Neutrino Physics

Neutrino masses and phenomenology

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Neutrinoless double beta decay

In standard beta decay:

$$(A, Z) \rightarrow (A, Z+1) + e^{-} + \bar{\nu}_e$$

This arises from the weak decay of a bound d-quark:

$$d \rightarrow u + e^{-} + \overline{\nu_e}$$

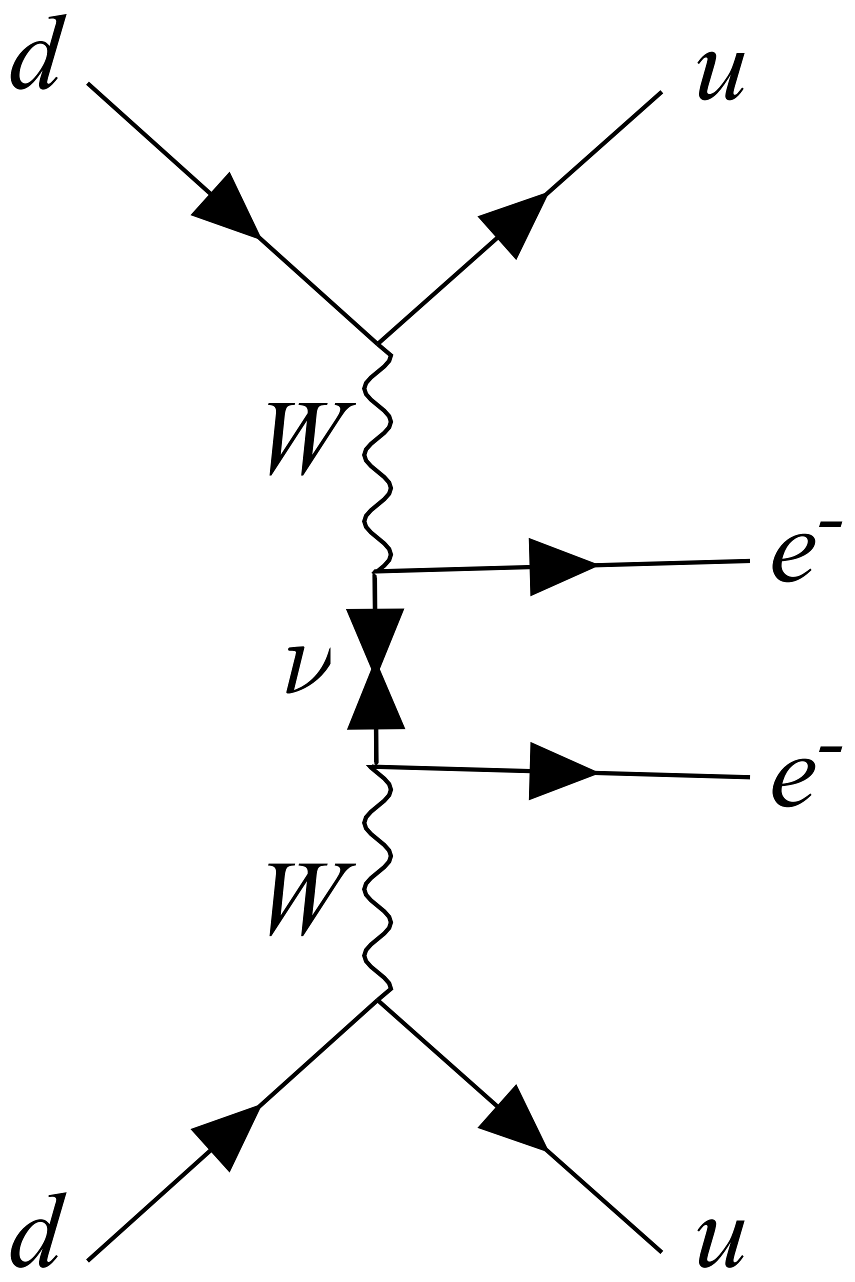
Double beta decay is far more rare (probabilistically need beta decay happen twice simultaneously)

