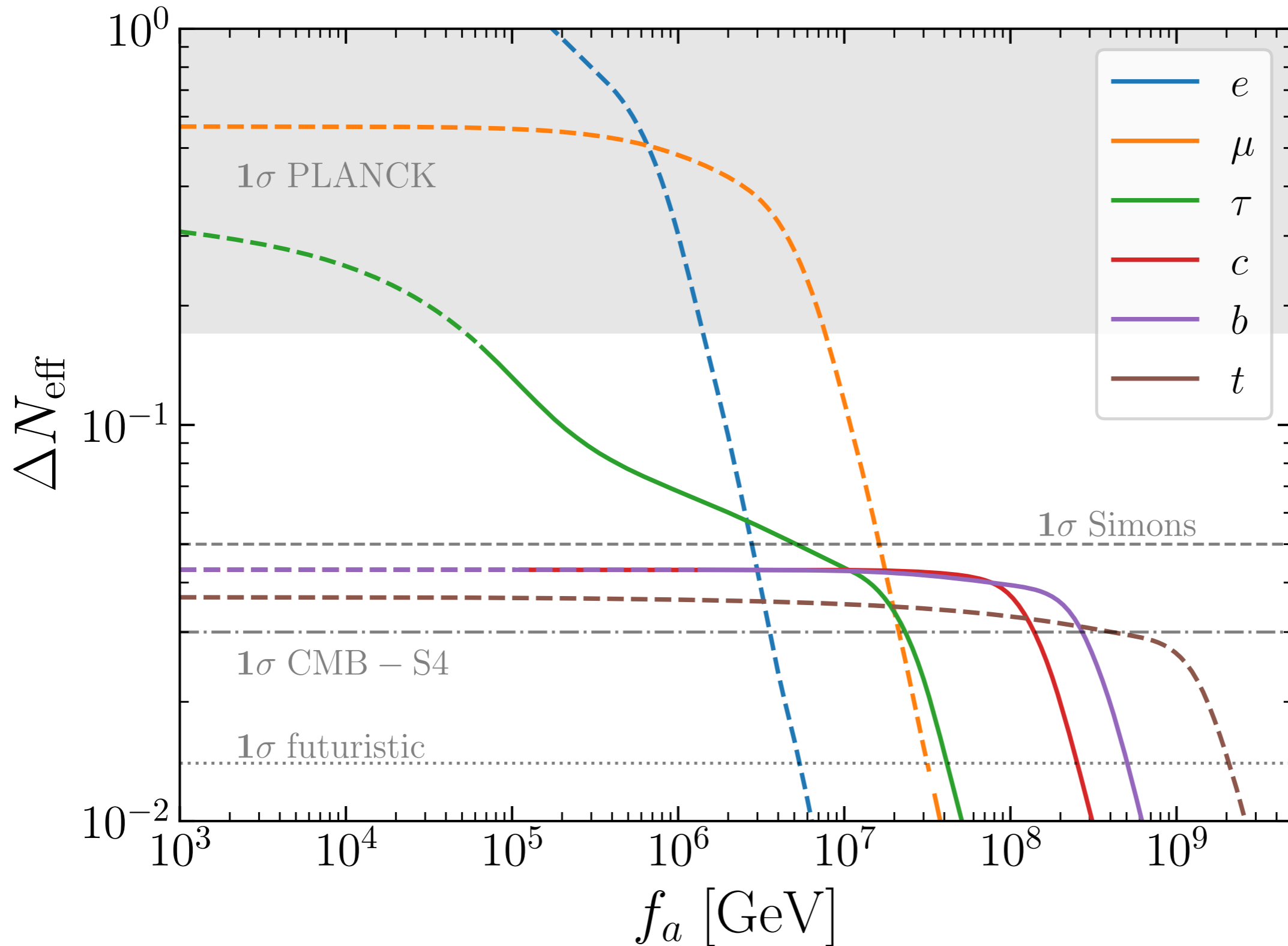
PRELIMINARY

# Axion-Fermion Interactions



PRELIMINARY

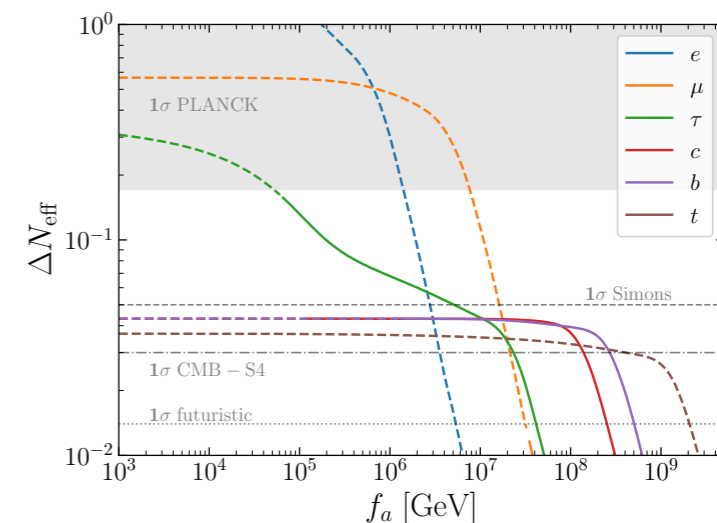
