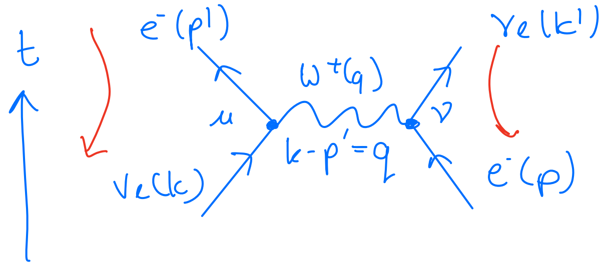
neutrino elastic scattering

$$\nu_e(k) + e^-(p) \longrightarrow \nu_e(k') + e^-(p')$$

$$\mathcal{L}_{cc} = \frac{-g}{2\sqrt{2}}\, \bar{e}(p')\, \gamma_\mu \,(1-\gamma^5)\, \nu_e(k)\, W^{+\mu}$$

$$(1-\gamma^5) = 2P_L$$



$u(p)$ incoming fermion
$\bar{u}(p)$ outgoing fermion

$$-i\mathcal{M}_{cc} = \left[\bar{u}(p')\left(\frac{-g}{\cancel{2}\sqrt{2}}\right)\gamma_\mu \cancel{2}P_L\, u(k)\right] \frac{-ig^{\mu\nu}}{M_W^2} \left[\bar{u}(k')\left(\frac{-g}{\cancel{2}\sqrt{2}}\right)\gamma_\nu \cancel{2}P_L\, u(p)\right] \times i$$

$$\mathcal{M}_{cc} = \frac{g^2}{2M_W^2}\left[\bar{u}_{p'}\gamma^\nu P_L u_k\right]\left[\bar{u}_{k'}\gamma_\nu P_L u_p\right]$$

where $u(k) \equiv u_k$ as short hand

since $\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$

$$\Rightarrow \quad \frac{g^2}{2M_W^2} = \frac{4G_F}{\sqrt{2}}$$

$$\mathcal{M}_{cc} = \frac{4G_F}{\sqrt{2}}\left[\bar{u}_{p'}\gamma^\nu P_L u_k\right]\left[\bar{u}_{k'}\gamma_\nu P_L u_p\right]$$

$$\mathcal{M}_{cc}\cdot\mathcal{M}_{cc}^* = 8G_F^2 \sum_s \left[\bar{u}_{p'}\gamma^\nu P_L u_k\right]\left[\bar{u}_{k'}\gamma_\nu P_L u_p\right] \left[\bar{u}_k P_L \gamma^\mu u_{p'}\right]\left[\bar{u}_p P_L \gamma_\mu u_{k'}\right]$$

$$\sum_s u_p \bar{u}_p = \not{p}' + m_e.$$

$$\left[\bar{u}_{p'}\gamma^\nu P_L u_k\right]\left[\bar{u}_k P_L \gamma^\mu u_{p'}\right] = \left[\bar{u}_{p'}\gamma^\nu P_L u_k\right]\left[\bar{u}_k \gamma^\mu P_R u_{p'}\right]$$

$$= \mathrm{Tr}\left[(\not{p}' + m_e)\gamma^\nu P_L \not{k} \gamma^\mu u_{p'}\right].$$

$P_L \cdot P_L = P_L$

$$\Rightarrow \mathrm{Tr}\left[(\not{p}' + m_e)\gamma^\nu P_L \not{k} \gamma^\mu\right] \; \mathrm{Tr}\left[\not{k}' \gamma_\nu P_L (\not{p} + m_e)\gamma_\mu\right]$$

perform trace
see Mathematica notebook

$$|\mathcal{M}|^2 = 128\, G_F^2 \,(k \cdot p')(k' \cdot p)$$

$$\Rightarrow t = (k - p') = k^2 + p'^2 - 2k \cdot p'$$
$$= 0 + m_e^2 - 2k \cdot p'$$

$$\Rightarrow -\left(\frac{t - m_e^2}{2}\right) = k \cdot p'$$

$$t = (p - k') \Rightarrow k' \cdot p = -\left(\frac{t - m_e^2}{2}\right)$$

$$|\mathcal{M}_c|^2 = 128\, G_F^2 \cdot \frac{1}{4} (t - m_e^2)^2$$
$$= 32 (t - m_e^2)^2$$

$$|\mathcal{M}_{cc}|^2 = 32 (t - m_e^2)^2$$

$$\frac{d\sigma}{dt} = \underbrace{\frac{1}{4}}_{\text{sum final states \& average incoming}} \; \frac{1}{16\pi} \; \frac{1}{\lambda(s, m_a^2, m_b^2)} \int_{t_1}^{t_f} dt\, |\mathcal{M}|^2$$

phase space

$$t_\pm = m_a^2 + m_1^2 - \frac{1}{2s}\Big[(t + m_b^2 - m_2^2)(t + m_a^2 - m_1^2)$$
$$\mp \lambda(s, m_a^2, m_b^2)^{1/2} \lambda(s, m_1^2, m_2^2)^{1/2}\Big]$$

see particle kinematics [Byckling & Kajantie Chp 2]

$$m_a^2 = m_2^2 = m_\nu^2 = 0$$
$$m_1^2 = m_b^2 = m_e^2$$

$$\sigma \sim \frac{s\, G_F^2}{6\pi}$$

(note $s+t+u = \sum m_i^2$
$\Rightarrow s+t+u = 2m_e^2$)