Tutorial 1: Neutrino and Astroparticle Physics

Exercise 1

The cross section for an electron neutrino to undergo a charged current interaction with an electron is

$$\sigma_{CC}(\nu_e e \to \nu_e e) = \frac{2m_e E_\nu G_F^2}{\pi}$$

where m_e is the mass of the electron, E_ν is the energy of the neutrino and G_F is Fermi's constant. Use this cross section to find the probability that a 10 MeV solar electron neutrino will undergo a charged-current weak interaction with an electron in the Earth if it travels along a trajectory passing through the centre of the Earth. Take the Earth to be a sphere of radius 6400 km and uniform density $\rho \sim 5520\,\mathrm{kg\,m^{-3}}$.

Exercise 2

Show that when L is given in km and Δm^2 is given in eV^2, the two-flavour oscillation probability expressed in natural units becomes:

$$\sin^2(2\theta)\sin^2\left(\frac{\Delta m^2[\mathrm{GeV}^2]\mathrm{L}[\mathrm{GeV}^{-1}]}{4E_\nu[\mathrm{GeV}]}\right) \to \sin^2(2\theta)\sin^2\left(\frac{1.27\Delta m^2[\mathrm{eV}^2]\mathrm{L}[\mathrm{km}]}{E_\nu[\mathrm{GeV}]}\right)$$

Tutorial 2: Neutrino and Astroparticle Physics

Exercise 1

Consider an extension to the Standard model in which sterile neutrinos are added. The massive basis ν_1, ν_4 is related to the flavour basis of the active, ν_a, and sterile neutrino, ν_s, as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} . \qquad [1.1]$$

We are interested in case in which $m_4 \gg m_1, m_2, m_3$. Assume that these massive neutrinos are Majorana particles.

Their existence can be tested in neutrinoless double beta decay.

(a) What is the contribution of the sterile neutrinos to the effective Majorana mass parameter (assume that the sterile neutrino mass is $m_4 \ll 100$ MeV)?

(b) Choose now some reference values for $m_4 = 1\text{eV}$ and $|U_{e4}|^2 = 0.02$. For which type of neutrino mass spectrum (NH, IH, QD) can there be a complete cancellation with the standard light neutrino contribution? [Note: Use the following values for the neutrino parameters: $\sin^2\theta_{12} = 0.31$, $\cos^2\theta_{13} \simeq 1$, $\sin^2\theta_{13} = 0.02$, $\Delta m^2_{21} = 7.9 \times 10^{-5}$ eV^2 and $|\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}$ eV^2. Alternatively, you can refer to the expected values for m_{ee} presented in the lecture notes.]

(c) If, instead $M \gg 100$ MeV, what will be the dependence of the decay rate on $M \simeq m_{\text{heavy}}$? [Hint: Recall the form of the fermionic propagator and require lepton number violation.]

Exercise 2

(a) Discuss the relation between the existence of Majorana neutrinos and the fundamental symmetries of elementary particles.

(b) Show that $P_L(\nu_L)^c = 0$ where $P_L = (1 - \gamma_5)/2$ and the other symbols have the usual meaning.

(c) Discuss why the Majorana mass term

$$\mathcal{L}_M = -\frac{1}{2}\nu_L^T C m_M \nu_L + \text{h.c.}$$

is forbidden and show that it breaks lepton number by two units.

(d) Consider a mass term which, in the simplest case of one active (e.g. ν_e) and one sterile neutrino, in this basis reads

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} ,$$

with $m_D \ll M$. What is the dependence of the light neutrino mass on the mixing angle between the flavour and the mass eigenstates, $\sin^2\theta \simeq \tan^2\theta$?

(e) Consider neutrinoless double beta decay. If $M \ll 100$ MeV, (where 100 MeV is the typical momentum exchanged in the process), what is the total contribution due to both the heavy and light mass states to the effective Majorana mass parameter?

Extend the see-saw type I to the inverse-see-saw by including one additional sterile neutrino N_c. Now the mass matrix in the ν_a, N_b, N_c basis reads

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_N \\ 0 & M_N & \mu \end{pmatrix} .$$

(f) Compute the resulting masses in the limit of $M_N \gg m_D \gg \mu$.