The TMD Parton Branching & Cascade

Monte Carlo based on TMDs

- **Ola Lelek** on behalf of the Parton Branching & Cascade team

Universiteit Antwerpen

Research Foundation Flanders
Opening new horizons

Motivation

- Monte Carlo (MC) generators crucial for HEP predictions
- Precision of theory predictions lower than experimental precision
- Improvement of theory precision crucial to find the BSM physics
- Intensive MC developments before High Lumi & EIC...
- Baseline MCs based on collinear factorization
- Hot topic: 3D hadron structure
- Recently new developments to include physics of Transverse Momentum Dependent (TMD) factorization in MCs

  Today: TMD PB method

  a MC approach to obtain QCD predictions based on **TMD PDFs**

  Hautmann, Jung, Lelek, Radescu, Zlebcik, Phys.Lett.B 772 (2017) 446 & JHEP 01 (2018) 070

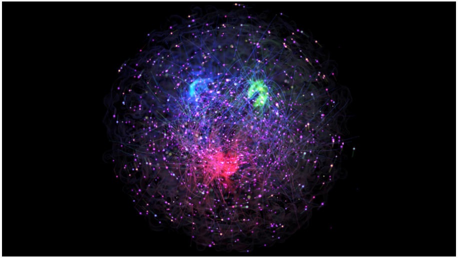


Image: James LaPlante/Sputnik Animation, MIT CAST & Jefferson Lab

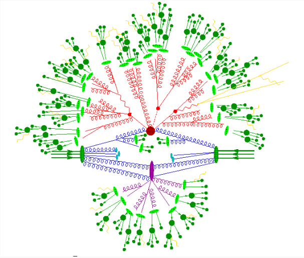
Standard MC generators: what can be improved?

- Every element of event generation has its uncertainty:

  ME, PS, PDFs, non-perturbative models, EW corrections, multi parton interactions, underlying events, hadronization, ...

- The way how we combine different generation stages has also uncertainties

  matching, merging...

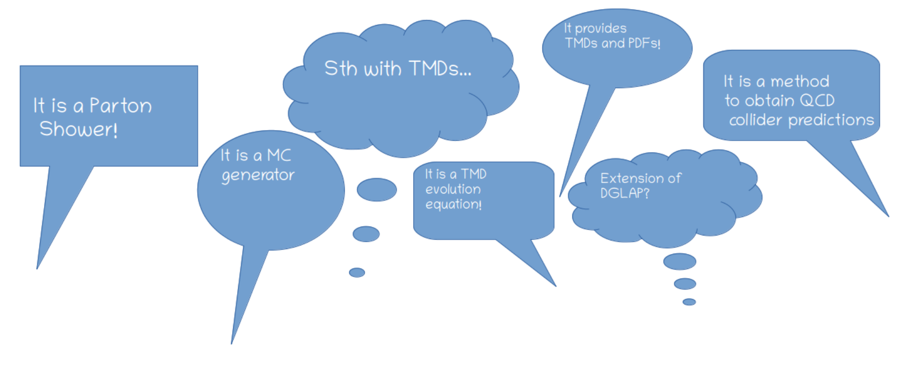


The accuracy of each element can be improved

but fundamental problem remains:
mismatch in kinematics originating from collinear assumption

TMD PB method addresses this issue

What is the TMD Parton Branching method?



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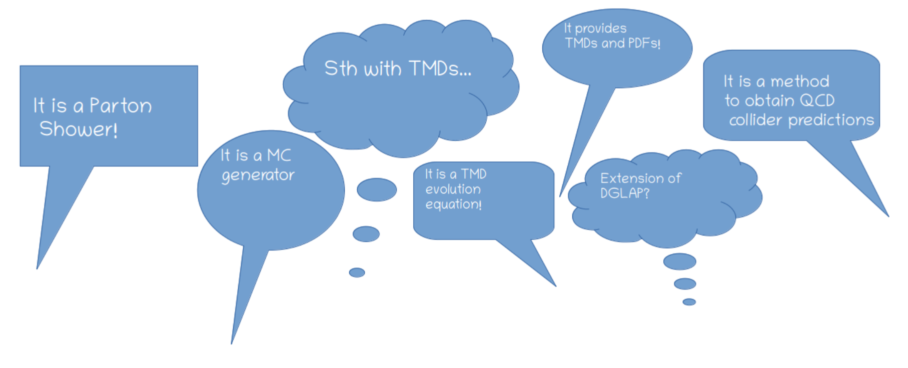
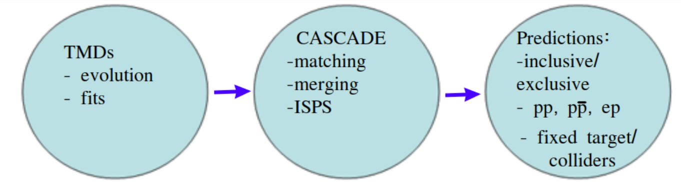


All this is true!

TMD PB method in a nutshell



Plan for today:

1. **Why a new method?**
2. **Intro** to TMD PB method
3. **limits/similarities** to other approaches
4. **Phenomenology & technical developments**

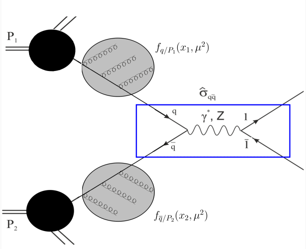
Motivation

Collinear factorization theorem in MCs

$$\sigma = \sum_{q\bar{q}} \int \mathrm{d}x_1 \mathrm{d}x_2 f_q(x_1, \mu^2) f_{\bar{q}}(x_2, \mu^2) \hat{\sigma}_{q\bar{q}}(x_1, x_2, \mu^2, Q^2)$$

Basis of many QCD calculations BUT

- proton structure in 1D only
- for some observables also the transverse degrees of freedom have to be taken into account



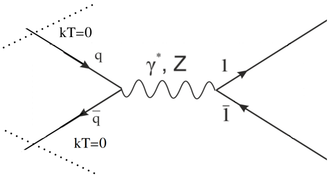
$\rightarrow$ **soft gluons need to be resummed:**

- **Transverse Momentum Dependent (TMD) factorization theorems**
  baseline: low $q_\perp$ Collins-Soper-Sterman (CSS)
- In practice **Monte Carlos** needed: **Parton Showers (PS) issues**:
  - treatment of $k_\perp$ in the evolution
  - consistency of the forward (i.e. this from which PDFs are being obtained) and backward (i.e. PSs) approaches

MC predictions

Collinear factorization: base assumption for MC generators

$$\sigma = \sum_{q\bar{q}} \int \mathrm{d}x_1 \mathrm{d}x_2 f_q(x_1, \mu^2) f_{\bar{q}}(x_2, \mu^2) \hat{\sigma}_{q\bar{q}}(x_1, x_2, \mu^2)$$



- **kinematics** of **ME** according to **PDFs** → incoming **partons do not have transverse momenta**
- PS applied. Transverse momentum generated

  PDFs extracted from approaches based on forward evolution PSs done in terms of backward evolution with PDFs as an input

In practical applications the evolution in forward and backward calculations doesn't match

- 4-momenta of incoming partons adjusted to compensate for k_T → partons' kinematics does not correspond to initial PDF