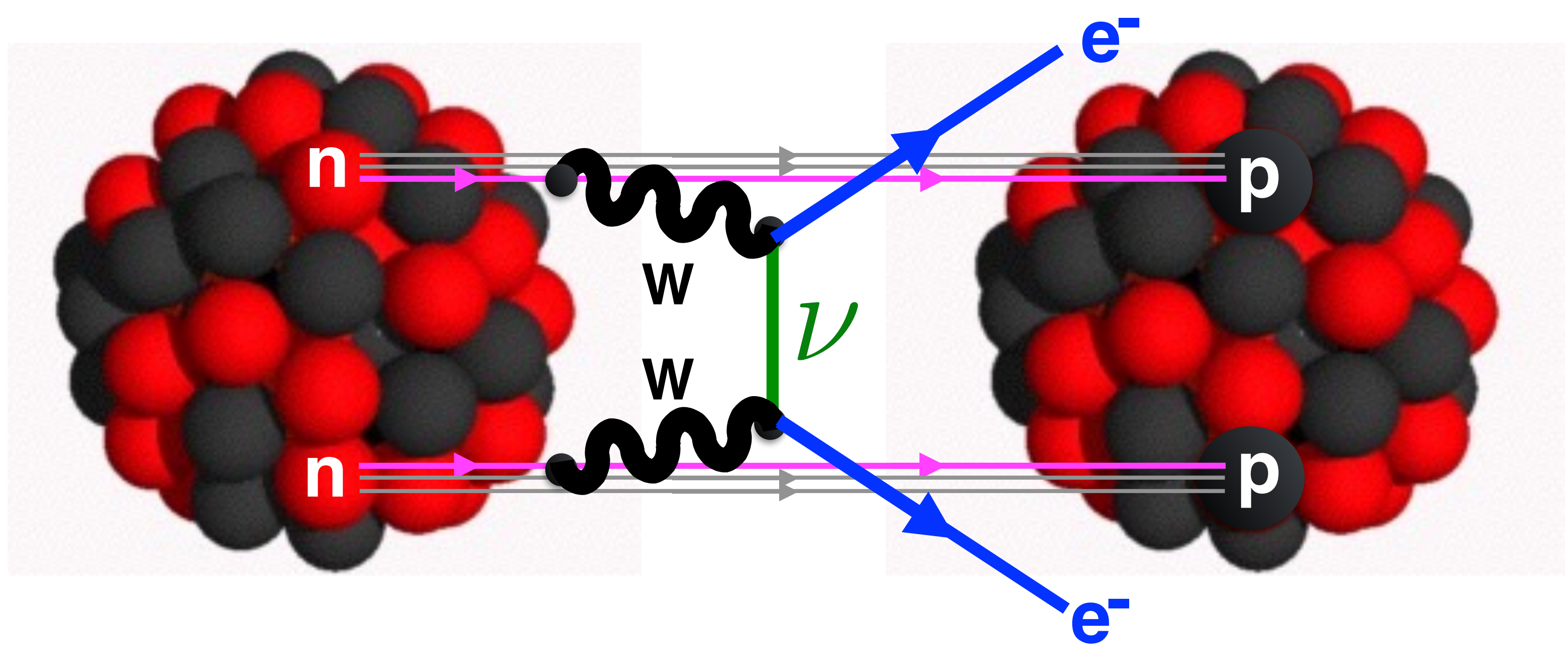
$$(A, Z) \rightarrow (A, Z+2) + e^{-} + e^{-} + \bar{\nu}_e + \bar{\nu}_e$$

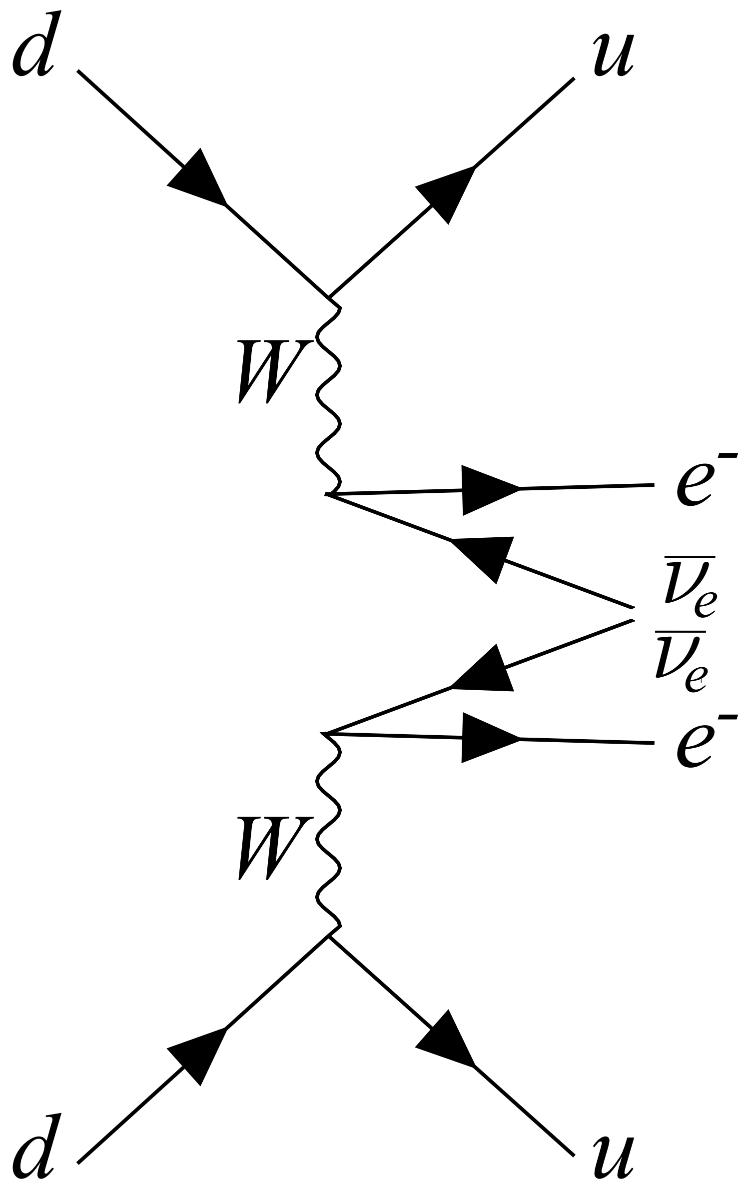
We have 0 leptons before (L=0) and we have 0 leptons after (-2+2=0). This clearly conserved lepton number

Neutrinoless double beta decay

Neutrinoless double beta decay, (A, Z) → (A, Z+2) + 2 e, will test the nature of neutrinos.





