Neutrino Physics

Neutrino Oscillations in matter

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Solar neutrinos - PP chain

- The Sun shines by making hydrogen into helium. There are two main ways of doing this: pp chain or CNO cycle

pp-chain I

<u>Step 1</u> $p + p \rightarrow d + e^{+} + \nu_e$

Sun most loses its energy (98.5 %) through pp-chain. This is pp-I chain and occurs around 85 % of the time

<u>Step 2</u> $p + d \rightarrow {}^{3}He + \gamma$

<u>Step 3</u> repeat steps 1 & 2

<u>Step 4</u> ${}^{3}He + {}^{3}He \rightarrow {}^{4}He + d + d$

Solar neutrinos - PP chain

- The Sun shines by making hydrogen into helium. There are two main ways of doing this: pp chain or CNO cycle

pp-chain II

Step 1 $^{4}He + {}^{3}He \longrightarrow {}^{7}Be + \gamma$

pp-II chain occurs around 15 % of the time

Step 2 $^{7}Be + e^{-} \longrightarrow {}^{7}Li + \nu_e$

Step 3 $^{7}Li + p \longrightarrow 2\,{}^{4}He$

Solar neutrinos - PP chain

- The Sun shines by making hydrogen into helium. There are two main ways of doing this: pp chain or CNO cycle

pp-chain III

Step 1 $^4He + {}^3He \rightarrow {}^7Be + \gamma$

pp-III chain occurs around 0.3 % of the time

Step 2 $^7Be + p \rightarrow {}^8B + \gamma$

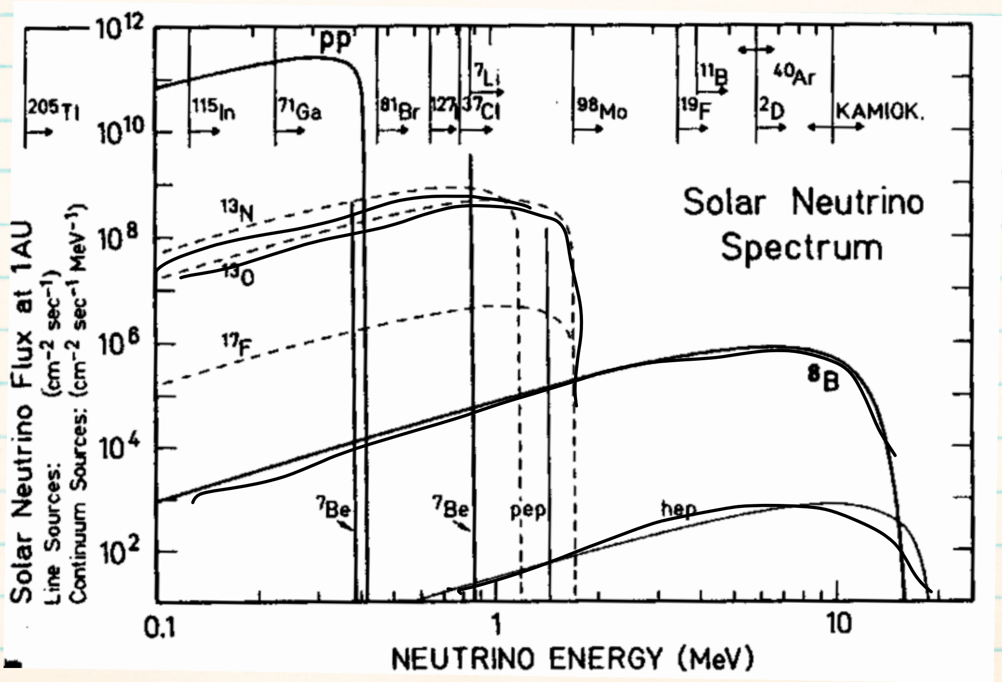
Step 3 $^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

$^8Be^* \rightarrow 2\,{}^4He$

Solar neutrinos - CNO cycle

$$
\begin{aligned}
{}^{12}C + {}^{1}H &\rightarrow {}^{13}N + \gamma \\
{}^{13}N &\rightarrow {}^{13}C + e^{+} + \nu_e \\
{}^{13}C + {}^{1}H &\rightarrow {}^{14}N + \gamma \\
{}^{14}N + {}^{1}H &\rightarrow {}^{15}O + \gamma \\
{}^{15}O &\rightarrow {}^{15}N + e^{+} + \nu_e \\
{}^{15}N + {}^{1}H &\rightarrow {}^{12}C + {}^{4}He
\end{aligned}
$$

- Many nuclear processes (pp chain and CNO cycle) produces electron neutrinos
- Energies of the neutrinos will differ, depending on the reaction.



