Collider Phenomenology (2)

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Plan for the lectures

- Basics of collider physics
- Basics of QCD
  - DIS and the Parton Model
  - Higher order corrections
  - Asymptotic freedom
  - QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT

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Master formula for LHC physics

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Important aspect of a Monte Carlo generator

Parton density functions

Universal:

~Probabilities of finding given parton with given momentum in proton

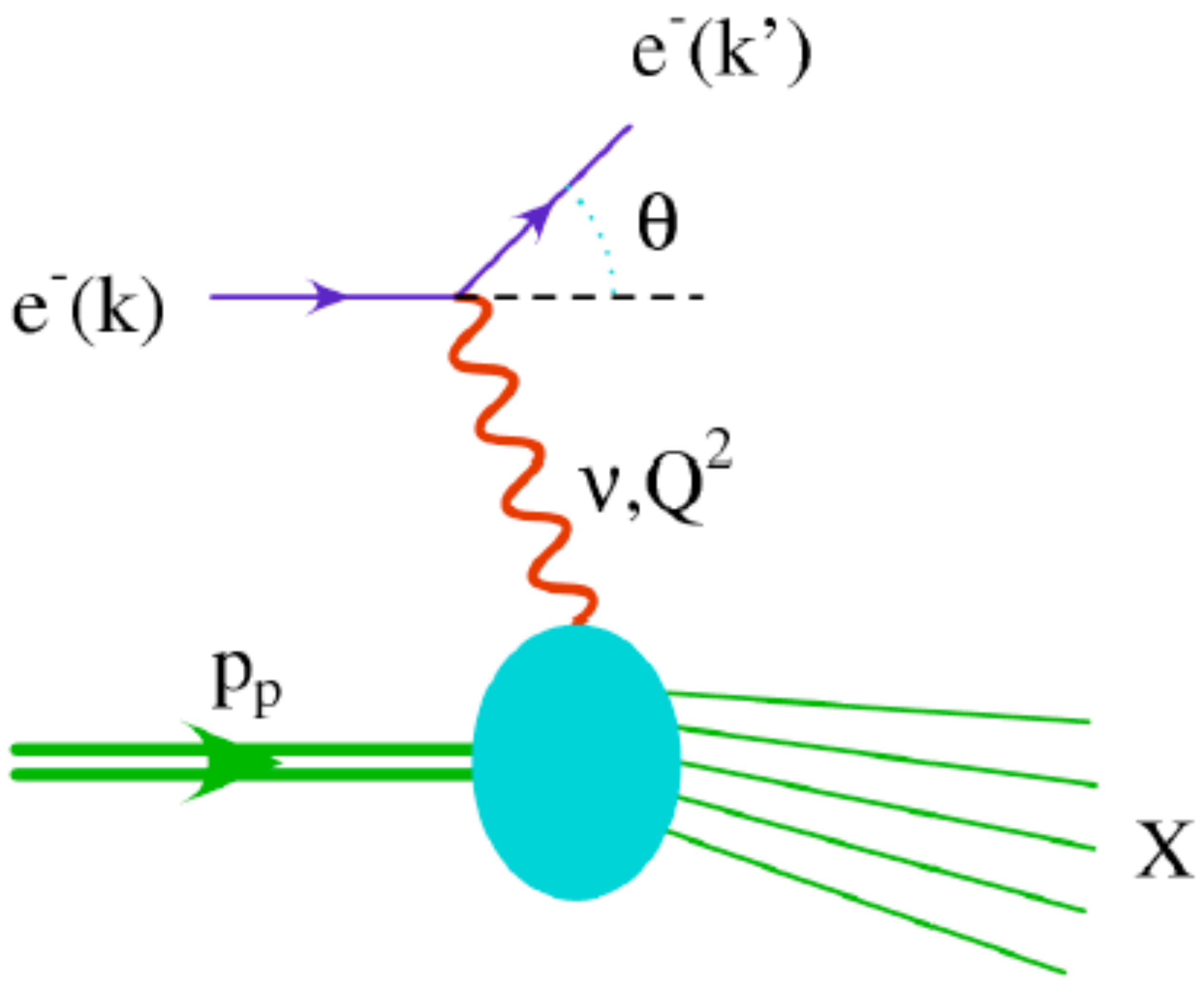
Extracted from data

Parton-level cross section

Subject of huge efforts in the LHC theory community to systematically improve this

The parton model of QCD

Deep Inelastic Scattering



$$s = (P+k)^2$$ CoM energy

$$Q^2 = -(k-k')^2$$ momentum transfer^2

$$x = Q^2/2(P\cdot q)$$ scaling variable

$$\nu = (P\cdot q)/M = E - E'$$ energy loss

$$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$$ relative energy loss

$$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x}Q^2$$ recoil mass

$$\frac{d\sigma_{\rm elastic}}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)_{\rm point} \cdot F^2_{\rm elastic}(q^2)\,\delta(1-x)\,dx$$

$$\frac{d\sigma_{\rm inelastic}}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)_{\rm point} \cdot F^2_{\rm inelastic}(q^2,x)\,dx$$

Can we guess what F looks like?

Deep Inelastic scattering

What can $F^2(q^2)$ look like?

1. Proton charge is smoothly distributed (probe penetrates proton like a knife through butter)

$$F^2_{elastic}(q^2) \sim F^2_{inelastic}(q^2, x) \ll 1$$

2. Proton consists of tightly bound charges (quarks hit as single particles, but cannot fly away because tightly bound)

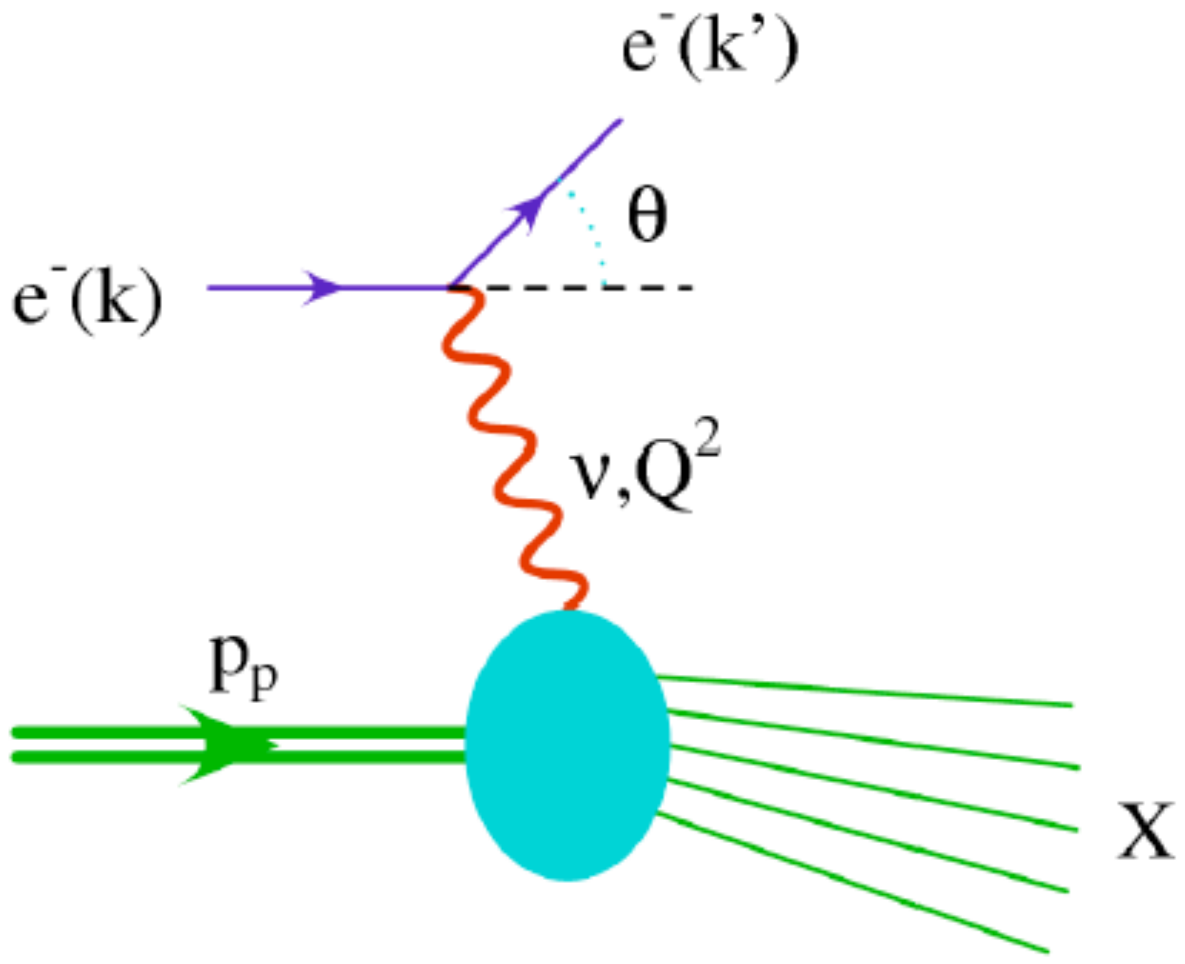
$$F^2_{elastic}(q^2) \sim 1 \qquad F^2_{inelastic}(q^2, x) \ll 1$$

!!!3. $F^2_{elastic}(q^2) \ll 1 \qquad F^2_{inelastic}(q^2, x) \sim 1$

Quarks are free particles which fly away without caring about confinement!

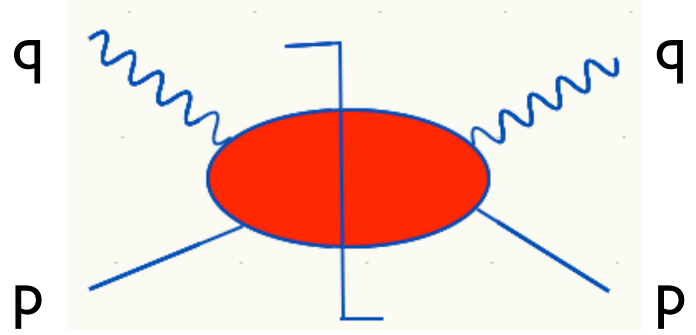
Parton Model

DIS cross-section



$$d\Phi = \frac{d^3k'}{(2\pi)^3 2E'} d\Phi_X = \frac{ME}{8\pi^2} y\, dy\, dx\, d\Phi_X$$

$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{e^4}{Q^4} L^{\mu\nu} h_{X\mu\nu}$$



Why $1/Q^4$?

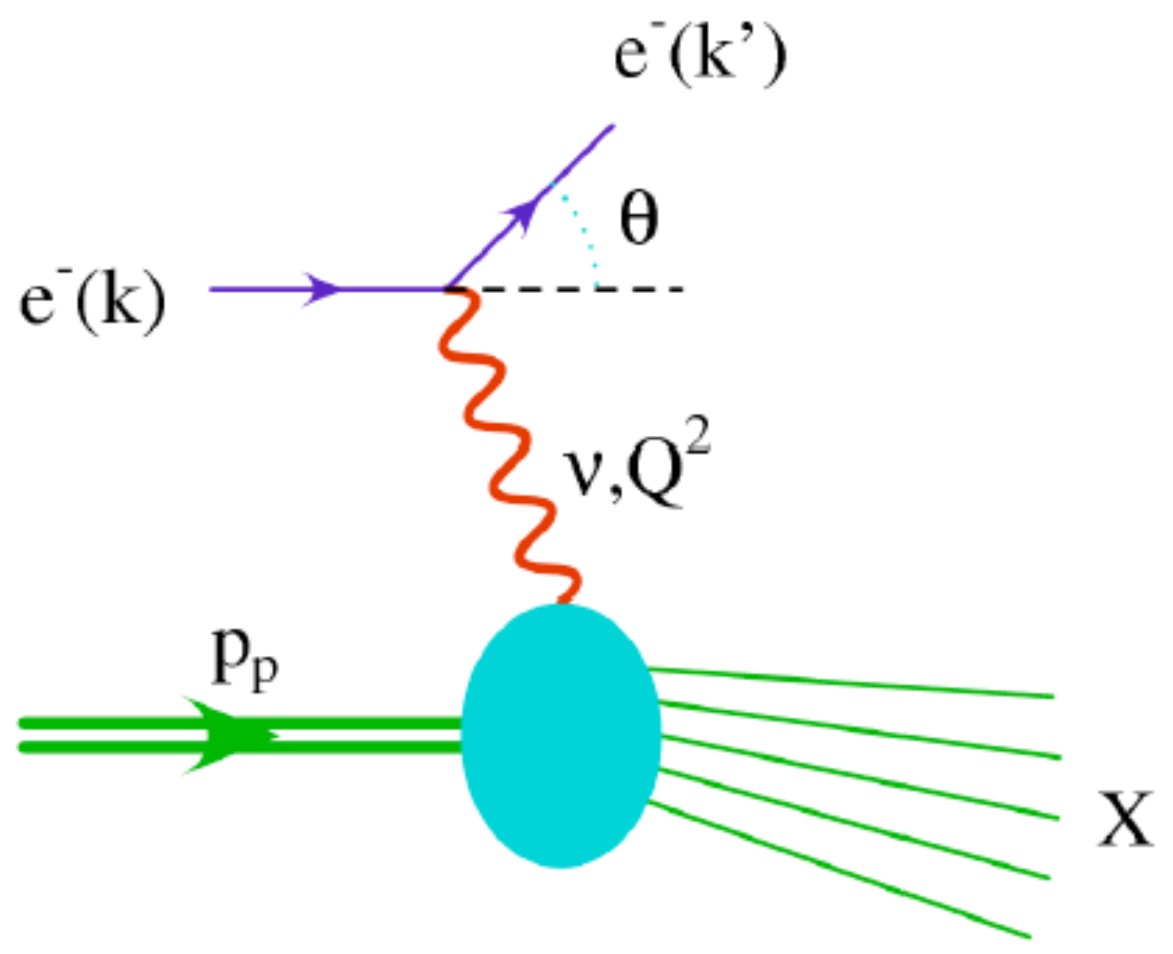
$$L^{\mu\nu} = \frac{1}{4}\mathrm{tr}[\not{k}\gamma^\mu \not{k}'\gamma^\nu] = k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k\cdot k'$$

Based on Lorentz and gauge invariance

$$W^{\mu\nu} = \sum_X \int d\Phi_X h_{X\mu\nu}$$

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)\frac{1}{p\cdot q} F_2(x,Q^2)$$

Parton Model



$$\sigma^{ep\to eX} = \sum_X \frac{1}{4ME}\int d\Phi\,\frac{1}{4}\sum_{\text{spin}}|\mathcal{M}|^2$$

After a bit of maths (good exercise!), we get:

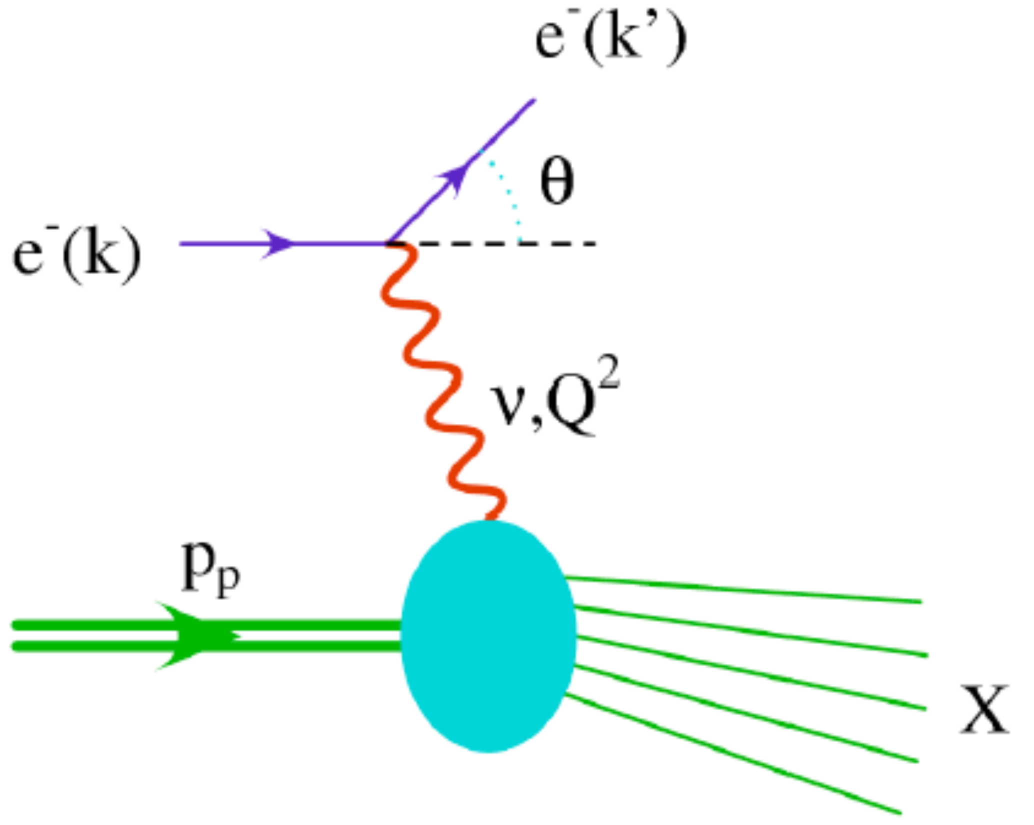
$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

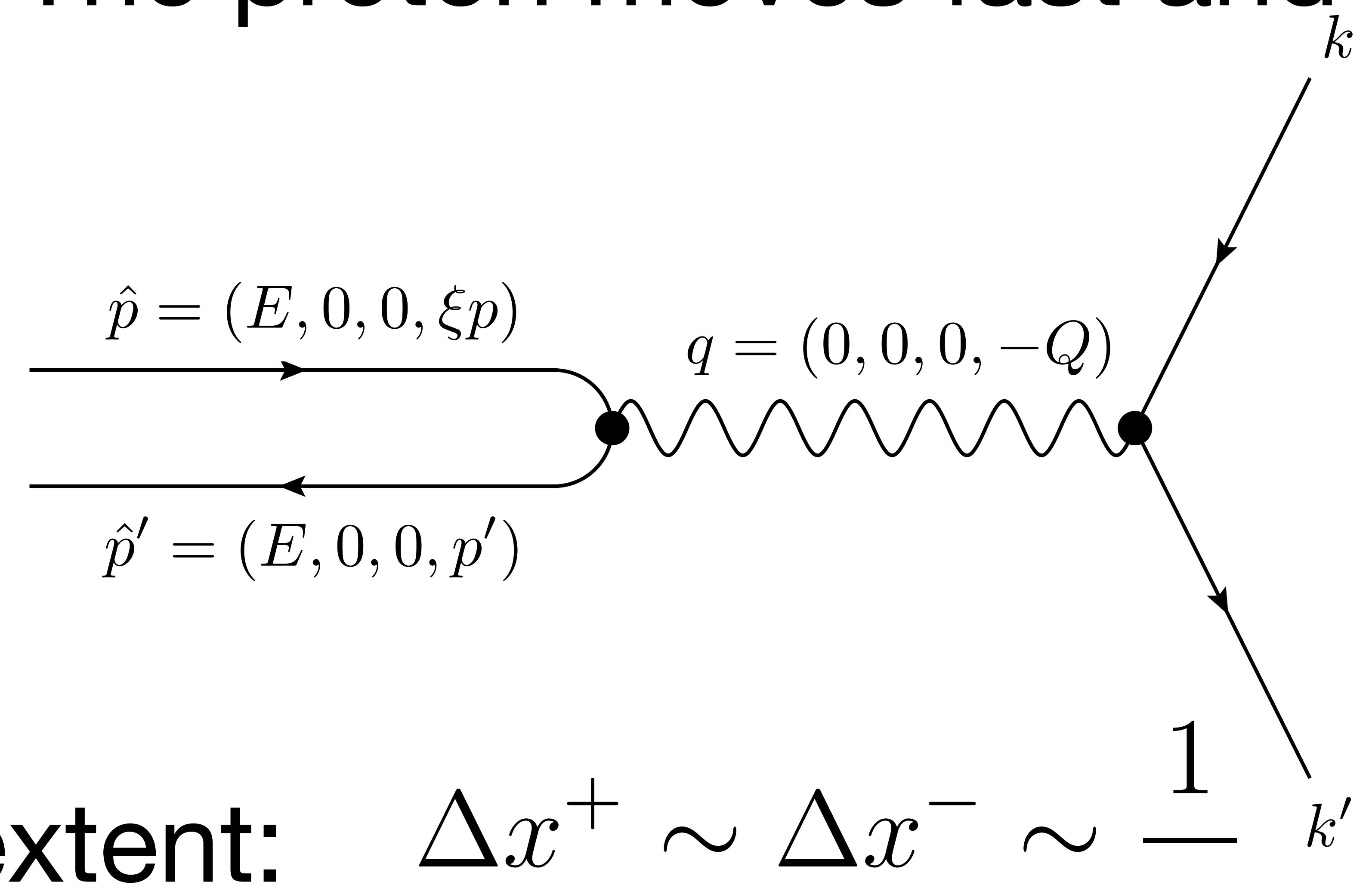
Transverse photon   Longitudinal photon

Parton Model

Breit frame

The proton moves fast and the photon has zero energy





$$p \equiv \left(\sqrt{\frac{Q^2}{4x^2} + m^2}, \frac{Q}{2x}, \vec{0}_\perp\right) \approx \left(\frac{Q}{2x} + \frac{xm^2}{Q}, \frac{Q}{2x}, \vec{0}_\perp\right)$$

$$q \equiv \left(0, -Q, \vec{0}_\perp\right).$$

Rest frame: Proton extent: $\Delta x^+ \sim \Delta x^- \sim \dfrac{1}{m}$

Breit frame: Proton extent: $\Delta x^+ \sim \dfrac{Q}{m^2}, \qquad \Delta x^- \sim \dfrac{1}{Q}$

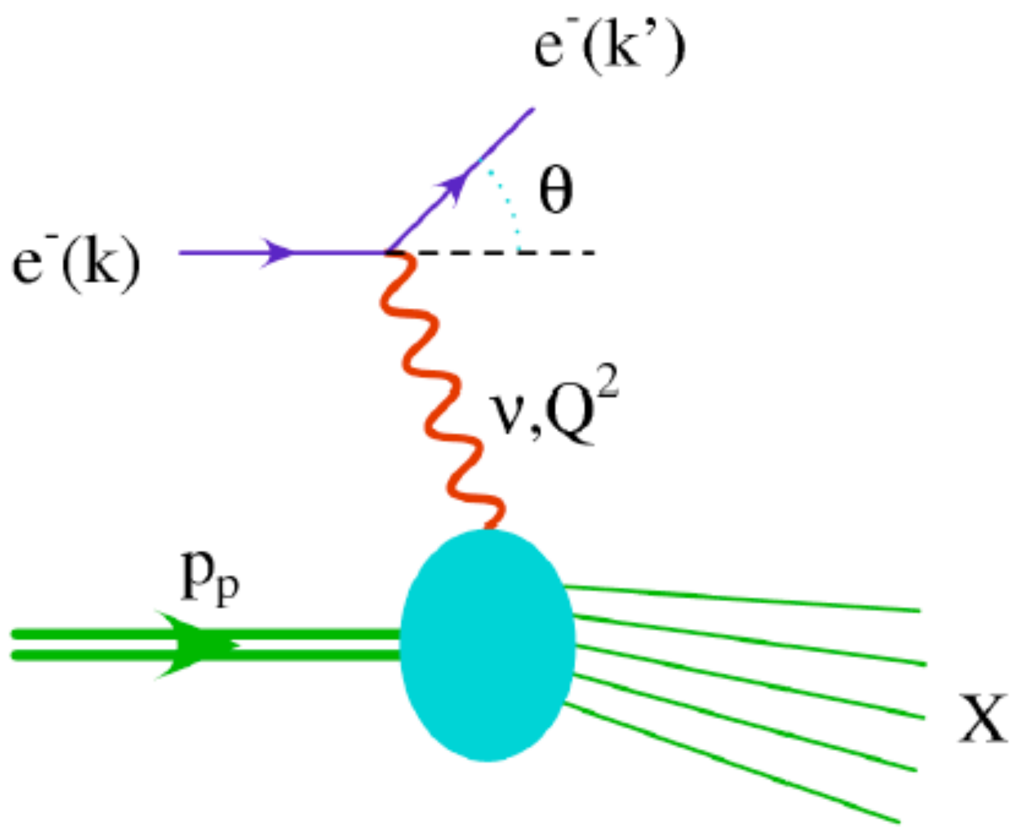
Photon extent: $\Delta x^+ \sim 1/Q,$

$\Rightarrow \quad (\Delta x^+)_{\text{photon}} \ll (\Delta x^+)_{\text{proton}}$