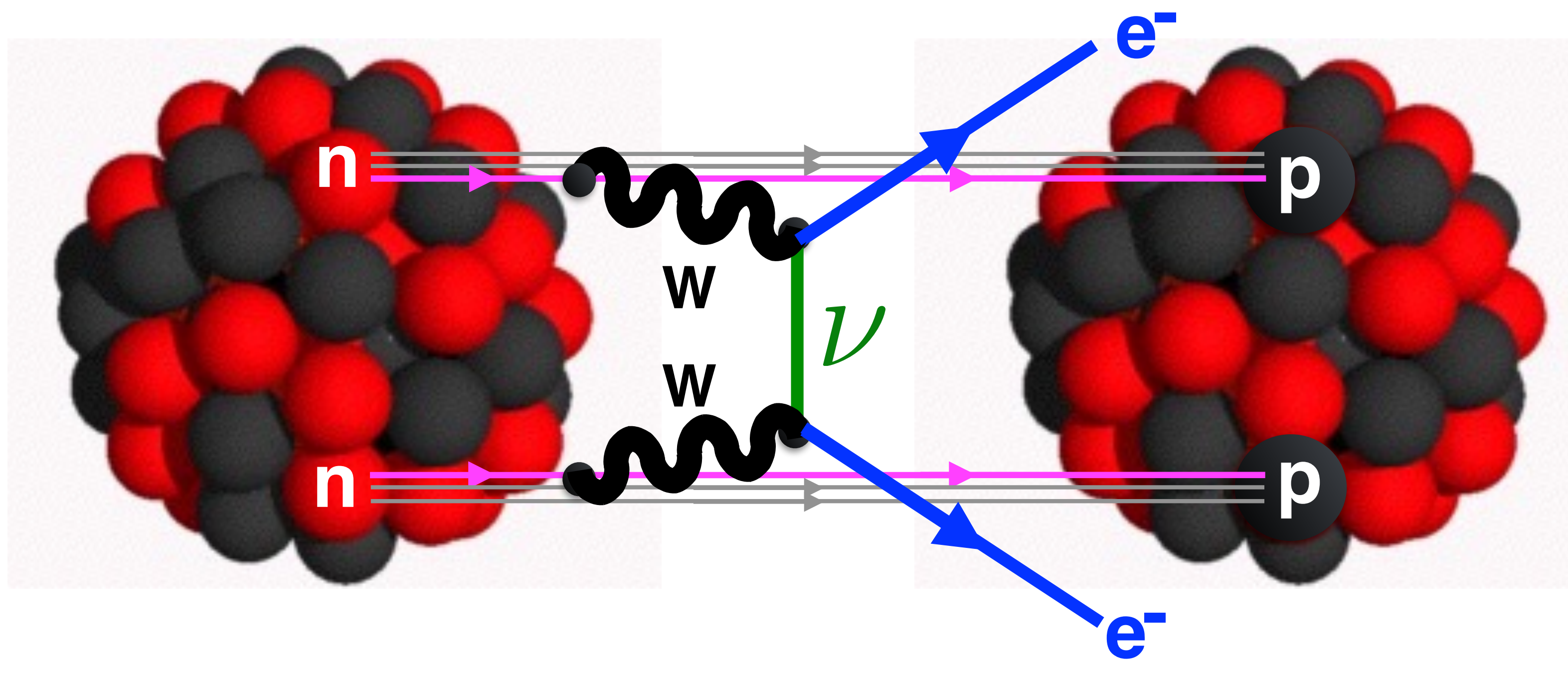


NDBD lepton number violating

double beta decay, lepton number conserving

Massive Majorana neutrinos mediate neutrino less double beta decay which violates lepton number by two units (L=0 before, L=2 after)

Decay Rate



$$\Gamma_{0\nu\beta\beta} = GM\,|m_{\beta\beta}|^2$$

$$\begin{aligned} G &= \text{ phase space factor} \\ M &= \text{ nuclear matrix element} \\ m_{\beta\beta} &= \text{ effective majorana mass} \end{aligned}$$

via the effective Majorana mass parameter:

$$\begin{aligned} |m_{\beta\beta}| &= \left|\sum_{i=1}^{3} m_i U_{ei}^2\right| \\ &= \left| m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13} e^{i\alpha_{21}} + m_3 \sin^2\theta_{13} e^{i(\alpha_{31}-2\delta)} \right| \end{aligned}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\frac{\alpha_{21}}{2}} & s_{13}e^{i\left(\frac{\alpha_{31}}{2}-\delta\right)} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}e^{i\alpha_{21}/2} - s_{12}s_{23}s_{13}e^{i\delta}e^{i\frac{\alpha_{21}}{2}} & s_{23}c_{13}e^{-i\delta}e^{i\frac{\alpha_{31}}{2}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}e^{i\frac{\alpha_{21}}{2}} - s_{12}c_{23}s_{13}e^{i\delta}e^{i\frac{\alpha_{21}}{2}} & c_{23}c_{13}e^{-i\delta}e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$m_1 \simeq 0 \qquad |U_{e1}| = \cos\theta_{13}\cos\theta_{12} \sim 0.84$$
$$m_2 \simeq \sqrt{\Delta m^2_{21}} \qquad |U_{e2}| = \cos\theta_{13}\sin\theta_{12} \sim 0.52$$
$$m_3 \simeq \sqrt{\Delta m^2_{31}} \qquad |U_{e3}| = \sin\theta_{13} \sim 0.1$$

$$|m_{\beta\beta}| = \left| m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13} e^{i\alpha_{21}} + m_3 \sin^2\theta_{13} e^{i(\alpha_{31}-2\delta)} \right|$$

- NO ($m_1 \ll m_2 \ll m_3$): $|\langle m_{\beta\beta}\rangle| \sim 1 - 5\,\mathrm{meV}$

$$|m_{\beta\beta}| \simeq \left| \sqrt{\Delta m^2_{21}} \cos^2\theta_{13}\sin^2\theta_{12} + \sqrt{\Delta m^2_{31}} \sin^2\theta_{13} e^{i(\alpha_{32}-2\delta)} \right|$$

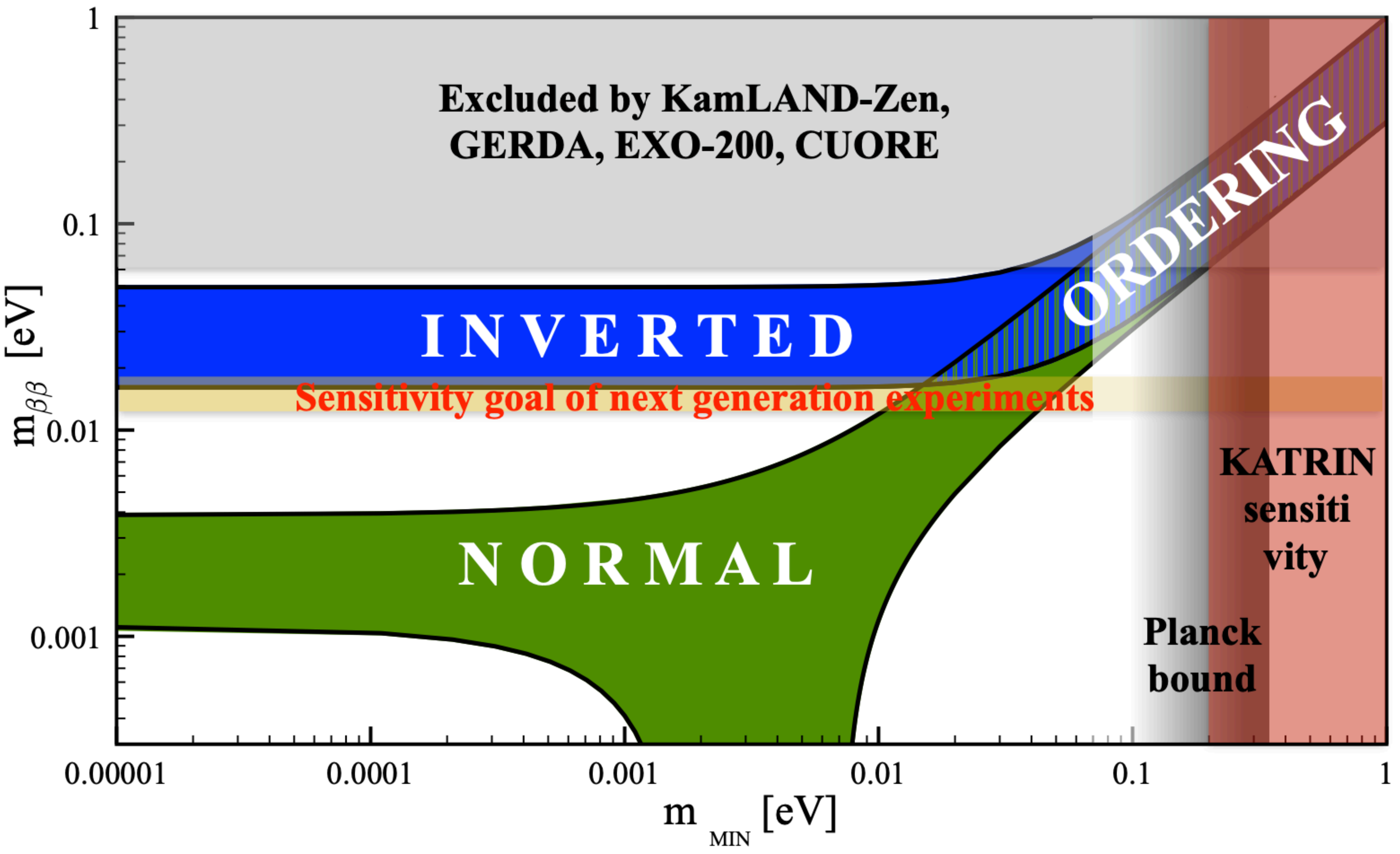
- IH ($m_3 \ll m_1 \sim m_2$): $15\,\mathrm{meV} \lesssim |\langle m_{\beta\beta}\rangle| \lesssim 50\,\mathrm{meV}$