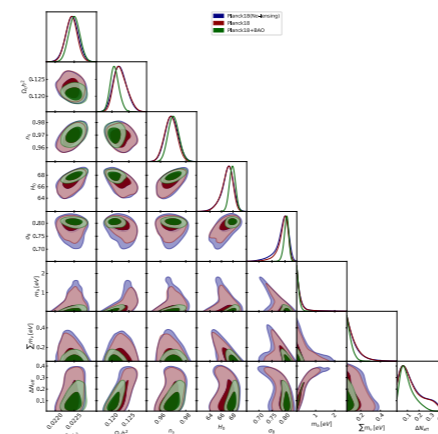
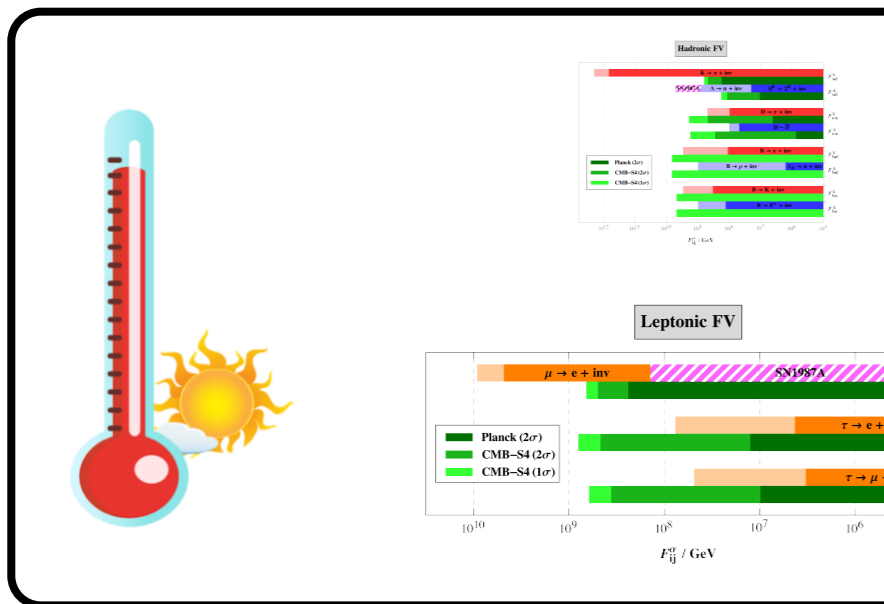
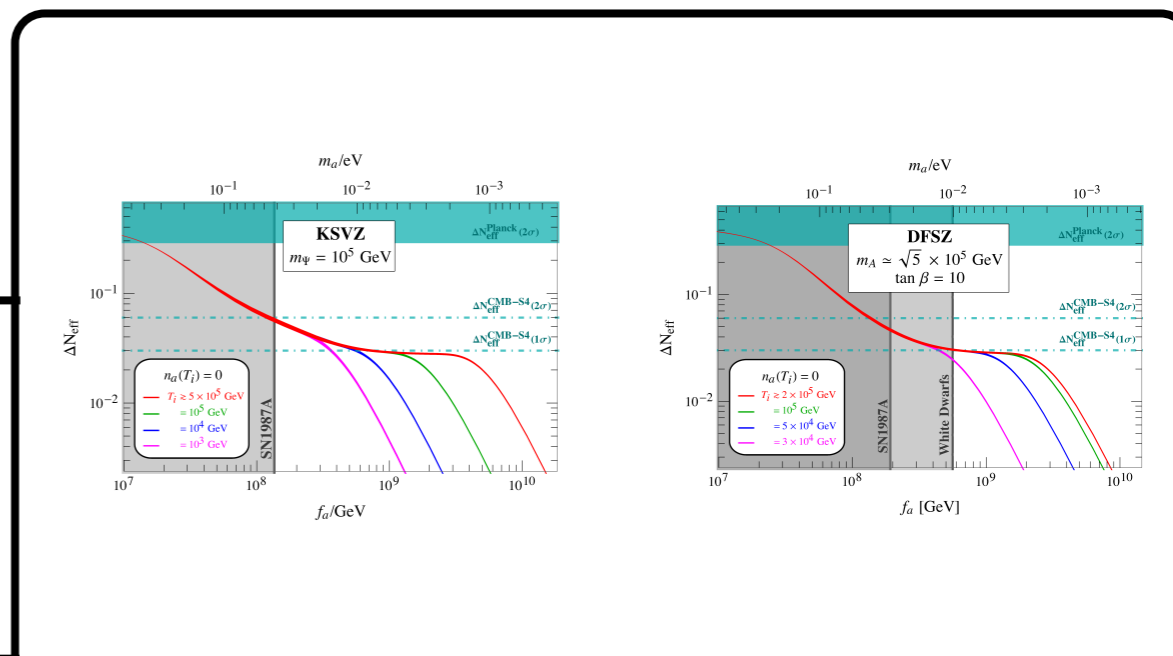
