### **STFC school, Durham 2-6/9/24**

# **Collider Phenomenology (2) Eleni Vryonidou**



# **Plan for the lectures**

- Basics of collider physics
- Basics of QCD
	- DIS and the Parton Model
	- Higher order corrections
	- Asymptotic freedom
	- QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT



# **Plan for the lectures**

- Basics of collider physics
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 $dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$ 

$$
\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)
$$
  
Phase-space integral Parton density functions Parton-level cross section

Universal:

~Probabilities of finding given parton with given momentum in proton Extracted from data

Important aspect of a Monte Carlo generator

Subject of huge efforts in the LHC theory community to systematically improve this





# **Master formula for LHC physics**

4





**Capacity**  $\mathcal{H}$  pomentum transfer $^2$  $x = Q^2/2(P \cdot q)$ <sup> $P$ </sup>scaling variable energy losse rel. toppergy rices recoil mass  $s = (D + k)^2 +$  $Q^2 = -(k-k')^2^{\frac{2}{\omega^2}}$  $\nu = (P \cdot q)/M = E - {E' \over L}$  $y = (P \cdot q)/(P \cdot k) = 1 - E'/E$  $W^2 = (P+q)^2 = M^2 + 3$  $\frac{-x}{6}$ *Q*<sup>2</sup> **Seache entry** mentum transfer  $\frac{1}{100}$ relationship in the contract of the contract o<br>The contract of the contract o recoil mass  $(k)^2 + k^2$  $-(k - k')^2$ <sup>2</sup> m  $\hat{P}$   $\hat{P}$   $\hat{B}$  $(a) / M = E = E'$ <sup> $\frac{P}{P}$ </sup>  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $\frac{1}{2}$   $\frac{1}{x}$   $\frac{1}{x}$ *x*<br>*x Q*<sup>2</sup> cms energy2 **momentum scaling vva egergy** *relative***ry** recoil d  $s = (P + k)^2$  $Q^2 = -(k - k')^2$  $x = Q^2/2(P \cdot q)$   $\int_S$  ealing variable  $\nu = (P \cdot q)/M = E - E'$  $y = (P \cdot q)/(P \cdot k) = 1 - E'/E$  $W^2 = (P+q)^2 = M^2 +$  $\frac{1-x}{x}$ *x*  $Q^2$ Convergery momentum transferisfer^2 energy losss relative energy loss réagil magss

### **The parton model of QCD Deep Inelastic Scattering** Scaling **BUSHEM DI MARY UNIVERSITY OF LONDON, JANUARY 2020 - COLLIDER PHENOMENOLOGY AREA DI SERVERSITY OF LONDON, A LON BUSSTEPP@50 - QUEEN MARY UNIVERSITY OF LONDON, JANUARY 2020 - COLLIDER PHENOMENOLOGY, M. UBIALI**

$$
\frac{\frac{d\theta_{\text{elastic}}}{dq\partial \sigma}}{\frac{dq\partial \sigma}{dq^{2}}}\n= \left(\n\frac{\frac{d\theta}{dq^{2}}}{dq^{2}}\n\right)\n\frac{F_{\text{elastic}}^{2}(q^{2})\delta(1-x) dx}{\frac{dq^{2}}{dq^{2}}d\sigma}\n\frac{F_{\text{elastic}}^{2}(q^{2})\delta(1-x) dx}{\frac{dq^{2}}{dq^{2}}}\n\cdot\nF_{\text{inelastic}}^{2}(q^{2}, x) dx
$$
\n
$$
\frac{dq_{\text{av}}^{2}(q^{2})}{dq_{\text{av}}^{2}(q^{2})}\n\cdot\n\frac{F_{\text{inelastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{inelastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{inelastic}}^{2}(q^{2}, x) dx}}\n\cdot\n\frac{F_{\text{inelastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{inelastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{inelastic}}^{2}(q^{2}, x) dx}}\n\cdot\n\frac{F_{\text{helastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{inelastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{relastic}}^{2}(q^{2}, x) dx}}\n\cdot\n\frac{F_{\text{relastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{relastic}}^{2}(q^{2}, x) dx}{\frac{F_{\text{relastic}}^{2}(q^{2}, x) dx}}
$$

# **Deep Inelastic scattering**

### What can  $F^2(q^2)$  look like?  $F^2(q^2)$

1. Proton charge is smoothly distributed (probe penetrates proton like a knife through butter)

2. Proton consists of tightly bound charges (quarks hit as single particles, but cannot fly away because tightly bound)

Quarks are free particles which fly away without caring about confinement!

$$
F_{elastic}^2(q^2) \sim 1 \qquad F_{inelastic}^2(q^2, x) \ll 1
$$
  
113.  $F_{elastic}^2(q^2) \ll 1 \qquad F_{inelastic}^2(q^2, x)$ 

$$
F_{elastic}^2(q^2) \sim F_{inelastic}^2(q^2, x) \ll 1
$$

, *x*) ∼ 1









 $\int d\Phi = \frac{d^3y}{dx^2} d\Phi = \frac{d\Phi}{dx^2} d\Psi dy dx d\Phi =$  $\mathbb{E} \left[ \frac{\partial \mathbf{v}}{\partial \mathbf{v}} \right]_{\Omega}^{2E} = \frac{8\pi}{\mathcal{C}} \mathcal{L} \left[ \frac{\mu v}{\mathbf{v}} \right]_{\mathcal{X}}^{2} \mu \nu$  defines by definition  $L^{\mu\nu} = -\text{tr}[k\gamma^{\mu}k^{\prime}\gamma^{\nu}] = k^{\mu}k^{\prime\nu} + 1$  $\mathbf{r}$  for  $\mu$  $\frac{q_\mu q_\nu}{F_1(r Q^2)}\Big|_{\mathcal{H}} = q \frac{p \cdot q}{m} \Big|_{\mathcal{H}} = q \frac{p \cdot q}{m} \Big|_{\mathcal{H}}$  $\frac{1}{(2\pi)^{32}E'}d\Phi_{X_{Q^2}}$ *ME*  $\frac{4}{8\pi k}y_c^2$   $\frac{y_c}{k}$  $\int_a^{\mu\nu} h_X^2 \frac{d}{\mu}$  **L**<br>*L*<br>*C*  $\int_a^{\mu\nu} k \cdot k'$  $k$ <sup>*w*</sup>  $\left(k^{\mu}k^{\nu} + k^{\prime\mu}k^{\nu} - g^{\mu\nu}k^{\nu}k^{\prime}\right)$  $\frac{d\Phi_X}{dx}$  $\overline{a}$ momentum transfer2 scaling variable energy loss rel.<br>Energy loss in the set of the set<br>Set of the set of  $k^{\nu}$  $s$   $\cancel{H}$  ( $\cancel{P}$   $\overset{A}{+}$   $\cancel{l}$  $\mathfrak{c}$  $Q^2 = 8\pi k^2 - k$  $\frac{dy}{dx} \frac{d\Phi_X}{dx} \mu \nu$  $=$   $\frac{Q}{\sqrt{4}}L^{\mu\nu}h_{X\mu\nu}$  ?  $y_1 = \frac{1}{\omega^4} L^r$   $\frac{n_X \omega}{\omega + n_L}$   $\frac{n_{H+1} \omega}{\omega + n_L}$   $\frac{1}{\omega + n_L}$   $\frac{1}{\omega + n_L}$  $=$   $\frac{1}{2}$   $\int_{0}^{\infty}$   $\int_{0}^{\infty}$  $\mu$   $\mu^{\prime}$ *x Q*<sup>2</sup>  $(2\pi)^{3}2E'$   $4\pi^2Q^2=8\pi k^2-k$  $\Gamma \stackrel{X}{=} k^{\mu}k^{\prime\nu} + k^{\prime\mu}k^{\nu} - q^{\mu\nu}k\cdot k^{\prime}$  $\mu \nu$  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  $\overline{p}$  a  $\sqrt{p}$  $\frac{d\Phi_{XY}}{(2\pi)^3 2E'} d\Phi_{X} = \frac{1}{2}$ *ME*  $\frac{2}{3}\pi^{2}v_{h}$  dard $\frac{d\pi}{2}$ = *e*4  $\frac{\sqrt{4}}{24} L^{\mu\nu} \hat{h}_{X\mu\nu}^{\nu}$  $\frac{1}{4}$  /  $\frac{1}{4}$   $\frac{1}{4}$  /  $\frac{1}{4}$  $\frac{q^2}{r^2}$ GGI Florence - 2017 Fabio Maltoni Fabio Maltoni 115  $\mathbf{Y} = \overline{(0.30 \,\mathrm{m})}$ W $\mathbf{Y} = \overline{(0.30 \,\mathrm{m})}$  $\mathbf{S} = \mathbf{S} + \mathbf{A}$  $\overline{\mathcal{M}}$  Combine the hadronic part of the amplitude and phase space space into "hadronic tensor" and phase space in  $q \gamma_{\alpha} = 1 \gamma_{\alpha} q$ p p  $d\Phi = \left| \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \frac{\partial}{\partial z} \frac{\partial}{\partial x} \frac{\partial}{\partial z} \frac{\partial}{\$  $d^3k'$ <sub>2 is</sub>  $d^4E$  $=\frac{e}{Q_{\pm}^4}L^{\mu\nu}h_X^{\nu}$  $-\mathrm{tr}[k\gamma^{\mu}k^{\prime}\gamma^{\nu}] = k^{\mu}k^{\prime}$  $2E'$  The parton model  $d\Phi = \frac{w-w}{\sqrt{2\pi}} d\Phi_x = \frac{s\pi u}{2\pi} \frac{w+v}{u} d\Phi_y$  $\frac{1}{\sqrt{2}}$   $\left|\mathcal{M}\right|^2 = \frac{e^2}{Q_1^4} L^{\mu\nu} h_X^2 \alpha^2$  $\sup_{\mathcal{U}}\argsup_{\mathcal{U}}\frac{1}{2}\|\mathcal{U}\|_{\mathcal{U}}^2\mathcal{U}\mathcal{U}^2\|_{\mathcal{U}}^2$  and  $\mathcal{U}\mathcal{U}^2\|_{\mathcal{U}}^2$  and  $\mathcal{U}^2\|_{\mathcal{U}}^2$  and  $\mathcal{U}^2\|_{\mathcal{U}}^2$  and  $\mathcal{U}^2\|_{\mathcal{U}}^2$  and  $\mathcal{U}^2\|_{\mathcal{U}}^2$  and  $\mathcal{U}^2\|_{\$  $U$  $\sum_{i=1}^{n}$  $F_1(x, Q^2) +$  $\int_{-\infty}^{\infty}$  $p$ <sub>p</sub> $\alpha$  Maltoni  $p\cdot q$  $\overline{q}$  $\bigwedge$ *ME*  $\frac{\partial \mathcal{L}}{\partial x}$   $\frac{\partial y}{\partial y} dx$   $\frac{\partial \Phi}{\partial r}$ *e*4  $\frac{Q}{24} L^{\mu} \frac{dy}{h} \frac{d^{\Phi}x}{X^{\mu}}$  $\int_{0}^{\infty} \int_{0}^{\infty} k \, \frac{d^{2}y}{y^{2}} \, dy = k^{\mu} k^{\prime \nu} + k^{\prime \mu} k^{\nu} - g^{\mu \nu} k \cdot k^{\prime \nu}$  $-\frac{p\cdot q}{q\cdot}$  $T^2 \nu$   $\blacksquare$ p p  $p \cdot q$  $\overline{q^2}$  $\bigwedge$  $p_{\nu} - q_{\nu}$  $p \cdot q$  $\overline{q^2}$  $\setminus$  1  $\overline{p\cdot q}$  $F_2(x,Q^2)$ Why 1/*Q*4?