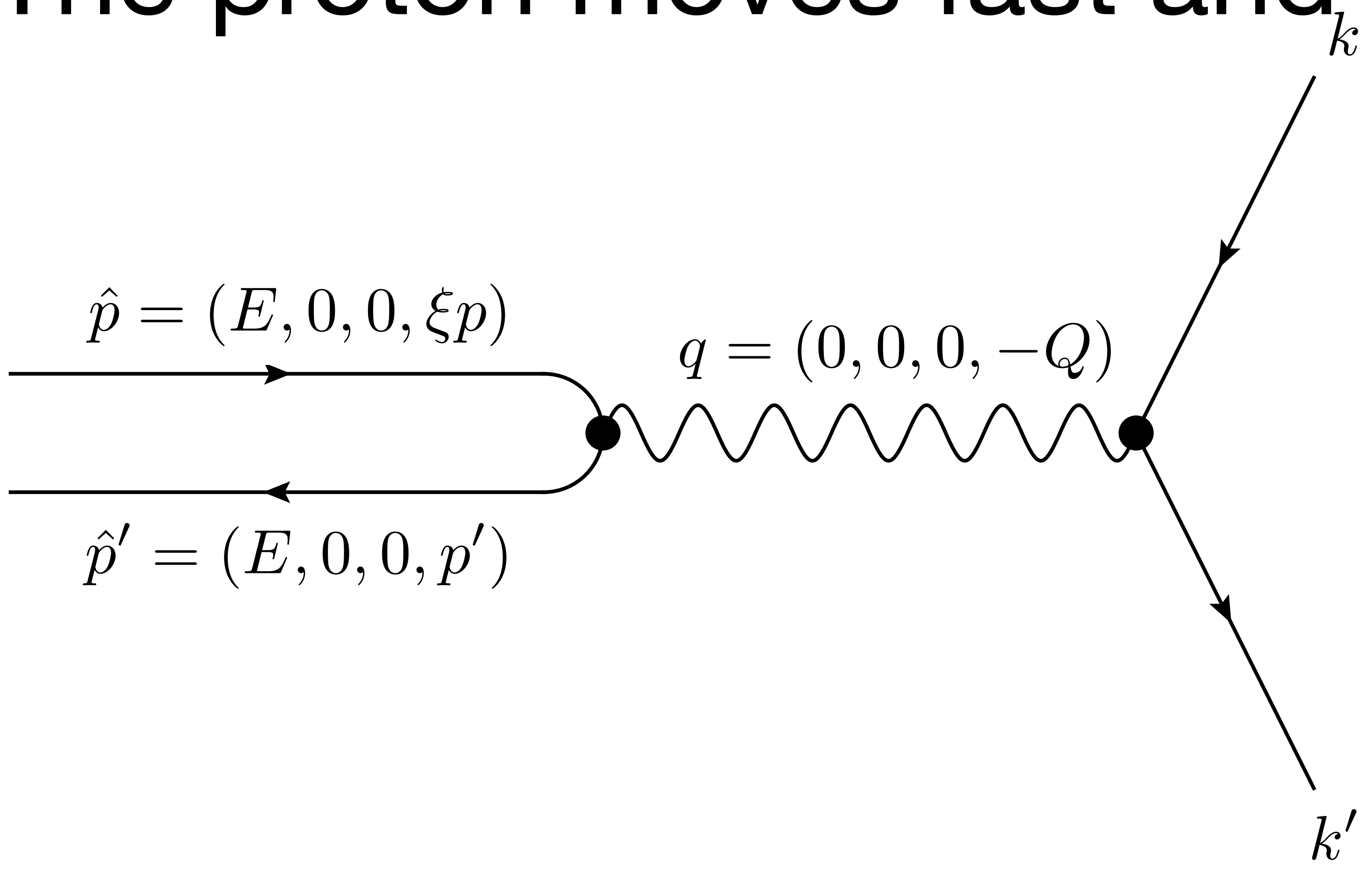
The time scale of a typical parton-parton interaction is much larger than the hard interaction time.

Parton Model

Breit frame



The proton moves fast and the photon has zero energy



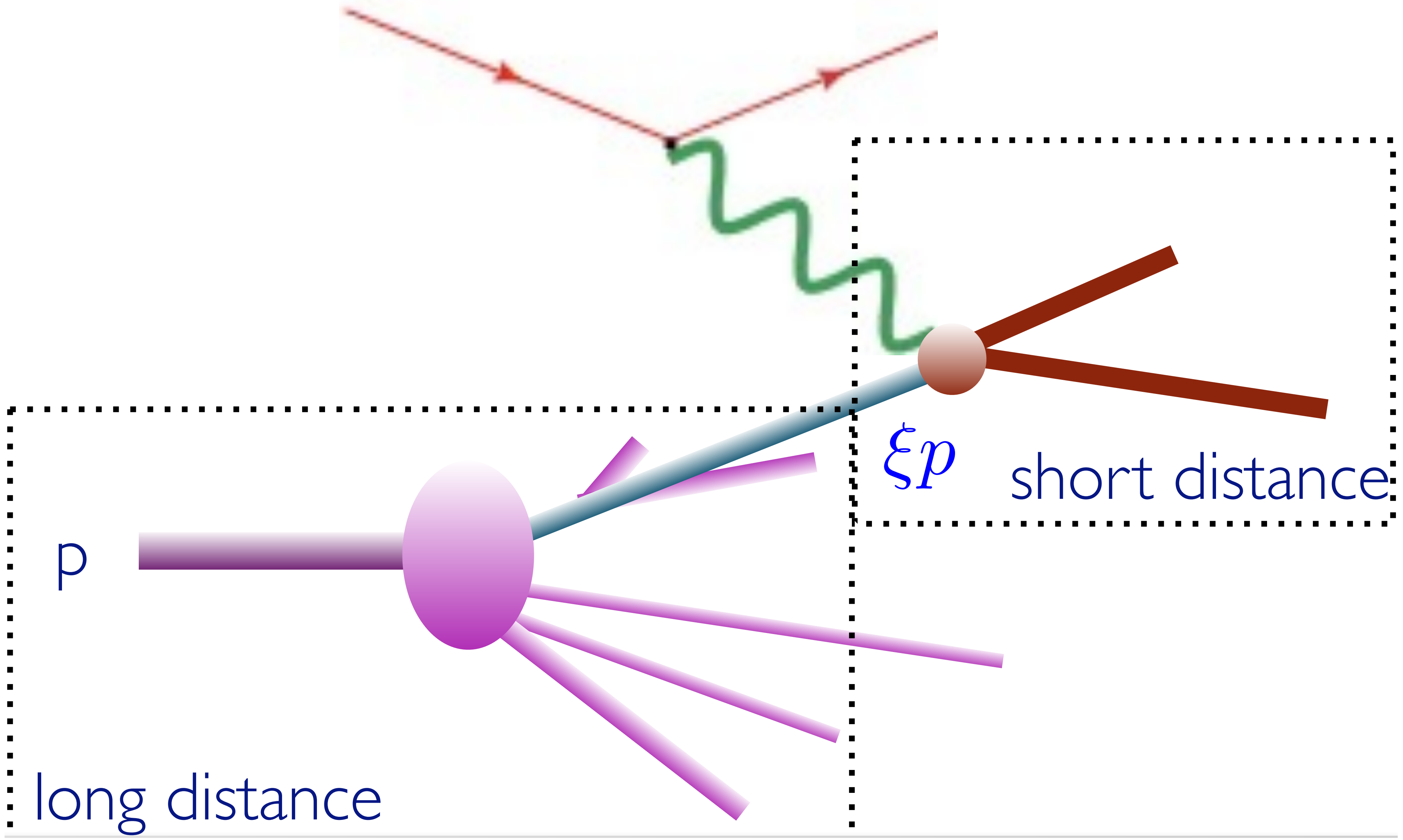
$$\left(\Delta x^{+}\right)_{\text{photon}} \ll \left(\Delta x^{+}\right)_{\text{proton}}$$

- The time scale of a typical parton-parton interaction is much larger than the hard interaction time.
- Schematically: in the Breit frame the proton moves very fast towards the photon, and is therefore Lorentz contracted to a kind of pancake.
- The photon interaction then takes place on the very short time scale when the photon passes that pancake.
- During the short interaction time, the struck quark thus does not interact with the spectator quarks and can be regarded as a free parton.

Factorisation

Breit picture frame allows us to assume partons are free within proton:

$$\frac{d^2\sigma}{dxdQ^2} = \int_0^1 \frac{d\xi}{\xi} \sum_i f_i(\xi) \frac{d^2\hat{\sigma}}{dxdQ^2}(\frac{x}{\xi}, Q^2)$$



$f_i(\xi)$ Probability of finding parton i in hadron carrying momentum fraction ξ

$\frac{d^2\hat{\sigma}}{dxdQ^2}$ Cross-section for parton-photon scattering

DIS cross-section

Comparing our inclusive cross-section:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

Factorised cross-section in the parton model:

$$\frac{d^2\sigma}{dx dQ^2} = \int_0^1 \frac{d\xi}{\xi}\sum_i f_i(\xi)\frac{d^2\sigma}{d\hat{x}dQ^2}\left(\frac{x}{\xi},Q^2\right) \quad \text{with} \quad \frac{d^2\hat{\sigma}}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}\left[1+(1-y)^2\right] e_q^2\,\delta(x-\xi)$$

We can express the structure functions as:

$$F_2(x) = 2xF_1 = \sum_{i=q,\bar{q}}\int_0^1 d\xi\, f_i(\xi)\, x e_q^2 \delta(x-\xi) = \sum_{i=q,\bar{q}} e_q^2\, x f_i(x)$$

DIS cross-section

We can express the structure functions as:

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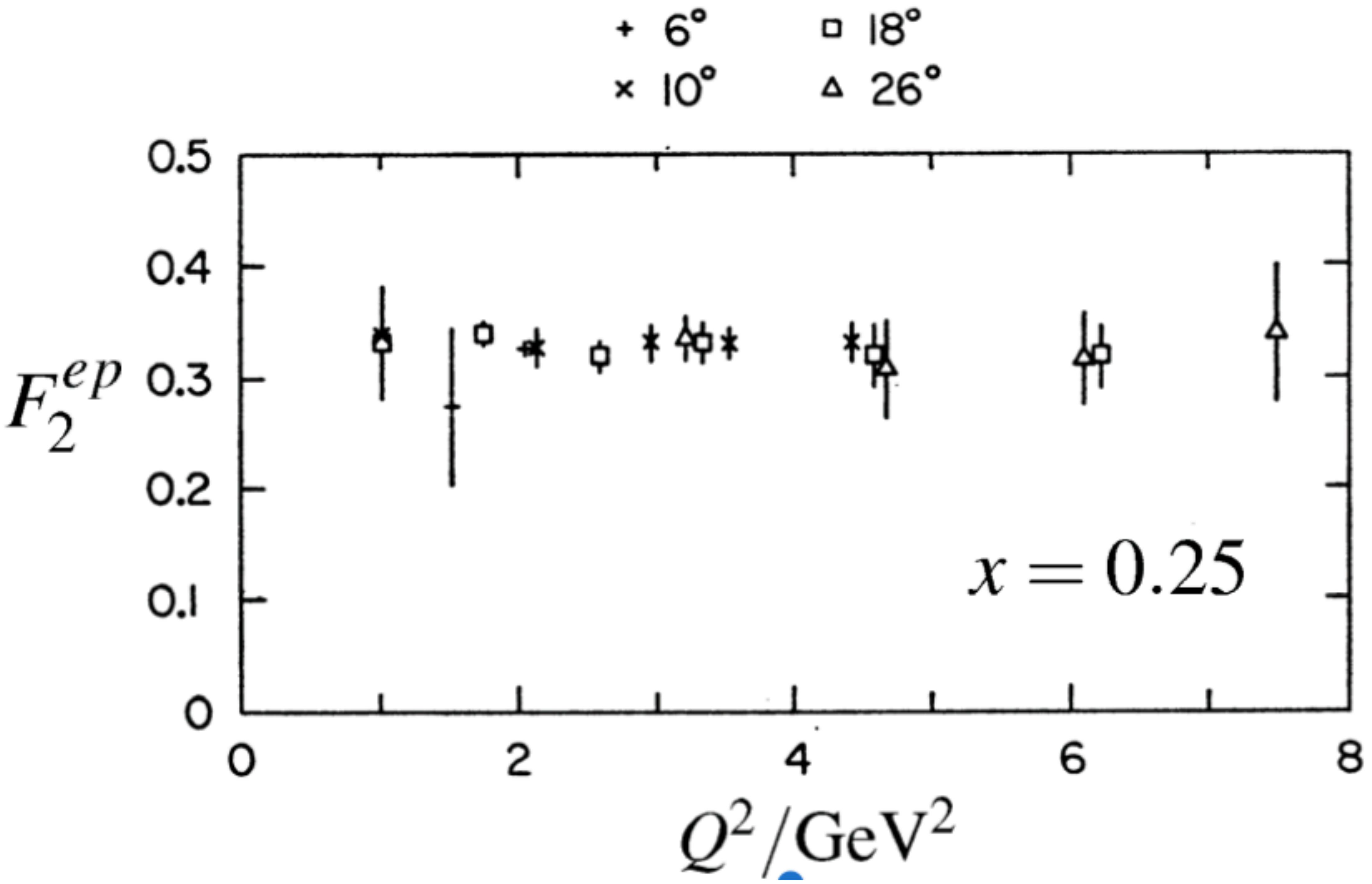
Quarks and anti-quarks enter together.

How can we separate them?

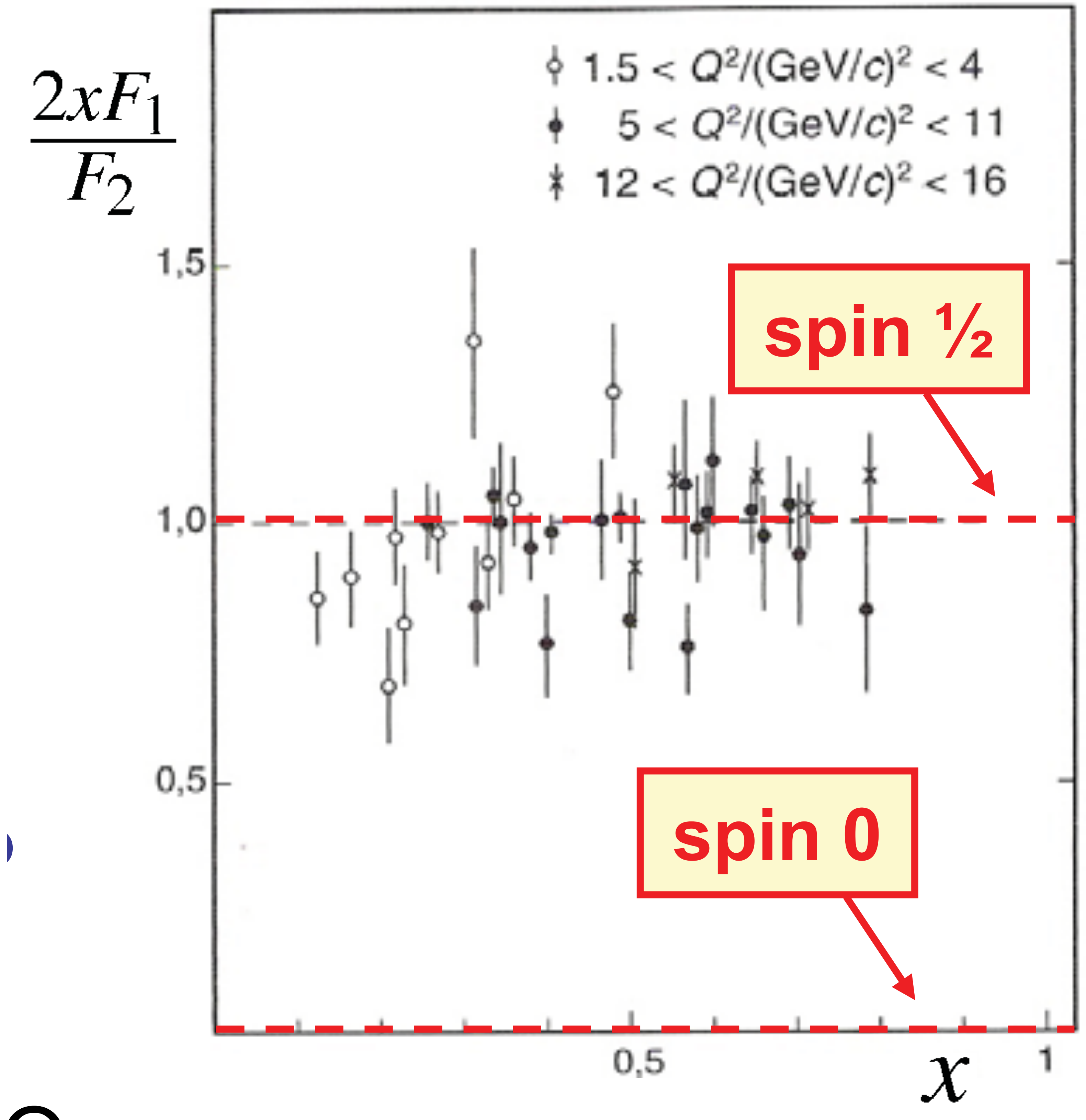
No dependence on Q: **Scaling**

$f_i(x)$ are the parton distribution functions which describe the probabilities of finding specific partons in the proton carrying momentum fraction x

Scaling and Callan-Gross relation



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203

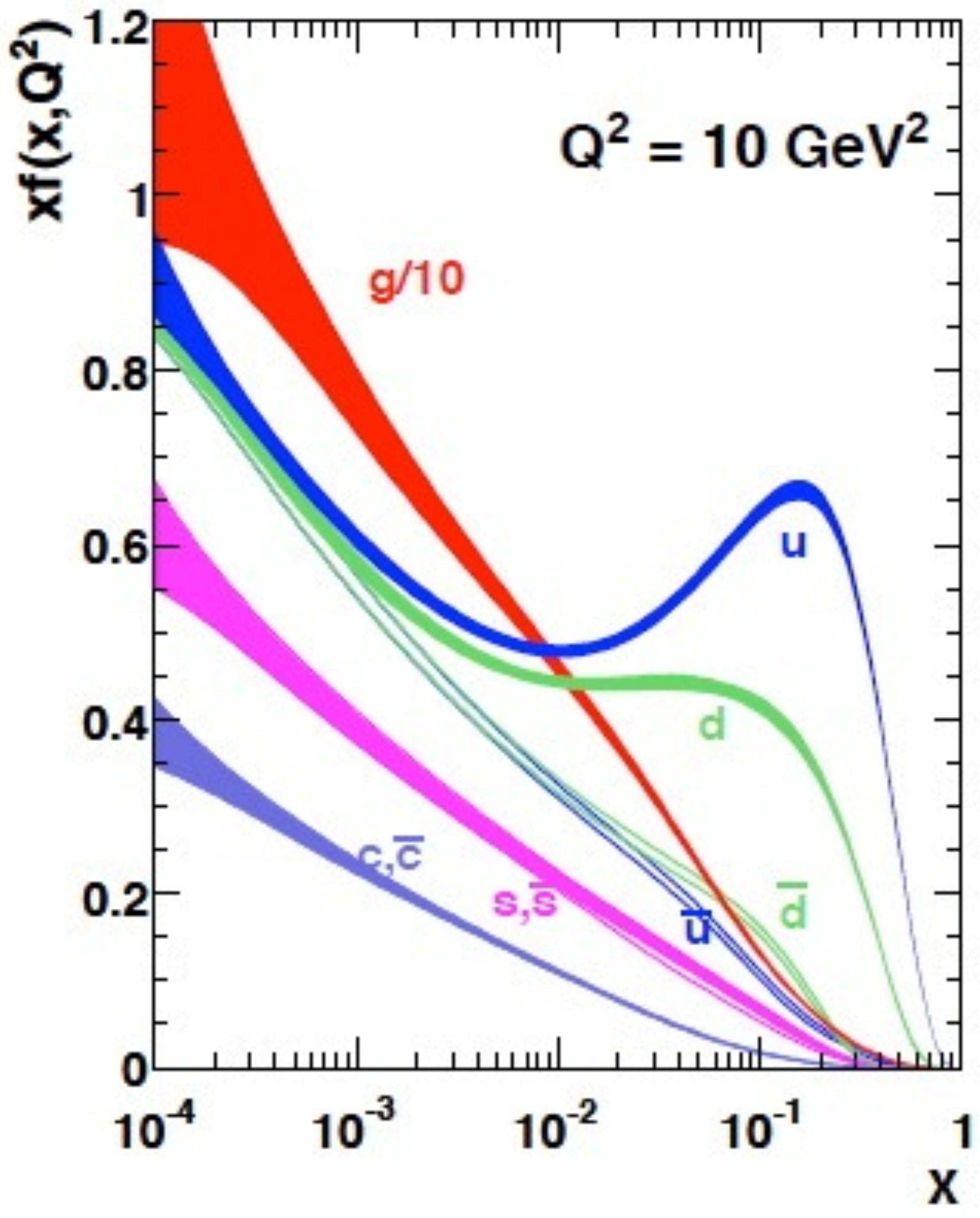


Scaling: Structure function does not depend on Q

Callan-Gross relation

Quarks are spin-1/2 particles!

Parton distribution functions



$$
\begin{aligned}
u(x) &= u_V(x) + \bar{u}(x) \\
d(x) &= d_V(x) + \bar{d}(x) \\
s(x) &= \bar{s}(x)
\end{aligned}
$$

$$\int_0^1 dx\, u_V(x) = 2\,, \quad \int_0^1 dx\, d_V(x) = 1$$

$$\sum_q \int_0^1 dx\, x[q(x) + \bar{q}(x)] \simeq 0.5$$

Quarks carry only 50% of the proton momentum

Evidence for gluons!

Parton model summary

DIS experiments show that virtual photon scatters off massless, free, point like, spin-1/2 quarks

One can **factorise** the short- and long-distance physics entering this process. Long-distance physics absorbed in PDFs. Short distance physics described by the hard scattering of the parton with the virtual photon.

