Neutrino Physics

Neutrino Oscillations in vacuum

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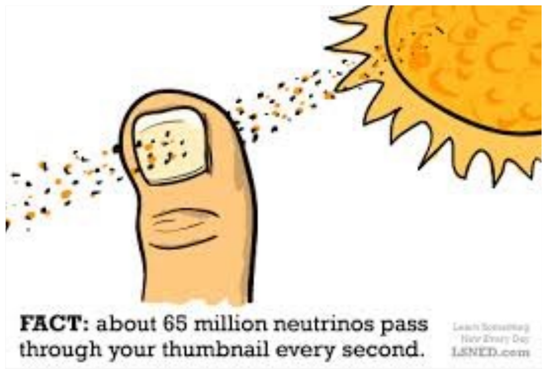
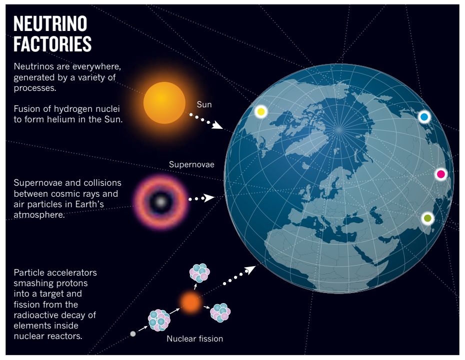
Summary of what we learned from the last lecture

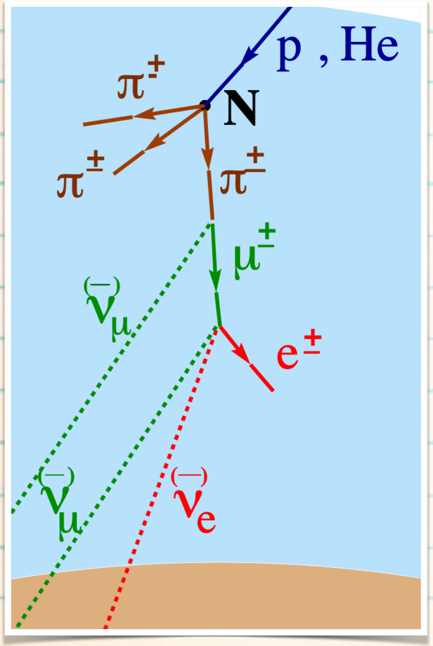
- Neutrinos are very weakly interacting fermions and are part of the SU2L doublet
- There are three “types” of neutrinos: $\nu_e \; \nu_\mu \; \nu_\tau$
- Electrically neutral fermion of the SM and there are no right-handed neutrinos in SM
- While we don’t understand neutrinos very well they are omnipresent!





Atmospheric neutrinos

- Neutrinos get produced via cosmic rays (accelerated protons, He) interacting with the atmosphere



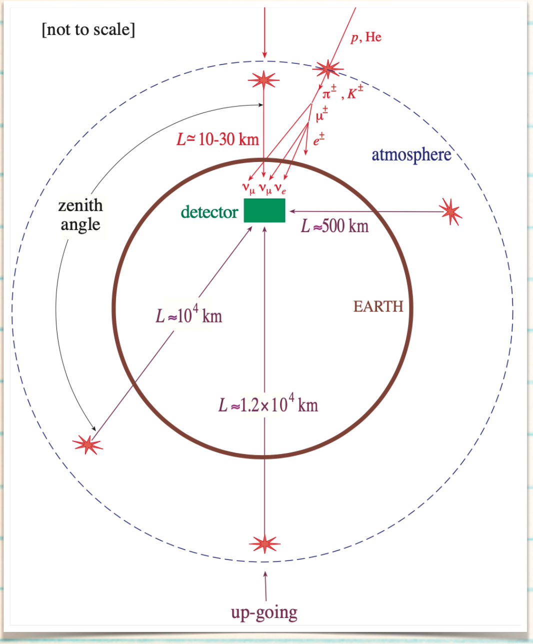
$$R_{\frac{\mu}{e}} \approx \frac{N_{\nu_\mu} + N_{\overline{\nu_\mu}}}{N_{\nu_e} + N_{\overline{\nu_e}}} \sim 2$$

