Primordial Black Holes (PBHs) as a dark matter candidate

Anne Green

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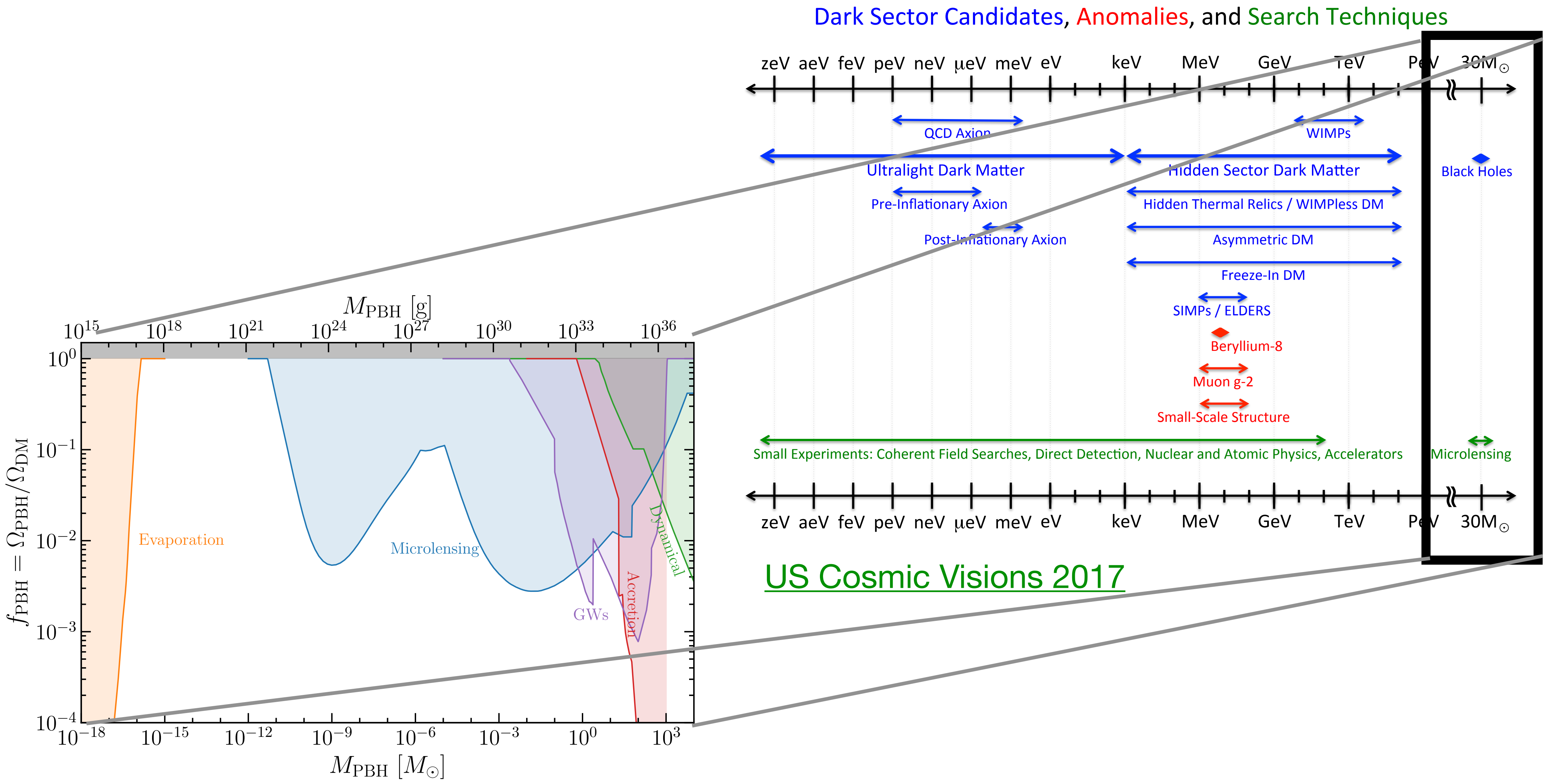

US Cosmic Visions 2017

https://github.com/bradkav/PBHbounds

Primordial Black Holes (PBHs) as a dark matter candidate

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Motivation & history

Formation

Observational constraints/signatures

Open questions

Reviews

Green & Kavanagh, J. Phys. G. arXiv:2007.10722, ‘PBHs as a dark matter candidate’

Carr & Kuhnel, Ann. Rev. Nuc. Part. Sci. arXiv:2006.02838, ‘PBHs as dark matter: recent developments’

Future prospects

Bird et al., Phys. Dark. Univ. arXiv:2203.08967, ‘Snowmass2001 Cosmic Frontier White Paper: PBH dark matter’

Motivation & history

Primordial Black Holes (PBHs) may form from over densities in the early Universe (before nucleosynthesis) and are therefore non-baryonic. Zel'dovich and Novikov; Hawking

PBHs evaporate (Hawking radiation), lifetime longer than the age of the Universe for $M \gtrsim 10^{15}\,\mathrm{g}$.
MacGibbon

A DM candidate which (unlike WIMPs, axions, sterile neutrinos,…) isn't a new particle, however their formation does usually require Beyond the Standard Model physics, e.g. inflation.

Was realised that PBHs are a cold dark matter (DM) candidate in the 1970s Hawking; Chapline

Wave of interest in ~Solar mass PBHs as DM in late 1990s, generated by excess of LMC microlensing events in MACHO collaboration's 2 year data set.

Nakamura et al. (1997): PBH binaries form in the early Universe and (**if** they survive to the present day) GWs from their coalescence detectable by LIGO.

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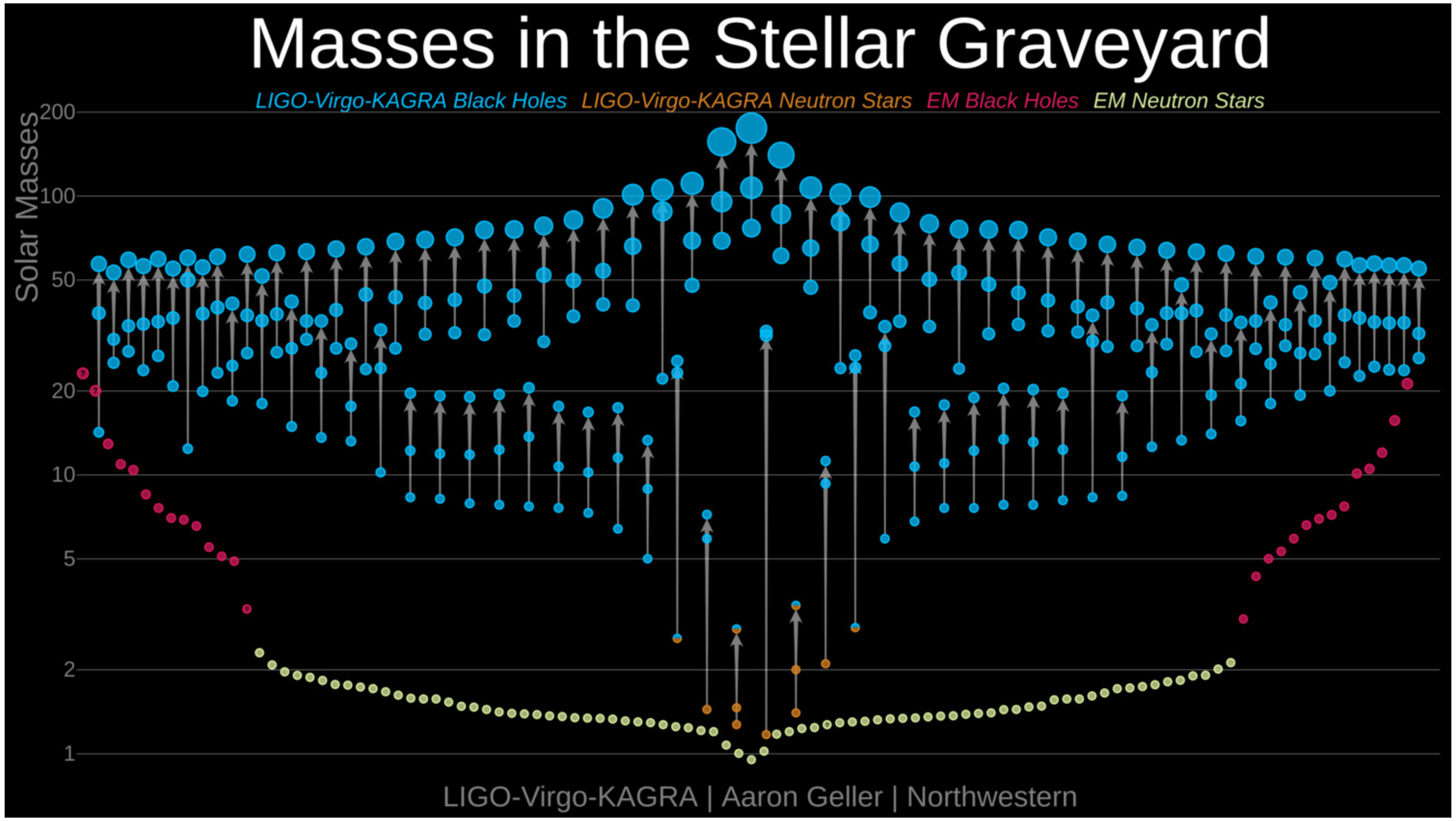
Could (some of) the BHs in the LIGO-Virgo BH binaries be primordial? (and also a significant component of the DM?) Bird et al.; Clesse & Garcia-Bellido; Sasaki et al.



$100M_\odot$

$10M_\odot$

LIGO-Virgo-KAGRA, Geller

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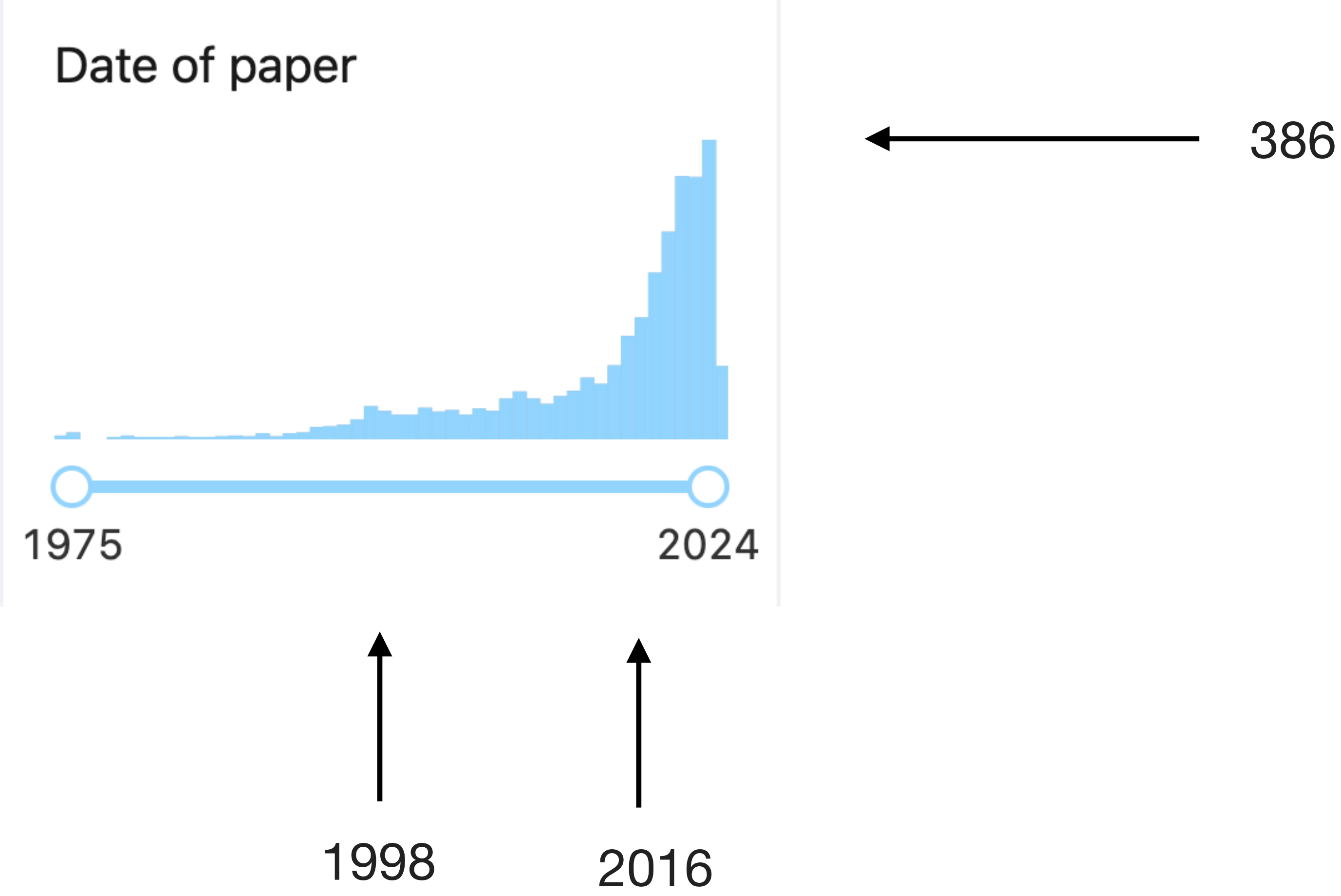
result of an inSPIRE search for 'primordial black hole'



Formation

Most ‘popular’ mechanism*: collapse of large density perturbations during radiation domination. Zeldovich & Novikov; Hawking; Carr & Hawking

If a region is sufficiently over-dense, gravity overcomes pressure and it collapse to form a BH shortly after ‘horizon entry’.

* other mechanisms: collapse of cosmic string loops Hawking; Polnarev & Zemboricz, bubble collisions Hawking, Moss & Stewart, fragmentation of inflaton scalar condensate Cotner & Kusenko, collapse of density perturbations during matter domination Khlopov & Polnarev, …

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‘zero-th order’ analysis: Carr

threshold for PBH formation:

$$\delta \geq \delta_{\rm c} \sim w = \frac{p}{\rho} = \frac{1}{3}$$

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

density contrast (at horizon crossing)

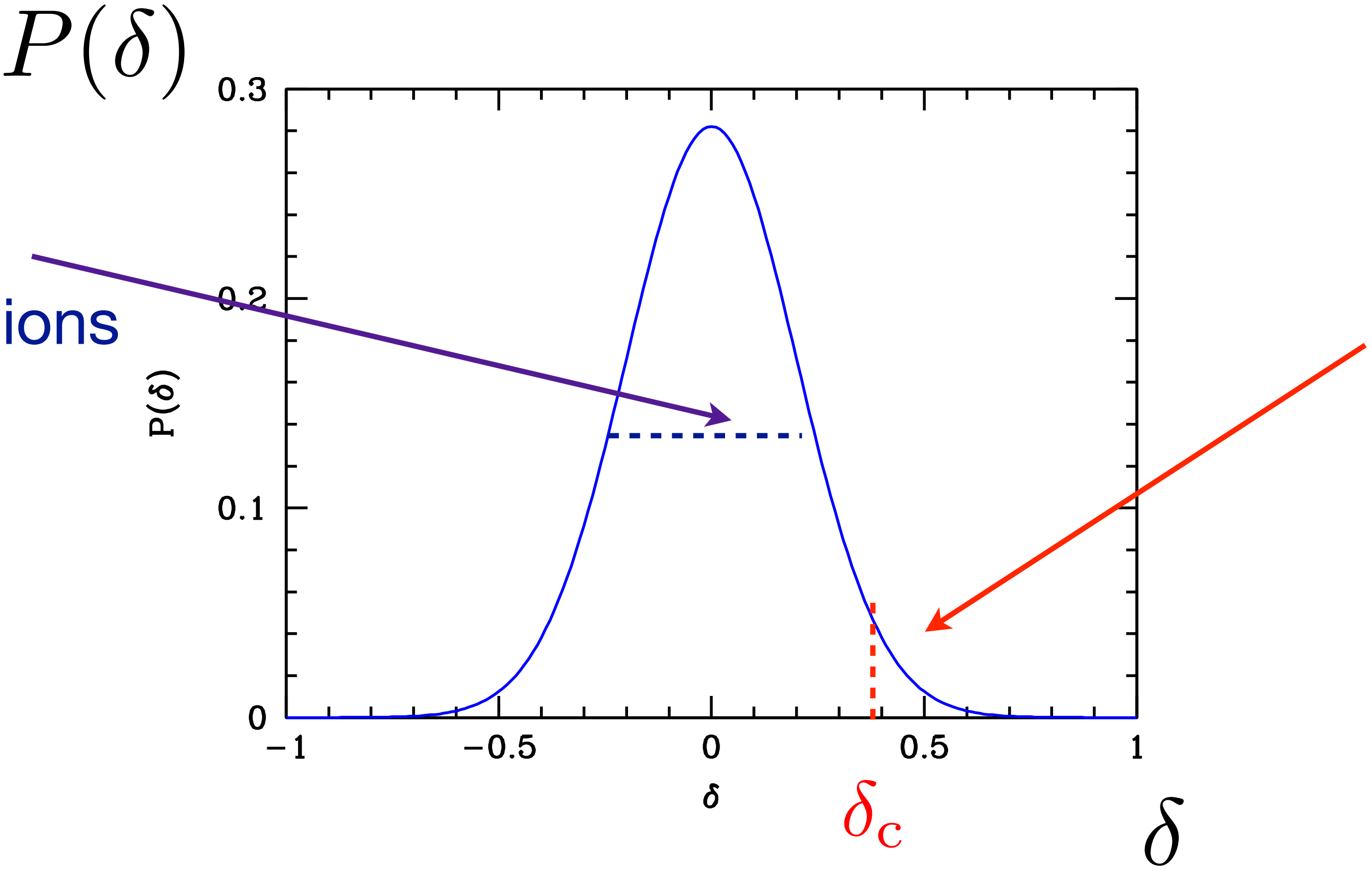
PBH mass roughly equal to horizon mass:

$$M_{\rm PBH} \sim 10^{15}\,{\rm g} \left(\frac{t}{10^{-23}\,{\rm s}} \right) \sim M_{\odot} \left(\frac{t}{10^{-6}\,{\rm s}} \right)$$

Threshold in fact depends on shape of perturbation (which depends on primordial power spectrum) and is best specified in terms of compaction function. Harada,Yoo & Kohri; Germani & Musco; Musco; Escriva, Germani & Sheth. For overview see Escriva, Kuhnel & Tada initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs):

$$\beta(M) \sim \int_{\delta_{\rm c}}^{\infty} P(\delta(M_{\rm H}))\, {\rm d}\delta(M_{\rm H})$$

assuming a gaussian probability distribution: $\beta(M) = {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M_{\rm H})}\right)$



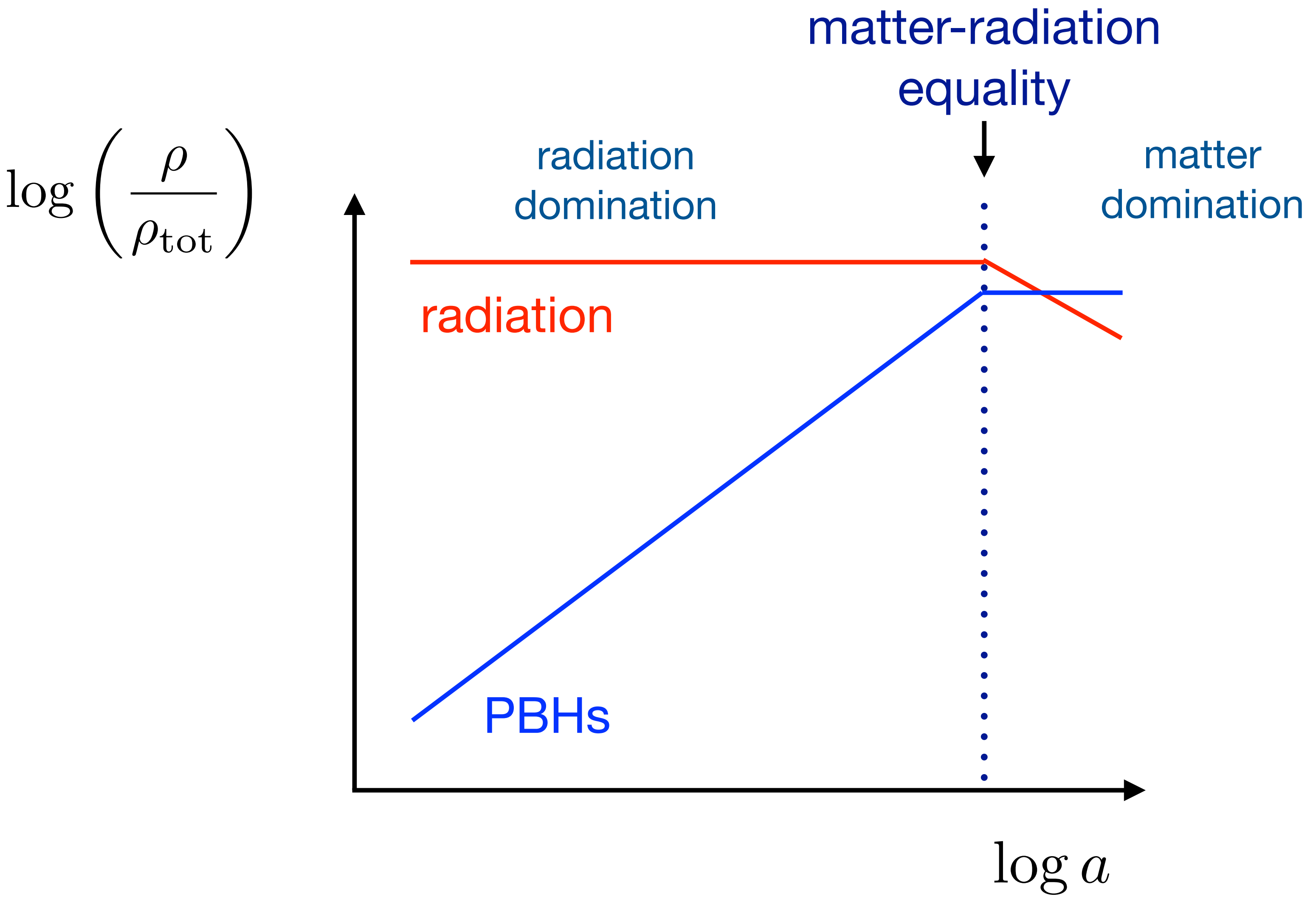
$\sigma(M_{\rm H})$ (mass variance)
typical size of fluctuations

PBH forming fluctuations

but in fact β must be small, hence $\sigma \ll \delta_{\rm c}$ and $\beta(M) \sim \sigma(M_{\rm H}) \exp\left(-\frac{\delta_{\rm c}^2}{2\sigma^2(M_{\rm H})}\right)$

Since PBHs are matter, during radiation domination the fraction of energy in PBHs grows with time:

$$\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$



Relationship between **PBH initial mass fraction, β**, and **fraction of DM in form of PBHs, $f_{\rm PBH}$**:

$$\beta(M) \sim 10^{-9} f_{\rm PBH} \left(\frac{M}{M_\odot}\right)^{1/2}$$

i.e. initial mass fraction must be small.

On CMB scales the primordial perturbations have amplitude $\sigma(M_{\rm H}) \sim 10^{-5}$

If the primordial perturbations are very close to scale-invariant the number of PBHs formed will be completely negligible:

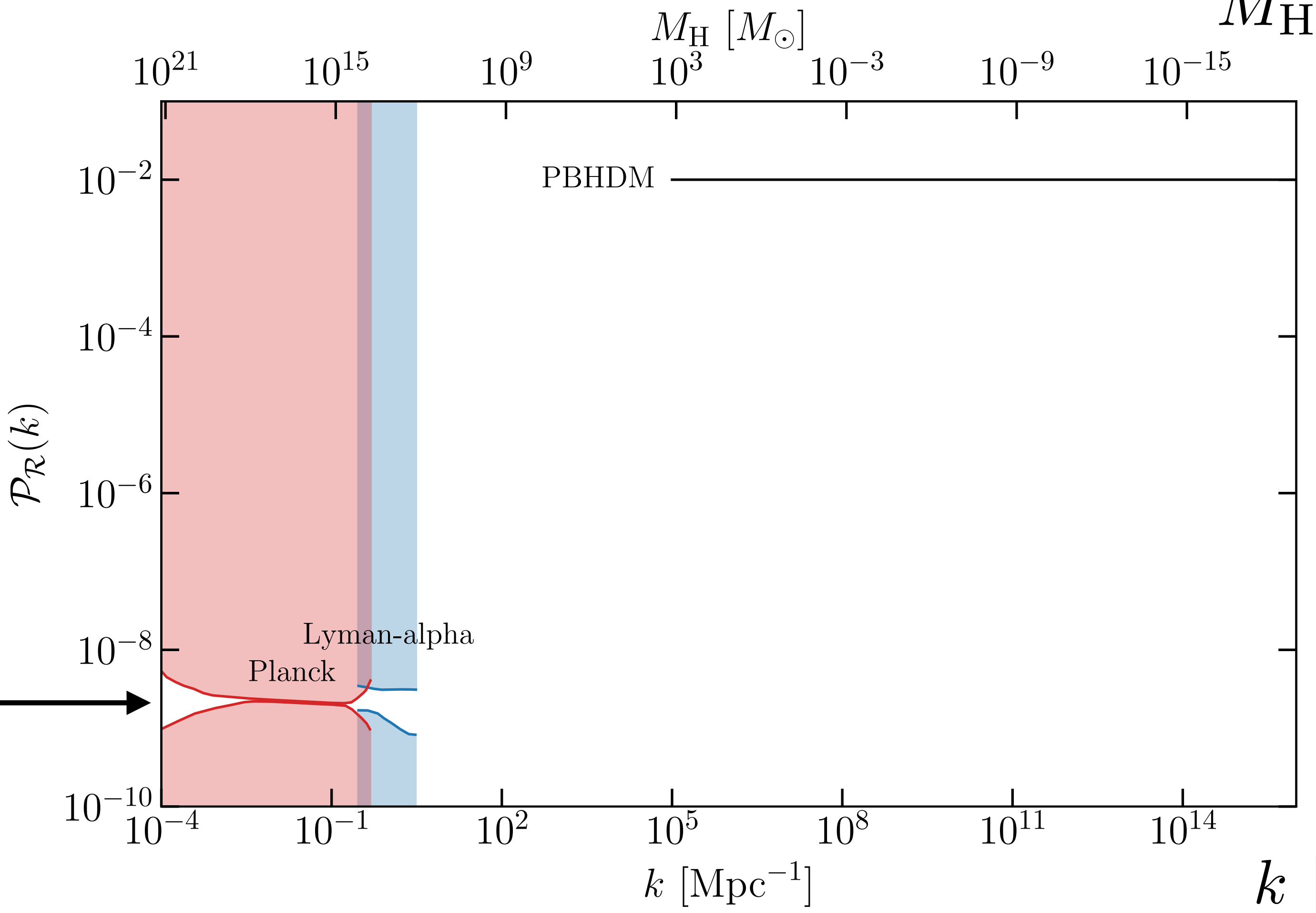
$$\beta(M) = {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M_{\rm H})}\right)$$

$$\beta(M) \sim {\rm erfc}(10^5) \sim \exp\left(-10^{10}\right)$$

To form an interesting number of PBHs the primordial perturbations must be significantly larger ($\sigma^2(M_{\rm H})$~0.01) on small scales than on cosmological scales.

$\mathcal{P}_{\mathcal{R}}(k) \sim \sigma^2$

$M_{\rm H}\ [M_\odot]$ ~ PBH mass



amplitude required for $f_{\rm PBH} \sim 1$

measured amplitude on cosmological scales

$k\ [{\rm Mpc}^{-1}]$

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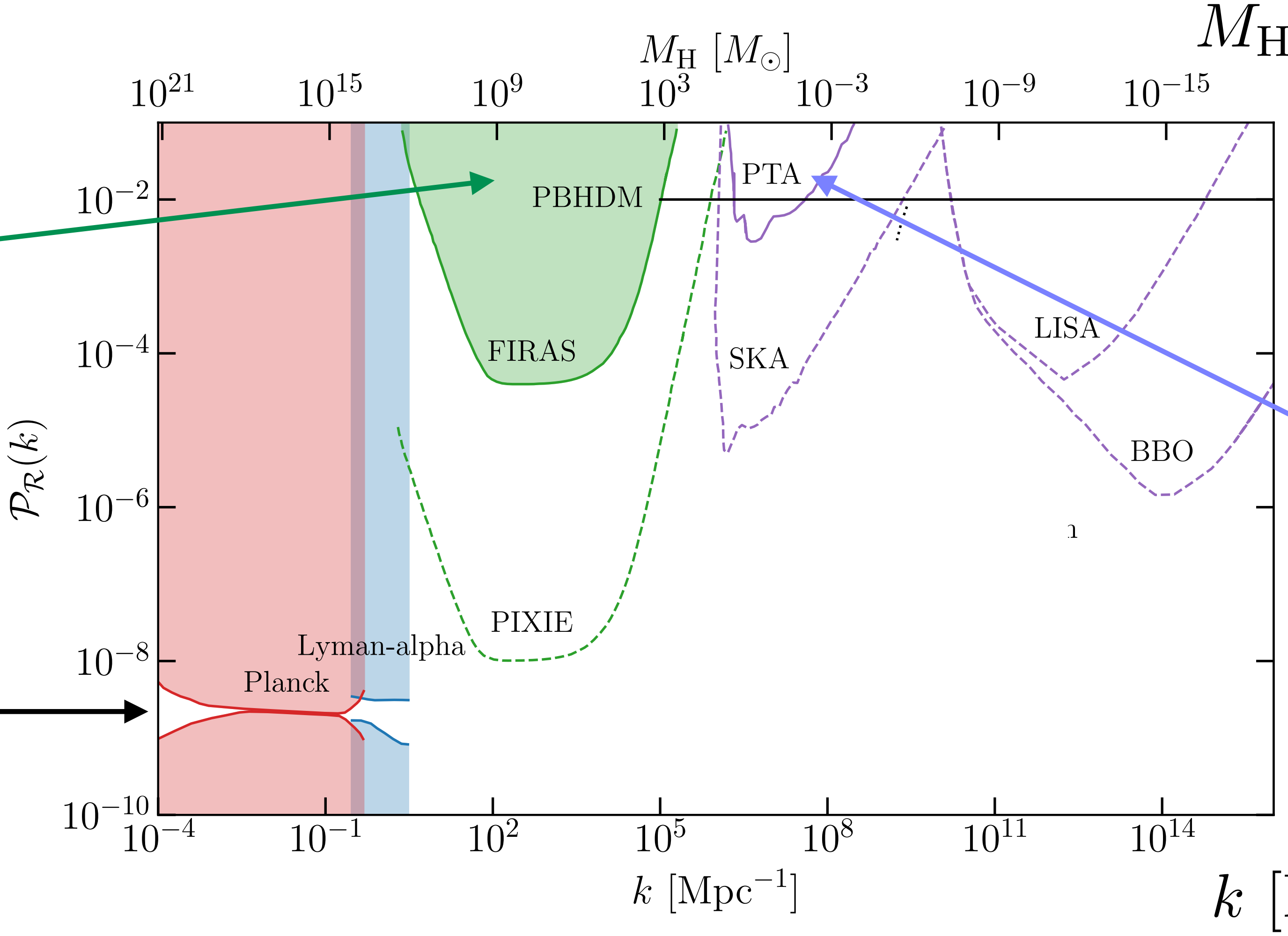


amplitude required for $f_{\rm PBH} \sim 1$

CMB spectral distortions
Carr & Lidsey

scalar induced gravitational waves
Ananda, Clarkson & Wands
Saito & Yokoyama

measured amplitude on cosmological scales

$k\ [\mathrm{Mpc}^{-1}]$

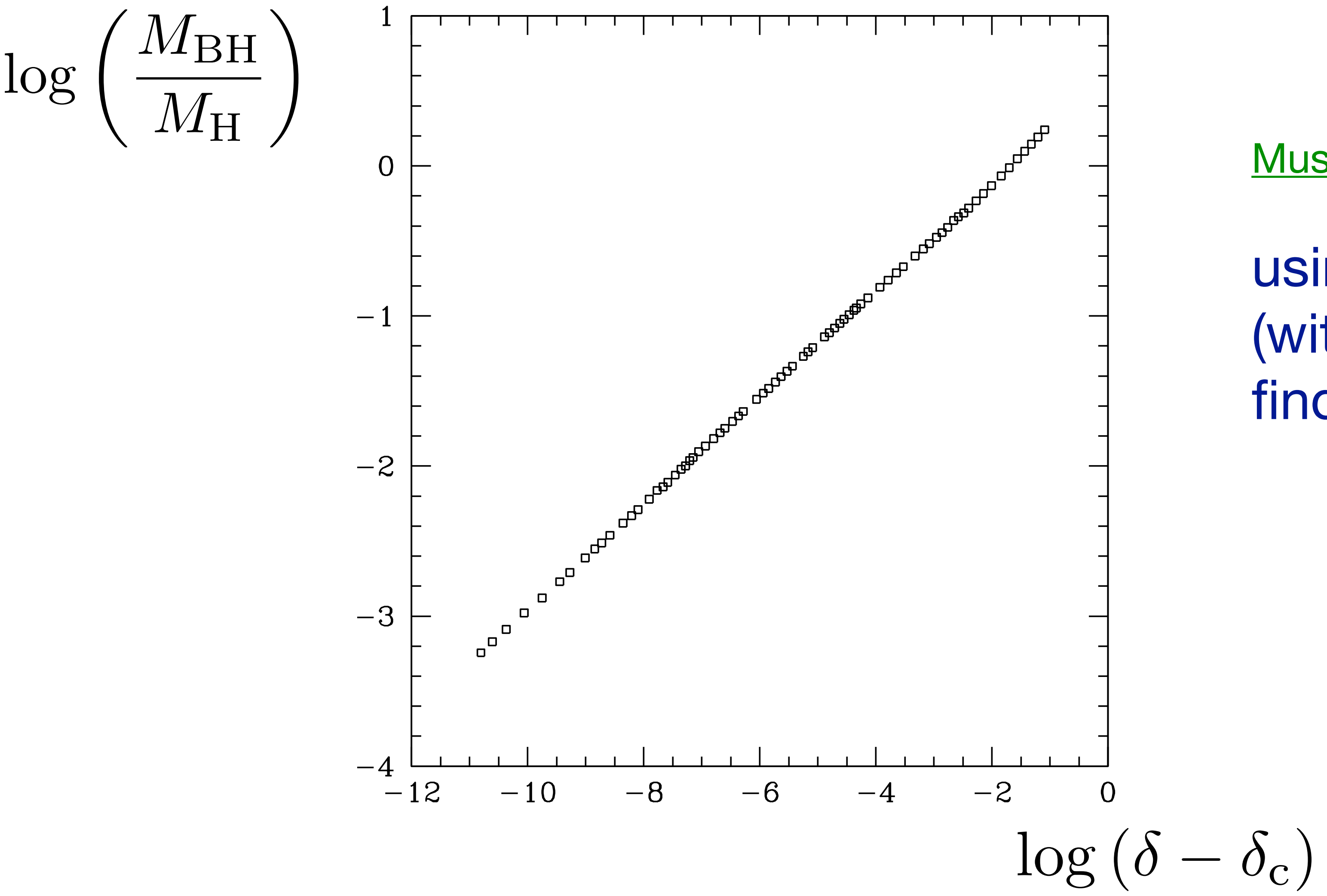
deviations from simple scenario:

i) critical collapse

Niemeyer & Jedamzik

BH mass depends on size of fluctuation it forms from:

$$M = k M_{\mathrm{H}} (\delta - \delta_{\mathrm{c}})^{\gamma}$$



Musco, Miller & Polnarev

using numerical simulations (with appropriate initial conditions) find k=4.02, γ=0.357, $\delta_{\mathrm{c}} = 0.45$

Get PBHs with range of masses produced even if they all form at the same time
i.e. we don't expect the PBH MF to be a delta-function

ii) non-gaussianity

Since PBHs are formed from rare large density fluctuations, changes in the shape of the tail of the probability distribution (i.e. non-gaussianity) can significantly affect the PBH abundance. Bullock & Primack; Ivanov;… Francolini et al.