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$$

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Phase-space integral Parton density functions Parton-level cross section

Master formula for LHC physics

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Important aspect of a Monte Carlo generator

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Universal:

~Probabilities of finding given parton with given momentum in proton

Extracted from data

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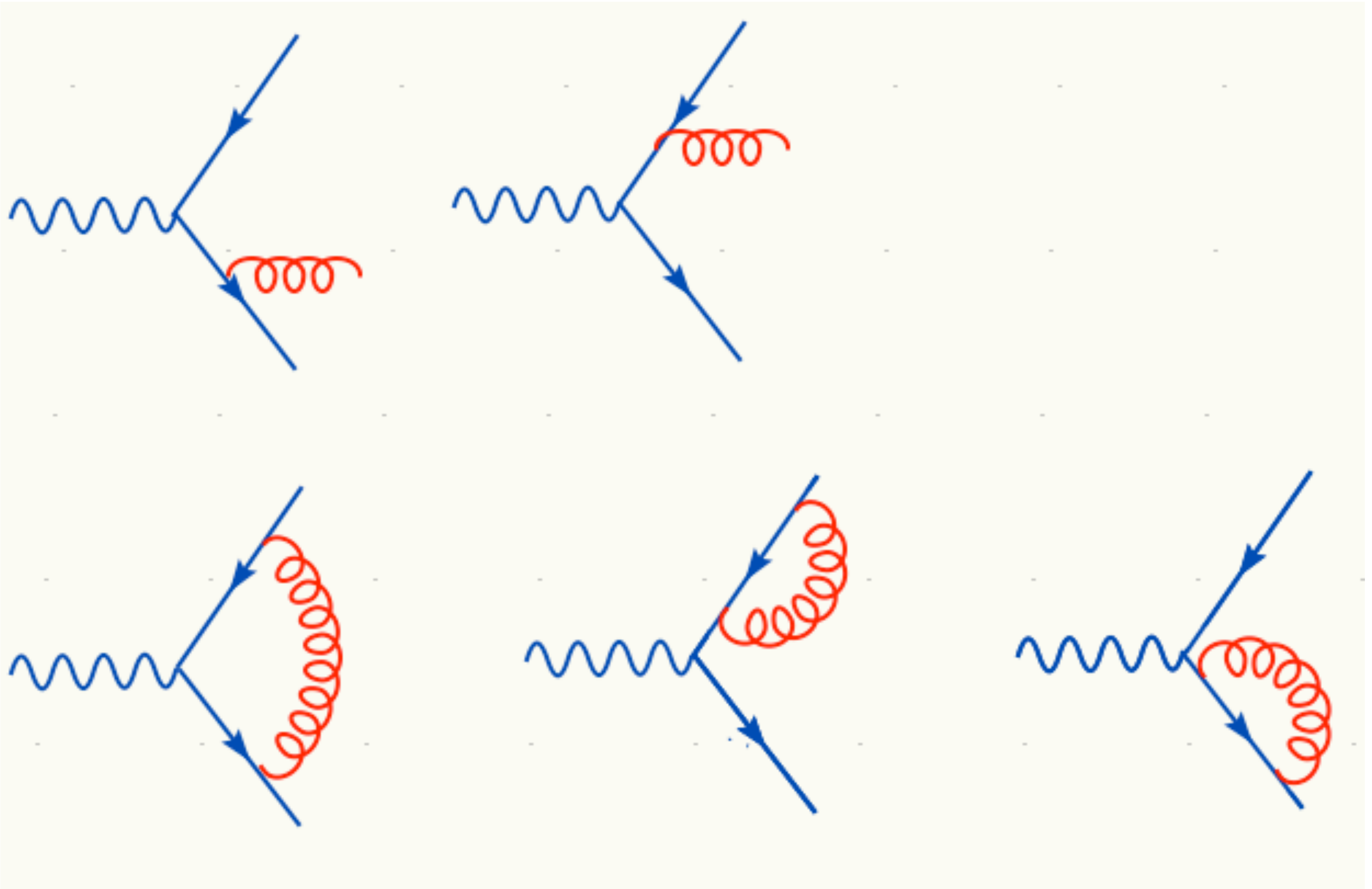
Higher order corrections

R-ratio@NLO



Real

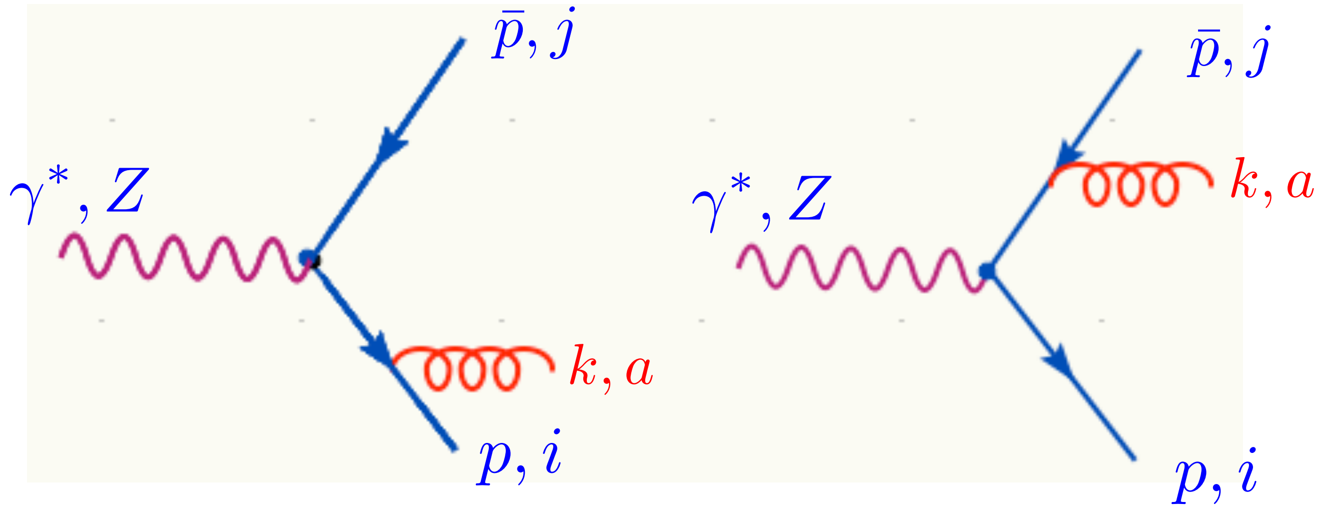
Virtual

$$\sigma_{NLO} = \sigma_{LO} + \int_R |M_{real}|^2 \, d\Phi_3 + \int_V 2\text{Re}(M_0 M^*_{vir}) \, d\Phi_2$$

QCD in the final state

R-ratio@NLO

Real corrections:



$$A = \bar{u}(p)\not{\epsilon}(-ig_s)\frac{-i}{\not{p}+\not{k}}\Gamma^\mu v(\bar{p})t^a + \bar{u}(p)\Gamma^\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s)\not{\epsilon}v(\bar{p})t^a$$

$$= -g_s\left[\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma^\mu v(\bar{p})}{2p\cdot k} - \frac{\bar{u}(p)\Gamma^\mu(\not{\bar{p}}+\not{k})\not{\epsilon}v(\bar{p})}{2\bar{p}\cdot k}\right]t^a$$

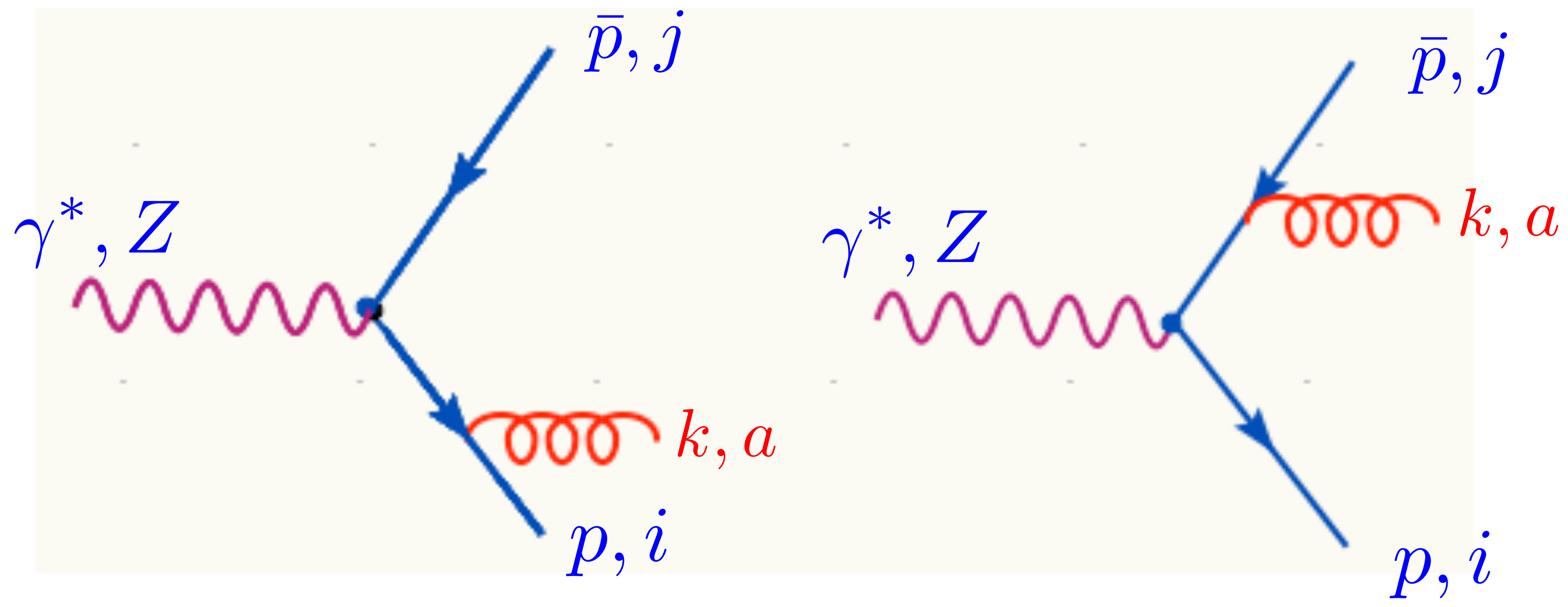
What are those denominators?

$$p \cdot k = p_0 k_0 (1 - \cos\theta)$$

What happens when the gluon is soft ($k_0 \to 0$) or collinear ($\theta \to 0$) to the quark?

QCD in the final state

R-ratio@NLO



What happens when the gluon is soft ($k_0 \to 0$) or collinear ($\theta \to 0$) to the quark?

$$A_{soft} = -g_s t^a \boxed{\left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right)} A_{Born}$$

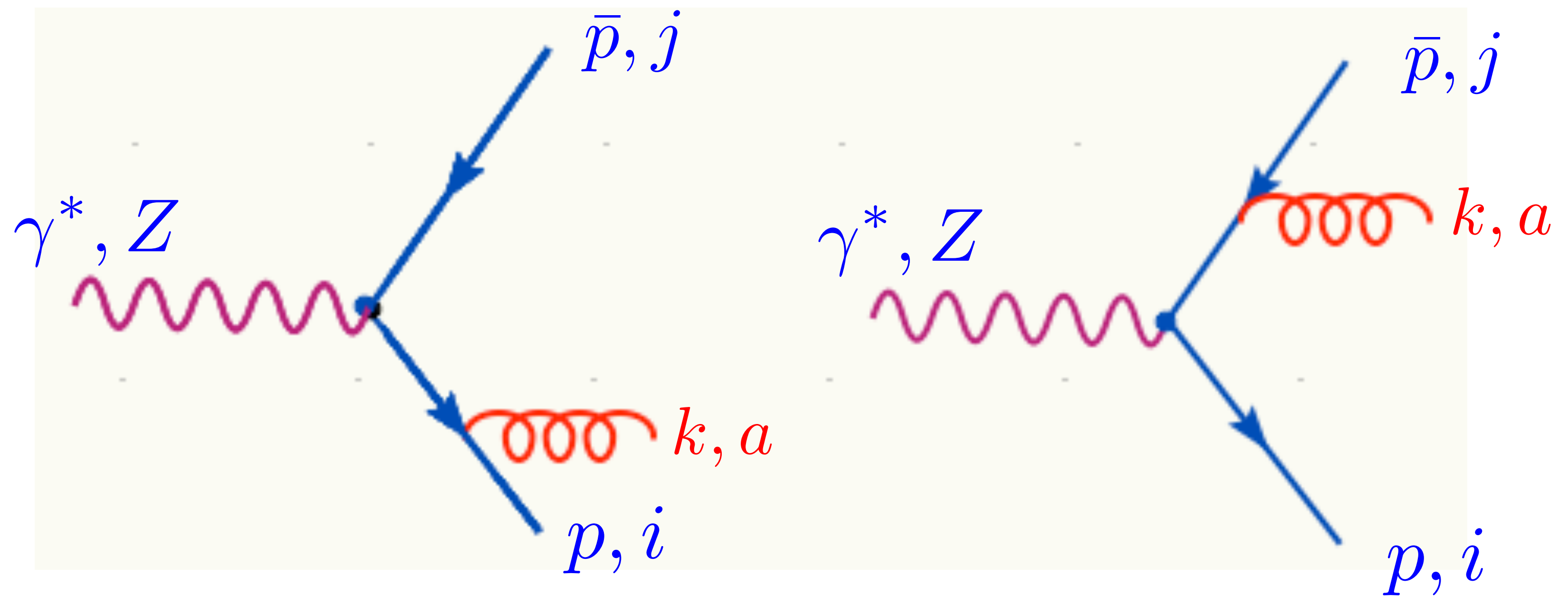
Very important property of QCD

Factorisation of long-wavelength (soft) emission from the short-distance (hard) scattering!

Soft emission factor is universal!

QCD in the final state

R-ratio@NLO



$$\sigma_{NLO} = \sigma_{LO} + \int_R |M_{real}|^2 \, d\Phi_3 + \int_V 2\mathrm{Re}(M_0 M^*_{vir}) \, d\Phi_2$$

$$A_{soft} = -g_s t^a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) A_{Born}$$

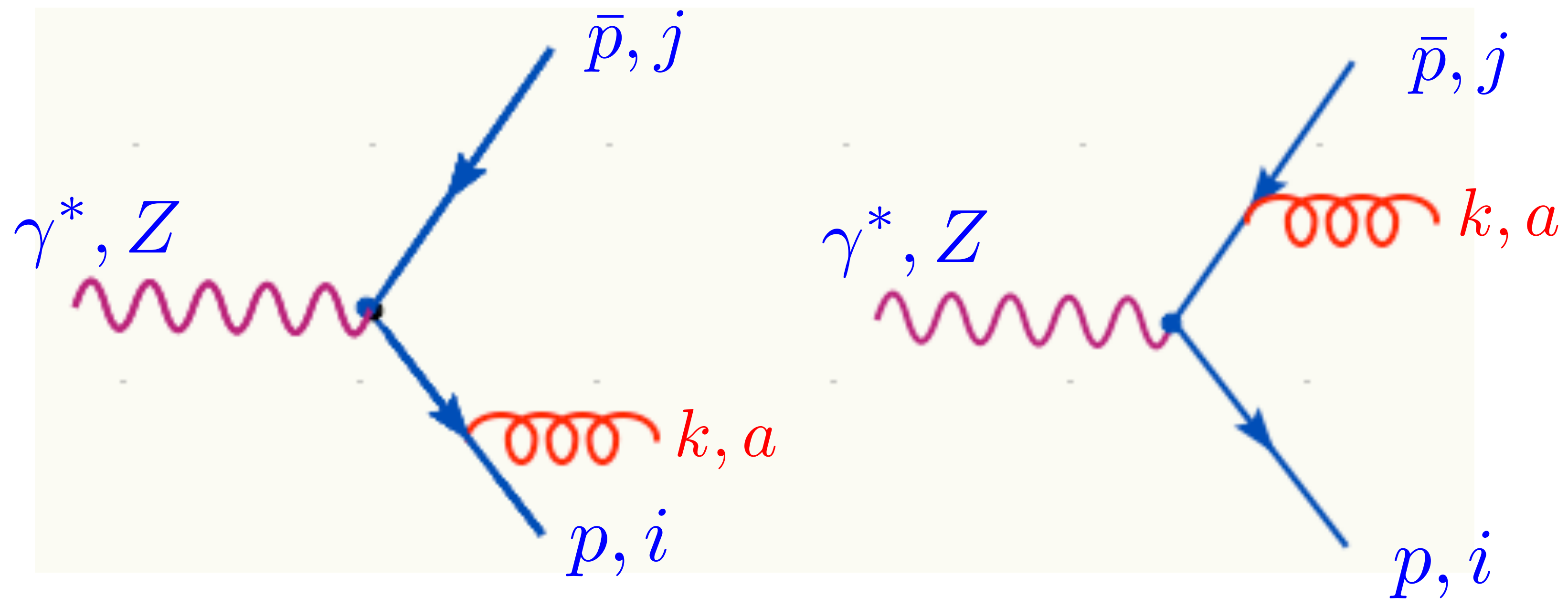
What does that mean for the NLO cross-section?

$$\sigma^{\text{REAL}}_{q\bar{q}g} = C_F g_s^2 \sigma^{\text{Born}}_{q\bar{q}} \int \frac{d^3k}{2k^0(2\pi)^3} 2 \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$$

$$= C_F \frac{\alpha_S}{2\pi} \sigma^{\text{Born}}_{q\bar{q}} \int d\cos\theta \, \frac{dk^0}{k^0} \frac{4}{(1 - \cos\theta)(1 + \cos\theta)}$$

QCD in the final state

R-ratio@NLO

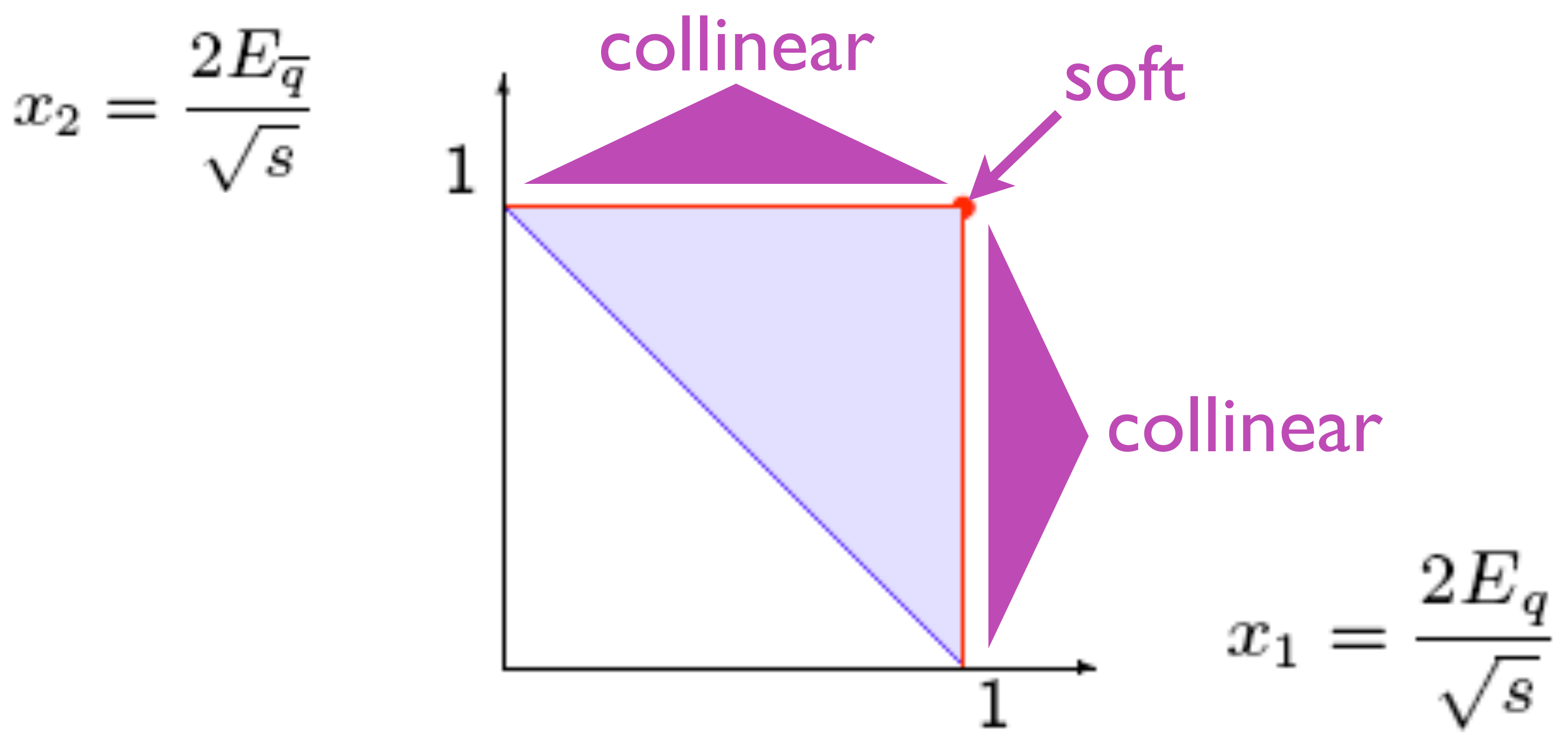


$$\sigma^{\text{REAL}}_{q\bar{q}g} = C_F g_s^2 \sigma^{\text{Born}}_{q\bar{q}} \int \frac{d^3k}{2k^0(2\pi)^3} 2 \frac{p\cdot\bar{p}}{(p\cdot k)(\bar{p}\cdot k)}$$

$$= C_F \frac{\alpha_S}{2\pi} \sigma^{\text{Born}}_{q\bar{q}} \int d\cos\theta \frac{dk^0}{k^0} \frac{4}{(1-\cos\theta)(1+\cos\theta)}$$

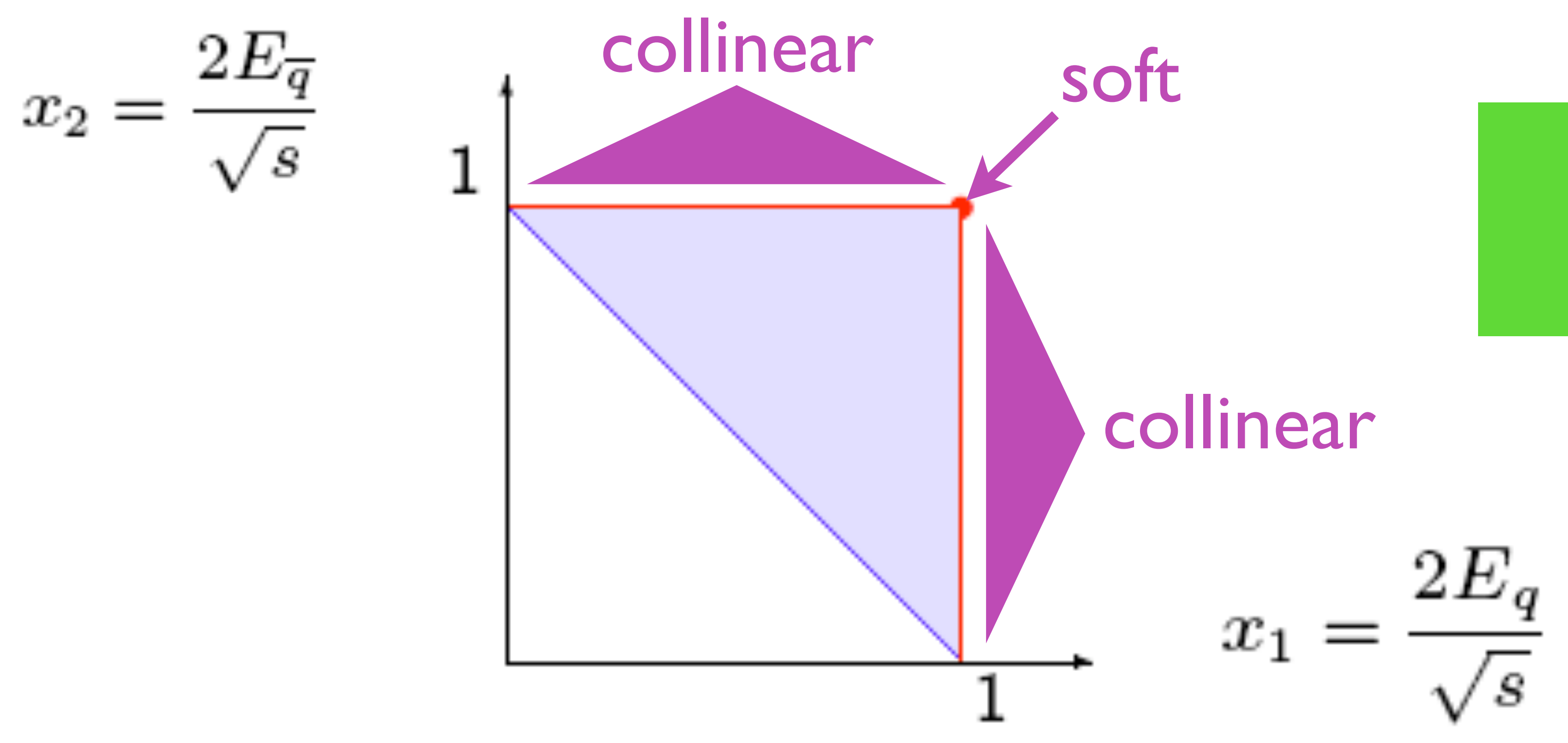
Soft divergence   Collinear divergence

$$x_1 = 1 - x_2 x_3 (1-\cos\theta_{23})/2$$
$$x_2 = 1 - x_1 x_3 (1-\cos\theta_{13})/2$$
$$x_1 + x_2 + x_3 = 2$$
$$0 \le x_1, x_2 \le 1, \quad \text{and} \quad x_1 + x_2 \ge 1$$



Divergences

$$x_1 = 1 - x_2 x_3 (1 - \cos\theta_{23})/2$$
$$x_2 = 1 - x_1 x_3 (1 - \cos\theta_{13})/2$$
$$x_1 + x_2 + x_3 = 2$$
$$0 \leq x_1, x_2 \leq 1, \quad \text{and} \quad x_1 + x_2 \geq 1$$



Why is $x_1 = x_2 = 1$ the soft case?

$$\sigma^{q\bar{q}g} = \frac{4\pi^2}{3s} f_q^2 C_F \frac{\alpha_s}{2\pi} \int\int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Integral diverges if $x_1 \to 1$ or $x_2 \to 1$ or $x_1, x_2 \to 1$!

