



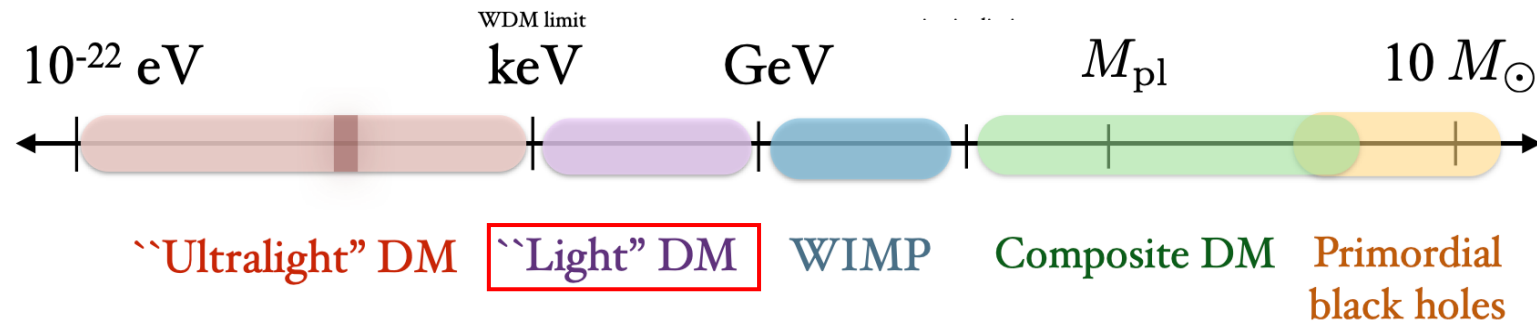
# Detection of Light Dark Matter

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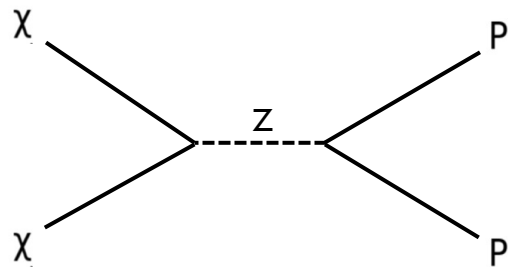
Louis Hamaide – Dark Matter Beyond The Weak Scale 03/24

# Motivating Searches of Light(er) Dark Matter

- Experiments show we need to add new *dark, stable, and relatively collisionless* particle (dark matter) to our description of physics. Large mass range available to explore experiments and theory



- Lee & Weinberg ('77) assumed weak interaction-generated (thermal, neutrino-like) WIMPs to set  $\sim 2\text{GeV}$  bound on WIMP mass:



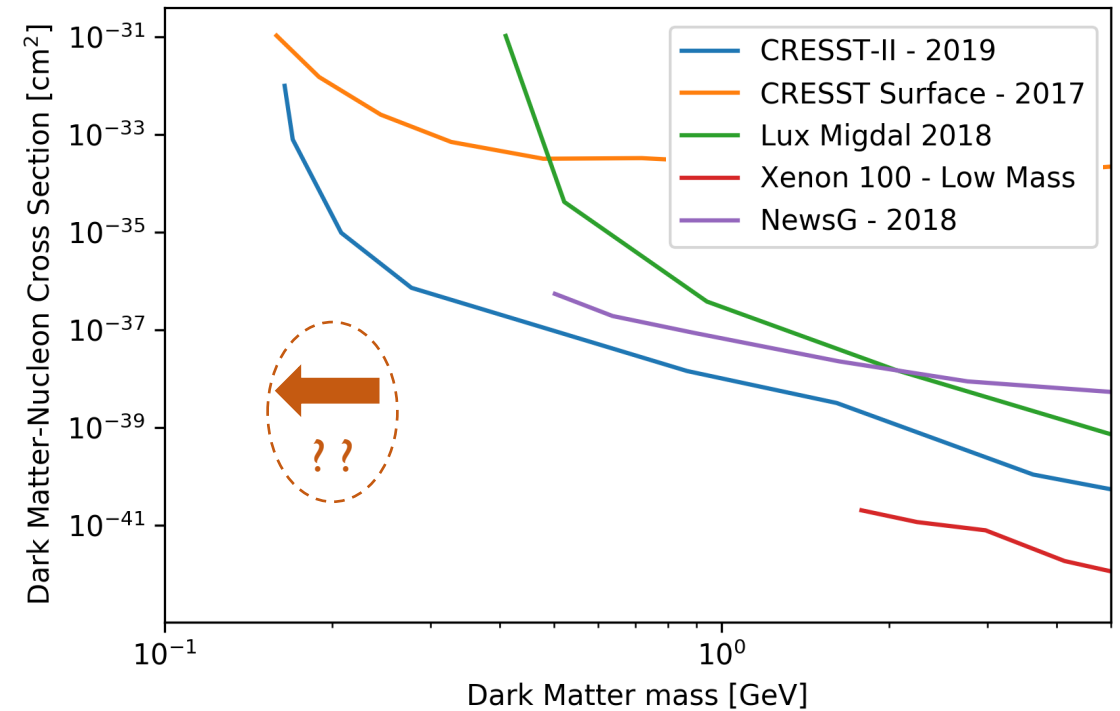
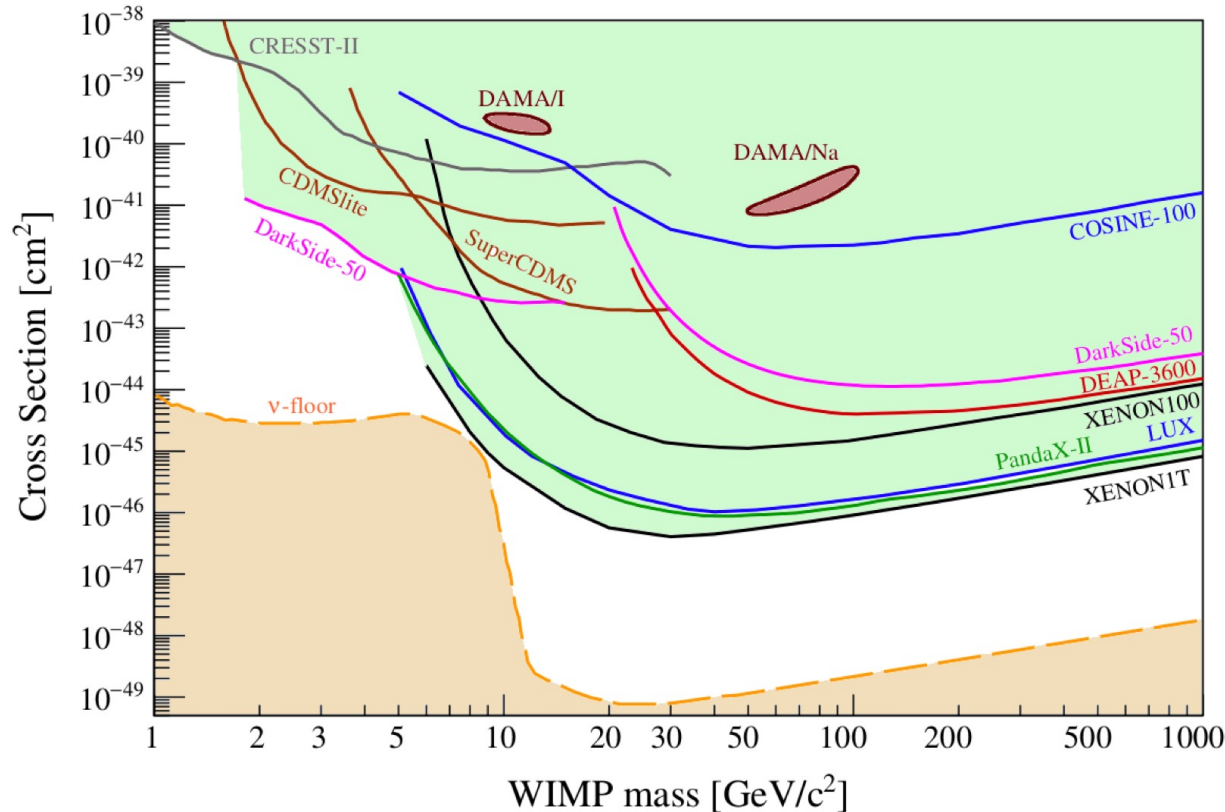
$$+ \quad \Omega_{\chi} h^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma_{\chi\chi} v_{\text{rel}} \rangle}$$

- Annihilation cross section  $\sim m_{\chi}^2 / m_Z^4$

- These assumptions can be relaxed  $\rightarrow$  **search for  $< 1 \text{ GeV}$  DM is motivated !**

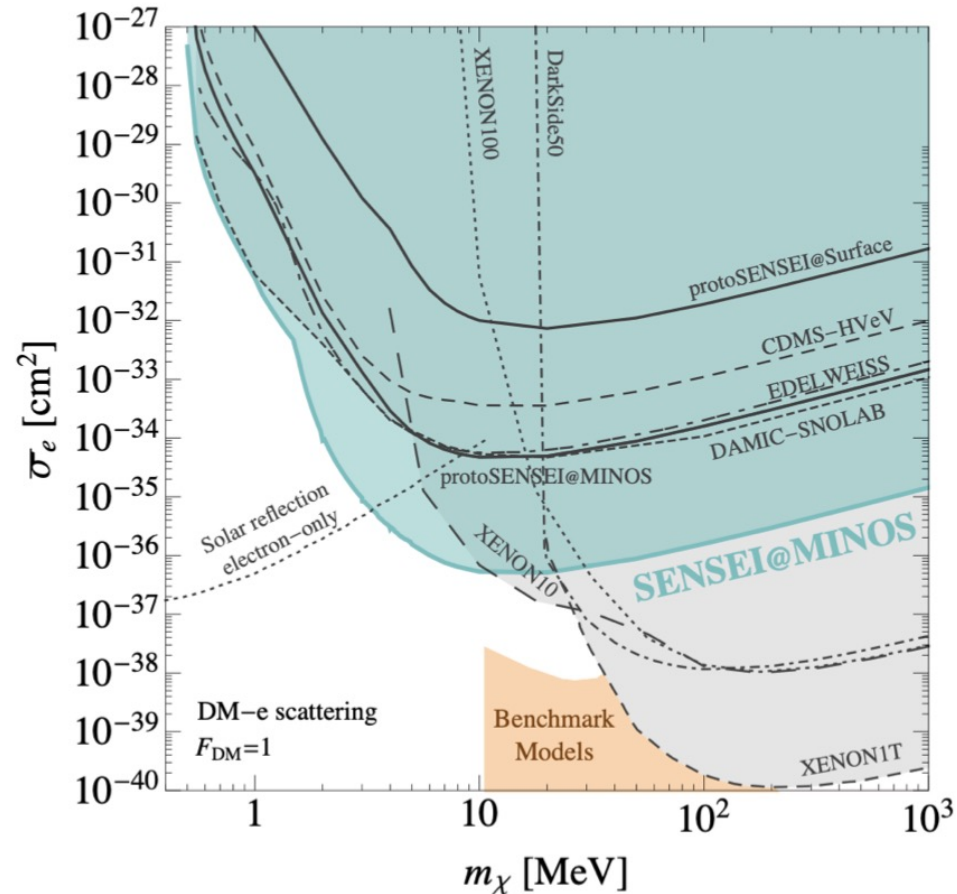
# Landscape of Light DM Searches (1/2)