 $\frac{1}{2}$   $\blacksquare$  $\frac{1}{2}$  W +  $\frac{1}{2}$  STFC HEP school 2024 2  $q$  $F_1(z) = \alpha z$  $\frac{1}{\sqrt{2}}$  $p_1$   $p_2$   $q_3$  $\overline{\mathcal{C}}$ 





### **Parton Model**  $\blacksquare$



 $\sigma^{ep\rightarrow eX}$ 





Transverse photon Longitudinal photon

Eleni Vryonidou STFC HEP school 2024 \* Bjorken scaling ⇒ F1 and F1 and F1 and F1 and F1 and F1 and F2 obey scaling themselves, i.e. the school 2024<br>The school 2024

$$
K = \sum_{X} \frac{1}{4ME} \int d\Phi \frac{1}{4} \sum_{\text{spin}} |\mathcal{M}|^2
$$

After a bit of maths (good exercise!), we get:

 $d^2\sigma$  $dxdQ^2$ =  $4\pi\alpha^2$  $\overline{Q^4}$  $\int$  $[1 + (1 - y)^2] \left( F_1(x, Q^2) \right) + \frac{1 - y}{2}$ 

### **Parton Model Breit frame**  $\text{UCL}_{\text{interactions}}$   $\overbrace{\left(\begin{smallmatrix} \hat{A} \\ \hat{B} \\ \hat{B} \\ \hat{C} \end{smallmatrix}\right)}^{\left(\hat{A} \right)}$ moving close to the light-cone. In this case, the light-cone. In this case, the light-cone. In this case, the l  $T_{\rm lo}$



### $\overline{x}$  $[F_2(x, Q_0^2)_{\tau}$  -  $2xF_1(x, Q^2)]$  $\bigcap$ *ep*!*eX*  $y = (0,$  $q = (0, 0, 0, -Q)$ <br>  $\bigwedge \bigwedge \bigwedge \bigwedge \bigwedge \bigwedge$ *<sup>d</sup>* <sup>1</sup>  $\frac{1}{\sqrt{2}}$  $\sum$ spin  $\phi$ <sup>2</sup>,  $T_{\rm eff}$  $\hat{p} = (E, 0, 0, \xi p)$  $\hat{p}' = (E, 0, 0, p')$ *k*  $k<sup>^{\prime}</sup>$  $\frac{1-\mathcal{Q}}{F_2(x,Q^2)-2xF_1(x,Q^2)}$   $\frac{1-\mathcal{Q}}{F_2(x,Q^2)-2xF_1(x,Q^2)}$ we calculate  $\alpha_{m2}$ , which also its direction of propagation (which is propagated in propagation of propagation (which is  $\alpha$ points to the proton) such that *q*<sup>0</sup> vanishes. This frame is called the oton extent:  $\Delta x^+ \sim 1/Q$ . The proton moves fast and the photon has zero energy  $k$  $p \equiv$  $\int \sqrt{Q^2}$  $\frac{q}{4x^2}+m^2,$  $\overline{Q}$  $2x$  $, \vec{0}_{\perp}$  $\sum_{i=1}^{n}$  $\approx$  $\bigcap$  $2x$  $+$  $xm^2$  $\frac{m}{Q},$  $Q$  $2x$  $, \vec{0}_{\perp}$  $q \equiv$  $\bigg\{$  $\left( \partial_{\mu}\mathcal{A}\partial_{\nu}^{2}\vec{0}_{\perp}\right)$ .  $\mathcal{S}$  in the rest frame, the proton has a space-time extension has a space-time extension has a space-time extension of  $\mathcal{S}$  $\sim$  1We shall see in addition, in addition, we must work in the light-cone gauge, where  $\sim$  $\vert F_2(x, Q_{\lambda}) - 2xF_1(x, Q^2) \vert$ he proton  $\overline{\phantom{a}}$ ....<br>. ,  $\overline{\mathsf{V}}$ ves fas  $\overline{a}$   $\overline{a}$  $\frac{2}{\sqrt{2}}$  $t$  $\overline{\phantom{a}}$ pho  $\overline{a}$  $\ddot{\phantom{0}}$ **1** !  $h$ .<br>D  $E, 0, 0,$  $\overline{n}$  $\hat{p}' = (E, 0, 0, p')$  $\Delta x^+$  ∼  $\Delta x^-$  ∼ 1  $\overline{m}$  $\mathsf{Bridt}$  is a meq. Proton extent:  $\langle u, \Delta x \rangle \sim \frac{1}{2}$ , we  $\Delta x^{-2}$  is the light-cone gauge  $\Delta x$  $p = (E, 0, 0, p)$  $\mathbf{A}^2$ <sup>+</sup>  $\sim$  $Q$  $\frac{d}{dx}$ <sup>r</sup><sub>2</sub>(x,  $\frac{d}{dx}$ <sup>-2xf</sup>)  $\frac{1}{2}$  $\frac{\tau x}{Q},$ Photon extent:  $\Delta x^+$  ∼ 1/Q,  $\mu = (\mu, 0, 0, p)$  $\frac{1}{2}F_2(x,\mathcal{Q})$  $\frac{1}{\sqrt{2}}$  $x^{-2}$  $\frac{1}{2}$  ,  $\frac{1}{2}$  $\left(\Delta x^+\right)_\text{photon} \ll \left(\Delta x^+\right)$  $q = (0,0,0,-Q)$   $\left(\begin{array}{ccc} 0 & 0 & -Q \\ 0 & 0 & 0 & 0 \end{array}\right)$ its extension in the moving frame is  $\frac{1-Q}{\sqrt{n-1}}[F_2(x,Q_1^2)]\left\{$ proton .  $\sim 1/Q,$



 $d^2\sigma$  $\partial x d\partial z$  $\overline{\overline{\Upsilon}}$  $4\pi\alpha^2$  $Q^4$  $\int$  $\frac{1 - y}{1 - y}$ Rest frame: Proton extent: Hest frame: Proton extent:  $\Delta x$ ie: Proton extent:  $\Delta x \sim \Delta x$ Breit frame Proton extenti

### er of the photon can resolve part that the photon can resolve that the photon can resolve part of  $\mathbf{L}$  differentiate can differentiate can differentiate can differentiate between  $\mathbf{L}$  $\sigma$  thrie scale of a typical parton-parton interaction interaction time.  $t$  moves with very large moves with very large momentum towards the virtual photon. The virtual photon  $\mathcal{L}$ **•** In this frame the incoming power with a 3-momentum with a 3-momentum  $\frac{1}{2}$ along the *z axis, where* <br>axis, where *is the fraction of the fraction of the proton 3-momentum 3-momentum 3-momentum 3-momentum 3-momentum*<br>Axis, where is the proton 3-momentum 3-momentum 3-momentum 3-momentum 3-momentu nton-parton interact The control of the photon can resolve partons.









on interaction is muc :h larg The time scale of a typical parton-parton interaction is much larger than the hard

 $\frac{1}{2}$  Bjoren scaling ⇒ F1 and F2 obey scaling themselves, i.e. the scaling themselves, i.e. the school  $2024$ **p**z. The virtual photon moves with a 3-mode  $\sigma$  2024

### **Parton Model Breit frame** DIS: The parton model its extension in the moving frame is the moving frame is not the moving frame is  $\mathcal{C}^{\text{max}}$



$$
\hat{p} = (E, 0, 0, \xi p)
$$
\n
$$
\hat{p}' = (E, 0, 0, p')
$$
\n
$$
\hat{p}' = (E, 0, 0, p')
$$