What happens now?

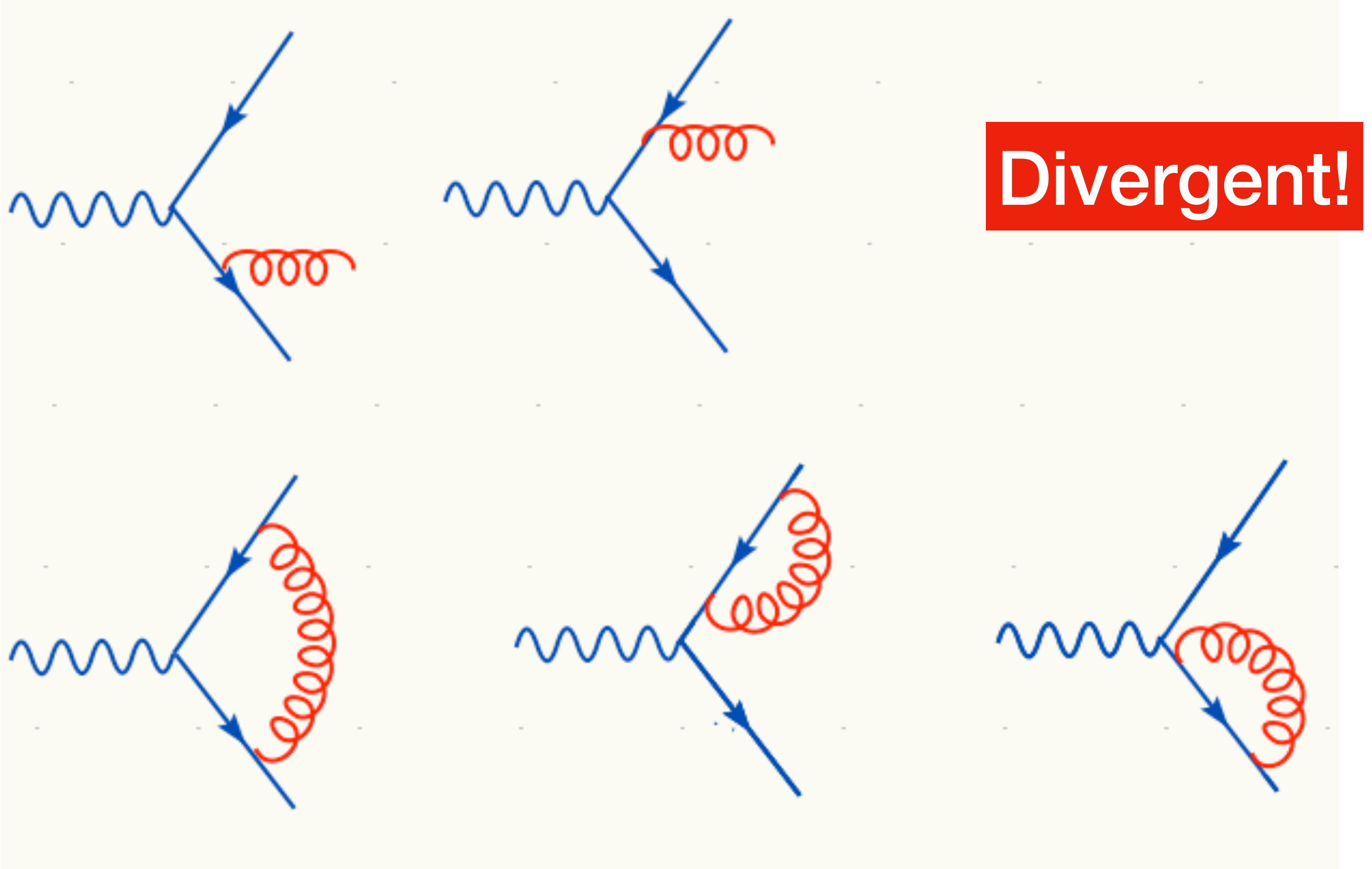
IR singularities

IR singularities arise when a parton is too soft or if two partons are collinear

- Infrared divergences arise from interactions that happen a long time after the creation of the quark/antiquark pair.
- When distances become comparable to the hadron size of ~1 Fermi, quasi-free partons of the perturbative calculation are confined/hadronized non-perturbatively.

How do we proceed with our calculation?

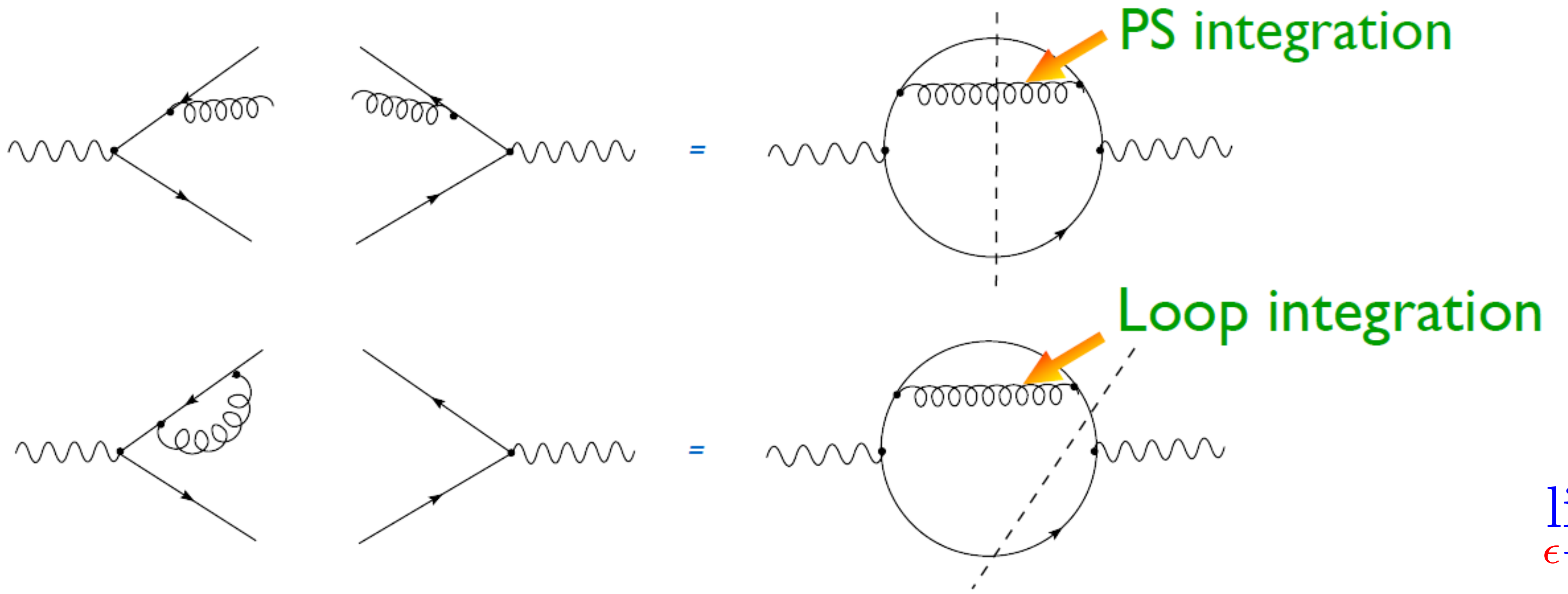
Cancellation of divergences



Real

Virtual

In practice: regularise both divergences (with either dimensional regularisation or mass regulator)



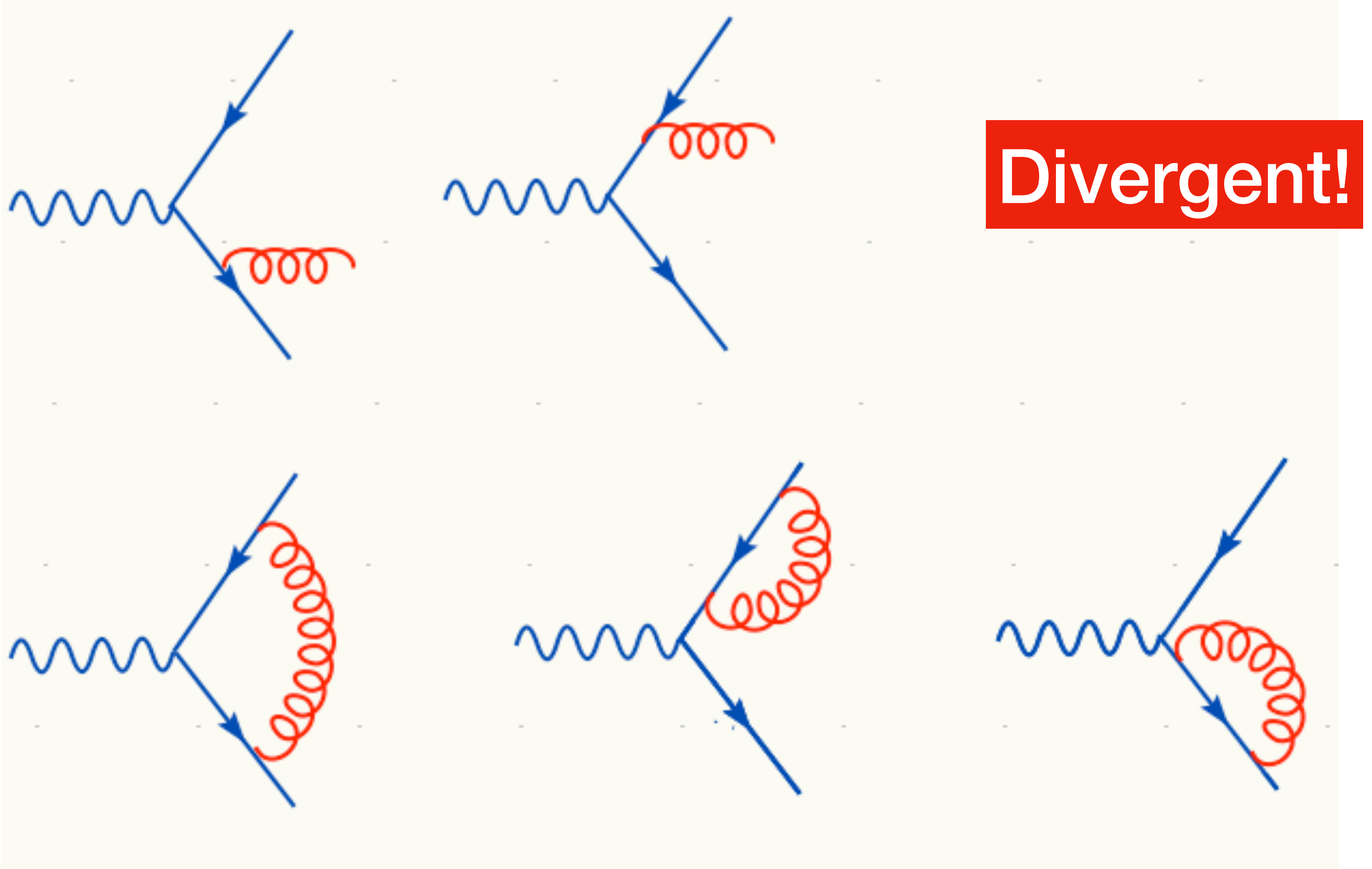
$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right)$$

$$\sigma^{\text{VIRT}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 \right)$$

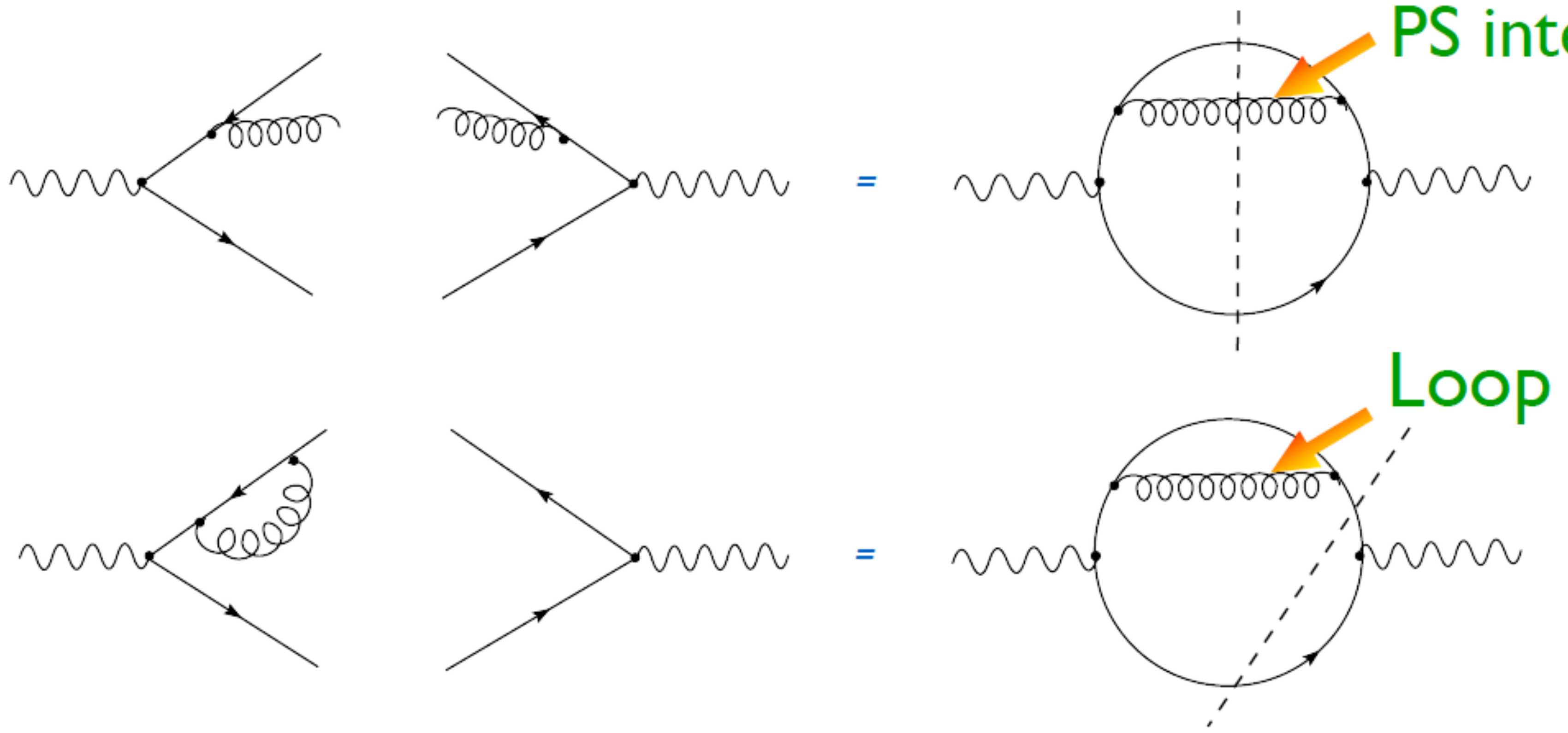
$$\lim_{\epsilon \to 0} (\sigma^{\text{REAL}} + \sigma^{\text{VIRT}}) = C_F \frac{3}{4} \frac{\alpha_S}{\pi} \sigma^{\text{Born}}$$

$$R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi} \right)$$

Cancellation of divergences



In practice: regularise both divergences (with either dimensional regularisation or mass regulator)



$$\sigma^{\mathrm{REAL}} = \sigma^{\mathrm{Born}} C_F \frac{\alpha_S}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2\right)$$

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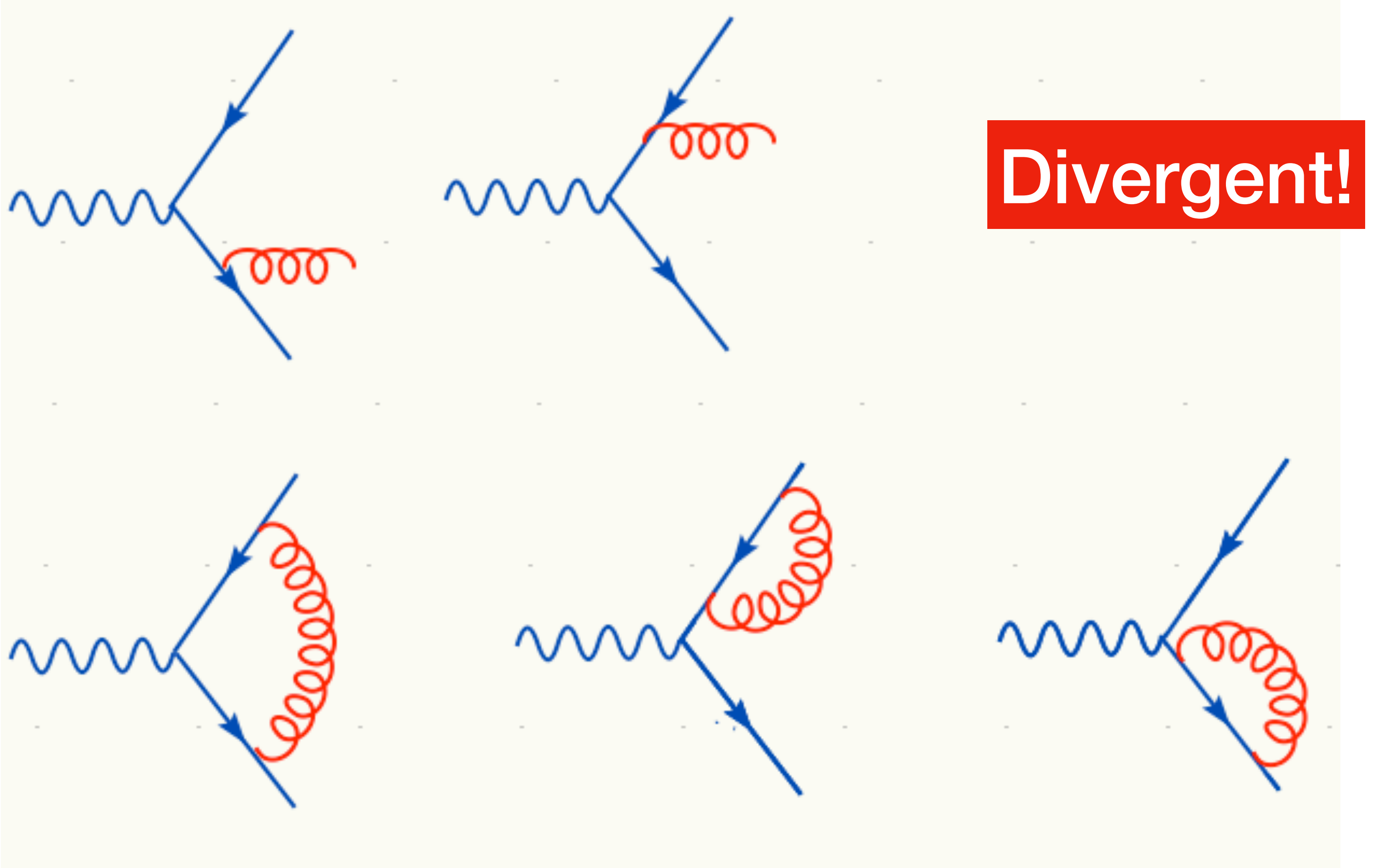
Cancellation of divergences

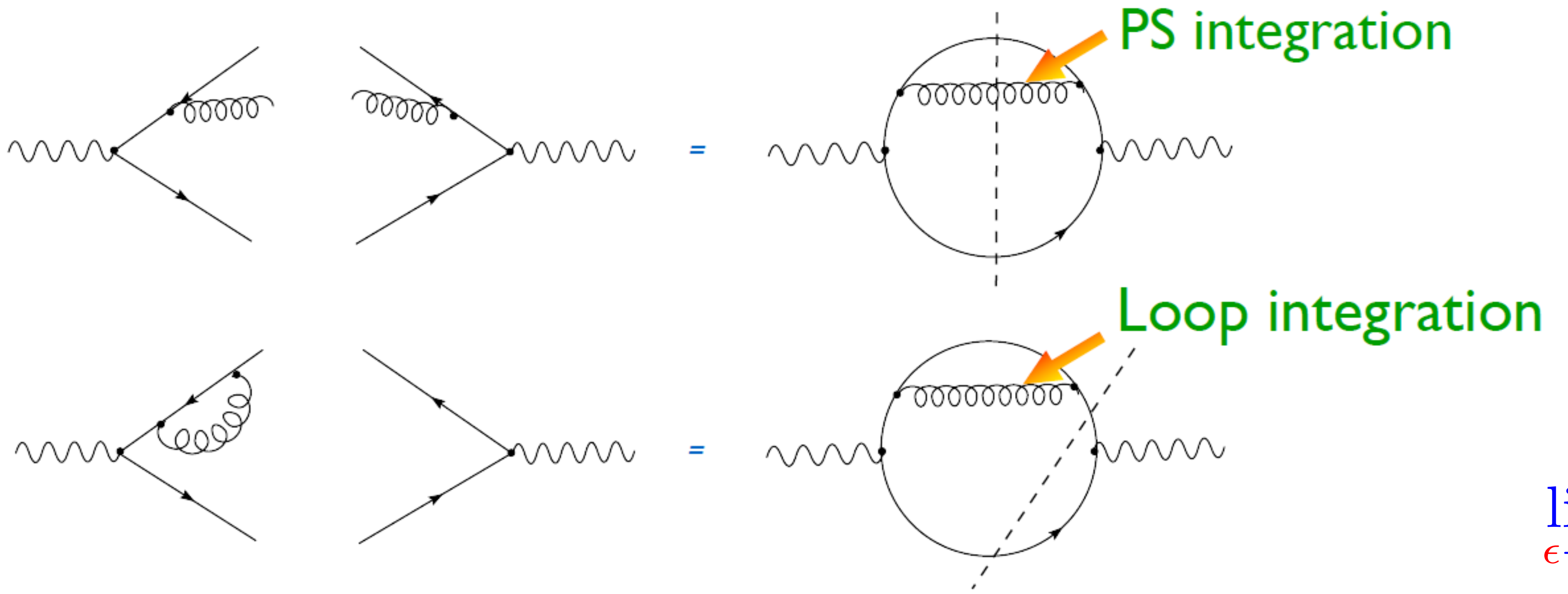


Real

Virtual

Also divergent!