- Historically, WIMP DM detectors using Xenon (and noble gas) to search for nuclear recoils have been the most competitive



# Landscape of Light DM Searches (2/2)

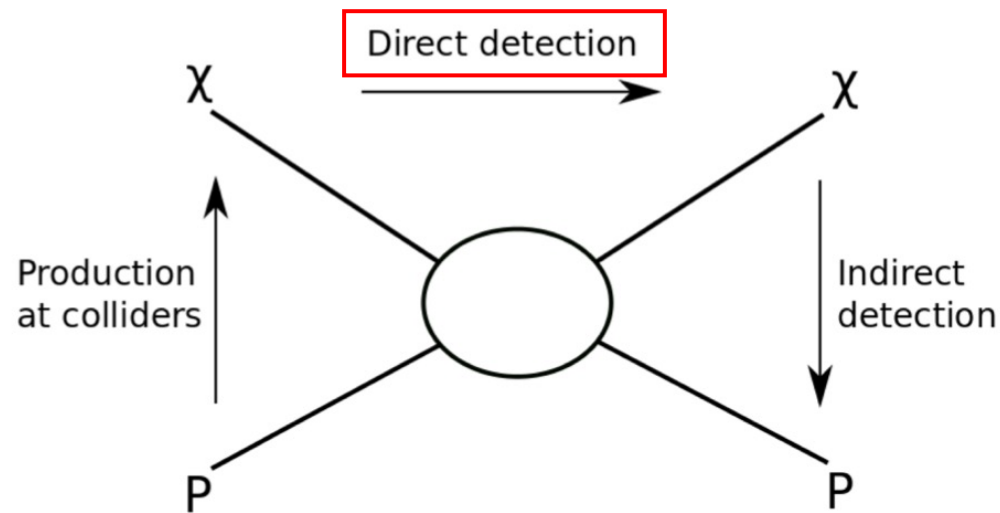
- The same seems to happen in electron scattering: experiments sensitive to single electron events should be competitive for lower mass electron scattering



# Where To Search For Light Dark Matter

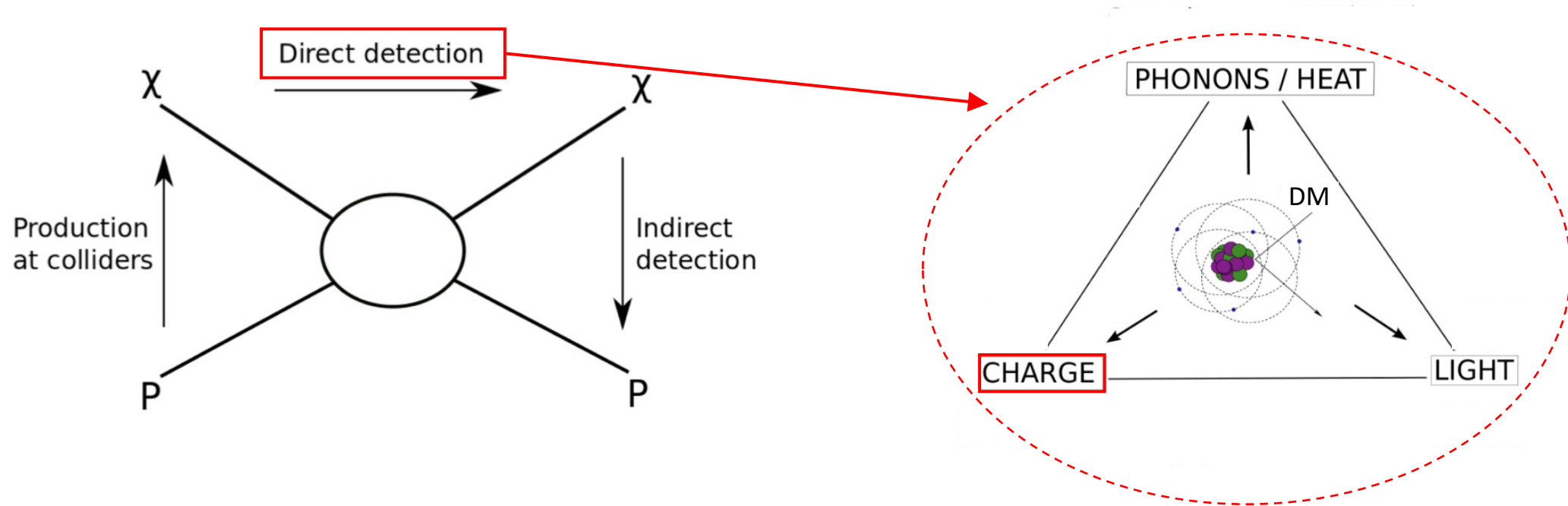
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- Virtually any interaction of DM with the Standard Model can be probed using one of the following methods/signals:



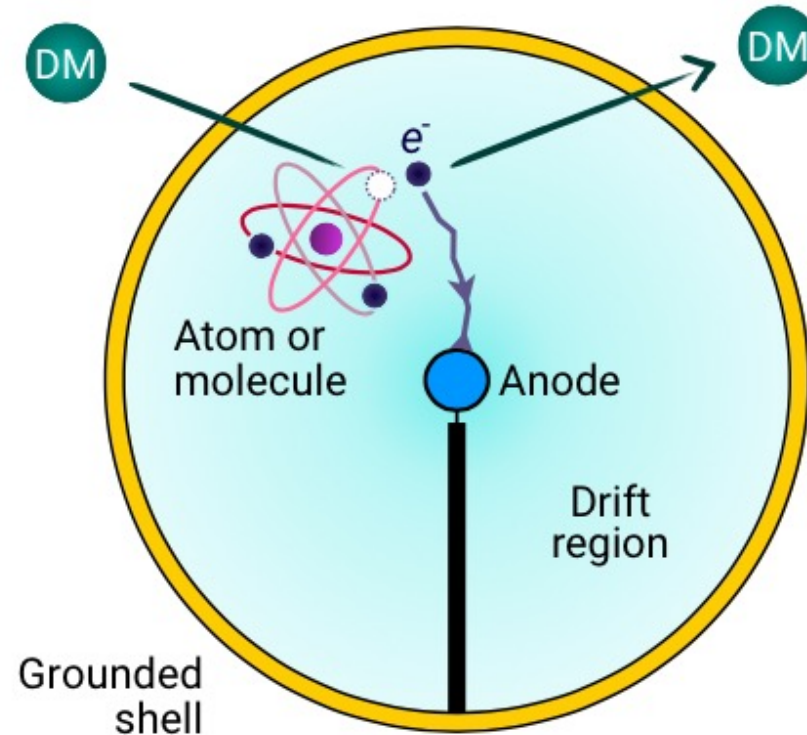
# Where To Search For Light Dark Matter

- Virtually any interaction of DM with the Standard Model can be probed using one of the following methods/signals:

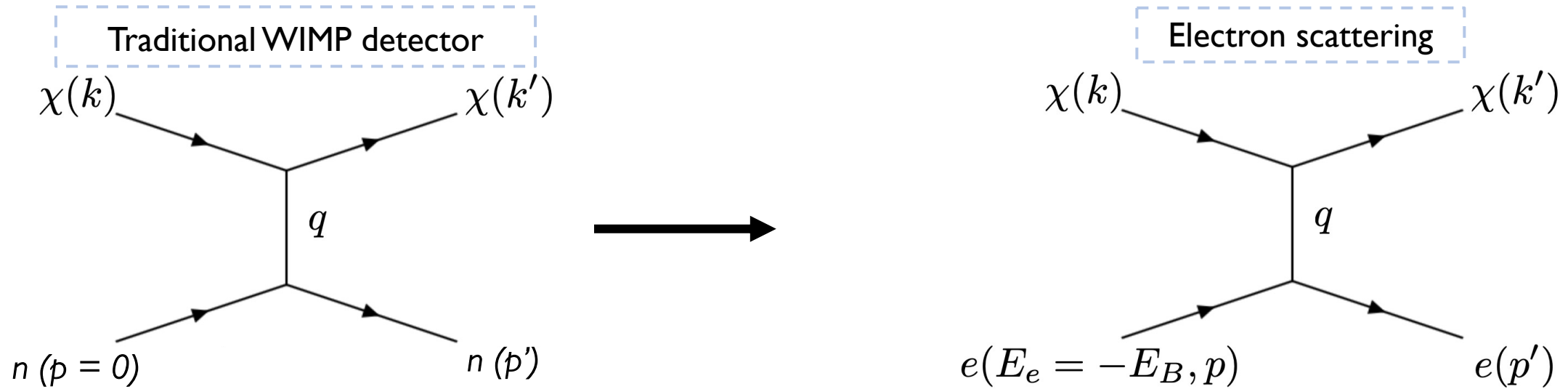


# Spherical Proportional Counters For DM-e Searches

- Electron gets kicked off by DM particle and drifts down, triggering an avalanche
- Molecular “quench gas” used to stabilize detector and search for  $1e$  events
- High voltage can be applied without spurious discharge and high signal gain