Relationship between density perturbations and curvature perturbations is non-linear, so even if curvature perturbations are gaussian (large) density perturbations won’t be. Kawasaki & Nakatsuka; De Luca et al.; Young, Musco & Byrnes

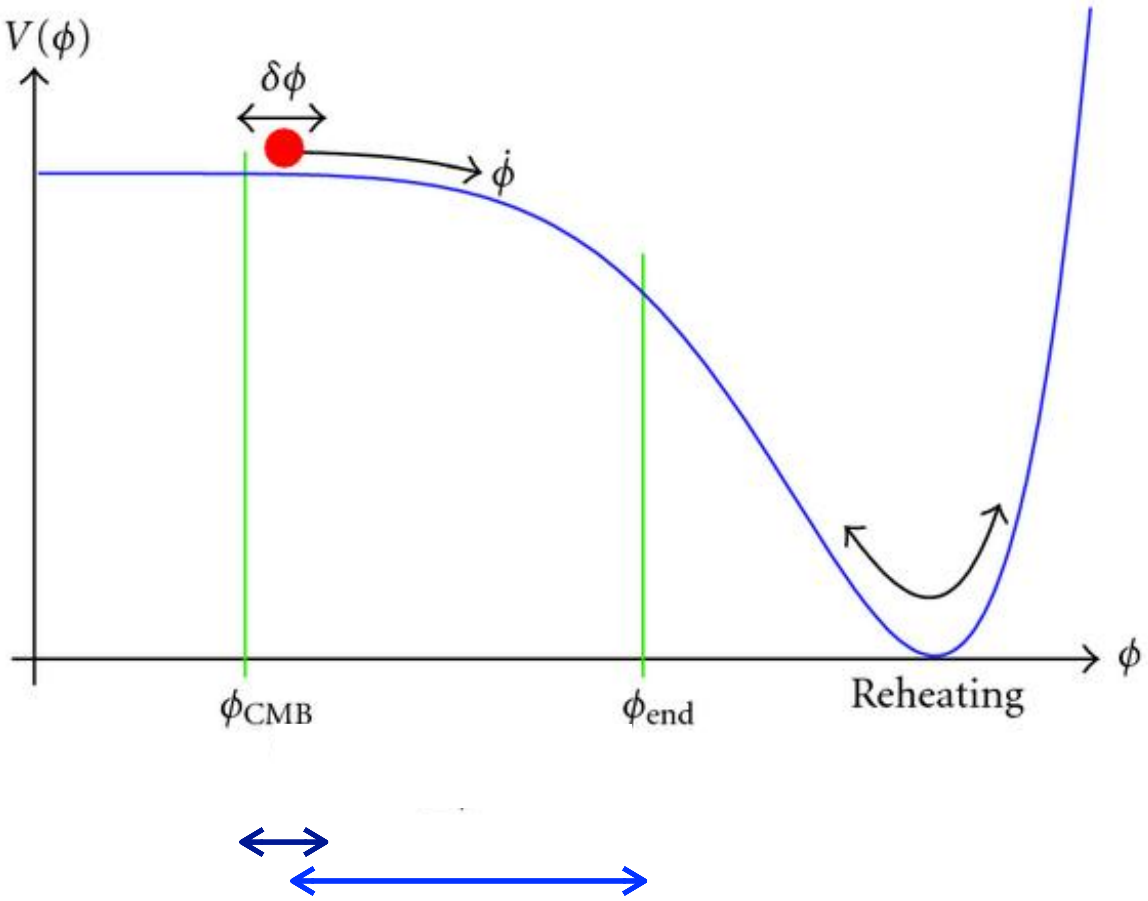
Inflation: a brief crash course/reminder

A postulated period of accelerated expansion in the early Universe, proposed to solve various problems with the Big Bang (flatness, horizon & monopole).

Driven by a 'slowly rolling' scalar field.

Quantum fluctuations in scalar field generate density perturbations.

Scale dependence of primordial perturbations depends on shape of potential:



Yadav & Wandelt

in slow-roll approx

$$\sigma^2(M_{\rm H}) \propto \frac{V^3}{(V')^2}$$

Scales probed by:

Large scale structure
& the CMB

Primordial Black Holes

inflation models that produce large perturbations

In slow-roll approx*:

$$\sigma \propto \frac{V^{3/2}}{V'}$$

A plateau in the potential can generate large perturbations which form an interesting abundance of PBHs. Ivanov, Naselsky, Novikov

* in 'ultra-slow-roll' limit, $V' \rightarrow 0$, this expression isn't accurate (and USR also affects probability distribution of fluctuations - more later).

Requirements for a PBH producing inflation model:

i) produce measured power spectrum (amplitude and scale dependence) on cosmological scales,

ii) amplitude of perturbations ~3.5 orders of magnitude larger on some smaller scale,

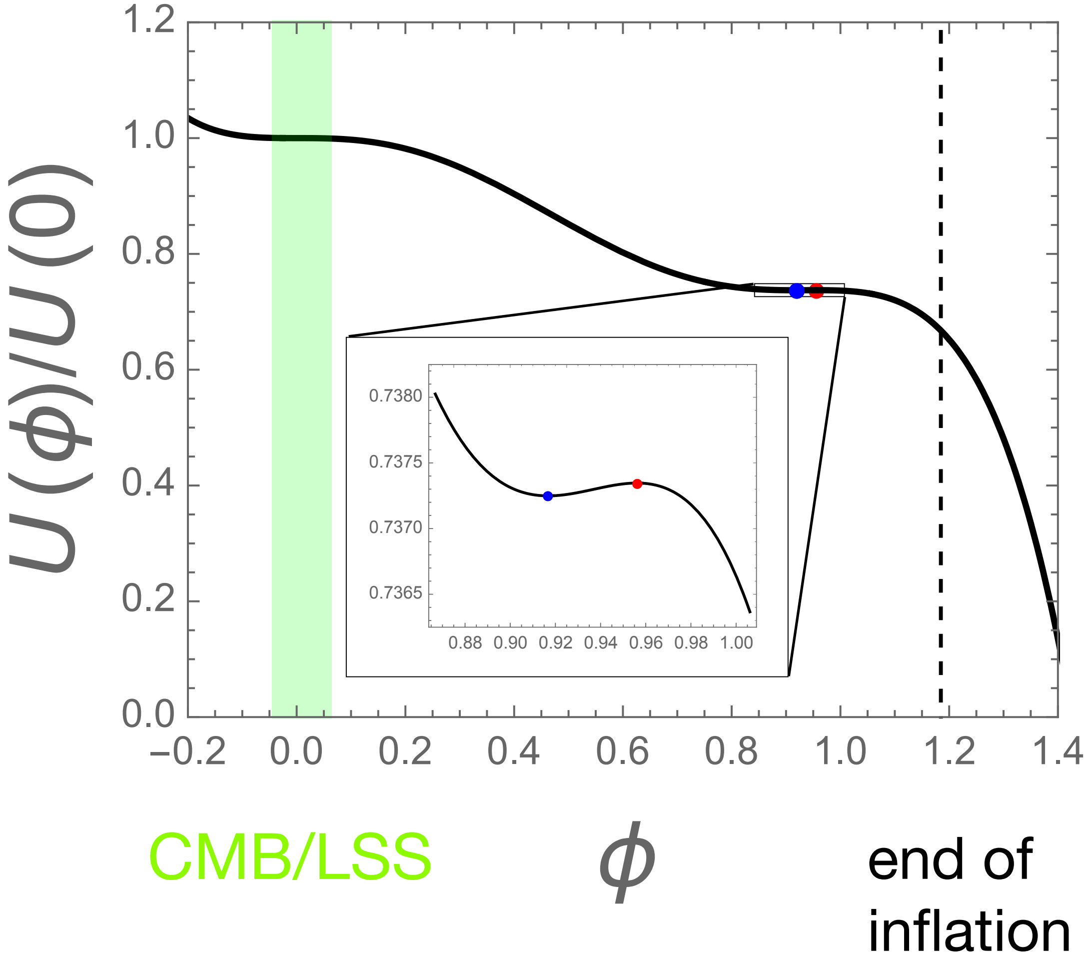
iii) inflation ends.

single field

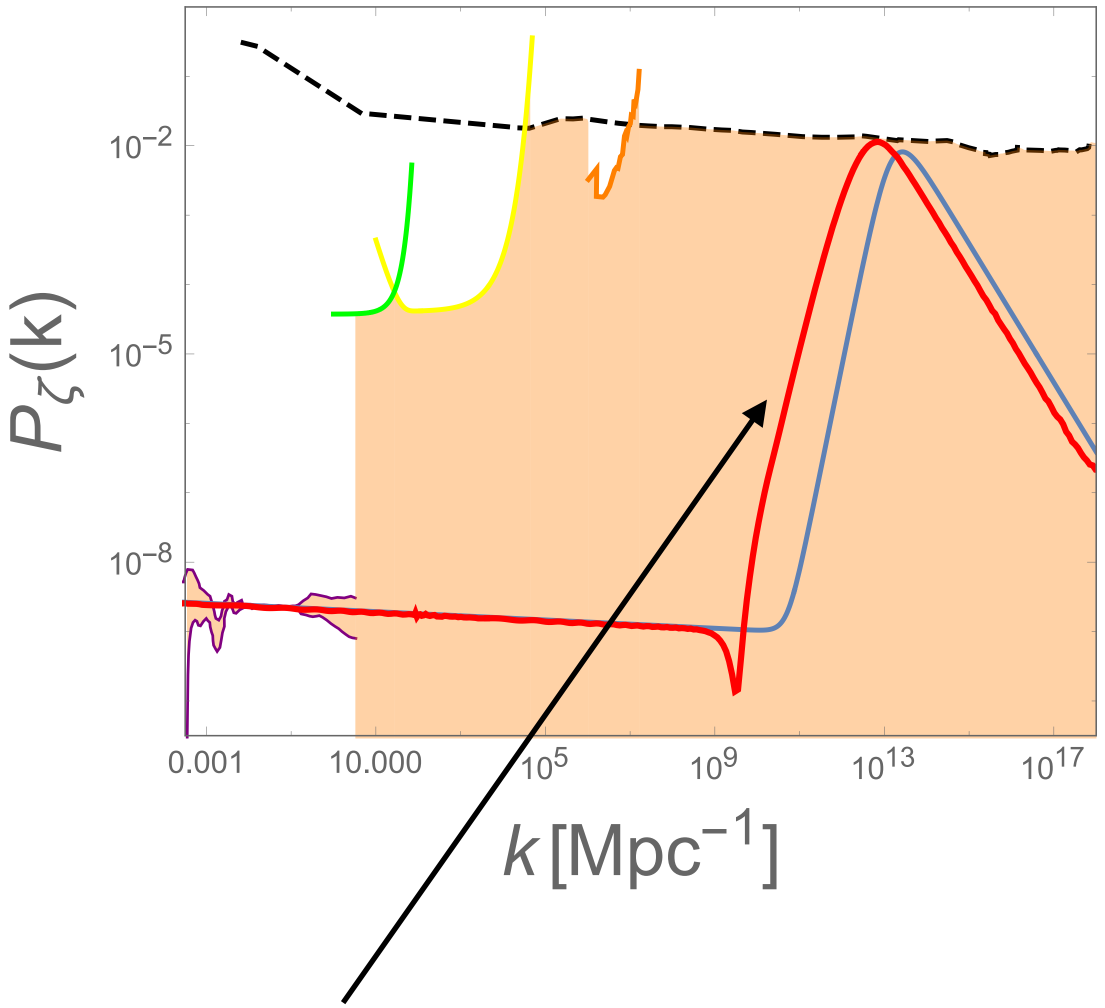
Potential needs to be fine-tuned so that field goes past local min, but with reduced speed.

Ballesteros & Taoso; Herzberg & Yamada

potential



primordial power spectrum

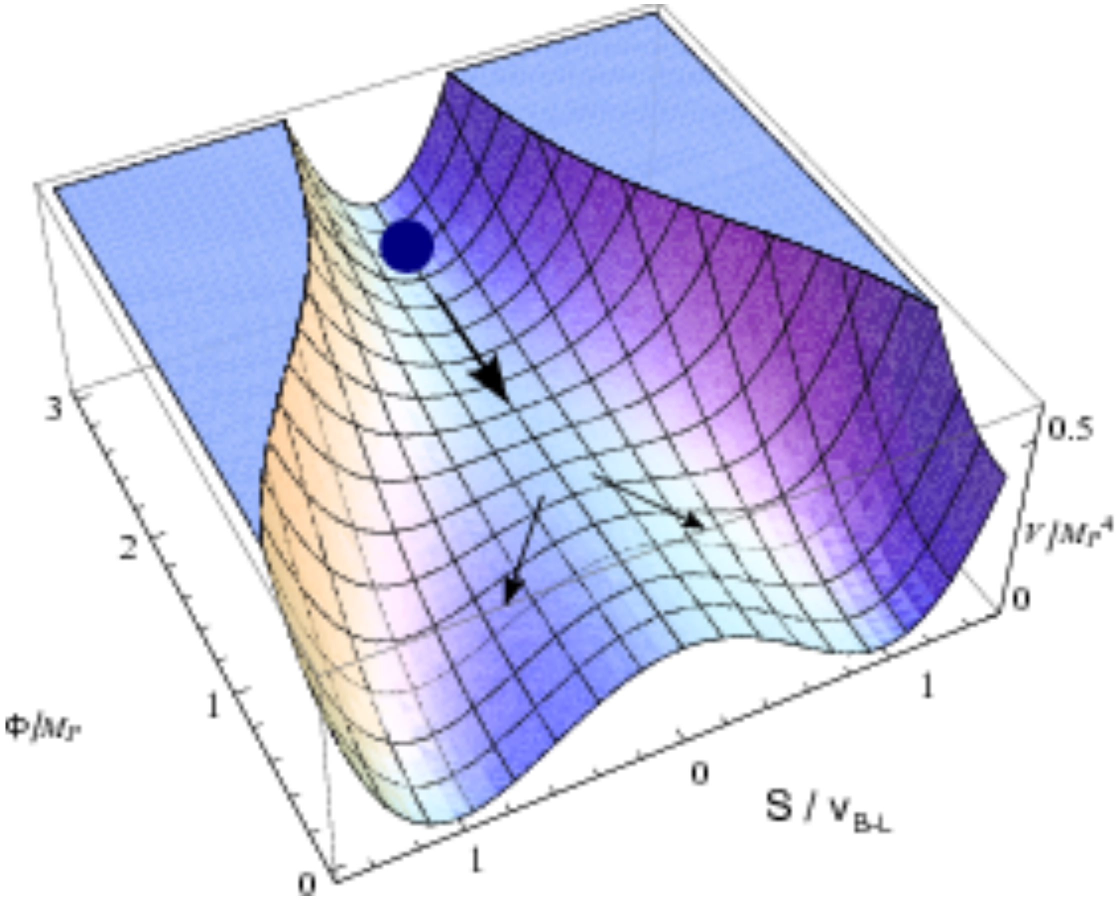


$\sim k^4$ Byrnes, Cole & Patil

multi-field models

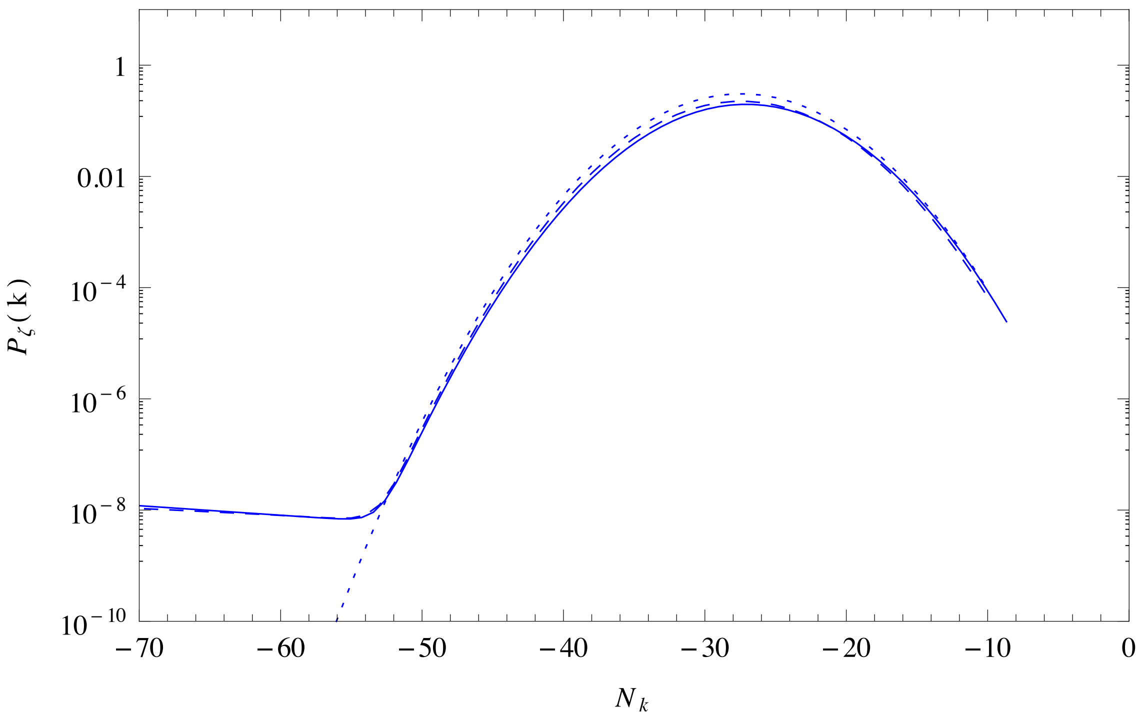
e.g. hybrid inflation with a mild waterfall transition Garcia-Bellido, Linde & Wands

potential



Buchmuller

primordial power spectrum



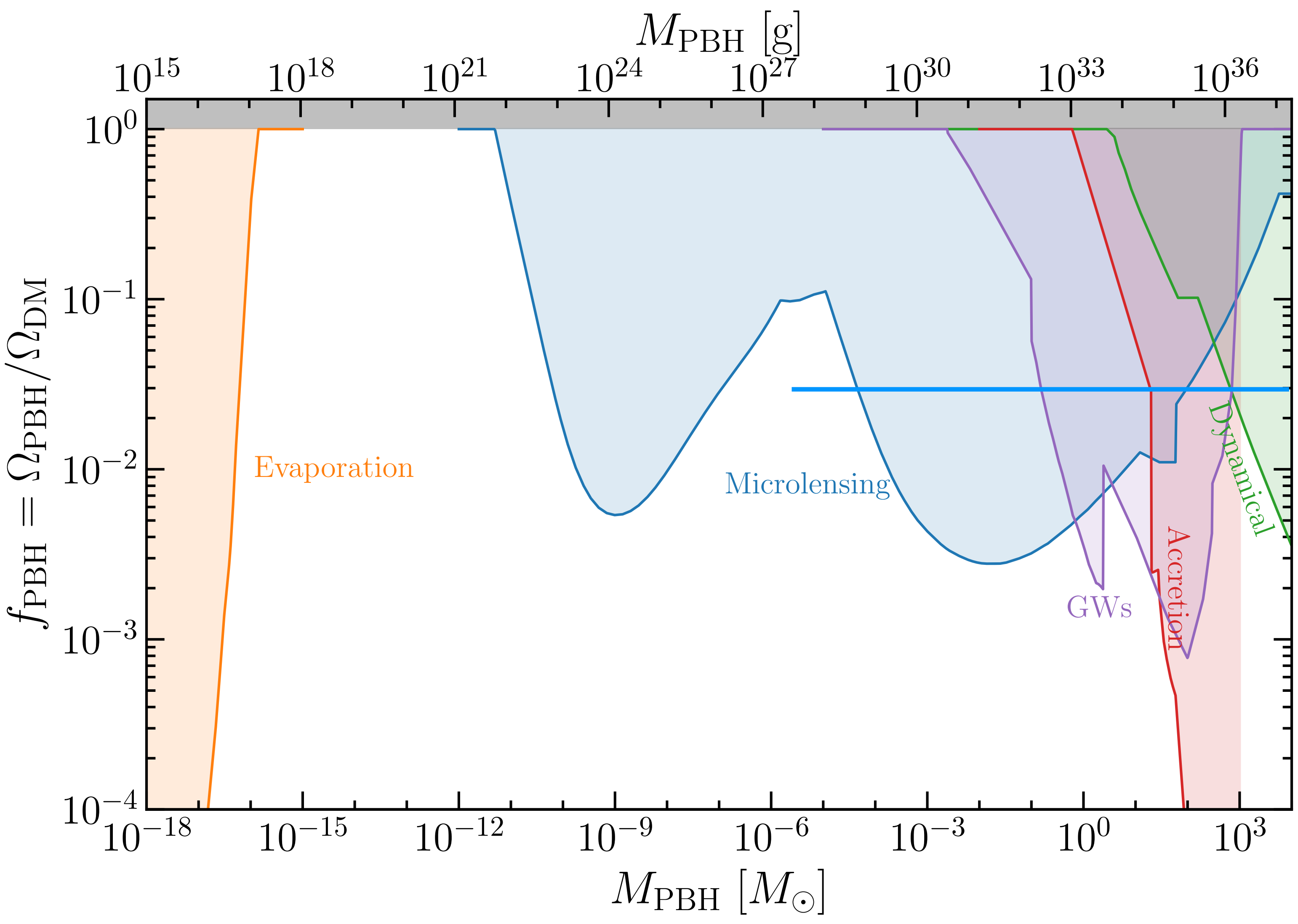
Clesse & Garcia-Bellido

various others for reviews see Öszoy & Tasinato; Escriva, Kuhnel & Tada

running mass, double inflation, axion-like curvaton, reduced sound speed, multi-field models with rapid turns in field space,…

Observational constraints

(assuming a delta-function PBH mass function)



evaporation

lensing

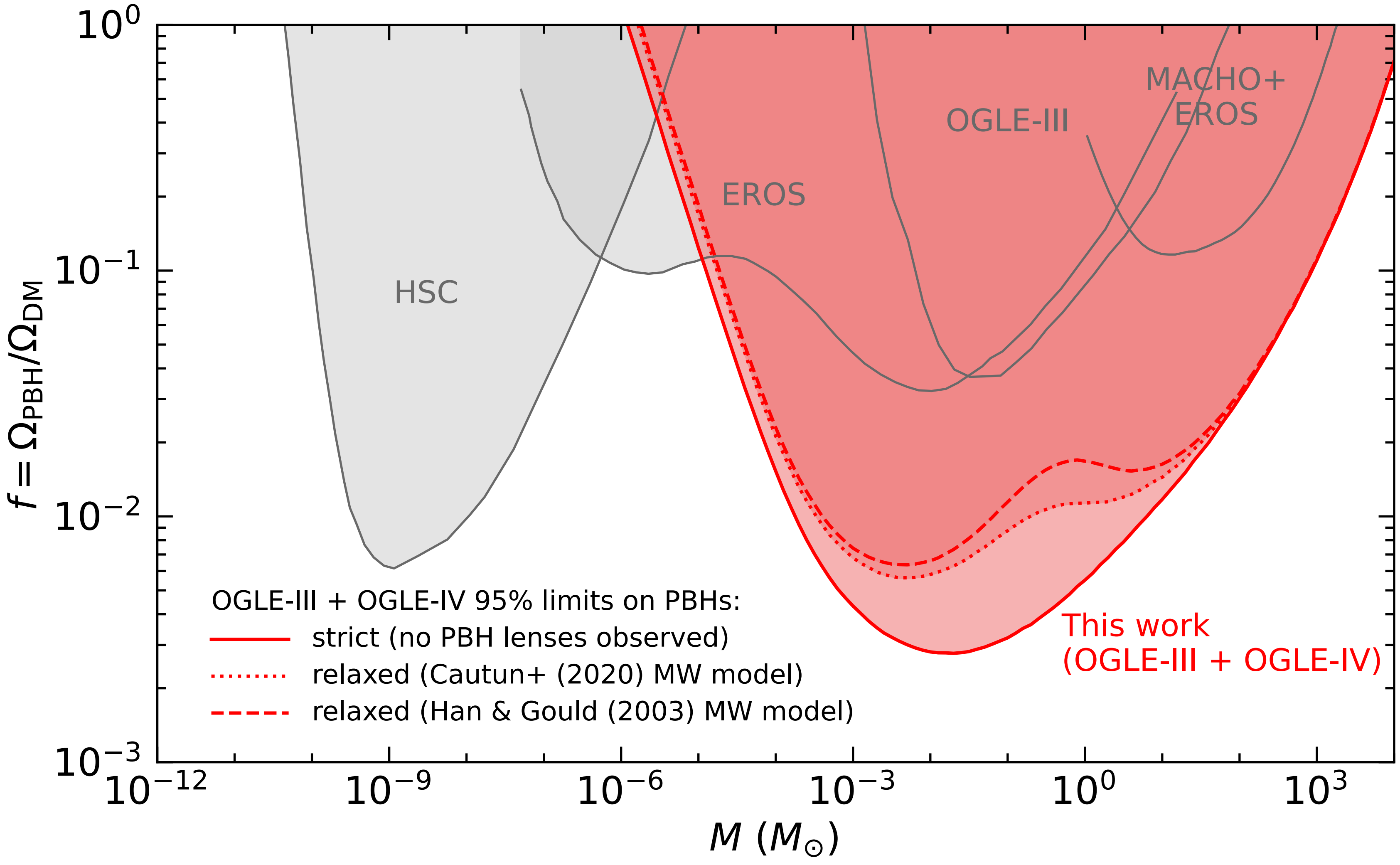
gravitational waves

accretion

dynamical

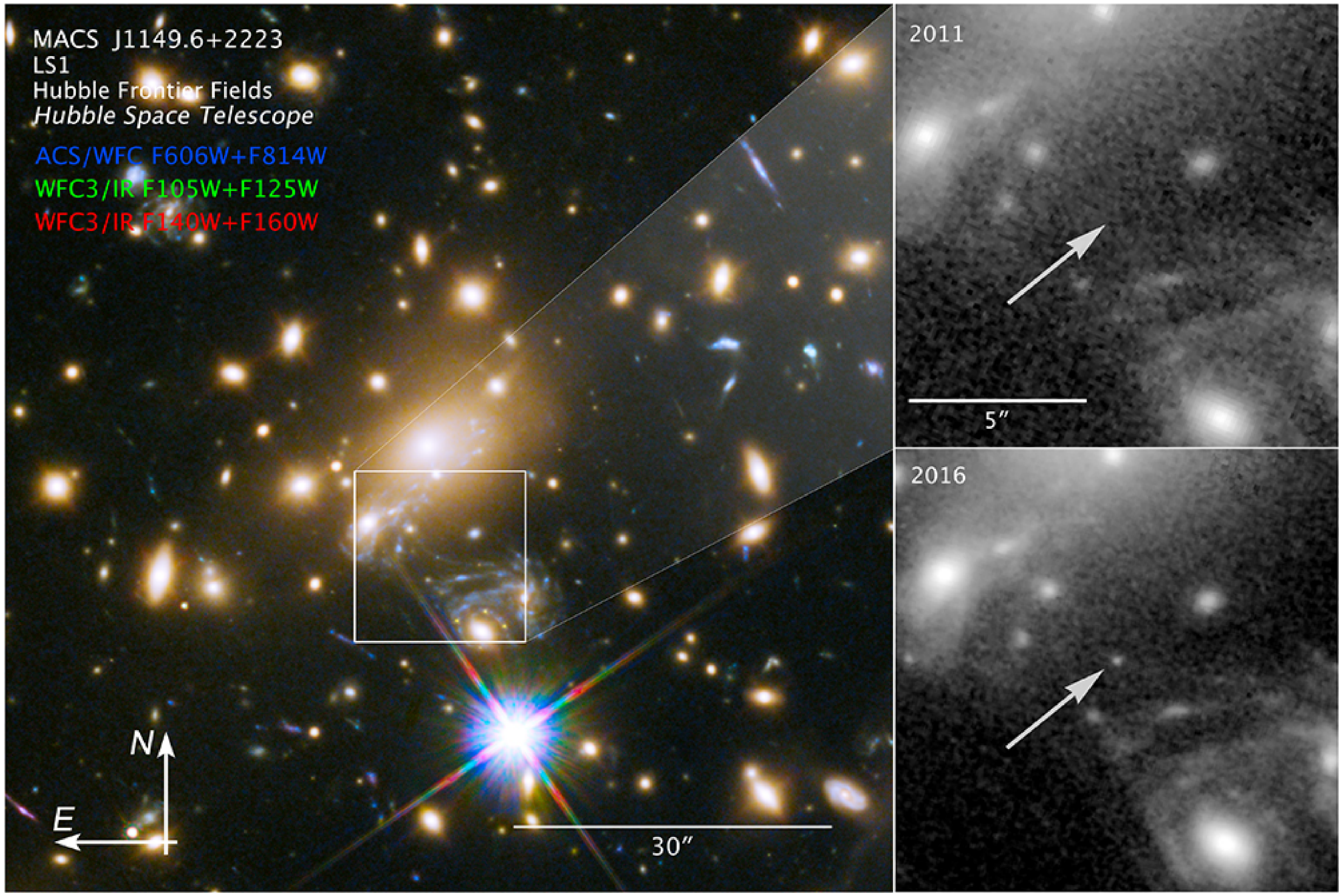
https://github.com/bradkav/PBHbounds

Recent tight constraints on planetary and stellar mass PBHs from 20 years of OGLE LMC microlensing observations: Mroz et al. arXiv:2403.02386



Supermagnified stars limit planetary mass and heavier PBHs to make up less than ~3% of the dark matter : Vall Müller & Miradla-Escudé arXiv:2403.16989

Icarus star at red-shift 1.5 (magnified x2000)! Kelly et al.