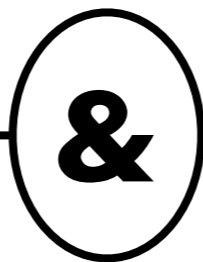
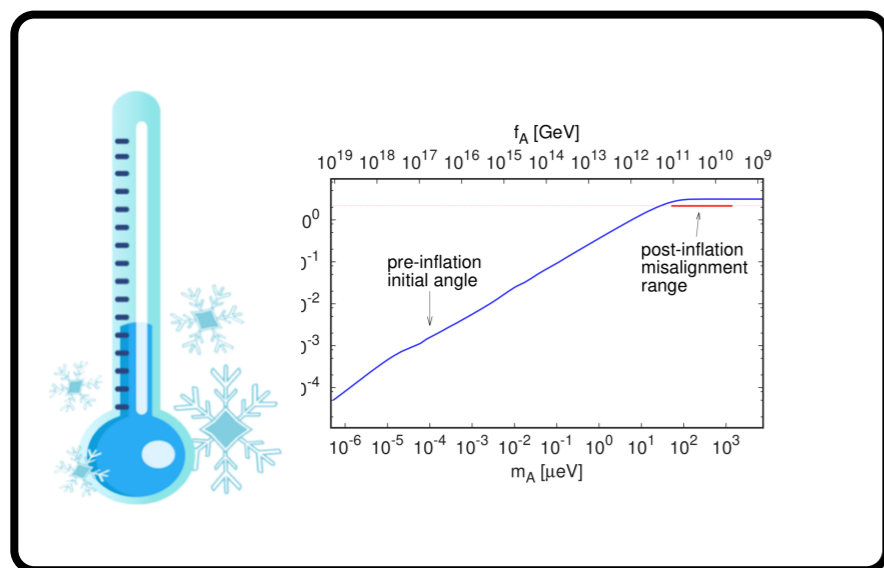
# Axion-Fermion Interactions



