

STFC HEP summer school 2024
Phenomenology Problems
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1 Basic Kinematics

The rapidity y and pseudo-rapidity η are defined as:

$$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$

$$\eta = -\log \left(\tan \left(\frac{\theta}{2} \right) \right)$$

where z is the direction of the colliding beams.

- a) Verify that for a particle of mass m

$$E = \sqrt{m^2 + p_T^2} \cosh y$$

$$p_z = \sqrt{m^2 + p_T^2} \sinh y$$

with $p_T^2 = p_x^2 + p_y^2$.

- b) Prove that $\tanh \eta = \cos \theta$
c) Prove that rapidity equals pseudo-rapidity for a relativistic particle $E \gg m$
d) Prove that the difference of two rapidities is Lorentz invariant.

2 $e^+e^- \rightarrow$ hadrons

By studying the shape of the Z-resonance in the R-ratio we can try to see the effect of some parameters in the electro-weak interactions. You can answer the questions qualitatively but you could also plot the function if you have time. The cross-section for fermion annihilation to another fermion pair takes the form:

$$d\sigma (f\bar{f} \rightarrow f'\bar{f}') = \alpha^2 \frac{\pi}{2s} d(\cos \theta) \left\{ (1 + \cos^2 \theta) \left(q_f^2 q_{f'}^2 + \frac{g_z^2}{4g_e^2} q_f q_{f'} v_f v_{f'} \chi_1 + \frac{g_z^4}{16g_e^4} (a_f^2 + v_f^2)(a_{f'}^2 + v_{f'}^2) \chi_2 \right) + \cos \theta \left(\frac{g_z^2}{2g_e^2} q_f q_{f'} v_f v_{f'} \chi_1 + \frac{g_z^4}{2g_e^4} a_f a_{f'} v_f v_{f'} \chi_2 \right) \right\} \quad (1)$$

where

$$\frac{g_z}{g_e} = \frac{1}{\cos \theta_w \sin \theta_w} \quad \chi_1 = \frac{s(s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \quad \chi_2 = \frac{s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

	q_f	a_f	v_f
u, c, t	2/3	1/2	$1/2 - 4/3 \sin^2 \theta_w$
d, s, b	-1/3	-1/2	$-1/2 + 2/3 \sin^2 \theta_w$
e, μ, τ	-1	-1/2	$-1/2 + 2 \sin^2 \theta_w$
ν_e, ν_μ, ν_τ	0	1/2	1/2

Table 1: EW couplings in the Standard Model.

and

$$g_z^2 = \frac{4\pi\alpha}{\cos^2\theta_w \sin^2\theta_w} \quad \Gamma_Z \approx \sum_l \Gamma_{Z \rightarrow l\bar{l}} + \sum_q N_c \Gamma_{Z \rightarrow q\bar{q}} \quad \Gamma_{Z \rightarrow f\bar{f}} = \frac{m_Z \alpha}{12 \cos^2\theta_w \sin^2\theta_w} (a_f^2 + v_f^2)$$

The axial and vector couplings in the Standard Model are given in Table 1.

- Look at the expression above (1) and identify the physical origin of each term, e.g. draw the relevant Feynman diagrams and discuss the link to the terms in the equation.
- What happens to the interference between the photon and the Z diagrams if $\sin^2\theta_w = 0.25$?
- Schematically plot $R(\frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-})$ like we saw in the lectures. If time allows one can use some plotting software to do a more precise job.

3 Jet Kinematics

At the LHC each beam has an energy of 7 TeV. Two partons collide and produce two jets with negligible mass, transverse momentum p_T and rapidities $y_{3,4}$.

- Show that

$$x_1 = \frac{p_T}{\sqrt{s}}(e^{y_3} + e^{y_4}), x_2 = \frac{p_T}{\sqrt{s}}(e^{-y_3} + e^{-y_4})$$

- Show that the invariant mass of the dijet system is

$$M_{JJ} = 2p_T \cosh\left(\frac{y_3 - y_4}{2}\right)$$

and the centre of mass scattering angle is:

$$\cos\theta^* = \tanh\left(\frac{y_3 - y_4}{2}\right)$$

- Discuss the regions of $x_{1,2}$, M_{JJ} and θ^* probed with a jet trigger of $p_T > 35$ and $|y_{3,4}| < 3$

4 Event shapes

The thrust is defined as

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|},$$

where the sum is over all the particles, the i th particle has 3-momentum \vec{p}_i , and \vec{n} is a unit-vector.

Explain why the value of the thrust is given by

- 1 for back-to-back configurations,
- $\frac{1}{2}$ for a perfectly spherical event (i.e. uniform distribution of momenta).
- Calculate the minimum possible value of the thrust for a $q\bar{q}g$ state. [Hint: This occurs for the ‘Mercedes’ configuration where all the particles have the same energy and the angle between any two particles is 120° .]

Consider the two event shape variables

$$S_{\text{lin}} = \left(\frac{4}{\pi}\right)^2 \min_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}\right)^2, \quad S_{\text{quad}} = \frac{3}{2} \min_{\vec{n}} \frac{\sum_i |\vec{p}_i \times \vec{n}|^2}{\sum_i |\vec{p}_i|^2}.$$

- Determine whether S_{lin} is infrared safe or not.
- What are the limiting values of S_{lin} for pencil-like (back-to-back) and spherical events?
- What is the value for the Mercedes configuration?