- de<sup>2</sup>  $\equiv$ **soat** <u>}∫(</u> pf a typical parton-parton in
- $d$  $\kappa$ d $Q^2$  $\overline{Q_1^4}$  $\boldsymbol{\mathcal{X}}$ *v*: In the Breit frame the proton moves very fast toward Lorentz contracted to a kind of pancake. • Schematically: in the Breit frame the proton moves very fast towards the photon, and is therefore
- that pancake. Breit frame or infinite momentum frame since the proton dipole and the proton and that  $\boldsymbol{\theta}$  is much and dipole interaction. If the lifetime of the dipole is much larger • The photon interaction then takes place on the very short time scale when the photon passes
- During the chart interaction time, the struck quark thus does **Danny are short interaction** quarks and can be regarded as a free parton. the interaction time the struck quark thus dees not int quarks and can be regarded as a free parton. along the *z axis, where*  $\alpha$  *axis, where*  $\alpha$  *is the fraction of the proton 3-momentum of the proton 3-momentum 3-momentum* • During the short interaction time, the struck quark thus does not interact with the spectator





orditz contracted to a ning or pancatte.<br>he photon interaction then takes place on the very short time scale when the photon passes

d.

dz





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*<i>ne* partgns are, f<br>  $\frac{d^2\sigma}{dx^2}$   $\frac{d^2\sigma}{dx^2}$   $\frac{d^2\sigma^2}{dx^2}$ The space-time picture suggests the possibility of separating short- and long-distance physics

 $\frac{\partial^2}{\partial^2}g^2$ ⇠ *, Q*<sup>2</sup>)  $dxd$  $4\pi\sqrt{\frac{2}{i}}$  $\hat{\bm{\sigma}}$  $\frac{d\mathcal{U}d\mathcal{V}}{dxdQ^2}$  $x^2$ ⇠ *, Q*<sup>2</sup>)  $f_i(\xi) \frac{d^2\alpha} {d\alpha d\Omega^2}$  $\frac{d}{dx}q$ ⇠  $Q^2$ 

ˆ



is the probability to find a in an t with the probability adjusted a **rt was mentum sup a rito in** is the probability to find a nnenium cpt cone momentum ξp+ parton with flavor i in an hadron h carrying a light-(*ξ*) Probability of finding parton in hadron *i* cone momentum ξp+ *f i* carrying momentum fraction *ξ*



is sections for epart of orelectron-parton scattering for electron-parton scattering ˆ  $1^0$ is the cross section for elation-parton sca Crossissertiggsforeparton-photon scatter filt

Fabio Maltoni<br>Fabio Maltoni



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## **DIS cross-section** DIS: The parton model DIS: The parton model

$$
\frac{1-y}{[4x_2(x_2^2 + 2x_1^2 + 2x_2^2 + 2x_2^2 + 2x_1^2 + 2x_2^2 + 2x_1^2 + 2x_1^2
$$

Comparing our inclusive cross-section: functions (which, BTW, are physical measurable quantities), Comparing our inclusive cross-section: functions (which, BTW, are physical measurable quantities), DIS: The parton model

 $d_2^2\sigma$  $\frac{du}{dx}$  $\frac{1}{2}$  $4\pi\alpha^2$  $\frac{24}{9}$  $\int$  $[1 + (1 - y)^2]F_1(x^2 + y^2 + y^2)$  $\mathbf{z}$ |
|-<br>|- $\frac{u}{\alpha}$  and  $\frac{u}{\alpha}$  are particular  $\alpha$  $u \overline{u} u \overline{u}$  for  $\overline{i}$  and  $\overline{u}$  $i = q, \overline{q} \cup$  $\frac{u}{1}$  $\frac{1}{2}$  $\frac{1}{2\,dx} =$  $\frac{1}{2}$  $2^{2}$  1  $\frac{1}{2}$  $\frac{1}{1}$ !<br>.<br>.  $d^2\sigma$  $\frac{1}{dQdQ^2}$  $\overline{\mathsf{L}}$  $d\dot{\mathcal{E}}$  $\frac{1}{2}$ dξ ξ e 2 57 i  $\frac{d^2\sigma d^2\sigma x}{dx^2}$  $\frac{d^2 \sigma x}{d^2 \sigma^2}$   $\left(\frac{d^2}{\epsilon}\right)^2$  $\ddot{\phantom{1}}$  $\mathcal{K}_{\dot{q}}$  $2\Omega^2$  $F_2(\mathbb{E}) (\vec{x})$  2xFin  $=2x$  $\frac{1}{2}$  $\frac{1}{2}$  $\mathbf{0}$  $\int_0^1$   $\int_0^1$   $\frac{1}{2}$   $\int$  $\zeta) =$ e<br>2  $\begin{array}{ccc} \hline \end{array}$  $d^2\frac{Q}{\sqrt{2}}$  $dx$ d $\partial x$  $\frac{1}{3}$  $4\pi$  $4\pi$  $\overline{Q^4}$ **)**  $\frac{1}{4} \left[ \frac{1}{4} + \frac{1}{4} \frac{1}{4$  $\overline{x}$  $\overline{\phantom{a}}$  $E_2(F_2 \backslash x) \geq 2xF_1(x) \geq 2xF_2(x)$  $\pmb{\mathfrak{c}}$  $\mathbf{b}$ Factorised cross-section in the parton model.  $\omega \omega \omega$  for  $\omega$  and  $\omega$  and  $\omega$  and  $\omega$  and  $\omega$  $i = q, \overline{q}$   $\overline{q}$   $\overline{q}$  $^{2}$ Wi  $d<sub>q</sub>$  $\overline{d\zeta} \, \overline{dQ^2}$  $\frac{1}{1}$  $47$  $\overline{a}$  $\cdot$  $\sum$ !<br>!<br>!  $\frac{1}{2}$   $\left[1 + \left(1 - \frac{1}{2}\right)\right]$  $\ell$  $\epsilon$  $d^2\sigma$  $dx d\widehat{d}\widehat{d}$  $\frac{1}{2}$  $\int_0^1$ 0 dξ ξ 5 .<br>i  $\int_{i}^{1}(\xi) \frac{d^{2} \sigma_{0}}{d\tau}$  $\left(\sum\limits_{\alpha\beta}^{\infty}\overline{d\alpha}\right)$  $\overline{x}$ ξ  $, (3^{2})$  $\zeta_2(x)$  :  $\overline{\mathcal{C}}$  $\frac{d}{dt}$  $\epsilon^2_{\alpha}\delta(x)$ e<br>Propinsi Ka  $\frac{d_2}{d^2}$  $\sigma$  $d$ æ $d$  $\sqrt{2}$  $\equiv$  $4\pi a^2$  $Q^4$  $-\int$  $\{ [1+ (1+2+2+1)] \frac{1}{2} \} \frac{1}{2} \$  $x^{\prime}$  $\sqrt{}$  $E_2(2^2)\sqrt[3]{2}$   $2^2E_4(x, \sqrt[3]{2})\sqrt[3]{2}$ **|
|**  $\mathbb Q$ actorised cross-section  $\frac{1}{100}$  find  $\frac{1}{100}$  find  $\frac{1}{100}$  find  $\frac{1}{100}$  find  $\frac{1}{100}$  $i = q, \overline{q}$   $\overline{q}$   $\overline{q}$ ITA.  $d^2\hat\sigma$  $\hat{\sigma}$  $dQ^2$ dx =  $4\pi\alpha^2$  $Q^4$ 1 2  $\sqrt{2}$  $1 + (1 - y)$  $\left| \frac{2}{\pi} \right|$  $d^2\sigma$  $d\widehat{d} \widehat{d} \widehat{Q^2}$  $\bm{\nparallel}$  $\mathbb{R}$  $\delta$ dξ ξ  $\sum_i$ '<br>Ú  $f_i\left(\frac{d^2}{d^2}\right)^2$  $\frac{1}{424}$  $\overline{\mathcal{X}}$ ξ  $\partial^2\partial^2\partial\hat{V}$ ith :)  $=2$ we find to the finite internet. 7  $\begin{array}{|c|c|c|c|}\hline \multicolumn{1}{c|c|}{\textbf{0}-\textbf{c}} & \multicolumn{1}{c|}{\textbf{0}-\textbf{c}} & \multicolumn{1}{c|}{\textbf{0}-\textbf{c}}\hline \multicolumn{1}{c|}{\textbf{0}-\textbf{c}} & \multicolumn{1}{c|}{\textbf{0}-\textbf{c}}\hline \multicolumn{1}{c|}{\textbf{0}-\textbf{c}} & \multicolumn{1}{c|}{\textbf{0}-\textbf{c}}\hline \multicolumn{1}{c|}{\textbf{0}-\textbf{c}}\hline \multicolumn{1}{c|$  $x-\xi$ e<sub>2</sub>  $\sqrt{ }$  $d^2\sigma$ dxdQ<sup>2</sup>  $\overline{\mathbf{S}}$  $4\pi\alpha^2$ Q4  $\int$  $\frac{x}{y}$  we have  $\frac{1-y}{y}$  $\overline{x}$  $\sqrt{2}$  $\chi^2$ ,  $\left($  $\frac{2}{41}$  functions (1,  $\frac{1}{21}$   $\frac{1}{2}$   $\frac{1$  $\partial^2 \sigma d^2 \sigma x$   $\partial^2 \rho x$  $Q\!\!\!\!{}^{\displaystyle 4} \hat{\sigma}$  $\overline{O}$  $\overline{dQ^2dx}$ =  $4\pi\alpha^2$  $\overline{Q^4}$ 1 2  $\begin{bmatrix} \phantom{-} \end{bmatrix}$  $\overline{d^2\omega}$  $\overline{dxdQ^2}$ =  $\mathcal{C}Q_{\mathcal{S}}$  $\overline{0}$ dξ  $\xi$  $\sum$ i  $f_i(\xi) \frac{d^2\sigma}{d^2\zeta}$  $\frac{d\hat{x}dQ^2}{d\hat{x}dQ^2} \big($  $\mathcal{X}$  $\xi$  $\binom{w}{2}$  $F_2(x)=2xF_1 = \sum$  $i=q,\bar{q}$  $\int_0^1$ 0  $d\xi f_i(\xi) \, x e_q^2 \delta(x-\xi) = \sum$  $i=q,\bar{q}$  $e_q^2\,x f_i(x)$ We can express the structure functions as:

 $\underbrace{\underbrace{\text{U} \text{C} \text{L}}_{\text{catholique}}}_{\text{de Louvain}} \left(\underbrace{\text{A}}_{\text{D}}\right)$ 

$$
\frac{d^2\hat{\sigma}}{d\hat{\sigma}^2} = \frac{4\pi\alpha^2}{\theta^4 \hat{\sigma}^2} \left[ 1 + (1 - y)^2 \right] e_q^2 \delta(x - \xi) - \xi
$$

$$
\frac{d^2\hat{\sigma}}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \frac{1}{2} \left[ 1 + (1 - y)^2 \right] e_q^2 \delta(x - \xi)
$$







[1 + (1 − y) <sup>2</sup>]F1(x, Q2) + <sup>1</sup> <sup>−</sup> <sup>y</sup>  $\overline{x}$  $\left| \frac{1}{2} \right| \left( \frac{1}{2} \right)$   $\left| \frac{1}{2} \right| \left( \frac{1}{2} \right)$ 

 $d^2\hat\sigma$  $dQ^2dx$ =  $4\pi\alpha^2$  $\overline{Q^4}$ 1 2  $[1 + (1 - y)^2] e_q^2 \delta(x - \xi)$  $, Q<sup>2</sup>)$ 

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# $f_{\vec{l}}(x)$  are the parton distribution functions which describe the probabilities of finding



 $Q^4$ 

 $dx dQ^2$ 

we find (be careful to distinguish  $\mathcal{L}(\mathcal{L})$  to distinguish  $\mathcal{L}(\mathcal{L})$  and  $\mathcal{L}(\mathcal{L})$ We can express the structure functions as:

 $s$  particles  $s$  and  $\Omega$  is fact if the scalar we want  $\Omega$ No dependence on Q: **Scaling** 

specific partons in the proton carrying momentum fraction  $x$ *i* (*x*)

and anti-quarks enter together : in How can we senarate them?  $\sim$  FL(x) and the facture at LO (Callange at LO (Callandon), which is a test that  $\sim$ Quarks and anti-quarks enter together. How can we separate them?

$$
F_2(x) = 2xF_1 = \sum_{i=q,\bar{q}} \int_0^1 d\xi f_i(\xi) x e_q^2 \delta(x - \xi) = \sum_{i=q,\bar{q}} e_q^2 x f_i(x)
$$



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14



The sea is NOT SU(3) flavor symmetric. The  $\Box$   $\int_0^1$ The  $\sum_{i=1}^{\infty}$  as  $\sum_{i=1}^{\infty}$ and  $\alpha$   $J_0$  $\mathbf{r}$ 

Note that there are uncertainty

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### **Parton distribution functions** Probed at scale Q, sea contains all quarks flavours with mq less than Q. For Q ∼1 we expect DIS: The parton model

For Q ∼1 we expect

 $T_{s,s}$   $\frac{1}{s}$   $\frac{1}{d}$  only  $\frac{1}{d}$  functional momentum. The rest is carried by  $\frac{1}{d}$ Although not directly measured in DIS gluons participate in other hard scattering  $p = \frac{p}{10^3}$   $p^2$   $p^1$   $p^1$  **processes such as large-pt and prompt photon production.** Quarks carry 10 nbyt 50% pot the proton momentum gluons. Although not directly measured in DIS, gluons participate in other hard scattering **Evidence for gluons!** 

Probed at scale Q, sea contains all quarks flavours with mq less than Q.

$$
\int_0^{\infty} dx \, u_V(x) = 2 \, , \, \int_0^{\infty} dx \, d_V(x) = 1 \, .
$$

$$
(\theta) + \bar{u}(x) \qquad \int_0^1 dx \ u_V(x) = 2 \ , \ \int_0^1 dx \ d_V(x) = 1
$$

$$
\sum_{b_i}^{f} \int_0^{f} \frac{1}{x} \, dx \, x[q(x) + \bar{q}(x)] \simeq 0.5 \; .
$$



# **Parton model summary**

15

## DIS experiments show that virtual photon scatters off massless, free,

point like, spin-1/2 quarks

One can **factorise** the short- and long-distance physics entering this process. Long-distance physics absorbed in PDFs. Short distance physics described by the hard scattering of the parton with the virtual photon.

 $\sum_{a,b} d x_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(s)$ 

∑



# **Parton model summary**

15

## DIS experiments show that virtual photon scatters off massless, free,

Phase-space integral Parton density functions Parton-level cross section

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 $\int_{a,b} dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(s)$ <br>
Phase-space integral Parton density functions Partor

∑

 $dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$ 

$$
\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)
$$
  
Phase-space integral Parton density functions Parton-level cross section

Universal:

~Probabilities of finding given parton with given momentum in proton Extracted from data

Important aspect of a Monte Carlo generator

Subject of huge efforts in the LHC theory community to systematically improve this





# **Master formula for LHC physics**

Parton density functions **Exercise 1** Parton-level cross section

$$
\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)
$$
  
Phase-space integral Parton density functions *Factor-level cross section*

Universal:

~Probabilities of finding given parton with given momentum in proton Extracted from data

Important aspect of a Monte Carlo generator

Subject of huge efforts in the LHC theory community to systematically improve this



# **Master formula for LHC physics**

## **Higher order corrections R-ratio@NLO**



:<br>∷ 1 with a final state with no gluon at all  $\mathbf{v}$  $\sigma_{NLO} = \sigma_{LO} + \int_R |M_{real}|$ 

 $|M_{real}|^2 d\Phi_3 +$ 

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Real

Virtual

Eleni Vryonidou a state state of the state of the state of the state of the STFC HEP school 2024 !  $\mathbf{V}$  $2Re(M_0M_{virt}^*)d\Phi_2 = \text{finite}!$  $M_{\phi ir}^*$ dk  $\frac{-\omega_i r^2}{(2\pi)^d}$  ...  $\boldsymbol{c}$  $\frac{2}{V}d\Phi_3 + \int_V 2\text{Re}(M_0^2M_0^*d) d\Phi_2$ 

σNLO

=

R





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$$
\frac{1}{\hbar} \text{th} \frac{\partial}{\partial \theta} \text{d} \theta \text{ in is soft}^*(k_0^{\overline{u}(p)\Gamma^{\mu}y(\overline{p})}) \text{ or collinear } (\theta \to 0) \text{ to the quark?}
$$

$$
\begin{aligned}[t] i g_s)&\frac{-i}{p\!\!\!/_+k}\!\Gamma^\mu v(\bar{p}) t^a+\bar{u}(p)\Gamma^\mu\frac{i}{\bar{p}\!\!\!/_+k}(-ig_s) \epsilon\!\!/\nu(\bar{p}) t^a\\&\frac{p)\epsilon\!\!\!/_\!\!\!/_+\!\!\!/_k)\Gamma^\mu v(\bar{p})}{2p\cdot k}-\frac{\bar{u}(p)\Gamma^\mu (\bar{p}\!\!\!/_+\!\!\!k)\epsilon\!\!\!/\nu(\bar{p})}{2\bar{p}\cdot k}\Big]t^a\end{aligned}
$$

$$
\int_{\text{es}}^{L} \mathbf{g} \cdot \mathbf{g
$$

$$
\begin{array}{ll}\n\sqrt{2} \left( \frac{p_i}{p_i} \right) \left( \frac{p_i}{p_i}
$$

 $\text{Sott emissi}$ Soft emission factor is universal!

Eleni Vryonidou and STFC HEP school 2024 Fabio Maltoni and STFC HE



$$
A_{soft} = -g_st^a\left(\frac{p\cdot\epsilon}{p\cdot k} - \frac{\bar{p}\cdot\epsilon}{\bar{p}\cdot k}\right)A_{Born} \qquad \qquad \mathbf{dist} \mathbf{g}_{orm} = \bar{u}(p)\Gamma^{\mu}v(p)\mathbf{g}_{\mu} \mathbf{g}_{\nu} \mathbf{q}_{\nu}
$$

 $(\theta \rightarrow 0)$  to the quark? Engerty of QCD of long-wavelength from the short-

distance (hard) scattering!

$$
\bar{u}(p)\bigotimes \bigodot \mathbf{G}\bigodot \mathbf{j}\bigodot \mathbf{j} \mathbf{A}^{\dagger} \mathbf{k}\bigodot \bigodot \mathbf{j}\bigodot \mathbf{A}^{\dagger} \mathbf{j}\bigodot \mathbf{k}\bigodot \mathbf{i} \mathbf{j}\bigodot \mathbf{j}\bigodot \mathbf{k}\bigodot \mathbf{j}\bigodot \mathbf{k}\bigodot \mathbf{k}\bigodot \mathbf{j}\bigodot \mathbf{k}\bigodot \mathbf{k}\
$$

k, and a structure of the structure of the

 $\mathbf{v}$ 

Eleni Vryonidou Two collinear divergences and a soft one. STFCHEP school 2024 Fabio Maltoni integration over phase space and a soft one. STFCHEP school 2024 Fabio Maltoni integration over phase space and a soft one. The st