$$\sqrt{\Delta m^2_{31}} \cos 2\theta_{12} \leq |m_{\beta\beta}| \simeq \sqrt{\left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{21}}{2}\right) \Delta m^2_{31}} \leq \sqrt{\Delta m^2_{31}}$$

- QD ($m_1 \sim m_2 \sim m_3$): $44\,\mathrm{meV} \lesssim |\langle m_{\beta\beta}\rangle| \lesssim m_1$

$$|m_{\beta\beta}| \simeq m_0 \left| \left(\cos^2\theta_{12} + \sin^2\theta_{12} e^{i\alpha_{21}}\right) \cos^2\theta_{13} + \sin^2\theta_{13} e^{i\alpha_{31}} \right|$$

Neutrinoless double beta decay



Neutrino Masses - Dirac Mass

Introduce a RHN (N) into the SM particle and **assume lepton number is conserved**

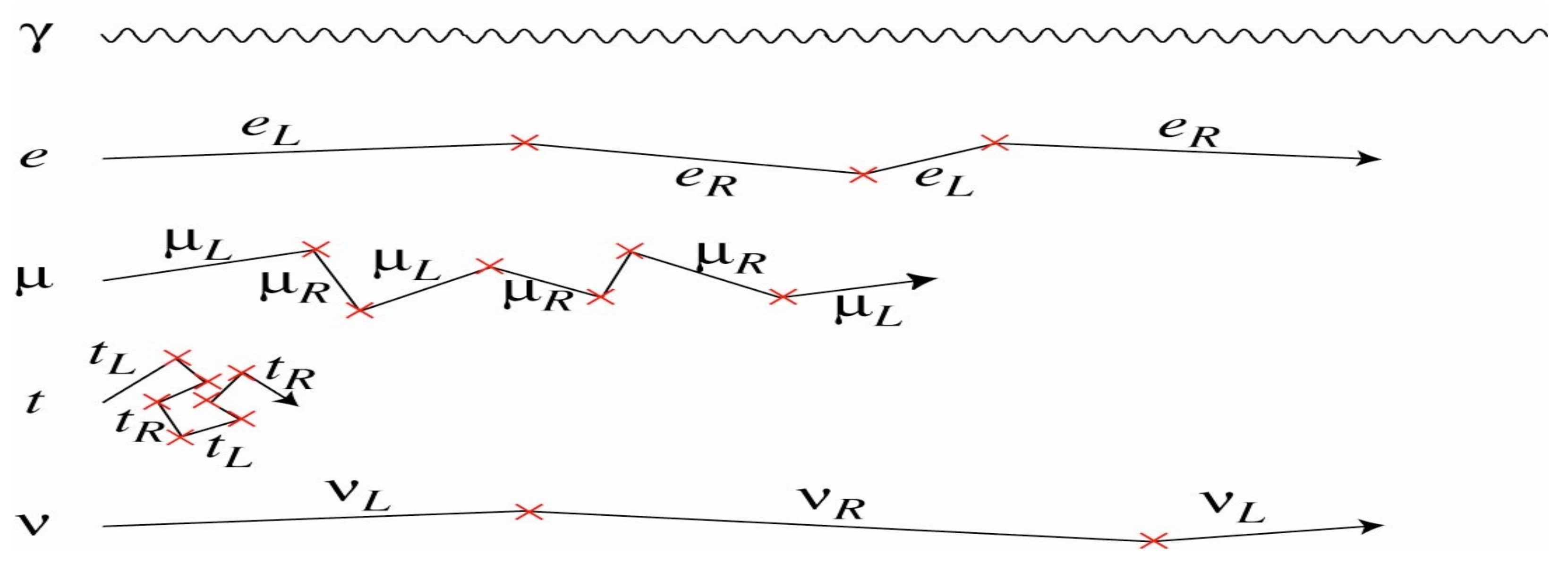
We find neutrinos are Dirac fermions. This term is $SU(2)_L$ invariant

$$\mathcal{L} \supset Y_\nu \overline{L_L} \tilde{H}^\dagger N + \text{ h.c.} \qquad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \tilde{H} = \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix}$$

$$\mathcal{L} \supset Y_\nu \left(\bar{\nu}_L, \bar{\ell}_L\right) \cdot \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} N + \text{ h.c.}$$

$$\supset Y_\nu \left(\bar{\nu}_L H^{0*} - \bar{\ell}_L H^-\right) N + \text{ h.c.}$$

