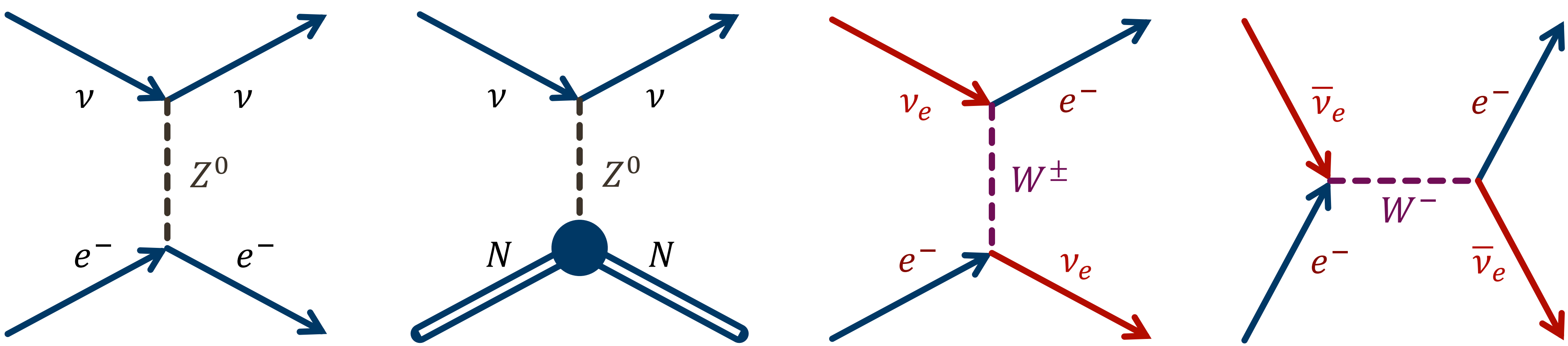
Neutrinos in matter

Neutrinos propagating in matter

Any neutrino propagating in matter can interact with the medium.

$\frac{d\sigma}{dQ^2}$ stays finite as $Q^2 \rightarrow 0$, so the neutrino can interact without scattering

- Analogue of optical refraction & the Higgs mechanism



$\sigma(Q^2 = 0)$ has additional CC contributions for ν_e and $\overline{\nu}_e$

- Extra mass for electron flavours (sign is opposite for ν_e and $\overline{\nu}_e$)

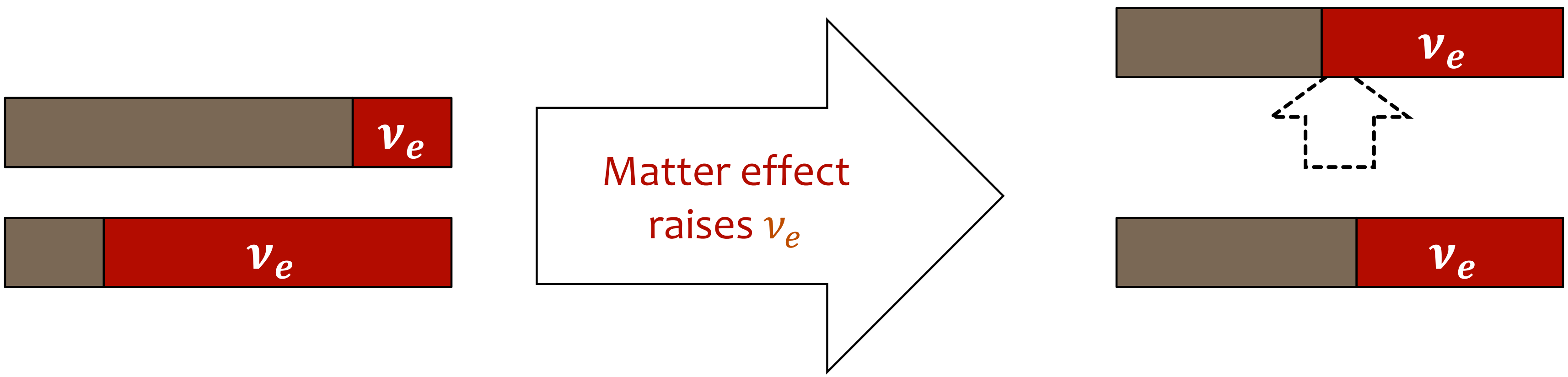
Matter effect hand-wave

Interactions with matter create a drag force on propagating neutrinos.

- Increase their effective *mass*…
- …but in a way that depends on their *flavour*

And we said neutrino flavour states look nothing like the mass states.