Solar neutrinos

- 1964 Homestake experiment (headed by Davis & Bahcall) detected solar neutrinos but there were approximately 2/3 less than expected from Bahcall's Standard Solar Model prediction.

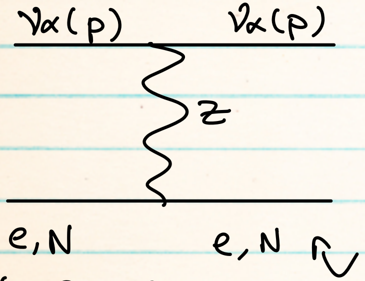
- It was initially proposed that the solar models were wrong.

- Or that two experiment Homestake and GALLEX were wrong!

- As you can guess, the resolution to this problem is neither!

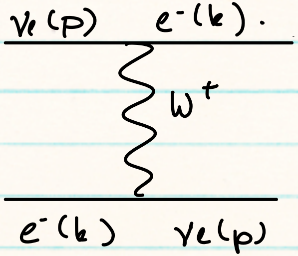
Neutrino Oscillations in matter

ν oscillation in matter is affected by the background.
For simplified two neutrino case; ν_e & ν_μ behave differently in matter



$\alpha = e, \mu, \tau$

NC same for all flavours



CC only present for ν_e.

- ν oscillations are affected by COHERENT SCATTERING with background (no momentum exchange between ν & background)
- coherent scattering is elastic scattering at $\theta = 0°$

Neutrino Oscillations in matter

The background creates a MATTER POTENTIAL for the ν.

$$V_{NC} = \frac{G_F N_n}{\sqrt{2}}$$

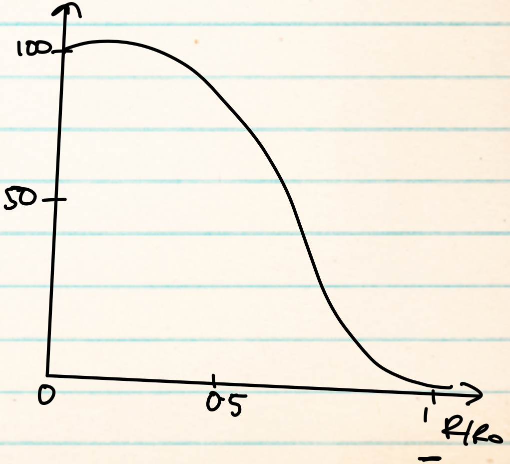
$N_n \equiv$ # density of nucleons

$$V_{CC} = \sqrt{2}\, G_F N_e.$$

$N_e =$ number density of electrons.

Medium	density (g/cm^3)	V_{cc} (eV)
solar	~ 100	$\sim 10^{-12}$
Earth's core	~ 10	$\sim 10^{-13}$
supernovae	$\sim 10^{14}$	~ 1

ρ density profile Sun.



Neutrino Oscillations in matter

$i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$

two neutrino case

$|\psi\rangle = \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix}$

$|\nu_e\rangle = c_\theta|\nu_1\rangle + s_\theta|\nu_2\rangle$

$|\nu_\mu\rangle = -s_\theta|\nu_1\rangle + c_\theta|\nu_2\rangle$

$\theta = \theta_{vac}.$

ie $\nu_\alpha = \sum_i U_{\alpha i}\nu_i$

Hamiltonian for ν-osc in vacuum:

$\langle\nu_\alpha|H_\nu|\nu_\beta\rangle = \langle\sum U_{\alpha i}\nu_i|H_\nu|\sum_j U_{\beta j}\nu_j\rangle$

