Quantum Field Theory – Lecture 1

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During these lectures we will
1. Recap some relevant classical physics
2. Quantise the free scalar field
3. Add interactions
4. Discuss matrix elements and Feynman diagrams

What is QFT?
- A mathematically consistent framework in which fields are understood as the basic degrees of freedom that participate in dynamical processes.

Why QFT?
- It is what we get when we try to combine QM with special relativity.
- It is highly successful in describing nature.
- Specific QFTs, like QED, have made predictions that match experiment to shocking precision.
- The theoretically computed value of the anomalous magnetic moment of the electron matches the experimentally measured value to more than 10

significant digits!

Notation

We live in 4 dimensions and special relativity has taught us that space and time are not absolute but that spacetime is. The need to treat space and time in equal footing gives rise to the notion of a 4-vector, which is used to define events:

$$x^\mu = (x^0, x^i) = (t, \vec{x}) \quad , \quad \mu = 0,1,2,3 \, , \quad i = 1,2,3 .$$

Spacetime is Minkowskian with metric

$$\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$$

spacetime is absolute

and all observers agree on the Minkowski distance between events.

The dot product of two vectors is defined with respect to the Minkowski metric:

$$k \cdot x \equiv \eta_{\mu\nu} k^\mu x^\nu = k^0 x^0 - \sum_{i=1}^{3} k^i x^i .$$

We can use the Minkowski metric to lower indices:

$$x_\mu = \eta_{\mu\nu} x^\nu \Rightarrow (x_0, x_i) = (\eta_{00} x^0, \eta_{ij} x^j) = (x^0, -x^i) .$$

The inverse metric, $\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ (so that $\eta_{\mu\nu}\eta^{\nu\rho} = \delta_\mu{}^\rho$) can be used to raise indices:

$$x^\mu = \eta^{\mu\nu} x_\nu .$$

Recap: classical mechanics

- Lagrangian mechanics

The central object here is the Lagrangian

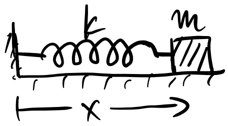
$$L = L(q, \dot{q}, t) .$$

q: generalised coordinates, e.g. $x, y, z, r, \theta, \phi, \ldots$

$\dot{q}$: $\dot{q} = dq/dt$.

The Lagrangian is typically given by

$$L = \underbrace{T}_{\text{kinetic energy}} - \underbrace{V}_{\text{potential energy}}$$

<u>Example</u>: SHO  $T = \frac{1}{2} m \dot{x}^2$, $V = \frac{1}{2} k x^2$

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$$

Equations of motion follow from principle of least action:

$$\delta S = 0, \qquad S = S(q) = \int_{t_1}^{t_2} dt \, L(q, \dot{q}, t).$$

$$\delta S = 0 \xRightarrow{\text{consider } q \to q + \delta q} \underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0}_{\text{Euler-Lagrange equations}}$$

<u>Example</u>: SHO $\frac{\partial L}{\partial \dot{x}} = \overbrace{m\dot{x}}^{\text{momentum}}$, $\frac{\partial L}{\partial x} = -kx$

E-L equations give $\overbrace{m\ddot{x}}^{ma} + \overbrace{kx}^{-F} = 0$

which is Newton's 2nd law.

- <u>Hamiltonian mechanics</u>

As we just saw, $\frac{\partial L}{\partial \dot{x}}$ = momentum. We may generalise this so that $\frac{\partial L}{\partial \dot{q}}$ is a generalised momentum:

$$p = \frac{\partial L}{\partial \dot{q}}.$$

In Hamiltonian mechanics we trade $\dot{q}$ for p:

$$H(p, q, t) = p\dot{q} - L(q, \dot{q}, t).$$

Now the equations of motion follow from Hamilton's equations:

$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$ follows from E-L equation

<u>Example</u>: SHO

$$p = \frac{\partial L}{\partial \dot{x}} = m\dot{x} \Rightarrow \dot{x} = \frac{p}{m}$$

$$H(p, x, t) = p\dot{x} - \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

$$= p\frac{p}{m} - \frac{1}{2}m\frac{p^2}{m^2} + \frac{1}{2}kx^2$$

$$= \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m}$$

$$\dot{p} = -\frac{\partial H}{\partial x} = -kx$$

These imply

$$\ddot{x} = \frac{\dot{p}}{m} = \frac{1}{m}(-kx) \Rightarrow m\ddot{x} + kx = 0.$$

Summary

Lagrangian $L = L(q, \dot{q}, t) = T - V$ (usually)

EOM: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0.$

Hamiltonian $H = H(p, q, t) = p\dot{q} - L(q, \dot{q}, t)$

with $p = \frac{\partial L}{\partial \dot{q}}$.

EOM: $\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$