PRELIMINARY



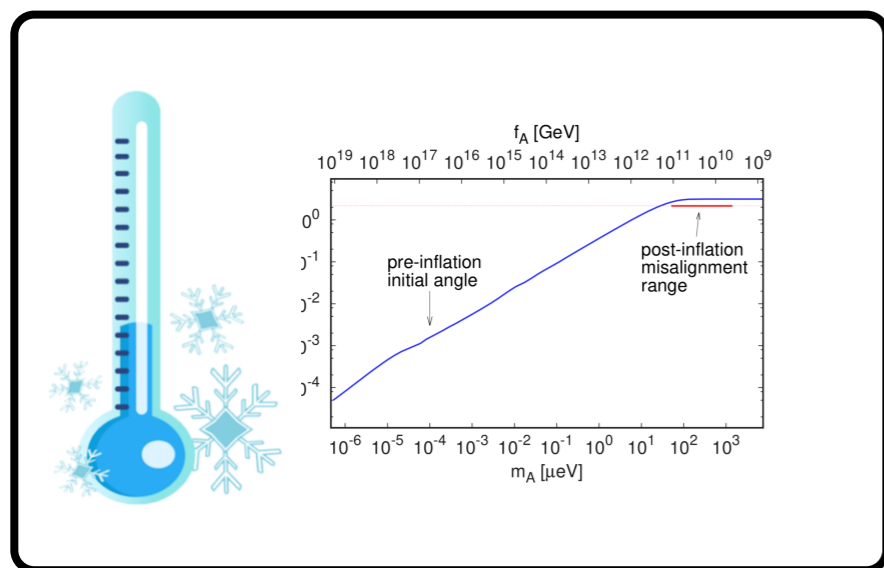
# Outlook



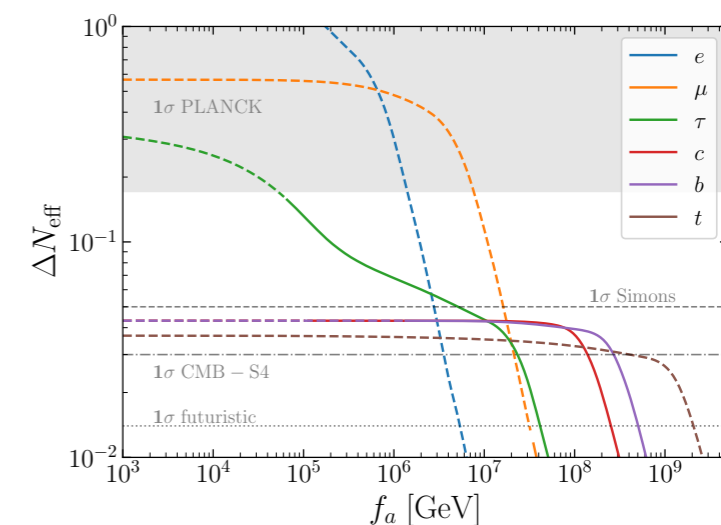
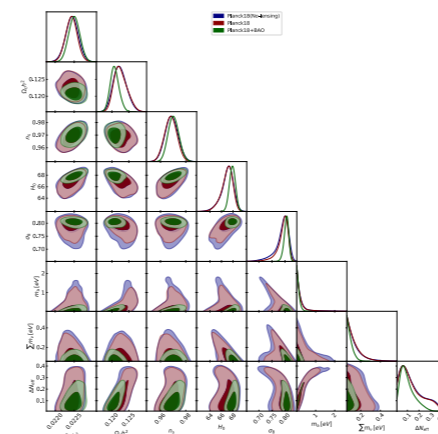
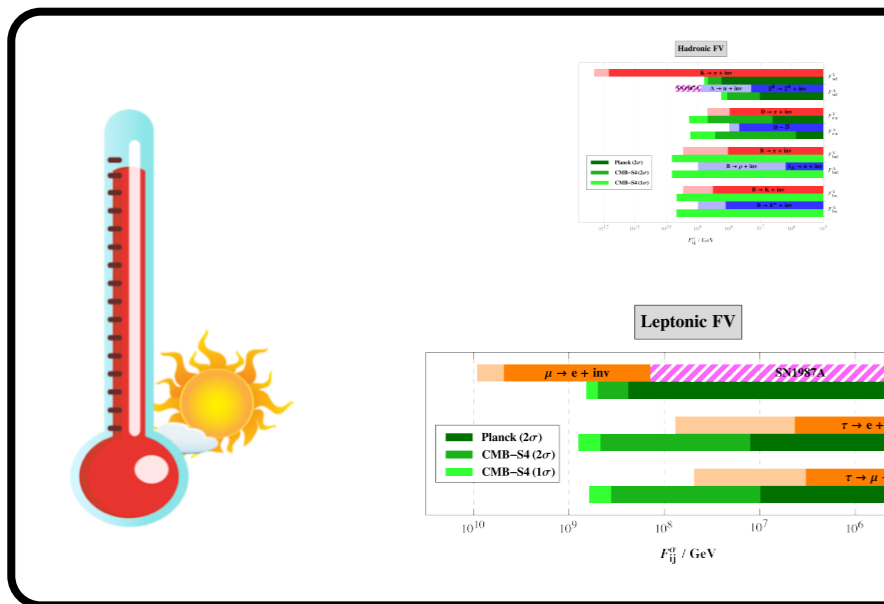