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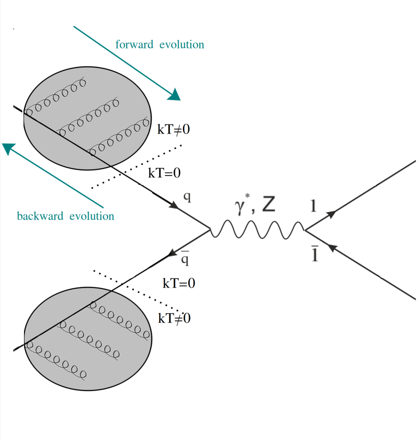

- **kinematics** of **ME** according to **PDFs** → incoming **partons do not have transverse momenta**
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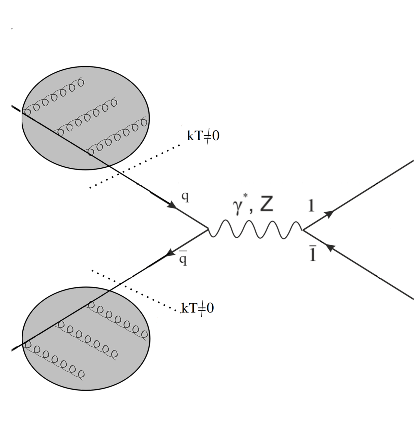

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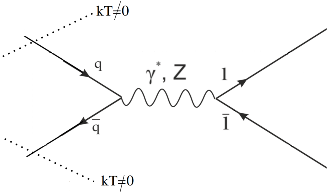
In practical applications the evolution in **forward and backward calculations doesn't match**

- **4-momenta** of incoming partons **adjusted** to compensate for k_T → partons' **kinematics does not correspond to initial PDF**

Parton Branching Method: Idea

Develop a **MC approach** in which transverse momentum kinematics will be treated without any mismatch between matrix element (ME) and Parton Shower (PS)
→ Transverse Momentum Dependent (TMD) factorization & **TMD PDFs (TMDs)**

$$\sigma = \sum_{q\bar{q}} \int \mathrm{d}^2 k_{\perp 1} \mathrm{d}^2 k_{\perp 2} \int \mathrm{d}x_1 \mathrm{d}x_2 A_q(x_1, k_{\perp 1}, \mu^2) A_{\bar{q}}(x_2, k_{\perp 2}, \mu^2) \hat{\sigma}_{q\bar{q}}(x_1, x_2, k_{\perp 1}, k_{\perp 2}, \mu^2)$$



- **kinematics** of **ME** generated according to **TMD PDFs** → incoming partons **have** transverse momenta
Enough to describe the inclusive spectra, e.g. Z $p_\perp$

Let's see how far the PB method has got in practice!

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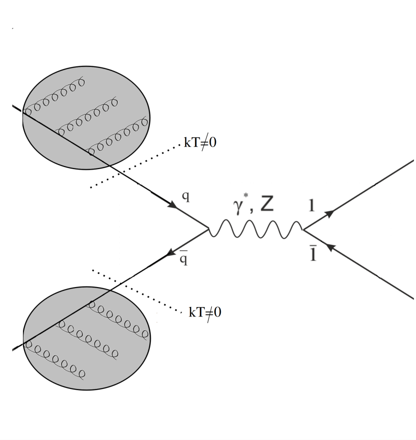


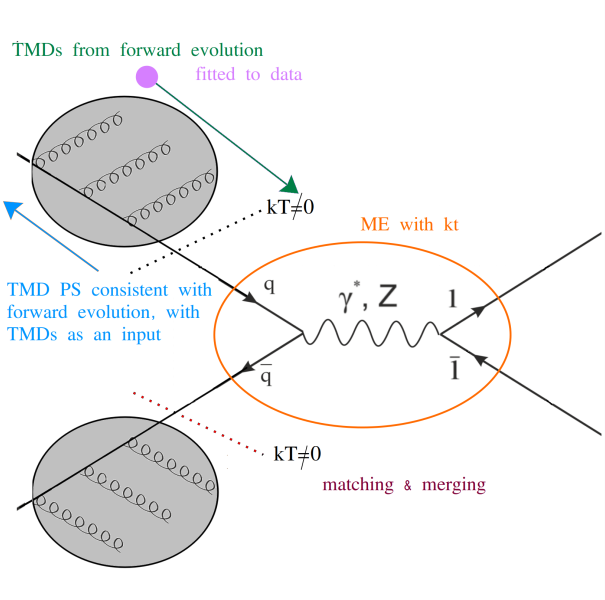
- **kinematics** of **ME** generated according to **TMD PDFs** → incoming partons **have** transverse momenta
Enough to describe the inclusive spectra, e.g. Z $p_\perp$

- For exclusive observables: **TMD PS**
consistent forward and backward evolution
k_T at each branching fixed by TMD PDF
→ **NO adjustment** of the kinematics in the ME needed after showering

Let's see how far the PB method has got in practice!

Required Blocks

To realize the PB idea, several elements developed



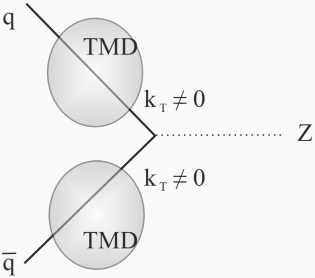
Educational power of the PB method

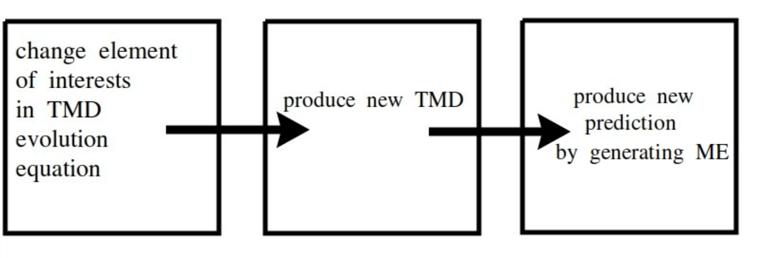
In PB, inclusive observables, e.g. Z $p_\perp$, generated without PS (because of $k_\perp$ in TMD)

→ **clear way of studying different evolution setups!**

i.e. enough to change the element of interest in evolution equation, produce new TMD and generate ME to get the prediction.

