Introduction to dark matter

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Contents

1 Introduction

Dark matter (DM) is a key part of our standard model of cosmology (ΛCDM). This model predicts the evolution of the Universe since the Big Bang at large scales, and depends on a few number of assumptions. Without dark matter, it is not possible to reconcile data with observations[1]. Recall that cosmological parameters are typically constrained at percent level.

Why do you need to know about DM? As physicist exploring the frontiers of the standard model of particle physics (SM), it is important to know which candidates beyond the SM (BSM) are well motivated. There are few corners where BSM is needed: neutrino masses, baryogenesis, DM and quantum gravity are the most discussed ones. Indeed, CERN itself has recognized the search for DM as one of its main targets in the near future. One of the goals of these lectures is to understand the link between DM and the weak scale, summarized in the WIMP paradigm. Furthermore, the searches for DM are going through a very interesting period: the WIMP paradigm is showing some of its limits, and there is an effort to find new ways to probe candidates with larger/smaller masses/cross-sections.

This is, by no means, a review, but a list of selected topics. Neither I am a professional dark matter particle physicist, though I have done some work in DM. For more detailed information I recommend:

- Lecture notes: Lecture notes by D. Cerdeño (STFC schools before 2018). Lecture notes by Lisanti [1], Gelmini [2].
- Book: This is a brief and up-to-date book by Profumo [3].
- Videos by S. Profumo (`https://www.youtube.com/watch?v=jviqfJPStCI`), T. Slatyer (`https://www.youtube.com/watch?v=ndS-w1XxQ6k`), M. Lisanti (`https://www.youtube.com/watch?v=gJem93SnnCA`, `https://www.youtube.com/watch?v=J5vcAZoxhAc`) and T. Volansky (`https://www.youtube.com/watch?v=5BR9dQa46AA`) at Florence HEP school/ICTP.

[1] I won't discuss any alternative to the dark matter paradigm. Maybe there is something about gravitation that we don't understand, but the leading explanation for the cosmological data is that there is an extra source of cold matter in the Universe with the properties we'll discuss.

- Videos by Neil Weiner, lecturing at IAS (`https://www.youtube.com/user/videosfromIAS/search?query=weiner`) and Mainz (`https://www.youtube.com/watch?v=9igHxDHs6Hs`).

- Slides, e.g. `https://www.studocu.com/en-gb/course/university-of-sheffield/dark-matter-and-the-universe/1677107`.

More advanced books include Kolb & Turner [4], Binney & Tremaine [5], Bertone [6], Mo, van den Bosch & White [7].

My presentation will be standard (except for some space for ultra-light dark matter candidates). It will consist on the following:

- Evidence of dark matter and properties of DM (sec. 2)
- Thermal production in the early universe (WIMP miracle and cousins) (sec. 3)
- Axions as DM candidates (sec. 4)
- Direct detection of WIMPs (sec. 5)
- The rest (Indirect detection/challenges of WIMPs/New ideas) (sec. 6)

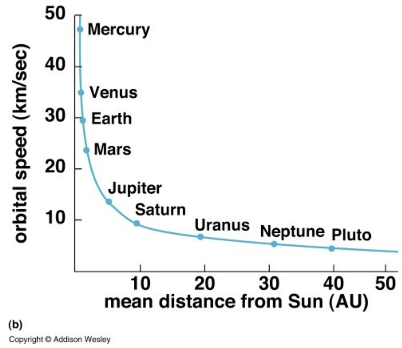
A good complementary to these lectures would be a discussion of DM at colliders, or the production of non-thermal relics. I invite you to check the material I suggested for more info.

I shall use $\hbar = c = 1$ in the lectures.

2 Evidence of dark matter and properties of DM

The modern evidence for DM dates back to the early years of the last century. However, it is interesting to note that astronomical data had already faced a DM crisis before: in the 1840s, the data of the orbits of the planets in the Solar System was not consistent with the mass observed. In particular, Uranus had an anomalous orbit. Le Verrier predicted a new source of matter that hadn't been detected before in the form of a new planet. He even predicted its orbit. This was done on the 31st of August of 1846. On the 23rd of September of 1846, Neptune was found. The Solar System had also an anomaly in the orbit of Mercury. Le Verrier predicted an inner planet (Vulcan), and some observers claimed detection. As we know, this was not a real detection. Indeed, the orbit of Mercury is anomalous in Newtonian dynamics because of the need to include General Relativity corrections. Hence, it was the theory of gravity that was failing!

Something that is important about these two examples is that Newtonian gravity was not being accurate in two situations with two very different gravitational potentials ($\phi_N \sim GM_\odot/r$) and that the solution to one of the problem couldn't fix the other one. For instance, General Relativity corrects Newtonian dynamics in situations where ϕ_N^2 corrections may be relevant. The latter are very small for the outer planets. In the case of DM, we'll also find a multi-scale phenomenology, that can be explained by a single hypothesis.



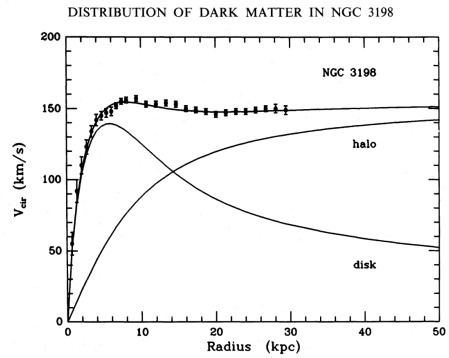


Figure 1: Left: Solar system orbits and law (1). Right: Orbits in the NGC 3198 [8]..

2.1 DM evidence

As I discussed above, the DM evidence comes from observations at very different scales, and can be explained by a single new particle with relatively natural properties. Let's discuss some of the evidences of DM.

2.1.1 Galactic Curves

In a spherical concentration of mass $M(r)$, the circular gravitational orbits satisfy

$$v^2(r) = \frac{GM(r)}{r}, \tag{1}$$

where $v(r)$ is the velocity of the orbit. The most clear example are the orbits of the planets in the Solar System, as shown in Fig. 1. In this case, all the mass is at the center, which yields a $1/r^{1/2}$ law.

Notice that in a mass distribution with constant density $M(r) \sim \rho r^3$, and hence $v \sim r$. Then, from the analysis of rotation curves in galaxies (assuming galaxies are axisymmetric objects of constant density), one expects a linear growth in $v(r)$, followed by $r^{-1/2}$ decay after leaving the region where most matter lives. However, the precise analysis of rotation curves in galaxies showed that the $\sim r$ behaviour was followed by a plateau. Vera Rubin was one of the main actresses in this discovery (in particular in the systematic analysis). You may already know that a galaxy is not a spherical or axisymmetric distribution of matter. Still, when one considers the observed distribution of matter, the picture is the same. A famous example is shown in Fig. 1 [8]. Notice, in particular, that the model of the matter in the disk works very well at small distances. However, to reproduce a flat velocity rotation curve at larger r one needs $\rho(r) \sim r^{-2}$ from (1). So, one can say that the data fits the theory only if there is an extended very spherical 'halo' of matter extending well beyond the limits of the disk of the observed galaxies, and with a profile such that $\rho \sim r^{-2}$. We will come back to it, but this kind of behaviour already tells us what DM can not be. For instance, in the Milky Way, most of the visible galaxy is confined into a flat disk. The reason why standard matter

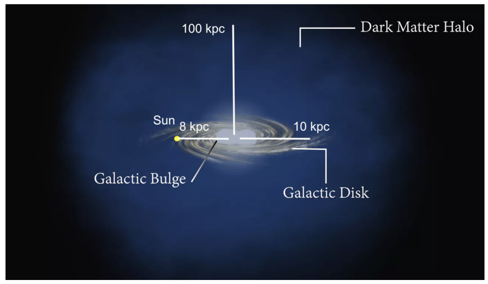


Figure 2: Milky Way

likes to form disks and not spherical configurations has to do with the cooling of the primordial halos due to emission of photons. Hence, if DM can cool efficiently, it can not be distributed as a sphere. Hence, DM can not be charged as much as SM particles. I also note that the visible matter in the Milky Way extends up to[2] 10 kpc, while the DM halo is supposed to extend up to 100 kpc. See fig. 2.

2.1.2 Dynamics of galaxy clusters

Galaxies are large structures. As I just mentioned, the radius of the DM halo in the Milky Way is 100 kpc. Still, in the universe we find even larger structures which are in a sort of equilibrium. Indeed, there are clusters of galaxies, where several galaxies are interacting with each other and form a gravitationally bound structure, where the typical distance between galaxies is 1 Mpc. I recommend the very beautiful video showing Hubble observations of the Coma cluster `https://hubblesite.org/video/1188/news`. The typical size of a cluster of galaxies is 10 Mpc, and one can consider each galaxy as a point for the large scale dynamics.

If the cluster of galaxies is in a stationary configuration, the virial theorem teaches us that the averaged kinetic and potential energies are related to each other by (**Ex.** *you'll derive this theorem in the problem session*)

$$2\langle T\rangle = -\langle V\rangle, \tag{2}$$

where

$$T = \sum_i m_i \frac{v_i^2}{2}, \qquad V = \sum_{i<j} \frac{G m_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|}, \tag{3}$$

and

$$\langle f(t)\rangle = \frac{1}{T}\int_{t_0}^{T+t_0} \mathrm{d}t f(t). \tag{4}$$

When analysing the data from the Coma cluster in the 1930s, Zwicky realized that the virial theorem was not satisfied. In astrophysics, it's always very difficult to make

[2] I'll use the parsec (pc) as a measure of distance. The rule of thumb is 1 pc $\approx$ 3 light years. The conversion to km is left as an exercise, if you want to know it.

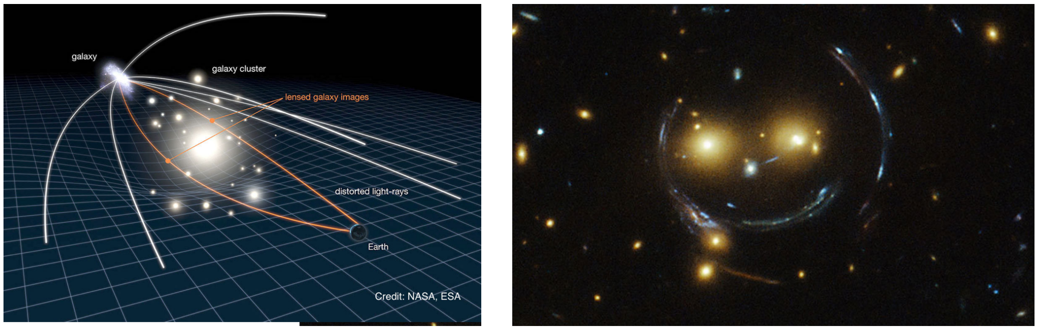


Figure 3: Light is lensed by an intervening gravitational potential. Given a field of sources, we can sometimes estimate the gravitational field (and from it, the mass) between the telescope and the field. The right panel is a famous image of a strongly lensed field `https://en.wikipedia.org/wiki/Einstein_ring`.