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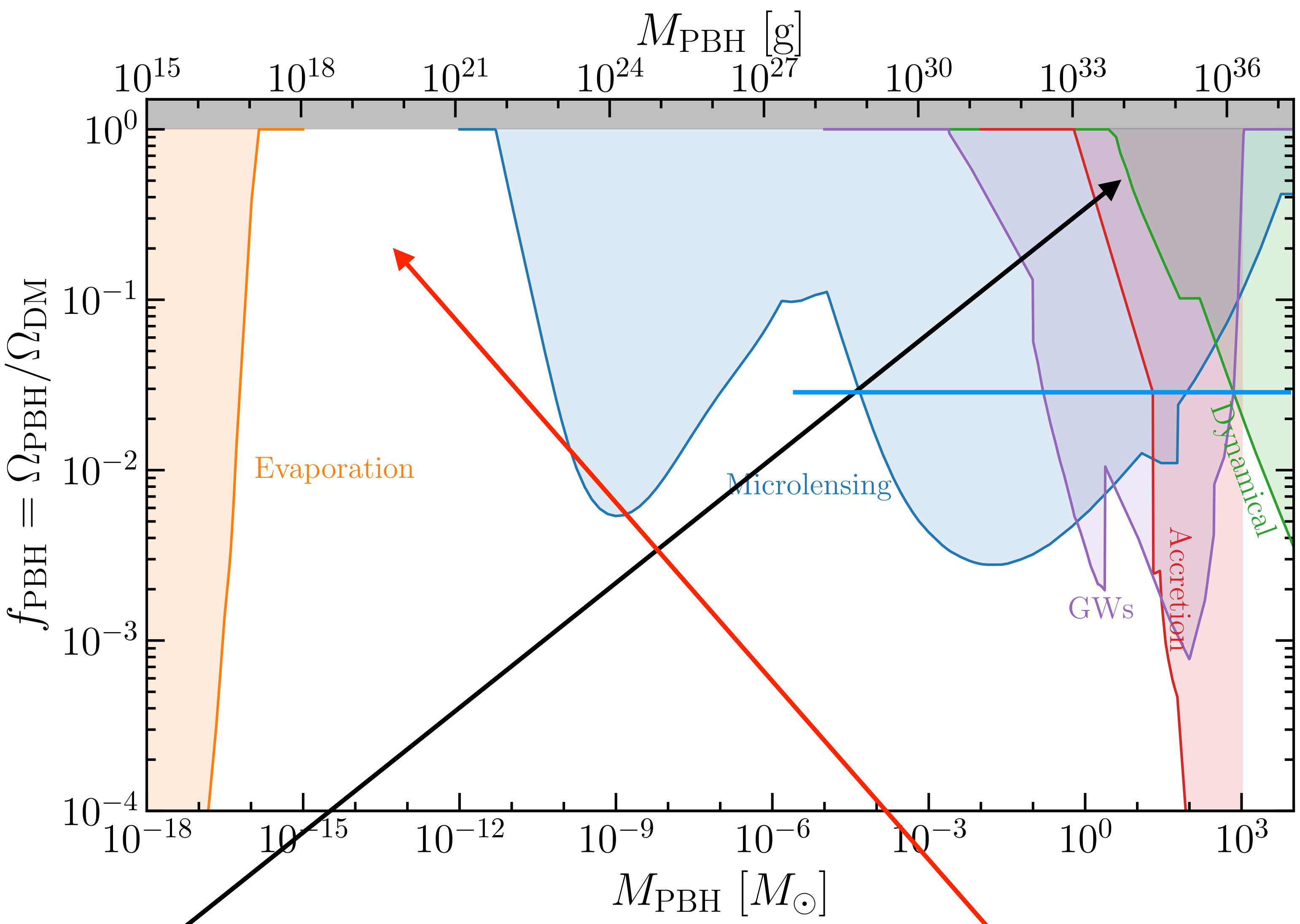


evaporation

lensing

gravitational waves

accretion

dynamical

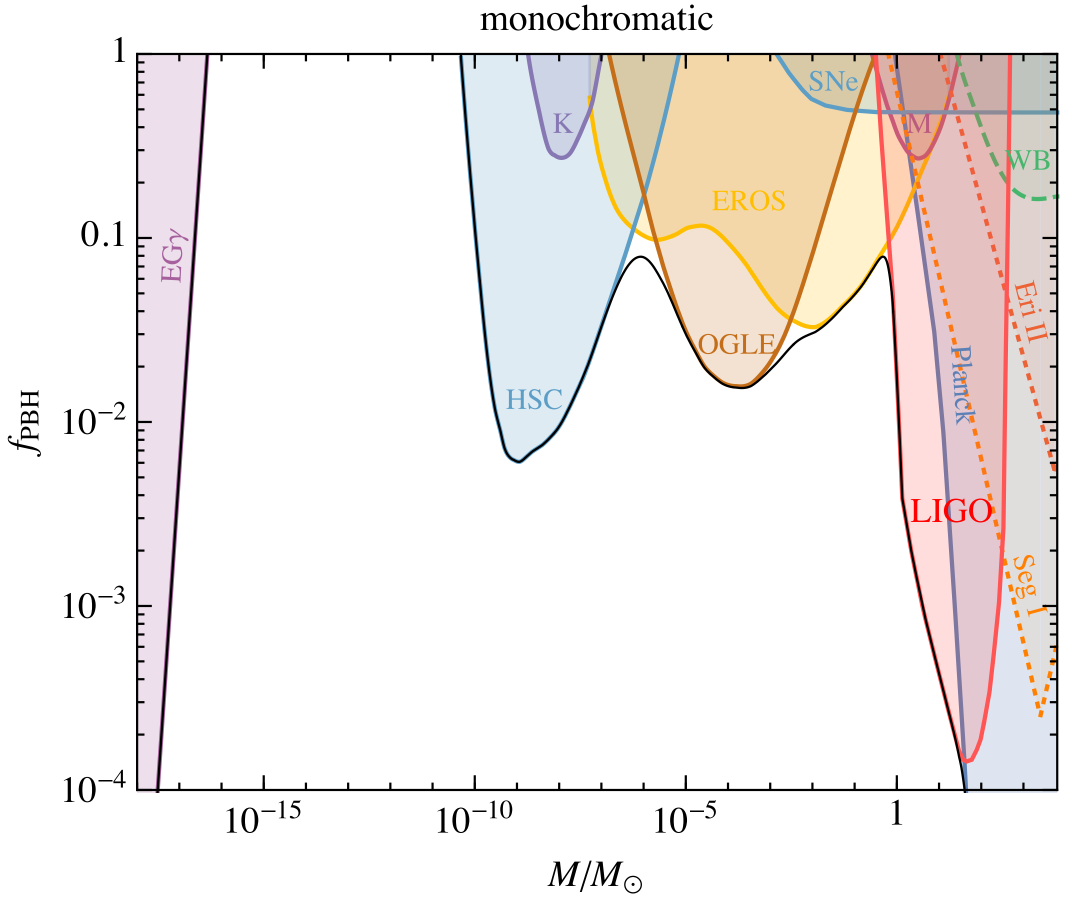
https://github.com/bradkav/PBHbounds

multi-Solar mass Primordial Black Holes making up all of the DM appears to be excluded.

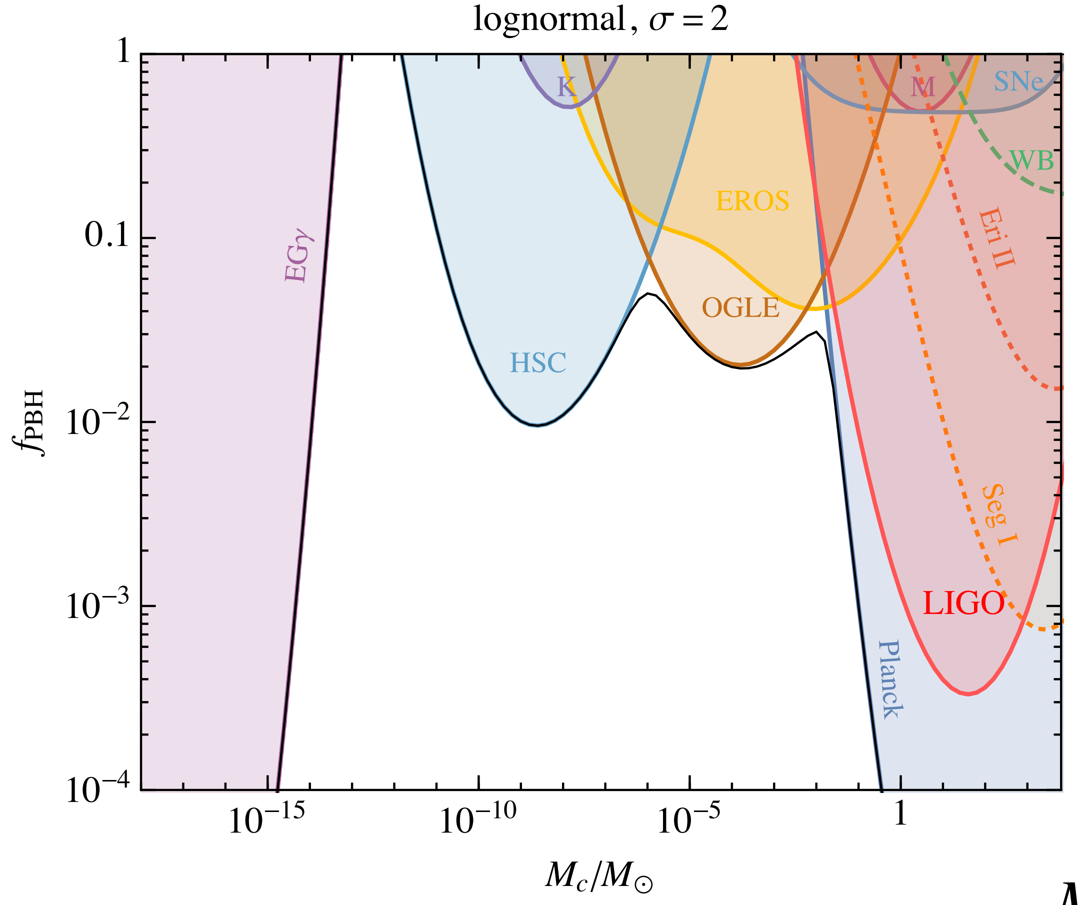
However there is a hard to probe, open window for very light (asteroid mass) PBHs.

For more realistic extended mass functions, constraints on f_{PBH} are smeared out, and gaps between constraints are ‘filled in’, but ‘asteroid mass’ window is currently still open even for broad mass functions: Green; Carr et al.; Gorton & Green

monochromatic



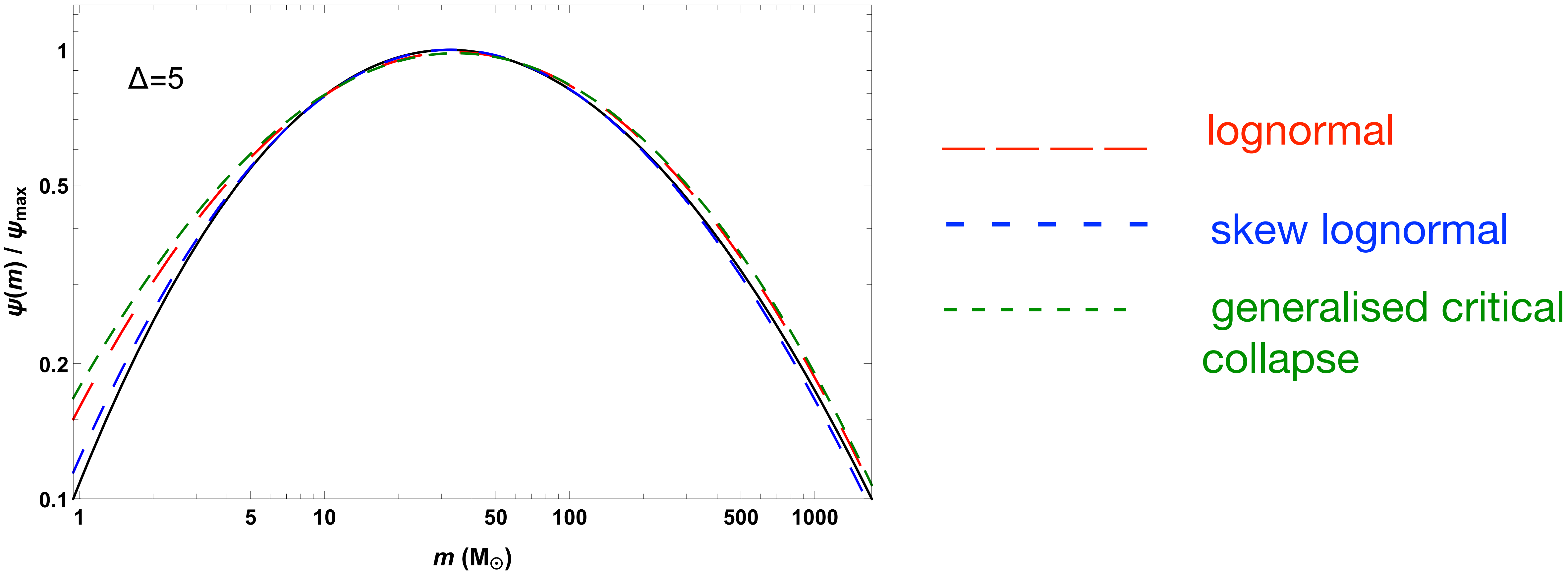
log-normal
(fixed width)



$$\frac{M_{\mathrm{c}}}{M_\odot}$$

Carr et al.

best fit MFs from a broad peak in the power spectrum Gow et al.



n.b. Germani and Sheth have calculated MF of PBHs, using the statistics of the compaction function.

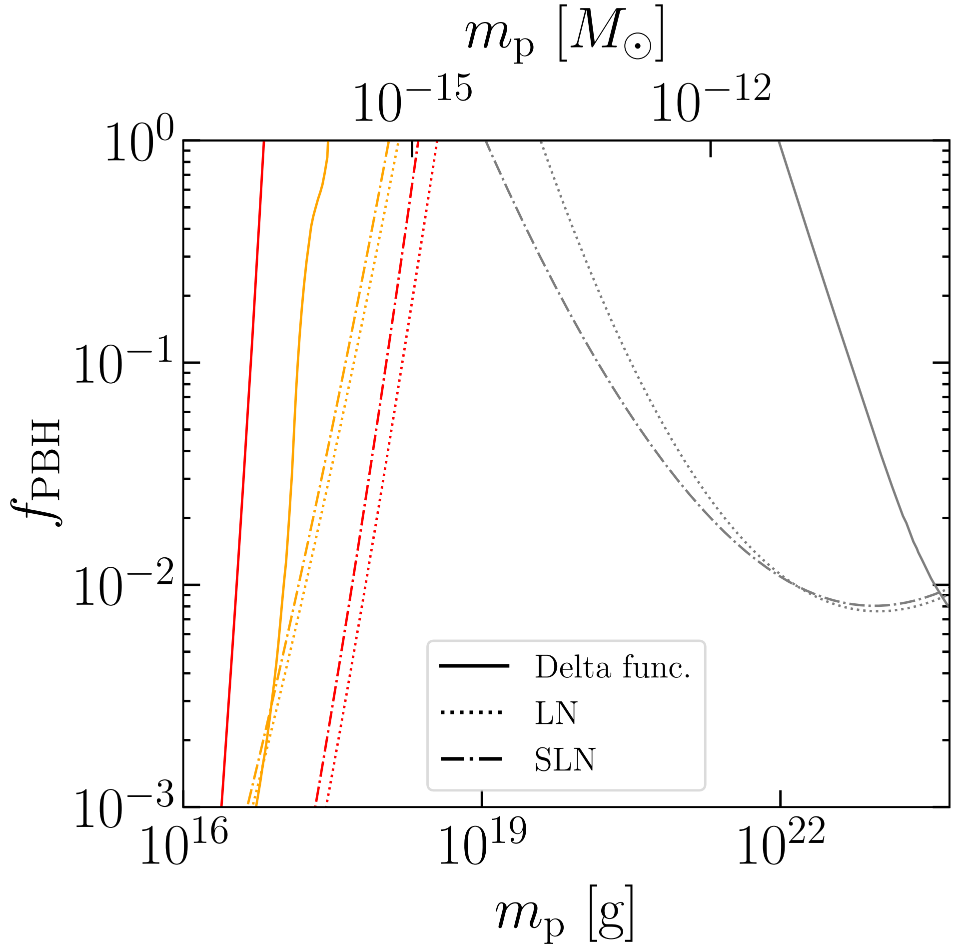
Find low mass tail generically $\psi(m) \propto m^{1/\gamma}$, high mass tail has cutoff which depends on shape & amplitude of power spectrum.

Constraints for best fit MF from a broad peak in the power spectrum Gow et al.

____ delta-function, …. lognormal _._._ skew lognormal

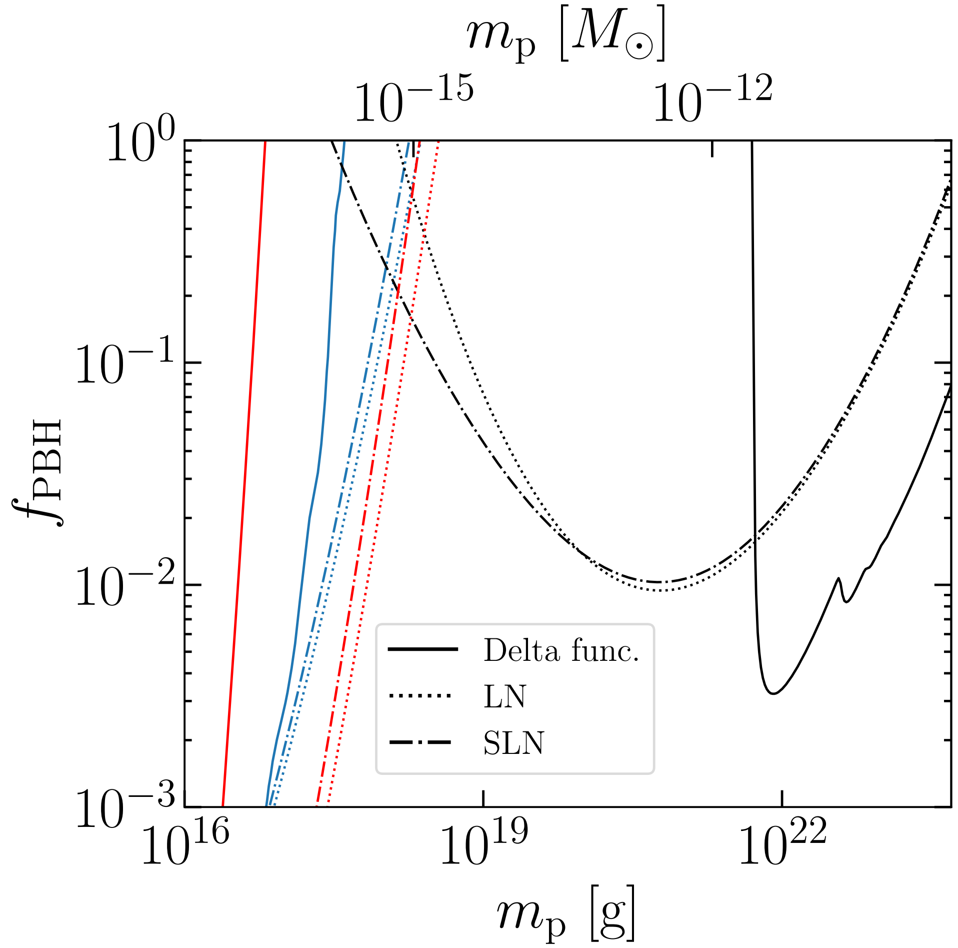
current

Voyager 1 $e^{\pm}$ Boudaud & Cirelli
MeV gamma rays Korwar & Profumo
Subaru-HSC M31 microlensing Niikura et al.; Croon et al.



future (+ Voyager 1 $e^{\pm}$)

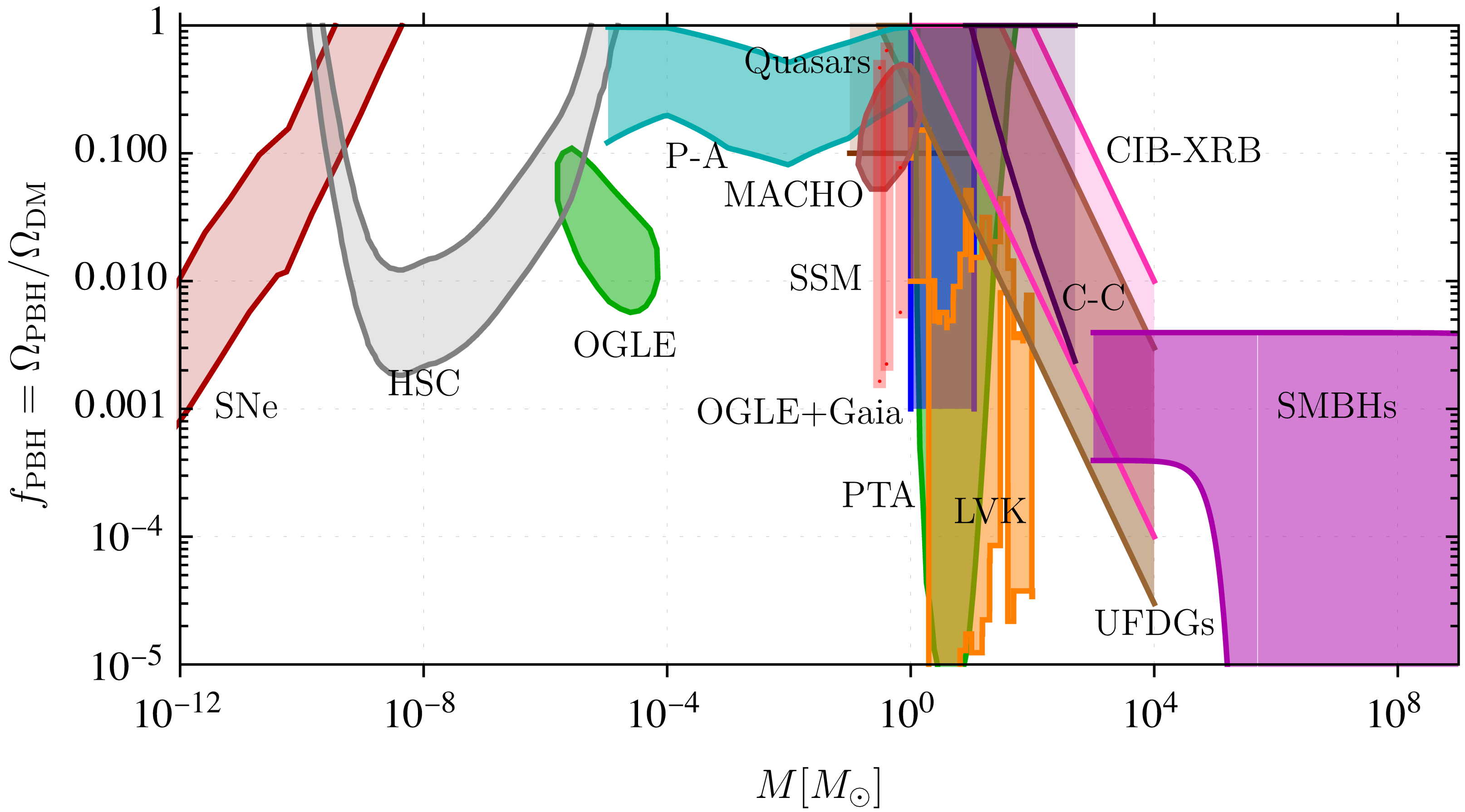
MeV gamma rays Coogan et al.
microlensing LMC white dwarfs Sugiyama et al.



Gorton & Green

Observational signatures ???

Carr, Clesse, Garcia-Bellido, Hawkins & Kuhnel, arXiv:2306.03903, ‘Observational evidence for PBHs: a positivist perspective’ and references therein.



SNe: trigger explosions of white dwarfs → calcium-rich supernovae

HSC, OGLE, PA, MACHO, OGLE-Gaia: microlensing

PTA: scalar induced gravitational waves detected by pulsar timing arrays

LVK: LIGO-Virgo-Kagra gravitational wave events

C-C: producing cores in density profiles of dwarf galaxies

CIB-XRB: accretion + clustering explains correlations in infra-red and X-ray backgrounds

UFDGs: clustering explains minimum mass & size of ultra faint dwarf galaxies

SMBHs: provide seeds for super massive black holes

Open questions

i) how to probe asteroid mass PBHs?

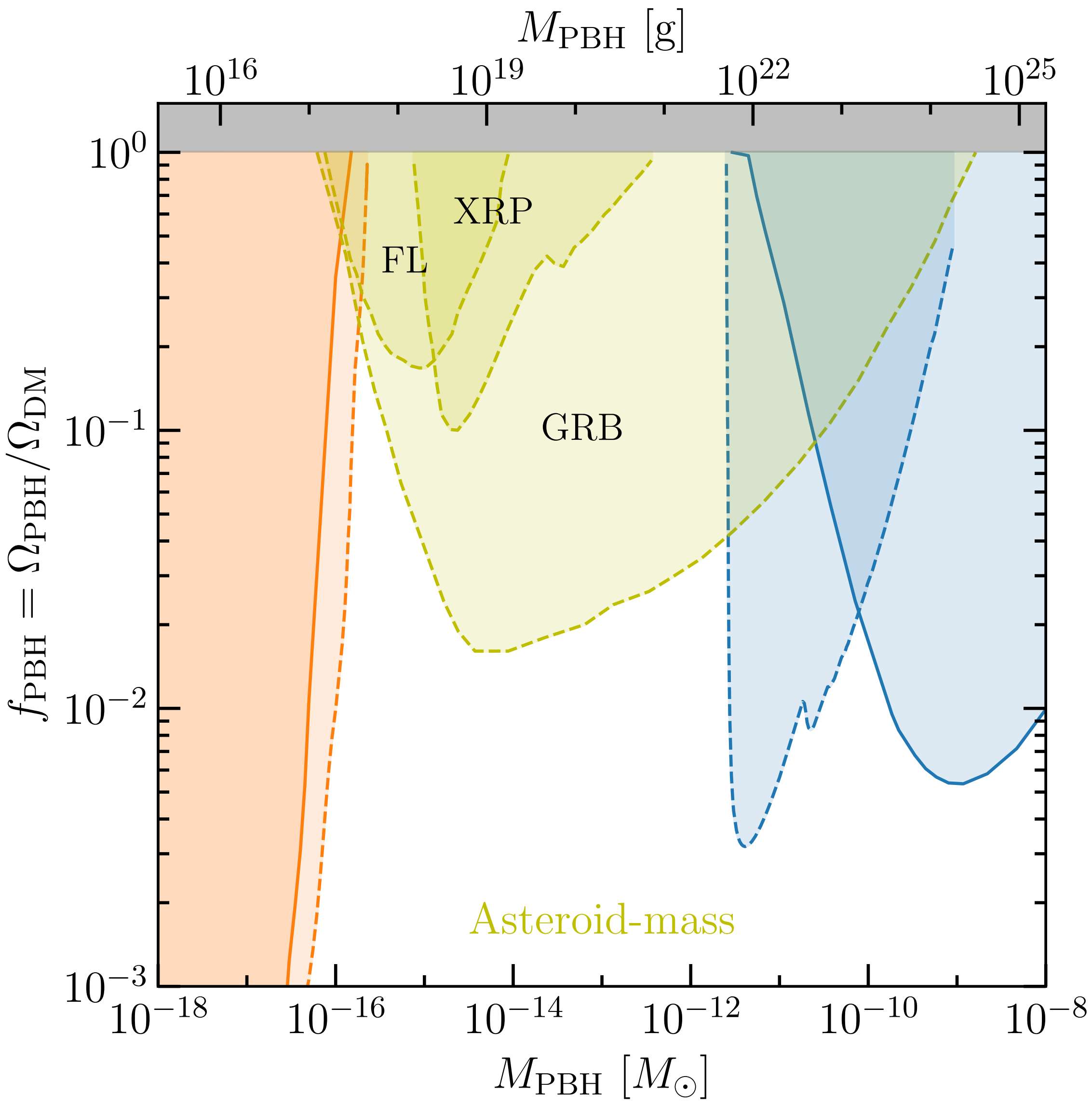
femtolensing of GRBs Gould need small GRBs Katz et al.