The effect **not blurred by PS!**

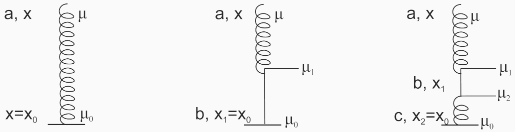




TMD evolution equation

Evolution in the TMD PB method

Hautmann, Jung, Lelek, Radescu, Zlebcik, Phys.Lett.B 772 (2017) 446 & JHEP 01 (2018) 070



$$\widetilde{A}_a\left(x,k_\perp^2,\mu^2\right)=\Delta_a\left(\mu^2,\mu_0^2\right)\widetilde{A}_a\left(x,k_\perp^2,\mu_0^2\right)+\sum_b\int\frac{\mathrm{d}^2\mu_{\perp 1}}{\pi\mu_{\perp 1}^2}\Theta\left(\mu_{\perp 1}^2-\mu_0^2\right)\Theta\left(\mu^2-\mu_{\perp 1}^2\right)$$

$$\times\Delta_a\left(\mu^2,\mu_{\perp 1}^2\right)\int_x^{z_M}\mathrm{d}zP_{ab}^R\left(z,\mu_{\perp 1}^2\right)\widetilde{A}_b\left(\frac{x}{z},|k_{\perp 1}|^2,\mu_{\perp 0}^2\right)\Delta_b\left(\mu_{\perp 1}^2,\mu_{\perp 0}^2\right)+\ldots$$

Intuitive **probabilistic interpretation** $\Longleftrightarrow$ easy to **solve by Monte Carlo** (MC) :

- Sudakov form factor $\Delta_a\left(\mu^2,\mu_0^2\right)=\exp\left(-\sum_b\int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\int_0^{z_M}\mathrm{d}z\; zP_{ba}^R\left(z,\mu'^2\right)\right)$

probability of an evolution without resorvable branchings between μ_0^2 and μ^2

- Splitting function $P_{ab}^R(z,\mu^2)$ - probability of $b\rightarrow a$

P_{qq}^R & P_{gg}^R- **divergent** for $z\rightarrow 1 \Leftrightarrow$ **soft gluons**: z_M defines **resolvable and non-resolvable** branchings

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$\widetilde{A}=xA$, z- splitting variable, $x=zx_1$, $z\in(0,1)$

Transverse momentum in PB

- **starting distribution** at μ_0^2:

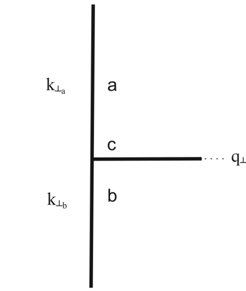
  $$\widetilde{A}_{a,0}(x, k_{\perp 0}^2, \mu_0^2) = \widetilde{f}_{a,0}(x, \mu_0^2)\frac{1}{\pi q_s^2}\exp\left(\frac{-k_{\perp 0}^2}{q_s^2}\right)$$

- Initial distribution $\widetilde{f}_{a,0}(x, \mu_0^2)$ obtained from **fits to inclusive DIS data**
- Intrinsic transverse momentum $k_{\perp 0}$ constraint from DY data
- transverse momentum **k calculated at each branching**

  $\mathbf{k}_a = \mathbf{k}_b - \mathbf{q}_c$,

  k of the propagating parton is a sum of intrinsic transverse momentum and all emitted transverse momenta

  $\mathbf{k} = \mathbf{k}_0 - \sum_i \mathbf{q}_i \rightarrow$ **TMD from parton branching**



How to relate $q_\perp$ and the evolution scale μ' ? $\rightarrow$ **Ordering condition**

Angular Ordering (AO)

PB implements **AO**

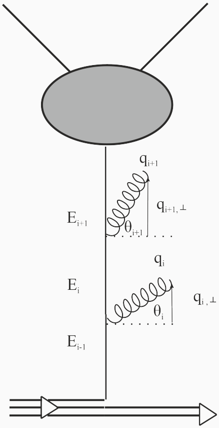
- angles of emitted partons increase from the hadron side towards hard scattering

S. Catani, G. Marchesini, B. Webber (CMW):

AO included when the **scale associated with the rescaled transverse momentum**

$q_{\perp} = (1 - z)\mu'$

AO assures PB TMDs do not have IR singularities

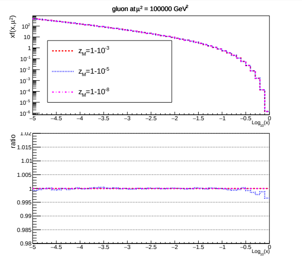


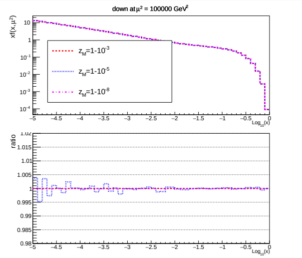
Effect of z_M on PDFs

PB integrated TMDs (iTMDs): $\widetilde{f}_a(x,\mu^2) = \int \mathrm{d}k_\perp^2\, \widetilde{A}_a(x,k_\perp,\mu^2)$

By introducing z_M, terms $\mathcal{O}(1-z_M)$ skipped compared to DGLAP

When $z_M \approx 1$, this effect not visible





Bigger z_M $\rightarrow$ more branchings

When a soft gluon is emitted:

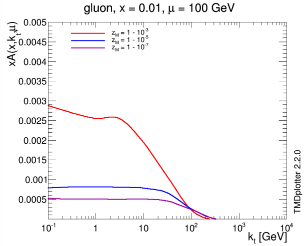
- x unchanged
- flavour unchanged

$\rightarrow$ this **emission unnoticeable in the integrated distribution**

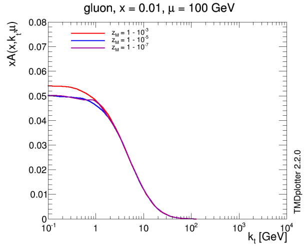
This is **not necessarily true at the level of TMDs!**

Effect of ordering & z_M on TMDs

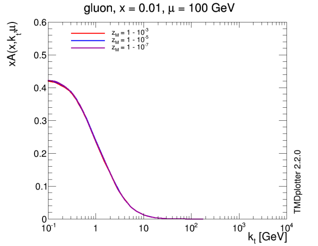
1704.01757, 1708.03279



$p_\perp$- ordering
$q_\perp^2 = \mathbf{1}\mu'^2$



virtuality ordering
$q_\perp^2 = \mathbf{(1\text{-}z)}\mu'^2$



angular ordering
$q_\perp^2 = (1 - z)^2\mu'^2$

Recall: $\mathbf{k} = \mathbf{k}_0 - \sum_i \mathbf{q}_i$

$p_\perp$- ordering: IR divergent TMDs

virtuality- and angular ordering: difference between z_M only in the small $k_\perp$ region at higher scales, with AO barely visible $\rightarrow$ **AO assures IR safe TMDs**

Note: All these TMDs after integration over $k_\perp$ give the same collinear PDF ✓

Soft gluon resolution scale z_M

Issues **related to ordering:**

1. soft gluon **resolution scale** z_M
   - DGLAP: $z_M = 1$
   - AO: q_0 - the minimal emitted transverse momentum for which a branching can be resolved

$$q_\perp = (1 - z)\mu' \to z_M(\mu') = 1 - q_0/\mu'$$

z_M **dynamical**, i.e. scale dependent

2. **scale in** α_s: $\alpha_s(\mu'^2)$ or $\alpha_s(q_\perp^2)$

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z_M **dynamical**, i.e. scale dependent

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PB limits for integrated TMDs (iTMDs): $\widetilde{f}_a(x,\mu^2) = \int \mathrm{d}k_\perp^2 \widetilde{A}_a(x,k_\perp,\mu^2)$

- $z_M = 1$ & $\alpha_s(\mu'^2) \rightarrow$ **DGLAP**
- $z_M(\mu') = 1 - q_0/\mu'$, LO P & $\alpha_s(q_\perp) \rightarrow$: **CMW**

Baseline MCs use PDFs obtained with **fixed** $z_M \approx 1$ and PS with **dynamical** z_M