## **QCD in the final state** R-ratio@NLO

By squaring the amplitude we obtain:  $\bar{p}$ , *i* REAL and Born  $\int$  $\bar{p}_{\rm \bf k}j$  $\int d^3k$  $\bar{p},\overline{j}$  $\frac{d^3k}{2k^0(2\pi)^3} 2 \frac{p\cdot \bar{p}}{(p\cdot k)(\bar{p})}$ REAL  $\alpha$  2 Born  $d^3k$  $\sigma_{q\bar{q}g} \equiv Q_{\bar{q}}^2 g^2$  $\sigma_{q\bar{q}g} \equiv_{\eta} Q_{\bar{q}\bar{g}}^2 g_s^2 \sigma_{q\bar{q}}^{\rm Born} \quad | \quad \frac{d\bar{q}}{d\bar{q}}$ By squaring the amplitude we obtain:  $\frac{1}{2}$  and the substantial  $\int_{0}^{1} k \, du^{3}k$  $\frac{1}{2} \frac{k_1}{2} \frac{d^3k_1}{d^3k_1^2} \frac{\sigma_{q\bar q g}}{p \cdot \phi} = \frac{C_f}{2} \frac{d^3k_1}{(p \cdot k_0)(q\bar p)}$  $\mathbf{R}$  it we assign to  $k$ ,  $d^{3}k$  $\frac{k}{2k}\frac{d^3k}{(p\cdot k)^3}$   $\frac{2}{(p\cdot k)(q)}$  $\overline{(p \cdot k)(\bar{p} \cdot k)}$ By squaring the amplitude we obtain e afmalityde we olatar<br>O gāg<sub>g</sub>āg Lichsg<sub>s</sub>gāgā  $k, d$ **TIRHAULGE AVE BORGH!**  $\frac{1}{3}$  $\frac{1}{2}$  $\frac{1}{\sqrt{2}}$  $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$  $\int \sqrt{2M^2(R)}$  $(p \cdot k)(\bar{p} \cdot k)$  $\frac{1}{2}$ k<sup>R</sup>(2 $\pi$ )<sup>3</sup>  $\frac{2}{(p \cdot k)}$  $(p \cdot k)$  $\frac{d^3}{dx^0_0} \frac{(p \cdot k) (\sqrt{p} \cdot k)}{dx^0_0}$  Born  $\int d \cos \theta \frac{dk^0}{dx^0_0}$ <u>rk</u>  $\alpha_{\mathbf{q}}$ <sub>g</sub> $\overline{\mathbf{q}}$  $\frac{8}{s}\frac{\sigma}{\Omega}$ Born 4  $\mathcal{D}$   $\mathcal{P}$   $\mathcal{$ !  $\Phi_{\alpha\overline{\alpha}}^{\rm Born}$  $\frac{1}{1-\kappa}\sqrt{\frac{1}{\kappa}}$  $d d c \cos \theta \frac{3}{100}$  $\sum\limits_{\mathcal{O}}\alpha$  $q\bar{q}$  $d\cos\theta \frac{3}{10}$  $\bar{\mathcal{A}}_S$  $\mathcal{E}(p = k)(E)$ 4  $k, a$ 2 **NH /j**  $\overline{q}\bar{\bar{q}}$  $S$  $B$  $B$ Bomn  $\frac{a\kappa}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\left(\frac{\sqrt{2\pi}}{2\sqrt{2\pi}}\right)$  $\stackrel{a}{=} \epsilon_f$  $k^0$  $\epsilon_F$  $(1 - \cos \theta)(1 + \cos \theta)$  $\sqrt[d]{q}$  $\overline{\vec q \vec q}$  $p, i$  $(1-\cos\theta)\sqrt[4]{1+\cos\theta}$  $2\pi$ **k**  $\frac{1}{2\pi}$ *kg*  $p, i$  $d \cos \theta$  $\overset{\cdot F}{\alpha_S}$  $\left\langle \right\vert$ !<br>!<br>!  $\sigma_{\lambda\pi}^{\rm Born}$  $=\mathbb{C}_F$  $\frac{1}{2} \int_{F} \frac{1}{\sqrt{2}} \sigma_{y}^{1} dx$ Γµ<br>ΠΗ sent price: Very offen vou til  $\overline{\dot{q}}\bar{q}$  $\frac{1}{\sqrt{1-x^2}}$  or  $\frac{1}{\sqrt{1-x^2}}$  or  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^2}}$  or  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^2}}$  of  $\frac{1}{\sqrt{1-x^$ **PAGE** Two collinear diverge hease and a spit one. We ty often you find the integration over phase space Two collinger divergences and a shirt one. Very often you find the integration of the integration of the space rt bile: Verworten von ting the free den divergies of space.<br>expressed in terms of the kuack and and the montgeliging and a colline and any englence  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$ pu<sub>r</sub>u unu expressed in terms of x1 and x2, the fraction of milenings of the kanack and antiexpressed in terms of x1 and x2, the fraction of energies of the quark and anti-quark: Vergences and des souls one. Were bitten you fried the ar divergences and or soul of Two collinear divergences and a soft one.  $\frac{\Psi_1(\mathbf{p}) \mathbf{F}(\mathbf{p})}{\mathbf{F}(\mathbf{p}) \mathbf{F}(\mathbf{p})}$  of  $\frac{\Psi_2(\mathbf{p})}{\mathbf{F}(\mathbf{p})}$  and  $\frac{\Psi_3(\mathbf{p}) \mathbf{F}(\mathbf{p})}{\mathbf{F}(\mathbf{p})}$  of  $\frac{\Psi_4(\mathbf{p})}{\mathbf{F}(\mathbf{p})}$  of  $\frac{\Psi_5(\mathbf{p})}{$  $x = 1$   $y = x$   $x = x + 3$   $y = x + 3$  $\frac{11004}{100}$  $\exp$ ressed in terms of  $x_0$  and  $x_0$  the fraction of energies of the collinear soft  $x_2$  terms  $x_2$ ,  $x_3$  and  $x_3$  the fra  $\frac{1}{2}$  $x_2 = 1 - \frac{2E_{\overline{q}}}{\overline{q}}$  and  $x_2$  and  $x_3$ soft  $x_2 = 1 - x_1 x_3 (1 - \cos \theta_{13})/2$ soft  $x_1 + x_2 + x_3 = 2$  $\frac{\overline{q}}{\overline{q}}$  $x_1 = 1 \frac{x_1}{2} \frac{x_2}{4} x_3^2 \frac{x_3}{4} \left(\frac{x_3}{2} \frac{x_3}{6} \frac{x_2}{6}\right)^2$  $\frac{1}{2}$   $\mathcal{L}_2^2$  ( $\pm \frac{\mathcal{L}_3}{2}$   $\overline{c}$   $\overline{6}$   $\overline{5}$   $\theta_{23}$  )  $T_x = \frac{1}{2} \frac{\sqrt{1}}{2} \frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2} \left( \frac{\sqrt{2}}{2} \frac{\sqrt{2}}$  $\overline{x}_1^1+\overline{x}_2^1\overline{x}_3^2$ e  $\frac{u_1}{1} + \frac{u_2}{1} + \frac{u_3}{1}$  and  $r_0 + r_0 > 1$  is the Dirac equation, the amplitude simplifies in the sim  $0 \leq x_1, x_2, y_2 \leq 1$   $\log \left( \frac{1}{2} \right)$  $x_2 = 1 + \frac{0}{2}x_1^2x_3^4$  =  $\frac{1}{2}cos^2\theta_{13}^1$   $/2$ collinear software the collinear software of the collinear software software that the collinear software softwa<br>Collinear software software software in the collinear software software software software software software so a  $x_2 = 1 - x$  fix  $3(4 - \cos 7)$ **collinear**  $0 \leq x_1, x_2 \leq 1, \text{ and } 13/(x_1 + x_2) \leq 1$  $2 \ge 1$   $1 \le x \le 1, \quad \epsilon$ <sup>A</sup>Born = ¯u(p)Γ<sup>µ</sup> A <sup>v</sup>(¯p) sof t = −gst a ! p · ! <sup>p</sup> · <sup>k</sup> <sup>−</sup> <sup>p</sup>¯ · ! p¯ · k  $x_1 + \overline{\mathcal{X}}_2^{\alpha_1, \alpha_2}$  and  $\overline{\mathcal{X}}_3^{\alpha_1}$ **M**  $\overline{S}$  we can now predict the divergent part of the virtual collinear ABorn  $c$  entribution, predict the dheer senite part and  $c$  the  $r$ So wentup unow, predict the divergent part of the virtual Some part of the divergent part of the divergent part of the virtual of the virtual of the virtual of the virtual order calculation is necessary:  $0 \leq$ ontfillition is necessary the finite pla  $x_1=\frac{2E_q}{\sqrt{s}}$  $\epsilon$ ontribution, l'while for  $x$ he finite part an explicit  $\frac{a_1}{b_1}$  is  $\frac{b_2}{b_1}$  and  $\frac{c_1}{b_1}$  is  $\frac{c_2}{b_1}$  is  $\frac{c_1}{b_1}$  is  $\frac{d\mathbf{r}}{d\mathbf{k}_0^T}$  $\frac{1}{\sqrt{16}}$   $\frac{1}{\sqrt{16$  $\alpha_S$ 1 !<br>|
| calculation is necessary  $C_F$ .<br>0  $2\delta(k_0')[\delta(1-\cos\theta')+\delta(1+\cos\theta')] + \dots$  $\log$ eale d'attion is necessary.  $\frac{1}{q\bar{q}}\frac{\log\log q}{q\bar{q}}$ **k!! 1 is neces 2**<br>2 **deale diaty**<br>gent partice cols<sup>2</sup> So we can  $\mu_{\rm p}$  predict, the  $\alpha$  vere part  $\kappa_{\rm p}$  the virtual  $\Phi$ 1866do fanas Babjerratult **d**  $26$ (k=)[sehdo|2025 Bb)e|} $31$  $\frac{C^{210}}{c}$  precision incorporation  $\frac{C^{210}}{c}$  for  $\frac{C^{210}}{c}$ 0  $)\Big]+...$ 1  $\log$  2028  $\beta$ b)  $\gamma$ b $(1_0$  $\mathcal{L} = \left[\frac{\text{E}}{\text{E}}\right]$   $\mathcal{$  $\frac{1}{2}$   $2\delta(k)$  $\frac{1}{2}$  $\delta(1-\cos \theta')$  $+\delta(1+\cos\theta^{\prime})$ 下承有5个 **h!!!** contribution, while  $q\bar{q}$  for the finite draw  $R$  and explane  $10 - 6 \times 12^2$  $\overline{\mathbf{h}\mathbf{e}}$ GGI Florence - 2017 Fabio Maltoni Fabio Maltoni k!  $\sim$ 1 − cos<del>2 de la 2 de la 2 de la 2 de la 2 d</del>