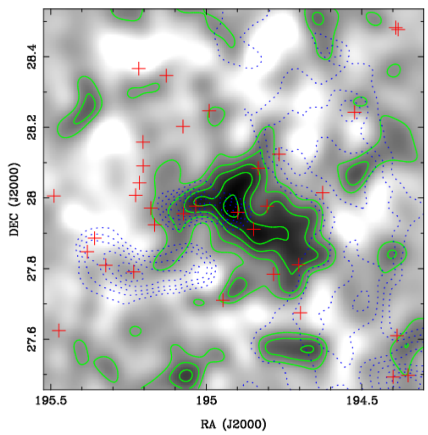


Figure 4: Matter in the Coma cluster [9]

definite statements with a single system, since there are many systematic factors which are not under control. One can perform the same analysis in different number of galaxy clusters and arrive to the same conclusion: there is matter missing if the virial theorem is satisfied (which should be the case for systems in stationary configurations).

Using more modern techniques, one can measure the gravitational potential in clusters through gravitational lensing. Gravitational lensing is based on the influence of gravitational potentials on light propagation. You can see an illustration in Fig. 3. Recent data of the Coma cluster is shown in Fig. 4. The red crosses represent the visible matter, while the contours represent where the matter responsible for gravitational fields hides. The conclusion is the same as Zwicky's: one needs more matter diffused in the cluster than what is observed in galaxies.

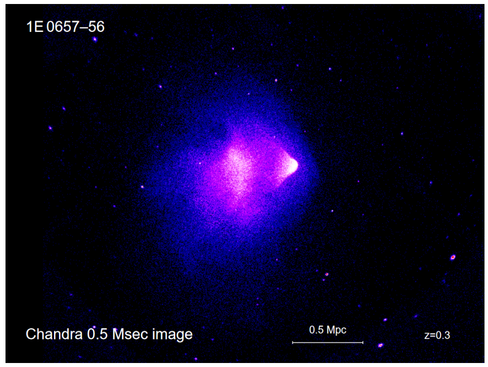


Figure 5: Bullet cluster from `https://en.wikipedia.org/wiki/Bullet_Cluster`

2.1.3 Collisions of galaxy clusters

Ex. *Watch the following video* ***https://www.youtube.com/watch?v=2DoPAeU3a6Y***. Galaxy clusters contain several galaxies, gas and DM. Let's imagine that these species are all distributed spherically. In the video, red is gas (sometimes called 'baryons', since most of the mass is in baryons anyway), blue the DM. When two clusters collide with each other, the gas of the two clusters interacts through SM cross-sections displaying a characteristic shock wave, and staying behind the DM component which goes through efficiently (only interacts gravitationally). The final result (as you can see in fig. 5) is an asymmetric configuration, with gas in the middle, and DM in the outskirts. The gas can be detected with X-ray surveys (it's hot, in part due to the collision). The DM halos are detected with gravitational lensing, that is sensitive to all the mass, not only to the mass that can generate photons. In this picture the stars also go through! But their number doesn't match with the observed matter through lensing.

When one tries to compare the amount matter in the different components, one again concludes that there is more matter that underwent the collision without interacting than gas or stars. This also informs us about the self-coupling of DM particles. A famous examples is the bullet cluster, the one shown in Fig. 5

Ex. *In the exercise you'll use the fact tat the mean-free path of a dark matter of the order of the size of a galaxy cluster to constrain the cross-section of DM self-interaction.*

2.1.4 Cosmological probes

The current cosmological standard model describes the evolution of the observed universe at high precision. Our universe was very homogeneous and isotropic in the past. Indeed, we know this with high precision from the detection of the cosmic microwave background (CMB), a picture of the universe from ~ 13 billion years ago. The existence of the CMB is a consequence of the Big Bang, that postulates a very hot primordial universe. At some point of evolution, this primordial universe was made of a plasma of protons, electrons and photons. As the universe expands, it cools down. At some point the temperature reaches $T \sim 0.1\,\mathrm{eV}$, and the electrons and protons recombine. From this moment, light

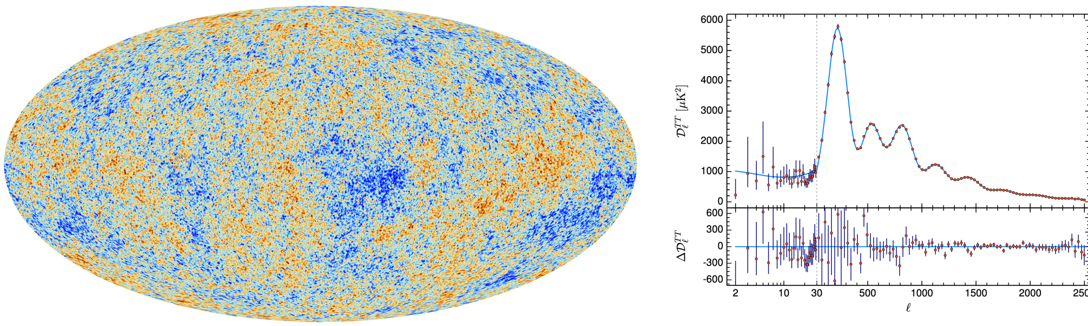


Figure 6: Planck CMB. The left panels shows a projection of sphere, where the colours represent deviations with respect to a homogenous distribution of temperature. The scale of the typical deviation is $\sim 10^{-5}$. The right panel is the decomposition of the two point correlation function of this temperature map of the left in spherical harmonics l. The dots are data points, the line is the ΛCDM model. `https://www.esa.int/Our_Activities/Space_Science/Planck/Planck_and_the_cosmic_microwave_background`

propagates almost freely, and this is the radiation that we detect today. This picture shows some irregularities in the energy density ρ of order $\delta\rho/\rho \sim 10^{-5}$, which you can see in fig. 6. To produce a model explaining the features observed in this primordial picture, and how it evolved to generate the complex universe that we see around us today, it is necessary to invoke a new form of matter that clusters. In other words, there is not enough baryonic matter in the early universe to make the primordial perturbations evolve into the dense matter environment that we see today. The constraints from CMB are very robust and precise, and inform us about a ratio of 5 between DM and baryonic matter. Even more, the existence of a more exotic component in the universe (dark energy) it is also required, but I won't touch this topic in these lectures.

Some recent numbers for the relative abundance of DM and baryons are [10] (recall that ρ represents the energy density of a species)

$$\Omega_b h^2 = 0.02233 \pm 0.00015, \qquad \Omega_{DM} h^2 = 0.1198 \pm 0.0012, \tag{5}$$

where $\Omega_X = \rho_X/\rho_c$, ρ_c is a fixed n critical density that we motivate later

$$\rho_c \approx 10^{-29}\,\mathrm{gr/cm^3} \approx 10^{-6}\ \mathrm{GeV/cm^3}. \tag{6}$$

Same for h, whose value is $h \approx 0.7$.

The PDG has a very nice intro to cosmology for particle physicists: `http://pdg.lbl.gov/2019/reviews/rpp2018-rev-bbang-cosmology.pdf`.

Ex. *Exercise to simulate the universe*

2.2 DM properties

All the different evidences that I mentioned point towards the existence of a single new component. Which is remarkable. Here is a list of the 'knowns' about what this new component should be.

2.2.1 Darkness

DM should be 'dark'. By this I mean that its interaction with SM particles should be small. For instance, as we will see, some of the leading candidates interact with SM with cross sections close to those of the weak scale.

This implies in particular that DM must be neutral to high degree. For instance, in [11] one can find the following constrain on the EM charge of DM for DM masses around the GeV,

$$q_{DM} \lesssim 10^{-4} \left(\frac{m_{DM}}{\text{TeV}} \right)^{1/2} . \tag{7}$$

It is normal to expect that the charge of DM will be constrained from different phenomena, from cosmology to collider physics. For instance, larger charge means larger cross-section, and hence equilibrium with SM in the early universe. Also, larger charge means that it will be present in the plasma in stars, and, maybe surprisingly, there are certain aspect about star dynamics that allow onr to put strong constraints on the presence of new particles. For more details on DM charges, see e.g. [12, 13]. Besides a fundamental charge, one could also look for dipole moments or other kind of charges.

Of course, if there is a fundamental reason beyond the quantization of charge in Nature, this kind of logic would not make sense.

2.2.2 Coldness (non-relativistic)

DM should have clustered for a large period of time. Indeed, whatever it is, its behaviour is the same as the one of a collection of non-relativistic particles that are attracted to each other by their mutual gravitational field. This is necessary for the DM to generate the growth of the small perturbations of the CMB. Hence, DM can't be massless (or relativistic). Indeed, it needs to be cold (small typical velocities) to accumulate and grow in the galactic halos of galaxies Relativistic (hot) dark matter has a larger free-streaming length (the average distance traveled by a dark matter particle before it falls into a potential well). This leads to inconsistencies with observations. This is one of the reasons why neutrinos can not be all the DM. It is remarkable that if neutrinos were heavier, they would be a good candidate of DM. However, since they are much lighter, they are produced in the big bang with large kinetic energies, that do not cool down fast enough to make them viable as DM (Also, they are too weakly coupled to the SM to make all of the DM. We will come back to this when discussion the thermal production of DM).

There is no reason to believe that DM was produced thermally, though thermal production in the early universe is one of the leading ways to generate DM. In this case, our requirement of DM to be cold simply means

$$T \ll m_{DM}. \tag{8}$$

When this is satisfied, the species of mass m_{DM} behaves as a cold gas of massive particles. Since $T(t)$, the important thing is that this happens already the comic times where we have evidence of the presence of DM. From this kind of arguments, one can put a robust bound on the mass of any thermally produced DM candidate [14],

$$m_{DM}^{thermal} \gtrsim \text{keV}. \tag{9}$$

This constrain does not apply if DM is produced non-thermally, but still has small velocities and clusters at cosmological scales. A very famous example of non-thermal DM is the axion (more in sec. 4)).