AO $z_M \Longleftrightarrow$ **soft gluon resummation**

Sudakov Resummation

Sudakov resummation in PB

Motivated by AO, **PB Sudakov factorized**:

$$\Delta_a(\mu^2,\mu_0^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\left[\int_0^{z_{\rm dyn}(\mu')} dz\,\frac{k_q(\alpha_s)}{1-z} - d_q(\alpha_s)\right]\right) \times\exp\left(-\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\int_{z_{\rm dyn}(\mu')}^{z_M\approx 1} dz\,\frac{k_q(\alpha_s)}{1-z}\right).$$

by introducing

$$z_{\rm dyn}(\mu') = 1 - q_0/\mu'$$

both perturbative and non-perturbative regions are taken into account:

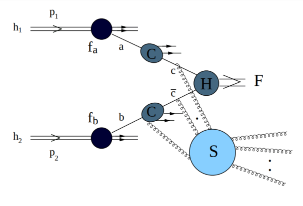
$$\Delta_a(\mu^2,\mu_0^2) = \Delta_a^{\rm (P)}\left(\mu^2,\mu_0^2,q_0\right)\cdot\Delta_a^{\rm (NP)}\left(\mu^2,\mu_0^2,\epsilon,q_0^2\right)\ .$$

P: $z < z_{\rm dyn} \Longleftrightarrow q_\perp > q_0$
NP: $z_{\rm dyn} < z < z_M$ ($z_M = 1-\epsilon$ with $\epsilon \ll 1$), $\Longleftrightarrow q_\perp < q_0$

Perturbative resummation in PB & CSS

PB Sudakov form factor **for AO**:

$$\Delta_a(Q^2, q_0^2)^{(P)} = \exp\left(-\int_{q_0^2}^{Q^2} \frac{dq_\perp^2}{q_\perp^2}\left(\int_0^{z_M=1-\frac{q_\perp}{Q}} dz \left(k_a(\alpha_s(q_\perp))\frac{1}{1-z}\right) - d(\alpha_s(q_\perp))\right)\right)$$



notice: $\int_0^{1-\frac{q_\perp}{Q}} dz\left(\frac{1}{1-z}\right) = \frac{1}{2}\ln\left(\frac{Q^2}{q_\perp^2}\right)$

Collins-Soper-Sterman (CSS) Sudakov form factor:

$$\sqrt{S^{(P)}} = \exp\left(-\frac{1}{2}\int_{c_0/b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[A_i\left(\alpha_s(\mu^2)\right)\ln\left(\frac{Q^2}{\mu^2}\right) + B_i\left(\alpha_s(\mu^2)\right)\right]\right)$$

$$\frac{d\sigma}{dq_\perp} \sim \int d^2b \exp(ib\cdot q_\perp)\int dz_1 dz_2 H(Q^2) F_1(z_1, b, \text{scales})F_2(z_2, b, \text{scales}) + Y$$

We can compare: $k_a \Longleftrightarrow A$ and $d \Longleftrightarrow B$, order by order in α_s

$F = f \otimes C \otimes \sqrt{S}$

where $\sqrt{S} = \sqrt{S^{(P)}S^{(NP)}}$

- LL (A_1), NLL (A_2, B_1) coefficients in Sudakov the same in PB and CSS

NNLL:

B_2:

Renormalization group transformations mix the B, C, and H
$\overline{MS}$ resummation scheme: B corresponds to d
Difference coming from different schemes proportional to β_0

A_3:

double logarithmic part in PB: $P_{aa} = \frac{1}{1-z} k_a + \ldots$ (part of the DGLAP P)

collinear anomaly: **at NNLL k_a and A_a do not coincide** Becher & Neubert
$\rightarrow$ NNLL resummation in the PB Sudakov not achievable by implementing NNLO P
BUT can be done with **effective coupling!**

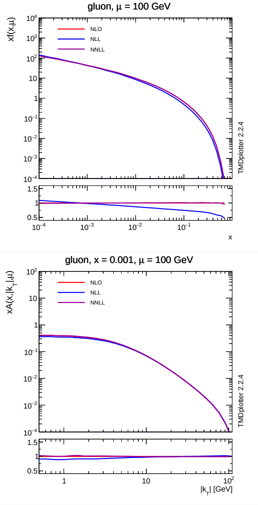
Banfi, El-Menoufi & Monni; Catani, de Florian & Grazzini:

$$\alpha_s^{\rm eff} = \alpha_s \left(1 + \sum_n \left(\frac{\alpha_s}{2\pi}\right)^n \mathcal{K}^{(n)}\right)$$

$\mathcal{K}^{(1)} = C_A \left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9} N_f$
$\mathcal{K}^{(2)} = C_A^2 \left(\frac{245}{24} - \frac{67}{9}\zeta_2 + \frac{11}{6}\zeta_3 + \frac{11}{5}\zeta_2^2\right) + C_F N_f \left(-\frac{55}{24} + 2\zeta_3\right) + C_A N_f \left(-\frac{209}{108} + \frac{10}{9}\zeta_2 - \frac{7}{3}\zeta_3\right) - \frac{1}{27} N_f^2 + \frac{\pi\beta_0}{2} \left(C_A \left(\frac{808}{27} - 28\zeta_3\right) - \frac{224}{54} N_f\right)$

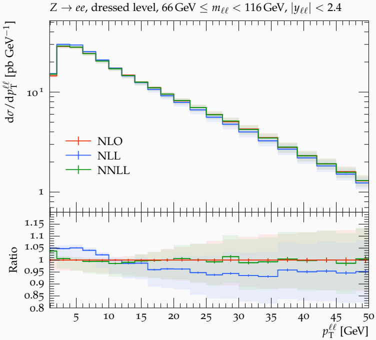
PB: recently implemented A_3 with $\alpha_s^{\rm eff}$

PB with A_3



NEW RESULTS

NLO: NLO P
NLL: LO P + α_s^{eff} with $\mathcal{K}^{(1)}$
NNLL: NLO P + α_s^{eff} with $\mathcal{K}^{(2)}$



Big effect between NLL and NLO
Effect between NLO and NNLL $\mathcal{O}(2\%)$

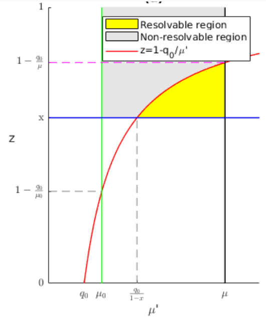
Non-perturbative Sudakov

if $z_M \approx$ 1- non - perturbative PB Sudakov included, similarly to CSS:

$$\Delta_a^{(\mathrm{NP})}(\mu^2, \mu_0^2, \epsilon, q_0) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{1-q_0\mu'}^{1-\epsilon} dz \frac{k_a(\alpha_s)}{1-z}\right) =$$
$$\exp\left(-\frac{k_a(\alpha_s)}{2} \ln\left(\frac{\mu^2}{\mu_0^2}\right) \ln\left(\frac{q_0^2}{\epsilon^2 \mu_0 \mu}\right)\right)$$

Logarithmic structure resembles CS kernel $\mathcal{D}$ of the modern CSS (CSS2)

$$\Delta_a^{\mathrm{CSS2}}(b, Q, Q_0, \mu_0) =$$
$$\exp\left(-\int_{\mu_0^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left(\gamma_k(\alpha_s)\ln\left(\frac{Q^2}{\mu'^2}\right) + \gamma_j(\alpha_s)\right)\right) \times \exp\left(\mathcal{D}(b, \mu_0) \ln \frac{Q^2}{Q_0^2}\right)$$