22

$$
=\frac{c_F \frac{cos_B}{2\pi r} \frac{1}{\sigma^2 \pi^2} \
$$



### **What happens now?**

# **IR singularities**



IR singularities arise when a parton is too soft or if two partons are collinear

• Infrared divergences arise from interactions that happen a long time after

- the creation of the quark/antiquark pair.
- When distances become comparable to the hadron size of  $\sim$ 1 Fermi, non-perturbatively.

quasi-free partons of the perturbative calculation are confined/hadronized

## **How do we proceed with our calculation?**



### **Cancellation of divergences** Anatomy of a NLO calculation Summary:

1 +

### In practice: regularise both c divergences, with either dimensional regularisation or mass regulator R Biotuce: regularizatis existentiate" divergences, by giv Solution: regularize the "intermediate" divergences, by giving a gluon a mass (se  $\mathbb{E} \mathbb{A}^1$  ,  $\mathbb{A}_1$  $\frac{1}{2}$ ,  $\frac{x}{\text{diverg}}$  $\text{H}$ **x give 50** regularization  $\int_0^1 \frac{(1-x)^{-2\varepsilon}}{x}$  $\frac{1}{\alpha}$   $\frac{x}{\alpha}$  $(\theta_1 - \theta_2)^{N} = C_F \frac{1}{4} \frac{1}{\pi} \sigma^{DOM}$  3  $R_1 = R_1$  $R_1 = R_0 \left(1 \frac{\epsilon - \lambda Q}{\epsilon}\right)$  $\overline{1}$  $\overline{4}$ ↵*<sup>S</sup>*  $\overline{\mathcal{U}}$  $\lim_{\epsilon \to 0} (\sigma^{\rm REAL} + \sigma^{\rm VIRT}) = C_F \frac{d}{4} \frac{dS}{d\pi} \sigma^{\rm Born}$  3  $R_1 = R_0 \left(1 + \frac{dS}{d\pi}\right)$  $\sqrt{2}$  $\alpha_{S}$  $\sum_{i=1}^{n}$  $\sigma^\mathrm{REAL} = \sigma^\mathrm{Boin} C_F^{-1}$ ↵*<sup>S</sup>*  $2\pi$  $\overline{C}$  $rac{\bar{\sigma}}{\epsilon^2}$  + 3 ✏  $\equiv \downarrow$ 19  $\frac{12}{2}$   $C_{\rm F}$   $\frac{2}{4}$  $\mathcal{C}_{\mathcal{F}}$  $\sigma_{\rm en}^{\rm VIFTT}(\frac{\epsilon^2}{2}\sigma_{3}^{\rm Bori}C_F)$  $\,\,\phi_{S}$  $2\pi$  $\left(\frac{2}{\frac{\text{tip}}{\epsilon}}\right)^2\left(\frac{3}{\epsilon}\right)^2 + \frac{3}{\epsilon^2}$  $\frac{\alpha_S}{R} \left( \frac{2}{\epsilon^2} + \frac{3}{\epsilon^2} + \frac{19}{\epsilon^2} - \pi^2 g \right)$ ularize the "intermediate" diverge  $\sigma^{\rm{REAL}}+\sigma^{\rm{V\!HRT}}$  these gives  $\beta$ Solution: regularize the "intermediate" divergences, by giving a gluon a mass (see later) or going to  $dx = -\log 0 \stackrel{\text{regularization}}{\rightarrow}$  $\int_0^1 \frac{(1-x)^{-2\epsilon}}{x}$  $1 - \alpha$ <sup>x</sup>  $dx = \frac{1}{2}$  $2\epsilon$  $(\sigma^{\rm REAL} + \sigma^{\rm VIRT}) = C_F$ 3 4  $\alpha_S$  $\pi$  $\sigma^{\rm Born}$   $\ge R_1 = R_0$ αS  $\frac{1}{\pi}$ "  $\sigma^{\rm REAL} = \sigma^{\rm Born} C_F$  $\alpha_S$   $\langle$  2  $\rangle$  3  $+$  $19\qquad \qquad 2)$  $\sigma^{\rm VIRT} = \sigma^{\rm Born} C_F$  $\alpha_S$  $2\pi$  $\left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right)$  $\overline{p_{\pi}}$   $\overline{\lim}$  $\overline{O}$  $\sigma^{\rm REL} + \sigma^{\rm VIRT}$ d=4-2ε dimensions. z grites  $1 - x$  $\partial x = 0$   $\partial y$  regularization  $\epsilon \rightarrow 0$  $(\sigma^{\rm REAB}_\epsilon + \tau^{\rm AV\!IRT}) = C_F$  $1 +$  $\alpha_S$  $\overline{\pi}$ "  $\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F$  $\alpha_S$  $2\pi$  $\frac{J}{\ell}$  2  $rac{1}{\epsilon^2}$  +  $\frac{ZV}{\sigma}$  +  $\frac{ZV}{\sigma}$  =  $\sigma$  Born $C_{\pi}$ ↵*<sup>S</sup>*  $2\pi$ This givest

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 $R_1$  =

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R<sub>1</sub> = R<sub>1</sub>  $\sqrt{2}$ 1 +  $\alpha_{S}$  $\sum_{i=1}^{n}$  $G_{\mathcal{A}}$  florence - 2017  $\mathcal{A}$  florence -

### **Cancellation of divergences** Anatomy of a NLO calculation Summary:



### 24



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R<sub>1</sub> = R<sub>1</sub>  $\sqrt{2}$ 1 +  $\alpha_S$  $\sum_{i=1}^{n}$ as presented before the present of the control of the present of the present of the present of the present of t  $G_{\mathcal{A}}$  florence - 2017  $\mathcal{A}$  florence -

### **Cancellation of divergences** Anatomy of a NLO calculation Summary:



2 collinear partons

Kinoshita-Lee-Nauenberg theorem: Infrared singularities in a massless theory cancel out after summing over degenerate (initial and final) states Kinoshita-Lee-Nauenberg theorem: Infrared singularities in a massless over degenerate (initial and final) states



## **KLN Theorem Why does this work?**



Physically a hard parton can not be distinguished from a hard parton plus a Physically a hard parton can not be distinguished from a hard parton plus a soft gluon or from two collinear partons with the same energy. They are degenerate states. A final state with a soft de verdielende ook Henale will a mial stale will no giuon al all (VII) gluon is nearly degenerate with a final state with no gluon at all (virtual) **Hence, one needs to add all degenerate states to get a physically sound observable** 

## **Infrared safety How can we make sure IR divergences cancel?**

We need to pick observables which are insensitive to soft and collinear radiation. These observables are determined by hard, short-distance physics, with long distance effects suppressed by an inverse power of a large momentum scale.

Schematically for an IR safe observable: *O*

whenever one of the k<sub>i</sub>/k<sub>i</sub> becomes soft or k<sub>i</sub> and k<sub>j</sub> are collinear

- $\mathcal{O}_{n+1}(k_1,k_2,\ldots,k_i,k_i,\ldots k_n) \to \mathcal{O}_n(k_1,k_2,\ldots k_i+k_i,\ldots k_n)$ 
	-

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## **Which observables are infrared safe?** Infrared safety: examples

- I energy of the hardest ‣ energy of the hardest particle in the event
- $\blacktriangleright$  multiplicity of gluons ‣ multiplicity of gluons
- if a momentum flow into a cone in rapidity and angle ‣ momentum flow into a cone in rapidity and angle
- ▶ jet cross-sections





## **Which observables are infrared safe?** Infrared safety: examples

- I energy of the hardest ‣ energy of the hardest particle in the event
- $\blacktriangleright$  multiplicity of gluons ‣ multiplicity of gluons
- momentum flow into a cone in rapidity and angle **ES** ‣ momentum flow into a cone in rapidity and angle
- → jet cross-sections DEPENI



ENDS DEPENDS NO NO **YES** 



# **Event shapes**







### pencil-like spherical

- widely used to
- · measure colo
- •test QCD
- learn about no physics

## Event shapes: describe the shape of the event, but are largely insensitive

to soft and collinear branching





## **Thrust Event-shape example Event-shape example**<br> $\blacksquare$





What happens in an  $e^+e^- \rightarrow q\bar{q}g$  event?