In practice: regularise both divergences (with either dimensional regularisation or mass regulator)



$$\sigma^{\text{REAL}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2\right)$$

$$\sigma^{\text{VIRT}} = \sigma^{\text{Born}} C_F \frac{\alpha_S}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right)$$

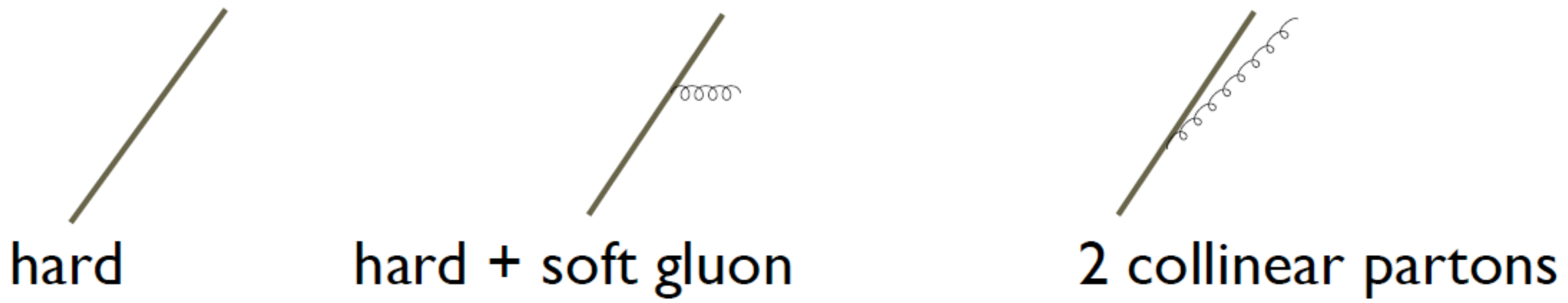
$$\lim_{\epsilon\to 0}(\sigma^{\text{REAL}} + \sigma^{\text{VIRT}}) = C_F \frac{3}{4}\frac{\alpha_S}{\pi}\sigma^{\text{Born}}$$

$$R_1 = R_0\left(1 + \frac{\alpha_S}{\pi}\right)$$ **Finite!**

KLN Theorem

Why does this work?

Kinoshita-Lee-Nauenberg theorem: Infrared singularities in a massless theory cancel out after summing over degenerate (initial and final) states



Physically a hard parton can not be distinguished from a hard parton plus a soft gluon or from two collinear partons with the same energy. They are degenerate states. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual)

Hence, one needs to add all degenerate states to get a physically sound observable

Infrared safety

How can we make sure IR divergences cancel?

We need to pick observables which are insensitive to soft and collinear radiation. These observables are determined by hard, short-distance physics, with long distance effects suppressed by an inverse power of a large momentum scale.

Schematically for an IR safe observable:

$$\mathcal{O}_{n+1}(k_1, k_2, \ldots, k_i, k_j, \ldots k_n) \to \mathcal{O}_n(k_1, k_2, \ldots k_i + k_j, \ldots k_n)$$

whenever one of the k_i/k_j becomes soft or k_i and k_j are collinear

Which observables are infrared safe?

- energy of the hardest particle in the event
- multiplicity of gluons
- momentum flow into a cone in rapidity and angle
- jet cross-sections

See exercises!

Which observables are infrared safe?

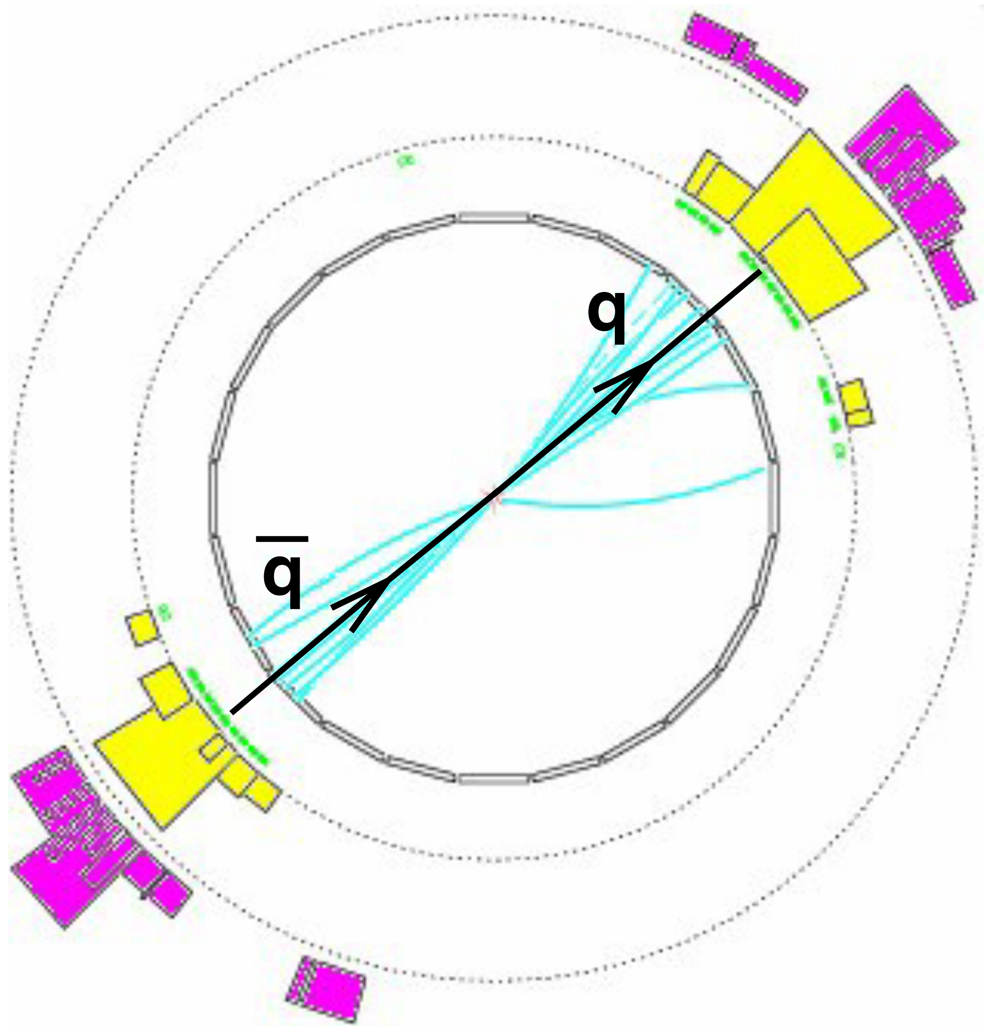
- energy of the hardest particle in the event — NO
- multiplicity of gluons — NO
- momentum flow into a cone in rapidity and angle — YES
- jet cross-sections — DEPENDS

See exercises!

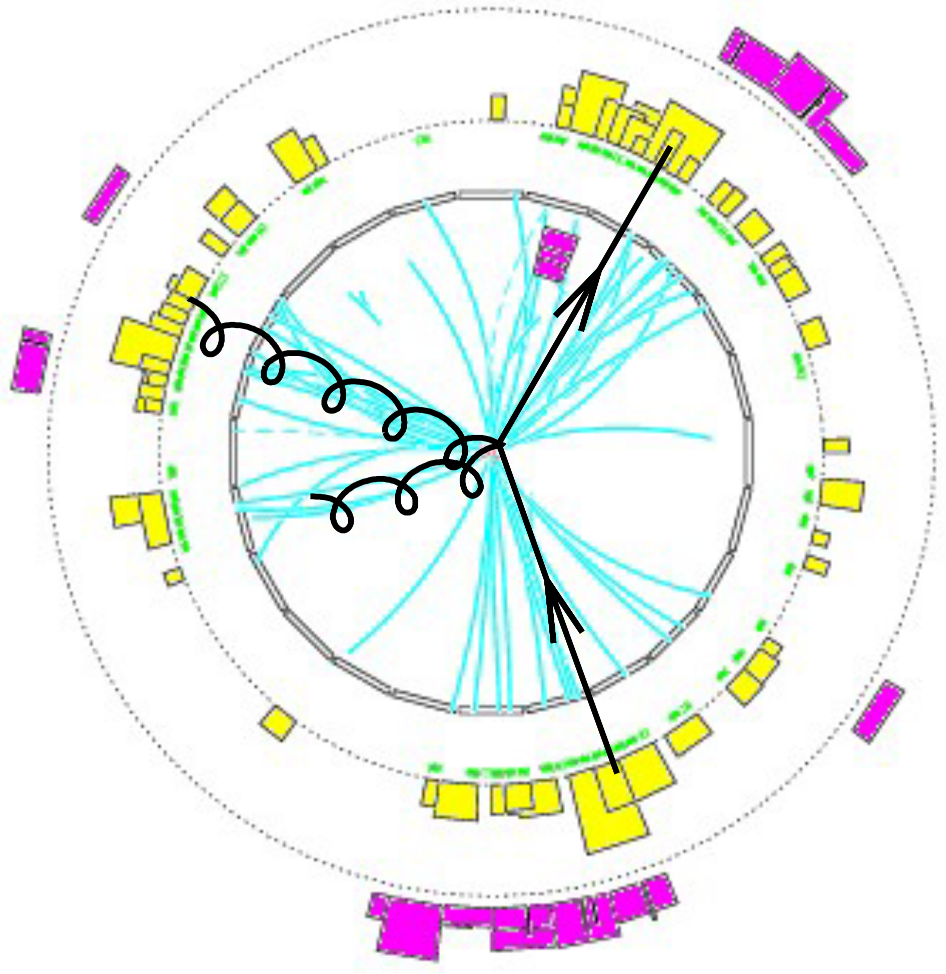
Event shapes

Event shapes: describe the shape of the event, but are largely insensitive to soft and collinear branching

- widely used to measure αs
- measure colour factors
- test QCD
- learn about non-perturbative physics



pencil-like



spherical

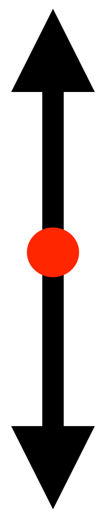
Thrust

Event-shape example

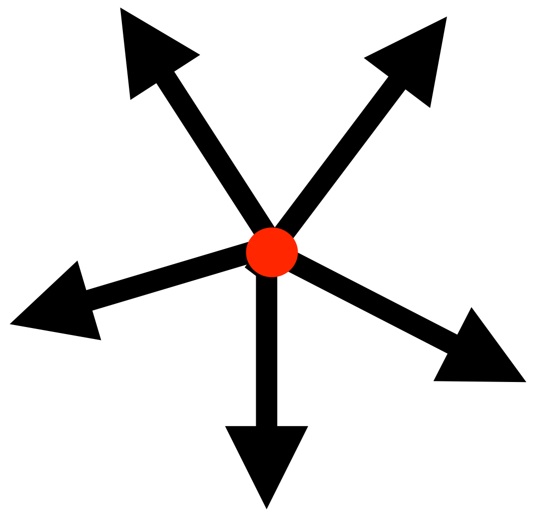
$$T = \max_{\vec{\hat{n}}} \frac{\sum_i |\vec{p_i} \cdot \vec{\hat{n}}|}{\sum_i |\vec{p_i}|}$$

Sum over all final state particles

Find axis n which maximises this projection



$T = 1$



$T = 1/2$

Noteby: if one of the partons emits a soft or collinear gluon the value of thrust is not changing. **IRC safe**

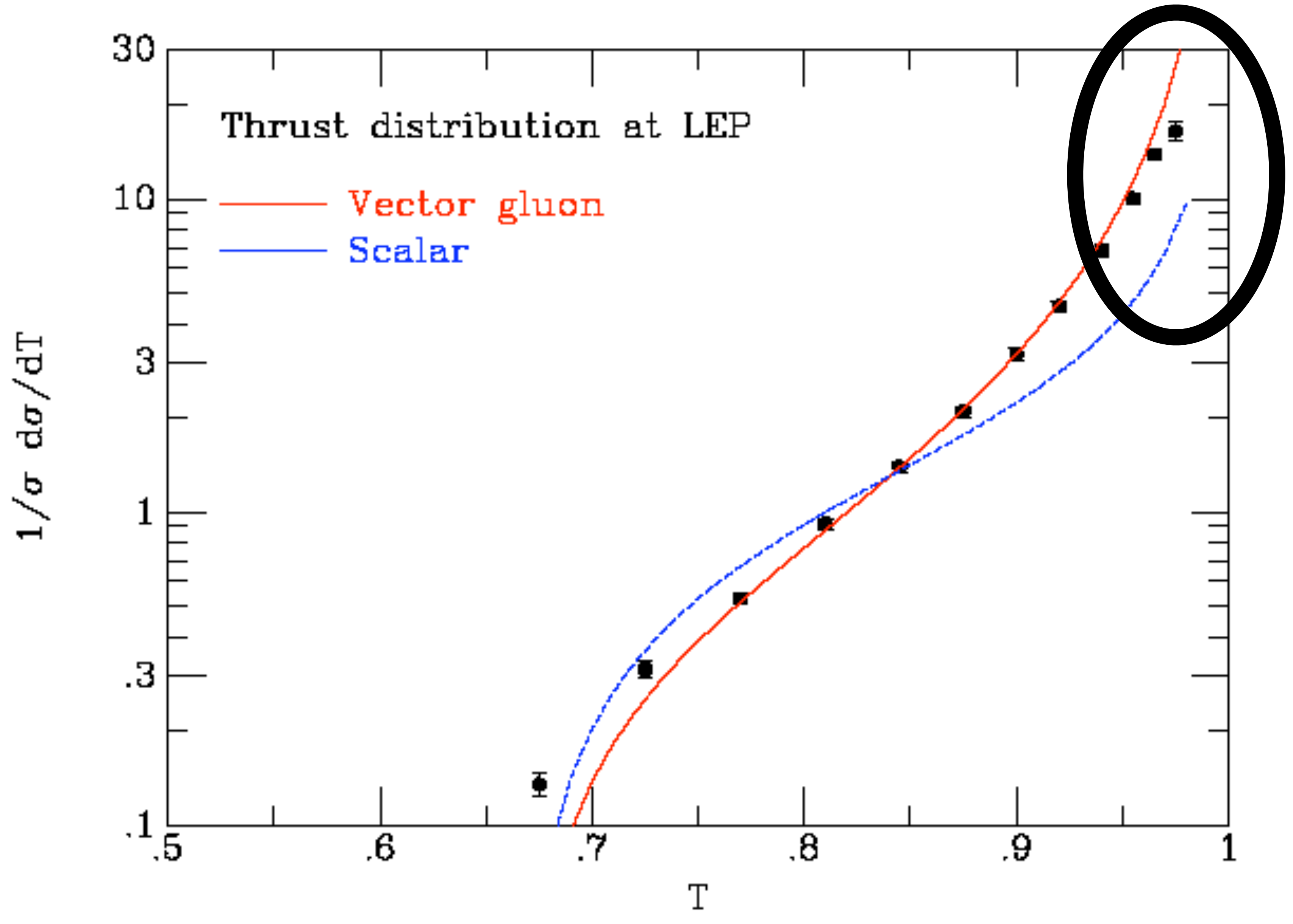
What happens in an $e^+e^- \to q\bar{q}g$ event?

Thrust

What happens in an $e^+e^- \to q\bar{q}g$ event?

$$T = \max_{\vec{\hat{n}}} \frac{\sum_i |\vec{p_i} \cdot \vec{\hat{n}}|}{\sum_i |\vec{p_i}|}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = C_F \frac{\alpha_S}{2\pi}\left[\frac{2(3T^2-3T+2)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{1-T}\right]$$



Divergent for T=1 Why?

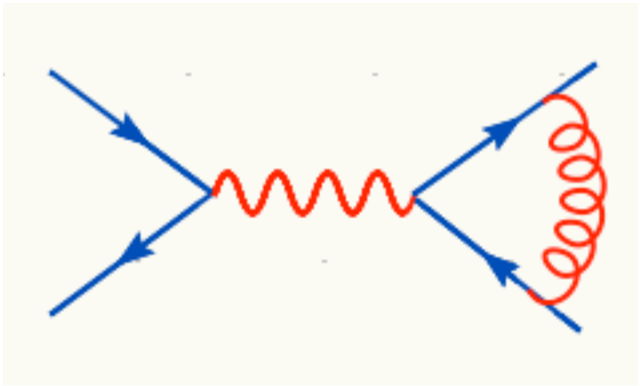
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}T} \xrightarrow{T\to 1} -C_F\frac{\alpha_S}{2\pi}\left[\frac{4}{(1-T)}\ln(1-T) + \frac{3}{1-T}\right]$$

Large higher order terms of the form $\alpha_S^N \frac{\mathrm{Log}^{2N-1}(1-T)}{1-T}$ need to be resummed.

Use either analytic resummation or the parton shower! See later!

Asymptotic freedom

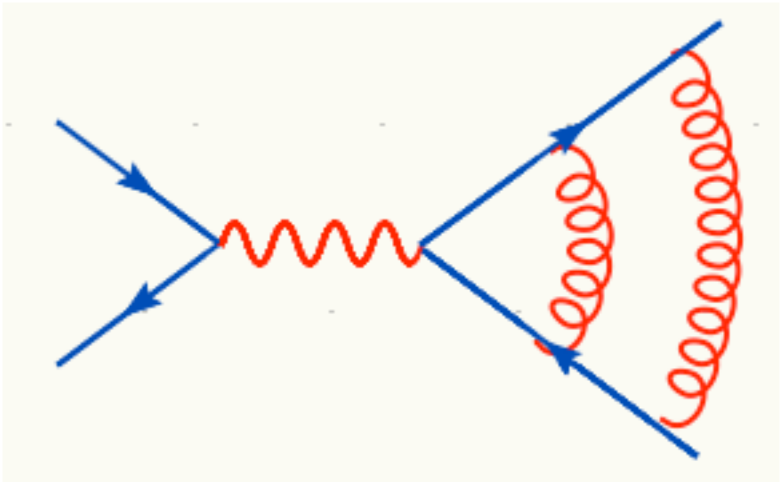
How about the UV?



$$R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi}\right)$$

No divergences!

What happens at higher orders?



$$R^{(2)} = R^{(0)} \left(1 + \frac{\alpha_S}{\pi} + \left(\frac{\alpha_S}{\pi}\right)^2 \left(c + \pi b_0 \log\left(\frac{M_{\text{UV}}^2}{Q^2}\right)\right)\right) \qquad b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

UV divergences don't cancel! We need renormalisation!

Renormalising the bare coupling we have:

$$\alpha_S(\mu) = \alpha_S^{\text{bare}} + b_0 \log\left(\frac{M_{\text{UV}}^2}{\mu^2}\right)(\alpha_S^{\text{bare}})^2$$

$$R_2^{\text{ren}}(\alpha_S(\mu), \frac{\mu^2}{Q^2}) = R_0 \left(1 + \frac{\alpha_S(\mu)}{\pi} + \left[c + \pi b_0 \log\frac{\mu^2}{Q^2}\right]\left(\frac{\alpha_S(\mu)}{\pi}\right)^2\right)$$

Asymptotic freedom

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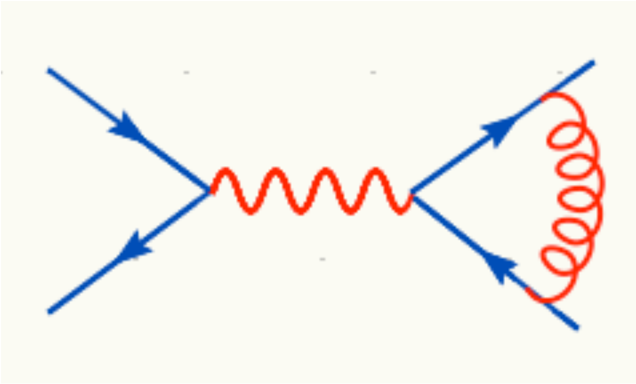

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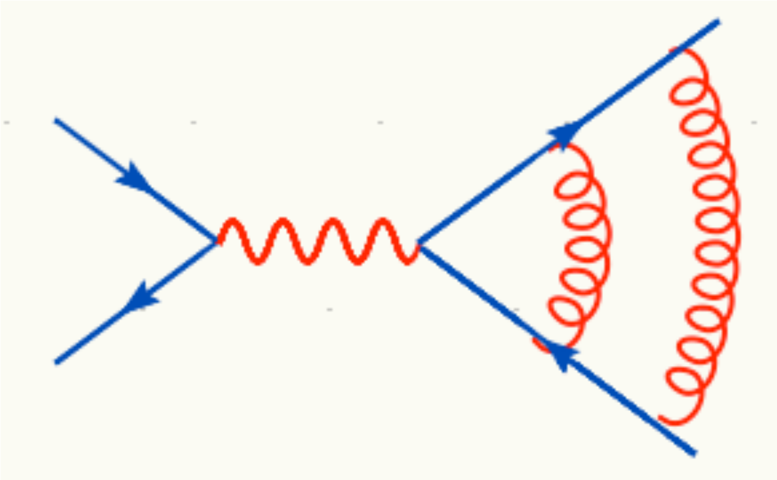

$$R^{(2)} = R^{(0)}\left(1+\frac{\alpha_S}{\pi}+\left(\frac{\alpha_S}{\pi}\right)^2\left(c+\pi b_0\log\left(\frac{M_{\rm UV}^2}{Q^2}\right)\right)\right) \qquad b_0=\frac{11N_c-4n_fT_R}{12\pi}$$

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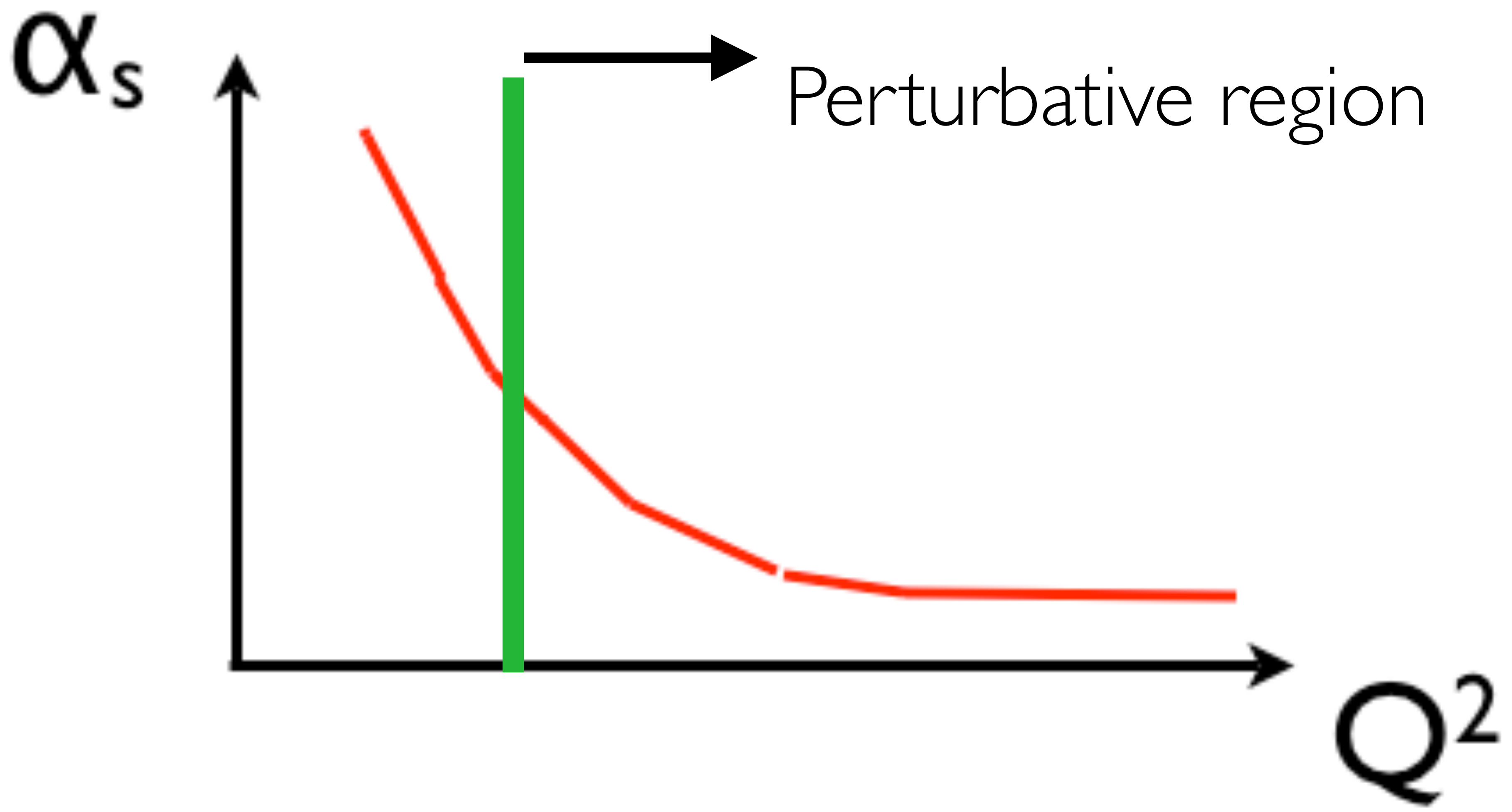
$$\alpha_S(\mu)=\alpha_S^{\rm bare}+b_0\log\left(\frac{M_{\rm UV}^2}{\mu^2}\right)(\alpha_S^{\rm bare})^2$$

$$R_2^{\rm ren}\left(\alpha_S(\mu),\frac{\mu^2}{Q^2}\right)=R_0\left(1+\frac{\alpha_S(\mu)}{\pi}+\left[c+\pi b_0\log\frac{\mu^2}{Q^2}\right]\left(\frac{\alpha_S(\mu)}{\pi}\right)^2\right)$$

Finite but scale dependent!