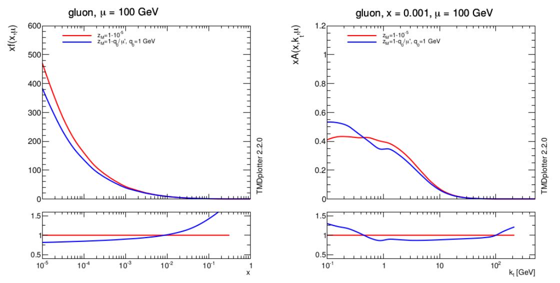
Later on: **extract the CS kernel from the PB approach**

but first:
Using dynamical $z_M = 1 - \frac{q_0}{\mu'}$ $\Longleftrightarrow$ skipping the non-perturbative Sudakov in the evolution has interesting consequences

Effect of z_M on TMDs and PDFs

Effect of z_M on TMDs and PDFs

$z_M = 1 - 10^{-5}$, $z_M = 1 - \frac{q_0}{\mu'}$ & $q_0 = 1$ GeV



Remark: toy model: $\mathbf{k} = -\mathbf{q}_{i=n}$
kt from last branching only
(a la Kimber-Martin-Ryskin-Watt)

Recall:

$\mathbf{k} = \mathbf{k}_0 - \sum_i \mathbf{q}_i$

$\Delta_a = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \int_0^{z_M} \mathrm{d}z\; z\; P_{ba}^R(z, \alpha_s)\right)$

- **bigger z_M → more branchings**

Notice bump around $k_\perp = 1$ GeV with dyn z_M

F. Hautmann, L. Keersmaekers, A. Lelek, A. M. van Kampen
Nucl.Phys.B 949 (2019) 114795

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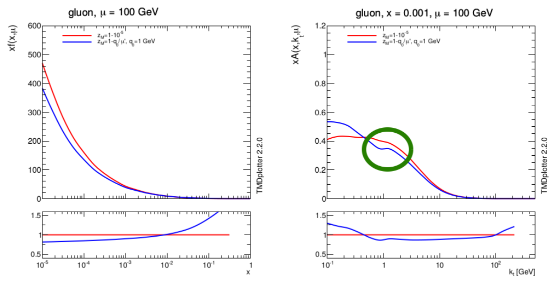

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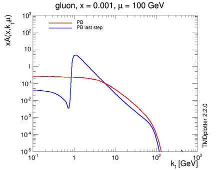

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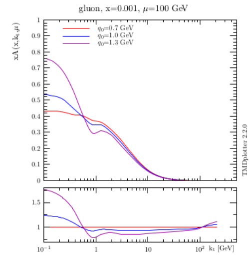
Let's focus on dyn $z_M = 1 - \frac{q_0}{\mu'}$

$q_s = 0.5$ GeV & :$q_0 = 0.7$ GeV, $q_0 = 1.0$ GeV, $q_0 = 1.3$ GeV



- large $q_0 \rightarrow$ less branchings
- large q_0: matching of intrinsic distribution with the evolution visible
- **low q_0: intrinsic $k_\perp$ distribution smeared** by the evolution

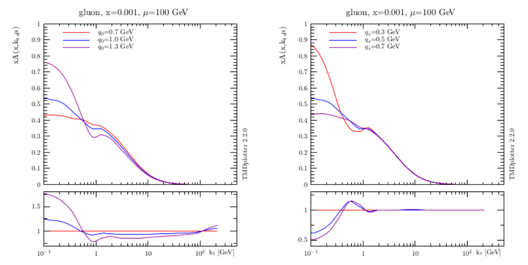
Interplay of the perturbative and non-perturbative region

Let's focus on dyn $z_M = 1 - \frac{q_0}{\mu'}$

$$\widetilde{A}_{a,0}(x, k_{\perp 0}^2, \mu_0^2) = \widetilde{f}_{a,0}(x, \mu_0^2) \frac{1}{\pi q_s^2} \exp\left(\frac{-k_{\perp 0}^2}{q_s^2}\right)$$

$q_s = 0.5$ GeV & :$q_0 = 0.7$ GeV, $q_0 = 1.0$ GeV, $q_0 = 1.3$ GeV

$q_0 = 1.0$ GeV & :$q_s = 0.3$ GeV, $q_s = 0.5$ GeV, $q_s = 0.7$ GeV



- large q_0 → less branchings
- large q_0: matching of intrinsic distribution with the evolution visible
- **low q_0: intrinsic $k_\perp$ distribution smeared** by the evolution

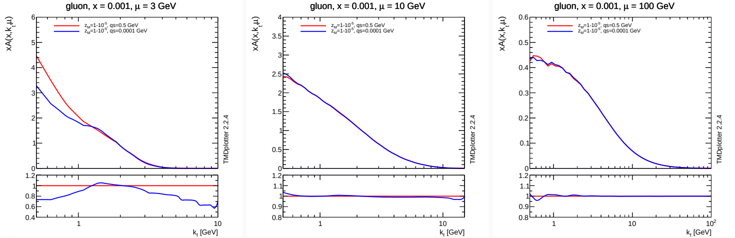
What if we **change intrinsic $k_\perp$?**

- with large intrinsic $k_\perp$ smooth distributions
- intrinsic $k_\perp$ affects only the low $k_\perp$ region & does not affect iTMD → problem for measurements

Interplay between pert. and non-pert. effects in the low $k_\perp$ with dyn z_M

And what about $z_M \approx 1$?

$z_M = 1 - 10^{-5}$ & $q_s = 0.5$ GeV, $z_M = 1 - 10^{-5}$ & $q_s = 0.0001$ GeV



bla

- effect of **intrinsic** $k_\perp$ visible only for small scales in the low $k_\perp$ region
- For higher scales it is **completely smeared by the evolution** effects
- exception: very large-x where there is no space for evolution

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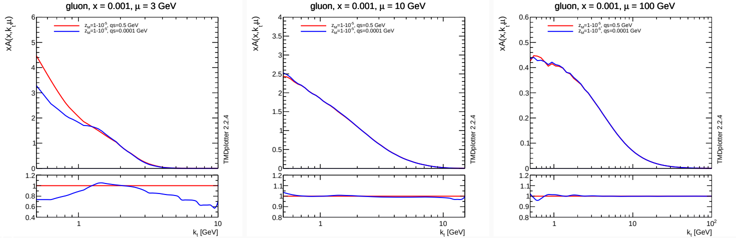

- effect of **intrinsic $k_\perp$** visible only for small scales in the low $k_\perp$ region
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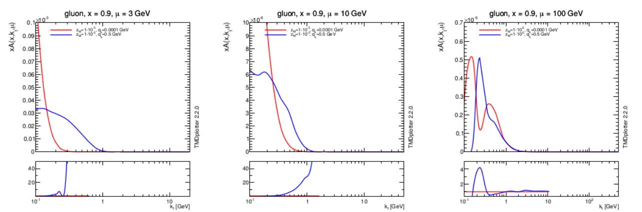
→ large-x data should be used to fit intrinsic kt



Fits of iTMDs in PB

Fit method

Bermudez Martinez, Connor, Hautmann, Jung, Lelek, Radescu, Zlebcik, Phys.Rev.D 99 (2019) 7, 074008

The parameters of the initial parton distributions have to be obtained from the fits to the experimental data $\rightarrow$ `xFitter` S. Alekhin, Eur.Phys.J.C 75 (2015) 7, 304

First **iTMDs** are fitted:

- kernel $K_{ab}(x'', \mu^2, \mu_0^2)$ from PB
- convolution with the starting distribution $f_{0,b}$
  $\widetilde{f}_a(x, \mu^2) = \int \mathrm{d}x' f_{0,b}(x', \mu_0^2) \frac{x}{x'} K_{ba}\left(\frac{x}{x'}, \mu^2, \mu_0^2\right)$
- $\widetilde{f}_a(x, \mu^2)$ convoluted with ME to obtain the F_2
- the procedure repeated with different $f_{0,b}$ until the minimal χ^2 is found.