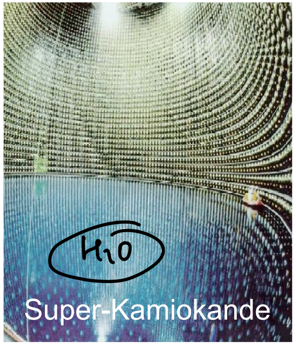
Exercise: show that

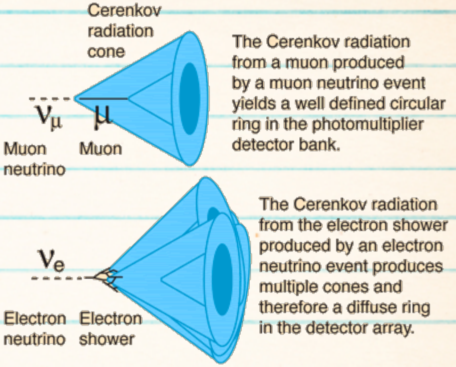
$$\frac{\Gamma\left(\pi^+ \to e^+ \nu_e\right)}{\Gamma\left(\pi^+ \to \mu^+ \nu_\mu\right)} \approx 2 \times 10^{-4}$$

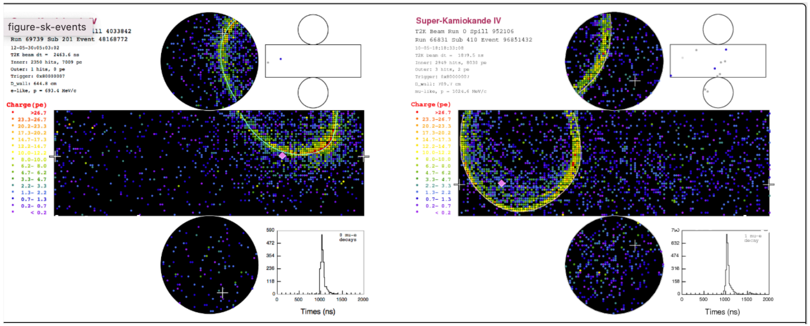
Atmospheric neutrinos

- 1998 Super-Kamiokande experiment: 40 kton water cherenkov experiment detected atmospheric neutrinos

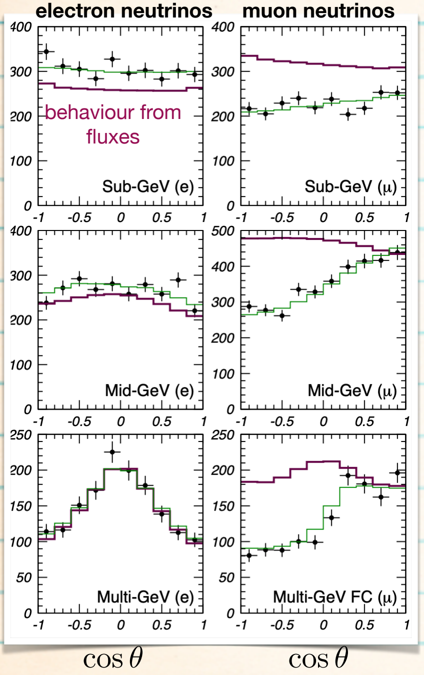


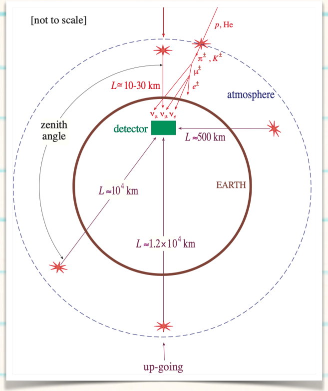






Atmospheric neutrinos





$$\nu_\mu \to \nu_\tau$$

T2K could not "see" the tau neutrinos so it appears there is a deficit of muon neutrinos

Exercise: What is the minimum energy a neutrino must have to create a charged lepton when it interactions with a neutron i.e. $\nu_\alpha n \to p\ell_\alpha$ calculate this for each neutrino flavour

Neutrino Oscillations

Two neutrino oscillation

$$\hat{H}\psi = i\frac{d\psi}{dt} = \underset{\uparrow\ \text{eigenvalue}}{E}\psi$$

Time evolution of the eigenstate :