2.2.3 Stability

As I mentioned before, the traces of DM are seen from $\sim 13\,$Gyrs away till today. This means that if it decays, its decay rate must be very slow

$$\Gamma_d \times (13 \text{ Gyrs}) \gtrsim 1. \tag{10}$$

More stringent bounds can be derived from concrete annihilation/decay channels that one can look for in DM dominated environments. Indeed, that's a way to look for DM. Stable DM candidates are common in models in which a new discrete symmetry is imposed by ensuring that the DM particle is the lightest with an exotic charge (and therefore its decay is forbidden). This is the case, e.g., in Supersymmetry (when R-parity is imposed), Kaluza-Klein scenarios (K-parity) or little Higgs models.

2.2.4 Non-baryonic

We know the amount of baryonic matter in the universe not only from counting objects in the late time universe, but also from CMB and BBN (big bang nucleosynthesis) predictions. Indeed, the big bang paradigm includes a mechanism of generation of different relics (H, D, He, Li,...) whose final amount depends on the amount of baryonic DM (among other things). The current measurements agree very well with the idea that

$$\Omega_b \approx \Omega_c/5. \tag{11}$$

BBN happens at in the very early Universe, where the influence of DM is negligible (the Universe was dominated by radiation) and hence DM could be generated afterwards. But doing it within SM degrees of freedom is tricky. One can look for exotic SM states, that may not play a role in BBN [15]. A particularly fashionable possibility is that DM is made of black holes (BHs). These BHs may have an origin common with baryons, but this is a long discussion.

2.2.5 Collisionless

This is something we have already mentioned when we discussed the Bullet cluster. DM can't have large collision cross-section because this would lead to a different phenomenology in the collision of clusters. Also, the distribution of DM in the galactic halos should

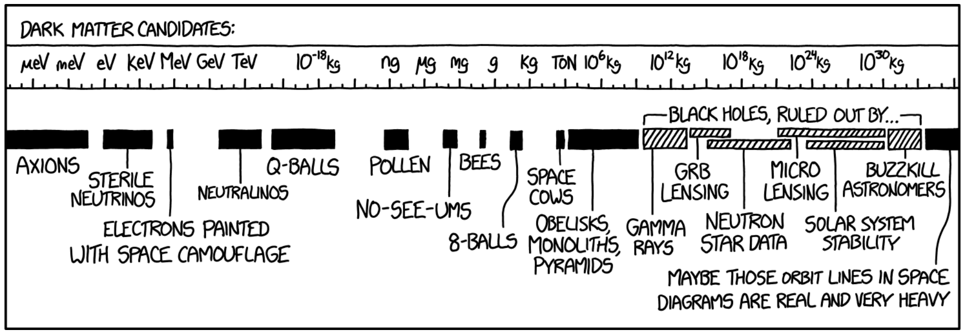


Figure 7: `https://imgs.xkcd.com/comics/dark_matter_candidates.png`

be close to spherical. Self-interactions and possibility to dissipate into lighter species typically imply a loss of sphericity in the halos[3]. The bound that appears from different probes is [16],

$$\sigma/m_{DM} \lesssim 1\,\mathrm{gr/cm}^2. \tag{12}$$

2.2.6 Is this enough to select a candidate?

No. The previous phenomenology can be reproduced by many models. We don't know anything about the fundamental properties of DM: the mass, the spin, the charges... Take the mass: it can be anything from 10^{-22} eV to several times the mass of the Sun (not as a fundamental particle, but as compact objects). Fig. 7 shows part of this puzzle (indeed it extends further in both directions). We have some bounds which are robust:

- As discussed, for thermally produced $m_{DM} \gtrsim$ keV.
- Also, if DM is a fermion, $m_{DM} \gtrsim 400\,\mathrm{eV}$ (Tremaine-Gunn bound [17]). This is because the DM candidate should be able to generate virialized halos of a certain size. The basic logic is the following: let's consider a galactic DM halo as a box in space. The smallest DM halos correspond to $d\sim$kpc (dwarf spheroidals). For a particle to be part of this bound structure, its velocity shouldn't be too large, or it would scape from it. Hence the momentum satisfies $p_{DM} = m_{DM} v \lesssim p_{esc}$. For v_{esc} in dwarf spheroidals, one can take $v_{esc} \sim 10^{-4}c$. Hence, a DM halo has a finite size in phase space. But phase space is quantized, and for fermions of spin 1/2 one can only put two states per phase space state. By trying to fill the states till we explain the mass of the dwarf spheroidals, you can derive this bound. **Ex.** *You'll do it in the problem class.*
- For bosonic DM, one can find a bound using related logic. The de Broglie wavelength of the candidate should be smaller than the radius of the virialized structure (d), or otherwise one can't localize the wavepackets of the given momentum ($< p_{esc}$)

[3] Think about the Milky Way: the reason why it's a disk and not a sphere has to do with SM particles interactions and dissipation into photons.

in the DM halo. This yields a bound $m_{DM} \gtrsim 10^{-22}\,\mathrm{eV}$. **Ex.** *You'll do it in the problem class.*

3 Thermal production in the early universe (WIMP miracle and other models)

We have seen that there is a strong case to introduce a new particle in our description of Nature. Where did it come from? The big bang paradigm is very precise in predicting the amount of SM matter (baryons, neutrinos and radiation) as a relic of a primordial hot Universe expanding. At first order, the only relevant quantity is the primordial baryonic asymmetry[4], and from there we use standard physics, based on

- thermodynamics (with some statistical mechanics)
- particle physics
- general relativity

It is hence natural to use the same idea for dark matter. As we will see, this idea is even more appealing because rates similar to the weak rates make this idea successful. We will now give a crash course on how to compute the relic abundance of DM, based on the previous items.

3.1 Physics in an expanding Universe (the basics of GR)

In general relativity, a homogeneous and isotropic universe filled with matter either expands or contracts.The universe at the largest scales is very much homogeneous and isotropic. Here we talk about scales larger than Mpc, and hence the galaxies are consider as points, and only the macroscopic properties of a 'medium of galaxies' are relevant. We can consider the coordinates of these galaxies x_i. The expansion of the universe means that $x_i(t)$. If we consider the medium to be homogeneous and isotropic, the expansion should happen at the same rate in the whole universe, so

$$x_i(t) = \frac{a(t)}{a(t_0)} x_i(t_0). \tag{13}$$

In this formula $a(t)$ is the growth factor, that encapsulate the way the universe expands at large scales: two galaxies at distance $l_1 = a(t_1)l$ at t_1 are at distance $l_2 = a(t_2)l$ at t_2. See fig. 8. This simple law is true on average since the universe is homogeneous and isotropic on average. Notice that if we compute the velocity of recession of galaxies, given by the previous law (called Hubble law) on finds

$$v = \left(\frac{\dot{a}}{a}\right)_{\text{today}} d. \tag{14}$$

[4]Whose value is a mystery in itself.

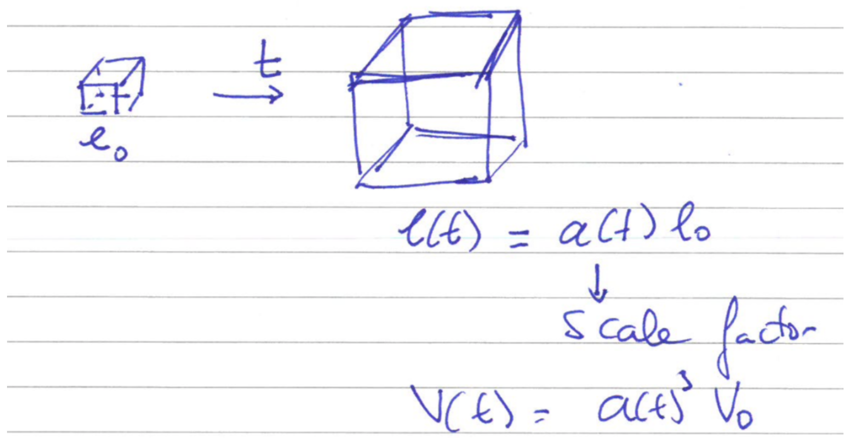


Figure 8: Expanding in a homogeneous and isotropic universe: growth factor

Hence, the furthest galaxies move faster with respect to each other. To give you an idea of this number,

$$\left(\frac{\dot{a}}{a}\right)_{\text{today}} = h \times 100\text{km/s/Mpc}, \tag{15}$$

and $h \approx 0.7$.

The evolution of $a(t)$ evolves in time according to the matter content in the universe (the expansion of the universe tends to be slowed down by matter). The equation behind it follows from general relativity,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho. \tag{16}$$

This is called Friedman equation. It is customary to define the Hubble function as

$$H(t) = \frac{\dot{a}}{a} \tag{17}$$

This equation informs us about the average properties of the Universe. From the previous equation, we can now understand what we mean by ρ_c in (6). It is the density corresponding to the expansion rate of the Universe today

$$\rho_c = \frac{3}{8\pi G_N}\left(\frac{\dot{a}}{a}\right)^2_{\text{today}} \tag{18}$$

To have a useful and physical way of characterizing time evolution in cosmology, we use the concept of 'cosmological redshift' defined as

$$1 + z = \frac{a(t)}{a(t_0)}, \tag{19}$$

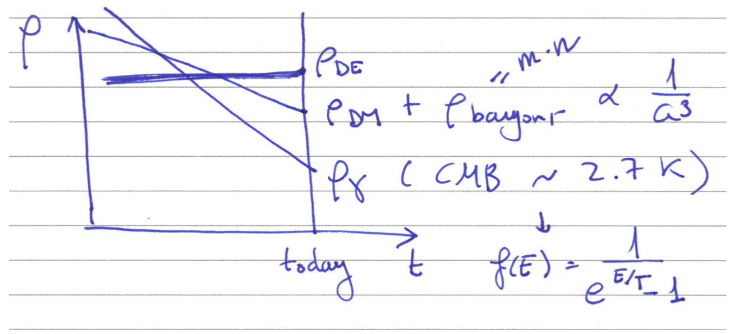


Figure 9: Time evolution of ρ for different components: DE, DM and radiation γ. We see that photons always dominate in the early universe, while DE does dominate at late times.

where t_0 is today. The universe has been expanding in the last 14 billion years, so z grows in the past. To give an example, the time where the CMB was produced (~ 13 billion years ago) was at $z \sim 10^3$.