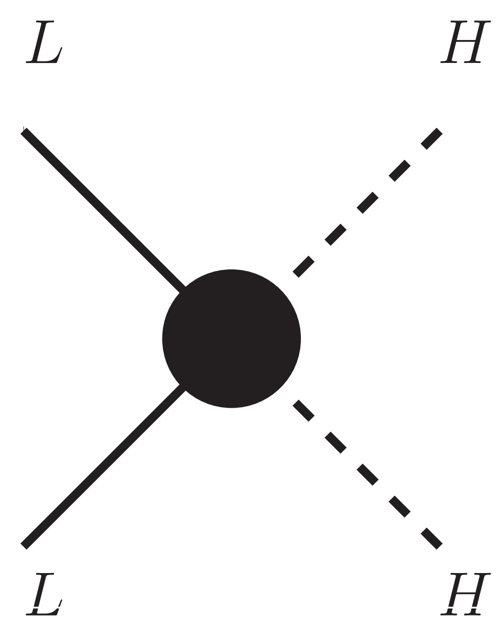
$$\supset \underbrace{\frac{Y_\nu v}{\sqrt{2}}}_{m_\nu} \bar{\nu}_L N + \text{ h.c.}$$



$$Y_\nu \sim \frac{\sqrt{2} m_\nu}{v} \sim \frac{\sqrt{2} \times 0.1\text{eV}}{246\text{GeV}} \sim 5 \times 10^{-13}$$

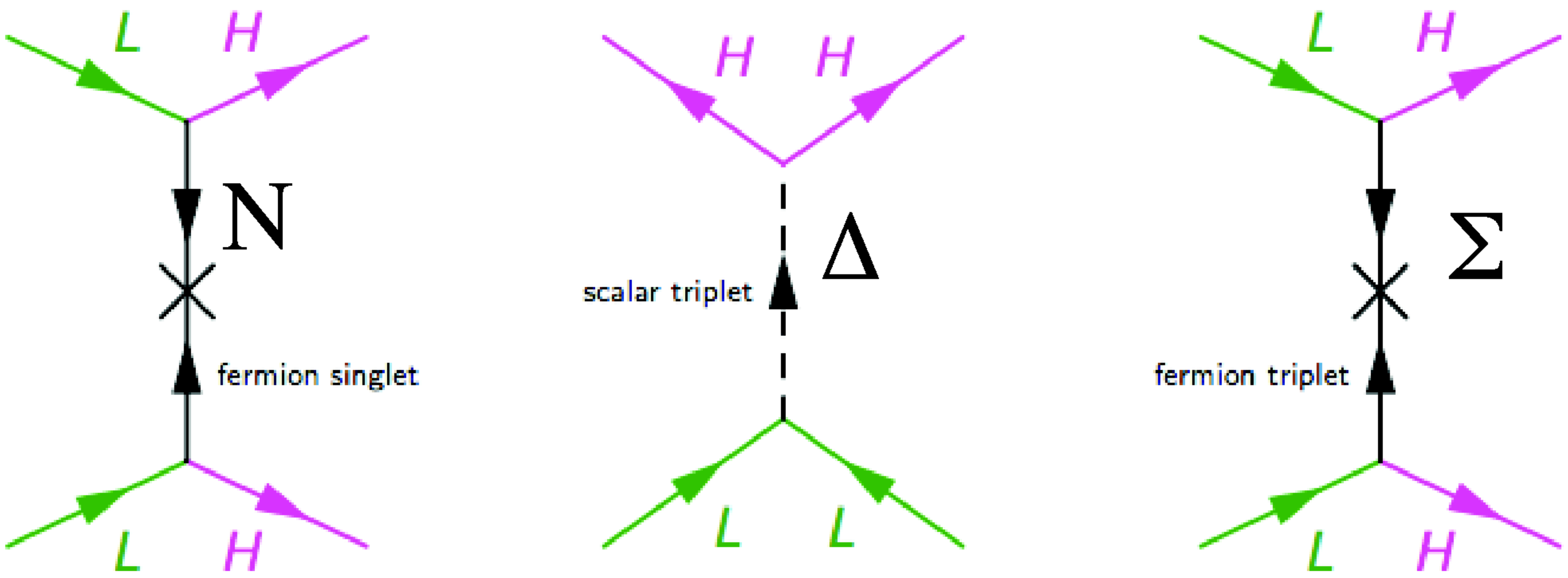
Hypothesise that lepton number is violated and form $\mathrm{SU}(2)_L$ invariant term mass term for neutrinos

$$\mathcal{L}_{d=5} = \frac{\left(Y^T Y\right)_{\alpha\beta}}{\Lambda_{\mathrm{NP}}} \left(\overline{L_\alpha} H\right) \left(H^T L^C_\beta\right)$$



Need to form gauge invariant interaction to “complete” the Weinberg operator

$$2 \otimes 2 = 1 \oplus 3$$



$N \sim (\mathbf{1}, \mathbf{1}, 0)$ $\Delta \sim (\mathbf{1}, \mathbf{3}, 2)$ $\Sigma \sim (\mathbf{1}, \mathbf{3}, 0)$