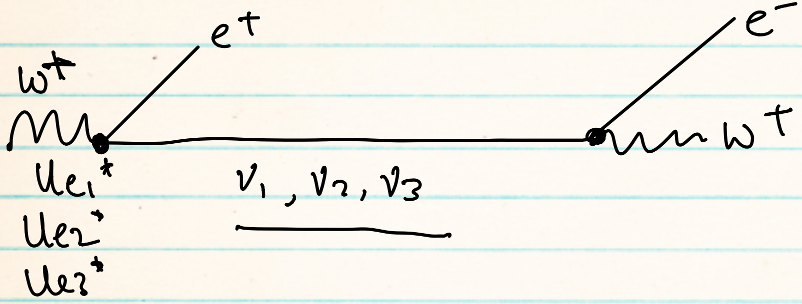
$$\psi(x,t) = \underbrace{\phi(x)}_{\text{amplitude}}\ \underbrace{e^{iEt}}_{\text{phase}}$$

massive states : ν_1, ν_2, ν_3

interaction states : ν_e, ν_μ, ν_τ

Neutrino Oscillations

charged current interaction $\quad W e^+ \nu_e$



we can't know which eigenstate was produced
$\Rightarrow$ system is described by a linear superposition of
ν_1, ν_2, ν_3

$$\mathcal{L}_{cc} \supset \frac{-ig}{\sqrt{2}} \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha = \frac{-ig}{\sqrt{2}} \bar{l}_\alpha \gamma^\mu P_L \underline{U_{\alpha i}} \nu_i$$

flavor index $\alpha = e, \mu, \tau$

$i = 1, 2, 3$

Neutrino Oscillations

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{\substack{\text{PMNS matrix} \\ \text{LMM}}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

- $|\nu_e\rangle$ propagates as a linear superposition until it is measured. then wavefunction collapses $\Rightarrow$ produces the charged lepton
- the wavefunction evolves in time due to the phase shift and this allows for ν-oscillations ie the phenomenon of neutrino flavour transformation.

Neutrino Oscillations

Two neutrino mixing

- (ν_e, ν_μ) $\qquad$ (ν_1, ν_2)

$$|\nu_1(t)\rangle = |\nu_1\rangle\, e^{i(\underline{p_1}\cdot\underline{x} - \underline{E_1}\cdot\underline{t})}$$

$$|\nu_2(t)\rangle = |\nu_2\rangle\, e^{i(p_2\cdot x - E_2\cdot t)}$$

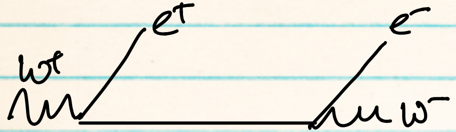
$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \underbrace{\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}}_{U} \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

$c_\theta \equiv \cos\theta$

$s_\theta \equiv \sin\theta$

Ex show from $U^\dagger U = \mathbb{1}_{2\times 2}$ that $U = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$

Neutrino Oscillations



$$|\psi(0)\rangle = |\nu_e\rangle = c_\theta|\nu_1\rangle + s_\theta|\nu_2\rangle$$

$$|\psi(\underline{\underline{x,t}})\rangle = c_\theta|\nu_1\rangle e^{-ip_1\cdot x} + s_\theta|\nu_2\rangle e^{-ip_2 x}$$

'L' 't'

$$|\psi(L,t)\rangle = c_\theta|\nu_1\rangle e^{-i\varphi_1} + s_\theta|\nu_2\rangle e^{-i\varphi_2}$$

$$\varphi_i = -\vec{p}_i L + E_i t$$

Neutrino Oscillations

$$\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix} c\theta & -s\theta \\ s\theta & c\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$|\psi(L,t)\rangle = c\theta\left[c\theta|\nu_e\rangle - s\theta|\nu_\mu\rangle\right]e^{-i\phi_1} + s\theta\left[s\theta|\nu_e\rangle + c\theta|\nu_\mu\rangle\right]e^{-i\phi_2}$$

$$= \left[c\theta^2 e^{-i\phi_1} + s\theta^2 e^{-i\phi_2}\right]|\nu_e\rangle + \left[-s\theta\, c\theta\, e^{-i\phi_1} + s\theta\, c\theta\, e^{-i\phi_2}\right]|\nu_\mu\rangle$$

$$\boxed{|\psi(L,t)\rangle = e^{-i\phi_1}\left[c\theta^2 + s\theta^2 e^{i\Delta\phi_{12}}\right]|\nu_e\rangle + \underline{cs\theta\left[e^{-i\Delta\phi_{12}} - 1\right]|\nu_\mu\rangle}}$$

$$\Delta\phi_{12} = \phi_1 - \phi_2$$

$$\Delta\phi_{12} = 0$$

$$|\psi(L,t)\rangle = e^{i\phi_1}\left(\underbrace{c\theta^2 + s\theta^2}_{1}\right)|\nu_e\rangle$$

$$= e^{i\phi_1}|\nu_e\rangle$$

Neutrino Oscillations

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L,t) \rangle|^2$$

Ex $P(\nu_e \to \nu_\mu)$ –

$$\langle \nu_\mu | \nu_e \rangle = 0 \qquad \langle \nu_\mu | \nu_\mu \rangle = 1$$