In an expanding universe, the distances between galaxies grow on average with the cosmic flow. Of course, on top of this cosmic expansion there may be local motion, and the position of a galaxy i will be

$$x_i(t) = a(t)d_i + \delta x_i. \tag{20}$$

Hence, the number density of massive objects will behave on average as

$$n(t) \propto a^{-3}. \tag{21}$$

What about the energy density that we need to solve the equation (16)? For massive, non-relativistic objects $E = m(1 + O(v/c)^2)$, and hence $\rho(t) \propto a^{-3}$. However, for a relativistic case (or for light), $E \approx p$. Since the universe expands, the physical momentum decays as $p \propto a^{-1}$. Hence, the energy density of radiation (or relativistic particles) evolves as $\rho_\gamma \propto a^{-4}$. This means that if the universe has some radiation today and some non-relativistic component, in the past the radiation part dominated the energy budget. Such radiation field exists: the CMB, which is a very homogeneous bath of photons that today are at ~ 2.7 K, fig. 6. In a logarithmic scale, this looks as shown in Fig. 9. Also, if one thinks about the thermal distribution of photons, from statistical mechanics we now that (We work in units such that $k_B = 1$)

$$f(E) = \frac{1}{e^{E/T} - 1}. \tag{22}$$

Since $E \propto a^{-1}$, this means that the physical effect of the expansion of the universe for photons (or relativistic species in general) is cooling or warming up the distribution as $T \propto 1/a(t)$. Also in fig 9 I have included the energy density of dark energy (DE). This is a 'material' whose energy density doesn't change as the universe expands. After reflecting about this property, you may now understand why is it so difficult to model it with any known source of matter. This also makes it dominate at late times, no matter how small ρ_{DE} is. There is another mysterious fact about cosmology: the size of the DE energy density is not only tiny when compared with EFT arguments, but it is also of the size that made it dominant in the recent universe...

What happens for a massive particle if the temperature is higher or smaller than the mass? This is the case of cosmic neutrinos! Imagine that they are produced thermally, and at $T \gg m_\nu$. That means that in the early universe, they behave as relativistic species. As the universe expands, they cool down, and when $T \sim m_\nu$, they start to behave as a cold, non-relativistic species.

3.2 Thermal equilibrium in an expanding Universe

Let's consider a patch of the universe where particles interact often (size smaller than mean free path) and for time scales larger than the time between collisions. Local equilibrium will hold in these patches, and we expect a state of maximum entropy locally. To characterize this state, we introduce the phase space distribution function, $f(\vec{x}, \vec{p}, t)$, such that

$$f(\vec{x}, \vec{p}, t) \frac{\mathrm{d}^3x \mathrm{d}^3p}{(2\pi\hbar)^3}, \tag{23}$$

is the number of particles with position $\vec{x} + \mathrm{d}\vec{x}$ and momentum $\vec{p} + \mathrm{d}\vec{p}$. If we know $f(\vec{x}, \vec{p}, t)$, the microscopic state will be characterized. In equilibrium it's enough to focus on the macroscopic properties, number density

$$n = \frac{N}{V} = \frac{1}{V} \frac{\int^V \mathrm{d}^3x \int \mathrm{d}^3p f(\vec{x}, \vec{p}, t)}{(2\pi\hbar)^3}, \tag{24}$$

where V is the volume of the patch we want to characterize, energy density

$$\rho = \frac{1}{V} \frac{\int^V \mathrm{d}^3x \int \mathrm{d}^3p E(p) f(\vec{x}, \vec{p}, t)}{(2\pi\hbar)^3}, \tag{25}$$

In the patch of interest, the distribution won't be space dependent (there are many scatterings). Furthermore, in equilibrium, the distribution can only be (for bosons $(-)$ or fermions $(+)$)

$$f(p; T, \mu) = \frac{1}{e^{(E-\mu)/T} \pm 1}, \tag{26}$$

where E is the energy and μ represents a chemical potential related to some conservation law. Let's forget about μ for simplicity (it also not very relevant if the DM particle is not associated with a conserved quantum number). If the particleve has extra degrees of freedom g, it's enough to multiply f by g. If the collection of particles is relativistic, $E = p$, and

$$\rho_\gamma = \frac{\pi^2}{15} T^4. \tag{27}$$

This is consistent with what we already discussed about radiation: $T \propto 1/a(t)$, and hence, $\rho \propto a^{-4}$. One can also compute the number density of relativistic species (as photons)

$$n_\gamma = 12\frac{g_{eff}}{\pi^2}T^3. \tag{28}$$

where $g_{eff} = 3/4$ for fermions and $g = 1$ for bosons.

For a massive particle, $E^2 = m^2 + p^2$. In the non-relativistic limit $T \ll m$,

$$\rho \approx m n_m, \qquad \text{where} \quad n_m = g_{eff}\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}. \tag{29}$$

The exponential suppressing this number is called Boltzmann factor. It basically represents the idea that in a thermal bath satisfying $m \gg T$, a massive particle may decay into other lighter components of the bath, but it is exponentially hard to find energy to generate the particle from the other species in the bath. In other words, massive particles in thermal equilibrium disappear very fast! Compare this with what happens with a particle that evolves freely (after it's decoupled from the bath), for which $N = nV$ is conserved, and hence $n \propto a^{-3}$, independently of the mass.

In the context of DM, since we want it to be produced at the observed levels, if it is a massive particle that was in equilibrium in the early universe, it should have decoupled from the thermal bath at large enough temperatures. Let's see how this is related to the SM-DM cross-section. Let's start discussing why a collection of particles in equilibrium get out of equilibrium in an expanding universe and generate a thermal relic. Recall that the equilibrium configuration is maintained by frequent scattering. Let's imagine a process (see also fig. 10)

$$\mathrm{DM_1} + \mathrm{DM_2} \to \mathrm{SM_A} + \mathrm{SM_B} \tag{30}$$

Given an annihilation cross-section for this process σ, the rate of this reaction per DM particle will be[5]

$$\Gamma = n_{DM}\sigma v_{DM}. \tag{31}$$

This process makes DM particles annihilate into SM states. The inverse process is in principle allowed, and also SM + DM scattering. As the universe expands, the number density gets smaller and the interaction rates decrease with time. At some point the previous reaction is not efficient to keep local chemical equilibrium. When does this happen? The important number to compare with is the rate of expansion of the universe $H(t)$. The intuition is that if the universe expands very fast, the particles do not have time to interact before being too far away from each other, and local equilibrium is lost (we'll be more explicit momentarily). In other words, there are not enough interactions to keep things in equilibrium in times H^{-1}, and the simple expansion of the universe dominates the behaviour of the particle distribution. **Ex.** *How much is H_0^{-1} in years?*

H also decays with time as the universe expands, but slower. Hence if $\Gamma \gg H$ held in the early universe, as the universe expands it will reach a moment when $\Gamma \sim H$

[5] One needs to average over the DM distribution, but the result will in principle be of the order of the following expression.

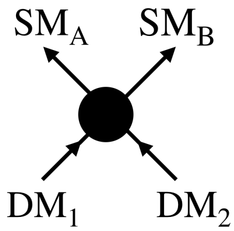


Figure 10: SM DM interaction

. Depending on when this happens, the particle may have already disappeared: if a particle stays in equilibrium and the temperature has dropped below its mass, it basically disappears, while if it decouples ($\Gamma \sim H$ satisifed) while the universe was still hot enough ($T \gtrsim m$), it number density was given by (28), and freely dilutes as a^{-3} from the decoupling moment.

To be able to predict the amount of DM one needs to be a bit more concrete. The number of particles in an expanding volume $V \sim a^3$ will change as

$$\frac{d(na^3)}{dt} = (\text{new particles per unit time}) - (\text{lost particles per unit time}). \tag{32}$$

The right hand side is called the 'collision operator'. If the process (30) and its inverse are possible, the previous equation can be written as

$$\begin{aligned} \frac{dn}{dt} + 3nH = -\int \mathrm{d}\Pi_1 \mathrm{d}\Pi_2 \mathrm{d}\Pi_A \mathrm{d}\Pi_B (2\pi)^4 \delta^{(4)}(p_A + p_B - p_1 - p_2) \\ \left[|\mathcal{M}_{12\to AB}|^2 f_1 f_2 (1 \pm f_A)(1 \pm f_B) - |\mathcal{M}_{AB\to 12}|^2 f_A f_B (1 \pm f_1)(a \pm f_2) \right], \end{aligned} \tag{33}$$

here $\mathrm{d}\Pi_i \equiv \frac{g\mathrm{d}^3 p_i}{(2\pi)^3 2E_p}$. The sign depends on the spin of the candidates. Fermions have the $-$ sign (one can't decay into occupied states) and bosons have the $+$ sign (there is an enhancement of the rates to states already occupied, Bose enhancement). The matrix elements $\mathcal{M}_{12\to AB}$ and $\mathcal{M}_{AB\to 12}$ can be computed given the Lagrangian responsible of the interaction between the DM and the SM using standard Feynman rules.

At high temperatures the occupation numbers are typically small. For the same reason, most of the available phase space is empty, hence we can ignore the Bose enhancement and blocking $1 \pm f \sim 1$. To simplify the expression, we can focus on CP invariant theories, for which both matrix elements coincide and will be denoted by $|\mathcal{M}|$. Finally, for the SM particles A and B we can assume an equilibrium distribution. This follows from the fact that at the cosmic times of interest the universe is very dense, and the interactions between SM particles and photons are very efficient to keep equilibrium. Also, the temperature is high enough to approximate them as (for both fermions and bosons)

$$f_A f_B \approx e^{-(E_A+E_B)/T}. \tag{34}$$

From energy conservation in the reaction, this is equal to

$$f_A f_B \approx e^{-(E_1+E_2)/T} = f_1^{eq} f_2^{eq}. \tag{35}$$

Finally, for f_1 and f_2, we will assume a configuration close to equilibrium,

$$f_{1,2} = f_{1,2}^{eq}(1+\delta f). \tag{36}$$

For simplicity, we assume the deviation to be p independent. With all these approximations we find

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle \left(n^2 - n_{eq}^2\right), \tag{37}$$

where

$$\langle\sigma v\rangle = \frac{1}{n_{eq}^2}\int \mathrm{d}\Pi_1 \mathrm{d}\Pi_2 \mathrm{d}\Pi_A \mathrm{d}\Pi_B (2\pi)^4 \delta^{(4)}(p_A + p_B - p_1 - p_2)|\mathcal{M}| f_1^{eq} f_2^{eq}. \tag{38}$$

This is a thermally averaged cross-section: cross-sections with energy of the states given by a thermal distribution.

The next step is to understand which was the main source of energy density in the universe (we need $H(t)$). We focus on the production of DM. From the fig. 9, we know that when DM was produced, the universe energy budget was dominate by radiation. This happens at redshift larger than $z \sim 3000$. Indeed, we have evidence that DM was there at around that time, so we need to create it before. At these early times, the Friedman equation reads (cf. (27))

$$H^2 = \frac{8\pi G}{3}\rho_\gamma = \left(1.66 g_\star^{1/2}\frac{T^2}{M_P}\right)^2, \tag{39}$$

where $M_P \sim 10^{19}$ GeV and $g_\star$ counts the number of degrees of freedom.