Type-I Fermionic SM i.e. right-handed neutrino (RHN)

$$\mathcal{L} = \frac{1}{2} Y_\nu \bar{N} L^c H + \frac{1}{2} Y_\nu \bar{L} H N + \frac{1}{2} \bar{N}^c M N + \text{ h.c.}$$

$$\mathcal{L} = \frac{1}{2} \left(\overline{\nu_L} \ \bar{N}^c \right) \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix} \qquad m_D = Y_\nu v$$

To find masses need to find eigenvalues of non-diagonal mass matrix:

$$\begin{vmatrix} \lambda & -m_D \\ -m_D & \lambda - M \end{vmatrix} = 0 \implies \lambda^2 - M\lambda - m_D^2 = 0$$

Minus
sign can be removed
by rephrasing fields

$$\lambda_{1,2} = \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2} \simeq \begin{matrix} M \\ \frac{M-M}{2} - \frac{4m_D^2}{4M} = -\frac{m_D^2}{M} \end{matrix}$$

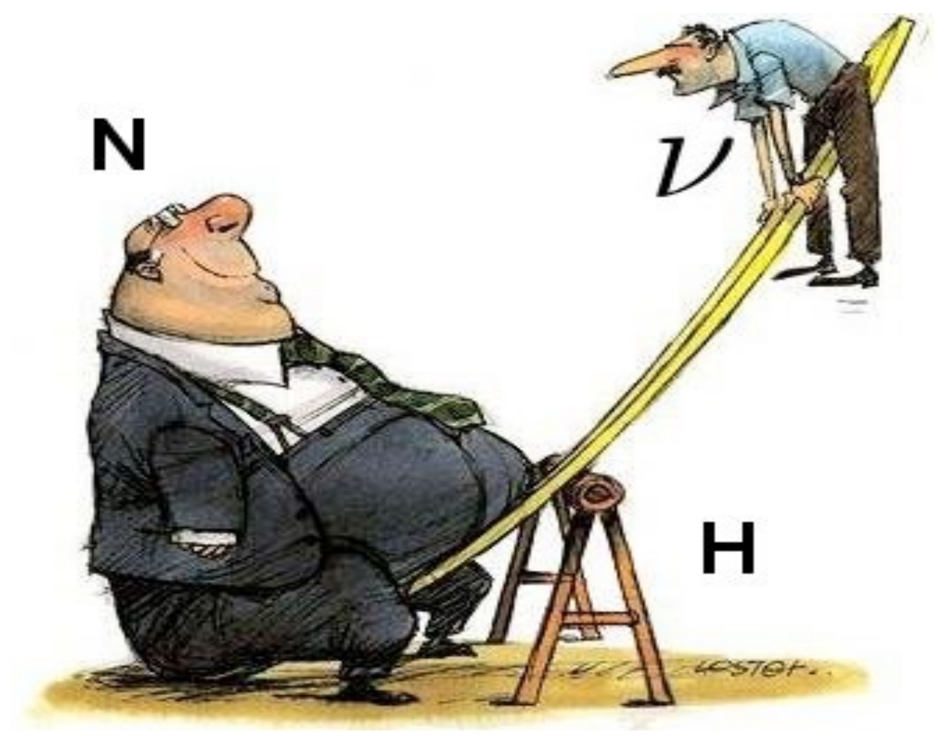
Type-I

One heavy state per one light state but we know from oscillation data there are at least two non-zero neutrino masses $\Longrightarrow$ two non-zero heavy RHN

$$m_\nu \simeq \frac{m_D^2}{M} = \frac{Y_\nu^2 v^2}{M} = \frac{1^2 \times (246\,\mathrm{GeV})^2}{M} \frac{1\mathrm{GeV}^2}{6 \times 10^{14}\mathrm{GeV}} \sim 0.1\mathrm{eV}$$

Mixing between active and very heavy state will occur but you can show (apply unitary matrix to non-diagonal mass matrix) that

$$\tan 2\theta = \frac{2m_D}{M}$$



This mixing is suppressed w.r.t the mass of the heavy RHN.

Heavy RHNs predicted by many Grand Unified Theories $SO(10)$

Pros and cons of type I see-saw models

Pros:

- they explain “naturally” the smallness of neutrino masses.
- can be embedded in GUT theories!
- neutrino masses are a indirect test of GUT theories
- have several phenomenological consequences (depending on the mass scale), e.g. leptogenesis, LFV

Cons:

- the new particles are typically too heavy to be produced at colliders (but TeV scale see-saws)
- the mixing with the new states are tiny
- in general: difficult to test

Type-II Add $SU(2)_L$ triplet scalar $\Delta \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 2)$

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad L_\alpha = \begin{pmatrix} \nu_{L,\alpha} \\ e_{L,\alpha} \end{pmatrix} \qquad \Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

Gauge invariant Yukawa potential:

$$\mathcal{L} = f_{\Delta_{ij}} \overline{L_{Li}} \Delta L^C_{L_j} + V(H, \Delta)$$

$$V(H, \Delta) = \lambda |H|^4 - \mu^2 |H|^2 - M_\Delta^2 |\Delta|^2 + \tilde{H}^\dagger \Delta H$$