GRB lensing parallax Nemiroff & Gould; Jung & Kim

microlensing of X-ray pulsars Bai & Orlofsky

evaporation

future:
  MeV gamma rays
  Coogan et al.



stellar microlensing

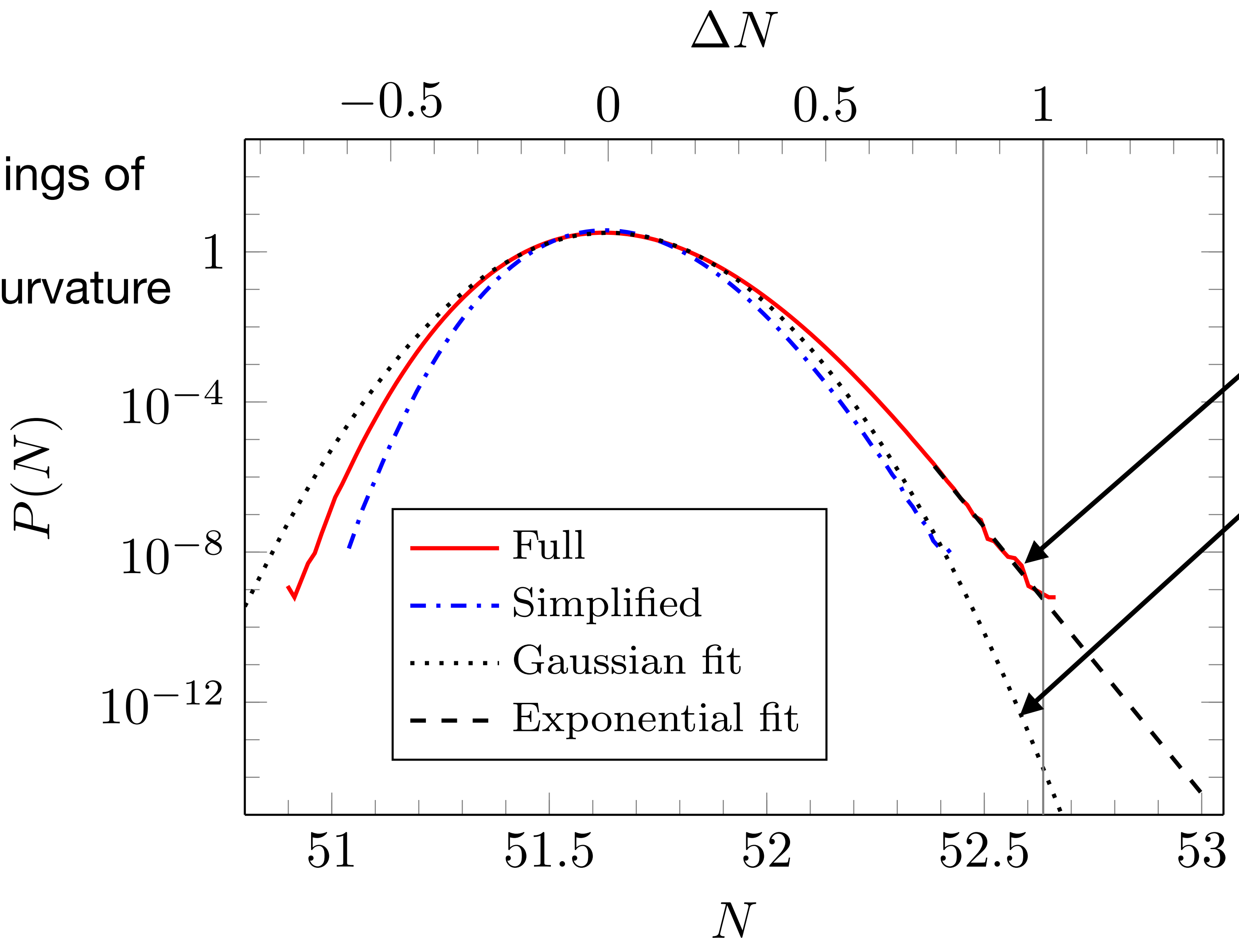
future:
  white dwarfs in LMC
  Sugiyama et al.

ii) probability distribution of density perturbations produced during ultra slow-roll inflation

Pattinson et al. … Figueroa et al.; Tada & Vennin…

In ultra-slow-roll inflation (i.e. for $V' \to 0$ as required in single-field inflation to produce large amplitude, PBH-forming, perturbations) stochastic effects are important, and can generate exponential rather than gaussian tail for probability distribution.

Prob. dist. of
number of e-foldings of
inflation
~ prob. dist. of curvature
perturbation



exponential

gaussian

Figueroa et al.

ongoing debate: do large amplitude small scale perturbations lead to significant one-loop corrections to perturbations on CMB scales?

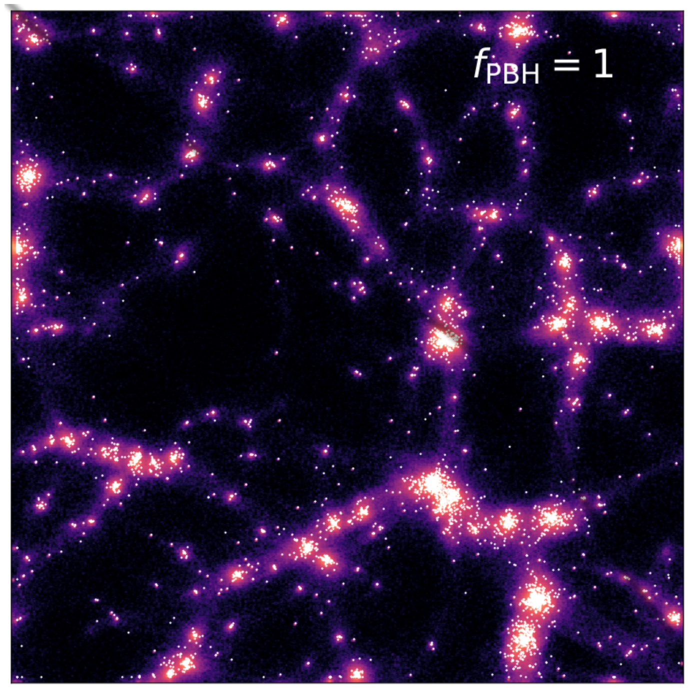
Kristiano & Yokoyama; Firouzjahi & Riotto; Fumagalli; Firouzjahi

iii) clustering

Potentially extremely important (affects PBH binary merger rate and potentially other constraints too).

If PBHs make up a large fraction of the DM, PBH clusters form shortly after matter-radiation equality.
Afshordi, Macdonald & Spergel;... Inman & Ali-Haïmoud

Evolution of PBH clusters (and in particular PBH binaries within them and hence the merger rate) through to the present day is a challenging open problem. e.g. Jedamzik; Trashorras et al….



PBH-DM dist at z=100

Inman & Ali-Haïmoud

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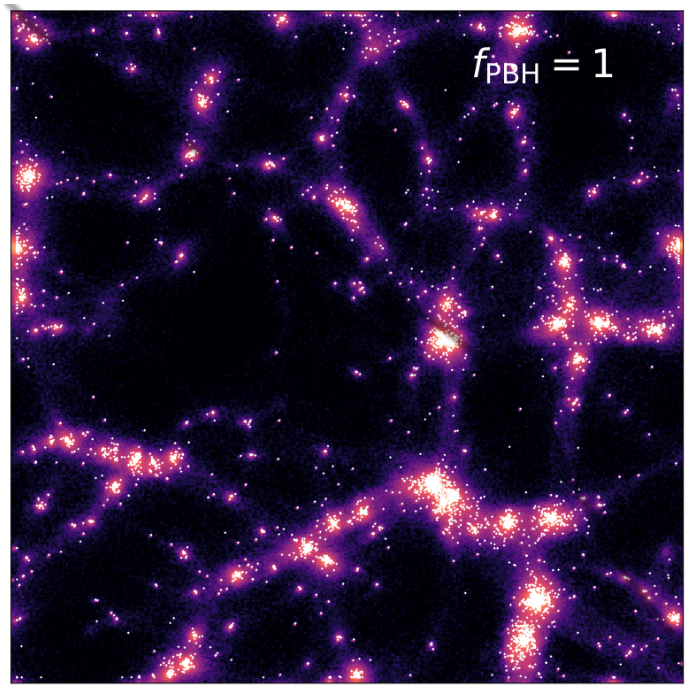


PBH-DM dist at z=100

Inman & Ali-Haïmoud

Clusters of PBH formed from collapse of gaussian density perturbations are sufficiently extended that PBHs microlens individually, & change in **microlensing** constraints is small. Petaĉ, Lavalle & Jedamzik; Gorton & Green.

Non-gaussianity can lead to more compact clusters, however while microlensing constraints would be weakened, other constraints (GWs, Lyman-alpha forest, dynamical) would be tightened. Bringmann et al.; Young & Byrnes; de Luca et al.

Summary

Primordial Black Holes can form in the early Universe, for instance from the collapse of large density perturbations during radiation domination.

- To produce an interesting number of PBHs, amplitude of perturbations must be ~3 orders of magnitude larger on small scales than on cosmological scales.
- This can be achieved in inflation models (e.g. with a feature in the potential or multiple fields). However it's not natural/generic.

There are numerous constraints on the abundance of PBHs from gravitational lensing, their evaporation, dynamical effects, accretion and other astrophysical processes.

- Solar mass PBHs probably can't make up all of the dark matter, but lighter, (10^{17}-10^{22})g, PBHs could.
- Limits are collectively tighter for (expected) extended mass functions than for delta-function which is usually assumed when calculating constraints.
- Clustering would weaken some constraints and tighten others.

Open questions: how to probe light PBHs,
perturbations in ultra-slow roll inflation,
clustering…

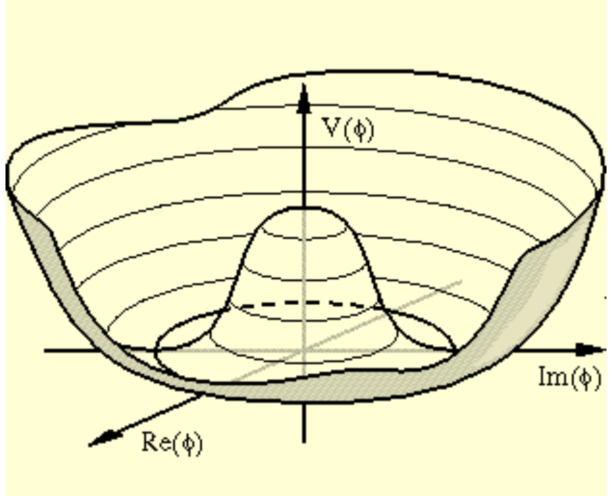
Back-up slides

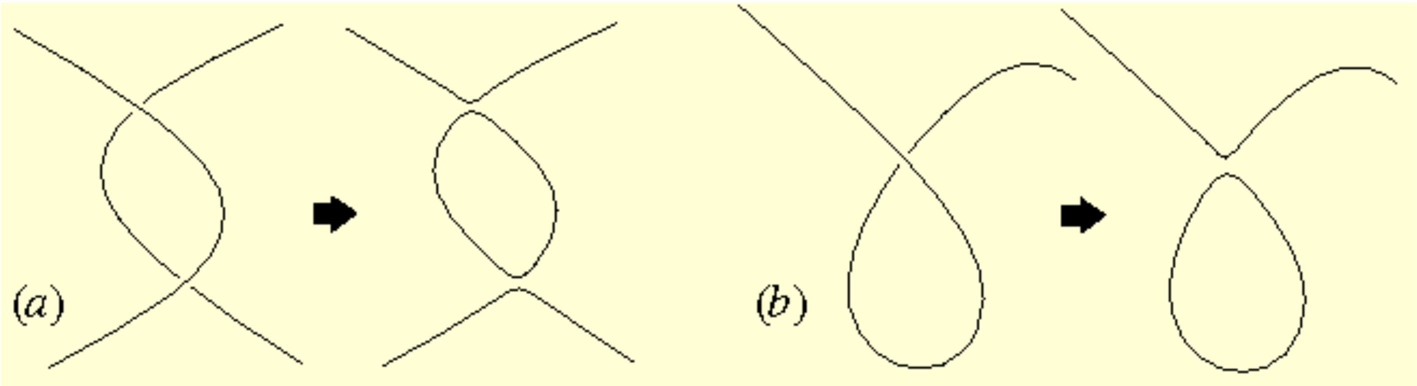
PBH formation: (some) other mechanisms

Collapse of cosmic string loops Hawking; Polnarev & Zemboricz;

Cosmic strings are 1d topological defects formed during symmetry breaking phase transition.

String intercommute producing loops.





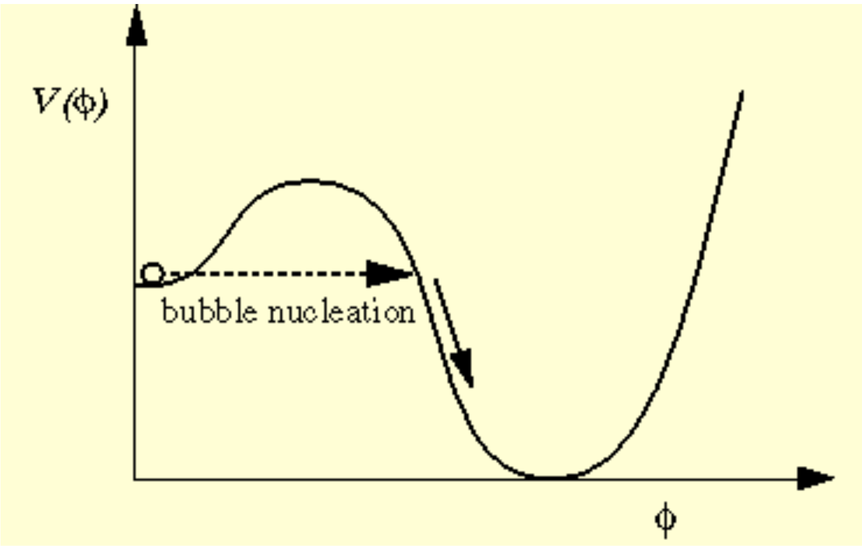
Small probability that loop will get into configuration where all dimensions lie within Schwarzschild radius (and hence collapse to from a PBH with mass of order the horizon mass at that time).

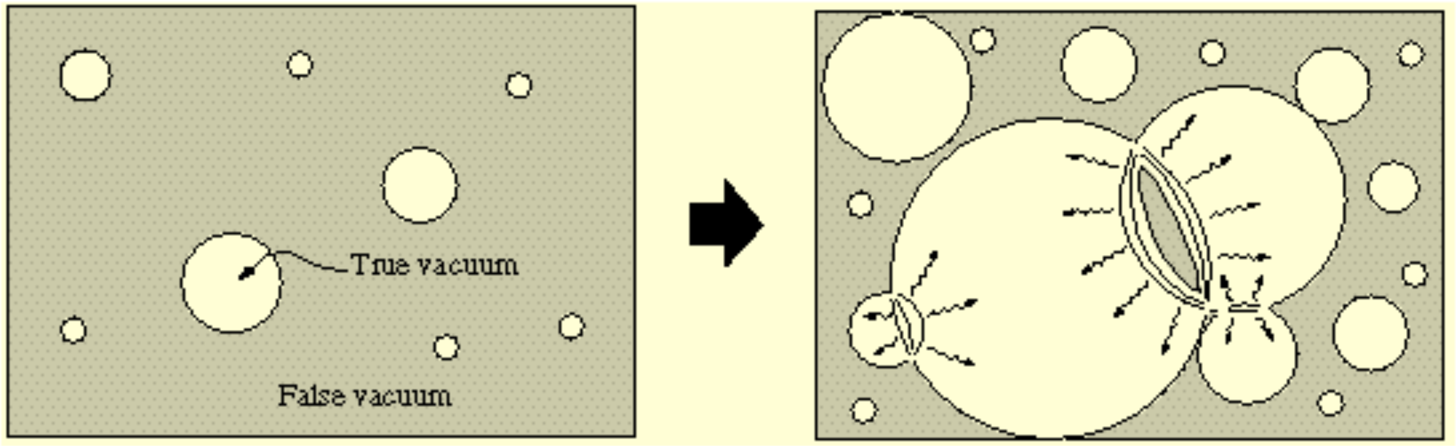
Probability is time independent, therefore PBHs have extended mass spectrum.

Bubble collisions

Hawking

1st order phase transitions occur via the nucleation of bubbles.





PBHs can form when bubbles collide (but bubble formation rate must be fine tuned).

PBH mass is of order horizon mass at phase transition.

Fragmentation of inflaton scalar condensate into oscillons/Q-balls

Cotner & Kusenko; Cotner, Kusenko & Takhistov

Scalar field with flat potential forms condensate at end of inflation, fragments into lumps (oscillons/Q-balls) which can come to dominate universe and have large density fluctuations that can produce PBHs.

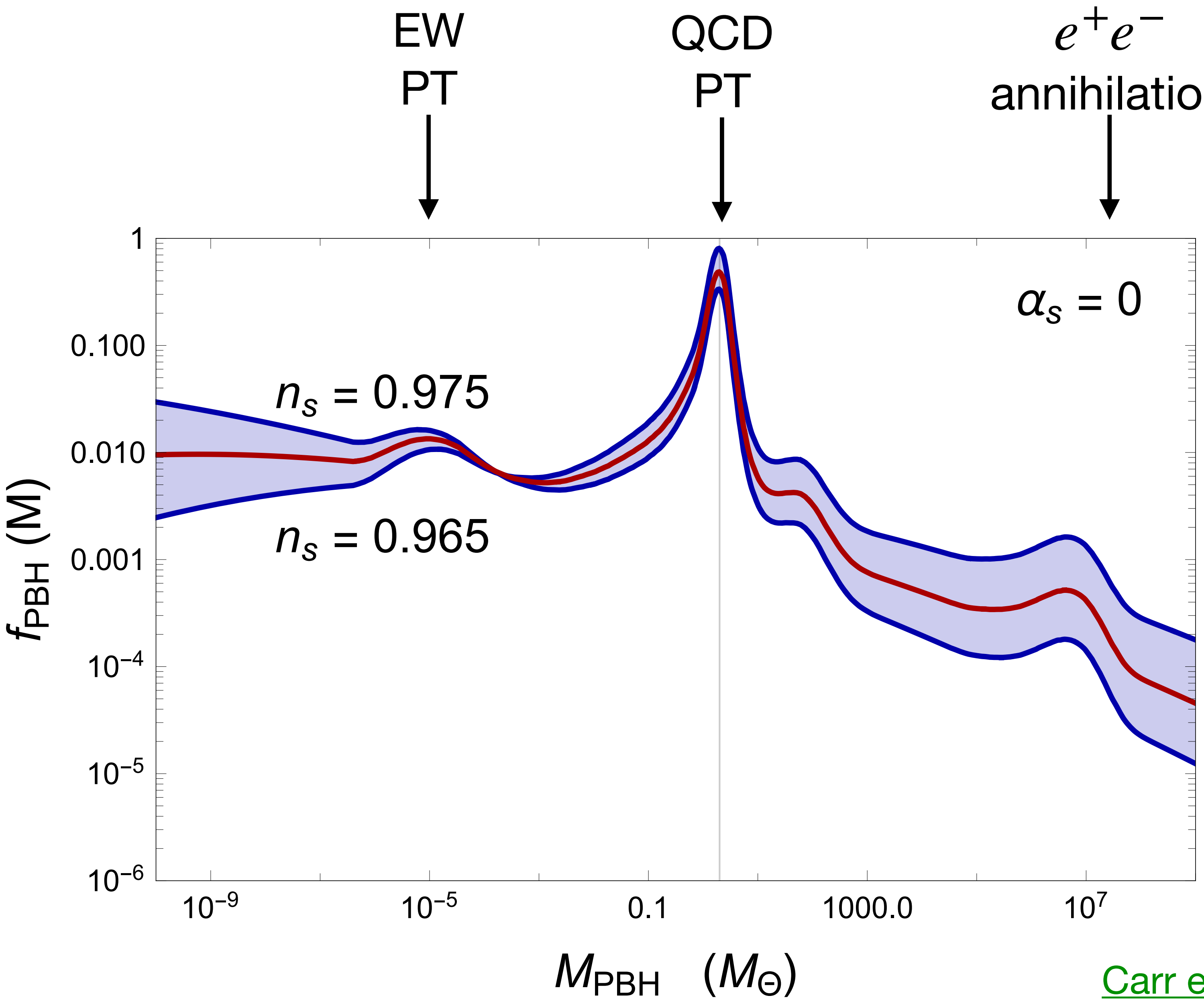
Mass smaller than horizon mass and spin can be of order 1.

ii) effect of phase transitions

Decrease in pressure leads to reduction in threshold for collapse and hence increase in PBH abundance

e.g. the QCD phase transition when the horizon mass is ~Solar mass. Jedamzik

fraction of DM
in PBHs



Carr et al.

n.b. amplitude of power spectrum A ≈ 0.02 assumed.

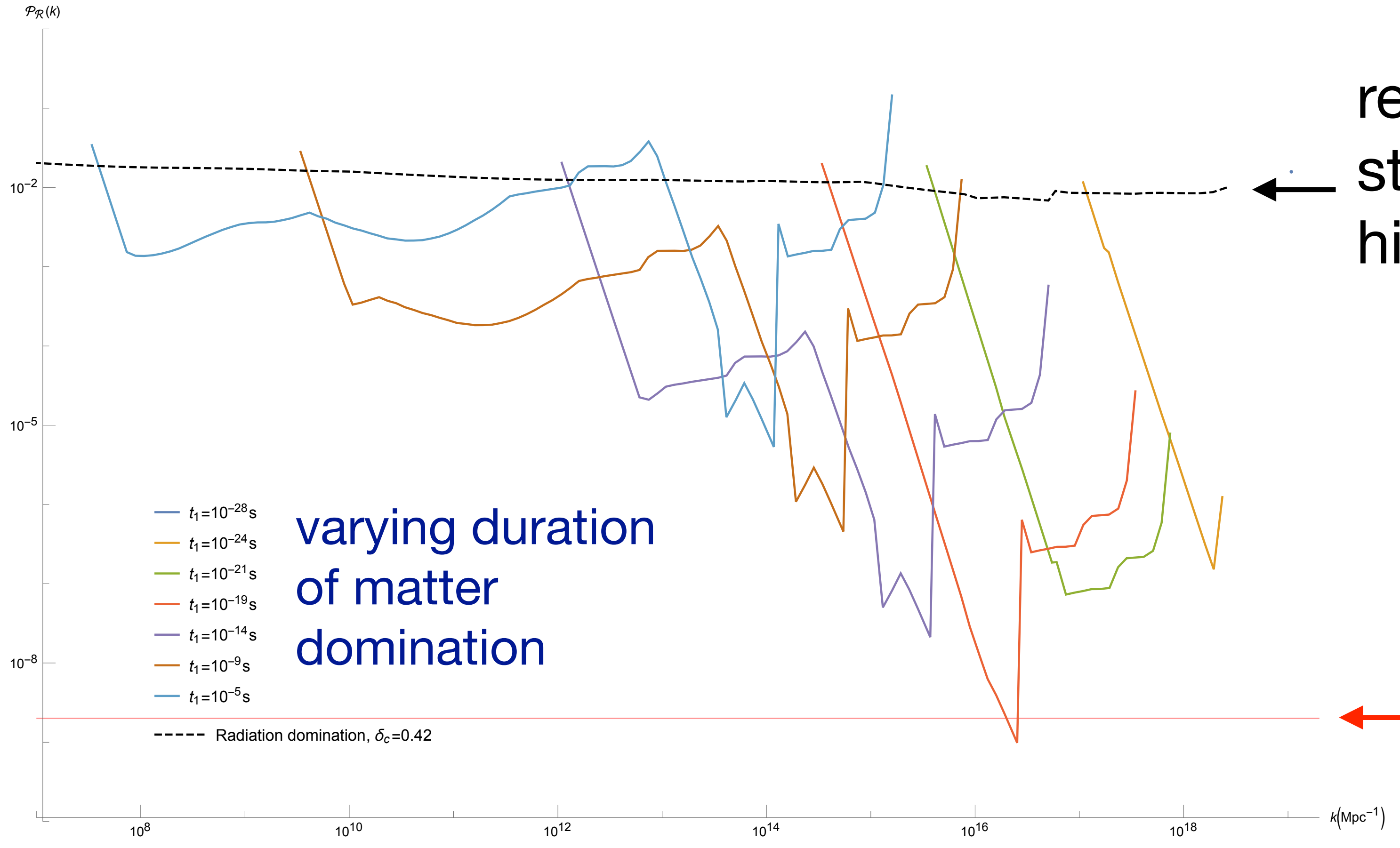
PBH formation during an early (pre nucleosynthesis) period of matter domination

During matter domination PBHs can form from smaller fluctuations (no pressure to resist collapse) in this case fluctuations must be sufficiently spherically symmetric Yu, Khlopov & Polnarev; Harada et al. and

$$\beta(M) \approx 0.056\sigma^{5(+1.5?)}$$

The required increase in the amplitude of the perturbations is reduced Georg, Sengör & Watson; Georg & Watson; Carr, Tenkanen & Vaskonen; Cole & Byrnes:

Primordial curvature perturbation power spectrum



varying duration of matter domination

required amplitude for standard cosmological history

amplitude on CMB scales

Cole & Byrnes

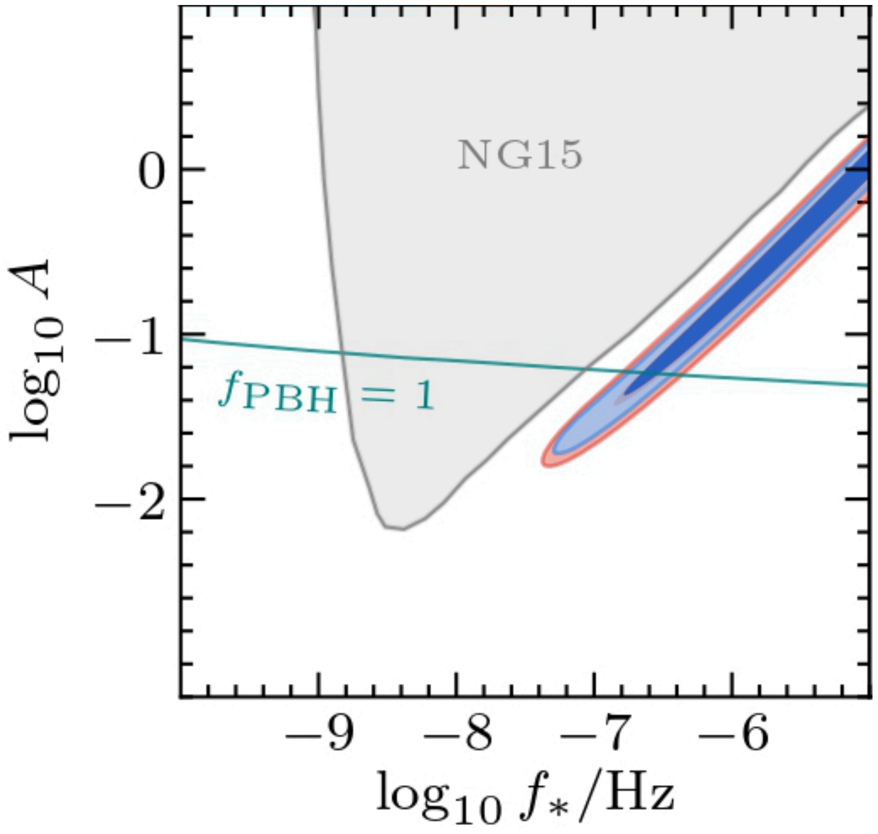
k

NANOGrav (pulsar timing array) 15 year data set

Interpretation in terms of scalar-induced gravitational waves Afzal et al.

for delta function primordial power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = A\,\delta(\ln k - \ln k_\star)$$



$$\underbrace{\qquad\qquad}_{(10^{-3} - 10^{-4})M_\odot}$$

axion-like curvaton

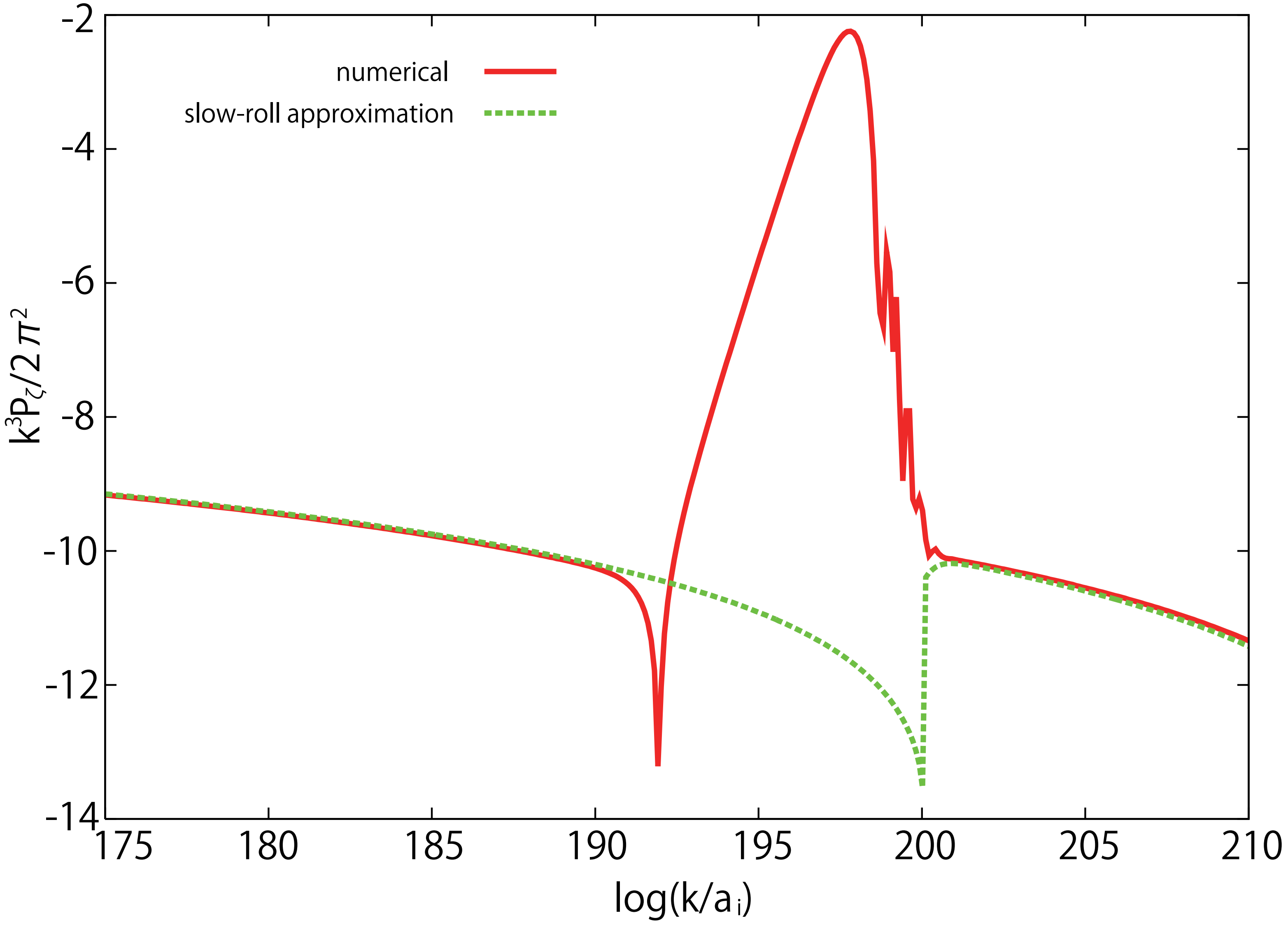
Kawasaki, Kitajima & Yanagida

Large scale perturbations generated by inflaton, small scale (PBH forming) perturbations by curvaton (a spectator field during inflation gets fluctuations and decays afterwards producing perturbations Lyth & Wands)

b) double inflation

Saito, Yokoyama & Nagata; Kannike et al.

Perturbations on scales which leave the horizon close to the end of the 1st period, of inflation get amplified during the 2nd period.