# Dark Matter Electron Scattering



$$\mathcal{M} = g_x g_y \frac{1}{q^2 - m^2} \int d^3x e^{-i\mathbf{p}\cdot\mathbf{r}} e^{i\mathbf{q}\cdot\mathbf{r}} e^{i\mathbf{p}'\cdot\mathbf{r}} \longrightarrow \mathcal{M}(n, l \rightarrow \text{free } e^-) = g_x g_y \frac{1}{q^2 - m^2} \int d^3x \tilde{\psi}_{p'l'm'}^*(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}} \psi_{nlm}(\mathbf{x})$$

where

$$d\sigma \propto |\mathcal{M}|^2$$

$F_{\text{DM}}(\mathbf{q})$



# Bound electron wavefunctions (1/5)

---

- Hartre-Fock approximation: mean field self-consistent bound states with energies correct to first order in non-relativistic single particle time independent perturbation theory:

$$H(\mathbf{x}_1, \dots, \mathbf{x}_N) = \sum_i \left( -\frac{1}{2} \nabla_i^2 - \frac{Z}{|\mathbf{x}_i|} \right) + \sum_{i \neq j} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \quad \rightarrow \quad H(\mathbf{x}) = \sum_i \left( -\frac{1}{2} \nabla_i^2 - \frac{Z}{|\mathbf{x}_i|} + U(\mathbf{x}_i) + V(\mathbf{x}) \right) ,$$

$$H\psi(\mathbf{x}_1, \dots, \mathbf{x}_N) = E\psi(\mathbf{x}_1, \dots, \mathbf{x}_N) \quad \rightarrow \quad \left( \sum_i (H_i^{(0)} + H_i^{(1)}) \right) \psi_1(\mathbf{x}_1) \dots \psi_N(\mathbf{x}_N) = (\epsilon_1^{(0)} + \epsilon_1^{(1)} + \dots + \epsilon_N^{(0)} + \epsilon_N^{(1)}) \psi_1(\mathbf{x}_1) \dots \psi_N(\mathbf{x}_N)$$

$$\text{where } H^{(0)}(\mathbf{x}_i) = -\frac{1}{2} \nabla^2 - \frac{Z}{|\mathbf{x}_i|} + U(\mathbf{x}_i) \text{ and } H_i^{(1)}(\mathbf{x}) = V_i(\mathbf{x}) = \sum_{j \neq i} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} - U(\mathbf{x}_i)$$

$$\text{and } \epsilon_i^{(1)} = \langle \psi^{(0)} | H_i^{(1)} | \psi^{(0)} \rangle$$

- HF wavefunctions variationally minimize energy: guarantees correct result (to first order). However does not guarantee orthogonality

# Bound electron wavefunctions (2/5)

- Hartree-Fock approximation: mean field self consistent bound states:

$$-\frac{1}{2} \frac{d^2 P_{n_a l_a}}{dr^2} + \frac{l_a(l_a + 1)}{2r^2} P_{n_a l_a}(r) - \frac{Z}{r} P_{n_a l_a}(r) + \sum_{n_b l_b} (4l_b + 2) \left( v_0(n_b l_b, r) P_{n_a l_a}(r) - \sum_l \Lambda_{l_a l_b} v_l(n_b l_b, n_a l_a, r) P_{n_b l_b}(r) \right)$$
$$= \epsilon_{n_a l_a} P_{n_a l_a}(r) + \sum_{n_b \neq n_a} \epsilon_{n_a l_a, n_b l_a} P_{n_b l_a}(r)$$

Self-consistent  
approach  
required

Expect accuracy of  
O(30%) in event  
rates/bounds

# Bound electron wavefunctions (2/5)

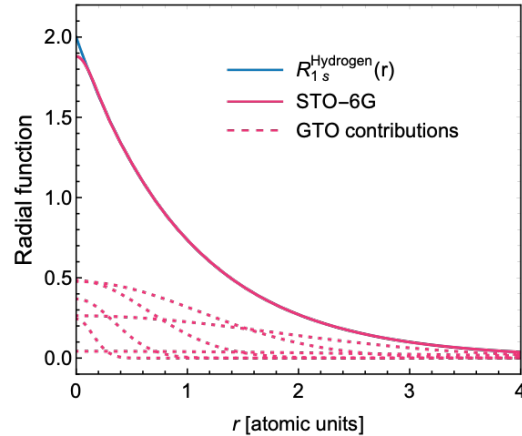
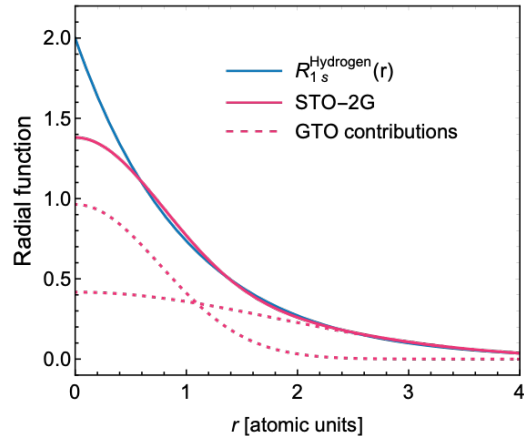
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$$= \epsilon_{n_a l_a} P_{n_a l_a}(r) + \sum_{n_b \neq n_a} \epsilon_{n_a l_a, n_b l_a} P_{n_b l_a}(r)$$

Self-consistent approach required

Expect accuracy of O(30%) in event rates/bounds



Gaussian basis choice important at small/large  $r$

# Bound electron wavefunctions (2/5)

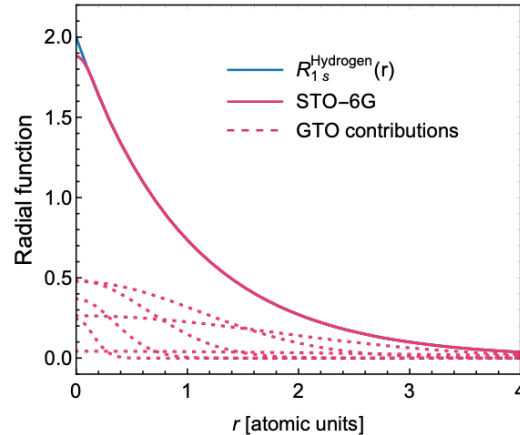
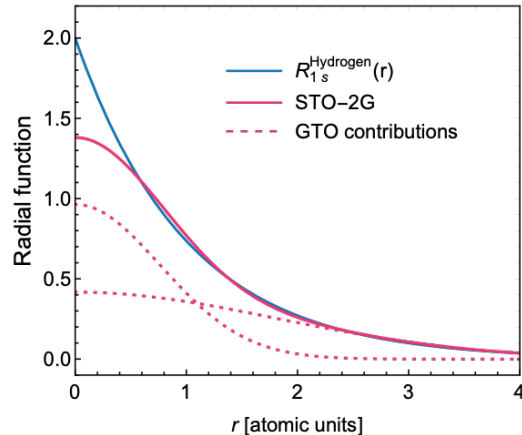
- Hartree-Fock approximation: mean field self consistent bound states:

$$-\frac{1}{2} \frac{d^2 P_{n_a l_a}}{dr^2} + \frac{l_a(l_a + 1)}{2r^2} P_{n_a l_a}(r) - \frac{Z}{r} P_{n_a l_a}(r) + \sum_{n_b l_b} (4l_b + 2) \left( v_0(n_b l_b, r) P_{n_a l_a}(r) - \sum_l \Lambda_{l_a l_b} v_l(n_b l_b, n_a l_a, r) P_{n_b l_b}(r) \right)$$

$$= \epsilon_{n_a l_a} P_{n_a l_a}(r) + \sum_{n_b \neq n_a} \epsilon_{n_a l_a, n_b l_a} P_{n_b l_a}(r)$$

Self-consistent approach required

Expect accuracy of O(30%) in event rates/bounds



Gaussian basis choice important at small/large r



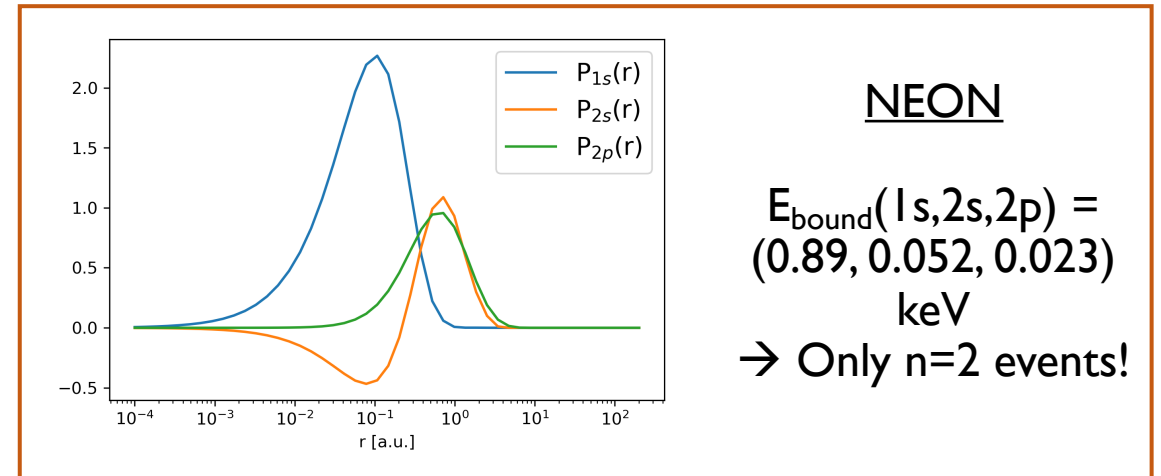
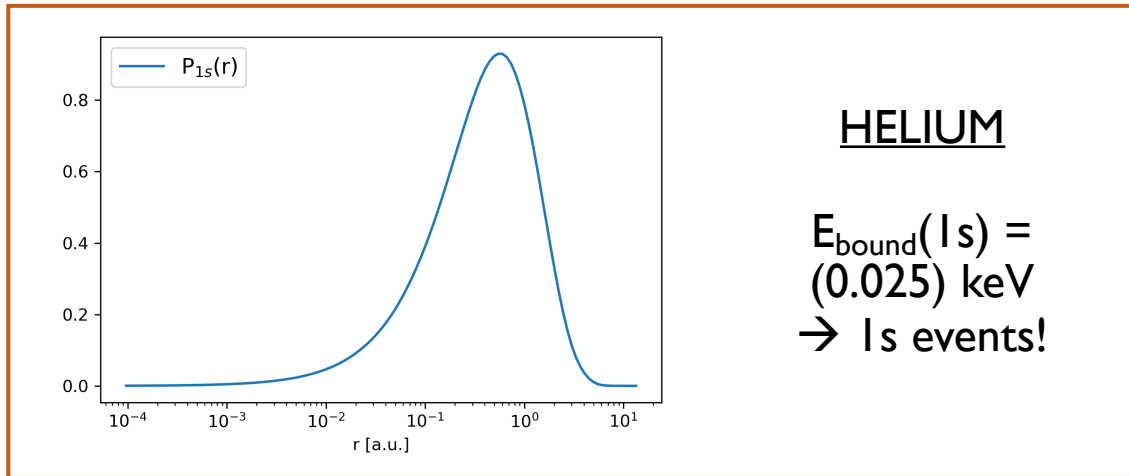
$$AO_{nlm} \propto \sum_i c_i^n \exp(-\alpha_i^n r^2) Y_{lm}(\theta, \phi)$$