$$P(\nu_e \to \nu_\mu) = c_\theta^2 s_\theta^2 (e^{-i\Delta\phi_{12}} - 1)(e^{i\Delta\phi_{12}} - 1)$$

use $2\cos\theta \sin\theta = \sin 2\theta$

$$P(\nu_e \to \nu_\mu) = \frac{\sin^2 2\theta}{4} (1 - e^{-i\Delta\phi_{12}})(1 + e^{i\Delta\phi_{12}})$$

$$e^{ix} = \cos x + i \sin x$$

mixing angle contained in the unitary matrix

phase

$$\boxed{P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta\phi_{12}}{2}\right)}$$

Neutrino Oscillations

1. assume that $\vec{p}_1 = \vec{p}_2 = \vec{p}$

$$\Rightarrow \Delta\Phi_{12} = (E_1 - E_2)t - \underline{(\vec{p}_1 - \vec{p}_2)}x$$

$$= (E_1 - E_2)t$$

$$E_i = \sqrt{p_i^2 + m_i^2} \Rightarrow \Delta\Phi_{12} = \vec{p}\left[\left(1 + \frac{m_1^2}{2p^2}\right)^{1/2} - \left(1 + \frac{m_2^2}{2p^2}\right)^{1/2}\right]$$

- neutrinos are relativistic, $m \ll \vec{p}$

$\Rightarrow$ use a Taylor expansion: $(1+x)^{1/2} \sim 1 + \frac{x}{2} + \theta(x^2)$

$$\Delta\Phi_{12} = \left(\frac{m_1^2 - m_2^2}{2E_\nu}\right)L \qquad L \approx t$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{12}^2 L}{4E_\nu}\right)$$

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{12}^2 L}{4E_\nu}\right)$$

Neutrino Oscillations

Three neutrino mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{U_{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad - (1)$$

$$U^\dagger U = U U^\dagger = \mathbb{1}_{3\times 3}$$

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

$$U_{e1} U_{\mu 1}^* + U_{e2} U_{\mu 2}^* + U_{e3} U_{\mu 3}^* = 0$$

$$|\psi(x,t)\rangle = U_{e1}^* |\nu_1\rangle e^{-i\phi_1} + U_{e2}^* |\nu_2\rangle e^{-i\phi_2} + U_{e3}^* |\nu_3\rangle e^{-i\phi_3}$$

$$|\nu_1\rangle = U_{e1} |\nu_e\rangle + U_{\mu 1} |\nu_\mu\rangle + U_{\tau 1} |\nu_\tau\rangle$$

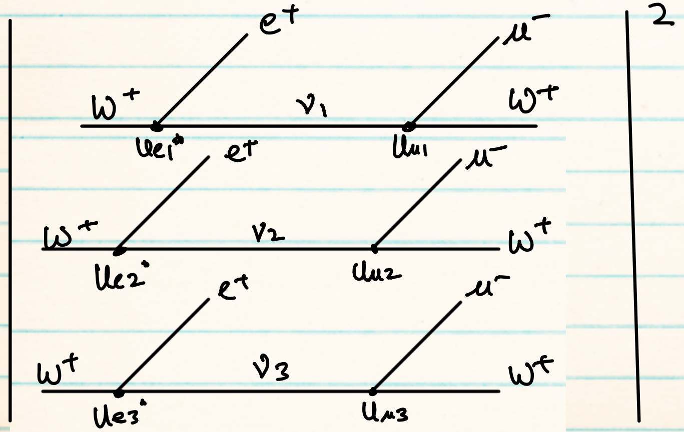
$$|\nu_2\rangle = U_{e2} |\nu_e\rangle + U_{\mu 2} |\nu_\mu\rangle + U_{\tau 2} |\nu_\tau\rangle$$

Neutrino Oscillations

$$
\left.
\begin{aligned}
|\psi(x,t)\rangle = &\left[U_{e1}^{*}U_{e1}\,e^{-i\varphi_1} + U_{e2}^{*}U_{e2}\,e^{-i\varphi_2} + U_{e3}^{*}U_{e3}\,e^{-i\varphi_3}\right]|\nu_e\rangle \\
&+\left[U_{e1}^{*}U_{\mu 1}\,e^{-i\varphi_1} + U_{e2}^{*}U_{\mu 2}\,e^{-i\varphi_2} + U_{e3}^{*}U_{\mu 3}\,e^{-i\varphi_3}\right]|\nu_\mu\rangle \\
&+\left[U_{e1}^{*}U_{\tau 1}\,e^{-i\varphi_1} + U_{e2}U_{\tau 2}^{*}\,e^{-i\varphi_2} + U_{e3}^{*}U_{\tau 3}\,e^{-i\varphi_3}\right]|\nu_\tau\rangle
\end{aligned}
\right\}
$$