In a non-expanding universe, $n = n_{eq}$ solves the equation (37). In other words, the processes of particle creation and annihilation generate the equilibrium configuration, as expected. But as the universe expands, the Hn term becomes important, and n decays as $n \propto a^{-3}$. In eq. (37) we see explicitly that the behaviour of n depends on which term dominates, and that $\Gamma = \langle\sigma v\rangle n$ vs H are the important rates to compare.

In cosmology it's customary to present the results in terms of the 'yield' Y, a quantity that remains constant in situations where entropy and particle number are conserved:

$$Y = \frac{n}{s}, \tag{40}$$

where s is the *total* entropy per unit volume. The entropy density is derived by noting that, from first law of thermodynamics

$$T\mathrm{d}S = \mathrm{d}E + p\mathrm{d}V = \mathrm{d}[\rho + p)V] - V\mathrm{d}p, \tag{41}$$

and, from,

$$\frac{\partial^2 S}{\partial V \partial T} = \frac{\partial^2 S}{\partial T \partial V} \tag{42}$$

it follows that $T\mathrm{d}p = \frac{\rho+p}{T}\mathrm{d}T$. Hence,

$$s = \frac{\rho + p}{T}. \tag{43}$$

The pressure of a collection of particles with phase-space distribution f is given by (e.g. [18])

$$p = \frac{1}{(2\pi)^3} \int d^3p \frac{|p|^2}{3E} f(p). \tag{44}$$

If entropy is conserved, $s \propto a^{-3}$, no matter what's going on in the distribution. This is not true if there are sources of heat, or departure from equilibrium. One can now compute the value of entropy for a relativistic particle

$$s = \frac{2\pi^2}{45} g_* T^3, \tag{45}$$

where $g_* = \frac{7}{8}$ for fermions. In terms of Y, the equation of evolution (37) reads (**Ex.** *Show it*)

$$\frac{dY}{dt} = -s\langle\sigma v\rangle \left(Y^2 - Y_{eq}^2\right). \tag{46}$$

One can introduce the 'time' variable $x = m/T$. Notice that if $x \ll 1$ a particle in equilibrium has a considerable yield. In this variable, one finds

$$\frac{1}{Y_{eq}} \frac{dY}{d\log x} = -\frac{\Gamma}{H} \left(\frac{Y^2}{Y_{eq}^2} - 1\right), \tag{47}$$

where $\Gamma = n\langle\sigma v\rangle$. Using now $Y_{eq}(x)$ and $H(x)$ from (39), we can solve this equation. This expression can also be written as

$$\begin{aligned} \frac{dY}{dx} &= \frac{-sx\langle\sigma v\rangle}{H(m)} \left(Y^2 - Y_{eq}^2\right) \\ &= \frac{-\lambda\langle\sigma v\rangle}{x^2} \left(Y^2 - Y_{eq}^2\right) \end{aligned} \tag{48}$$

where we have used (45) and

$$\begin{aligned} \lambda &\equiv \frac{2\pi^2}{45} \frac{M_P g_{*s}}{1.66 g_*^{1/2}} m \\ &\approx 0.26 \frac{g_{*s}}{g_*^{1/2}} M_P m \end{aligned} \tag{49}$$

.

Ex. *When do neutrinos decouple from the thermal expansion?*

3.2.1 Digression: Kinetic equilibrium

The previous formulae are connected to the process shown in (30) and fig. 10. There may be other processes, as for instance elastic scattering,

$$SM_A + DM_1 \to SM_B + DM_2 \tag{50}$$

This process will have a rate $\Gamma_s \sim n_{SM}$, which will make it relevant when Γ of (30) is below H. Thus, kinetic energy will be still redistributed and one can still keep equilibrium even after losing 'chemical equilibrium'. This should happen at fixed N_{DM}, since DM is conserved, which means that the final distribution will develop a chemical potential μ.

3.2.2 Digression: Lioville equation

Finally, we may worry also about the distributions (no only the macroscopic quantities). The equation that $f(\vec{p}, t)$ solves can be written in a more convenient form, based on the Boltzmann equation, which is the manifestation in phase-space of the conservation of particles in time:

$$\frac{Df(\vec{p}, t)}{dt} = (\text{new } \vec{p} \text{ particles per unit time}) - (\text{lost } \vec{p} \text{ particles per unit time}). \tag{51}$$

This can be written in an homogeneous and isotropic expanding universe as

$$E\partial_t f - Hp^2 \frac{\partial}{\partial E}\rho = C[f]. \tag{52}$$

The collision term $C[f]$represents the balance of created and annihilated particles. **Ex.** *Show that if $C[f] = 0$, the total number of particles is conserved.*

3.3 Freeze out

3.3.1 Freeze out of relativistic species

The equation (47) can be solved numerically, and the evolution with x of Y is shown in fig. 11. One sees that depending on the cross-section, the decoupling happens at different times, impacting the final yield. This is called 'freeze-out'. One can estimate this analytically: let's consider a species that decouples while being relativistic $m \ll T$. At $\Gamma_A \sim H$,

$$Y_{rel} = \frac{0.278 g_{eff}^{DM}}{g_{eff}^{plasma}}. \tag{53}$$

This comes from the entropy being a function of all the elements in the plasma. **Ex.** *Show the previous result.*

This yield remains constant as the universe expands afterwards. If this species is non-relativistic today,

$$\Omega_0 = \frac{\rho_0}{\rho_c} = \frac{mn_0}{\rho_c} = \frac{m}{\rho_c} Y_0 s_0 = \frac{m}{\rho_c} Y_{rel} s_0. \tag{54}$$

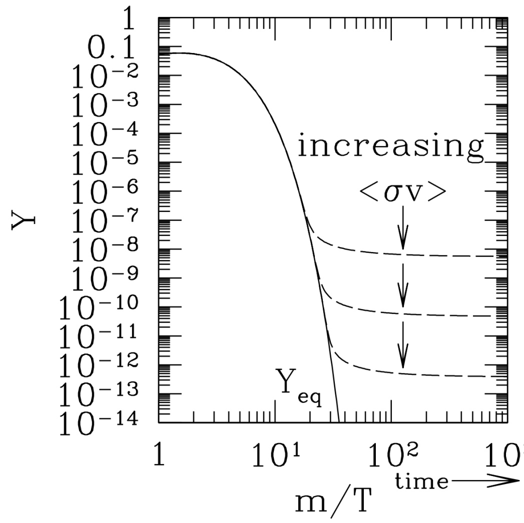


Figure 11: Yield as a function of time for different annihilation cross sections, from [19]. Notice the fact that higher cross-sections give lower yields.

The entropy today is stored in the 2 polarizations of the CMB, so

$$s_0 = 2\frac{2\pi^2}{45}(2.7\,\mathrm{K})^3. \tag{55}$$

From this we find,

$$\Omega_{today}h^2 = h^2\frac{g^{DM}m}{136\,\mathrm{eV}}, \tag{56}$$

where we used as input the number of species in the primordial plasma g_{eff}, assuming that the decoupling happens at few MeV: $e^\pm$, γ, ν, $\bar{\nu}$.

Notice that this doesn't depend on the cross-section!! The only relevant quantity is the number of degrees of freedom in the plasma at decoupling, and how the rest of degrees of freedom evolved afterwards. We already see a reason why neutrinos can not be a good DM candidate: $m_\nu < 0.6\,\mathrm{eV}$.

3.3.2 Freeze out of non-relativistic species

The calculation of freeze out of non-relativistic species is more complex, but allows for more flexibility in the final result. If $m \gtrsim T$ at freeze-out time, $n \sim e^{-m/T}$. So, one can forget about $m \gg T$ for this mechanism to be efficient, and focus on $m \sim T$. One can make a relatively complete analytic derivation (see e.g. Cerdeño's notes or [1]). Here I use an estimate, which gives the right order of magnitude **Ex.** *Do the same for* $3 \to 2$ *processes.* Let's assume that $\Gamma \sim H$ at $m \sim T$. This means that

$$n\langle\sigma v\rangle \sim H \sim \frac{T^2}{M_P} \sim \frac{m^2}{M_P}. \tag{57}$$

Now, since freeze-out happens when the DM is non-relativistic $\rho_f = mn \sim \frac{m^3}{\langle\sigma v\rangle M_P}$. From this moment, to any future cosmic time ρ behaves as

$$\rho_t = \rho_f \left(\frac{T_t}{T_f}\right)^3. \tag{58}$$

In our universe $\rho_{DM} = \rho_{CMB}$ at $z_{rme} \sim 3000$ (radiation-matter equality), which corresponds to $T_{rme} \sim$ eV. So,

$$\rho_f \left(\frac{T_{rme}}{T_f}\right)^3 \sim T_{rme}^4. \tag{59}$$

From this

$$\langle\sigma v\rangle \sim \frac{1}{T_{rme} M_P} \sim \frac{\alpha^2}{10^{-4}} \frac{1}{(10^2\,\mathrm{GeV})^2}. \tag{60}$$

Hence, a thermally averaged cross-section around the weak scale (still perturbative at this scale, since $\alpha \sim 10^{-2}$) produces the right amount of DM through this mechanism. The precise calculation of $\langle\sigma v\rangle$ is model dependent and has some subtleties. In these lectures, I only discuss order of magnitude estimates. If you want a deeper understanding, you are invited to look at Cerdeño's notes or at [20, 21, 3].

Another way to write this is as

$$\Omega_{DM} h^2 \approx 0.1 \left(\frac{0.01}{\alpha}\right)^2 \left(\frac{m_{DM}}{100\,\mathrm{GeV}}\right)^2. \tag{61}$$

This result is known as the 'wimp miracle'. The reason why people like this name is that it is very natural to produce the required amount of DM with an extra species with weak interactions, and not much heavier than GeV. Hence, if new physics is responsible for the weak scale, it may come with a related DM candidate. The reason why people do not like this name is that hidden in the previous logic there is an exponential sensitivity to the mass. Hence if the decoupling happens a bit earlier or later, one obtains totally different results.

4 Axions as DM candidates

Most of what we have done in the last lecture applies for WIMP-like particles . However, there are other candidates for physics BSM that also provide lighter DM candidates. The most famous one is the axion. As we discussed before, the DM phenomenology could be explained with bosons as light as 10^{-21} eV, as long as they are cold enough to behave as CDM! See the discussion after eq. (21). Clearly, very light particles can not behave like this if they are thermally distributed: DM existed when the universe was at $T \sim 1$ eV. So nothing thermal and lighter than eV can behave as DM when it should[6].