$$MO_{nlm} = \bar{v}_{nlm} AO_{nlm}$$

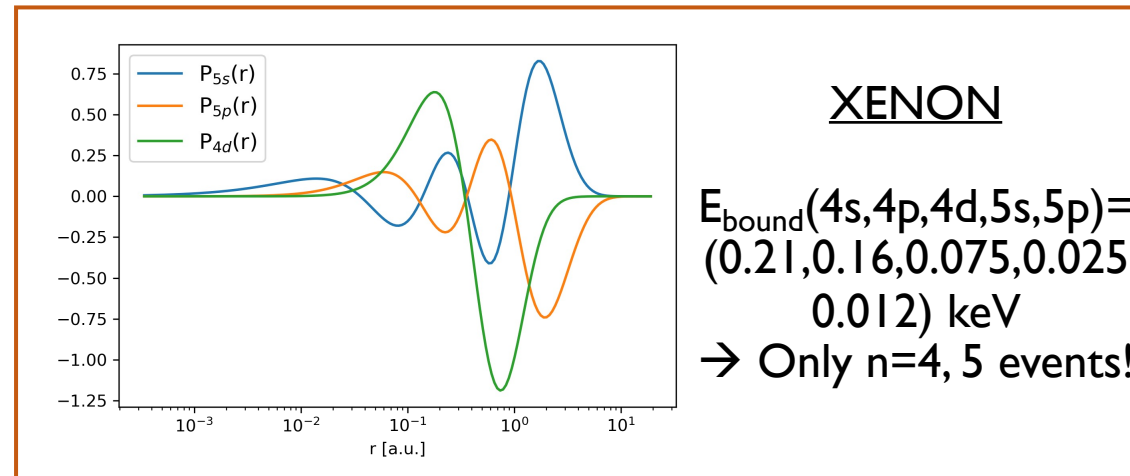
$$H_{ij} = \int AO_i(\mathbf{r}) H_j(\mathbf{r}) AO_j(\mathbf{r}) d^3 \mathbf{r} .$$

Treats atoms and molecules, relativistic treatments, molecular dipoles, and more.

# Bound electron wavefunctions (3/5)



Cutoff condition:  
 $KE_{\text{DM}} > E_{\text{B}}$



# Bound electron wavefunctions (4/5)

- Molecular gas can **quench itself** → part of target
- Molecular orbitals are no longer eigenfunctions (spherical harmonics) of the SO(3) generators.

$$\psi(\mathbf{r}) = \frac{P(r)}{r} Y_{lm}(\theta, \phi) \rightarrow \psi(x, y, z)$$

- We need new classification of orbitals :  
irreducible representations of SO(3) → irreducible representation of point group

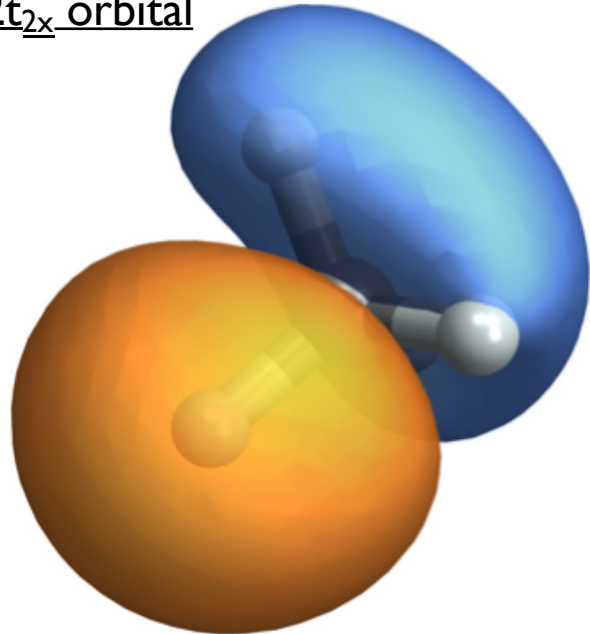
$T_d$	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	linear functions, rotations	quadratic functions	cubic functions
$A_1$	+1	+1	+1	+1	+1	-	$x^2+y^2+z^2$	xyz
$A_2$	+1	+1	+1	-1	-1	-	-	-
E	+2	-1	+2	0	0	-	$(2z^2-x^2-y^2, x^2-y^2)$	-
$T_1$	+3	0	-1	+1	-1	$(R_x, R_y, R_z)$	-	$[x(z^2-y^2), y(z^2-x^2), z(x^2-y^2)]$
$T_2$	+3	0	-1	-1	+1	$(x, y, z)$	$(xy, xz, yz)$	$(x^3, y^3, z^3) [x(z^2+y^2), y(z^2+x^2), z(x^2+y^2)]$

Tetrahedral group (Methane):  $T_d$

# Bound electron wavefunctions (5/5)

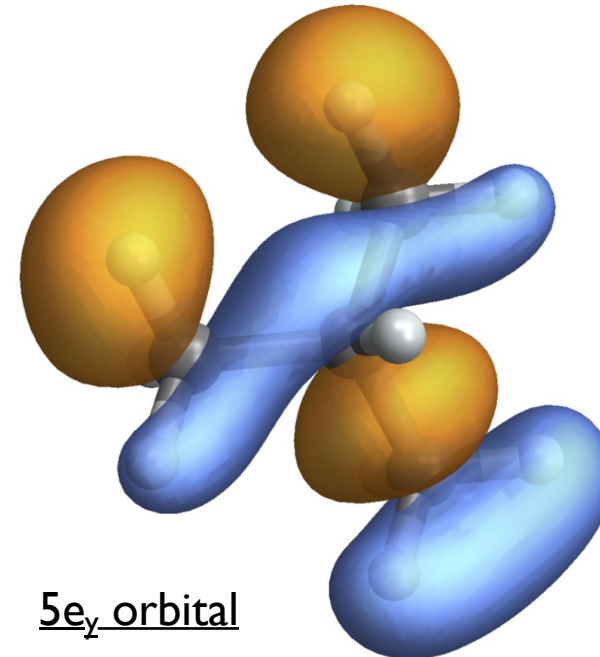
- Molecular orbitals don't enjoy the spherically symmetric Hamiltonian of noble gas. Instead the gaussian basis is multi-centered:

2t<sub>2x</sub> orbital



METHANE (CH<sub>4</sub>)

$E_{\text{bound}} =$   
(-304.9 eV, -24.8 eV, -14.8 eV)  
→ Only 2a and 2t events!



5e<sub>y</sub> orbital

ISOBUTHANE (C<sub>4</sub>H<sub>10</sub>)

$E_{\text{bound}} =$   
(-305.429 eV, -304.941 eV,  
...  
-12.4723 eV, -12.318 eV)  
→ Only  $E > E_{2a}$  events!

# Unbound electron wavefunctions

- Continuum limit: Hartree-Fock integrated, approximate the self-consistent piece of the potential:

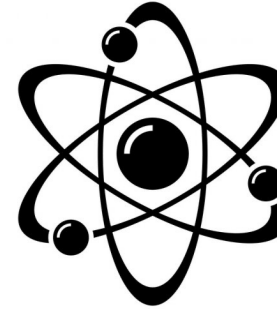
$$-\sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} = V_{dir}(r) + V_{exch}(r)$$

$$V_{dir}(r) = \sum_{n_b l_b} (4l_b + 2) \int_0^\infty \left( \frac{P_{n_b l_b}^2(r_1)}{\max(r_1, r)} \right) dr_1$$

$$V_{exch} = k_x \left( \frac{24\rho(r)}{\pi} \right)^{1/3}$$

→ « Hartree-Plus-Statistical-Exchange » potential (Cowan) gives accurate wavefunctions and energies (checked against bound WFs).

- Use frozen core approximation: electrons don't have time to react (ie change wavefunction) to ejected electron.

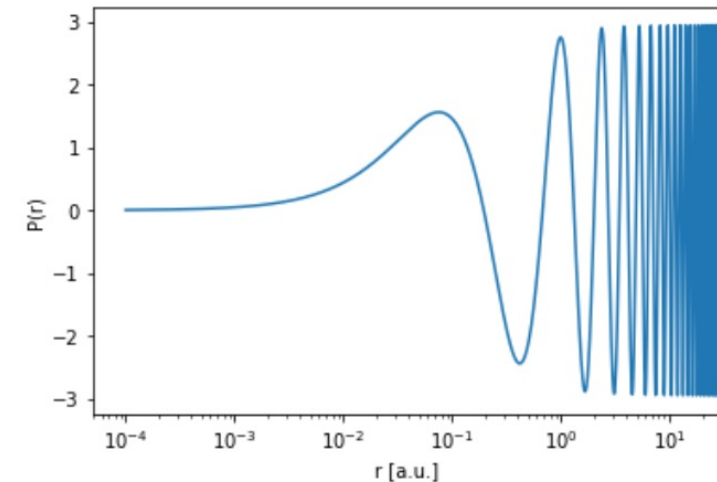


## Potential:

- Point nucleus :  $-Z/r$
- Electron "cloud":  $V_{dir}(r)$
- Electron "exchange":  $V_{exch}$