$= \sum_i U^*_{\alpha i}\langle\nu_i|H_\nu|\nu_i\rangle U_{\beta i} = \sum_i U^*_{\alpha i}E_i U_{\beta i}$

$\alpha = e,\mu.$

Neutrino Oscillations in matter

$$\langle \nu_\alpha | H_\nu | \nu_\beta \rangle = \begin{pmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{pmatrix} \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}$$

ex.

$$= \begin{pmatrix} H_{\alpha\alpha} & H_{\alpha\beta} \\ H_{\beta\alpha} & H_{\beta\beta} \end{pmatrix}$$

let

$$|\vec{p}_1| = |\vec{p}_2| = |\vec{p}|.$$

$$H_{\alpha\alpha} = E_1 c\theta^2 + E_2 s\theta^2 = \sqrt{|\vec{p}|^2 + m_1^2}\, c\theta^2 + \sqrt{|\vec{p}|^2 + m_2^2}\, s\theta^2$$

$$= |\vec{p}| \left(\left(1 + \frac{m_1^2}{|\vec{p}|^2}\right)^{\frac{1}{2}} c\theta^2 + \left(1 + \frac{m_2^2}{|\vec{p}|^2}\right)^{1/2} s\theta^2 \right)$$

$$= |\vec{p}| \left(c\theta^2 + \frac{m_1^2}{2|\vec{p}|^2} c\theta^2 + s\theta^2 + \frac{m_2^2}{2|\vec{p}|^2} s\theta^2 \right)$$

Neutrino Oscillations in matter

$$= |\vec{p}| \left(1 + \frac{m_1^2 c_\theta^2}{2|\vec{p}|^2} + \frac{m_2^2 s_\theta^2}{2|\vec{p}|^2} \right)$$

$$= |\vec{p}| + \frac{m_1^2}{2|\vec{p}|} \left(\frac{1 + \cos 2\theta}{2} \right) + \frac{m_2^2}{2|\vec{p}|} \left(\frac{1 - \cos 2\theta}{2} \right)$$

$$= |\vec{p}| + \frac{m_1^2 + m_2^2}{4|\vec{p}|} + \frac{m_1^2 - m_2^2}{4|\vec{p}|} \cos 2\theta$$

$$H_{\alpha\alpha} = |\vec{p}| + \frac{m_1^2 + m_2^2}{4|\vec{p}|} - \frac{\cos 2\theta \, \Delta m^2}{4|\vec{p}|} \qquad \Delta m^2 = m_2^2 - m_1^2$$

Neutrino Oscillations in matter

Ex

$$H_{\alpha\beta} = H_{\beta\alpha} = -\frac{\Delta m^2}{4|\vec{p}|}\sin 2\theta .$$

$$\Rightarrow \quad H_\nu = \frac{\Delta m^2}{4|\vec{p}|}\begin{pmatrix} -\cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{pmatrix} + \underbrace{\left(|\vec{p}| + \frac{m_1^2+m_2^2}{4|\vec{p}|}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\mathbb{1}_{2\times 2}.}$$

consider only this term · as relative phase matter.

Since $E \approx |\vec{p}| \Rightarrow$

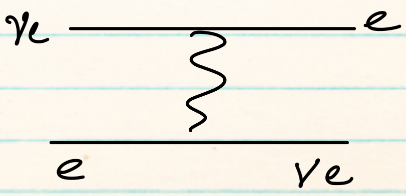
$$\boxed{H_\nu = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{pmatrix}}$$

Neutrino Oscillations in matter

$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix} = H_V\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix} = \frac{\Delta m^2}{4E}\begin{pmatrix}-\cos(2\theta) & -\sin(2\theta)\\ -\sin(2\theta) & \cos(2\theta)\end{pmatrix}\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix}$$

In matter :

$$\boxed{H_M = \underbrace{\frac{\Delta m^2}{4E}\begin{pmatrix}-\cos(2\theta) & -\sin(2\theta)\\ -\sin(2\theta) & \cos(2\theta)\end{pmatrix}}_{H_V} + G_F N_e\sqrt{2}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}}$$



NB we only account for the CC potential as the NC potential affects ν_e & ν_μ equally.