- Sum over all final state particles
- Find axis *n* which maximises this projection
	- $\int$  ( ; el<br>hrı Noteby: if one of the partons emits a soft or collinear gluon the value of thrust is not changing. **IRC safe**
	-













$$
\sum_{n=1}^{n} \frac{1}{\sqrt{2\pi}} \sum_{\substack{n=1 \text{odd } n}}^{n} \frac{1}{\sqrt{n}} \sum_{\substack{n=
$$







$$
\sum_{\substack{a,b \text{ odd}}}^{n} x^{\frac{1}{2}}
$$
\n
$$
\sum_{\substack{a,b \text{ odd}}}^{n} x^{\frac{1}{2}} = R^{(0)} \left( 1 + \frac{\alpha_S}{\pi} - \left( \frac{\alpha_S}{\pi} \right)^2 \left( c + \pi b_0 \log \left( \frac{M_0^2 v}{Q^2} \right) \right) \right)
$$
\n
$$
\sum_{\substack{a,b \text{ odd}}}^{n} x^{\frac{1}{2}}
$$
\n
$$
\sum_{\substack{a,b \text{ odd}}}^{n} x^{\frac{1}{2}} + \left( \frac{1}{M} \right) \left( c + \pi b_0 \log \left( \frac{1}{\pi} \right)^2 \right) \left( c + \pi b_0 \log \left( \frac{M_0^2 v}{Q^2} \right) \right) \right)
$$
\n
$$
\sum_{\substack{a,b \text{ odd}}}^{n} x^{\frac{1}{2}}
$$
\n
$$
\sum_{\substack{a,b \text{ odd}}}^{n} x^{\frac{1}{2}} \log \left( \frac{1}{\pi} \log \left( \frac{M_0^2 v}{Q^2} \right) \right) \left( \frac{M_0^2 v}{Q^2} \right)^2 = R_0 \left(
$$



## **Asymptotic freedom**  $\tau$  and the possibility that the color neutrality of the color neutrality of the hadrons could have a dynamical have a dynamical hadrons control  $\alpha$





**OT**(2) 
$$
\alpha_S(\mu) = \alpha_S + b_0 \log \frac{M^2}{\mu^2} \alpha_S^2
$$
  $b_0 = \frac{11N_c - 2n_f}{12\pi}$ 



$$
\mu^{2} \frac{d\alpha_{\text{max}}}{\beta(\mu_{S})} = \mu^{2} \frac{\partial \alpha_{S} - (b_{0}\alpha^{2} + b_{1}\alpha^{3} + b_{2}\alpha_{S}^{4}(\mu))}{\partial \mu^{2}} = -b_{0}\alpha_{S}^{2}
$$
\n
$$
\mu^{2} \frac{d\alpha_{\text{max}}}{\beta(\mu_{S})} = \mu^{2} \frac{\partial \alpha_{S} - (b_{0}\alpha^{2} + b_{1}\alpha^{3} + b_{2}\alpha_{S}^{4}(\mu))}{\partial \mu^{2}} = -b_{0}\alpha_{S}^{2}
$$
\n
$$
\mu^{2} \frac{d\alpha_{\text{max}}}{\beta(\mu_{S})} = \mu^{2} \frac{\partial \alpha_{S}}{\beta(\mu_{S})} = \mu^{2} \frac{\partial \alpha_{S}}{\beta(\mu_{S})} = -b_{1}\alpha_{S}^{3}(\mu) - b_{2}\alpha_{S}^{4}\mu^{2}\pi \sum_{\text{max}} \alpha_{S}^{2}(\mu) + \dots
$$
\n
$$
b_{0} = \frac{\beta(\alpha_{S})}{\alpha_{S}} = \mu^{2} \frac{\partial \alpha_{S}}{\partial \mu^{2}} = -b_{0}\alpha_{S}^{2} \implies \alpha_{S}(\mu) = \frac{1}{b_{0}\log\frac{\mu^{2}}{\Lambda^{2}}}
$$
\n
$$
\mu^{2} \frac{2\text{-loop}}{\Lambda^{2}}
$$
\n
$$
\mu^{2} \frac{\alpha_{\text{max}}}{\lambda} = \frac{1}{b_{0}\log\frac{\mu^{2}}{\Lambda^{2}}} \left[1 - \frac{b_{1}\log\log\mu^{2}/\Lambda^{2}}{b_{0}^{2}\log\mu^{2}/\Lambda^{2}}\right]
$$
\n
$$
\frac{\alpha_{\text{max}}}{\lambda^{2}}
$$
\n
$$
\frac{\alpha_{\text{max}}}{\lambda^{2}}
$$

### **Running of** *α<sup>s</sup>* **THE STRONG COUPLING**





Eleni Vryonidou STFC HEP school 2024 Many measurements at different scales all leading to very consistent results once evolved to the same reference scale,  $M_{z}$ .





 $\sum_{a,b} d x_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(s)$ ̂





 $\sum_{a,b} d x_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(s)$ ̂





  $\blacktriangledown$ *a,b*

 $\sum_{a,b} d x_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(s)$ ̂

 $dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$ 







  $dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$  $\blacktriangledown$ *a,b*  $\sum_{a,b} d x_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(s)$  $\sum_{a,b} d x_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(\hat{s}, \mu_R)$ ̂ **???**

# **QCD improved parton model**





 $\text{Ledl}$ eg<sub>d!</sub>



### Scaling violations of the control of The parton model predicts scaling. Experiment shows:

Scaling violation

 $Q^2$  (GeV<sup>2</sup>) STFC HEP school 2024

# **QCD improved parton model**



### Scaling violations of the control of The parton model predicts scaling. Experiment shows:



 $\text{Ledl}$ eg<sub>d!</sub>





DIS in QCD Scaling violation

What are we missing?



 $\left(\begin{matrix}\n\mathbf{UCL} \\
\hline\n\text{iniversite} \\
\text{catholique} \\
\text{de Louvain}\n\end{matrix}\right)$ 

Given the computation In order to make the intermediate steps of the calculation finite, we introduce a Given the computation of R at NLO, we expect IR divergences

We need to regulate these, and hope that they cancel! However for illustrative purposes other regulators (that cannot be easily used beyond  $\sum_{i=1}^{n}$ 





**CE** 









OUIL ANU UV UIVEIYENC  $\bigcap$  $r = \frac{\Delta}{\omega}$  removed at the end.  $\frac{1}{2}$  give  $\frac{1}{2}$  and  $\frac{1}{4}$  and  $\frac{1}{2}$  $H\ell$  is in the purpose other regulators (that cannot be easily used beyond beyon  $\overline{S}$  corrections to the LO process photon-gluon functions photon-gluon functions photon-gluon functions photon-gluon functions  $\overline{S}$ *F* ̂ *q* 2  $= e_q^2 x [\delta(1-x) +$ *αs* 4*π*  $P_{qq}$ log  $\frac{Q^2}{m^2}$  $m_{\tilde{g}}^2$  $+ C_2^q$ Soft and UV divergences cancel but a collinear divergence arises:

What are functions  $P_{qq}$  and  $P_{qg}$ ?

Splitting functions  $P_{ij}\!(x)$ : they give the probability of parton j splitting into parton i which carries momentum fraction x of the original parton

### GGI Florence - 2017 Fabio Maltoni Fabio Maltoni Fabio Maltoni Fabio Maltoni Fabio Maltoni Fabio Maltoni Fabio  $\binom{q}{2}(x)$  *F* ̂ *g* 2  $= e_q^2 x [0 +$ *αs* 4*π*  $P_{qg}$ log  $\frac{Q^2}{m^2}$  $m_{\tilde{g}}^2$  $+ C_2^g$  $\binom{2}{2}(x)$ ] IR<sup>ni</sup> Cut-O



$$
+(1-z)^2], \qquad P_{g\to gg}(z) = C_A \left[ z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right]
$$

*z z*

 $P_{g \to q\bar{q}}(z) = T_R$  $[z^2 + (1-z)^2]$ 1 + *z*<sup>2</sup>  $\overline{D}$ 

 $Z$ *P<sup>q</sup>*!*qg*(*z*) = *C<sup>F</sup>*  $\left\langle \right\rangle$ *<sup>P</sup>qq* <sup>=</sup> *<sup>C</sup>* <sup>+</sup> *<sup>z</sup> z z*

纂

### $\frac{1}{a}$   $\frac{1}{a}$  | 2  $\mathbf{t}$ θ *M*antribution to the (n+1)-bagy cross & b c a  $\frac{2}{a}$ | 2 *M<sup>n</sup>* TSI Relear Factorization  $|M_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n$ *dt t*  $dz\frac{d\phi}{d\phi}$  $2\pi$  $\alpha_{\text{\tiny S}}$  $2\pi$  $P_{a\rightarrow bc}(z)$ Notice that what has been roughly called 'branching probability' is a tually a singular factor, so one will need to make sense precisely of this definition. At the leading contribution to the (n+1)-body cross section the Altarelli-Parisi Branching has ta universal form given by the Altarelli-Parisi splitting **Altarelli-Parisi Splitting functions** functions

 $P_{q \to qg}(z) = C_F \left[ \frac{1}{1-z} \right], \qquad P_{q \to gq}(z)$ *n*<sup>2</sup>  $\left| \begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array} \right|$  $\mathcal{L}$  $P_{q \to qg}(z) = C_F$  $\sqrt{1 + z^2}$  $1 - z$  $\overline{\phantom{a}}$ *,*  $P_{q \to gq}(z) = C_F$ 

$$
P_{q \to qg}(z) = C_F \left[ \frac{1+z^2}{1-z} \right], \qquad P_{q \to gq}(z) = C_F \left[ \frac{1+(1-z)^2}{z} \right].
$$



$$
(-z)^{2}, \qquad P_{g \to gg}(z) = C_{A} \left[ z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right]
$$

*z z*

纂

 $Z$  $P_{g \to q\bar{q}}(z) = T_R$  $[z^2 + (1-z)^2]$ *P<sup>q</sup>*!*qg*(*z*) = *C<sup>F</sup>* 1 + *z*<sup>2</sup>  $\left\langle \right\rangle$ *<sup>P</sup>qq* <sup>=</sup> *<sup>C</sup>* <sup>+</sup> *<sup>z</sup>*  $\overline{D}$ *z z*

### $\frac{1}{a}$   $\frac{1}{a}$  | 2  $\mathbf{t}$ θ *M*antribution to the (n+1)-bagy cross & b c a  $\frac{2}{a}$ | 2 *M<sup>n</sup>* TSI Relear Factorization  $|M_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n$ *dt t*  $dz\frac{d\phi}{d\phi}$  $2\pi$  $\alpha_{\text{\tiny S}}$  $2\pi$  $P_{a\rightarrow bc}(z)$ Notice that what has been roughly called 'branching probability' is a tually a singular factor, so one will need to make sense precisely of this definition. At the leading contribution to the (n+1)-body cross section the Altarelli-Parisi Branching has ta universal form given by the Altarelli-Parisi splitting **Altarelli-Parisi Splitting functions** functions