In practice what this means that matter effects distort both the mass spectrum and the flavour composition of each mass state



Hamiltonian picture

Oscillations are driven by non-uniform phase advance of different mass components:

$$|\nu(t)\rangle = e^{-iE\cdot t}|\nu(0)\rangle \rightarrow \exp[-i\mathcal{H}\cdot t]\,|\nu(0)\rangle$$

For ultra-relativistic particles (as before):

$$\mathcal{H}\cdot t \rightarrow \frac{m^2 L}{2E}$$

When acting on a mixture of state with different masses this becomes:

$$\mathcal{H}|\nu\rangle = \frac{M^2}{2E_\nu}|\nu\rangle$$

where M^2 is a matrix. [In the mass basis, $M^2 = \mathrm{diag}(m_1^2, m_2^2, \ldots)$]

Can again subtract a uniform component, e.g. m_1^2

$$\left|e^{i\phi(z)}f(z)\right|^2 = |f(z)|^2$$

$$\Longrightarrow M^2 = m_1^2\mathbb{I} + \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}$$

Aside: exponentials & $\mathcal{H}$

Pulling out uniform phases using $\left|e^{i\phi(z)}f(z)\right|^2 = |f(z)|^2$ seems straightforward enough, but why can we do this here?

Remember the Hamiltonian $\mathcal{H}$ is actually the time evolution operator:

$$\mathcal{H}\,|\psi(x,t)\rangle = i\frac{\partial}{\partial t}|\psi(x,t)\rangle$$

And from even further back, complex exponentials appear natuarally as solutions to the equation: $\frac{\partial\psi}{\partial t} = -iE\psi$

So with a little generalization (scalar E to matrix operator $\mathcal{H}$) it follows that

$$\mathcal{H}\,|\psi\rangle = i\frac{\partial}{\partial t}|\psi\rangle \quad \Longrightarrow \quad |\nu(t)\rangle = \exp[-i\mathcal{H}\cdot t]\,|\nu(0)\rangle$$

Hamiltonian in flavour basis

In flavour basis, M^2 transforms via PMNS matrix:

$$M^2 \to UM^2U^\dagger$$

and propagation Hamiltonian looks like:

$$\mathcal{H}|\nu\rangle = \frac{1}{2E_\nu} \underbrace{UM^2U^\dagger}_{\text{This part is just some matrix}} |\nu\rangle$$

Trick to understanding matter effects:

It doesn't matter what the matrix $\mathbf{X} = UM^2U^\dagger$ is.

If you **diagonalize it** (an eigenvalue problem), then:

- **Eigenvalues** are the propagation 'effective **mass**' **eigenstates**
- Matrix of **eigenvectors** is the effective **mixing matrix**

Aside: common phases

Howcome we can pull out a common phase before transforming the mass matrix?

Specifically, we have in the flavour basis $\mathcal{H} \propto UM^2U^\dagger$. **Is it okay to do:**

$$M^2 = m_1^2\mathbb{I} + \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix} = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_1^2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}$$

and discard the $m_1^2\mathbb{I}$, before the $UM^2U^\dagger$ transformation?

Check by writing it out:

$$UM^2U^\dagger = U\left[m_1^2\mathbb{I} + \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}\right]U^\dagger = Um_1^2\mathbb{I}U^\dagger + U\begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}U^\dagger$$

$\mathbb{I}$ commutes with everything, so $Um_1^2\mathbb{I}U^\dagger = UU^\dagger m_1^2\mathbb{I} = m_1^2\mathbb{I}$ and

$$UM^2U^\dagger = m_1^2\mathbb{I} + U\begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}U^\dagger \qquad \textit{as required.}$$

Matter effect

Comes from the coherent forward scattering ($Q^2 \to 0$) amplitude.

- 'Flavour independent' component V_A affects all active neutrinos
- Extra CC components $+V_e$ for ν_e and $-V_e$ for $\overline{\nu}_e$
- The potentials are $V_e = \pm\sqrt{2}\mathrm{G_F}n_e$ and $V_A = \frac{1}{2}\sqrt{2}\mathrm{G_F}n_e$

$$\mathcal{H}|\nu\rangle = \left[\frac{1}{2E_\nu}UM^2U^\dagger + V_A + \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}\right]|\nu\rangle$$

But again, any common phase like V_A can be taken out front and put on the pile, so the effective Hamiltonian becomes (for 2ν):

$$\frac{1}{2E_\nu}UM^2U^\dagger + \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}$$

Matter effect in 2 neutrinos

In 2 neutrino case:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \text{ and } M^2 = \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix} = \Delta m^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{H} = \frac{\Delta m^2}{2E_\nu} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} + \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathcal{H} = \frac{\Delta m^2}{2E_\nu} \begin{pmatrix} \xi - \sin^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \quad \text{where} \quad \xi = \frac{2E_\nu V_e}{\Delta m^2}$$

Eigenvalue equation: $\begin{vmatrix} \xi - \sin^2\theta - \lambda & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta - \lambda \end{vmatrix} = 0$

gives $\lambda_2 - \lambda_1 = \sqrt{(\xi - \cos 2\theta)^2 + \sin^2 2\theta}$

Matter effect in 2 neutrinos

So we can write:

$$\mathcal{H} = \frac{1}{2E_\nu} U_\mathrm{m} \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \sqrt{(\xi - \cos 2\theta)^2 + \sin^2 2\theta} \end{pmatrix} U_\mathrm{m}^T$$