Neutrino Oscillations in matter

We can diagonalise H_m to find energy eigenstates in matter

$$U_m^{+} E_{DIAG_m} U_m = H_m$$

$\theta_m \equiv$ mixing angle in matter.

$$\Rightarrow E_{DIAG_m} = U_m H_m U_m^{+}$$

$$\begin{pmatrix} E_{1m} & 0 \\ 0 & E_{2m} \end{pmatrix} = \begin{pmatrix} C\theta_m & S\theta_m \\ -S\theta_m & C\theta_m \end{pmatrix} \begin{pmatrix} \frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2} G_F N_e & -\frac{\Delta m^2}{4E}\sin 2\theta \\ -\frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \begin{pmatrix} C\theta_m & -S\theta_m \\ S\theta_m & C\theta_m \end{pmatrix}$$

$\Rightarrow$ the $(1,2)$ & $(2,1)$ entry must be zero.

Neutrino Oscillations in matter

(1,2) entry :

$$-\frac{\Delta m^2}{4E}\cos^2\theta_m \sin 2\theta - \sqrt{2}\,G_F N_e \underbrace{\cos\theta_m \sin\theta_m}_{\frac{1}{2}\sin 2\theta_m} + \frac{\Delta m^2}{2E}\cos 2\theta \underbrace{\cos\theta_m \sin\theta_m}$$

$$+ \frac{\Delta m^2}{4E}\sin 2\theta \sin\theta_m^2 \qquad = 0$$

$$= -\frac{\Delta m^2}{4E}\sin 2\theta \cos 2\theta_m \underbrace{- \frac{\sqrt{2}}{2} G_F N_e \sin 2\theta_m + \frac{\Delta m^2}{4E}\cos 2\theta \sin 2\theta_m} = 0$$

$$= -\frac{\Delta m^2}{4E}\sin 2\theta \cos 2\theta_m + \sin 2\theta_m\left(\frac{\Delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}}G_F N_e\right) = 0$$

$$\Rightarrow \frac{\Delta m^2}{4E}\sin 2\theta \cos 2\theta_m = \sin 2\theta_m\left(\frac{\Delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}}G_F N_e\right)$$

Neutrino Oscillations in matter

$$\frac{\Delta m^2}{4E} \sin 2\theta \cos 2\theta_m = \sin 2\theta_m \left(\frac{\Delta m^2}{4E} \cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e \right)$$

$$\Rightarrow \quad \tan 2\theta_m = \frac{\frac{\Delta m^2}{4E} \sin 2\theta}{\left(\frac{\Delta m^2}{4E} \cos 2\theta - \frac{G_F}{\sqrt{2}} N_e \right)}$$

$$= \frac{\frac{\Delta m^2}{2E} \sin 2\theta}{\left(\frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e \right)}$$

Neutrino Oscillations in matter

Show that the difference in the eigenvalues.

$$E_{1m} - E_{2m} = \sqrt{\left(\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}\,G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin(2\theta)\right)^2}.$$

To do this $\det(\lambda \mathbb{I} - H_m) = 0$ and rearrange a bit.

Neutrino Oscillations in matter

$$\boxed{\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\left(\frac{\Delta m^2}{2E}\cos 2\theta - \frac{G_F N_e}{\sqrt{2}}\right)}.}$$

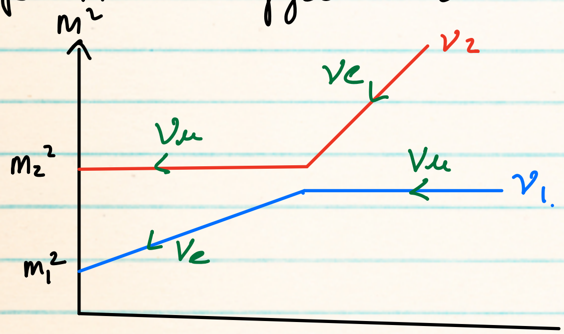
- $G_F N_e = 0 \Rightarrow \theta = \theta_m$. ie vacuum case is recovered.
- $\frac{G_F N_e}{\sqrt{2}} = \frac{\Delta m^2}{2E}\cos 2\theta \Rightarrow \theta_m = \frac{\pi}{4}$ (maximal mixing independent of vacuum mixing!)
- $\frac{G_F N_e}{\sqrt{2}} \gg \frac{\Delta m^2}{2E}\cos 2\theta \Rightarrow \theta_m = \frac{\pi}{2}$.