 $P_{q \to qg}(z) = C_F \left[ \frac{1}{1-z} \right], \qquad P_{q \to gq}(z)$  $\delta^{6^t}$ *n*<sup>2</sup>  $\left| \begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array} \right|$  $\mathcal{L}$  $P_{q \to qg}(z) = C_F$  $\sqrt{1 + z^2}$  $1 - z$  $\overline{\phantom{a}}$ *,*  $P_{q \to gq}(z) = C_F$ 

$$
P_{q \to qg}(z) = C_F \left[ \frac{1+z^2}{1-z} \right], \qquad P_{q \to gq}(z) = C_F \left[ \frac{1+(1-z)^2}{\frac{1}{\sqrt{1-\frac{1}{\sqrt
$$

*<sup>P</sup>gq* <sup>=</sup> *<sup>C</sup> <sup>P</sup>qg* <sup>=</sup> *<sup>T</sup>* (*<sup>z</sup>* + ( *<sup>z</sup>*) ) + ( *<sup>z</sup>*)

These functions are universal for each type of splitting

## **What does this collinear divergence mean? i** COMMed

- Residual long-distance physics, not disappearing once real and virtual corrections are added. These appear along with the universal splitting functions. Can a physical observable be divergent? No, as the physical observable is the hadronic structure function: are auueu. These appear alung with the universal spiltury functions.  $S$  the natural question is: what is it that is it that is going wrong? Do we have IR sensitiveness in a se **Residual iong-distance**  $\frac{1}{2}$  in section  $\frac{1}{2}$
- $F_2^q(x,Q^2)=i\text{E}q,\bar{q}$  $i=q,\bar{q}$  $e_{q}^{i}$  $\int_{i,0}^{x} \frac{1}{2\pi} \int_{\mathbb{R}} \frac{\overline{c_{s}}}{2}$  $2\pi$  $i$ , of  $\xi$  $\overline{x}$ dξ  $\sum_{\xi} f_{i,0}(\xi)$ )<br>)<br>|  $\mathcal{P}_{qq}^{g}(% \mathcal{Q}_{q}^{f}(\mathcal{Q})\mathcal{Q}_{q}^{f}(\mathcal{Q}))=\mathcal{P}_{qq}^{f}(\mathcal{Q}_{q}^{f}(\mathcal{Q}))$  $\boldsymbol{x}$  $\begin{equation} \boldsymbol{\xi} \end{equation}$  $\sqrt{\frac{q}{2}(\frac{c}{\xi})}$   $\sqrt{2}$  $\overline{m_g^2}$  $+ C_2^q ($  $\overline{x}$ ξ )  $11$  $i = q, \bar{q}$  defined on  $\bar{u}$  and do not do n  $F_2^q(x,Q^2) = x$  $i$   $\cancel{\pi}q,\bar{q}$  $e_q^2$  $\int f_{i,0}(x) + \frac{\alpha s}{2}$  $2\pi$  $\int_0^1$  $\boldsymbol{x}$ dξ  $\frac{1}{\xi S} f_{i,0}(\xi)$  $\sqrt{ }$  $\cancel{P}_{q q} ($  $\overline{x}$  $\boldsymbol{\xi}$ )  $\log \frac{Q^2}{2}$  $\overline{\mathcal{P}_{g}^2}$  $+ \mathcal{L}_2^q($  $\overline{x}$ ξ )  $\mathbf{1}$

We can absorb the dependence on the IR cutoff into the PDF:





$$
f_q(x,\mu_f) \equiv f_{q,0}(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) P_{qq}(\frac{x}{\xi}) \log \frac{\mu_f^2}{m_g^2} + z
$$
  

$$
f_q(x,\mu_f) \equiv f_{q,0}(x) + \frac{\mu_S}{2\pi} \text{Phonym4lised} \text{PBrg}(\frac{x}{\xi})
$$



## **Factorisation**

Structure function is a measurable object and cannot depend on scale

- Long distance physics is universally factorised into the PDFs, which now depend
	- $\mathcal{L}_1$  . Can we exploit that physical quantities have to be scale  $\mathcal{L}_2$  , that physical quantities have to be scale  $\mathcal{L}_3$





on  $\mu_f$  PDFs are not calculable in perturbation theory. PDFs are universal, they don't depend on the process.

Factorisation scale  $\ \mu_f$  acts as a cut-off, emissions below  $\mu_f$  are included in the PDFs. sation scale  $\ \mu_f$  acts as a cut-off, emiss

at all orders (renormalisation group invariance) Factorization

$$
F_2^q(x, Q^2) = x \sum_{i=q, \bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi, \mu_f^2) \left[ \delta(1 - \frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi} \left[ P_{qq}(\frac{x}{\xi}) \log \frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi}) \right] \right]
$$

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# ̂

Parton model

Renormalisation

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Parton model

Renormalisation

QCD improved parton model



  $dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$  $\blacktriangledown$ *a,b*  $\sum_{a,b} d x_1 dx_2 d \Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(s)$ *a*,*b*  $\int dx_1 dx_2 d\Phi_{PS} f_a(x_1) f_b(x) \hat{\sigma}(\hat{s}, \mu_R)$ ̂ Parton model Renormalisation QCD improved parton model





### **DGLAP** in scale: DGLAP equation FS IN P

 $\mu^2 \frac{\partial f(\bm{x}, \mu^2)}{2}$ 

 $\partial \mu^2$ 

We can't compute PDFs in perturbation theory but we can predict their evolution



=

 $\int_0^1$ 

*x*

*dz*

*z*

### Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process em as an input for another process

$$
\mu^2 \frac{\partial f(x, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right)
$$

 *Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77* 

$$
P_{ab}(\alpha_s, z) = \frac{\alpha_s}{2\pi} P_{ab}^{(0)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ab}^{(2)}(z) + \dots
$$
 **perturbation t**  
\n
$$
10 \text{ Dokshitzer; Gribov, NLO (1974)}
$$
\n
$$
10 \text{ Dokshitzer; Gribov, NLO (2004)}
$$
\n
$$
10 \text{ Floratos, Ross, Sac Gonzalez-Arrovo, Lopez}
$$





## **PDF evolution**



![](_page_56_Picture_8.jpeg)

**input to the LHC**

# *fi*(*x, µ*) *fi*(*x, µ*)

![](_page_56_Figure_6.jpeg)

# **PDF extraction**

DGLAP equations to evolve them to different scales.

- Choose **experimental data** to fit and include all info on correlations **Theory settings**: perturbative order, EW corrections, intrinsic heavy quarks,  $\alpha_{\rm s}$ , quark masses value and scheme
- Choose a starting scale  $Q_0$  where pQCD applies
- **Parametrise** independent quarks and gluon distributions at the starting scale
- Solve **DGLAP equations** from initial scale to scales of experimental data and build up observables
- **Fit** PDFs to data
- Provide PDF **error sets** to compute PDF uncertainties

## We can't compute PDFs in perturbation theory but we can extract them from data, and use

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![](_page_58_Picture_5.jpeg)

## **Data for PDF determination DISENTANGLING PDFS WITH EXPERIMENTAL DATA**

![](_page_58_Figure_1.jpeg)

## LHC kinemat **How can we tell wh**

For the production of a particle of  $\blacksquare$ : E2= x2 Ebeam  $\alpha^{w}$  $y = \frac{1}{2} \log \frac{w_1}{r_2}$  $\alpha$  and  $\alpha$  particle of mass  $\alpha$  $e^y$   $x_2 = \frac{1}{1}e^{-y}$  $\boldsymbol{y}$  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $x_2$  $x \in R$  $\boldsymbol{\Lambda}$ ISE  $e_1$   $x_2 =$  $\overline{M}$ √ S  $\epsilon$  production of a part  $\epsilon$ and into a particle of particle of mass  $\mathcal{P}$  $M^2 = x_1 x_2 S = x_1 x_2 4 E_{\text{beam}}^2$  $\frac{1}{2} \log \frac{x_1}{x_2}$  $x_1 =$ M √ S  $e^y$   $x_2 =$  $\overline{M}$ √ S  $e^{-y}$ See exercises!

![](_page_59_Figure_4.jpeg)

Eleni Vryonidou **STFC HEP school 2024 STFC HEP school 2024 Fabio Maltoni** 46

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## **Data complementarity** LUUTION<br>LUITESSE

![](_page_60_Picture_9.jpeg)

![](_page_60_Picture_10.jpeg)

Low and high mass Drell-Yan WW production

High  $p_T W(+jets)$  ratios  **(medium/large x)**  W and Z production  **(medium x)**  Low and high mass Drell-Yan  **(small and large x)**  Wc **(strangeness at medium x)**  {

 Inclusive jets and dijets **(medium/large x)**  Isolated photon and γ+jets  **(medium/large x)**  Top pair production **(large x)**  High p<sub>T</sub> V(+jets) distribution  **(small/medium x)**  {

**{**

**GLUON**

**QUARKS**

**PHOTON**

PHOTON

![](_page_60_Picture_6.jpeg)

![](_page_60_Figure_7.jpeg)

![](_page_61_Picture_5.jpeg)

![](_page_61_Figure_0.jpeg)

Interent collaborations, predictions usually J GALIQUE ATT UNIGHEATHLY GITTGIUPG. Different collaborations, predictions usually computed with different PDFs to extract an uncertainty envelope.

STFC HEP school 2024

![](_page_62_Picture_7.jpeg)

## Impact of PDF uncertainties. **PHENO IMPLICATION OF PDF UNCERTAINTIES**

reliew Report a (2013) limiting factor in the accuracy of theoretical predictions Yellow Report 3 (2013)

![](_page_62_Figure_1.jpeg)

![](_page_63_Figure_8.jpeg)

### Impact of PDF uncertainties. **PHENO IMPLICATION OF PDF UNCERTAINTIES PHENO IMPLICATION OF PDF UNCERTAINTIES**

![](_page_63_Figure_1.jpeg)

![](_page_63_Figure_7.jpeg)