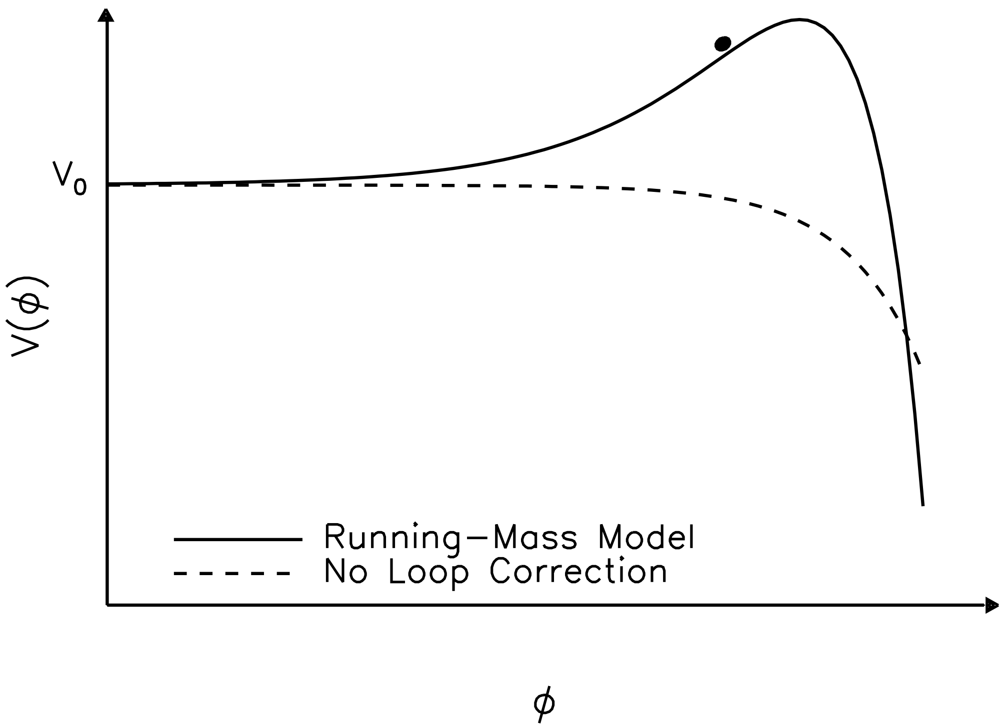
Also double inflation models where large scale perturbations are produced during 1st period, and small scale (PBH forming) perturbations during 2nd (Kawasaki et al.; Kannike et al.; Inomata et al.)

ii) monotonically increasing power spectrum

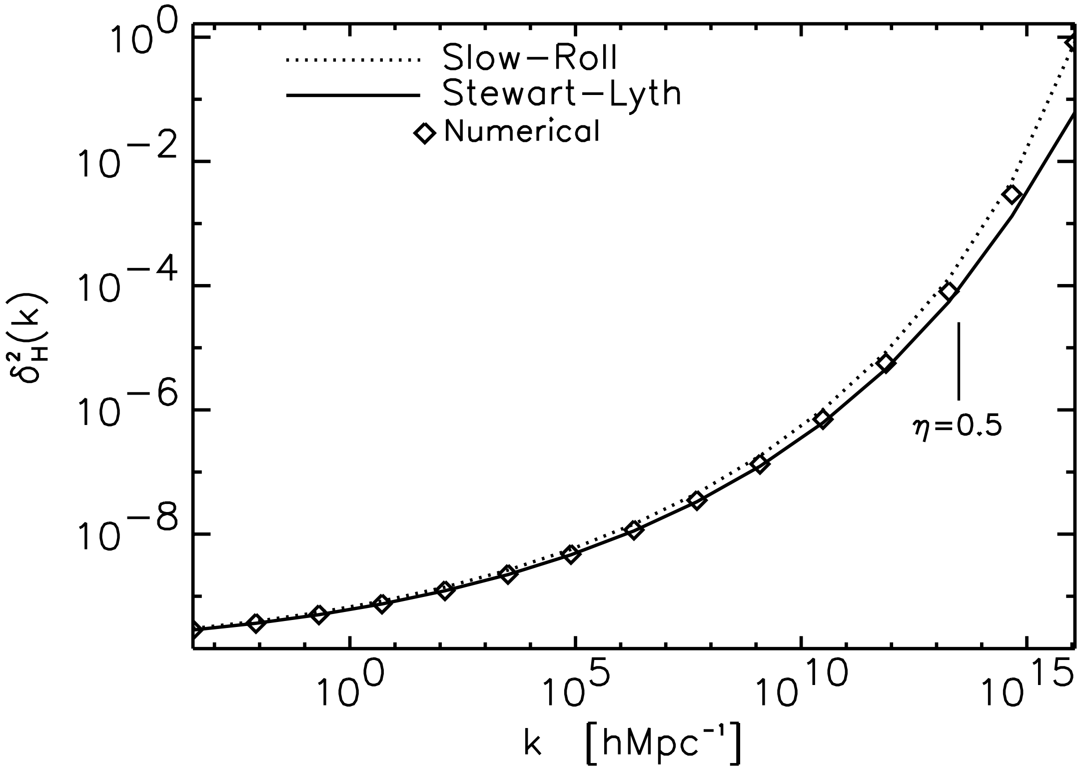
running-mass inflation Stewart

$$V(\phi) = V_0 + \frac{1}{2} m_\phi^2(\phi)\phi^2$$

potential



primordial power spectrum



Leach, Grivell, Liddle

An aside: ‘Pitfalls of a power-law parameterisation of the primordial power spectrum for primordial black hole formation’ 1805.05178

It is common to parameterise the primordial power spectrum as:

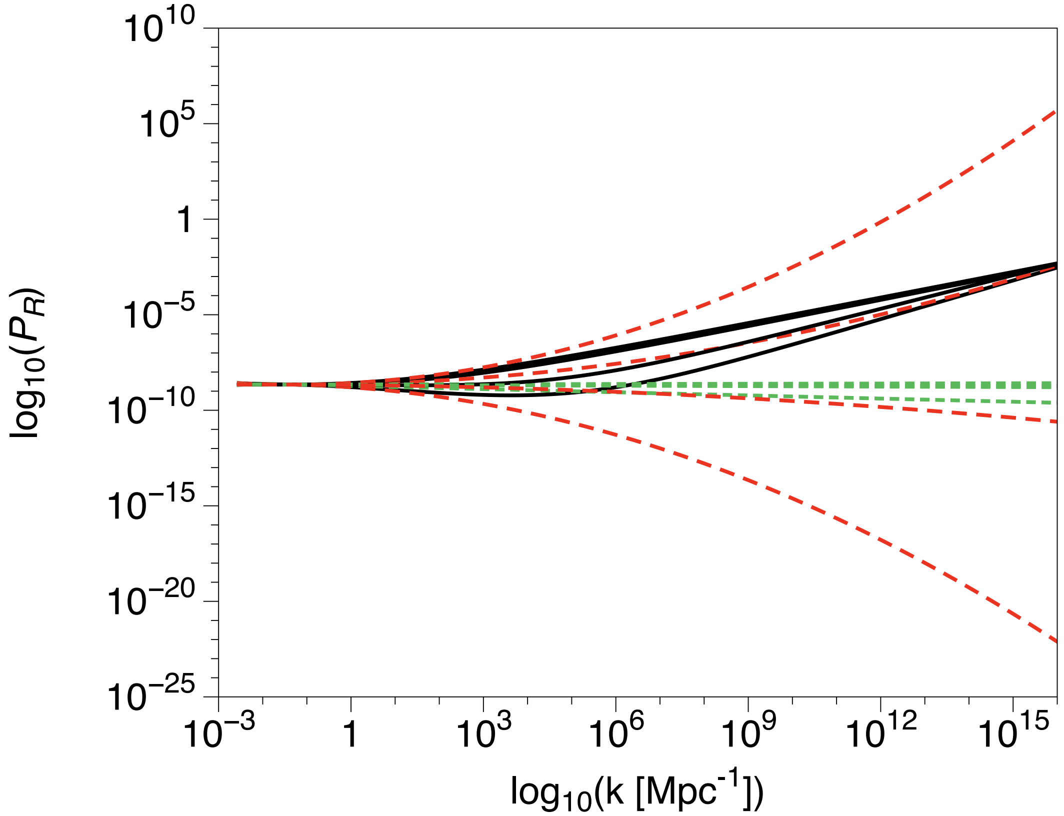
$$\mathcal{P}_\mathcal{R}(k) = A_\mathrm{s}\left(\frac{k}{k_0}\right)^{n_\mathrm{s}(k)-1} \quad \text{with} \quad n_\mathrm{s}(k) = n_\mathrm{s}|_{k_0} + \alpha_\mathrm{s}\ln\left(\frac{k}{k_0}\right) + \beta_\mathrm{s}\ln^2\left(\frac{k}{k_0}\right) + \ldots\,,$$

For slow-roll inflation $(n_\mathrm{s}-1) \sim \mathcal{O}(\epsilon)\,, \quad \alpha_\mathrm{s} \sim \mathcal{O}(\epsilon^2)\,, \quad \beta_\mathrm{s} \sim \mathcal{O}(\epsilon^3) \quad \text{where} \quad \epsilon < 1$

The expansion of n_s is therefore valid only if $\epsilon \ln\left(\frac{k}{k_0}\right) \ll 1$

This holds over cosmological scales, but not down to PBH forming scales:

Power spectra of some PBH producing inflation models:

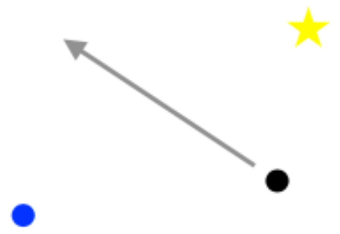


full calculation

1st order in expansion

2nd order in expansion

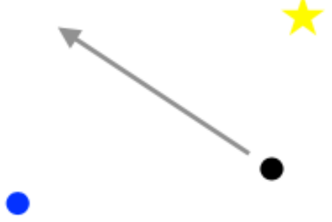
stellar microlensing



Gravitational lensing where separation of images is micro-arcsecond, too small to resolve, but can detect variations in magnification.

M31 (HSC, Croon et al.), Galactic bulge (OGLE),LMC/SMC (MACHO, EROS, OGLE, combined long duration).

other microlensing



Gravitational lensing where separation of images is micro-arcsecond, too small to resolve, but can detect variations in magnification.

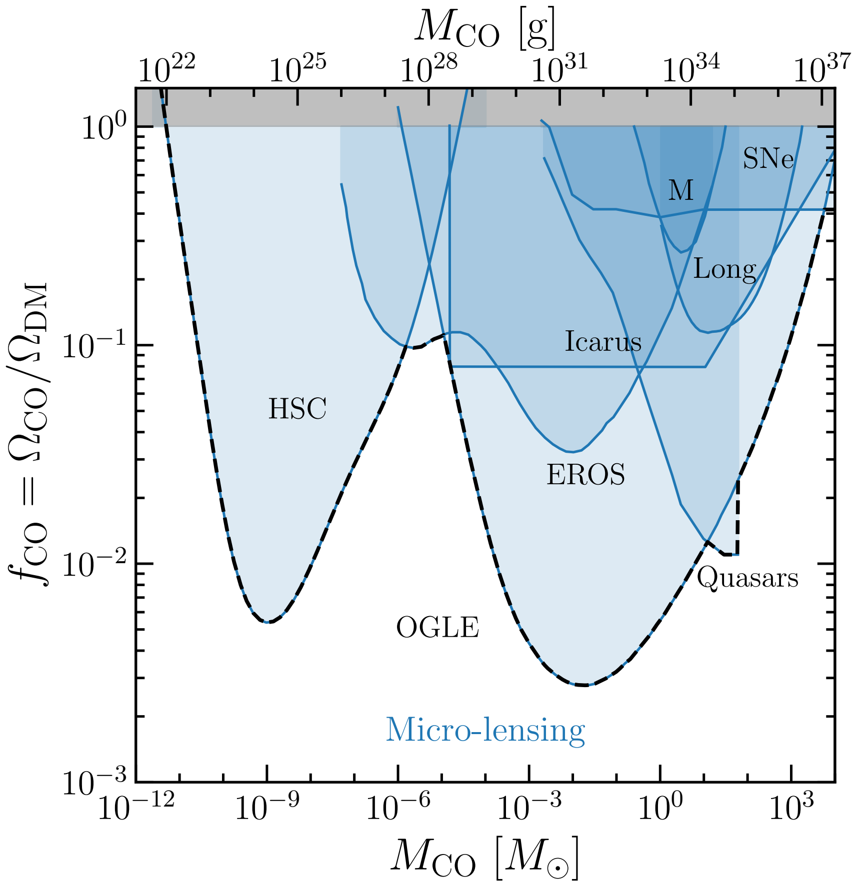
supernovae: magnification distribution Zumalacarregui & Seljak
luminosity-redshift relation Dhawan & Mörtsell

Icarus: caustic crossing event Oguri et al.

quasars: flux ratios of multiply-lensed systems Esteban-Gutierrez et al.

fraction of dark matter
in form of compact objects

$$f_{\rm CO} = \frac{\Omega_{\rm CO}}{\Omega_{\rm DM}}$$



mass in grams

mass in Solar masses

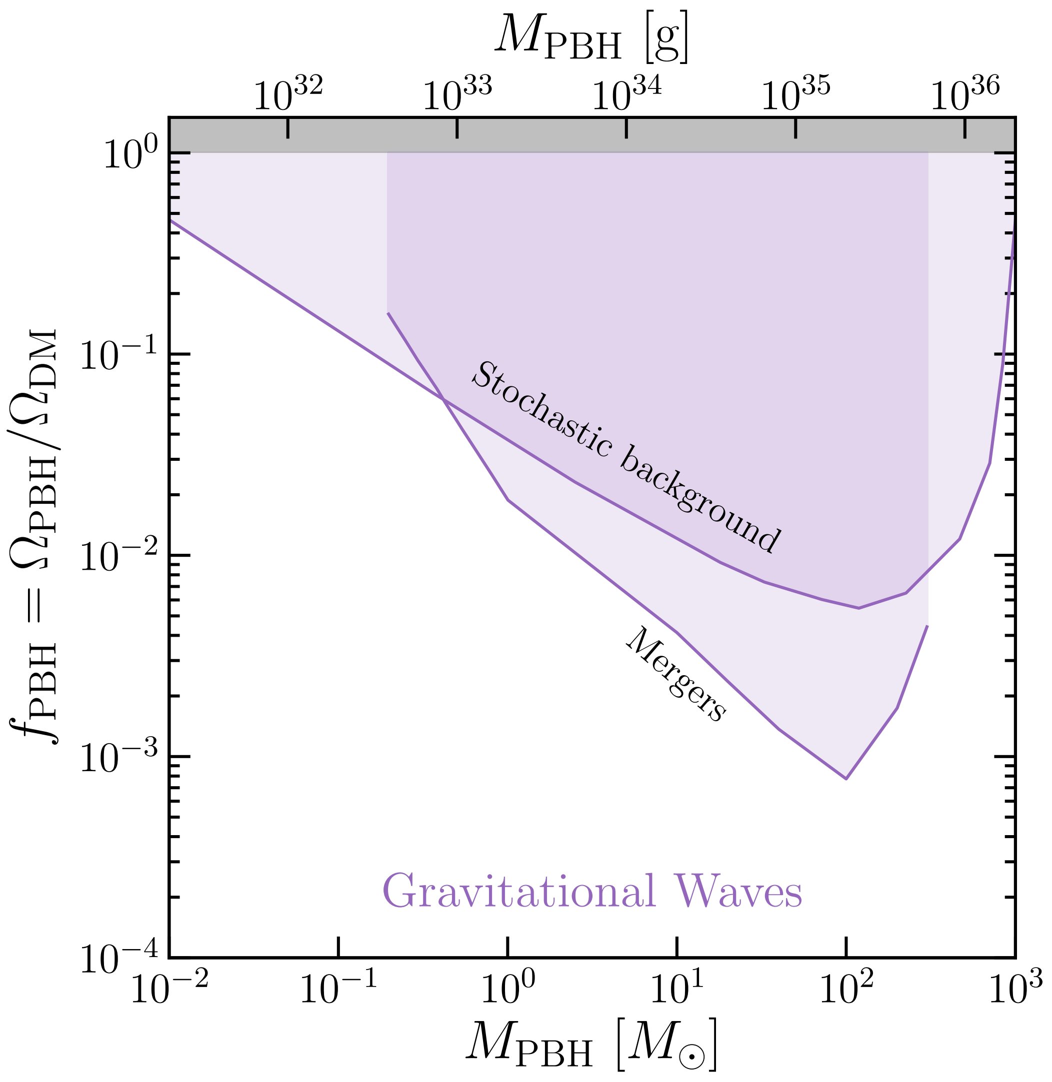
gravitational waves
from PBH-PBH binary mergers

PBH binaries can form at early times (from chance proximity). Nakamura et al.

If orbits aren’t significantly perturbed subsequently, then their mergers are orders of magnitude larger than the merger rate measured by LIGO. Ali-Haïmoud, Kovetz & Kamionkowski

Also comparable constraints from stochastic GW from mergers. Wang et al.

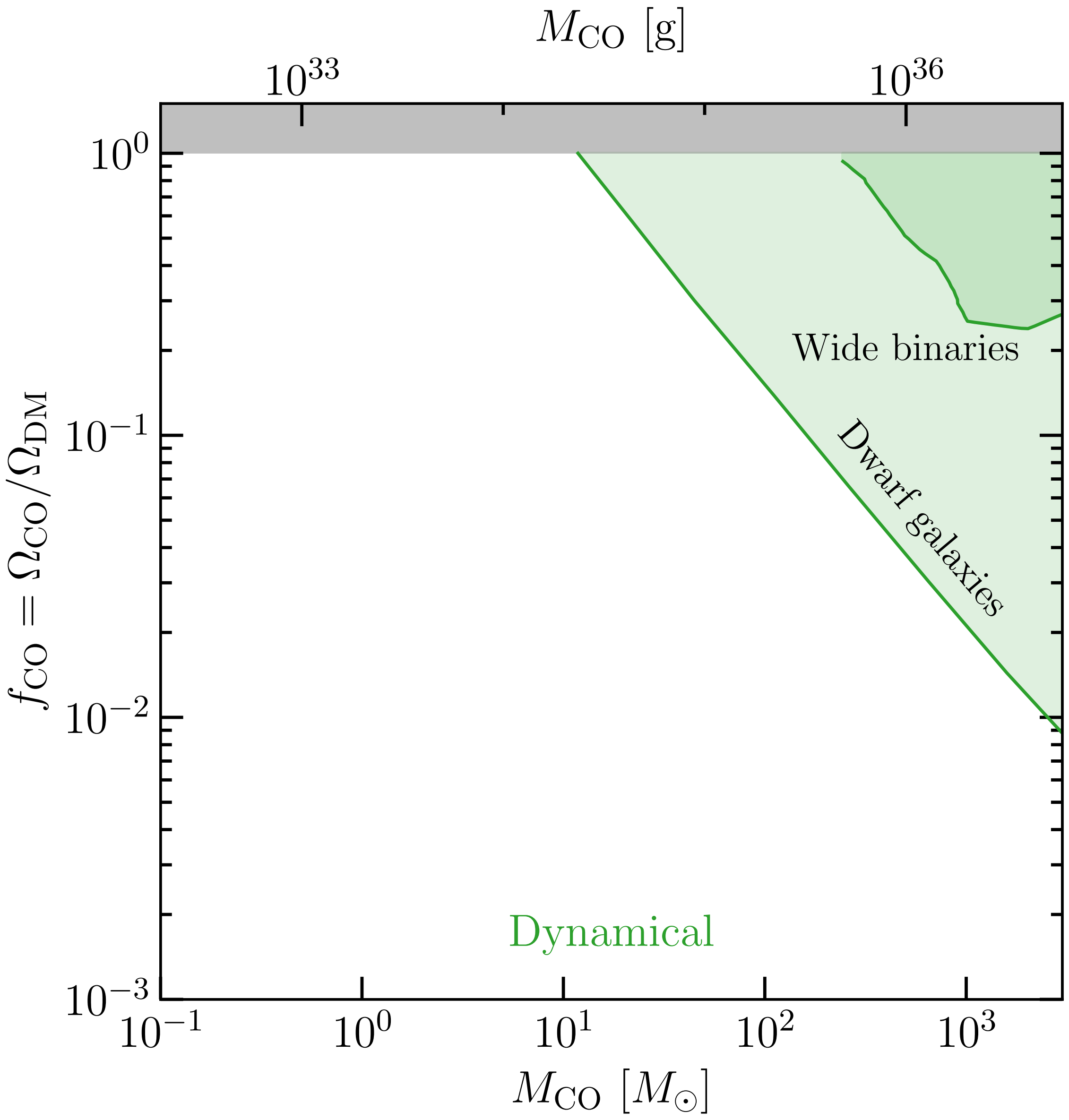
$$f_{\rm PBH} = \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}$$



dynamical effects

dwarf galaxies: stars are dynamically heated and size of stellar component increased
Brandt; Koushiappas & Loeb; Zhu et al.; Stegmann et al.

wide binaries: dynamically heated, separations increased, and widest binaries disrupted. Yoo, Chaname & Gould; … Monroy-Rodriguez & Allen; Tyler, Green & Goodwin



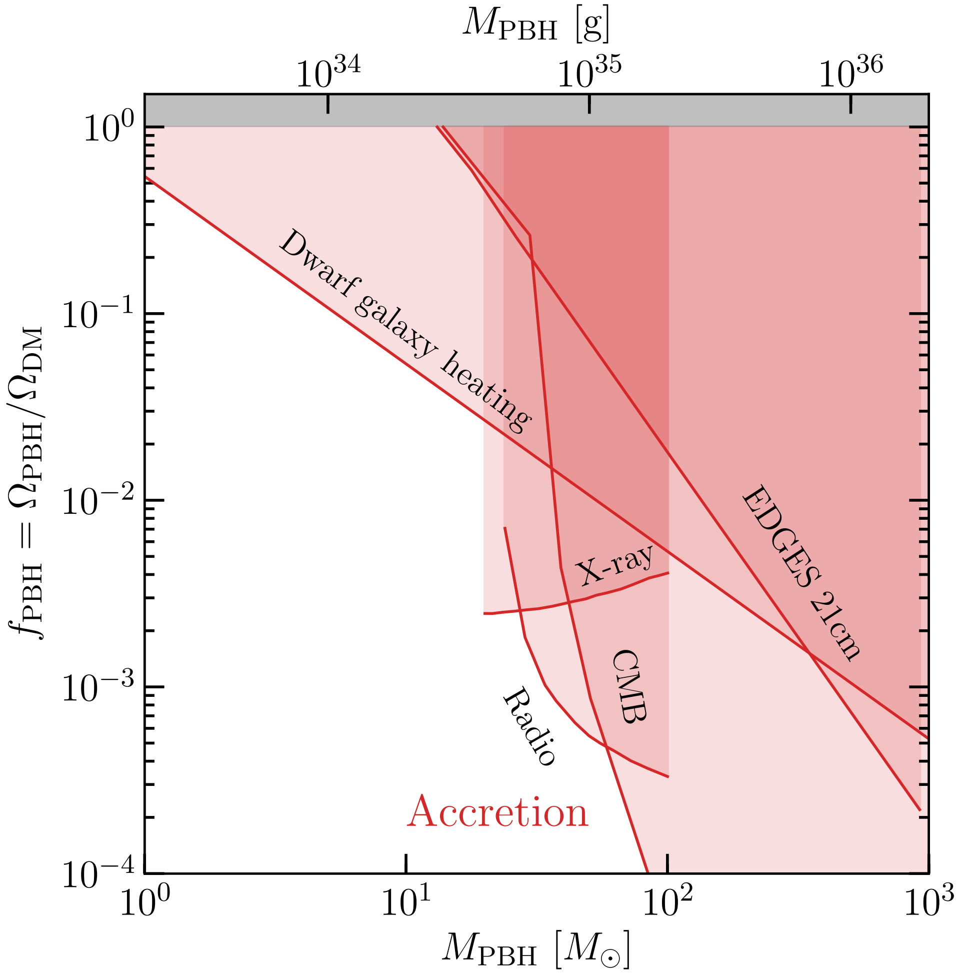
accretion

Radiation emitted due to gas accretion onto PBHs can modify the recombination history of the universe, constrained by

- distortion of CMB anisotropies Ricotti et al; Ali-Haïmoud & Kamionkowski; … Poulin et al.…
- EDGES 21cm measurements Hektor et al.;

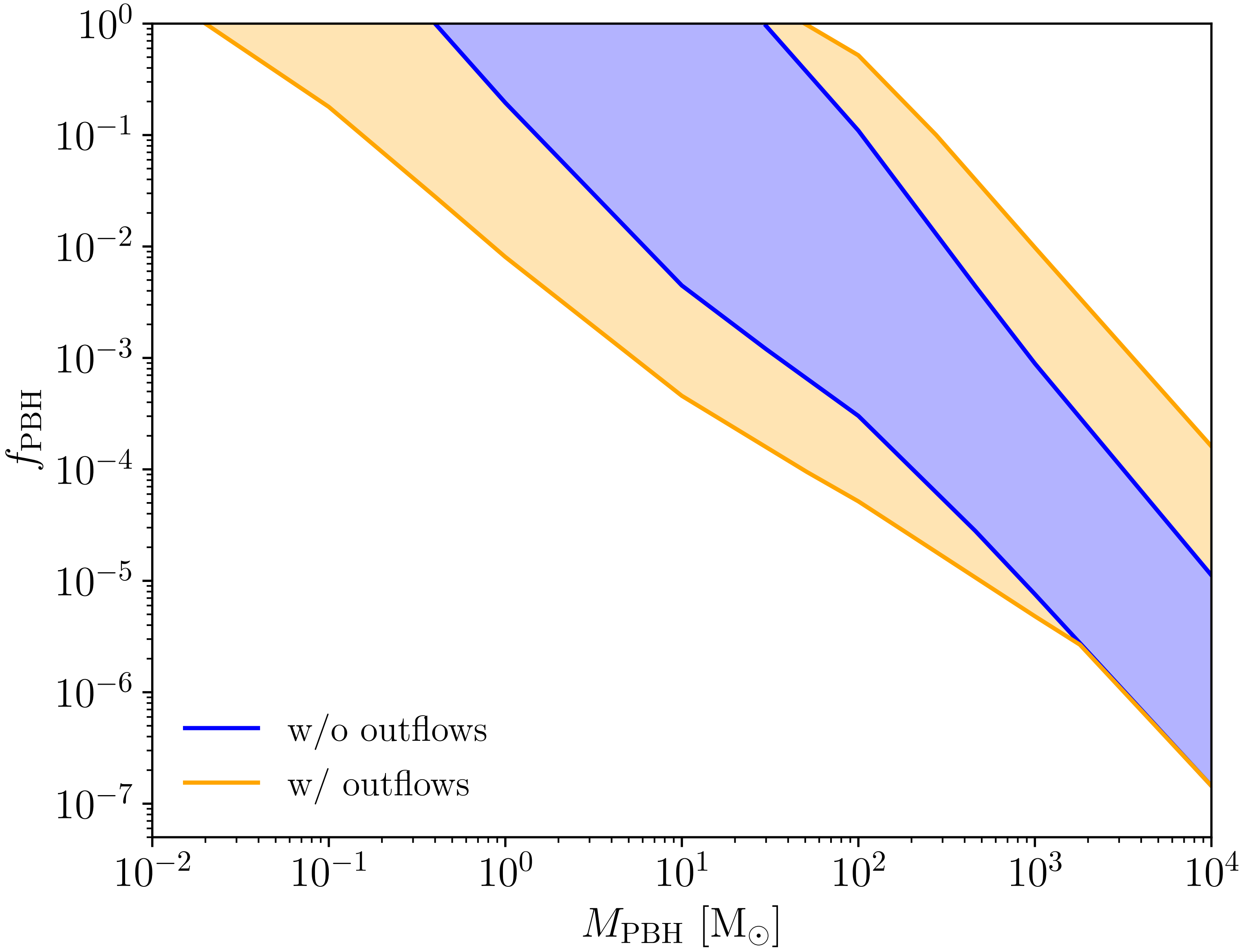
Accretion onto PBHs today constrained by

- X-ray and radio emission in MW Gaggero et al; Inoue & Kusenko; Manshanden et al.
- gas-heating in dwarf galaxies Lu et al.



uncertainty in constraint from distortion of CMB anisotropies

from geometry of accretion (spherical or disc) Poulin et al. and outflows Piga et al.



Piga et al.

constraints on asteroid mass PBHs from interactions with stars

Stars can capture asteroid mass PBHs through dynamical friction, accretion onto PBH can then destroy the star. Capela, Pshirkov & Tinyakov; Pani & Loeb; Montero-Camacho et al.

Transit of asteroid mass PBH through white dwarf heats it, due to dynamical friction, causing it to explode. Graham, Rajendran & Varela

Montero-Camacho et al. **No current constraints**, but potential future constraints from

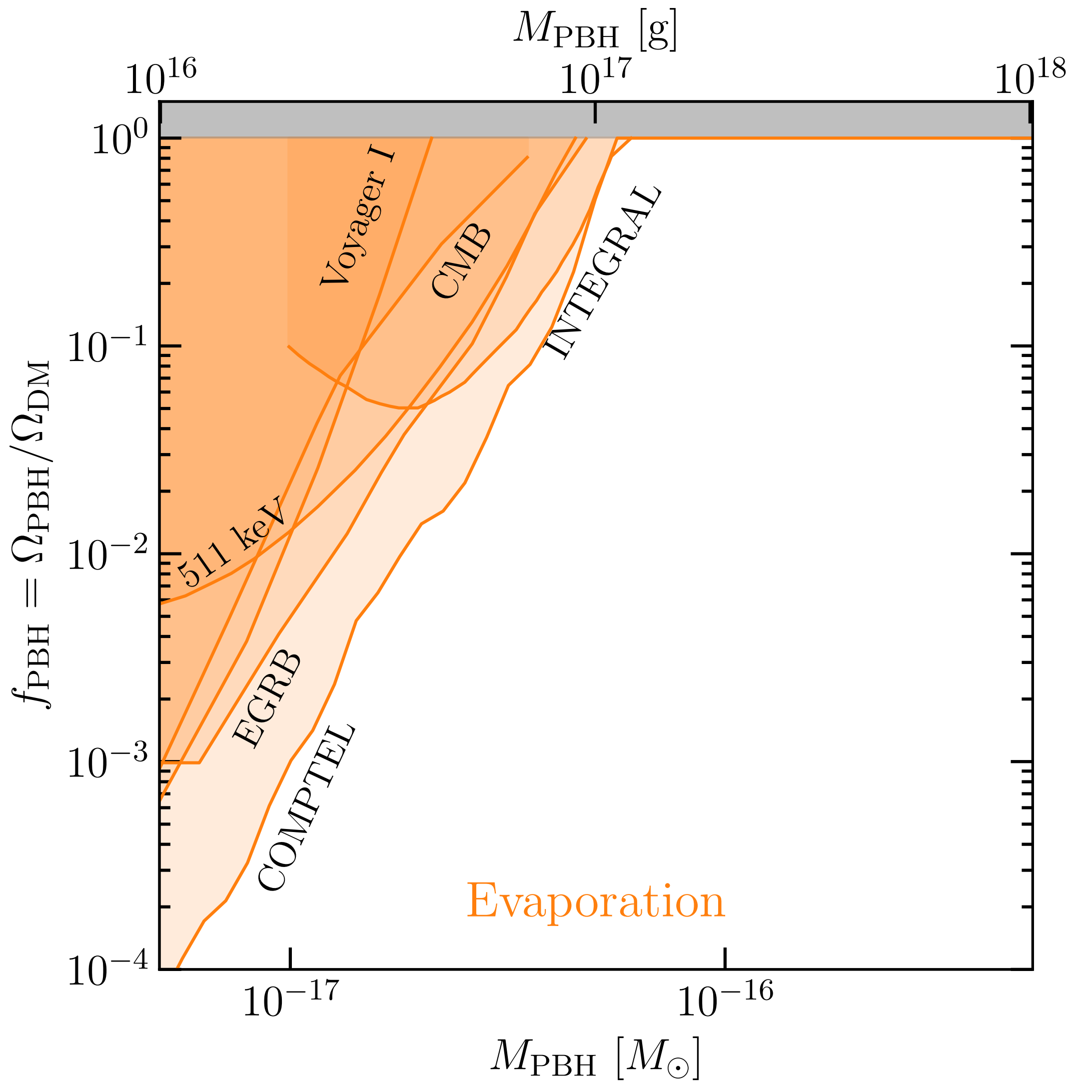
i) survival of neutron stars in globular cluster **if** it has DM halo (need high DM density, low velocity-dispersion environment),

ii) signatures of star being destroyed.