5 Infrared safety

Are these observables infrared safe at a hadron collider? If not, how would you modify them to make them infrared safe?

- Partonic center of mass energy (defined as the invariant mass of the sum of all final state particles in the event).
- The sum of the energies of all jets with transverse momentum above a given p_T threshold.
- The invariant mass of all jets in the event.
- The number of partons.

6 Jet algorithms

Recall the distance measure used by the anti- k_T jet algorithm is given by:

$$d_{i,j} = \min(p_{i,\perp}^{-2}, p_{j,\perp}^{-2}) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = p_{i,\perp}^{-2}.$$

where $\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$. Consider the clustering of a three particle system with one hard jet $j_H = (p_{H,\perp}, \eta, \phi)$ and two softer jets $j_1 = (p_{1,\perp}, 0, 0)$ and $j_2 = (p_{2,\perp}, \eta_0, 0)$ where $p_{H,\perp} \gg p_{i,\perp}$. You may also take $R = 1$.

- Find the conditions for the soft jets to be clustered with the hard jet.
- Show that anti- k_T jets are circular in the $\eta - \phi$ plane.
- What happens to the clustering when cones of jets 1 and 2 overlap?
- How does it depend on the relative size of the transverse momentum, $r = \frac{p_{1,\perp}}{p_{2,\perp}}$?

7 EFT and anomalous couplings

The following dimension-6 operators modify the couplings of the top quark to the weak gauge bosons (Q is the third generation left-handed doublet and t the right handed top quark field, φ is the Higgs field):

$$O_{\varphi Q}^{(3)} = i \frac{1}{2} y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu^I \varphi \right) (\bar{Q} \gamma^\mu \tau^I Q) \quad (2)$$

$$O_{\varphi Q}^{(1)} = i \frac{1}{2} y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi \right) (\bar{Q} \gamma^\mu Q) \quad (3)$$

$$O_{\varphi t} = i \frac{1}{2} y_t^2 \left(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi \right) (\bar{t} \gamma^\mu t) \quad (4)$$

$$O_{tW} = y_t g_w (\bar{Q} \sigma^{\mu\nu} \tau^I t) \tilde{\varphi} W_{\mu\nu}^I \quad (5)$$

$$O_{tB} = y_t g_Y (\bar{Q} \sigma^{\mu\nu} t) \tilde{\varphi} B_{\mu\nu} \quad (6)$$

- Explain qualitatively which particular top couplings will be modified by each operator.
- Write down the Feynman rules for the ttZ vertex including the impact of 2-fermion operators listed above: $\mathcal{O}_{tW}, \mathcal{O}_{tB}, \mathcal{O}_{\phi t}, \mathcal{O}_{\phi Q}^{(3)}, \mathcal{O}_{\phi Q}^{(1)}$ e.t.c. and compare with the typical anomalous coupling parametrisation of the ttZ vertex.

$$\mathcal{L}_{ttZ} = e \bar{u}(p_t) \left[\gamma^\mu (C_{1,V}^Z + \gamma_5 C_{1,A}^Z) + \frac{i \sigma^{\mu\nu} q_\nu}{m_Z} (C_{2,V}^Z + i \gamma_5 C_{2,A}^Z) \right] v(p_{\bar{t}}) Z_\mu$$

- What are the expressions for: $C_{1,V}^Z, C_{1,A}^Z, C_{2,V}^Z$ and $C_{2,A}^Z$ in terms of the dim-6 Wilson coefficients?
- Use this to explain why there are degeneracies between operators if one only looks at processes involving the ttZ interaction.

8 DGLAP splitting kernels (Optional)

Show that in the collinear limit $p_3 \rightarrow zp_{\bar{1}3}$, $p_1 \rightarrow (1-z)p_{\bar{1}3}$ the matrix element

$$\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}}, 3_g)|^2 \rangle = \frac{4e^2 e_q^2 g_s^2 N_c}{s} C_F \frac{s_{a1}^2 + s_{a2}^2 + s_{b1}^2 + s_{b2}^2}{s_{13}s_{23}}$$

factorizes to

$$|\mathcal{M}_{q\bar{q}g}|^2 \rightarrow |\mathcal{M}_{q\bar{q}}|^2 \times \frac{2g_s^2}{s_{13}} \times C_F \frac{1 + (1-z)^2}{z}$$

$$\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}}, 3_g)|^2 \rangle \xrightarrow{3||1} \langle |\mathcal{M}(a_{e^+}, b_{e^-}, (\tilde{1}3)_q, 2_{\bar{q}})|^2 \rangle \frac{2g_s^2 C_F}{s_{13}} \frac{1 + (1-z)^2}{z}$$

with

$$\langle |\mathcal{M}(a_{e^+}, b_{e^-}, 1_q, 2_{\bar{q}})|^2 \rangle = 2e^2 e_q^2 N_c \frac{s_{a1}^2 + s_{a2}^2}{s_{ab}}$$

9 Operators and EOMs (Optional)

Show that the two operators in:

$$\mathcal{O}_{gt} = \bar{t} T_A \gamma^\mu D^\nu t G_{\mu\nu}^A, \quad (7)$$

$$\mathcal{O}_{gQ} = \bar{Q} T_A \gamma^\mu D^\nu Q G_{\mu\nu}^A, \quad (8)$$

can be written as a sum of four fermion operators