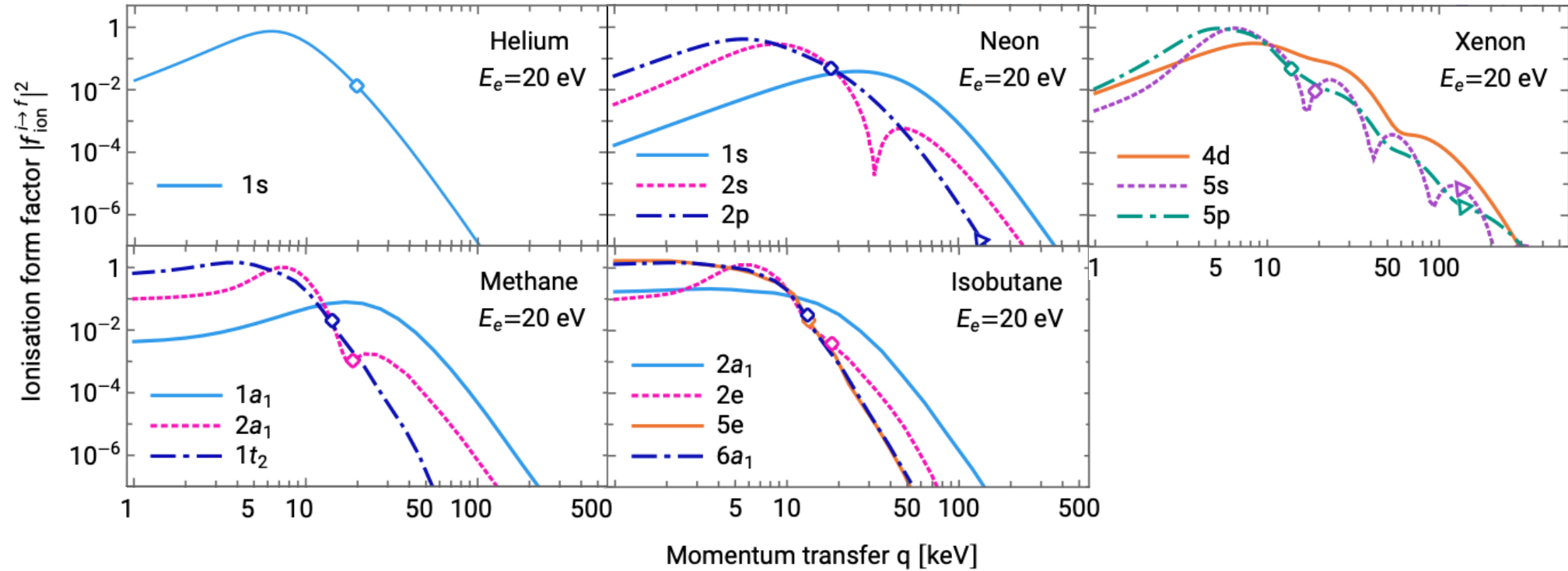
→  $1/r$  at large  $r$

Neon 2p →  $l_R=0, E = 250\text{eV}$





# Form Factors



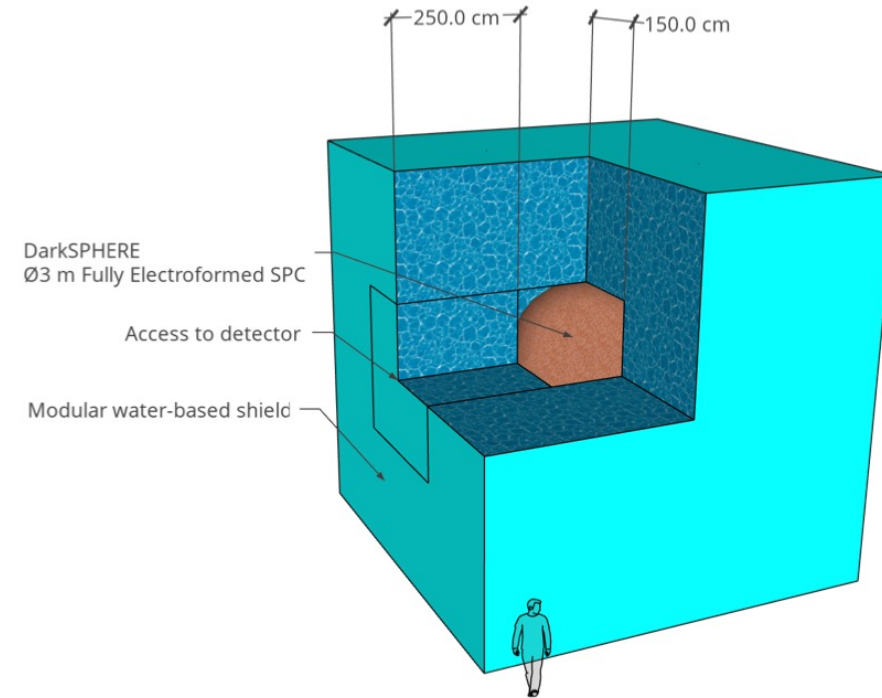
Form factor ( $\alpha |\mathcal{M}|^2$ ):

$$|f_{\text{ion}}^{nl}(q)|^2 = \frac{(2m_e E_R)^{3/2}}{\pi^2} \sum_{\text{states:i,f}} \left| \int d^3x \tilde{\psi}_{p'l'm'}^*(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}} \psi_{nlm}(\mathbf{x}) \right|^2$$

arxiv 2110.02985

# DarkSPHERE at Boulby

- Boulby Underground Laboratory hosts a 4000m<sup>3</sup> clean room space supporting DM experiments – and is looking for new experimental proposals!
- Boulby benefits from relatively low seismic noise and radioactivity (from rock)
- We propose DarkSPHERE : a 3m SPC fully electroformed underground, similar to ECUME in Canada, and will form the next stage of the NEWS-G experiment



Shielding Configuration	Environmental background rate $\leq 1$ keV [dru]			
	Photon-induced Photon	Neutron-induced Neutron	Neutron-induced Photon	Muon-induced
2.5 m water	$4.2 \times 10^{-3}$ (0.3)	$9 \times 10^{-5}$ (5)	$1.3 \times 10^{-4}$ (0.4)	$5 \times 10^{-3}$ (4)

# Event rates

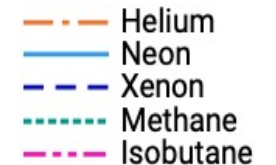
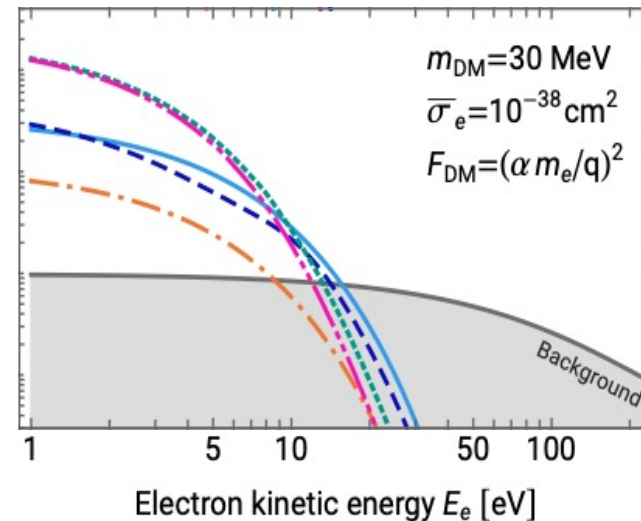
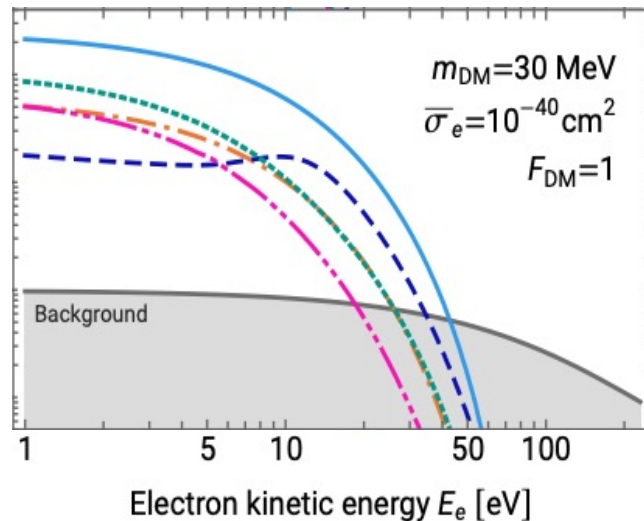
- The dark event rate can be calculated using:

$$\frac{dR}{dE_e} = \frac{1}{m_A} \frac{\rho_{\text{DM}}}{m_{\text{DM}}} \sum_{nl} w_{nl} \frac{d\langle\sigma_{\text{ion}}^{nl} v_{\text{DM}}\rangle}{dE_e}, \quad \text{where} \quad \frac{d\langle\sigma_{\text{ion}}^{nl} v\rangle}{d \ln E_e} = \frac{\sigma_e}{8\mu_e^2} \int_{q_-}^{q_+} q dq |f_{\text{ion}}^{nl}|^2 |F_{\text{DM}}|^2 g(v_{\text{min}}^{nl})$$

- Assume simulated+measured background provided by NEWS-G,  $F_{\text{DM}}=1$
- 10% Methane or Isobutane (Coulomb wave) contribution

Heavy mediator ←

→ Light mediator



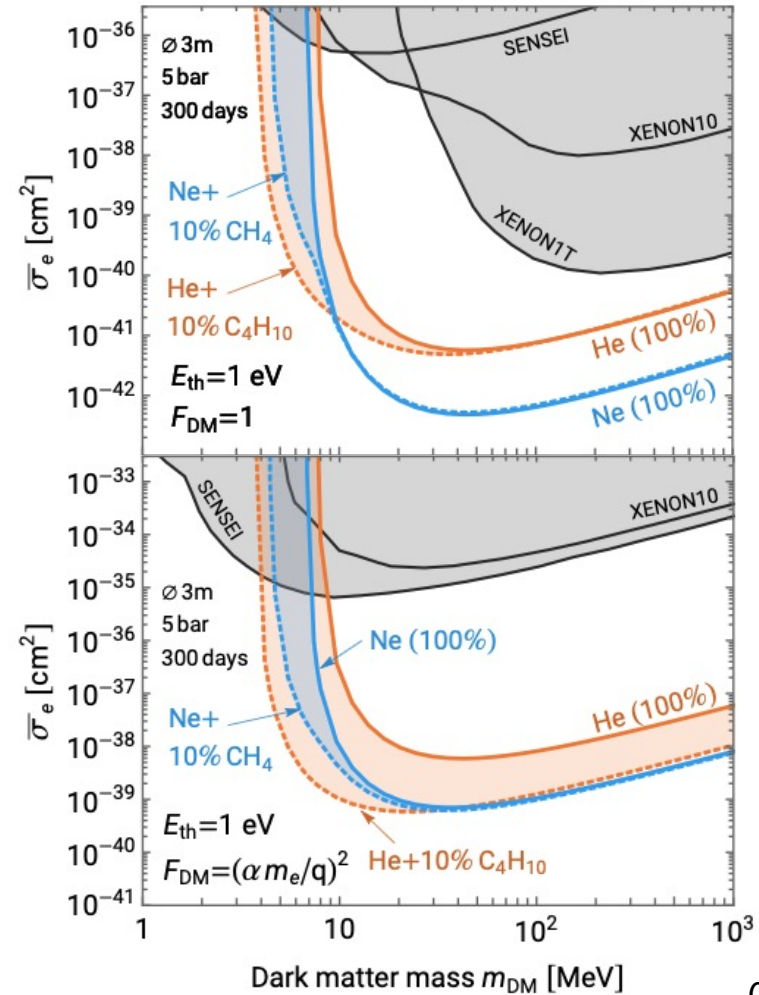
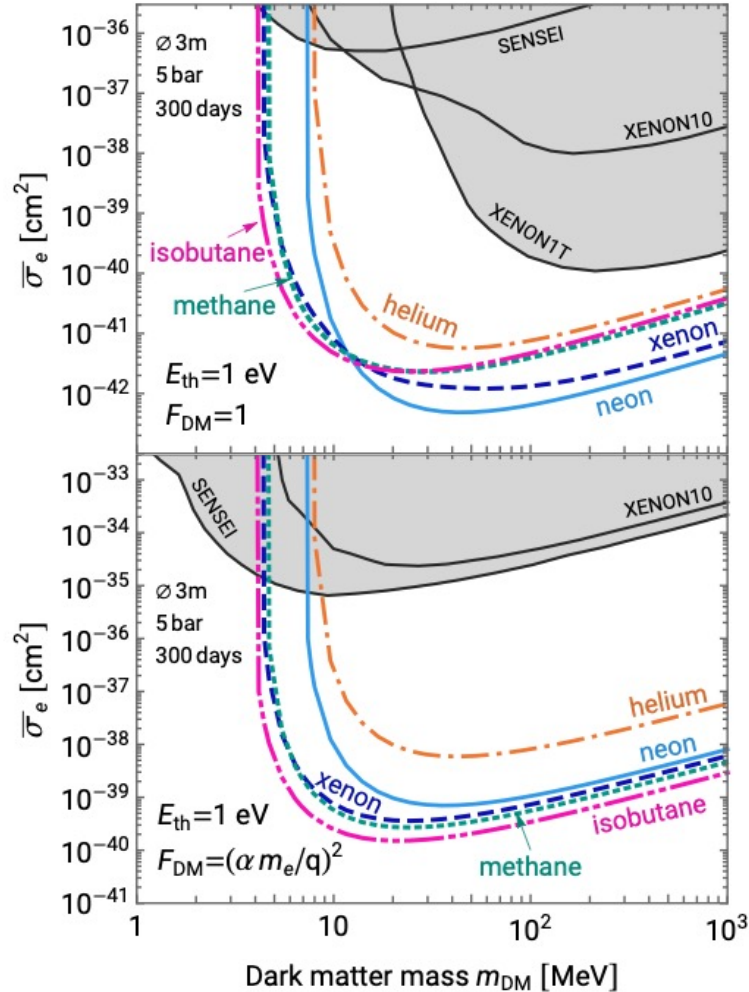
arxiv 2110.02985