Esser & Tinyakov potential constraints from disruption of main sequence stars in dwarf galaxies, due to PBH capture during star formation.

constraints on light PBHs from evaporation products

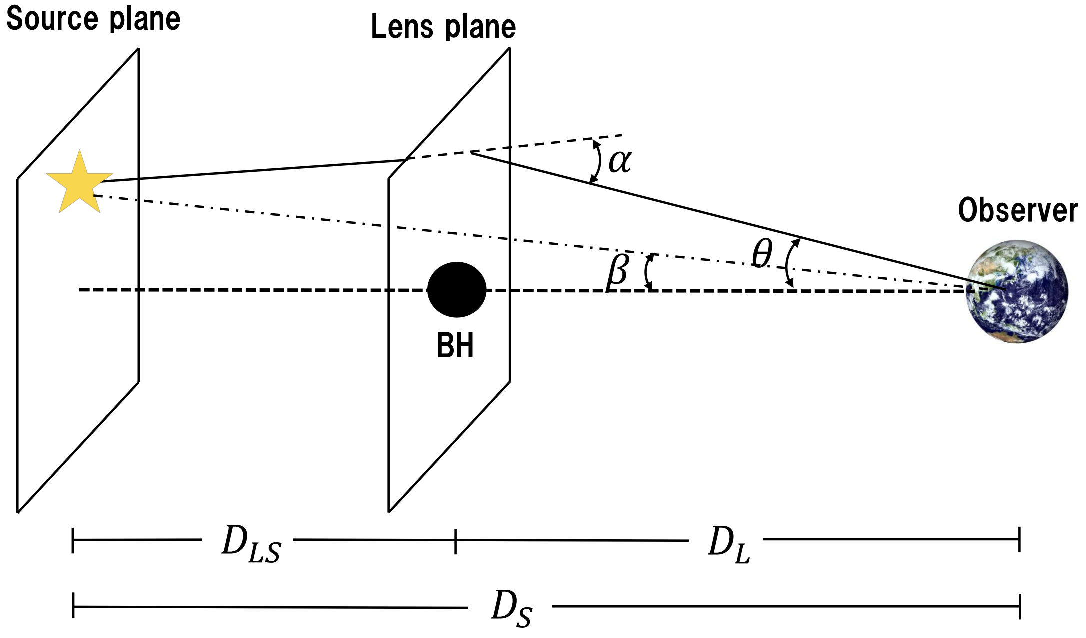
Evaporation products (gamma rays, $e^{\pm}$,…) from PBHs reaching the end of their lifetime would be detectable/have observable consequences.



See also Auffinger review.

gravitational lensing

for an intro see e.g. Sasaki et al.



$$x = \frac{D_L}{D_S}$$

Sasaki et al.

Lens equation: $$\theta D_{\rm S} = D_{\rm S}\beta + D_{\rm LS}\alpha$$

deflection $$\alpha = \frac{4GM_{\rm BH}}{D_{\rm L}\theta}$$

Lens equation on lens plane: $$r^2 - r_0 r - R_{\rm E}^2 = 0$$

$$r = D_L\theta \qquad r_0 = D_L\beta$$

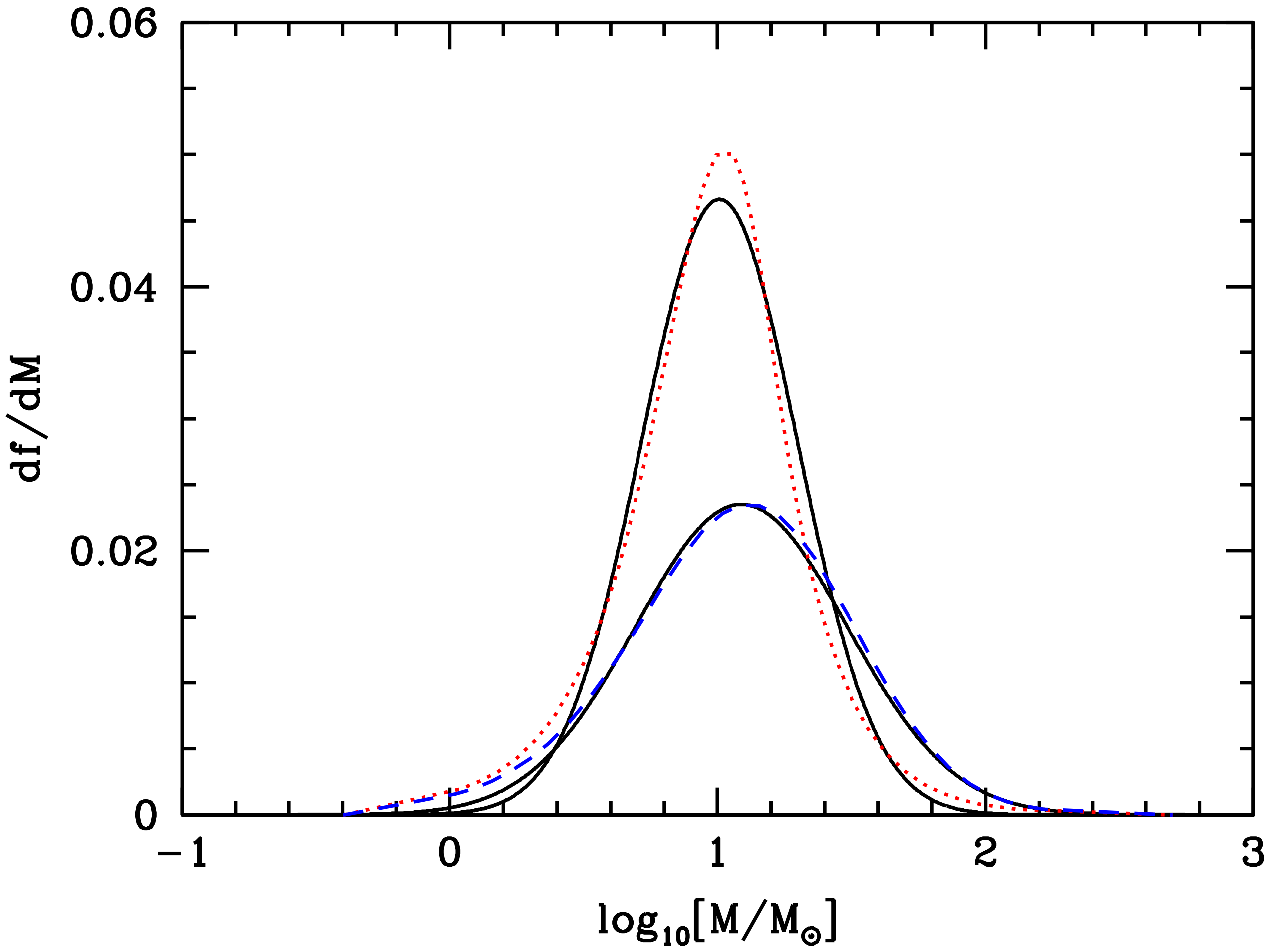
Einstein radius: $$R_E = \sqrt{\frac{4GMD_LD_{LS}}{D_S}}$$

Image positions: $$r_{1,2} = \frac{1}{2}\left(r_0 \pm \sqrt{r_0^2 + 4R_E^2}\right)$$

Angular separation: $$\Delta \sim \frac{R_E}{D_L} = 0.3\,{\rm mas}\left(\frac{M}{10\,M_\odot}\right)^{1/2}\left(\frac{D_S}{100\,{\rm kpc}}\right)^{-1/2}\sqrt{\frac{1-x}{x}}$$

Extended MFs produced by broad peak in power spectrum, moderately well approximated by a **log-normal distribution**: Green; Kannike et al.

$$M \frac{\mathrm{d}n}{\mathrm{d}M} \propto \exp\left\{ -\frac{\left[\log\left(M/M_{\mathrm{c}}\right)\right]^2}{2\sigma^2} \right\}$$



axion-like curvaton

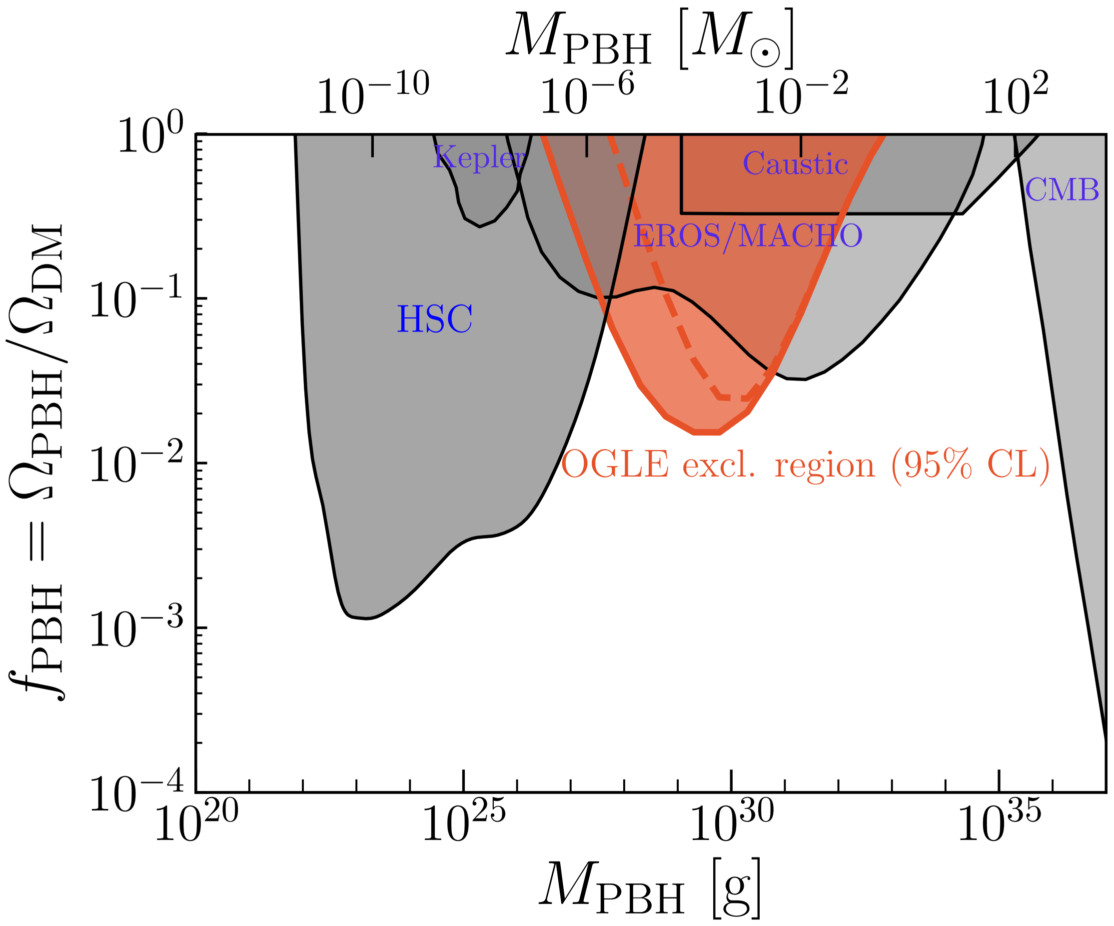
running mass inflation

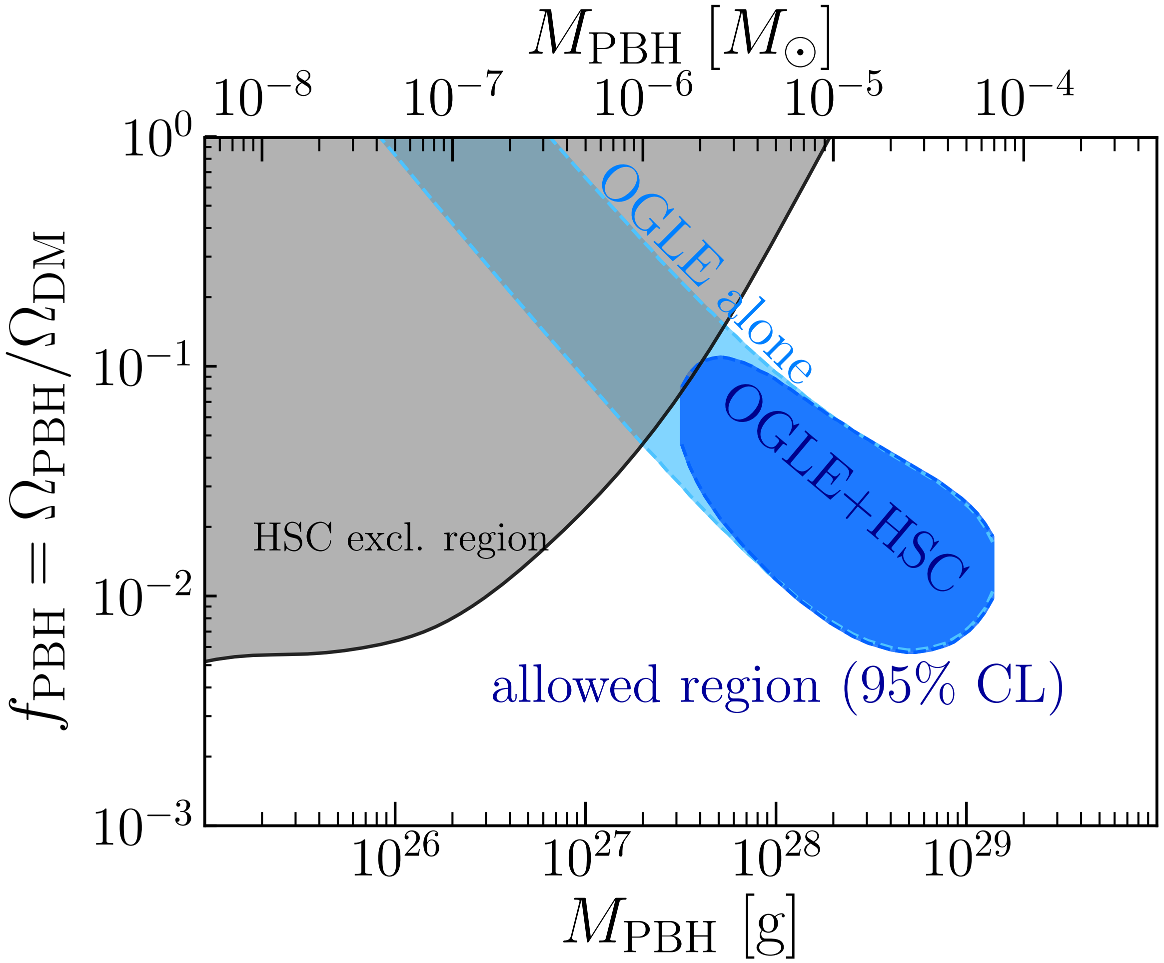
stars in Galactic bulge

Observed events consistent with expectations from stars (except for 6 ultra-short (0.1-0.3) day events)

Exclusion limit
assuming no PBH lensing observed

Allowed region
assuming 6 ultra-short events are due to PBHs

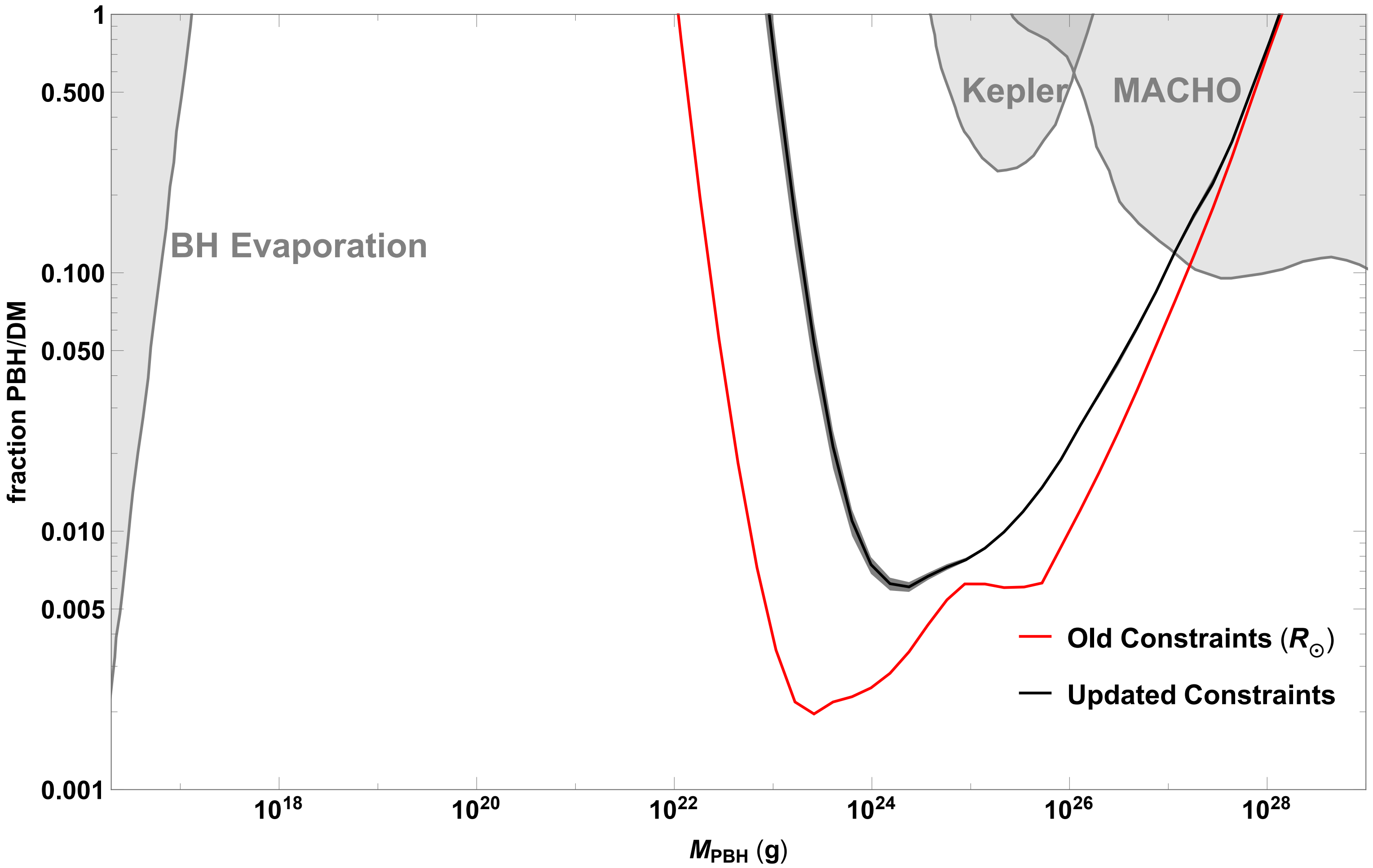




Niikura et al.

stars in M31

Subaru HSC observations have higher cadence than EROS/MACHO, so sensitive to shorter duration events and hence lighter compact objects. Niikura et al.



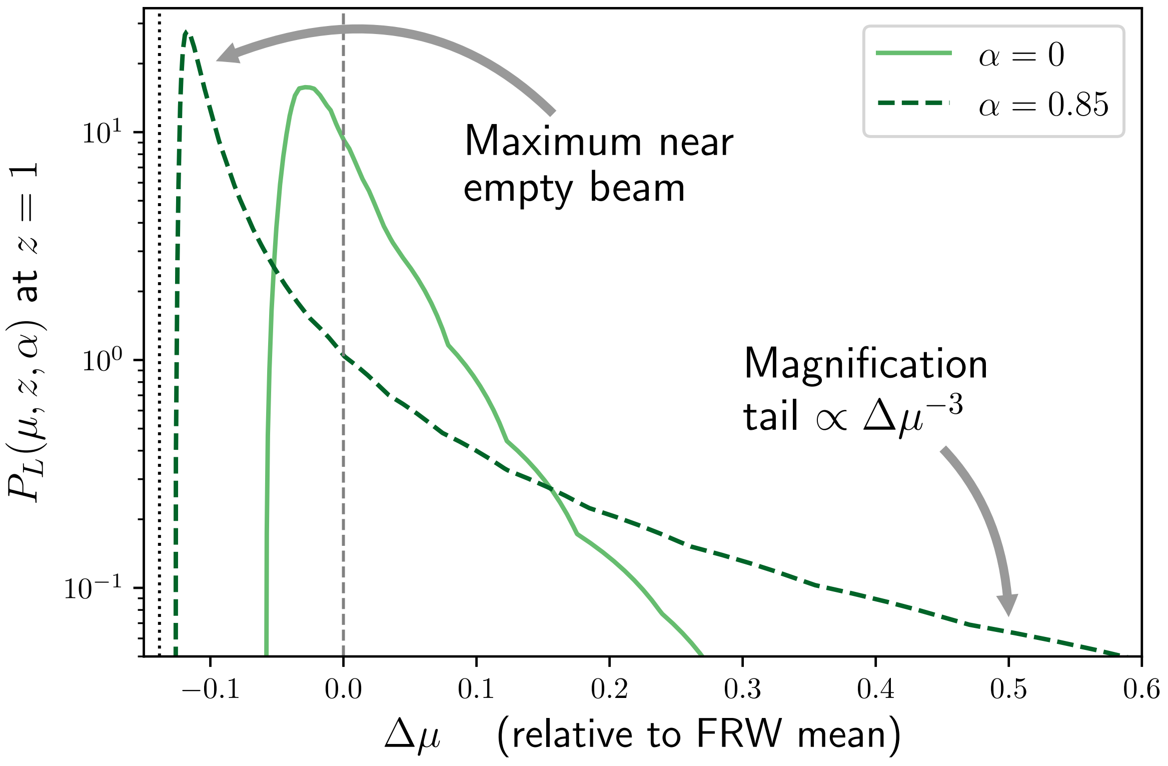
Smyth et al.

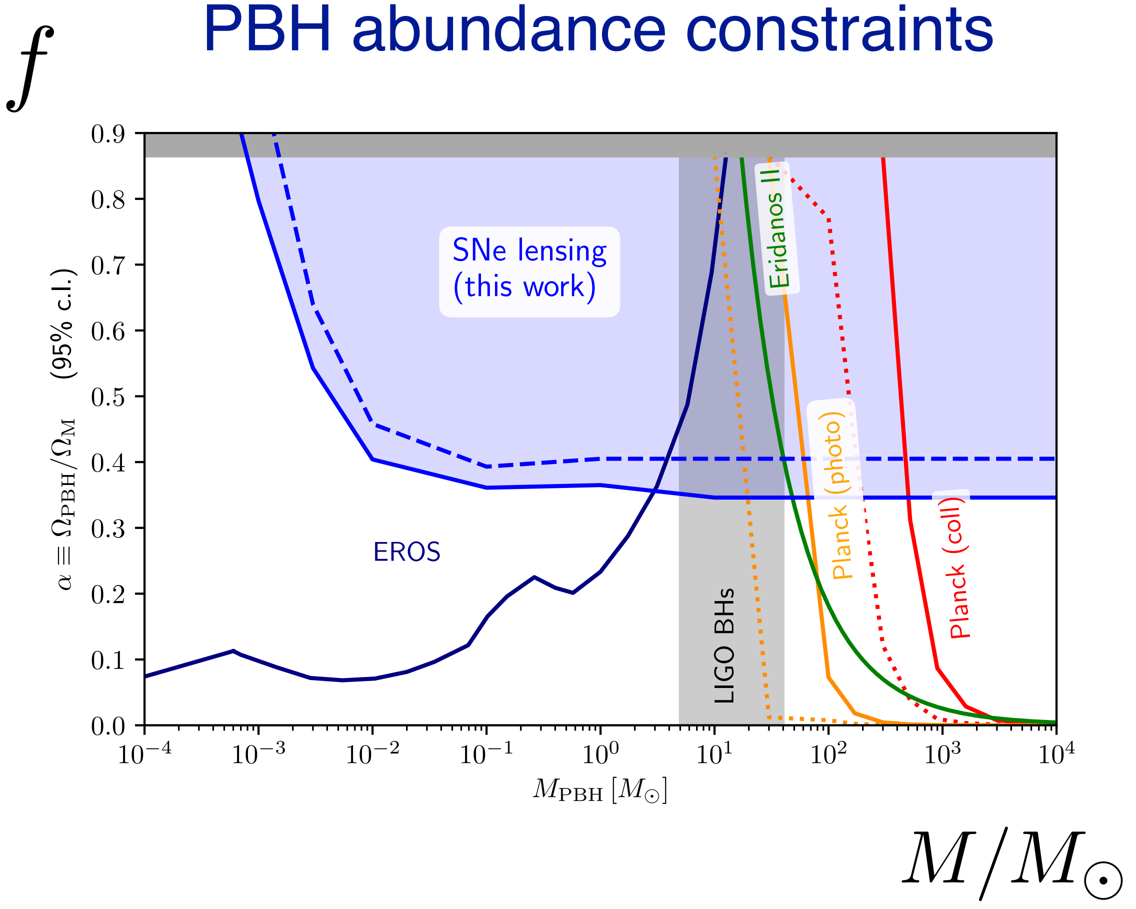
Finite size of source stars and effects of wave optics (Schwarzschild radius of BH comparable to wavelength of light) leads to reduction in maximum magnification for $M \lesssim 10^{-7} M_\odot$ and $M \lesssim 10^{-11} M_\odot$ respectively. Witt & Mao; Gould; Nakamura; Sugiyama, Kurita & Takada

And only large stars are bright enough for microlensing to be observed. Montero-Camacho et al.; Smyth et al.

supernova microlensing

Lensing magnification distribution of type 1a SNe affected (most lines of sight are demagnified relative to mean, plus long-tail of high magnifications): Zumalacarregui & Seljak

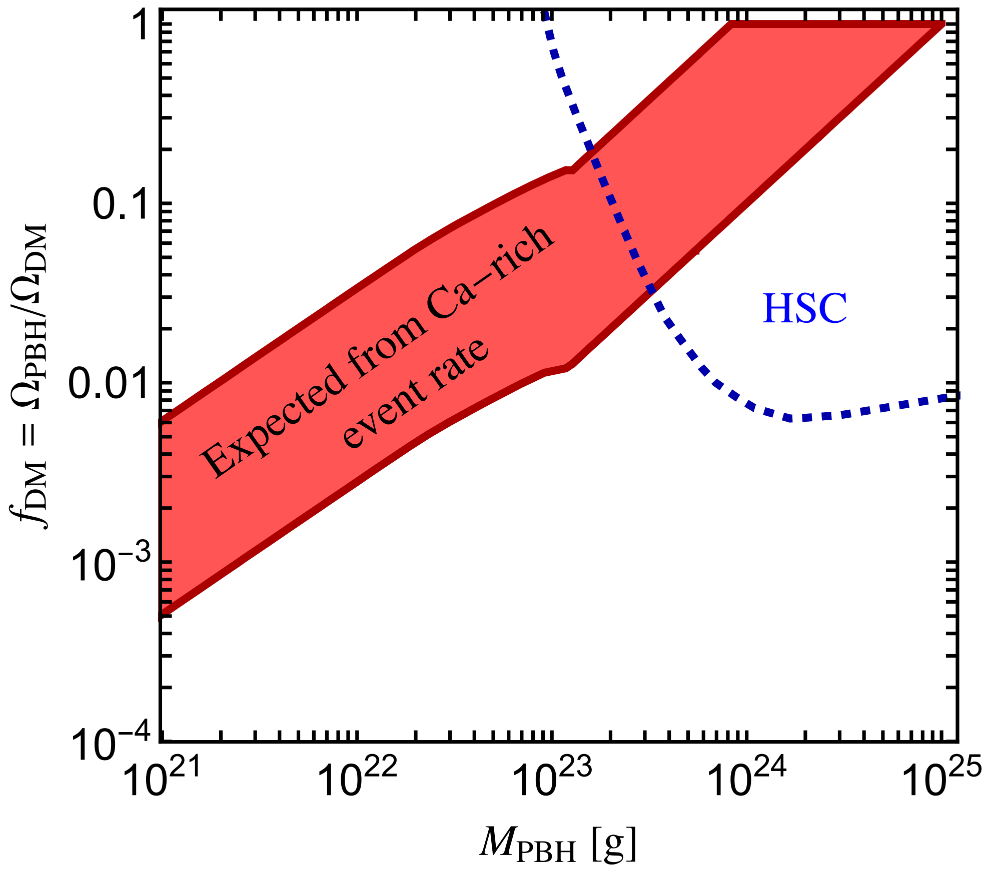

magnification distribution


PBH abundance constraints

Garcia-Bellido, Clesse & Fleury argue priors on cosmological parameters are overly restrictive and physical size of supernovae have been underestimated.

Transit of asteroid mass PBH through white dwarf heats it, due to dynamical friction, causing it to explode. Graham, Rajendran & Varela.

Population of faint, Calcium-rich supernovae mostly located at large distances from centre of host galaxy, could be due to PBHs interacting with low mass white dwarfs in dwarf galaxies??
Smirnov et al.



Smirnov et al.

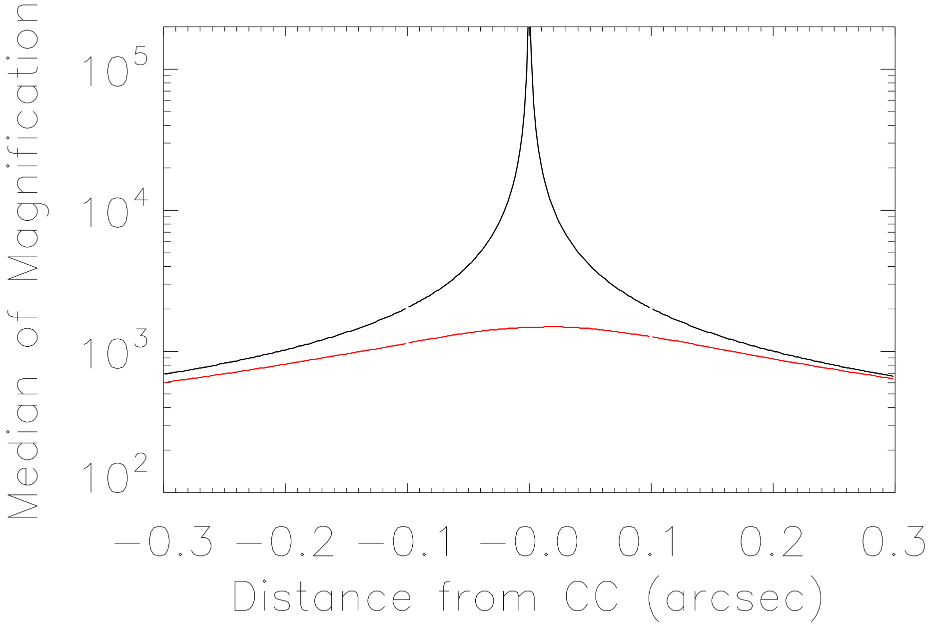
But observational signature of PBH-induced white dwarf explosion not yet reliably calculated. Montero-Camacho et al.

Icarus

When a distant star crosses a galaxy cluster caustic get huge magnification which can be increased by microlensing by compact objects (stars, black holes,..) in cluster. Miralda-Escude.

However if large fraction of DM is in compact objects magnification is reduced.

magnification Kelly et al.



smooth DM, compact DM

Icarus is first (serendipitously) observed event involving a star at red-shift 1.5. Kelly et al.