Neutrino Oscillations in matter

$|\nu_e\rangle = c_\theta |\nu_1\rangle + s_\theta |\nu_2\rangle$
$|\nu_\mu\rangle = -s_\theta |\nu_1\rangle + c_\theta |\nu_2\rangle$ in vacuum.

$|\nu_e\rangle = c_{\theta_m} |\nu_1\rangle + s_{\theta_m} |\nu_2\rangle$
$|\nu_\mu\rangle = -s_{\theta_m} |\nu_1\rangle + c_{\theta_m} |\nu_2\rangle$ in matter.

If matter effects dominate ie $\theta_m \approx \frac{\pi}{2} \Rightarrow |\nu_e\rangle \approx |\nu_2\rangle$



in vacuum for $\theta < 45^0$

$\Rightarrow |\nu_e\rangle \approx |\nu_1\rangle$

Neutrino Oscillations in matter

The matter potential creates a mixing angle in matter (θ_m)
It also creates a mass squared splitting in matter. To derive this recall the Hamiltonian in vacuum:

$$\boxed{H_v = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{pmatrix}}$$

$\Delta m^2 \equiv$ mass squared splitting in vacuum.

We can equally well parameterize the Hamiltonian in matter as:

① — $$\boxed{H_m = \frac{\Delta m_m^2}{4E}\begin{pmatrix} -\cos(2\theta_m) & -\sin(2\theta_m) \\ -\sin(2\theta_m) & \cos(2\theta_m) \end{pmatrix}}$$

$\Delta m_m^2 \equiv$ mass squared splitting in matter.

But from before we had:

② — $$\boxed{H_m = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{pmatrix} + G_F N_e \sqrt{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}$$

Neutrino Oscillations in matter

1 and 2 are equal. $\therefore$

difference in eigenvalues of 1 = difference in eigenvalues of 2

difference in eigenvalues of 2 $= \dfrac{\Delta m_m^2}{2E}$

difference in eigenvalues of 1 $= \dfrac{\Delta m^2}{2E}\sqrt{\sin^2 2\theta + \left(\cos 2\theta - 2\sqrt{2}G_F N_e\right)^2}$

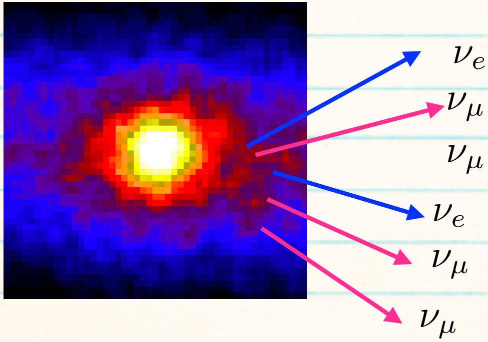
$$\Rightarrow \Delta m_m^2 = \Delta m^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - 2\sqrt{2}G_F N_e\right)^2}$$

$$\Rightarrow m_{1m}^2 = -\frac{1}{2}\Delta m^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - 2\sqrt{2}G_F N_e\right)^2}$$

$$m_{2m}^2 = +\frac{1}{2}\Delta m^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - 2\sqrt{2}G_F N_e\right)^2}$$

Solar neutrinos

- Confirmation of neutrino oscillations came in 2001 by the Sudbury Neutrino. They measured not only electron neutrino flux but all flavour neutrinos via NC interactions



$\nu_e + d \rightarrow p + p + e^-$	Charged current (CC)
$\nu_x + d \rightarrow p + n + \nu_x$	Neutral current (NC)
$\nu_x + e^- \rightarrow \nu_x + e^-$	Elastic scattering (ES)