Asymptotic freedom



$$\mu^2 \frac{d\alpha}{d\mu^2} = \beta(\alpha) = -(b_0\alpha^2 + b_1\alpha^3 + b_2\alpha^4 + \cdots)$$

QCD $b_0 = \frac{11N_c - 2n_f}{12\pi} \quad \Rightarrow \quad \beta(\alpha_S) < 0$

QED $b_0 = -\frac{n_f}{3\pi} \quad \Rightarrow \quad \beta(\alpha_{\rm EM}) > 0$

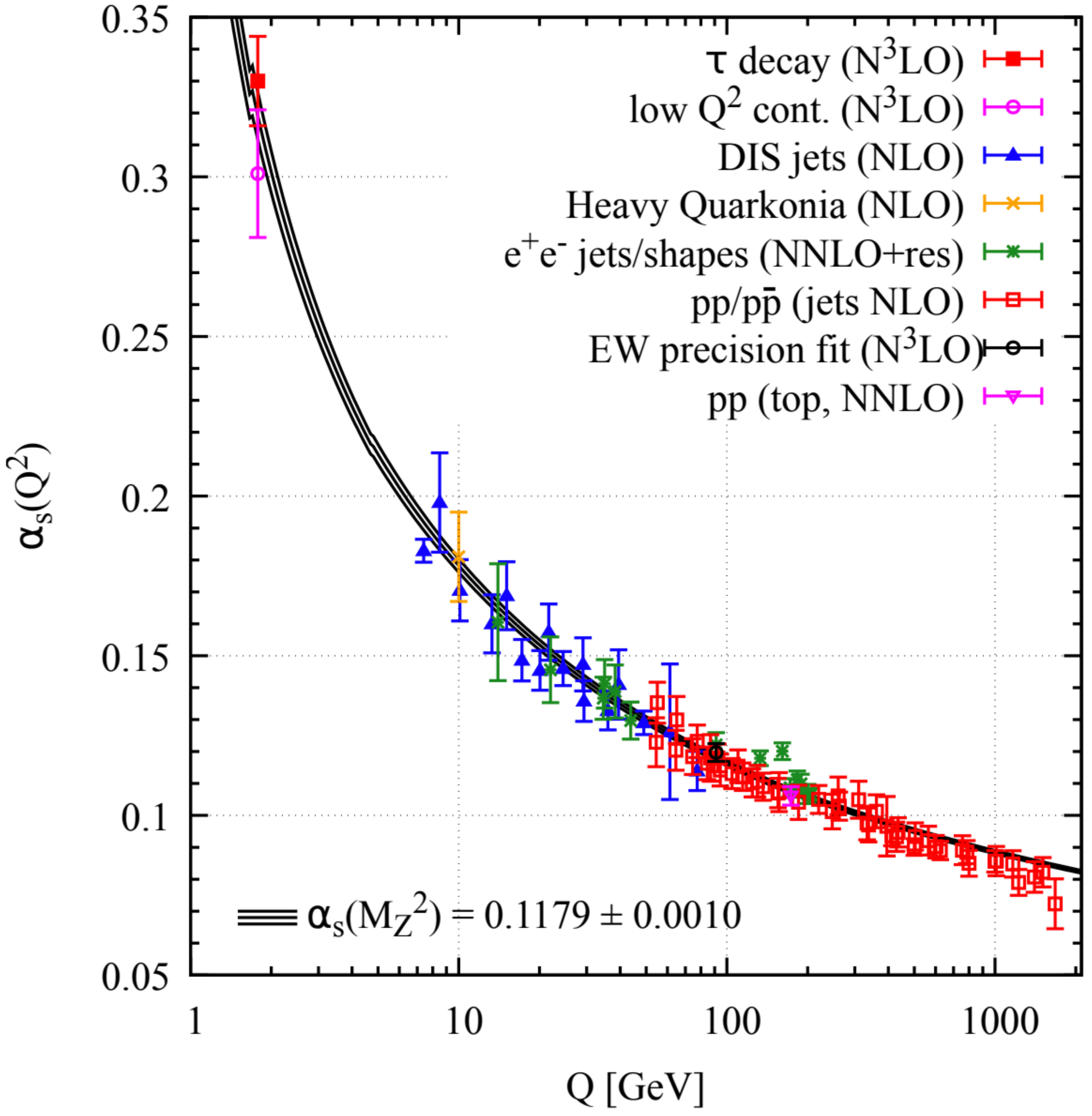
1-loop

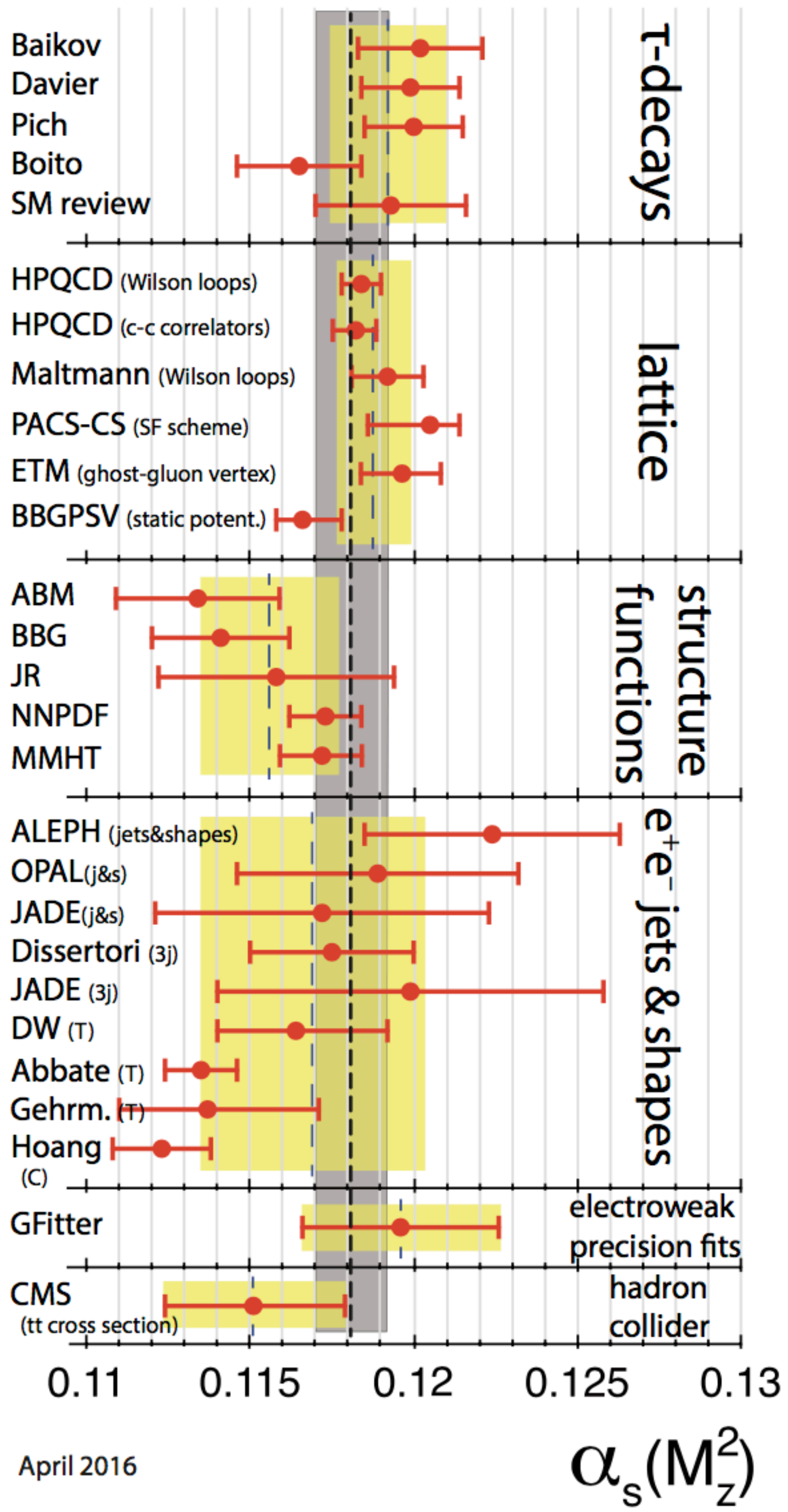
$$\beta(\alpha_S) \equiv \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} = -b_0 \alpha_S^2 \quad \Rightarrow \quad \alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}$$

2-loop

$$\alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \left[1 - \frac{b_1}{b_0^2} \frac{\log\log \mu^2/\Lambda^2}{\log \mu^2/\Lambda^2}\right]$$

Running of α_s





Many measurements at different scales all leading to very consistent results once evolved to the same reference scale, M_Z.

Going back to the Master formula

$$\sum_{a,b} \int dx_1 dx_2\, d\Phi_{PS}\, f_a(x_1) f_b(x)\, \hat{\sigma}(\hat{s})$$

$$\downarrow$$

$$\sum_{a,b} \int dx_1 dx_2\, d\Phi_{PS}\, f_a(x_1) f_b(x)\, \hat{\sigma}(\hat{s}, \mu_R)$$

Going back to the Master formula

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$$

↓

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s}, \mu_R)$$

↓

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$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s}, \mu_R)$$

$$\downarrow$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Going back to the Master formula

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$$

$$\downarrow$$

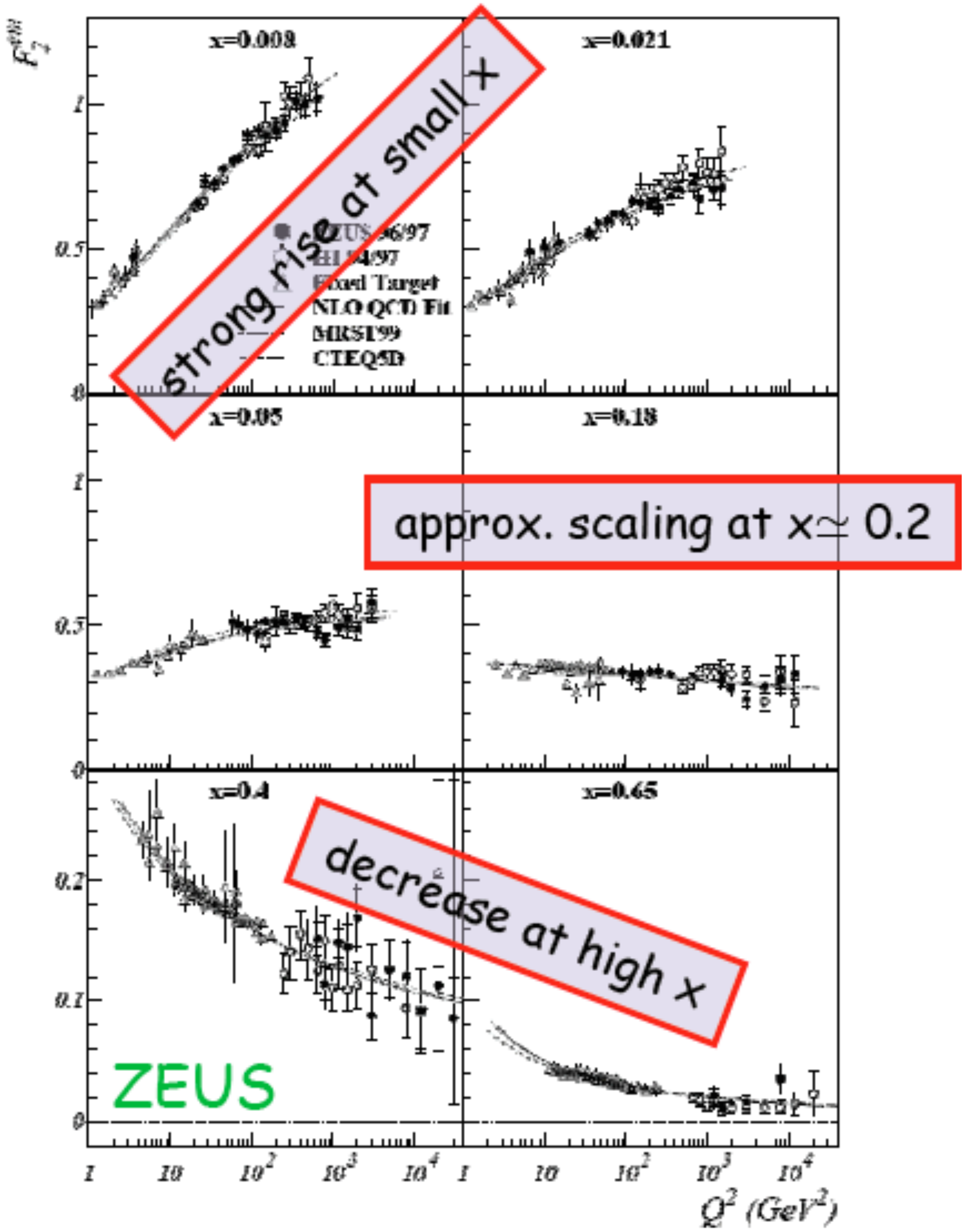
$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s}, \mu_R)$$

$$\downarrow$$???

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

QCD improved parton model

The parton model predicts scaling. Experiment shows:



Scaling violation

QCD improved parton model

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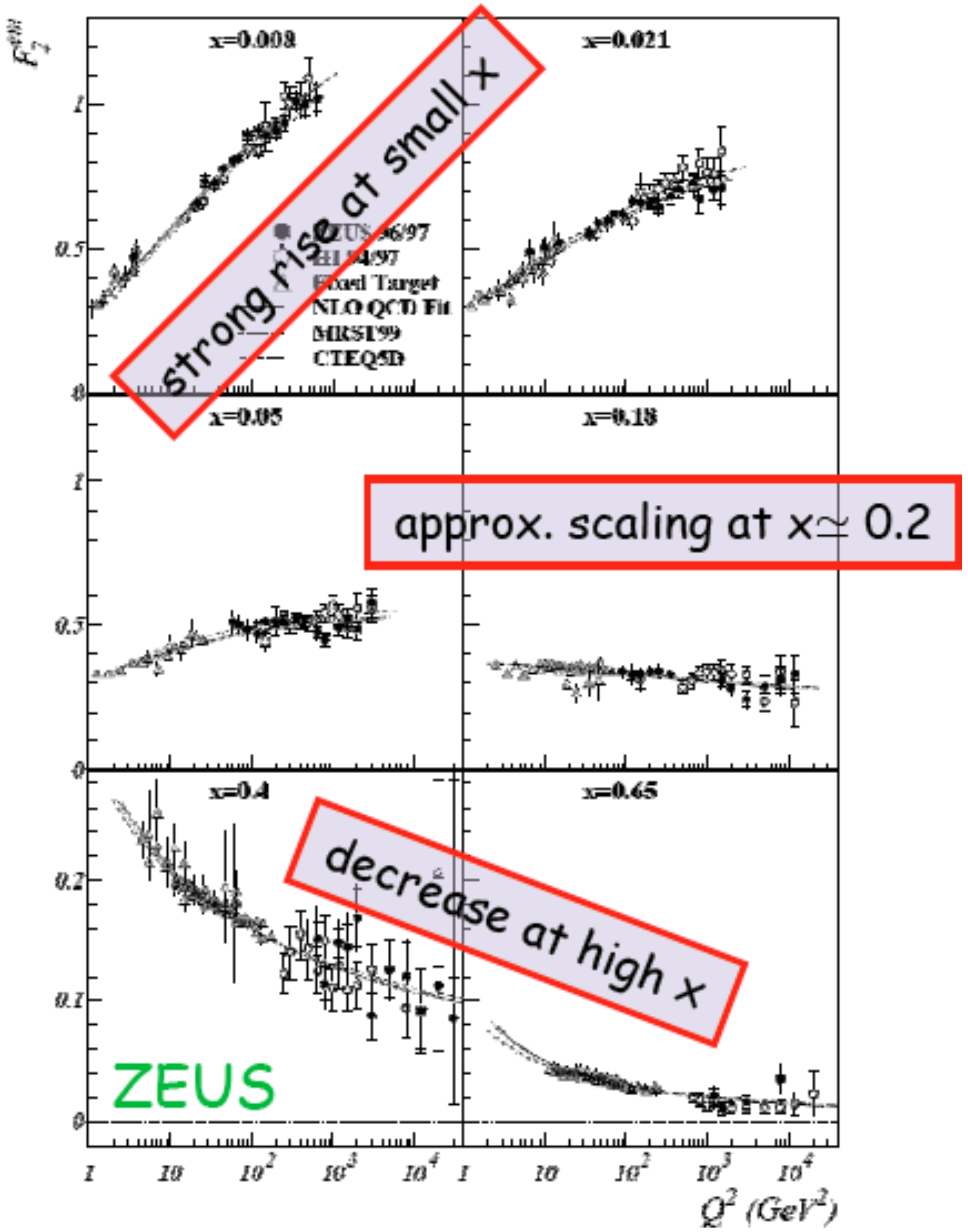
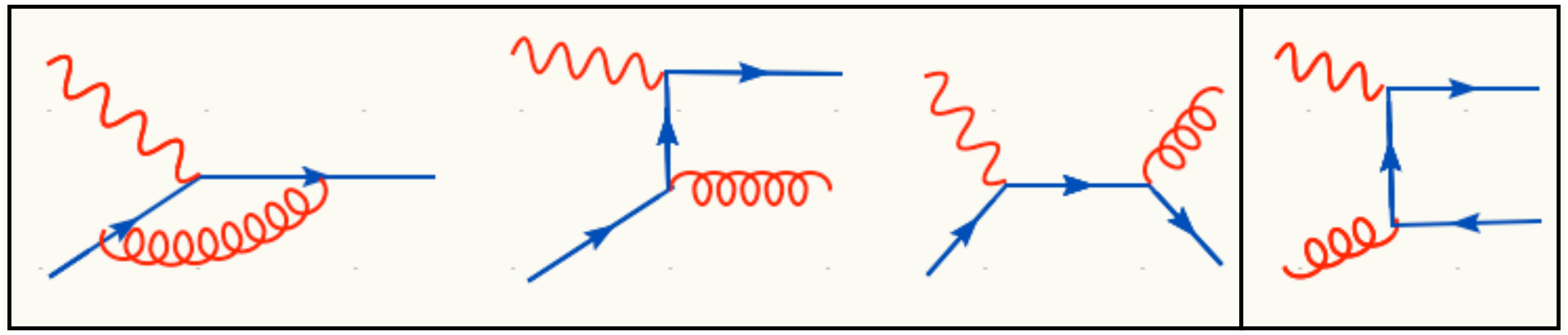


Scaling violation

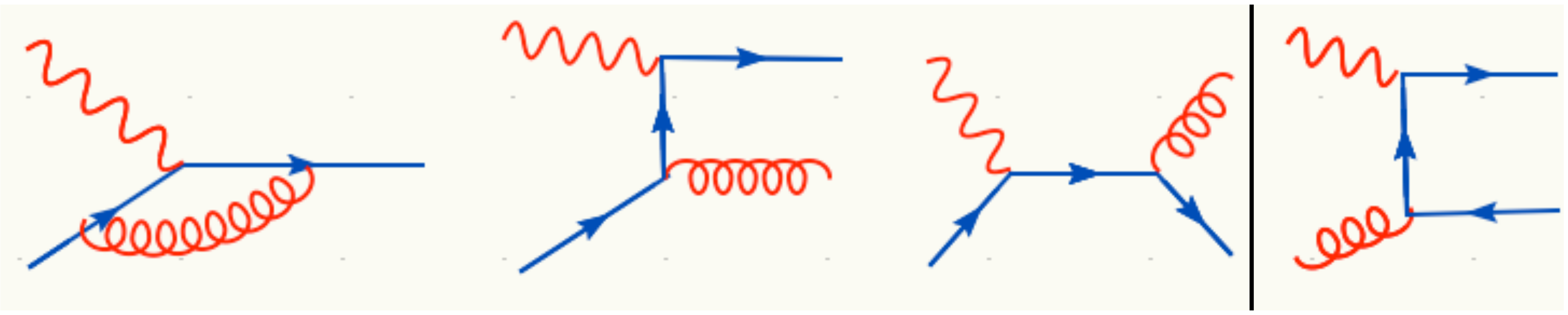
What are we missing?



α_S corrections to the LO process

photon-gluon fusion

QCD improved parton model

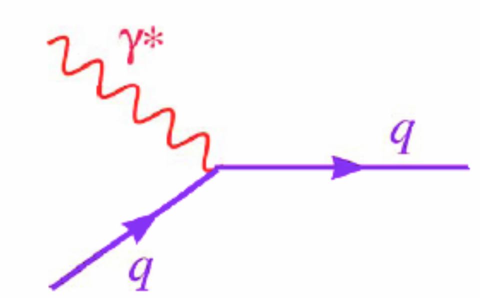


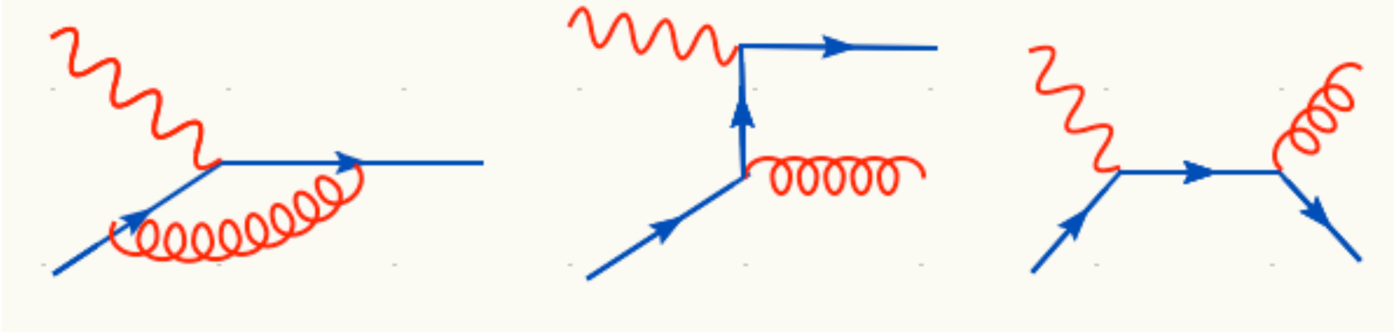
What do we expect?