Thus

$$\Delta m_\mathrm{m}^2 = \Delta m^2 \sqrt{(\xi - \cos 2\theta)^2 + \sin^2 2\theta}$$

And equating *off-diagonal* elements*:

$$[U_\mathrm{m} M_\mathrm{m} U_\mathrm{m}^T]_{12} = [U M U^T]_{12}$$
$$\Delta m_\mathrm{m}^2 \sin\theta_\mathrm{m} \cos\theta_\mathrm{m} = \Delta m^2 \sin\theta \cos\theta$$

So

$$\sin^2 2\theta_\mathrm{m} = \frac{\sin^2 2\theta}{(\xi - \cos 2\theta)^2 + \sin^2 2\theta}$$

* The on-diagonal elements are messy because of the pile of global phases like V_A that we did not keep track of.

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Limiting cases

Both the effective parameters Δm^2_{m} and $\sin^2 2\theta_{\mathrm{m}}$ depend on the extra factor $[(\xi - \cos 2\theta)^2 + \sin^2 2\theta]$ where $\xi = 2\sqrt{2}\mathrm{G_F} n_e E_\nu / \Delta m^2$

- Interestingly this means

$$\Delta m^2_{\mathrm{m}} \sin 2\theta_m \sim \quad \text{frequency}^2 \times \text{amplitude}$$

is **invariant** under the matter effect. We'll see this with 3ν as well.

For low energy (E_ν) or low density (n_e), then $\xi \ll 1$:

$$[(\xi - \cos 2\theta)^2 + \sin^2 2\theta] \rightarrow 1, \text{ and vaccum mixing is restored}$$

For high energy E_ν or high density n_e, then $\xi \gg 1$:

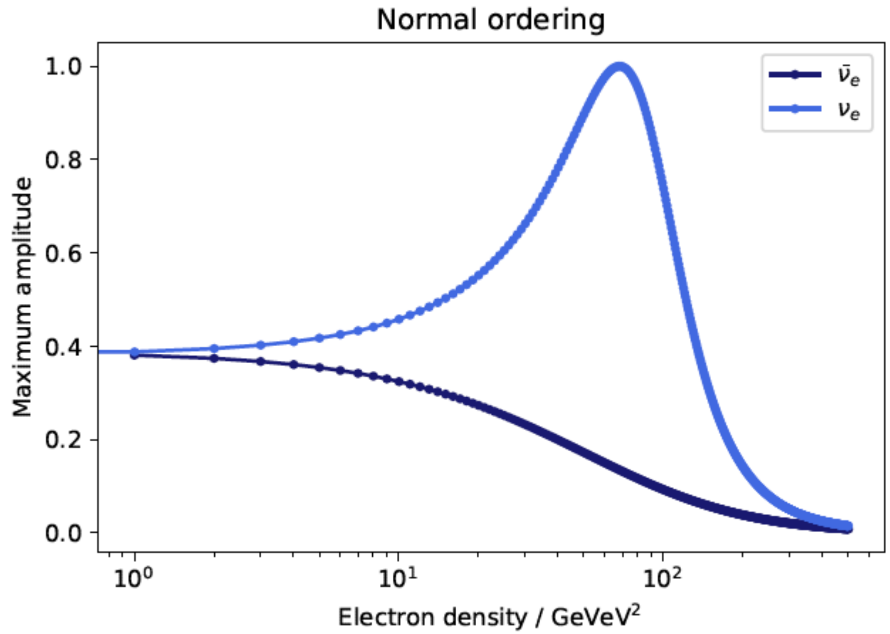
$$[(\xi - \cos 2\theta)^2 + \sin^2 2\theta] \rightarrow \xi^2 \text{ and mixing is heavily suppressed}$$

- **Physics:** the overlap $\langle \nu_e | \nu_2 \rangle \sim 1$ so $U_m \simeq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and flavour $\simeq$ mass

Resonance and antineutrinos

For the right value of ξ then $(\xi - \cos 2\theta)^2 = 0$ and

$$\sin^2 2\theta_\mathrm{m} = \frac{\sin^2 2\theta}{0+\sin^2 2\theta} = 1,$$ for any (non-zero) value of $\sin^2 2\theta$



- The oscillation amplitude can be large, *regardless* of the initial mixing.
- MSW resonance for **Mikheyev, Smirnov & Wolfenstein**.
- Was very interesting, if you believed the PMNS matrix ≃ CKM matrix. (or even exactly the same)

[But it turns out neutrino mixing is very large…]

Whether a resonance exists, depends on the sign of $\xi = 2E_\nu V_e / \Delta m^2$,

- Opposite for ν_e and ν_e, and **tells you the the sign of Δm^2 !**