[6] This bound is too naive. Doing it properly, which includes considering kinetic decoupling and other processes, yields the limit at keV.

Besides the axions, there are other DM candidates that can be produced via non-thermal processes. A well studied possibility is based on a thermally coupled particle decaying into the DM (freeze-in mechanism). These mechanisms extend the DM parameter space [3, 6]. As you would expect, one of the main differences with freeze-out is that in the freeze-in, the higher the coupling, the higher the yield. The simplest DM model beyond[7] the WIMP is the axion, and I'll focus on it.

You may be aware that the standard model suffers from the strong-CP problem. This should be explained in an advanced module on QFT or in SM lectures, see e.g. [22]. In a nutshell, one can add the following term to the SM Lagrangian

$$\theta \int \mathrm{d}^4 x G_{\mu\nu} G_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}, \tag{62}$$

where $G_{\mu\nu}$ is the QCD field strength and $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor[8] . This term doesn't show up in Feynman rules, but contributes to non-perturbative calculations. In particular it generates CP violating contributions. For instance, it enters in the dipole moment of the neutron, that vanishes otherwise. Experimentally, this is measured to the level that requires $\theta \lesssim 10^{-11}$. As far as I know there is no reason for this number to be large or small. So, there is not really a hierarchy problem here. But its value is certainly very small.

The way the axion field solves this problem is by postulating the existence of a new pseudo-scalar, whose mass is protected by a scale invariant symmetry $\varphi \to \varphi + a$. It is then natural to associate it to the Nambu-Goldstone boson of a spontaneously broken symmetry of the early universe. (A famous mechanism is the Peccei-Quin mechanism, who postulated a $U(1)_{PQ}$ symmetry). This field will couple to QCD as

$$\int \mathrm{d}^4 x \frac{\varphi}{f_\varphi} G_{\mu\nu} G_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}, \tag{64}$$

since this is a scale invariant coupling. f_φ is a high energy scale related to the symmetry breaking scale. If φ was produced in the early universe after a phase transition, its expectation value will be fixed to minimize the energy functional. When one considers the two terms (62) and (64), one sees that the solution will be

$$\bar{\varphi} = -\theta f_\varphi, \tag{65}$$

and the bulk part of the CP violating contribution vanishes. The axion-framework also includes a way to generate a mass term to φ through some suppressed contributions. In practice, this means that it is natural to have very small masses in the axion scenario. It

[7]Maybe even the simplest DM model, period.

[8]

$$\epsilon^{0123} = 1 \tag{63}$$

and the rest of components are found by permuting indexes. Each permutation changes 1 to -1. `https://en.wikipedia.org/wiki/Levi-Civita_symbol#Four_dimensions`.

is also natural to have even more suppressed self-couplings, so one can focus on a simple low-energy Lagrangian

$$\mathcal{L}_\varphi = \frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi - m_\varphi^2\varphi^2\right). \tag{66}$$

The value of the mass in the PQ case (one of the concrete axion scenarios) is

$$m_\varphi \approx \frac{13\,\mathrm{MeV}}{f_\varphi/\mathrm{GeV}}. \tag{67}$$

One can try to produce axions though freeze-out, but it doesn't work (at least in the QCD axion. One can always extend the parameter space of 'axion-like' particles. Which, btw, are very common in string theory).

The other mechanism that people prefer is the 'misalignment' after a phase-transition. Let's consider the early universe as a very homogeneous and isotropic medium. In the beginning it was very hot. So hot that the vacuum (65) was not relevant: the field fluctuated to much larger values simply by thermal fluctuations. At some point, the plasma cools down enough to generate this field configuration. However, this process is not instantaneous or totally efficient, and there will be deviations with respect to $\bar{\varphi}$ of certain coherence length in the universe. How do they behave? It's enough to focus on $\varphi = \bar{\varphi} + \delta\varphi$. In each of these patches, $\delta\varphi$ is homogeneous, and one can simply worry about

$$\mathcal{L}_\varphi = \frac{1}{2}\delta\dot{\varphi}^2 - m^2\delta\varphi^2, \tag{68}$$

the Lagrangian density for a harmonic oscillator. The related energy density is given by

$$\rho_\varphi = \frac{1}{2}\delta\dot{\varphi}^2 + \frac{1}{2}m^2\delta\varphi^2. \tag{69}$$

How is the dynamics determined? In the WIMP case we focused on the kinetic picture given by a phase space distribution function. For the case of interest here, one simply solves the evolution of the field equations, as one does in classical electromagnetism, without worrying about particles, or quantum calculations. The reason is that once the $\delta\varphi$ is produced, it has large occupation numbers and they are not modified by creation or annihilation processes in this scenario. So, the classical evolution of the field (macroscopic) quantities is enough. We need to include the fact that this field is living in an expanding universe. One can show that the standard harmonic oscillator equations that follow from (68) are modified to

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + m^2\delta\varphi = 0. \tag{70}$$

From the Friedman equation (16) and once the different contributions to ρ are considered (e.g. (69)), the set of equations closes.

To solve the previous equation, we first consider the early universe where $H \gg m$. In this case, there is a decaying and a constant solution, (we take $H =$const. for simplicity)

$$\delta\varphi = \delta\varphi_0 e^{\omega t}, \quad \omega^2 + 3H\omega + m^2 = 0. \tag{71}$$

The almost constant solution $\omega \approx -\frac{2}{3}\frac{m^2}{H} \ll 1$ dominates the time evolution. During this period of time, ρ_φ is constant! So it doesn't behaves as a DM candidate.

After the universe expands enough, $H \ll m$. In that case, the solution to (71) is

$$\omega \approx -\frac{3H}{2} \pm im, \tag{72}$$

and

$$\delta\varphi \approx \delta\varphi_0 e^{-\frac{3}{2}Ht}\cos(mt+\phi_0). \tag{73}$$

Since we considered H constant,

$$\frac{\dot{a}}{a} = H, \qquad a = a_0 e^{Ht} \tag{74}$$

Now, from the previous expressions and (69) we find that

$$\rho \sim a^{-3}(\cos(mt+\phi_0))^2. \tag{75}$$

The oscillatory part averages to $1/2$ in times longer than $1/m$ and we find that the average energy density of this field theory behaves as a DM candidate! However, this was derived assuming $H \approx$constant. If DM dominates the universe, H evolves with time. To see that even in this case, the candidate behaves as DM as long as $H \ll m$ one can use the following logic. First, on short time scales, the field will oscillate at frequency m. The energy density averaged over longer time scales is

$$\rho_{av} = \langle \delta\dot{\varphi}^2 \rangle \tag{76}$$

To find the evolution equation for this averaged quantity, one can multiply (70) by $\delta\dot{\varphi}$ and average over time. Notice that the last term is a total derivative, and hence it will not contribute on long term averaging. The final answer is

$$\dot{\rho}_{av} + 3H\rho_{av} \approx 0, \tag{77}$$

for any H.

The relic abundance is fixed by $\delta\varphi_0$, since $\rho \approx m^2\delta\varphi_0^2$. From dimensional reason one expects to be $\delta\varphi_0 \sim f_\varphi$, but the prefactor is not fixed. In the PQ scenario, one finds expressions of the form

$$\Omega_\varphi h^2 \sim \left(\frac{m_\varphi}{10^{-5}\,\text{eV}}\right)^{-3/2}. \tag{78}$$

We may come back to axion phenomenology later on. A good summary of axion cosmology can be found in [4].

More on axions `https://static.ias.edu/pitp/2017/node/1381.html`.

5 Local DM and direct detection of WIMPs

A very relevant aspect of the WIMP paradigm (part of the miracle) is that the cross-section relevant for freeze-out are in the ballpark of those that could allow a direct detection. This is really a 'miracle', since the local abundance of DM has little to do with the generation mechanism or with the technologies of the XXIst century.

To understand the prospects for direct detection, we need *i)* the flux of DM on Earth $n_{DM}v$ and *ii)* the experimental sensitivities. For the first one, there are models that try to extract the DM density at the Sun's location, $n_{DM,\odot}$. It is instructive to have a closer look at the Milky Way (MW). The MW has 10^{11} stars. It has a mass in stars of around $5 \times 10^{10}\, M_\odot$ and 10% extra mass in gas. It also has a supermassive black hole at the galactic center of mass $\sim 10^6\, M_\odot$ (the galactic center is in Sg). The MW has a bulge in the center and spiral arms in the disk that extend up to 10 kpc. The Sun is located at 8 kpc from the galactic center. The disk has width ~ 0.5 kpc. See fig. 2. From this data, one can try to look for a DM halo in the MW, see e.g. [23]. There is an order of magnitude uncertainty of the value, but most studies cite the number $\rho_\odot \sim 0.3\,\mathrm{GeV/cm^3}$.

5.1 Digression: Kinetic picture of the Milky Way

One can try to estimate the phase space distribution of the DM particles in the MW halo. Let's start again from $f(\vec{x}, \vec{p}, t)$ (cf. eq. (23)). Recall that the DM should be distributed in a way that reproduces flat rotation curves at large radius. This means $M(r) \propto r$, and hence $\rho \propto r^{-2}$. If this corresponds to stationary situation, there is a theorem (Jean's theorem) that says that the corresponding phase space distribution function can only depend on conserved quantities. The most obvious conserved quantity is the energy per unit mass,

$$\epsilon = \frac{1}{2}v^2 - \psi, \tag{79}$$

where ψ is the gravitational potential. Hence, one expects $f(\epsilon)$. One can try

$$f \propto e^{-\epsilon/\epsilon_0}, \tag{80}$$

which could be justified by some equilibrium logic (Maxwell-Boltzmann like distribution). But the arguments about equilibrium are not so clean in a system with long-range interactions. In any case, with this first attempt one finds

$$\rho = mn \propto m \int \mathrm{d}v v^2 f(v) \propto m e^{\psi/\epsilon_0}. \tag{81}$$

Together with Newton's law (in the form of Poisson equation)

$$\nabla^2 \psi = 4\pi G \rho, \tag{82}$$

one finds

$$\rho(r) = \frac{\epsilon_0}{2\pi G r^2}. \tag{83}$$

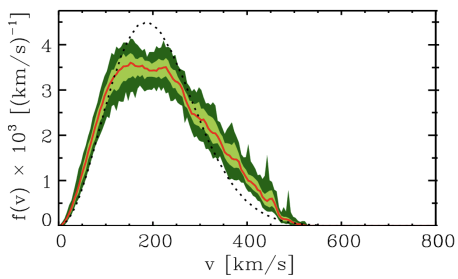


Figure 12: Phase space distribution of DM, from [24]

Which is what we needed. This is called 'isothermal' profile. Hence, the profile that is valid for the rotation curves seems to correspond to a distribution of the form (80).