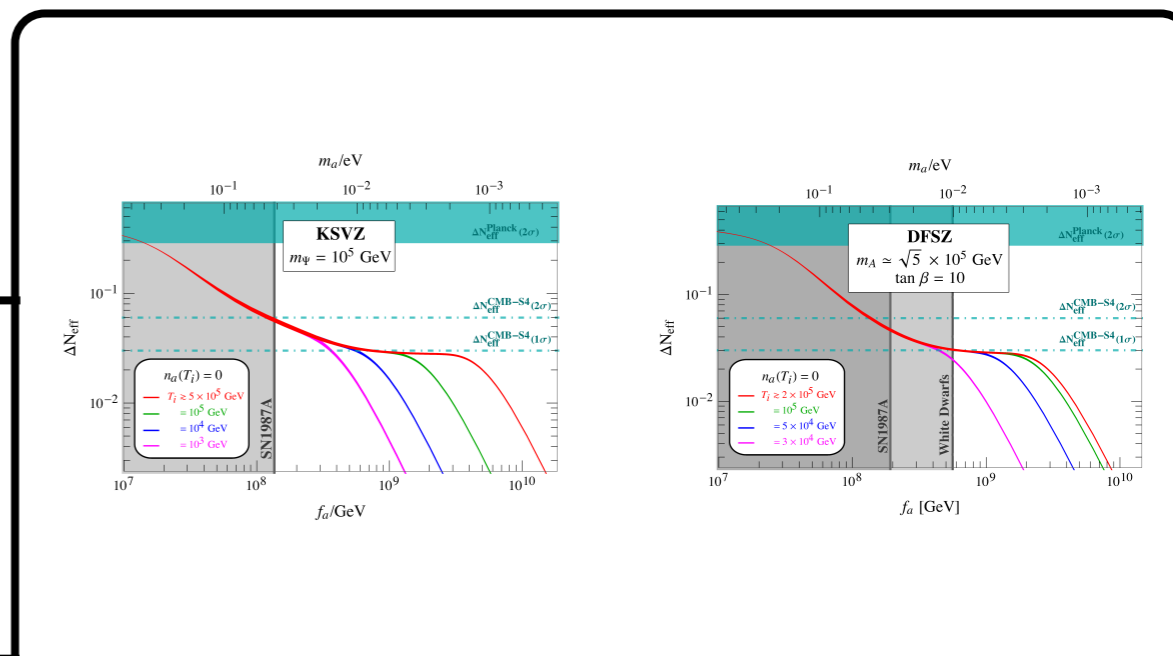
## Thermal Axions

Complementary to other probes of the PQ mechanism

# Outlook



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THANK YOU!