To obtain **TMDs**:

- TMD kernel $K_a^b(x'', k_\perp, k_{\perp 0}^2, \mu^2, \mu_0^2)$ from PB
- convoluted with the initial distribution $A_{0,b}$

$$xA_a(x, k_\perp, \mu^2) = \int \mathrm{d}x' A_{0,b}(x', k_{\perp 0}^2, \mu_0^2) \frac{x}{x'} K_{ba}\left(\frac{x}{x'}, k_\perp^2, k_{\perp 0}^2, \mu^2, \mu_0^2\right)$$

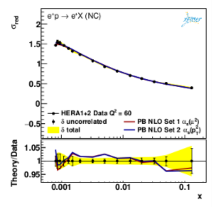
where $A_{0,b}(x', k_{\perp,0}^2, \mu_0^2) = \widetilde{f}_{b,0}(x, \mu_0^2) \frac{1}{\pi q_s^2} \exp\left(\frac{-k_{\perp 0}^2}{q_s^2}\right)$, with $f_{0,b}$ **from the fit of iTMDs**

The intrinsic transverse momentum is not constrained by the xFitter fit procedure, here fixed to $q_s = 0.5$ GeV recently constrained from DY $\rightarrow$ see later

Baseline PB distributions

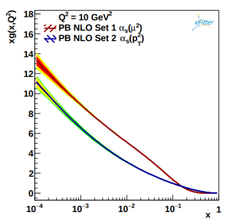
Bermudez Martinez, Connor, Hautmann, Jung, Lelek, Radescu, Zlebcik, Phys.Rev.D 99 (2019) 7, 074008

PB iTMDs are obtained from **HERAPDF2.0 recipe**: the same parametrization, heave flavour scheme, uncertainty calculation etc.
H1, ZEUS, Eur.Phys.J.C 75 (2015) 12, 580



Two scenarios, both very similar $\chi^2/\mathrm{d.o.f.} \approx 1.21$:

- PB-NLO-HERAI+II-2018-set1: $\alpha_s\left(\mu'^2\right)$, reproduces HERAPDF2.0 ✓
- PB-NLO-HERAI+II-2018-set2: $\alpha_s\left(q_\perp^2\right)$, different HERAPDF2.0 ✓



- data: **HERA H1 and ZEUS combined DIS** measurement
- range: $3.5 < Q^2 < 50000\ \mathrm{GeV}^2$, $4 \cdot 10^{-5} < x < 0.65$
- model uncertainties: variation of m_c, m_b, μ_0 (Set2: q_0 as a cut in α_s)
- initial parametrization in a form of HERAPDF2.0

TMDs and iTMDs available in **TMDlib**

TMDlib & TMDplotter

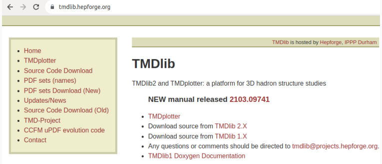
F. Hautmann et al., Phys.J.C 74 (2014) 3220

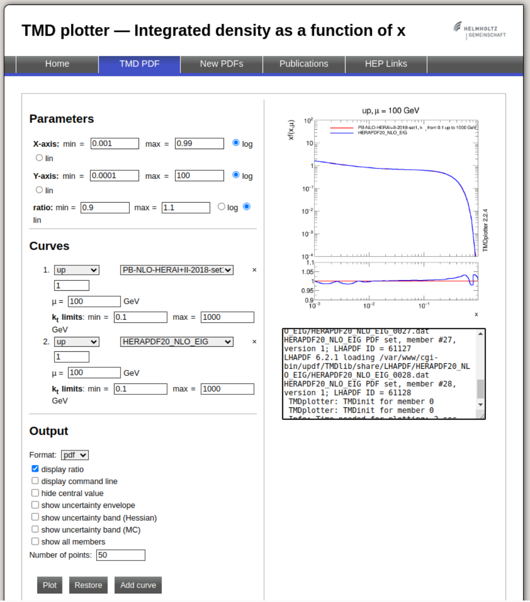
N.A. Abdulov et al., Eur.Phys.J.C 81 (2021) 8, 752

A library for TMDs, PDFs and unintegrated parton distributions (uPDFs)

allows for easy access to commonly used TMDs, PDFs and uPDFs

TMDplotter allows for web based plotting of distributions implemented in TMDlib & LHAPDF.

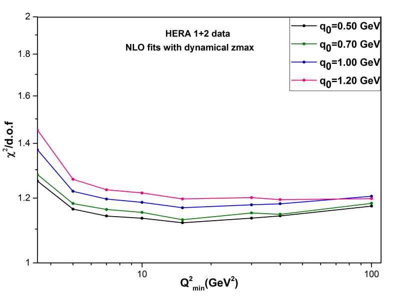


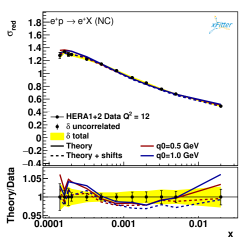


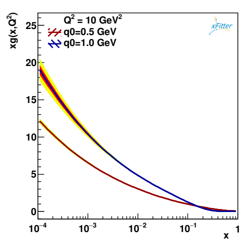
Fits with dynamical z_M

NEW RESULTS

- **Standard MCs use dynamical z_M in PSs but PDFs are fitted with fixed z_M**
  $\rightarrow$ Fits with dynamical z_M needed
- Is it possible to obtain **reasonable fit with dynamical z_M** within PB framework?
- **Which q_0** value to choose?