Ex: show that minimum occurs at

$$\langle H \rangle = \frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}} \text{ and } \langle \Delta \rangle = \frac{\kappa v^2}{2M_\Delta^2} \qquad \Rightarrow m_\nu = f_\Delta \frac{\kappa v^2}{M_\Delta^2}$$

SS2 can be tested at LHC

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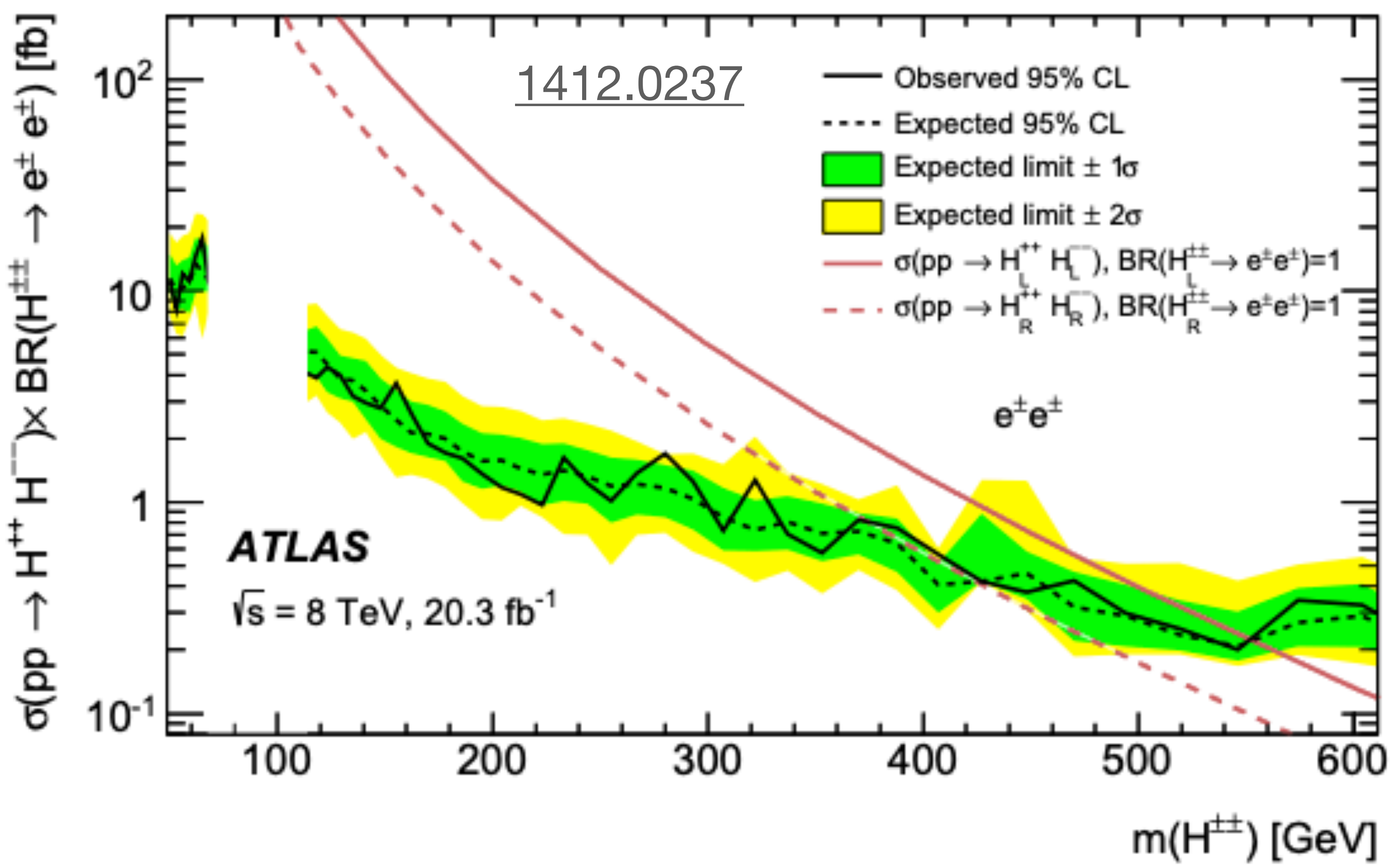

Type-III Add $SU(2)_L$ triplet fermion $$\Sigma \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)$$

$$\mathcal{L} \supset Y_{ij} \bar{L}_\alpha \sigma H \cdot \overline{\Sigma_j^c} + \frac{1}{2} M_{\Sigma,ij} \overline{\Sigma_\alpha^c} \Sigma_j + \text{ h.c.}$$

$$\overline{\Sigma^c} = \begin{pmatrix} \Sigma^0 & \Sigma^+ \\ \Sigma^- & -\Sigma^0 \end{pmatrix}$$

Again we extract the non-diagonal mass matrix

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_\Sigma \end{pmatrix} \qquad m_D = Yv$$

And find the eigenvalues:

$$m_\nu \simeq -\frac{Y^T Y v^2}{M_\Sigma}$$

SS2 can also be tested at LHC

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