Constraint from Icarus: $f < 0.08$ (but factor of 2 uncertainty in transverse velocity leads to similar uncertainty on f). Oguri et al.

constraints on light PBHs from evaporation products

Extragalactic gamma-rays background (EGRET/Fermi) Carr, Kohri, Sendouda & Yokoyama

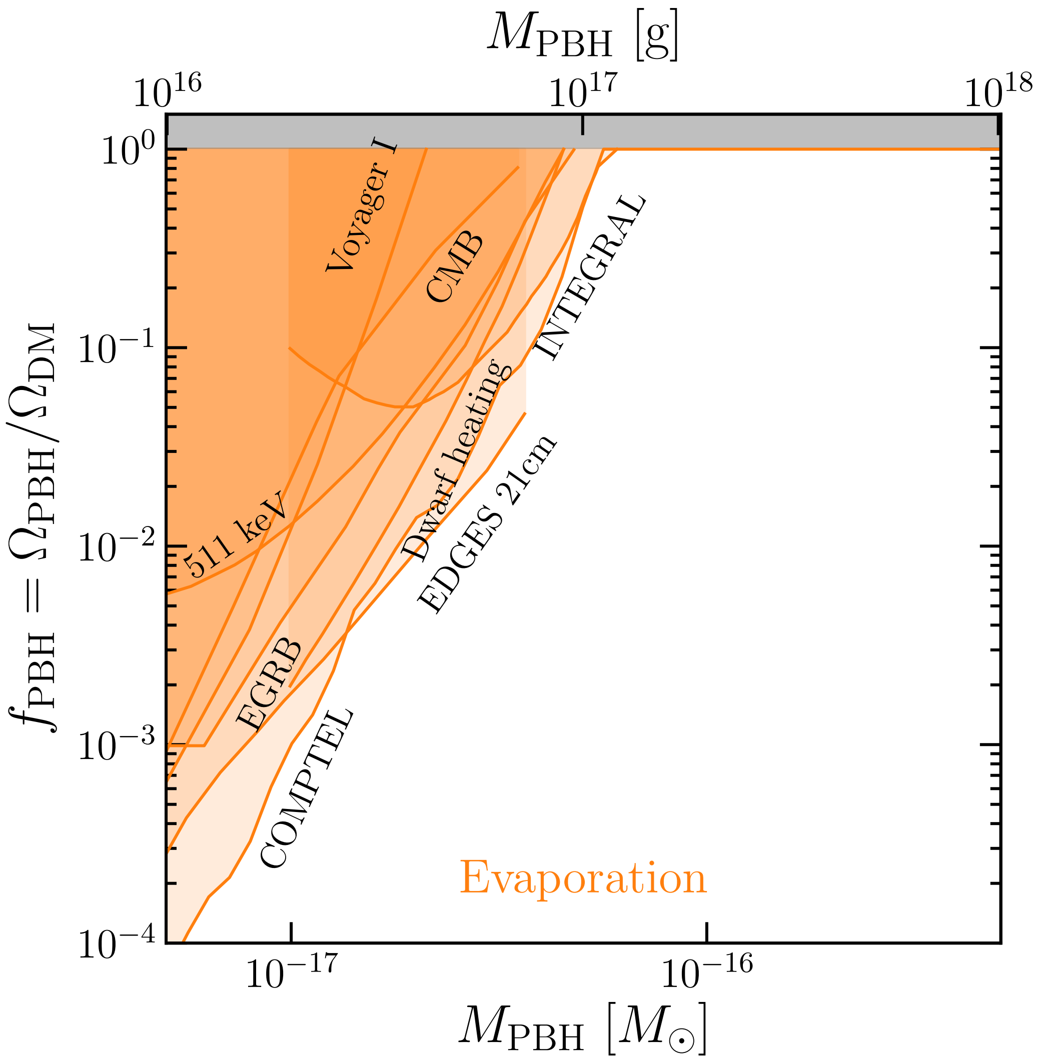
MeV galactic diffuse flux (INTEGRAL) Laha, Munoz & Slatyer (COMPTEL) Coogan, Morrison & Profumo

damping of CMB anisotropies during recombination (Planck) Poulin et al.; Clark et al.

$e^{\pm}$ flux (Voyager 1) Boudaud & Cirelli

511 keV line from $e^{\pm}$ annihilation (INTEGRAL) DeRocco & Graham; Laha

heating of ISM in dwarf galaxy Kim



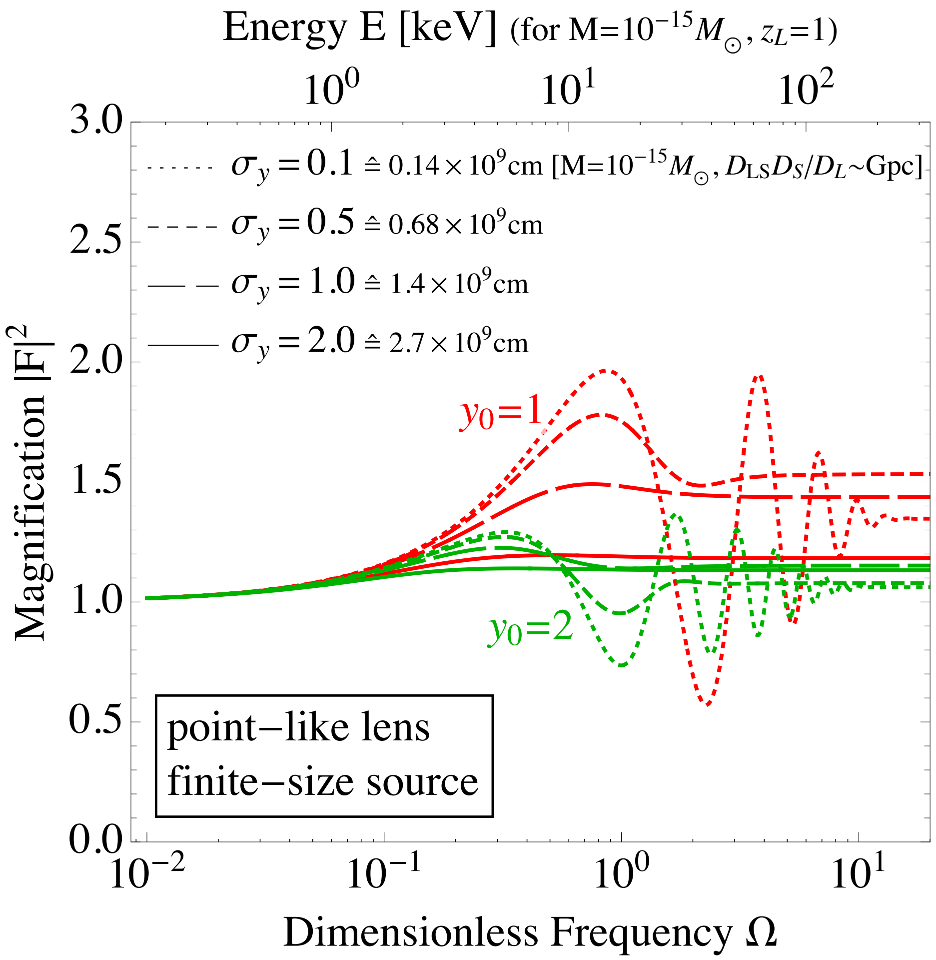
how to constrain asteroid mass PBHs??

Femtolensing of GRBs

Different path lengths lead to phase differences, and hence interference fringes in energy spectrum of lensed GRBs. Gould

Barnacka, Glickenstein & Moderski constraints from Fermi Gamma Ray Burst monitor.

BUT Katz, Kopp, Sibiryakov, Xue most GRBs not point-like, and (less significantly) geometric optics approximation also breaks down:

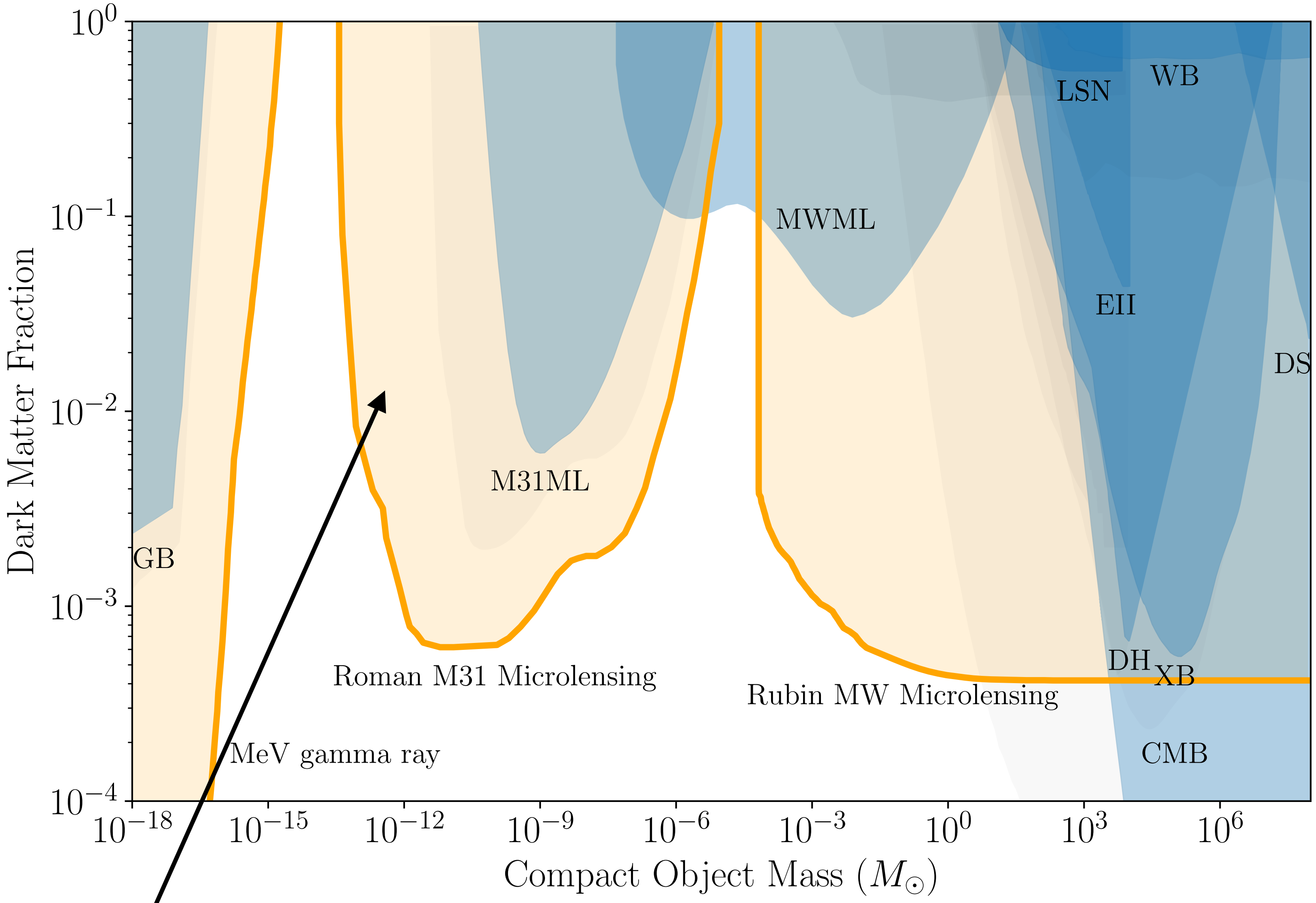


increasing angular size of source

y0 = position of source centre in terms of β/θ_E

Constraints could be achieved in a future with a sample of GRBs with well-measured red-shift and spectra, and small size (which is expected to correspond to sub-milli-second variability).

Future constraints



Bird et al. (Snowmass PBH white paper)

But for $M \lesssim 10^{-12} M_\odot$ microlensing amplification reduced due to:

i) finite source size

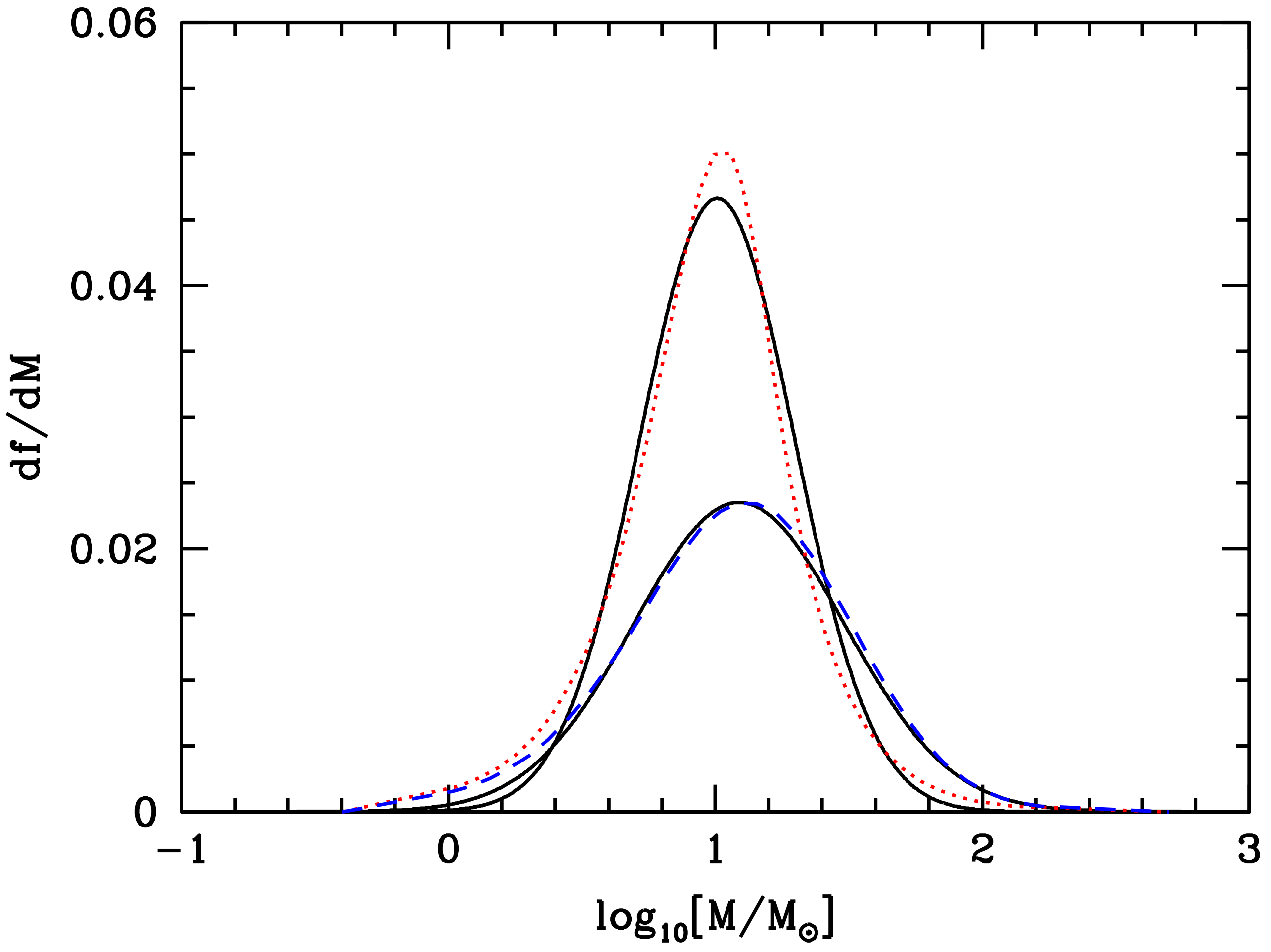
ii) wave optics (wavelength of light similar to Schwarzschild radius of PBH).

Sugiyama et al. and references therein.

(expected) extended mass functions

Extended MFs produced by broad peak in power spectrum, moderately well approximated by a **log-normal distribution**: Green; Kannike et al.

$$M \frac{\mathrm{d}n}{\mathrm{d}M} \propto \exp\left\{ -\frac{\left[\log\left(M/M_{\mathrm{c}}\right)\right]^2}{2\sigma^2} \right\}$$



axion-like curvaton

running mass inflation

Method for applying delta-function constraints to extended mass functions:

Carr, Raidal, Tenkanen, Vaskonen& Veermae, see also Bellomo, Bernal, Raccanelli & Verde:

If $f_{max}(M)$ is the maximum allowed PBH fraction for a delta-function MF, an extended mass function $\psi(M)$ has to satisfy:

$$\int \mathrm{d}M \, \frac{\psi(M)}{f_{\max}(M)} \leq 1$$

Probing origin of BH binaries using their spins

Farr, Holtz & Farr;… Fernandez & Profumo

Dimensionless spin of individual BH: $$\chi = \frac{|\mathbf{S}|}{GM^2}$$

Effective spin parameter: $$\chi_{\rm eff} = \frac{M_1\chi_1\cos\theta_1 + M_2\chi_2\cos\theta_2}{M_1 + M_2}$$

θ_i=tilt angle between $\mathbf{S}_i$ and orbital AM **L**

Astrophysical BH binaries:

i) formed in dense stellar environments, spins uncorrelated with orbit: $\chi_{\rm eff} \approx 0$

ii) formed in isolation, spins generally aligned with orbital AM: $\chi_{\rm eff} \approx 1$

Primordial BH binaries:

small intrinsic spins, $\chi_i \approx 0$ → $\chi_{\rm eff} \approx 0$

de Luca et al.

Structure formation with PBH dark matter

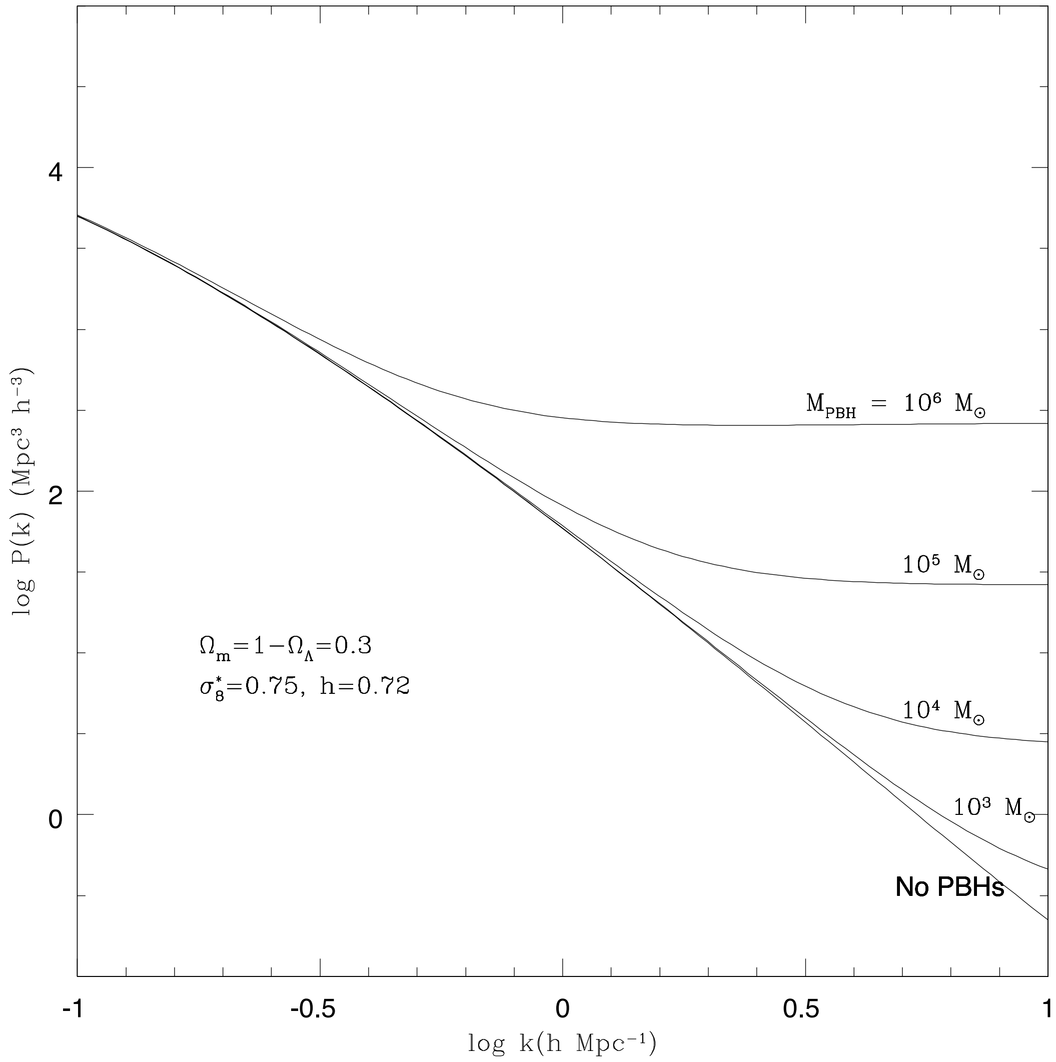
PBHs don't form in clusters Ali-Haïmoud (previous work Chisholm extrapolated an expression for the correlation function beyond its range of validity).

But if PBHs make up a large fraction of the DM, PBH clusters form shortly after matter-radiation equality. Afshordi, Macdonald & Spergel; Raidal et al.; Inman & Ali-Haïmoud; Jedamzik

power spectrum

$P(k)$

$(\propto k^{n_s})$



increasing PBH mass

no PBHs

k = comoving wavenumber

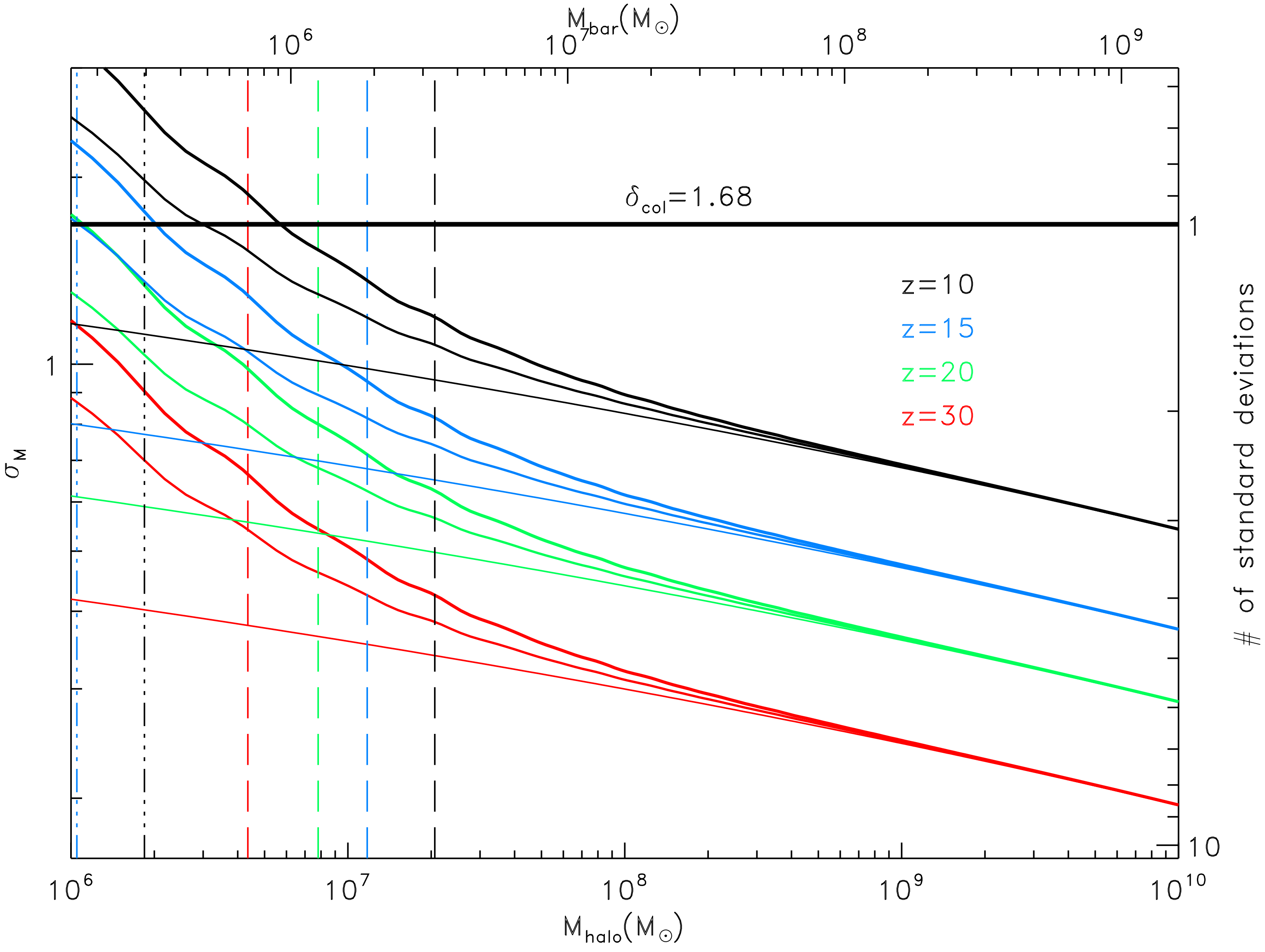
Afshordi, Macdonald & Spergel

Clustering of PBHs formed from collapse of large density perturbations

PBHs don’t form in clusters Ali-Haïmoud (previous work Chisholm extrapolated an expression for the correlation function beyond its range of validity).

However there are additional isocurvature perturbations (due to Poisson fluctuations in PBH distribution) and PBH clusters form shortly after matter-radiation equality.
Afshordi, Macdonald & Spergel; Inman & Ali-Haïmoud; Jedamzik

mass variance



threshold for collapse

z= 10, 15, 20, 30

— $M_{PBH}=30\ M_\odot$

— $M_{PBH}=15\ M_\odot$

— no PBHs

halo mass

Kashlinksy

Approximate analytic calculation

c.f. Afshordi, Macdonald & Spergel; Jedamzik

PBH DM has additional isocurvature perturbations due to Poisson fluctuations in their distribution:

$$\delta(N) = \frac{\Delta N}{N} = \frac{1}{\sqrt{N}}$$

growth factor for isocurvature perturbations:

$$D(a) \approx \left(1 + \frac{3}{2}\frac{a}{a_{\rm eq}}\right)$$

spherical top hat collapse:

collapse occurs when:

$$D(a_{\rm col})\delta(N) = \delta_{\rm critical} \approx 1.69$$

final halo/cluster density:

$$\rho_{\rm cl} \approx 178\rho_{\rm DM}(a_{\rm coll})$$

radius of cluster:

$$r_{\rm cl} \approx 0.01 \left(\frac{M_{\rm PBH}}{M_\odot}\right)^{1/3} N^{5/6}\,{\rm pc}$$

For $M_{\rm PBH} = M_\odot$, N=10 (100) clusters form at $z_{\rm coll}$ ≈1200 (320) and have $r_{\rm cl}$≈ 0.06 (0.5) pc.

N-body simulations

Inman & Ali-Haïmoud

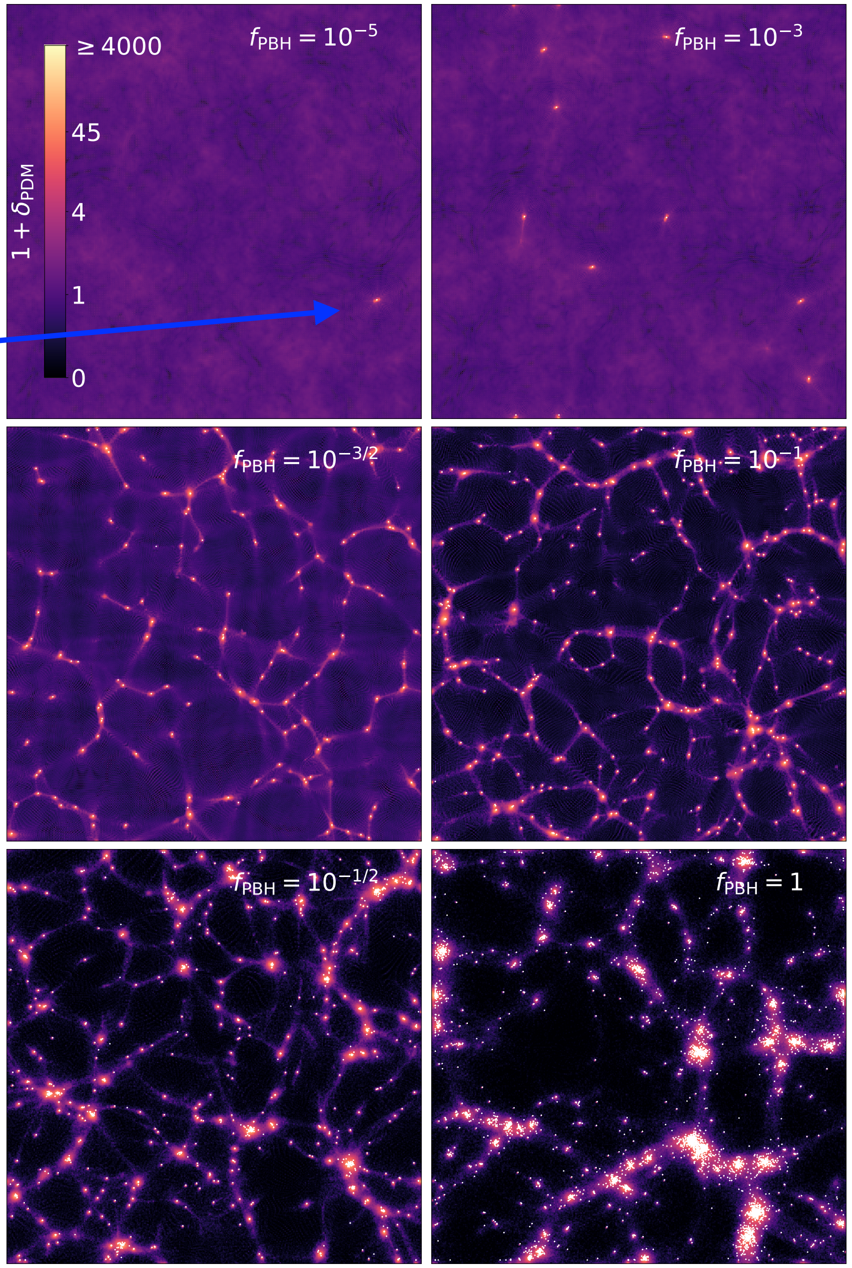
Simulate a L = 30 h^{-1} kpc box, with $M_{\rm PBH} = 20h^{-1}M_\odot$ from radiation domination to z = 99, for $f_{\rm PBH} = 1$ and also $f_{\rm PBH} < 1$ + particle dark matter.

matter field at z=100

$f_{\rm PBH} = 10^{-5}$

$f_{\rm PBH} = 10^{-3}$

a PBH!