Given the computation of R at NLO, we expect IR divergences

We need to regulate these, and hope that they cancel!

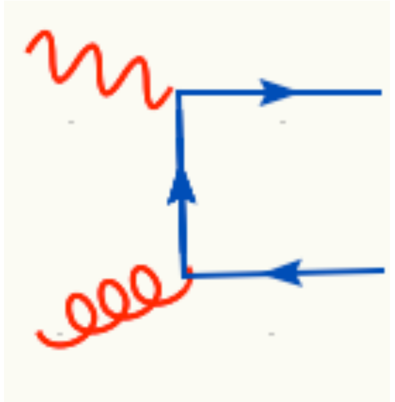
$$\frac{d^2\hat{\sigma}}{dx dQ^2}|_{F_2} \equiv \hat{F}_2^q$$



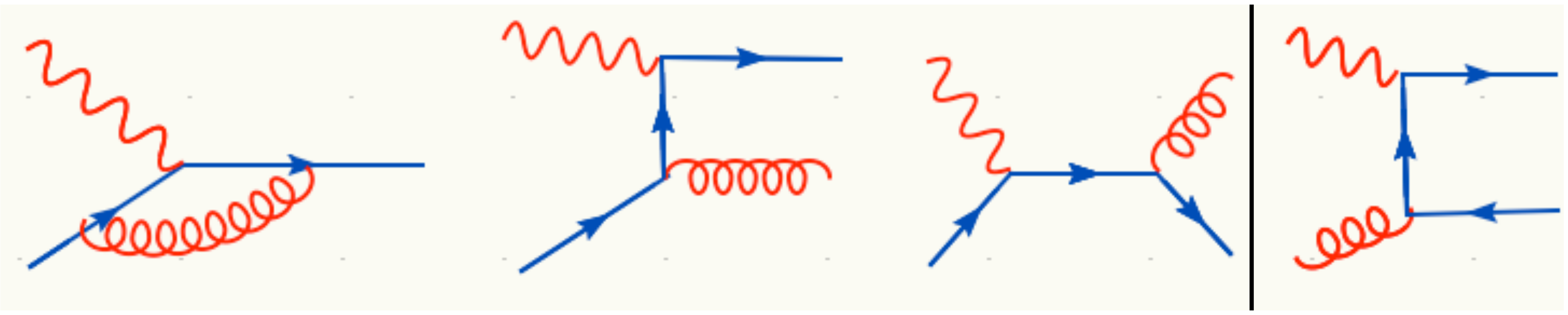


Soft and UV divergences cancel but a collinear divergence arises:

$$\hat{F}_2^q = e_q^2 x[\delta(1-x) + \frac{\alpha_s}{4\pi} P_{qq} \log \frac{Q^2}{m_g^2} + C_2^q(x)] \qquad \hat{F}_2^g = e_q^2 x[0 + \frac{\alpha_s}{4\pi} P_{qg} \log \frac{Q^2}{m_g^2} + C_2^g(x)]$$



QCD improved parton model



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IR cut-off

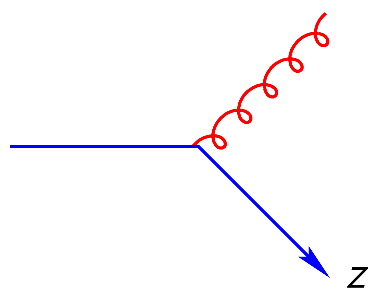
What are functions P_{qq} and P_{qg}?

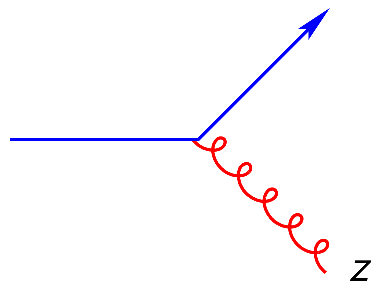
Splitting functions $P_{ij}(x)$: they give the probability of parton j splitting into parton i which carries momentum fraction x of the original parton

Altarelli-Parisi Splitting functions

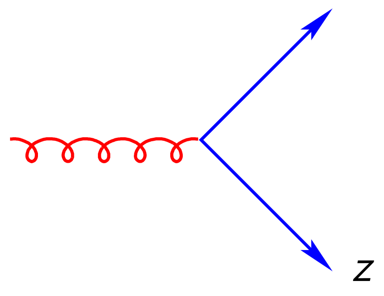
Branching has a universal form given by the Altarelli-Parisi splitting functions

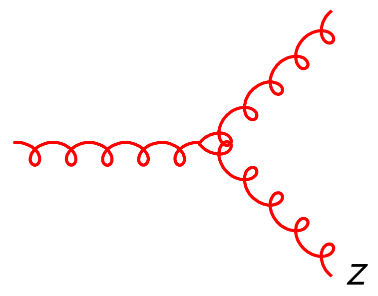
$$P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{1-z}\right], \qquad P_{q\to gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right].$$





$$P_{g\to qq}(z) = T_R\left[z^2+(1-z)^2\right], \qquad P_{g\to gg}(z) = C_A\left[z(1-z)+\frac{z}{1-z}+\frac{1-z}{z}\right]$$





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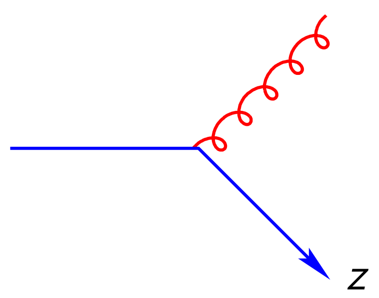

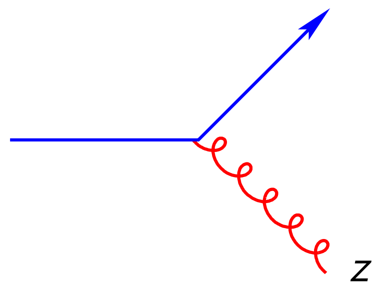

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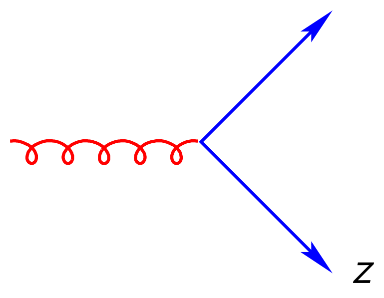

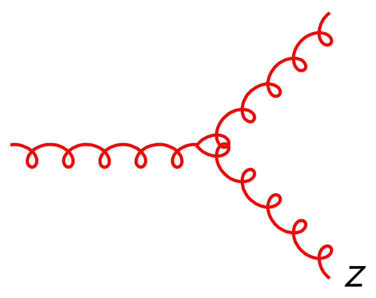

These functions are universal for each type of splitting

What does this collinear divergence mean?

Residual long-distance physics, not disappearing once real and virtual corrections are added. These appear along with the universal splitting functions.

Can a physical observable be divergent?

No, as the physical observable is the hadronic structure function:

$$F_2^q(x,Q^2) = x\sum_{i=q,\bar{q}} e_q^2 \left[f_{i,0}(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_{i,0}(\xi)\left[P_{qq}\left(\frac{x}{\xi}\right)\log\frac{Q^2}{m_g^2} + C_2^q\left(\frac{x}{\xi}\right)\right]\right]$$

We can absorb the dependence on the IR cutoff into the PDF:

$$f_q(x,\mu_f) \equiv f_{q,0}(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) P_{qq}\left(\frac{x}{\xi}\right)\log\frac{\mu_f^2}{m_g^2} + z_{qq}$$

Renormalised PDFs!

Factorisation

Structure function is a measurable object and cannot depend on scale at all orders (renormalisation group invariance)

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi}\left[P_{qq}(\frac{x}{\xi})\log\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi})\right]\right]$$

Long distance physics is universally factorised into the PDFs, which now depend on μ_f. PDFs are not calculable in perturbation theory. PDFs are universal, they don't depend on the process.

Factorisation scale μ_f acts as a cut-off, emissions below μ_f are included in the PDFs.

LHC Master Formula

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$$

Parton model

↓ Renormalisation

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s}, \mu_R)$$

LHC Master Formula

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↓ QCD improved parton model

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↓ QCD improved parton model

$$\sum_{a,b}\int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$ ✓

DGLAP

We can't compute PDFs in perturbation theory but we can predict their evolution in scale:

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

$$P_{ab}(\alpha_S, z) = \frac{\alpha_S}{2\pi} P_{ab}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ab}^{(1)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ab}^{(2)}(z) + ...$$

LO (1974) NLO (1980) NNLO (2004)

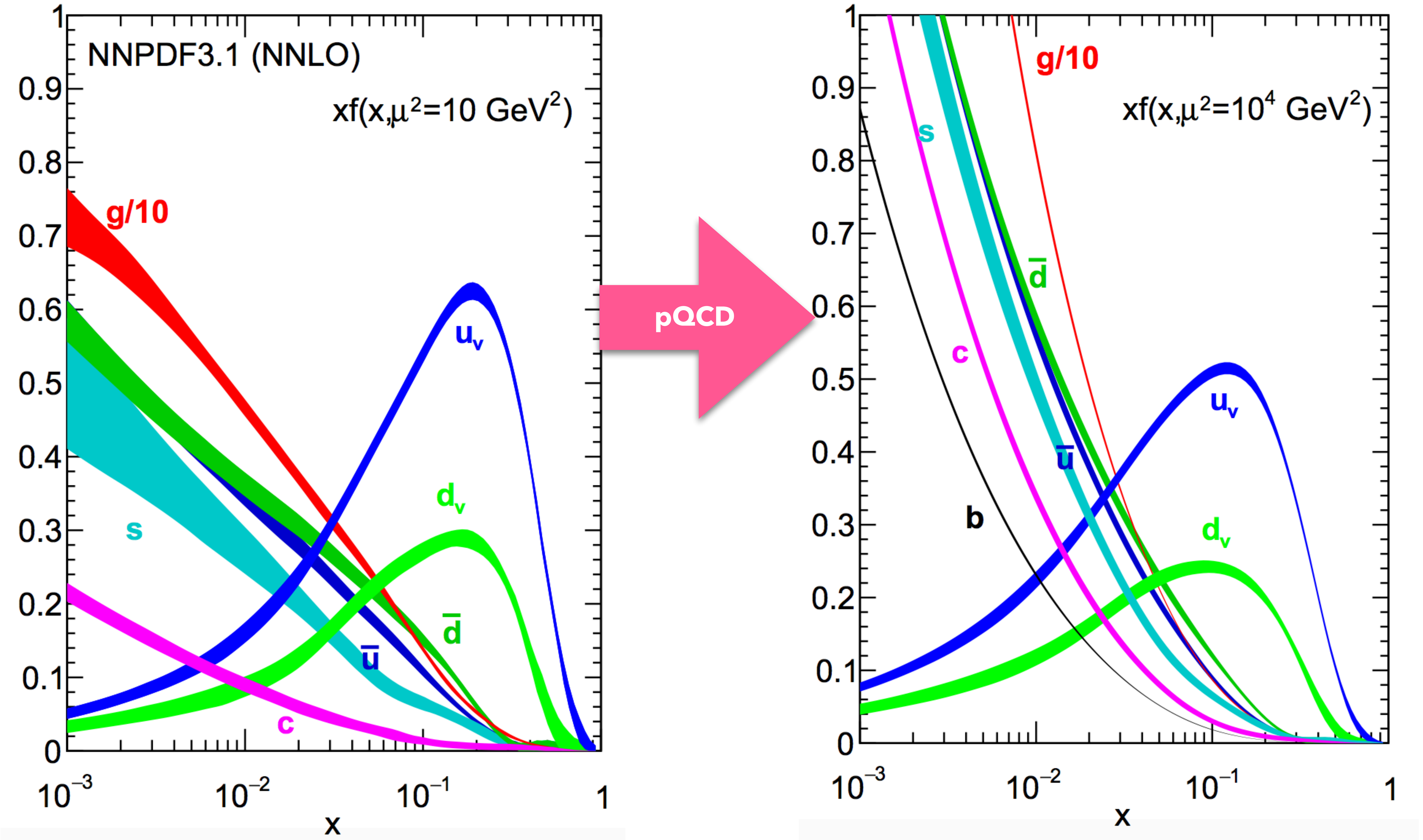
Splitting functions improved in perturbation theory!

LO Dokshitzer; Gribov, Lipatov; Altarelli, Parisi (1977)

NLO Floratos,Ross,Sachrajda; Floratos, Lacaze, Kounnas Gonzalez-Arroyo,Lopez,Yndurain; Curci,Furmanski Petronzio, (1981)

NNLO - Moch, Vermaseren, Vogt, 2004

PDF evolution

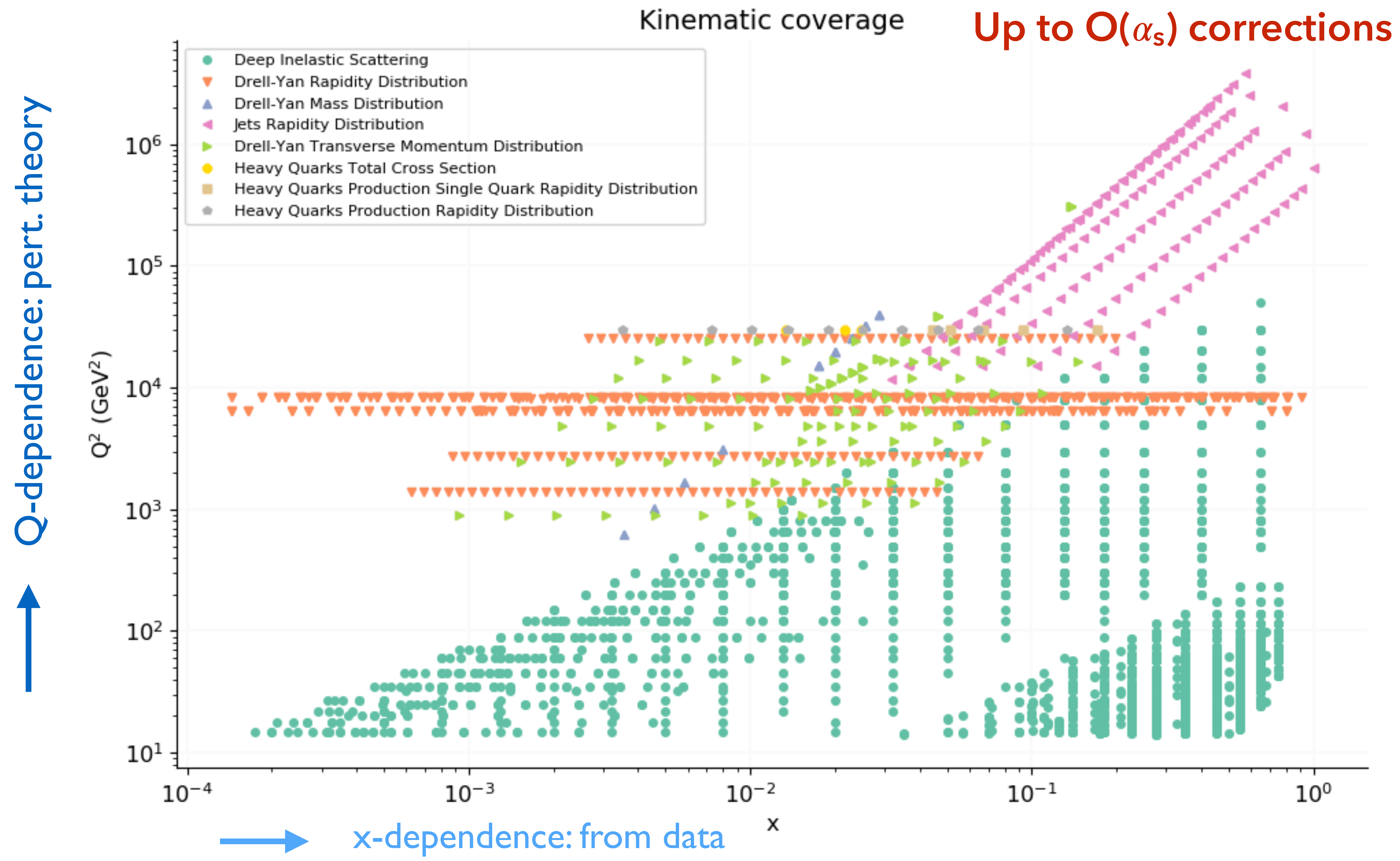


PDF extraction

We can't compute PDFs in perturbation theory but we can extract them from data, and use DGLAP equations to evolve them to different scales.

- Choose **experimental data** to fit and include all info on correlations
  Theory settings: perturbative order, EW corrections, intrinsic heavy quarks, α_s, quark masses value and scheme
- Choose a starting scale Q_0 where pQCD applies
- **Parametrise** independent quarks and gluon distributions at the starting scale
- Solve **DGLAP equations** from initial scale to scales of experimental data and build up observables
- **Fit** PDFs to data
- Provide PDF **error sets** to compute PDF uncertainties

Data for PDF determination



LHC kinematics

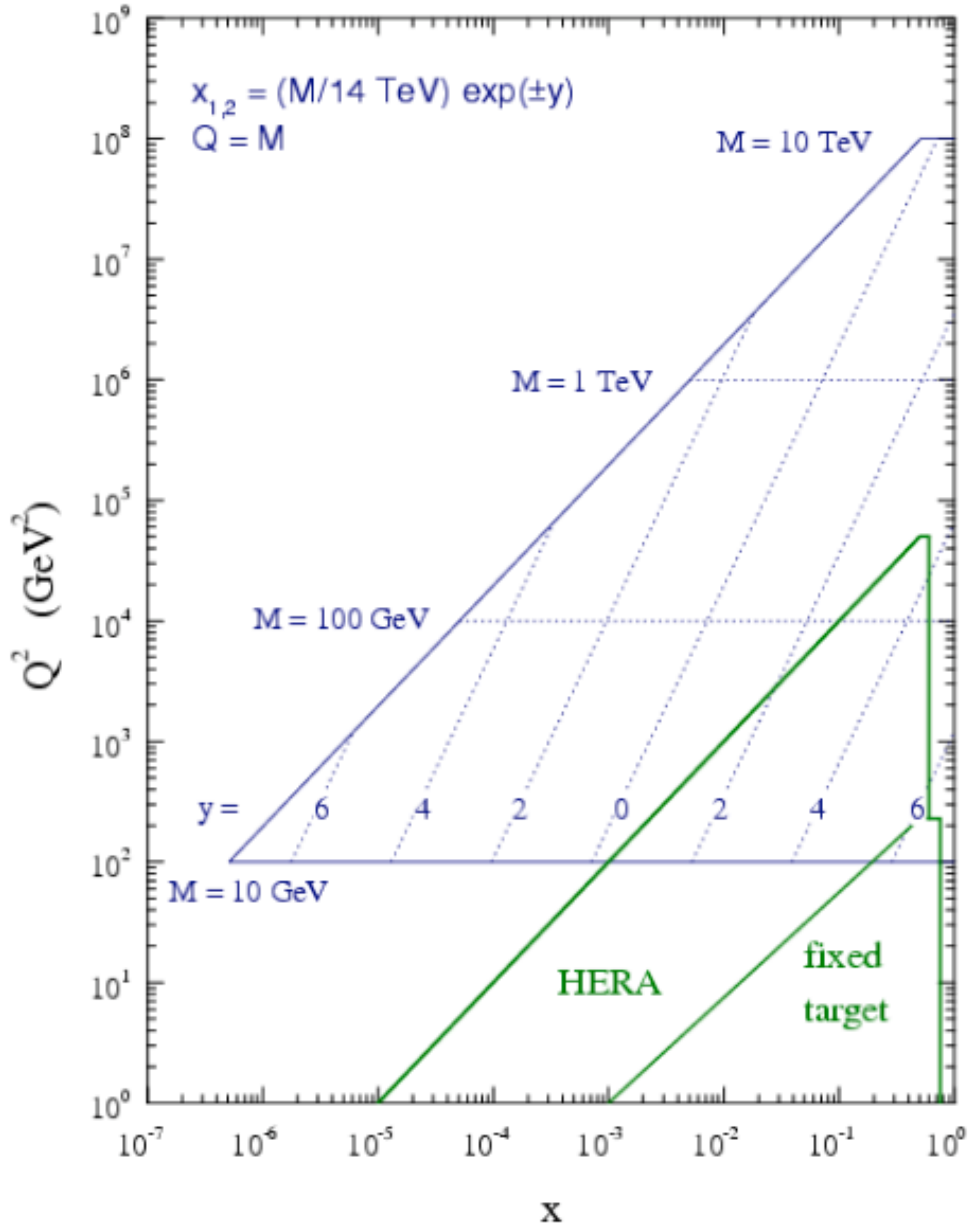
How can we tell which x data probes?

For the production of a particle of mass M:

$$M^2 = x_1 x_2 S = x_1 x_2 4E^2_{\rm beam}$$

$$y = \frac{1}{2}\log\frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{S}}e^{y} \quad x_2 = \frac{M}{\sqrt{S}}e^{-y}$$



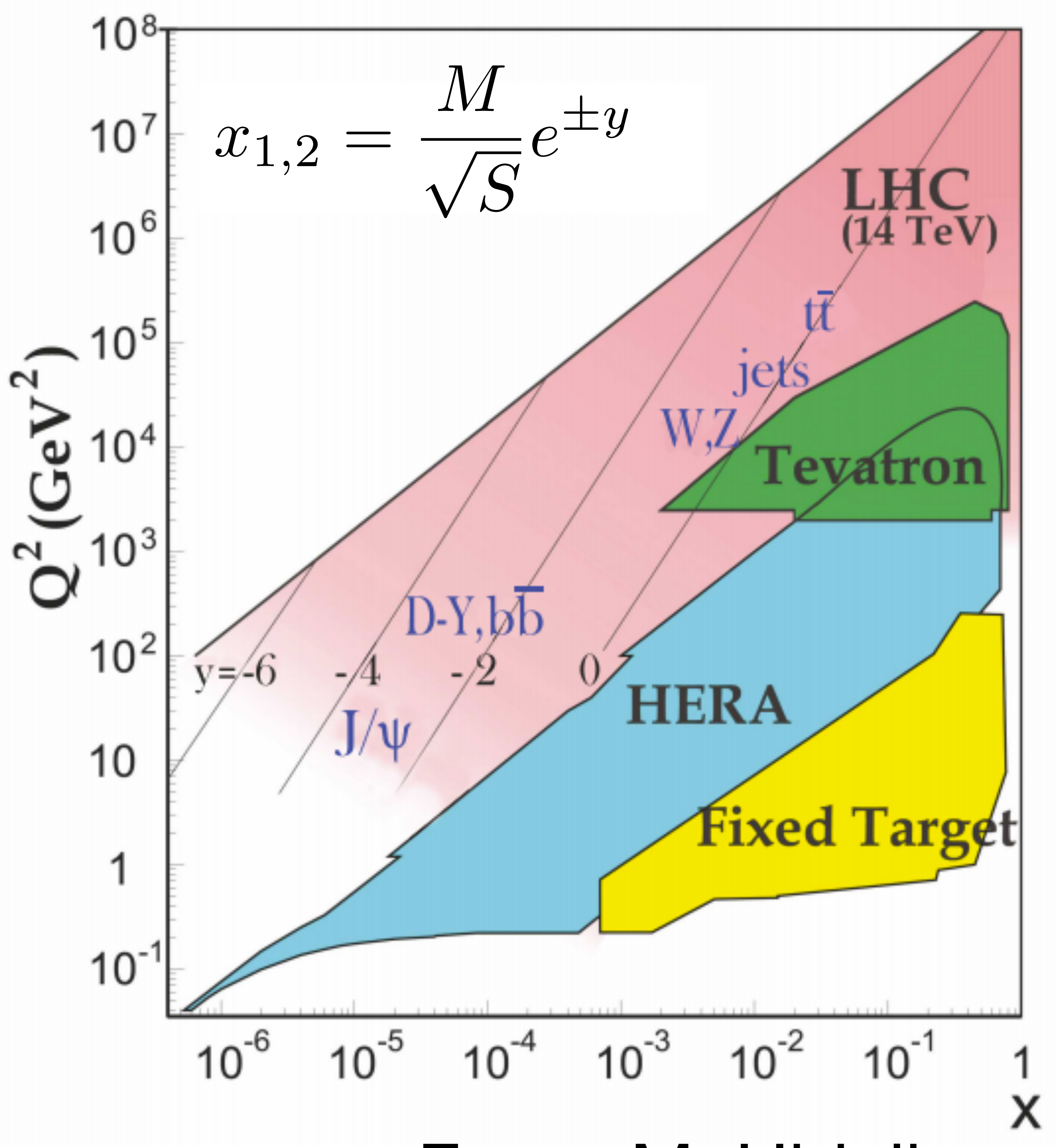
See exercises!