# Sensitivities

Heavy mediator



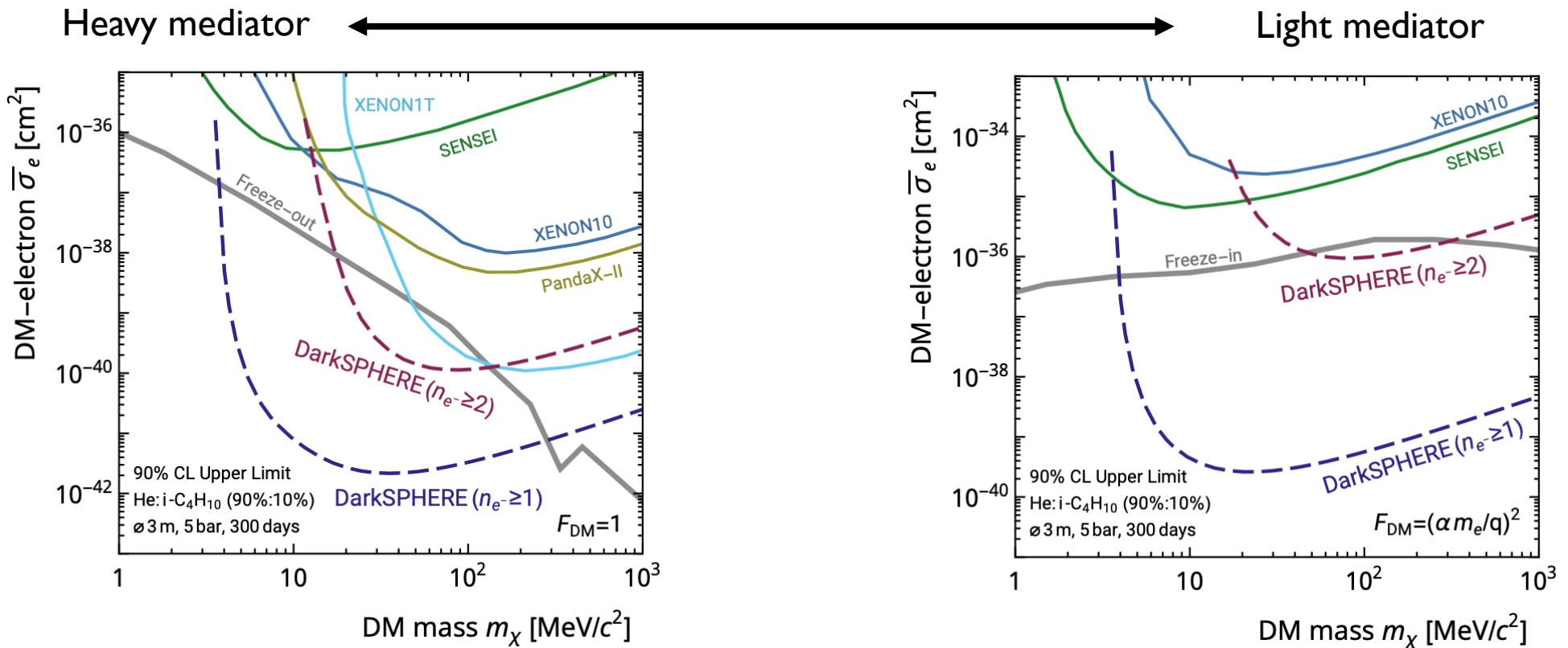
Light mediator



arxiv 2110.02985

# Sensitivities

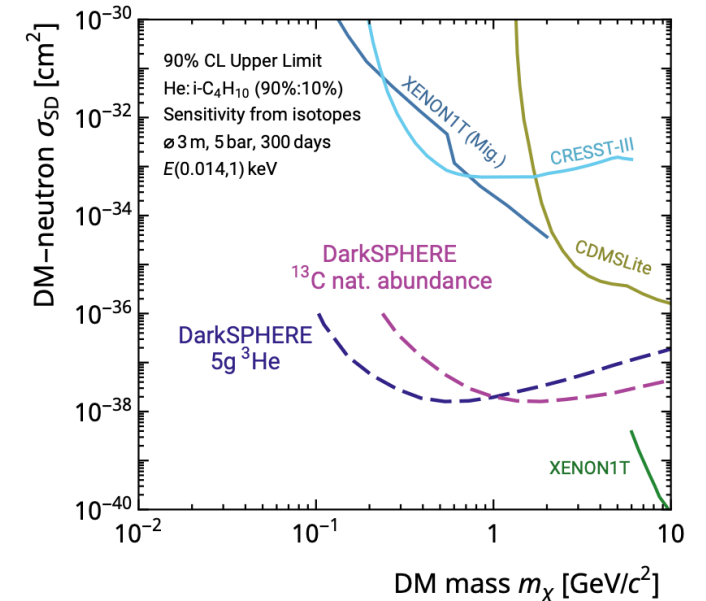
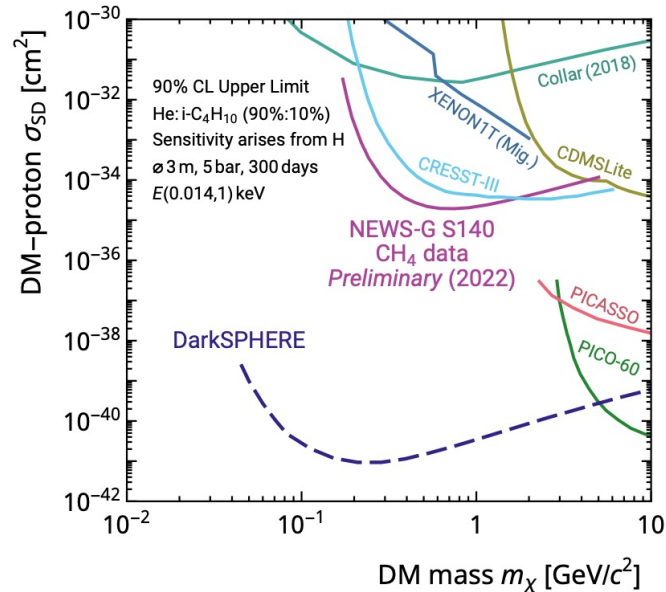
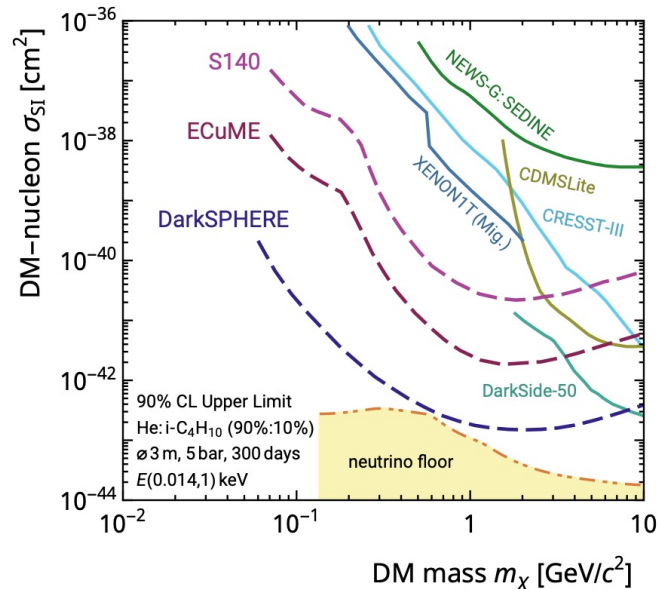
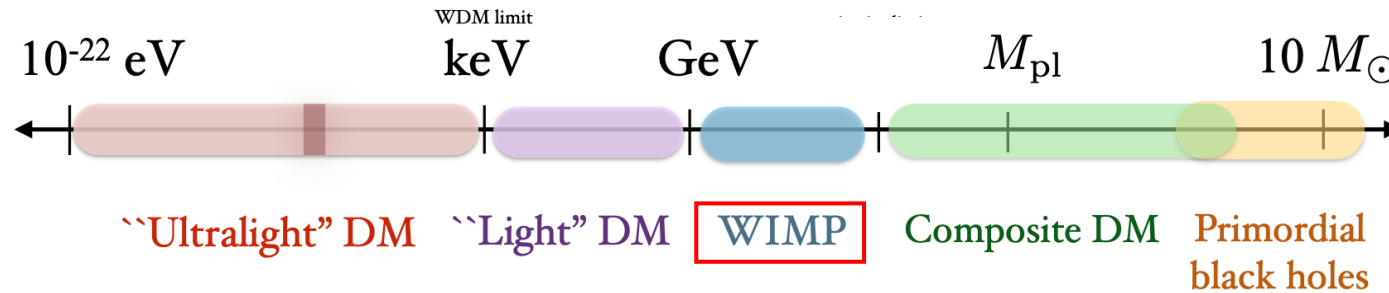
- Good sensitivity to freeze-in / freeze-out benchmark scenarios depends on ability to see 1e- events