$f_{\rm PBH} = 10^{-3/2}$

$f_{\rm PBH} = 10^{-1}$

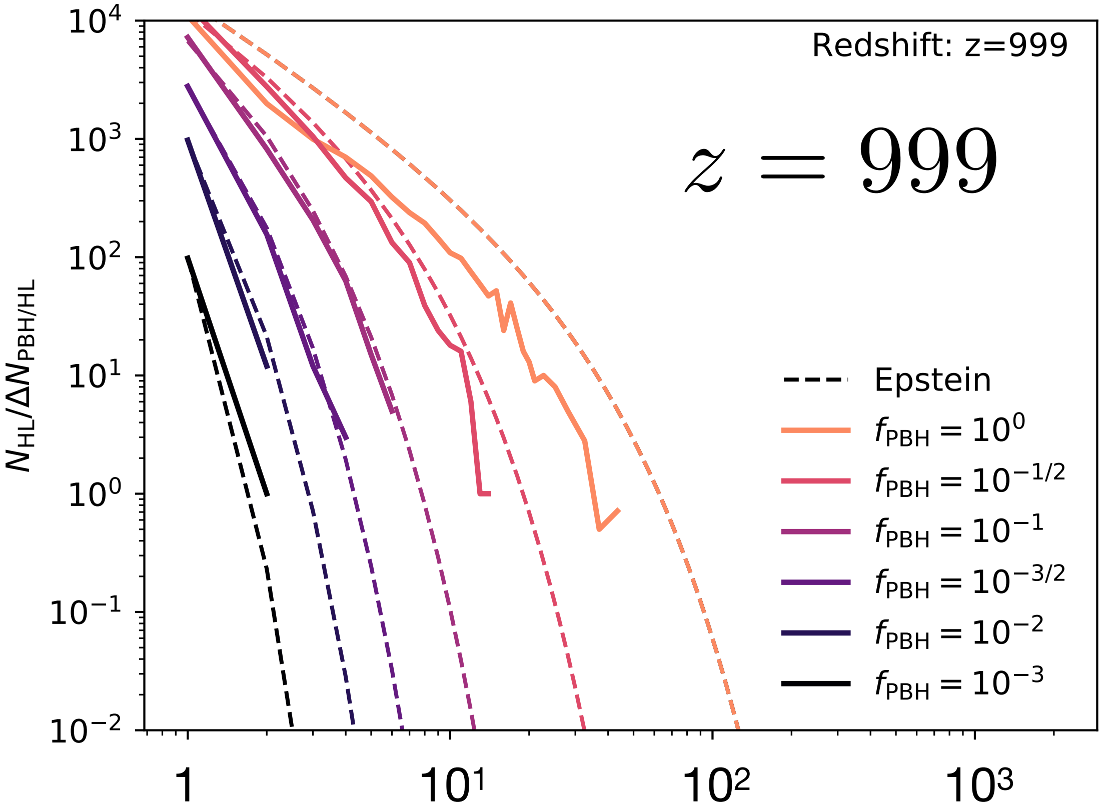
$f_{\rm PBH} = 10^{-1/2}$

$f_{\rm PBH} = 1$

Inman & Ali-Haïmoud

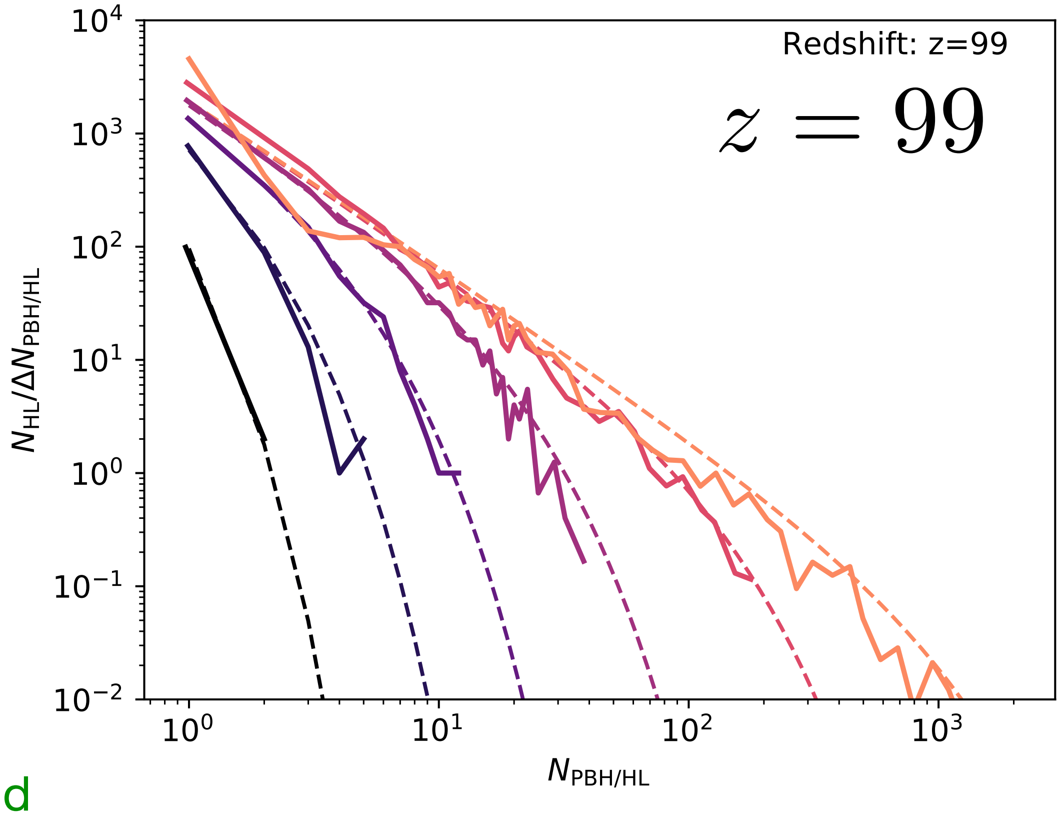
Clusters containing small numbers of PBHs always most abundant, but abundance of clusters containing large numbers of PBHs increases with time.

halo mass function (number of halos containing a given number of PBHs)



decreasing $f_{\rm PBH}$

Inman & Ali-Haïmoud



Evolution of PBH clusters (and in particular PBH binaries) through to the present day is a challenging open problem. e.g. Jedamzik; Trashorras et al….

Clusters containing $\lesssim 10^3$ PBHs will evaporate by present day. Afshordi, Macdonald & Spergel; Jedamzik

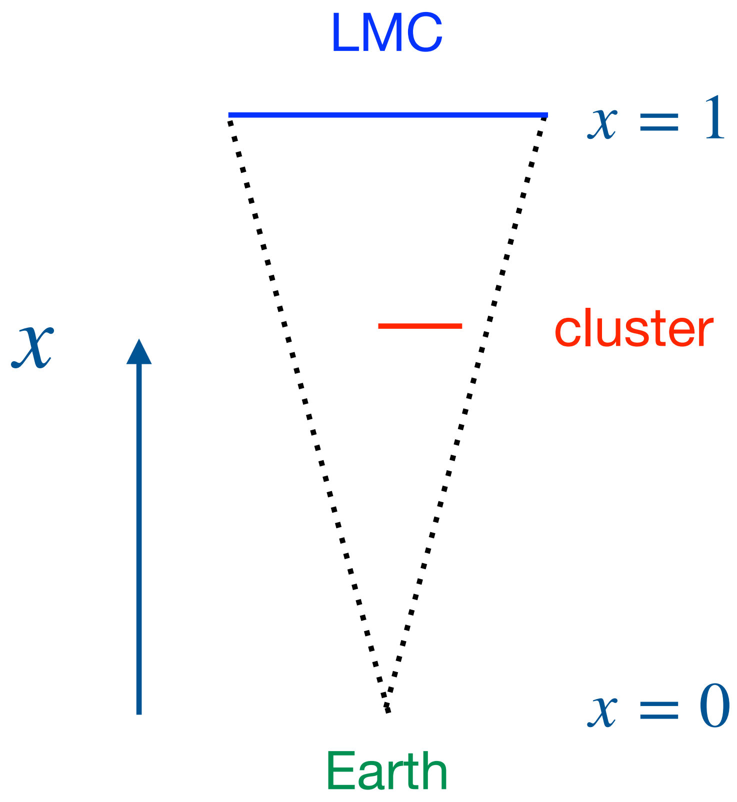
Effect of clustering on LMC microlensing constraints

Gorton & Green (see also Petaĉ, Lavalle & Jedamzik)

For PBHs formed from collapse of density perturbations during radiation, clusters are sufficiently extended that PBHs lens individually (separation of PBHs $\gg R_E$).

Microlensing from a single cluster:

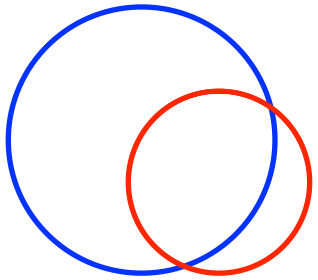
looking down on line of sight



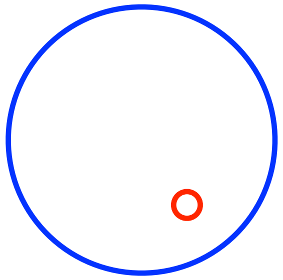
x = fractional
line of sight dist

looking along line of sight

cluster with small x



cluster with large x



probability of finding a cluster at line of sight distance x is proportional to cross sectional area of ‘cone’ to LMC $\propto x^2$

all the PBHs in a given cluster cause events with the same duration:

$$\hat{t} = \frac{2R_{\rm E}(x)}{v} \propto [x(1-x)]^{1/2}$$

rate at which cluster causes microlensing events is proportional to solid angle subtended by cluster times Einstein radius:

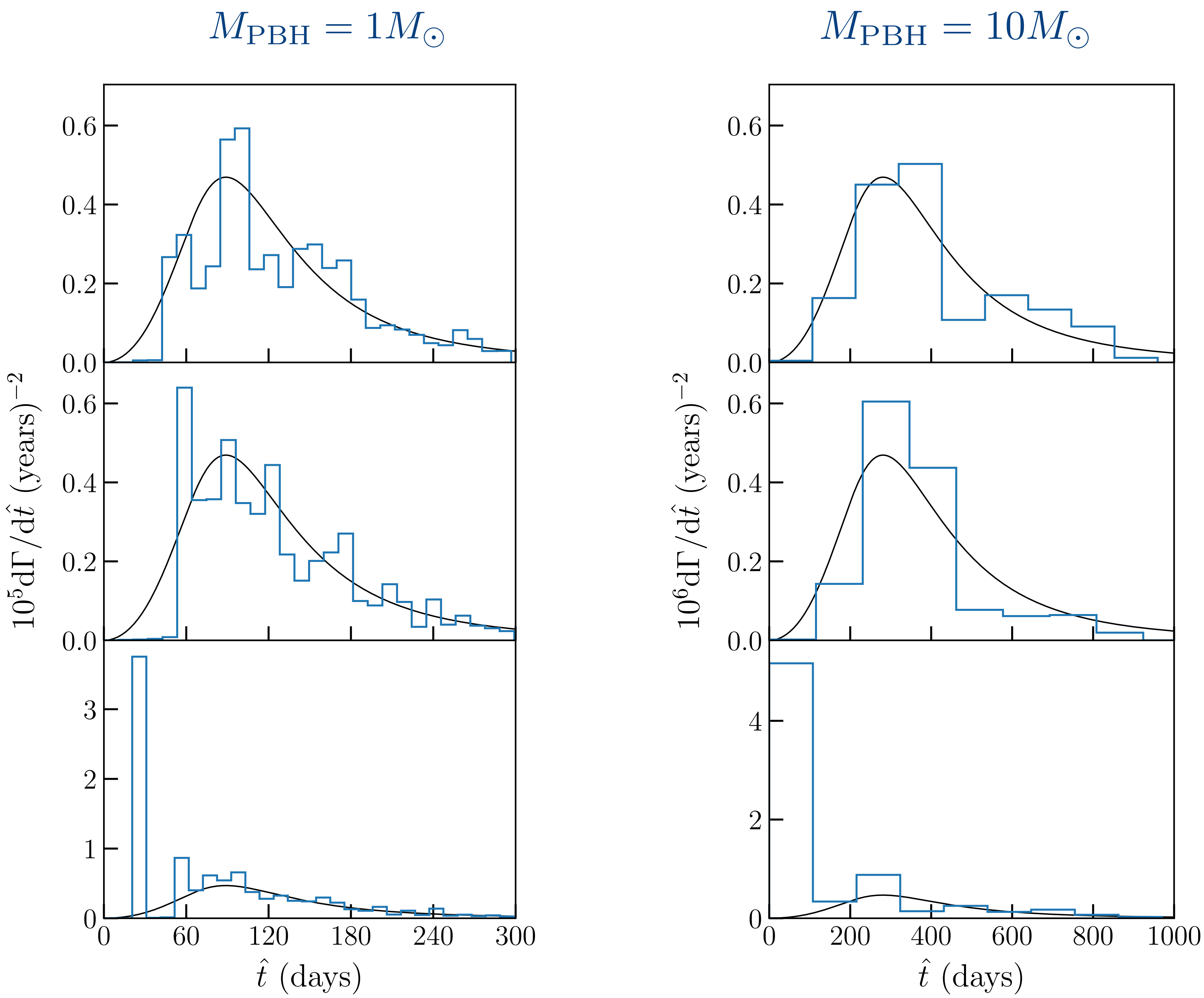
$$\propto \frac{[x(1-x)]^{1/2}}{x^2}$$

Close clusters (small x) are rare, but if one intersects the line of sight if produces short duration events at a high rate.

LMC microlensing differential event rate for clustered DM and standard smooth DM

all of the DM in clusters containing $N_{cl}=10^6$ PBHs

n.b. not realistic!



Typical realisations

No close cluster.
Deficit of short duration events.

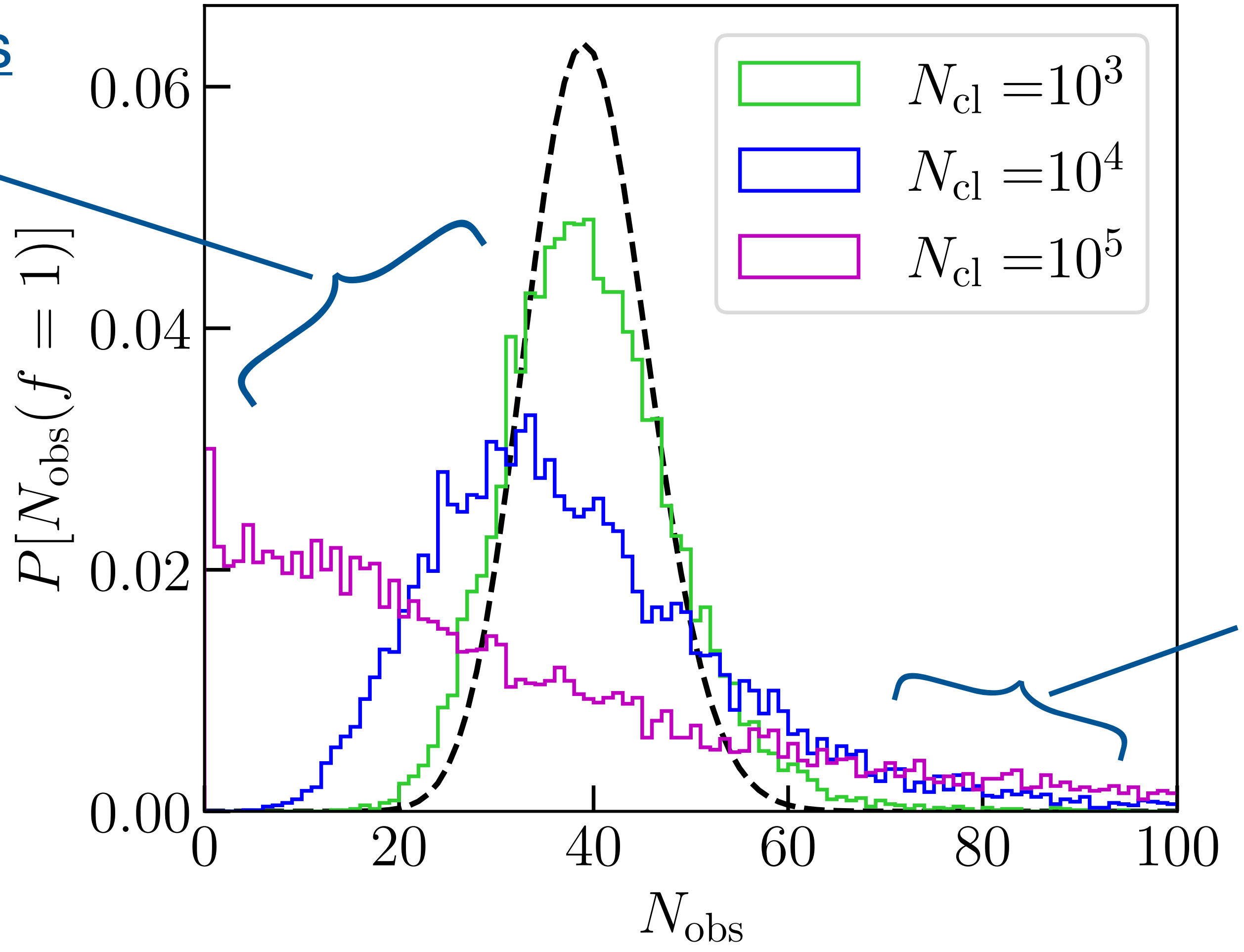
Rare realisation

Close cluster.
Excess of short duration events.

Probability distribution of number of events in a long duration microlensing survey if all of the DM is in PBHs clusters containing $N_{\rm cl}$ PBHs with mass $M_{\rm PBH} = 10^3 M_\odot$

Typical realisations

No close cluster.
Deficit of events.



Rare realisations

Close cluster.
Excess of events.

Change in constraints is negligible apart (possibly) from at largest $M_{\rm PBH}$ probed by stellar microlensing.

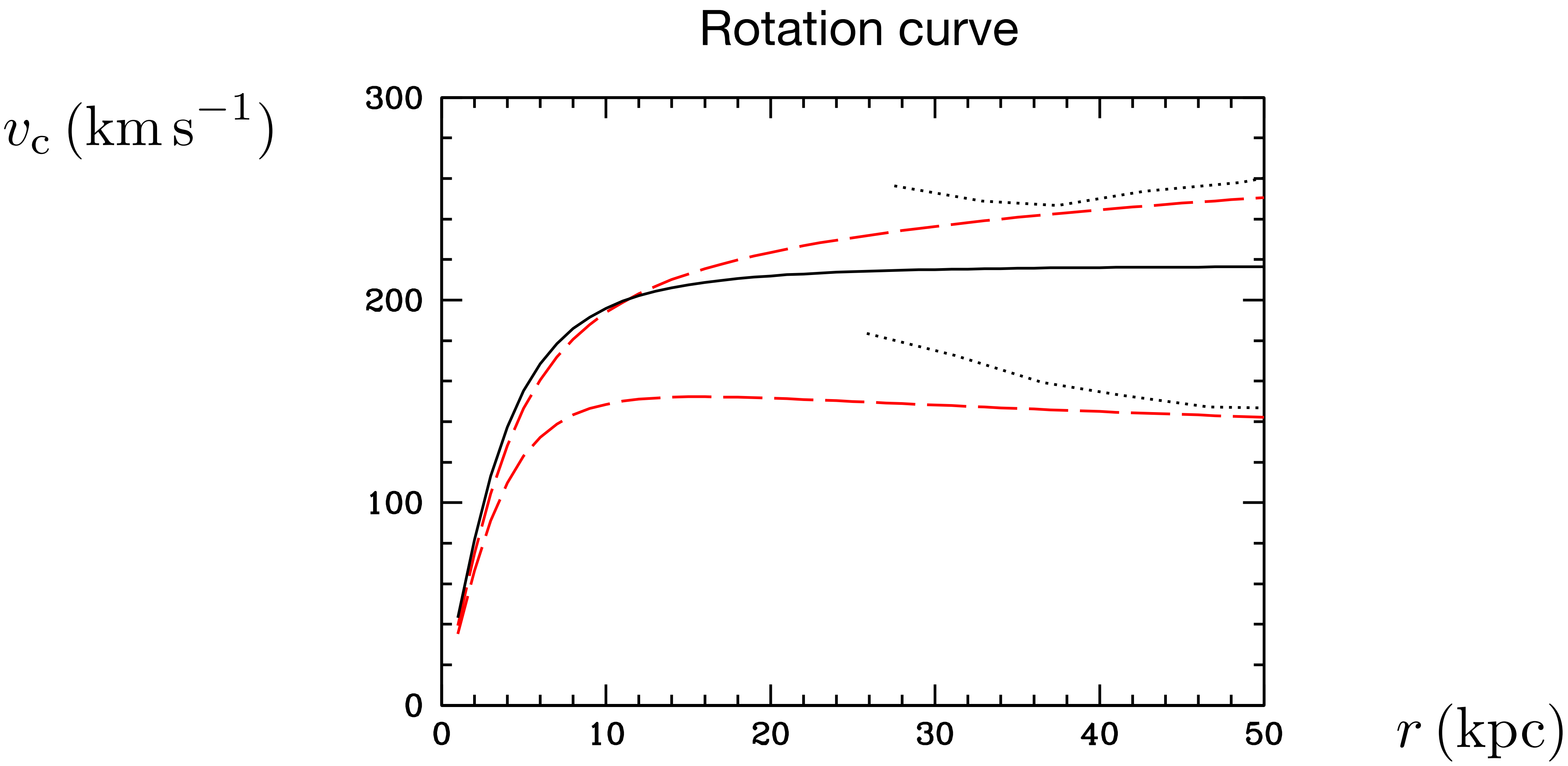
(**if** all of the DM is in PBH clusters containing $N_{\rm cl} = 10^3$ PBHs with mass $M_{\rm PBH} = 10^3 M_\odot$ constraint on $f_{\rm PBH}$ from long duration microlensing survey weakens from 0.076 to 0.096).

Petaĉ, Lavalle & Jedamzik; Gorton & Green.

Effect of MW DM distribution on constraints

Evans power law halo models: self-consistent halo models, which allow for non-flat rotation curves.

Traditionally used in microlensing studies [Alcock et al. MACHO collab.; Hawkins] since there are analytic expressions for velocity distribution.



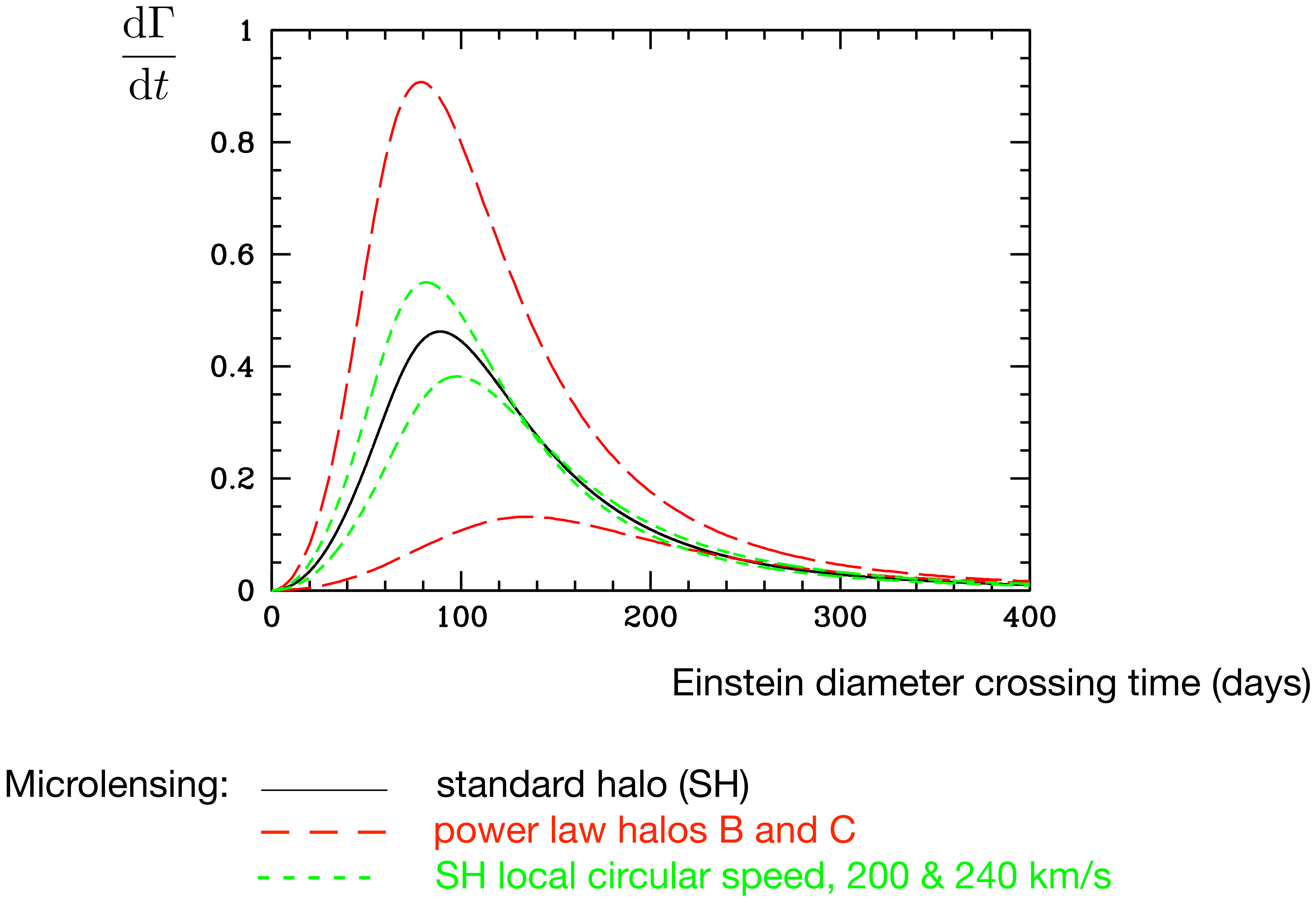
——— standard halo (SH)

— — — top: power law halo B (massive halo, rising rotation curve)
bottom: power law halo C (light halo falling rotation curve)

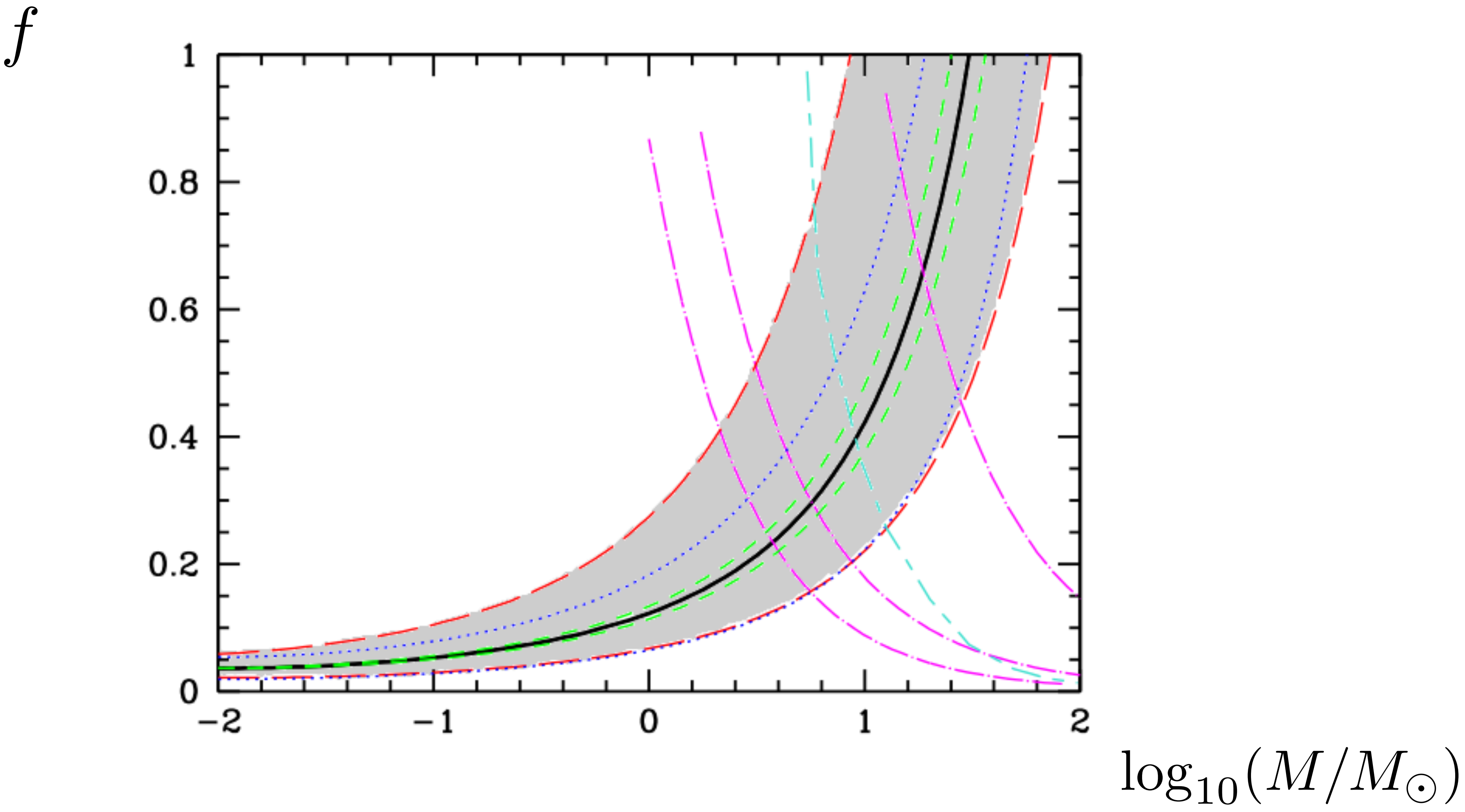
……….. envelope of MW rotation curve data [Bhattacharjee et al.]

Microlensing differential event rate

(f=1 M= 1 $M_\odot$, and perfect detection efficiency)



EROS constraints on halo fraction for delta-function MF



—— standard halo (SH)

- - - - - SH local circular speed, 200 & 240 km/s

………. SH local density, 0.005 and 0.015 $M_\odot\,\mathrm{pc}^{-3}$

— — — power law halos C and B

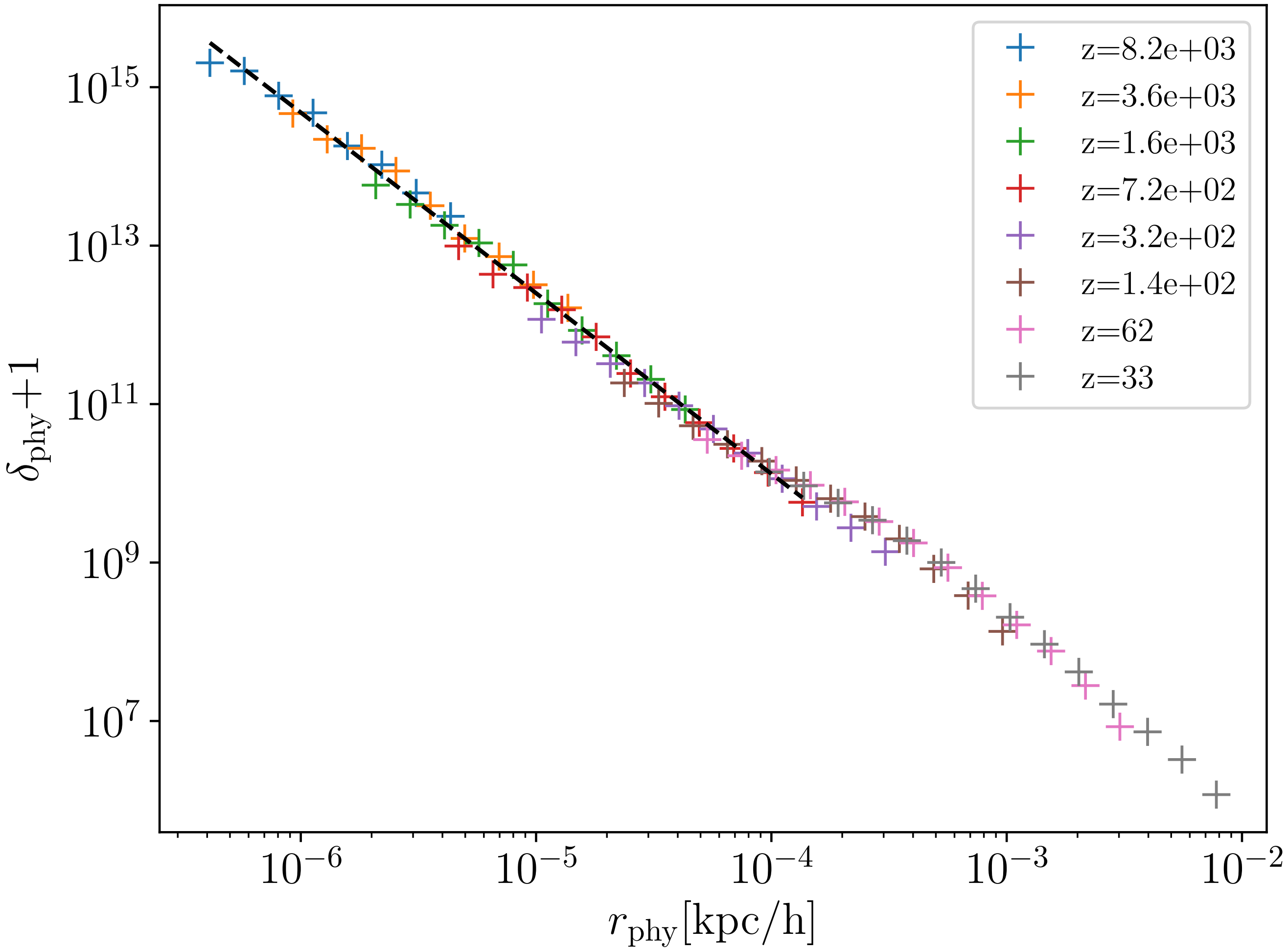
—— Brandt dwarf galaxy constraints

mixed PBH-particle dark matter

If PBHs don't make up all of the DM ($0 < f_{\rm PBH} < 1$) then isolated PBHs accrete a halo of particle DM with a steep density profile: $\rho(r) \propto r^{-9/4}$

Mack, Ostriker & Ricotti; Adamek et al.; Inman & Ali-Haïmoud

Density profile, in physical units, formed around a $30 M_{\odot}$ PBH



Adamek et al