When low Q^2 data included in the fit, the $\chi^2/\mathrm{d.o.f}$ of the fit gets worse with increasing q_0 but it's still reasonable

$q_0 = 0.5$ GeV: $\chi^2/\mathrm{d.o.f} = 1.25$
$q_0 = 1.0$ GeV: $\chi^2/\mathrm{d.o.f} = 1.37$

Possible to obtain **good fit with dynamical z_M** even with low Q^2 data

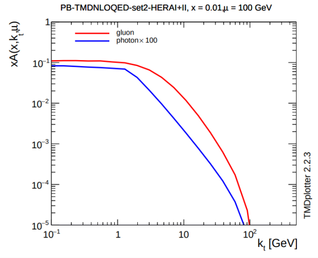
Photon TMD

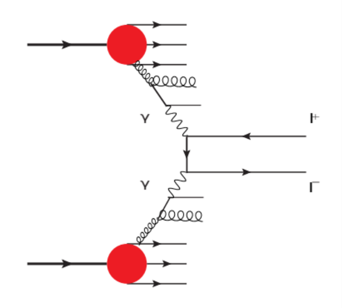
H. Jung, S. Taheri Monfared, T. Wening, Phys.Lett.B 817 (2021) 136299

$\alpha_s^2 \sim \alpha$ over a wide range of scales
$\rightarrow$ necessary to include electroweak (EW) corrections in the evolution

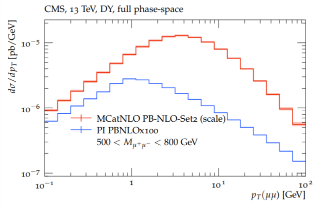
QED corrections included in the PB evolution by incorporating QED splitting functions for P_{qq}, $P_{q\gamma}$, $P_{\gamma q}$ and $P_{\gamma\gamma}$

PB (i)TMDs refitted & photon TMD obtained
other flavors in the "old" PBset2 not affected





difference in shape at large k_t from P_{gg} and no analogue in QED



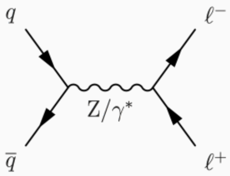
Contributions from $\gamma\gamma \rightarrow ll$ known to be sizable at high invariant mass with the photon TMD, the calculation possible with PB

PB predictions for DY

Predictions for DY

Drell-Yan process:

- is a "standard candle" for electroweak precision measurements at LHC
- helps to understand the QCD evolution, resummation, factorization (collinear, transverse momentum dependent (TMD))
- used for extraction of the PDFs
- at low mass and low energy gives access to partons' intrinsic $k_\perp$
- ...



The description of the DY data in a wide kinematic regime is problematic

DY with PB TMDs and Cascade3

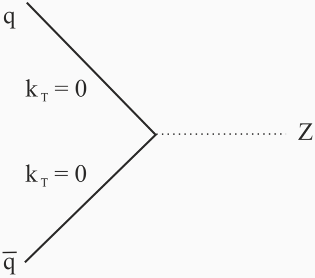
Bermudez Martinez et al., Phys.Rev.D 99 (2019) 7, 074008

S. Baranov et al., Eur.Phys.J.C 81 (2021) 5, 425

PB TMDs are used by **TMD MC generator CASCADE3** to obtain predictions

- ME obtained from standard automated methods used in collinear physics (Pythia, MCatNLO,...) with **k** added according to TMD

- DY collinear ME
- Generate $k_\perp$ of $q\bar{q}$ according to TMDs (m_{DY} fixed, x_1, x_2 change)
- compare with the 8 TeV ATLAS measurement



Phys. Rev. D 99, 074008 (2019)

In collinear MC transverse momentum comes from PS⇔ in PB method it is included in TMD

- For exclusive observables: Initial State TMD Parton Shower (PS)
- Final State PS, Hadronization via Pythia

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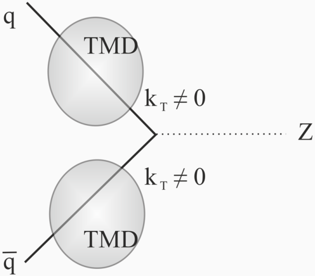

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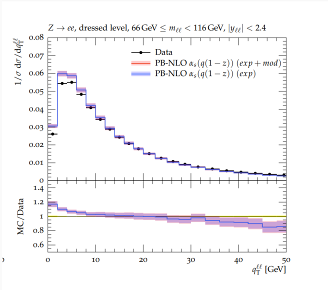

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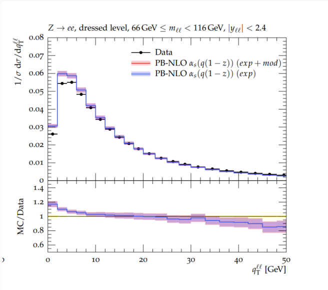

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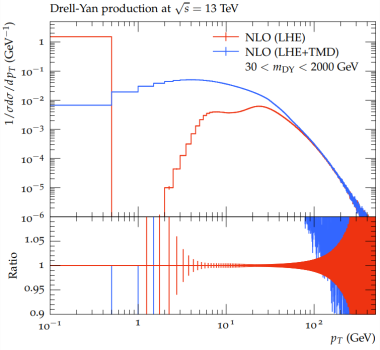
PB TMDs and MCatNLO for DY

A. Bermudez Martinez et al., Phys.Rev.D 100 (2019) 7, 074027

- standard MCatNLO: when ME matched with PS, **subtraction terms** (for soft and collinear contribution) must be used to avoid double counting
- Subtraction term depends on the PS to be used
- PB TMDs have similar role to PS
  $\rightarrow$ subtraction term has to be used to combine PB TMDs with NLO cross section
- PB uses AO, similar to Herwig6
  $\rightarrow$ **MCatNLO + Herwig6 subtraction used by PB TMD + MCatNLO** calculation

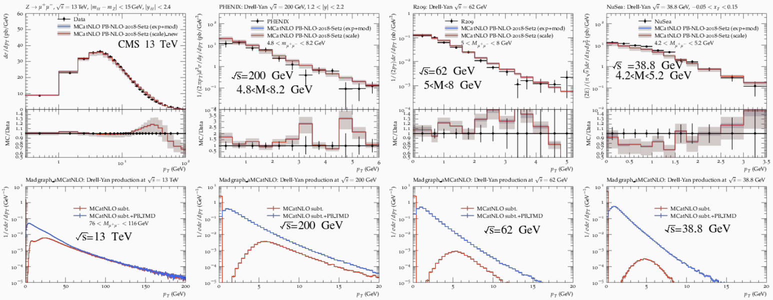


MCatNLO calculation with subtraction

k included in ME according to PB TMD

DY from fixed-target up to LHC

A. Bermudez Martinez et al., Eur.Phys.J.C 80 (2020) 7, 59



- Low and middle $p_\perp$ spectrum well described. At higher $p_\perp$ from Z+ jets important → see later
- **Good description of DY** from experiments **in different kinematic ranges**: NuSea, R209, Phenix, Tevatron, LHC. **No tuning/adjusting** of the method for different $\sqrt{s}$
- **"low $q_\perp$ crisis"** A. Bacchetta et al., Phys. Rev. D 100, 014018 (2019):
perturbative fixed order calculations in collinear factorization not able to describe DY p_T spectra at fixed target experiments for $p_T/m_{DY} \sim 1$ → we **confirm** this:
  - at larger masses and LHC energies the contribution from soft gluons in the region of $p_\perp/m_{DY} \sim 1$ is small and the spectrum driven by hard real emission.
  - at low DY mass and low $\sqrt{s}$ even in the region of $p_\perp/m_{DY} \sim 1$ the contribution of soft gluon emissions essential