arxiv 2301.05183

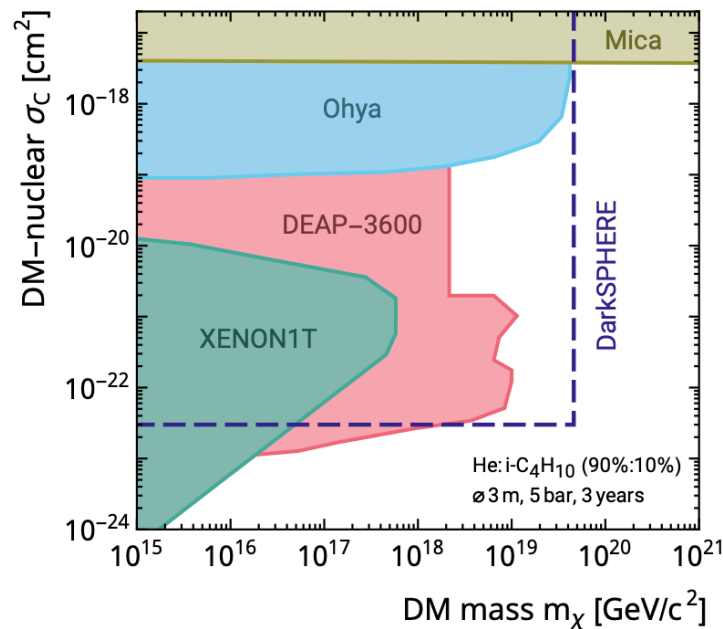
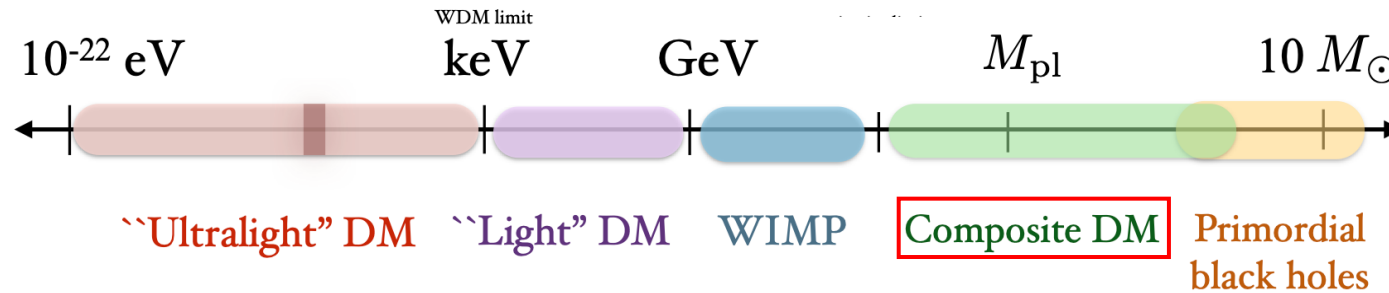
# Other bounds: DM-nucleon scattering

- At higher masses DarkSPHERE is also very competitive in the 0.1-5GeV region for SI and SD interactions



# Other bounds: DM-nucleon scattering

- And at very high masses DarkSPHERE is still competitive! Sensitivity here depends on size of experiment



# Summary

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- Dedicated direct detection of DM-electron scattering good probe of light DM ( $< 1 \text{ GeV}$ )
- Atomic calculation under control – good accuracy & understanding (vs. other HF & experiment).  
Molecular calculation more difficult (Coulomb approximation used), but can be used to set bounds (i.e. with mixing)
- More sensitive than current bounds and easier than other proposed experiments  $\rightarrow$  SPC good probe of light DM-electron and scattering !
- SPCs sensitive to large range of DM masses, with more interactions to explore (e.g. dark photons, ALPs).  
Could also search for molecules with better form factors (i.e. looking for large  $q$ -variance)
- Boulby is ideal for a world-leading large-scale proposal (DarkSPHERE) based in the UK



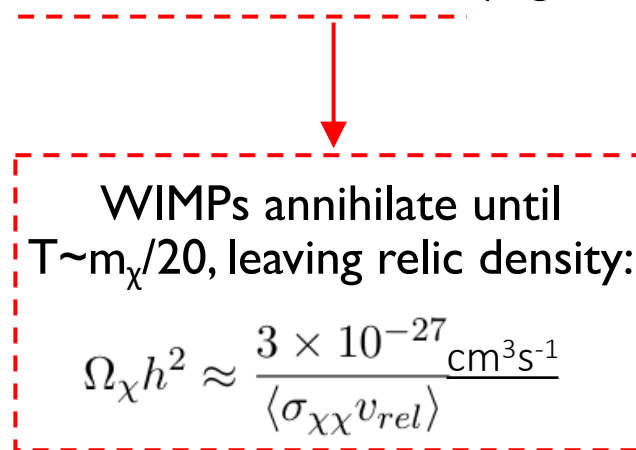
**Thank you!**

# Back up - Motivation for Light(er) Dark Matter

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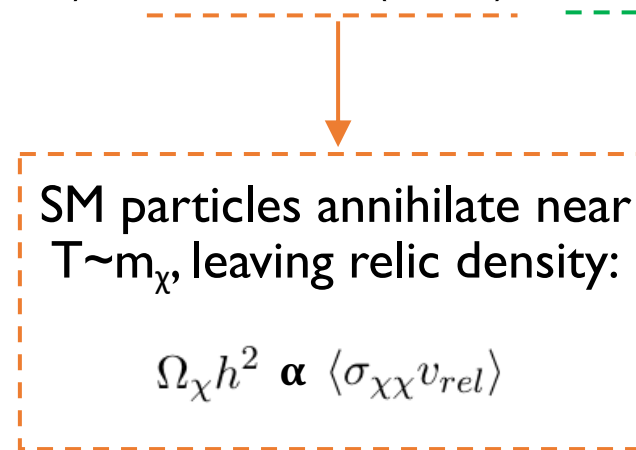
- Dark matter has many ways of appearing in the present day universe:

Thermal Production (e.g. freeze-out WIMP), Freeze-in (FIMP), Inflaton decay, Gravitino...



WIMPs annihilate until  $T \sim m_\chi/20$ , leaving relic density:

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma_{\chi\chi} v_{rel} \rangle}$$



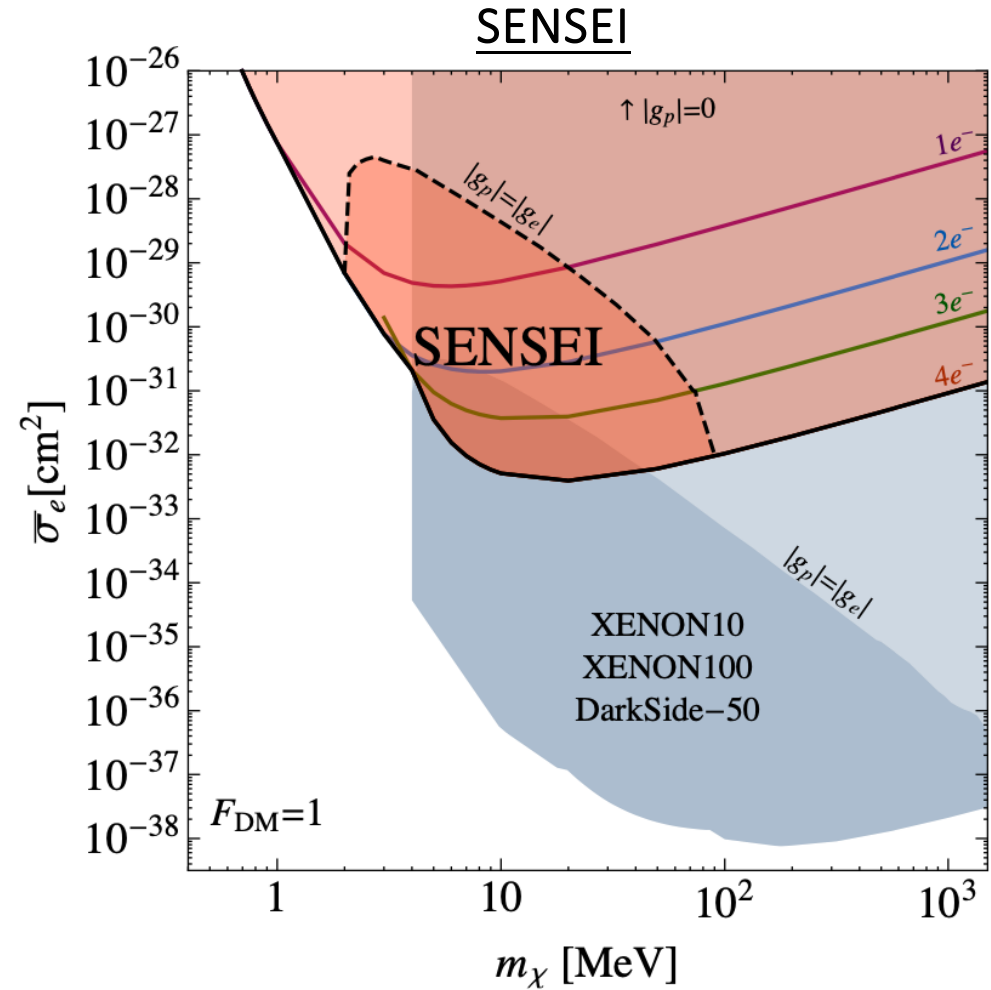
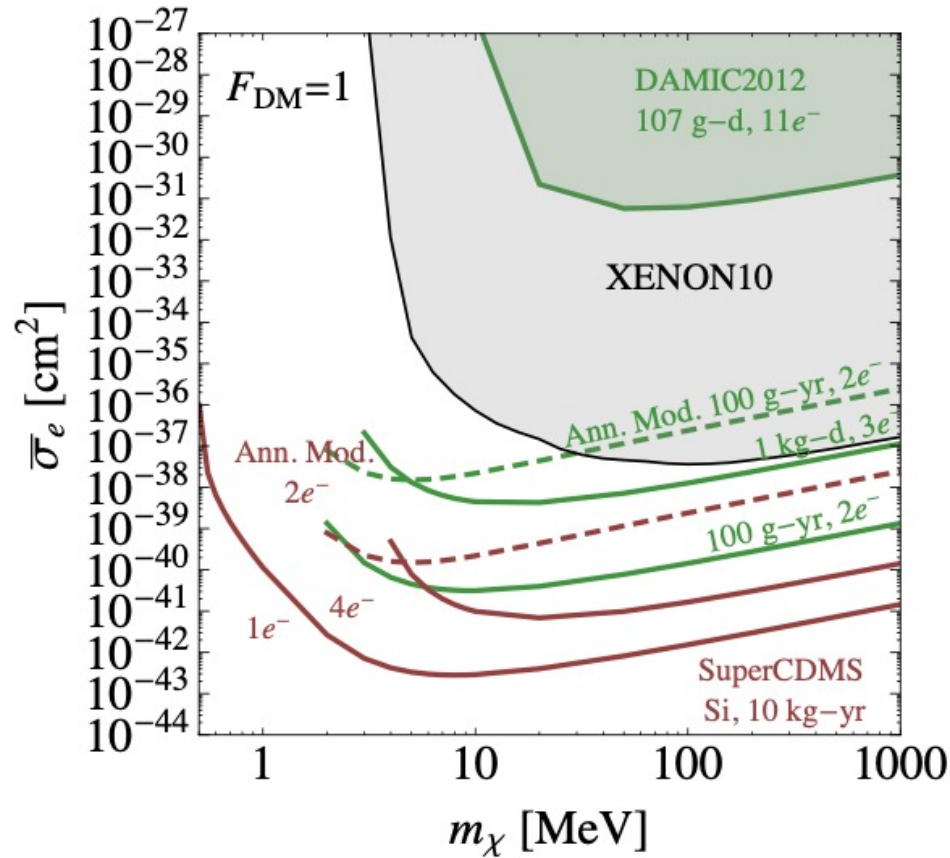
SM particles annihilate near  $T \sim m_\chi$ , leaving relic density:

$$\Omega_\chi h^2 \propto \langle \sigma_{\chi\chi} v_{rel} \rangle$$

- Individual models of freeze-out and or freeze-in DM can be fully tested (even for unknown details of UV cosmology).
- Without knowledge of  $T_R$ , we cannot fully test inflaton decay or gravitino.

# Back up – More On Constraints

Electron Recoil SuperCDMS projection (Essig et al.)



# Back up – Hartree Fock choice

- Hartree-Fock approximation: self-consistent bound states with energies correct to first order:

$$\begin{aligned}
 & -\frac{1}{2} \frac{d^2 P_{n_a l_a}}{dr^2} + \frac{l_a(l_a + 1)}{2r^2} P_{n_a l_a}(r) - \frac{Z}{r} P_{n_a l_a}(r) + \\
 & \sum_{n_b l_b} (4l_b + 2) \left( v_0(n_b l_b, r) P_{n_a l_a}(r) - \sum_l \Lambda_{l_a l_b} v_l(n_b l_b, n_a l_a, r) P_{n_b l_b}(r) \right) \\
 & = \epsilon_{n_a l_a} P_{n_a l_a}(r) + \sum_{n_b \neq n_a} \epsilon_{n_a l_a, n_b l_a} P_{n_b l_a}(r), \quad (3.71)
 \end{aligned}$$

Easy to solve  
Hamiltonians for  
full shell atoms

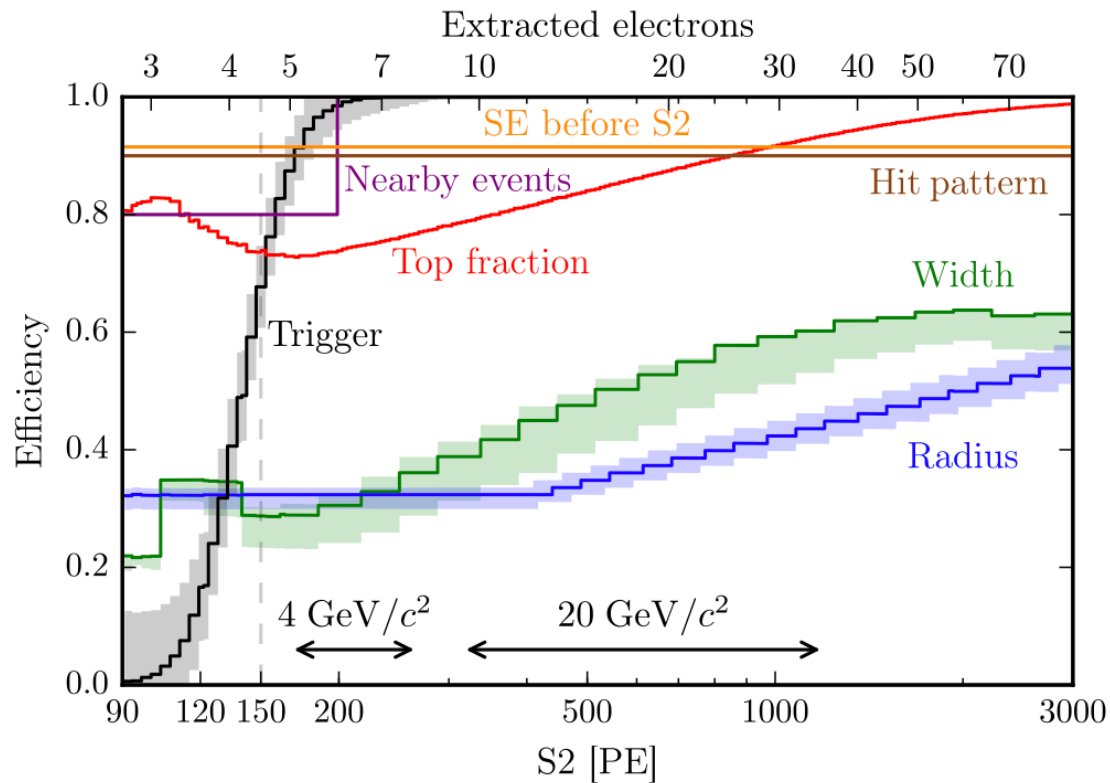
Non-Rel approx.  
good for  
 $q < 200\text{keV}$

Accuracy of  
~30% in event  
rates/bounds

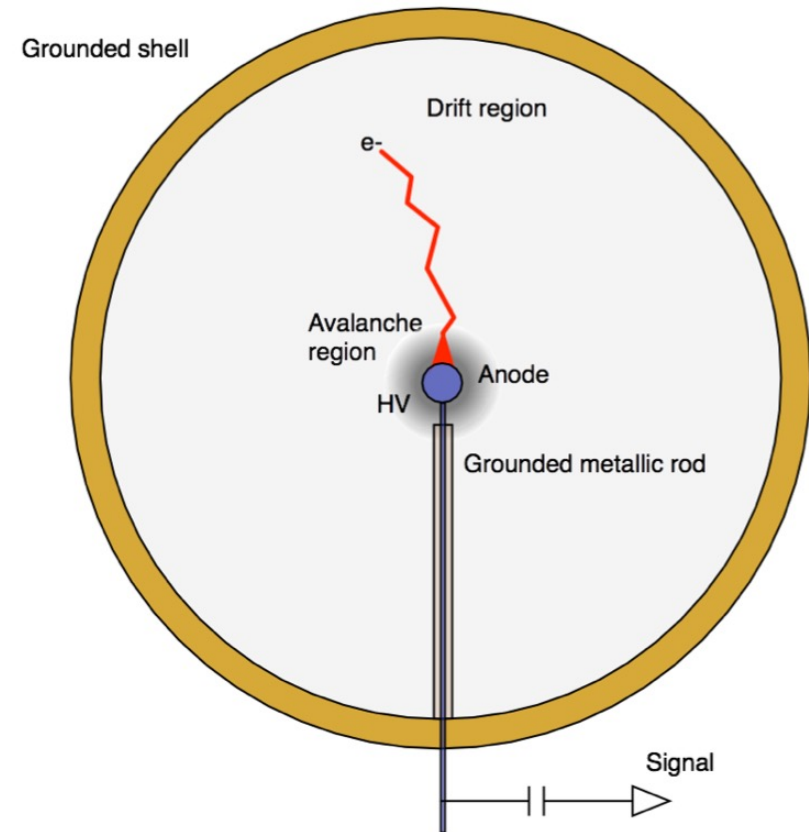
- Sensitivities of bounds to choices:
  - ~30-50% Gaussian basis choice
  - ~50-100% exchange potential choice, orthogonalization
  - ~10-20% analysis of recoil energy profile vs. deposited energies
  - ~30% astrophysical parameter choices
  - Linear with background

# Back up – More On Detector

(liquid) Xenon 1T threshold ( $>4.5e^-$ )

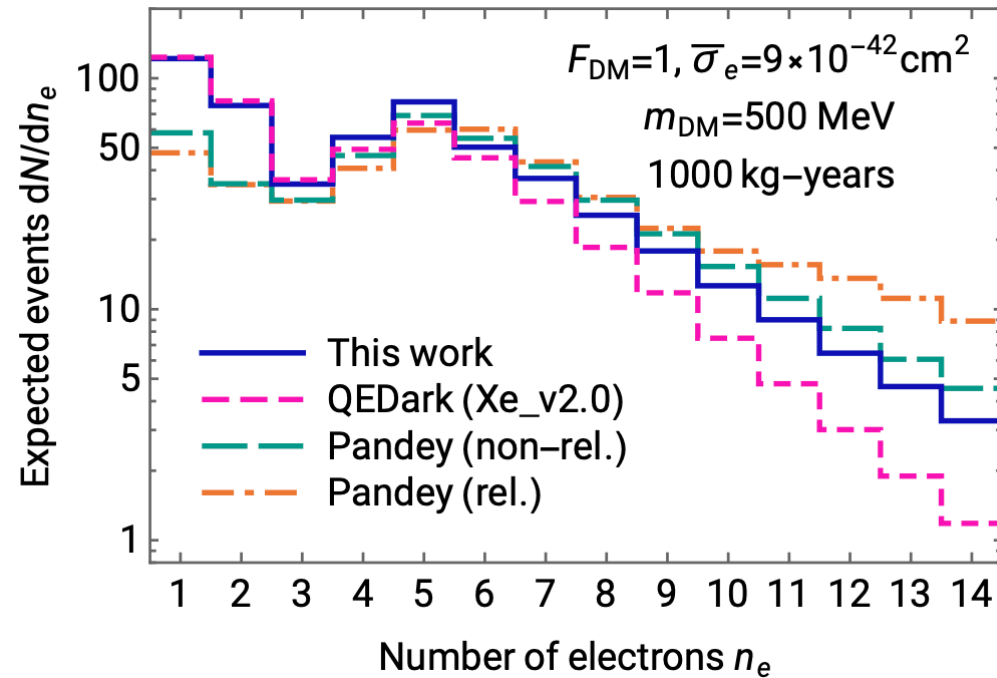


Spherical Proportional Counter (SPC, as proposed in DarkSPHERE)



# Back-up

- Not all previous calculations (e.g. QEDark) exhibit modelling of atomic structure ( $V_{\text{exch}}$ )



- $\text{CH}_4/\text{C}_4\text{H}_{10}$  are quite symmetric so monocenter calculation sufficient