Things are more complicated, and the final universal profile that emerges from numerical simulations of DM particles generating halos is the Navarro-Frenk-White (or a related one called Einasto)

$$\rho_{NFW} = \frac{\rho_0}{r/r_s(1+r/r_s)^2}, \tag{84}$$

which corresponds to isothermal profile at large distances. Similarly, the distribution that is used is a truncated version of the Maxwellian

$$f(v) = \begin{cases} \frac{1}{N_{esc}} \left(\frac{3}{2\pi\sigma_0^2 v}\right)^{3/2} e^{-\frac{3}{2}\frac{v^2}{2\sigma_0^2}} & , \; v^2 < v_{esc} \\ 0 & , \; v > v_{esc} \end{cases} \tag{85}$$

where v_{esc} is the escape velocity of the MW, $v_{esc} \approx 600\,\mathrm{km/s}$ and the dispersion is $\sigma_0 \approx 300\,\mathrm{km/s}$.

On top of the previous simple distribution, there are different substructures, from debris to streams, coming from collisions with other galaxies or other processes. There is a nice discussion about this in [1]. A more realistic distribution is shown in fig. 12. It was recently discovered that there is a DM hurricane coming towards us [25].

The previous distribution has a preferred frame, where $\bar{v} = 0$. However, the Sun moves with respect to this frame. In models of galactic halo, the baryonic disk rotates faster, and hence, the angular velocity of the Sun is a good proxy for this relative velocity. So, we expect the relative velocity of DM and Sun to be $v \sim 10^{-3}$. On top of this, one can add the velocity of the Earth around the Sun $v_\oplus \sim 10^{-4}$. **Ex.** *Compute the mean velocity in the lab frame.* In the laboratory frame, one gets

$$\bar{v}_{DM} \sim 10^{-3}, \tag{86}$$

and an annual modulation of order 10^{-4}. The DM flux is hence

$$\phi \sim 10^7 \frac{\mathrm{GeV}}{m} \mathrm{cm}^{-2} s^{-1}. \tag{87}$$

5.2 DM scattering on Earth

Let us consider a scattering process as the one shown in fig. 10, read horizontally. We focus on the scattering with nuclei. The kinetic energy of DM in the lab frame is

$$E_{DM} = \frac{1}{2} m_{DM} v^2 \sim 10^{-6} m_{DM}. \tag{88}$$

Hence, if $m_{DM} \lesssim \text{GeV}$, E_{DM} is smaller than the binding energy in nuclei (MeV) and the scattering is elastic. In this case, one can compute the recoil energy of the nucleus,

$$E_R = \frac{1}{2} m_{DM} v^2 \frac{4 m_{DM} m_N}{(m_{DM} + m_N)^2} \frac{1 + \cos\theta}{2} = \mu v^2 \frac{\mu}{m_N} (1 + \cos\theta), \tag{89}$$

where θ is the scattering angle **Ex.** *Check the kinematic*. Here

$$\mu = \frac{m_{DM} m_N}{m_{DM} + m_N} \tag{90}$$

is the reduced mass. So, at most $E_R \sim E_{DM}$,

$$E_R^{max} \sim 10\,\text{keV} \left(\frac{m_{DM}}{20\,\text{GeV}} \right)^2 \frac{100\,\text{GeV}}{m_n}. \tag{91}$$

This amount of energy per nucleus needs to leave a trace in the detector. For the standard detector (e.g. Xenon) there is a threshold for signals below keV. So either one finds a way to heat up the DM, or these detectors do not see anything below $\sim$GeV masses[9]. Another way out is to consider scattering with electrons that can get ionized, or in materials where they are free. But this is whole new world...`https://indico.cern.ch/event/676835/contributions/3008408/`.

The scattering is normally coherent in the atom, since the de Broglie wavelength of the DM packet

$$\lambda \sim \frac{1}{mv} \gg \text{size of the atom}. \tag{92}$$

Let us now consider the rate of events per recoil energy in a detector with N_T targets of mass m_N

$$\frac{dR}{dE_R} = N_T n_{DM} \left\langle \frac{d\sigma}{dE_R} v \right\rangle. \tag{93}$$

In the previous formula, $\frac{d\sigma}{dE_R}$ is the differential cross section per recoil energy. The brackets represent the averaging in the DM distribution,

$$\left\langle \frac{d\sigma}{dE_R} v \right\rangle = \int \mathrm{d}^3 v f(v) \frac{d\sigma}{dE_R} v. \tag{94}$$

[9]There are always some events, but this is very suppressed as we will compute.

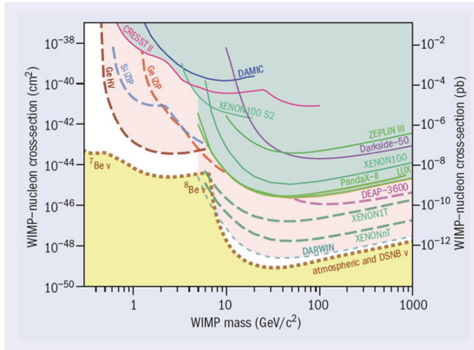


Figure 13: Sensitivity curves for DM cross section with nucleons as as function of the DM mass `https://cerncourier.com/a/testing-wimps-to-the-limit/`

An important point is that this integral starts at the minimum velocity that generates a observable recoil in the detector $v_{min} = \sqrt{m_N E_R/(2\mu^2)}$. We first note from (89) that

$$\frac{dE_R}{d\cos\theta} = \frac{\mu^2 v^2}{m_N}. \tag{95}$$

Furthermore, since we are in a non-relativistic regime, we expect

$$\frac{d\sigma}{d\cos\theta} = ct. + O(v), \tag{96}$$

and we keep only the leading order. As a result (integrating the previous equation)

$$\frac{d\sigma}{d\cos\theta} = \frac{\sigma}{2}. \tag{97}$$

From the previous equations we can explain some features of the sensitivity curves of DM searches. First,

$$\int_{v_{min}}^{v_{esc}} \mathrm{d}^3 v f(v) \frac{d\sigma}{dE_R} v \propto \int_{v_{min}}^{v_{esc}} \mathrm{d}v v e^{-v^2} \sim e^{-E_R/E_0}, \tag{98}$$

where $E_0 = 2\mu^2 v_0^2/m_N$. For a 100 GeV DM scattering off a Xenon target, $E_0 \sim 50$ keV [1]. This means that the expected recoil spectrum for the nucleus is exponentially falling, for typical assumptions about the cross section and velocity distribution. In other words, only the tail of the distribution has enough energy at low m_{DM} to generate the E_R.

Similarly, at high masses, the previous expression behaves as $\sim n \sim \rho/m_{DM}$. Hence, at fixed $\rho_\odot$, it will decay as $1/m_{DM}$. This behaviour is also seen in the final sensitivity curves, cf. fig. 13.

5.3 Differential Scattering Cross Section (sketched)

The missing point to understand the sensitivity to DM scattering is to compute σ. We have already seen that we expect it to be v independent at first order. One can be relatively generic and assume an effective vertex SM-DM (we only discuss DM coupling to quarks for simplicity)

$$\mathcal{L}_{\text{eff}} = g\left(q^2, m_\phi\right) \overline{DM}\Gamma_{DM} DM \overline{Q}\Gamma_Q Q, \tag{99}$$

where Q are the quarks, $\Gamma_{DM,Q} = \{I, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}, \sigma^{\mu\nu}\gamma^5\}$, and $g(q^2, m\phi)$ is an effective coupling, where q is the momentum transferred and m_ϕ represents a new mediator. See e.g. [26]. The next step is to relate this fundamental Lagrangian to the cross-section in atoms[10]. What one needs to evaluate are matrix elements of the form $\langle n|\overline{Q}\Gamma_Q Q|n\rangle$, where $|n\rangle$ are the states corresponding to the nucleons[11]. The later are either measured or computed with advanced techniques. The final step is to build the target nucleus as a collection of Z protons and $(A-Z)$ neutrons. There are two paradigmatic examples that differ mostly in the last step: spin-independent and spin-dependent cross-section. The reason why they are different is that 'spin-independent' scattering in the nucleus adds up from different nucleons (e.g. if the coupling is the mass). This generates enhancements of order $O(Z^2)$. However, the spin of the nucleus is of the order of the spin of the nucleons, so $\sigma_{Ncl} \approx \sigma_n$.

Let's discuss these two examples in parallel,

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= g_{s.i}\overline{DM}DM\overline{Q}Q, \\ \mathcal{L}_{\text{eff}} &= g_{s.d}\overline{DM}\gamma_\mu\gamma^5 DM\overline{Q}\gamma^\mu\gamma^5 Q. \end{aligned} \tag{100}$$

For the first case (s.i.), one can compute or measure the following form factors $\left\langle p\left|m_q\overline{Q}Q\right|p\right\rangle = m_p f^p_{T_q}$, from which the coupling to a proton reads [27]

$$f_p = \sum_{q=u,d,s} m_p \frac{g_{s.i.}}{m_q} f^p_{T_q} + \frac{2}{27} f^p_{T_G} \sum_{q=c,b,t} m_p \frac{g_{s.i.}}{m_q}, \tag{101}$$

with $f^p_{T_G} = 1 - \sum_{q=u,d,s} f^p_{T_q}$. And similarly for the neutron. The scattering amplitude then reads

$$\mathcal{M} = \langle f_p\overline{DM}DM\overline{p}p + f_n\overline{DM}DM\overline{n}n\rangle. \tag{102}$$

On the field of a nucleus (Z, A), the previous evaluates to

$$\mathcal{M} = \left[Zf_p + (A-Z)f_n\right]\overline{DM}DM\overline{N}NF(q), \tag{103}$$

where the unknown piece $F(q)$ has to do with the coherence of the scattering process. The larger the momentum transfer, the less coherent the process (see more in [1]). $\bar{N}N$

[10] Notice that for electrons this is simpler

[11] These matrix elements are in principle q dependent, where q is the momentum transferred to the nucleon.

has to be evaluated in the Dirac spinors of a nucleon. Putting all together,

$$\frac{d\sigma}{dE_R} = \frac{2m_N}{\pi v^2} \left[Z f_p + (A-Z) f_n\right]^2 F^2(q). \tag{104}$$

The spin dependent case yields

$$\frac{d\sigma}{dE_R} = \frac{16m_N}{\pi v^2} g_{s.d}^2 J(J+1)\Lambda^2 F_{\mathrm{SD}}^2(q), \tag{105}$$

where

$$\Lambda \equiv \frac{1}{J}\left(a_p \langle S_p\rangle + a_n \langle S_n\rangle\right), \tag{106}$$

$a_{p(n)}$ is the effective coupling of the DM to the proton(neutron), and $\langle S_{p(n)}\rangle$ is the average spin contribution of the proton(neutron). As I mentioned, the spin-dependent form factor is different in order of magnitude from the spin-independent form factor. The reason is that the spin-dependent interaction is no longer coherent with the nucleus, and hence the result does not scale as A^2. As a result, spin-dependent interactions are more challenging to observe experimentally and the current bounds are weaker than those from spin-independent interactions.