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(x,t)\rangle|^2 \qquad \langle \nu_\mu | \nu_\mu \rangle = 1$$

$$\Rightarrow \quad P(\nu_e \to \nu_\mu) = |U_{e1}^{*}U_{\mu 1}e^{-i\varphi_1} + U_{e2}^{*}U_{\mu 2}e^{-i\varphi_2} + U_{e3}^{*}U_{\mu 3}e^{-i\varphi_3}|^2 \leftarrow$$



Neutrino Oscillations

recall

$$|z_1 + z_2 + z_3|^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\mathrm{Re}\left[z_1 z_2^* + z_1 z_3^* + z_2 z_3^*\right]$$

$$\Delta_{31} = \Delta_{32} + \Delta_{21}$$

$\Rightarrow$ two independent mass squared differences.

$$P(\nu_e \to \nu_\mu) = |U_{e1}^* U_{\mu 1}|^2 + |U_{e2}^* U_{\mu 2}|^2 + |U_{e3}^* U_{\mu 3}|^2$$
$$+ 2\mathrm{Re}\left[(U_{\mu 1}^* U_{\mu 1} U_{e2} U_{\mu 2}^*)\, e^{-i(\phi_1 - \phi_2)}\right]$$
$$+ 2\mathrm{Re}\left[(U_{e1}^* U_{\mu 1} U_{e3} U_{\mu 3}^*)\, e^{-i(\phi_1 - \phi_3)}\right]$$
$$+ 2\mathrm{Re}\left[(U_{e2}^* U_{\mu 2} U_{e3} U_{\mu 3}^*)\, e^{-i(\phi_2 - \phi_3)}\right]$$

$$U^\dagger U = U U^\dagger = \mathbb{1}_{3\times 3}$$

$$P(\nu_e \to \nu_\mu) = 2\mathrm{Re}\left[U_{e1}^* U_{\mu 1} U_{e2} U_{\mu 2}^* \left(e^{i(\phi_2 - \phi_1)} - 1\right)\right]$$
$$+ 2\mathrm{Re}\left[U_{e1}^* U_{\mu 1} U_{e3} U_{\mu 3}^* \left(e^{i(\phi_3 - \phi_1)} - 1\right)\right]$$
$$+ 2\mathrm{Re}\left[U_{e2}^* U_{\mu 2} U_{e3} U_{\mu 3}^* \left(e^{i(\phi_3 - \phi_2)} - 1\right)\right]$$

$$\Delta_{ji} = \frac{\phi_j - \phi_i}{2} = \frac{m_j^2 - m_i^2}{4E_\nu} L$$

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) - P(\nu_e \to \nu_\tau)$$
$$= 1 - 4|U_{e1}|^2 |U_{e2}|^2 \sin^2 \Delta_{21} - 4|U_{e1}|^2 |U_{e3}|^2 \sin^2 \Delta_{31}$$
$$- 4|U_{e2}|^2 |U_{e3}|^2 \sin^2 \Delta_{32}$$

Neutrino Oscillations

$$\mathcal{L}_{CC} = \frac{ig}{\sqrt{2}} \left(\bar{\nu}_{iL} \, U_{ei}^{*} \gamma^{\mu} e_L W_{\mu} + h.c \right)$$

- PMNS matrix only appears in C.C interactions not N.C interactions. $(U^{+}U = \mathbb{1}_{3\times3})$

$$(\bar{\nu}_1 \; \bar{\nu}_2 \; \bar{\nu}_3) \; e^{i\psi} = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \text{CKM type} \\ f(\theta_{12}, \theta_{23}, \theta_{13}) \\ \delta \end{pmatrix} \begin{pmatrix} e^{i\phi_e} & & \\ & e^{i\phi_\mu} & \\ & & 1 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L$$

$e_L' \to e^{-i(\phi_e+\psi)} e_L$

$\mu_L' \to e^{-i(\phi_\mu+\psi)} \mu_L$

$\tau_L' \to e^{-i\psi} \tau_L$

- Kinetic terms
- NC terms
- Dirac mass terms: $\bar{\psi}_L \psi_R$
- $\nu^c = \nu$