Data complementarity

GLUON
- Inclusive jets and dijets **(medium/large x)**
- Isolated photon and γ+jets **(medium/large x)**
- Top pair production **(large x)**
- High p_T V(+jets) distribution **(small/medium x)**

QUARKS
- High p_T W(+jets) ratios **(medium/large x)**
- W and Z production **(medium x)**
- Low and high mass Drell-Yan **(small and large x)**
- Wc **(strangeness at medium x)**

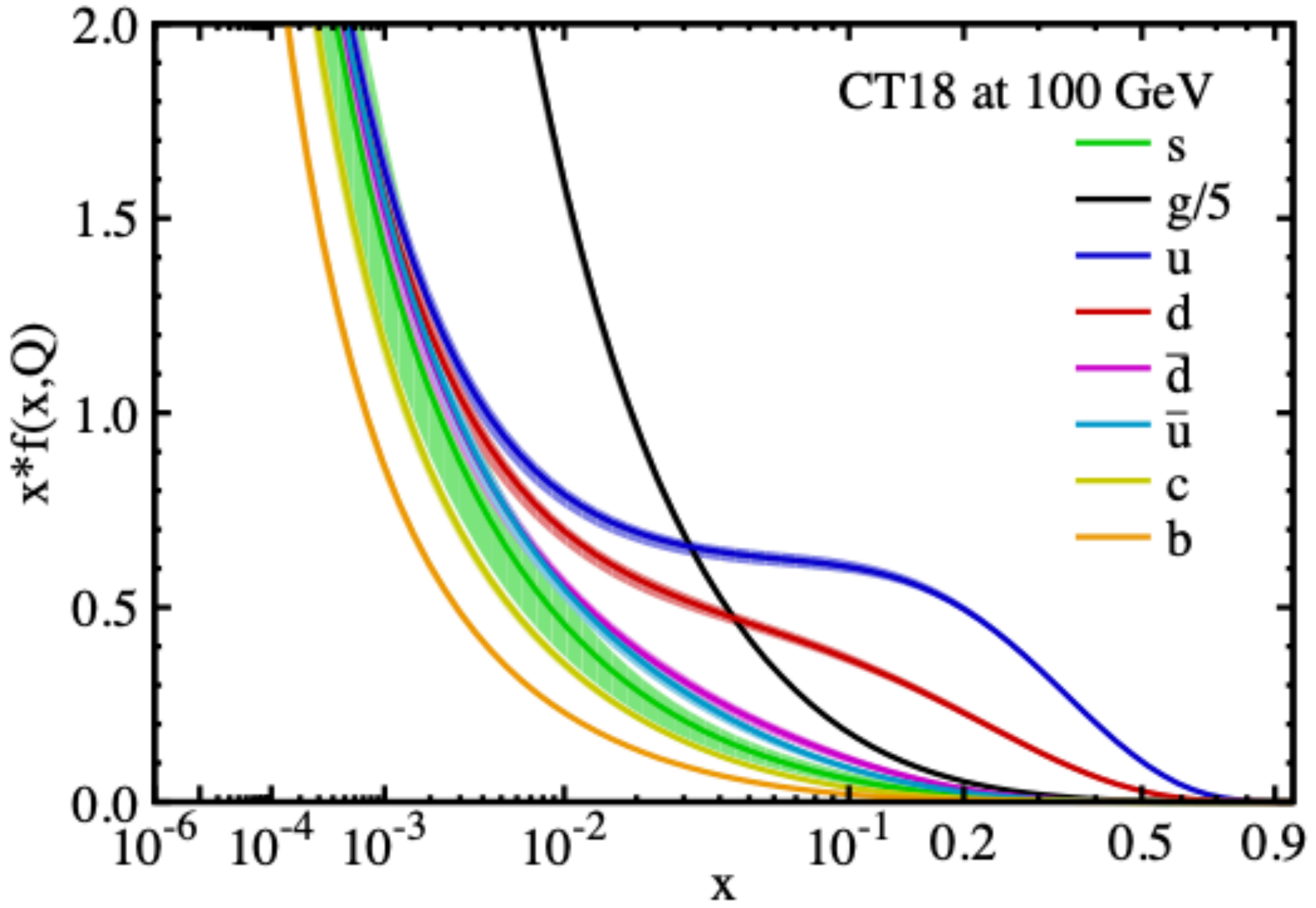
PHOTON
- Low and high mass Drell-Yan
- WW production



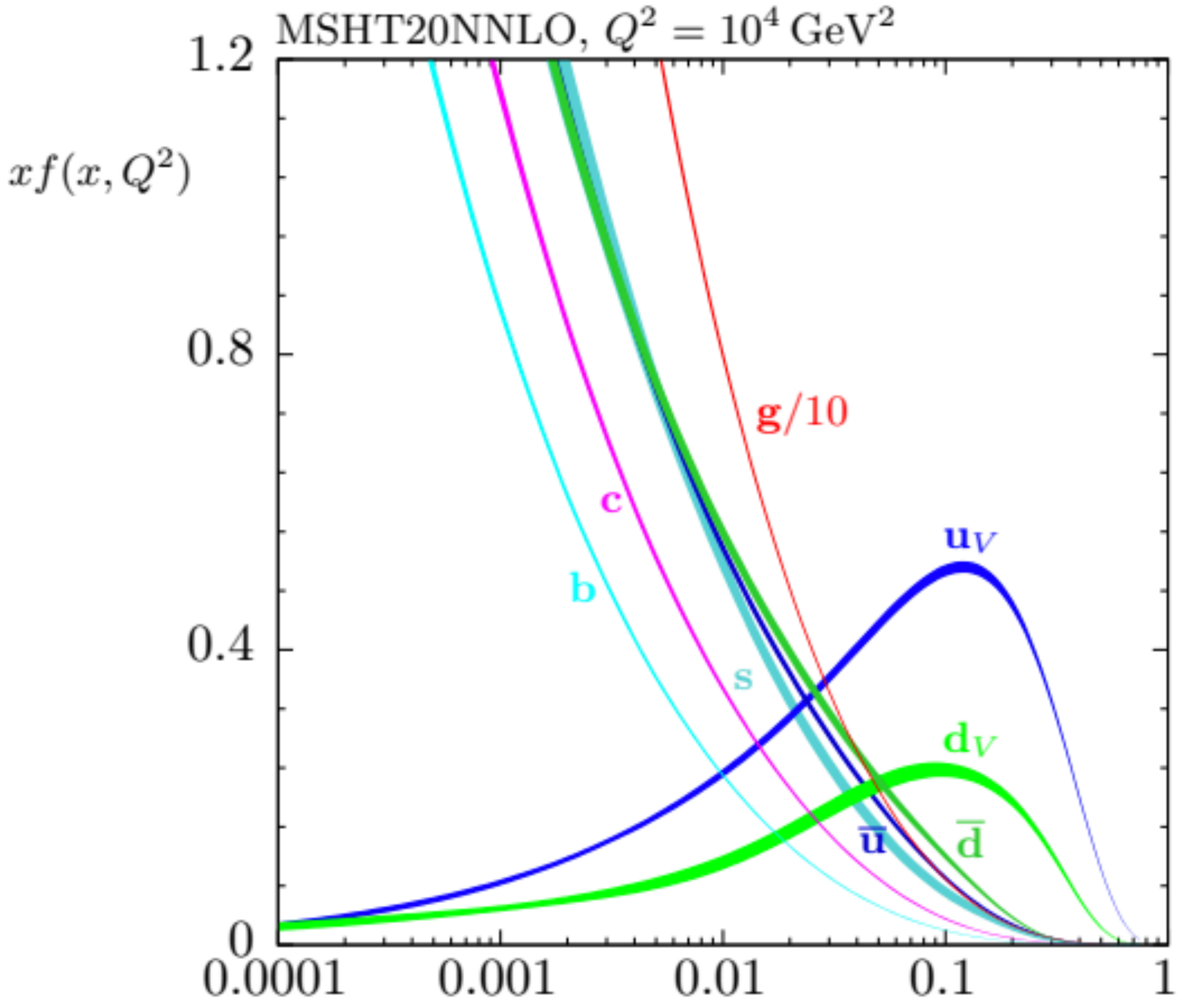
From. M. Ubiali

Modern PDFs

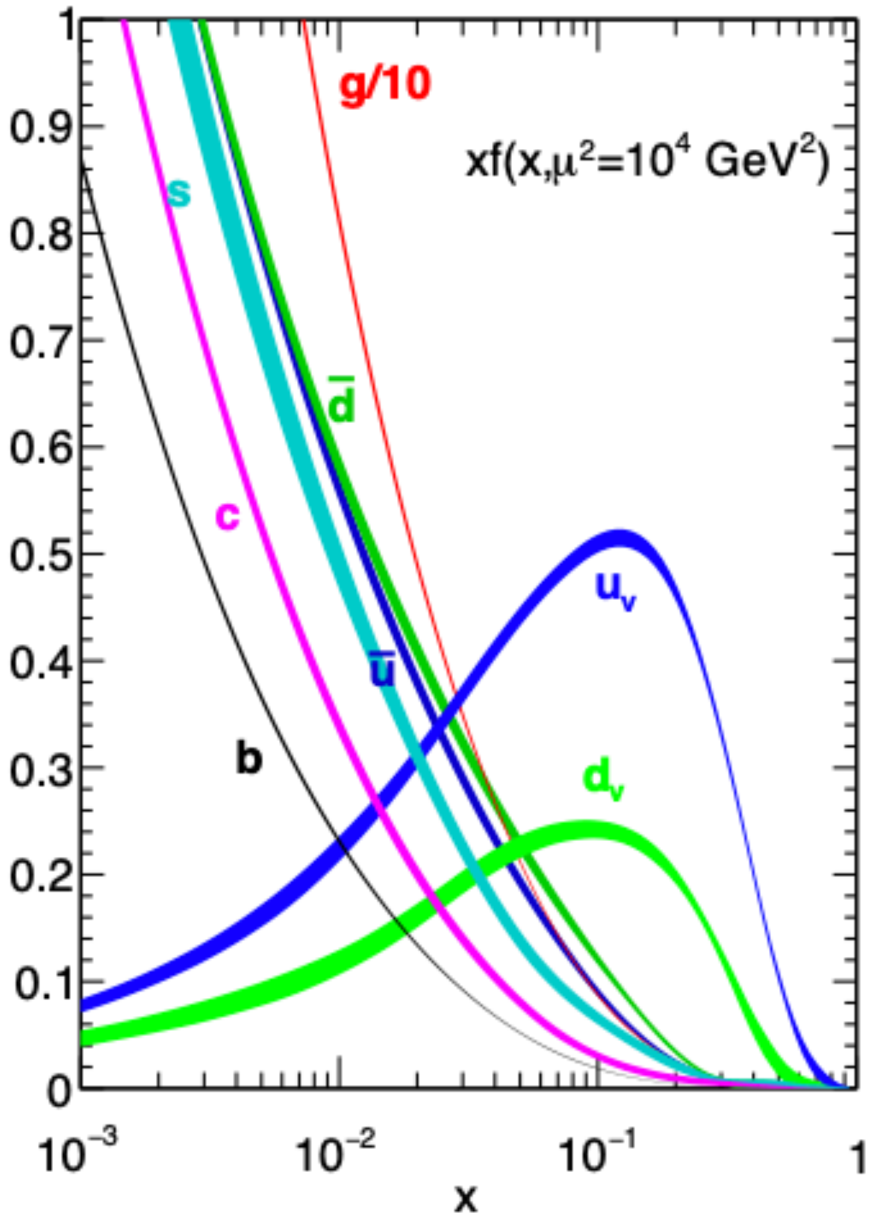
CT18



MSTH20



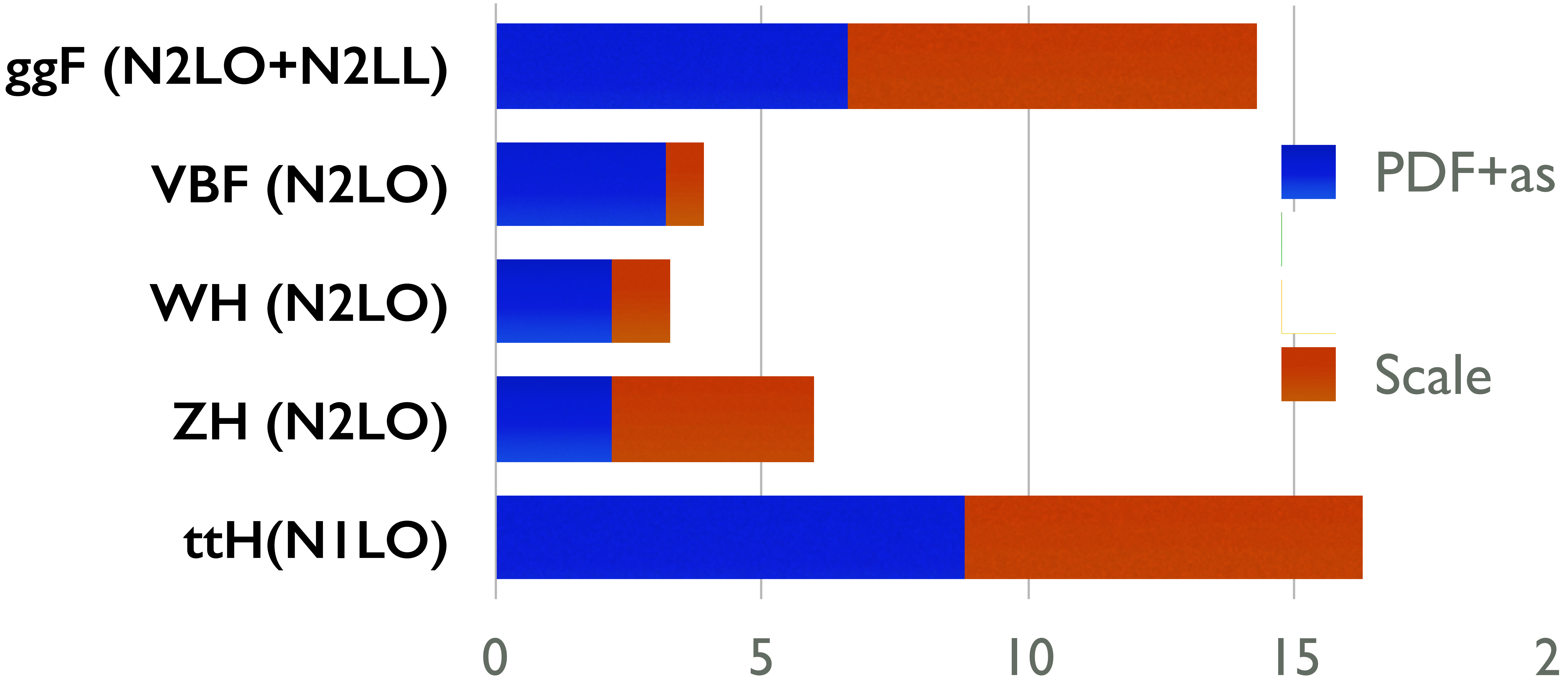
NNPDF3.1



Different collaborations, predictions usually computed with different PDFs to extract an uncertainty envelope.

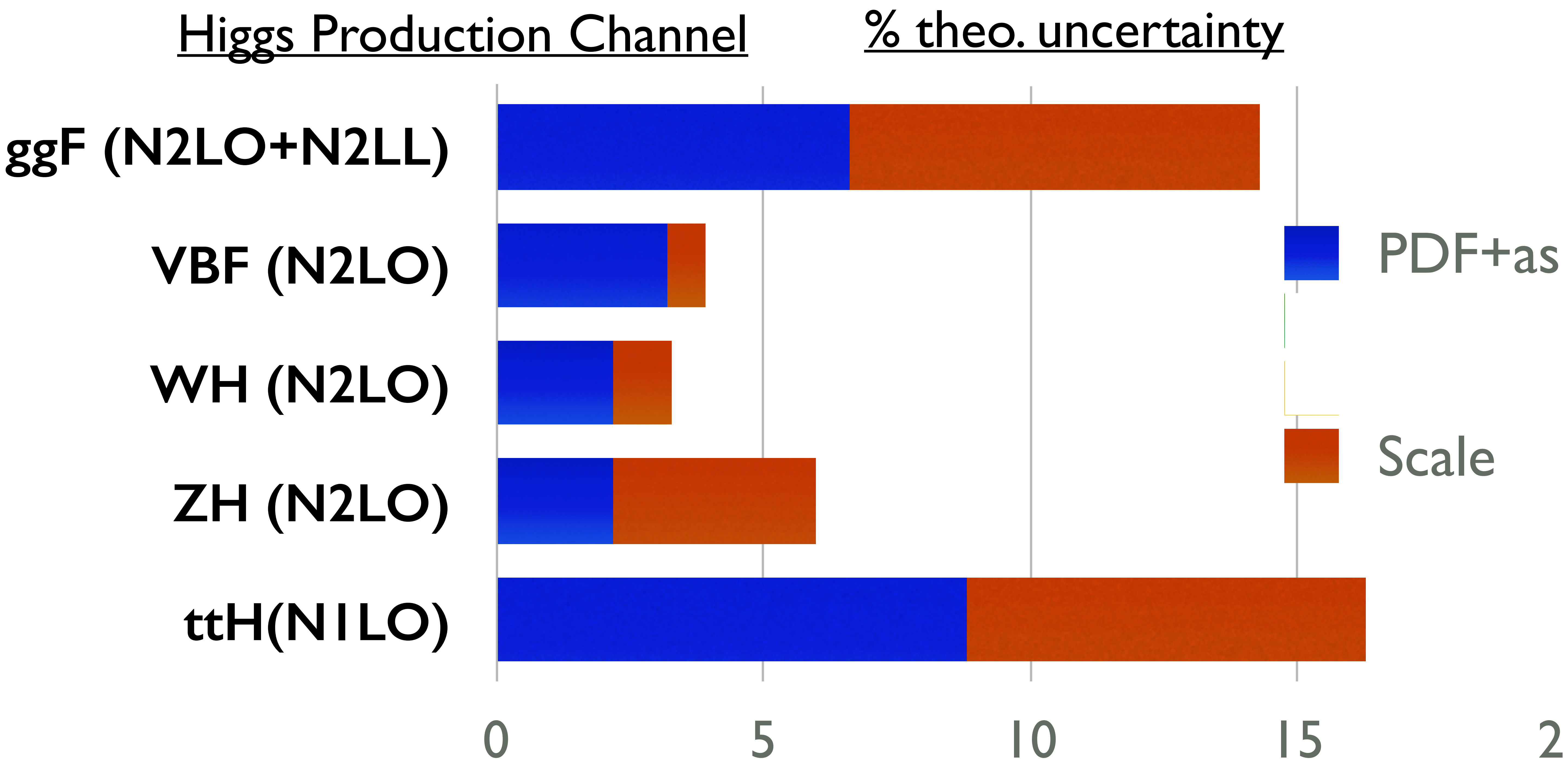
Impact of PDF uncertainties

Higgs Production Channel   % theo. uncertainty

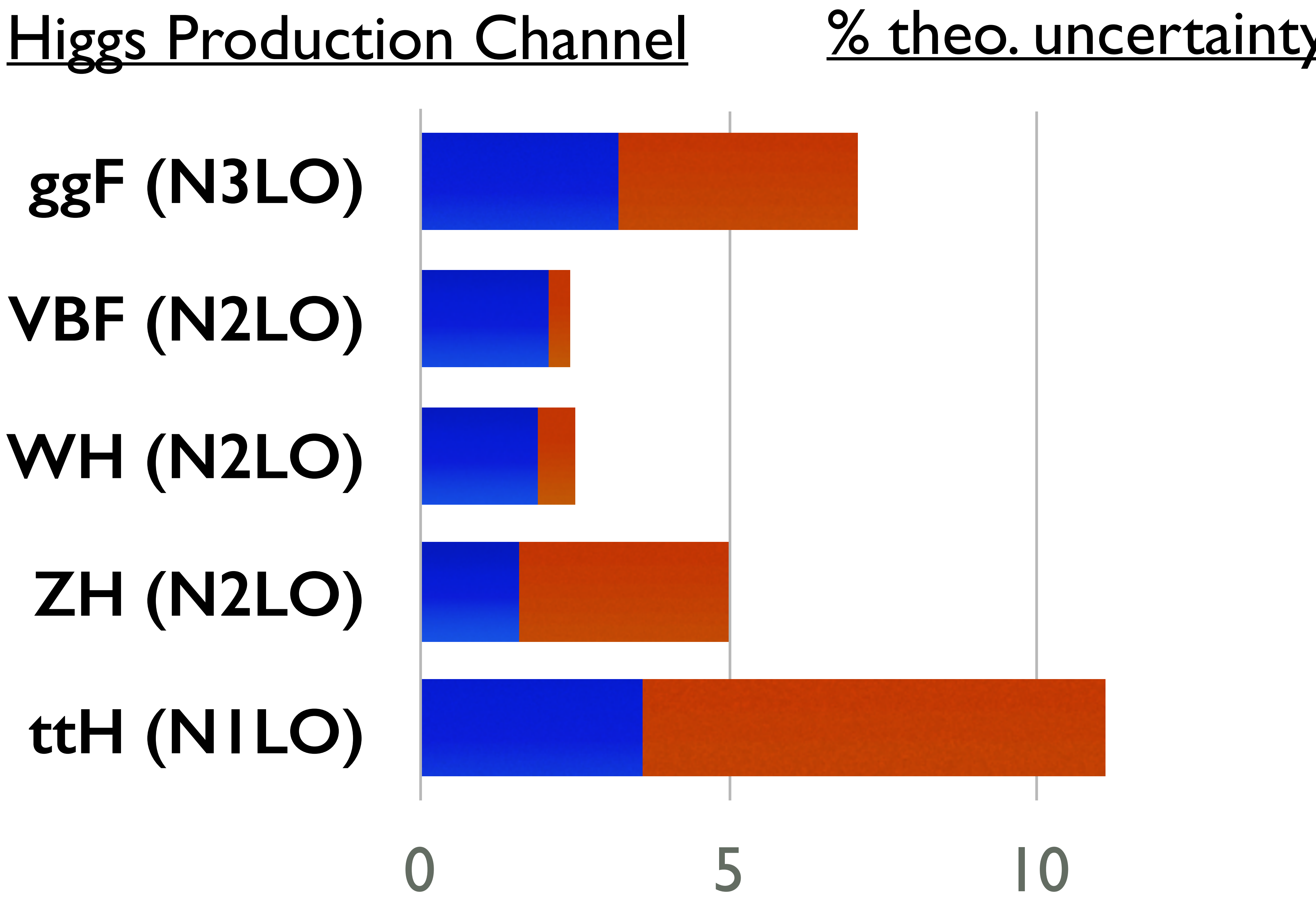


Yellow Report 3 (2013)

Impact of PDF uncertainties



Yellow Report 3 (2013)



Yellow Report 4 (2016)

Progress in PDFs!