The previous is the cross-section with the nuclei. To compare different detectors, one normally chooses to plot DM-nucleon cross sections. The conversion one can do (which is valid as an order of magnitude conversion is)

$$\mu_p^2 \sigma_N = \mu_N^2 A^2 \sigma_p. \tag{107}$$

This quantity σ_p is the one that appears in exclusion plots such as in fig. 13.

Before closing this section, it's worth commenting on one of the headaches of DM direct searches: the yellow band in fig. 13. This represents the background of solar and terrestrial neutrinos in the detector. The Sun generates a flux of neutrinos, whose cross section per recoil energy with nuclei reads

$$\frac{d\sigma_{\nu N}}{dE_R} = \frac{G_F G^2}{4\pi} Q^2 m_N \left(1 - \frac{m_N E_R}{3E_\nu^2}\right) F^2(E_R), \tag{108}$$

where $F^2(E_R)$ is the nuclear form factor, for which we have taken the parametrisation given by Helm [28]. Q parametrises the coherent interaction with protons (Z) and neutrons $N = A - Z$ in the nucleus:

$$Q = N - (1 - 4\sin\theta_W) Z. \tag{109}$$

See D. Cerdeño's notes. The important point is that this background can't be reduced, and distinguishing DM signals will be very difficult at these small cross-sections.

5.4 Shall we build a DM detector?

Now that we have all the numbers, let us make some optimistic guesses. The first people who realized that building a DM detector made sense are [29, 30]. Let's consider an interaction such that

$$\sigma_p \propto \frac{\mu_p^2 g^2}{\pi} \propto \frac{\mathrm{GeV}^2}{\pi(300\,\mathrm{GeV})^2}. \tag{110}$$

If one can detect the recoil for these candidates, the number of events per day per kilogram will be

$$N = nv\sigma t N_T \sim 10^{-2} \text{ events/kg/day}, \tag{111}$$

assuming $N_T \sim kg/(100$ GeV) and $m_{DM} \sim 1$ GeV, which means recoils in the keV. To understand the relevance of these rates, one needs to understand how well the background of any contaminant can be reduced to generate less signal. It is hard [3]. Still DM has an extra handle, which is the annual modulation and daily modulation.

6 The rest

After the previous material, it is customary in DM lectures to present another aspect of the DM problem. The most popular ones are indirect detection and collider searches. Indirect detection tries to find DM in astrophysical processes beyond the gravitational phenomenology. The most natural processes are the DM annihilating into SM particles (as in (30)) or DM decay. The WIMP paradigm generates an interesting phenomenology also in this regard. This is another part of the WIMP miracle.

Collider searches proceed as any other collider search, though this time the cross-sections are mapped to particular models of DM. Needless to say, there are no big news here, though, again, the WIMP candidate likes the weak scale, and as such the LHC is a great machine to test WIMP scenarios.

Another way to go beyond the previous topics is to discuss the challenges of WIMP candidates, and alternatives (including modifying the laws of gravitation). In fairness, the WIMP paradigm works well where it should (pure DM environments), while there are some challenges when the physics get more complicated. For instance, in the center of galaxies, where there are many other SM related effects. So, I won't discuss more about this.

I think it is more interesting (and timely) to discuss some ideas about detecting light DM in the lab. Recall from the previous section that DM particles of low mass do not leave detectable traces in current detectors looking for recoils (since they do not carry enough momentum). Furthermore, DM can be as light as keV if it is a fermion, and 10^{-21} eV if it is a boson. How do we detect it? For concreteness I will focus on some aspects of direct detection of axions. You can read more about this in [31] and .`https://indico.cern.ch/event/676835/contributions/3008408/`.

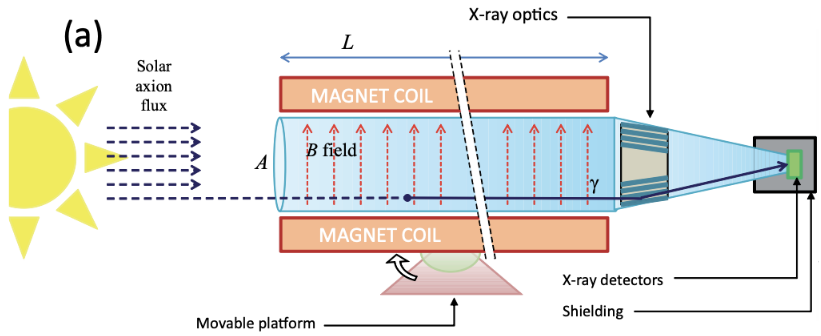


Figure 14: Scheme behind CAST helioscope. From [31]

6.1 Detection of axions in the lab

Axions are generally coupled to SM fields by suppressed operators. We already discussed the interaction term (64). This generates an EDM for neutrons, that one can look for. For the case of very light axions φ will behave as a classical field (the reason being that in the MW the occupation numbers for the DM states will be macroscopic). This field has coherent oscillations of period m,

$$\varphi = \frac{\sqrt{2\rho}}{m}\cos(mt + \phi_0) \tag{112}$$

An experiment looking for time variations due to the coupling (64) and with the previous background is the Casper experiment described in [31].

The axion also couples to light through the operator

$$\mathcal{L}_{\gamma\gamma\varphi} = g_{\gamma\gamma\varphi}\varphi F_{\mu\nu}F_{\alpha\beta}\epsilon^{\mu\nu\alpha\beta} = g_{\gamma\gamma\varphi}\varphi\vec{E}\cdot\vec{B}. \tag{113}$$

This kind of coupling is constrained from a plethora of phenomena. I describe a couple of important ones having to do with the mixing of photons and axions in the presence of magnetic fields. If $g_{\gamma\gamma\varphi} \neq 0$, the Sun generates axions though the $\gamma\gamma \to \varphi$ process. These axions travel through space and reach the Earth. If on Earth one has an intense magnetic field, part of this axionic flux will convert back to photons. This is the philosophy behind the CAST telescope at CERN (described also in [31]). This scheme is shown in Fig. 14

Another way to look for axion is to create them on Earth. For this, one starts with a strong source of light. As it goes though a magnetic field, it will generate axions. For large occupation numbers this can be treated as a classical propagation problem **Ex.** *Solve the system of two coupled oscillators to show this.* Light will satisfy a modified equation of motion, that makes it oscillate to axions, and back. If you are familiar with neutrino oscillations, this is a similar phenomenon. The probability of conversion into axions for light going through constant B at distance L is

$$P = \frac{4\Delta_M^2}{\Delta_m^2 + 4\Delta_M^2} \sin^2\left(\frac{1}{2} L \Delta_{\rm osc}\right), \tag{114}$$

where

$$\begin{aligned} \Delta_M &= 540 \left(\frac{B}{1\text{G}}\right)\left(\frac{10^{10}\text{GeV}}{g_{\gamma\gamma\varphi}^{-1}}\right) \text{pc}^{-1}, \\ \Delta_m = \frac{m^2}{2\omega} &= 7.8 \times 10^{-11} \left(\frac{m}{10^{-7}\text{eV}}\right)^2 \left(\frac{10^{19}\text{eV}}{\omega}\right) \text{pc}^{-1} \end{aligned} \tag{115}$$

and

$$\Delta_{\rm osc}^2 = (\Delta_m)^2 + 4\Delta_M^2. \tag{116}$$

Hence if we have an intense light beam going though an intense B, we may generate enough φ such that if a wall blocks all the light, the axions will still propagate and reconvert into photons in an intense B after the wall. This is the logic behind 'light shining through wall' (LSW) experiments. See again [31].

Quite amazingly, the strongest constraints some times come from phenomenology in stars. The way this works has analogies with the phenomenology of neutrinos in stars. Neutrinos are important to cool down stars and transport energy. If there is a new particle produced efficiently in the star plasma (which can be as hot as MeV), it may leave the star easily and cool it too fast. Remarkably, the theory of star evolution is mature enough to put constraints on deviations from the standard picture of order $O(1)$ [32, 33]. The first rule of thumb is that anything with mass below MeV can't be produced more efficiently than neutrinos. Even more, there are some situations where the production of axions is more efficient, and the bounds are as strong as

$$g_{\gamma\gamma\varphi} < 10^{-10}\text{GeV}^{-1}. \tag{117}$$

A relatively up-to-date summary of bounds on this coupling is shown in fig. 15

There are many new ideas now emerging. If you want to know more, come talk to me! And look at `http://qsfp.physics.ox.ac.uk/`.

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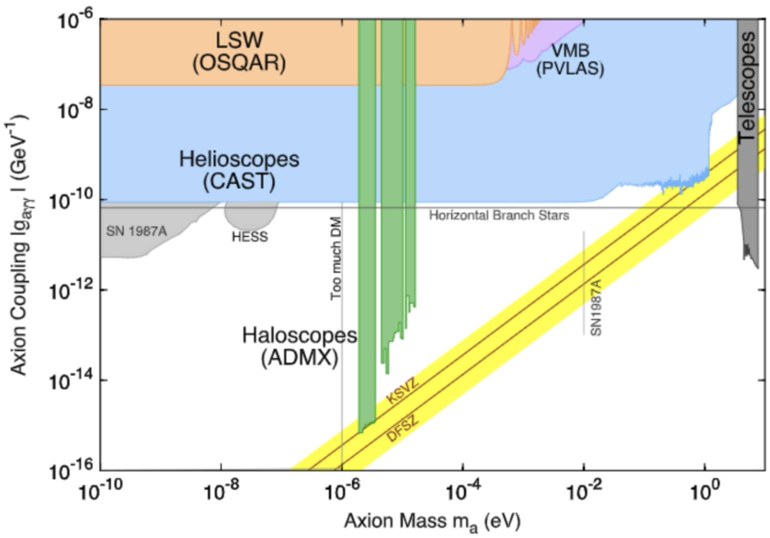

Figure 15: Constraints on the coupling of axions to light as a function of axion mass. From [34]

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