Fitting of intrinsic kt

Intrinsic kt vs center-of-mass energy & DY mass

NEW RESULTS

Pythia, Herwig: the intrinsic $k_\perp$ is center-of-mass dependent

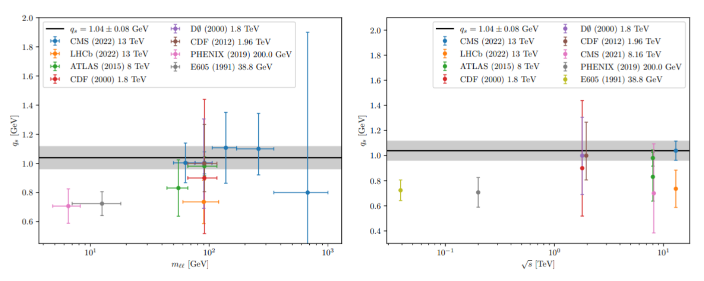
T. Sjostrand, Peter Z. Skands, JHEP 03 (2004) 053

Stefan Gieseke, Michael H. Seymour, Andrzej Siodmok, JHEP 06 (2008) 001

In PB/Cascade the situation different when **PB-NLO-HERAI+II-2018-set2** is used

Method:

- replicas of PB-NLO-HERAI+II-2018-set2 created with q_s scanned scanned between $q_s = 0.1$ and $q_s = 2.0$ GeV with a step of 0.1 GeV;
- prediction for each DY measurement obtained with each replica;
- for each measurement, the q_s providing the best χ^2 was extracted.



I. Bubanja et al., Eur.Phys.J.C 84 (2024) 2, 154

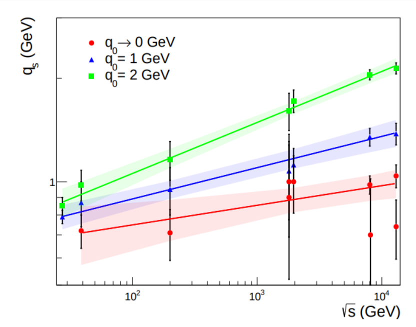
In PB the $\sqrt{s}$ and $m_{\rm DY}$ dependence of intrinsic kt much weaker than in other MCs

Intrinsic kt vs center-of-mass energy for dynamical z_M

NEW RESULTS

The center-of-mass **dependence of the intrinsic kt** comes from the treatment of **soft gluons**

study with models with $z_M = 1 - \frac{q_0}{\mu'}$ for different q_0 values & $\alpha_s(q_\perp)$
fixed $z_M \approx 1 \leftrightarrow q_0 \to 0$



I. Bubanja et al., 2404.04088

When q_0 $O(1\mathrm{GeV})$ is used, intrinsic kt depends on center-of-mass energy
The **slope increases with increasing q_0**
Including **non-perturbative Sudakov** ($z_M \to 1$) & $\alpha_s(q_\perp)$ **crucial for intrinsic kt (almost) independent of $\sqrt{s}$**

CS kernel

Recall: Non-perturbative Sudakov

In CSS formalism:

- Evolution of the TMD with respect to ζ given by CS kernel
  $\frac{\partial \ln \tilde{f}_{f/H}(x, b_t, \zeta, \mu)}{\partial \ln \sqrt{\zeta}} = \mathcal{D}(b_t, \mu)$
- **Sudakov** form factor of the modern CSS (CSS2)

$$\Delta_a^{\mathrm{CSS2}}(b, Q, Q_0, \mu_0) = \exp\left(-\int_{\mu_0^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left(\gamma_k(\alpha_s)\ln\left(\frac{Q^2}{\mu'^2}\right) + \gamma_j(\alpha_s)\right)\right) \times \exp\left(\mathcal{D}(b, \mu_0)\ln\frac{Q^2}{Q_0^2}\right)$$

The logarithmic structure in PB the same when non-perturbative Sudakov included

$$\Delta_a(\mu^2, \mu_0^2) = \Delta_a^{(\mathrm{P})}\left(\mu^2, \mu_0^2, q_0\right) \cdot \Delta_a^{(\mathrm{NP})}\left(\mu^2, \mu_0^2, \epsilon, q_0^2\right) .$$

if $z_M \approx 1$- non - perturbative PB Sudakov included, similarly to CSS:

$$\Delta_a^{(\mathrm{NP})}(\mu^2, \mu_0^2, \epsilon, q_0) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{1-q_0\mu'}^{1-\epsilon} dz \frac{k_a(\alpha_s)}{1-z}\right) = \exp\left(-\frac{k_a(\alpha_s)}{2}\ln\left(\frac{\mu^2}{\mu_0^2}\right)\ln\left(\frac{q_0^2}{\epsilon^2\mu_0\mu}\right)\right)$$

CS kernel:

- contains non-perturbative information
- can be extracted from measurements
- is the only QCD function which is largely unknown

Models for numerical studies

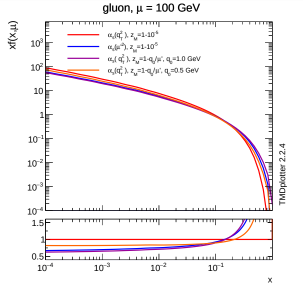
We study 4 PB models which **differ in the amount of (soft) radiation**.
Amount of radiation **modelled in terms of α_s and z_M**

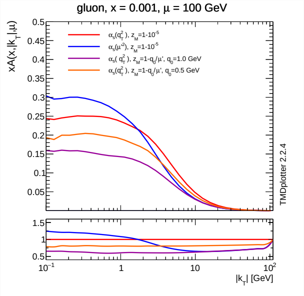
Models with fixed $z_M \approx 1$:

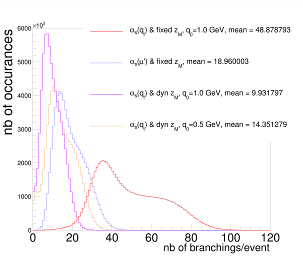
- $\alpha_s(q_\perp^2)$, $\alpha_s = \alpha_s(\max(q_0^2, q_\perp^2))$, $q_0 = 1.0$ GeV (red)
- $\alpha_s(\mu'^2)$ (blue)

Models with $\alpha_s(q_\perp^2)$ and dynamical $z_M = 1 - q_0/\mu'$ (i.e. no non-perturbative Sudakov):

- $q_0 = 1.0$ GeV (purple)
- $q_0 = 0.5$ GeV (orange)



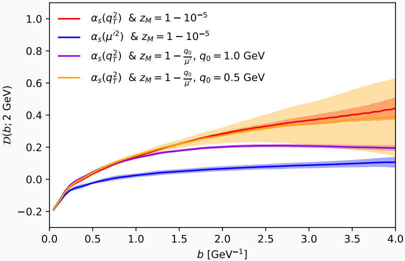




CS kernel

NEW RESULTS

The method of A. Bermudez Martinez and A. Vladimirov (Phys.Rev.D 106 (2022) 9, L091501) used to extract CS kernel from PB DY predictions



$$\mathcal{D}(b, \mu_0) = \frac{\ln(\Sigma_1(b)/\Sigma_2(b)) - \ln Z(Q_1, Q_2) - 2\Delta_R(Q_1, Q_2; \mu_0)}{4\ln(Q_2/Q_1)} - 1$$

Σ_1 and Σ_2 - Hankel transformed DY cross sections

$$\Delta_R(Q_1, Q_2; \mu_0) = \int_{Q_2}^{Q_1} \frac{d\mu}{\mu} \gamma_F(\mu, Q_1) - 2\ln\frac{Q_1}{Q_2} \int_{\mu_0}^{Q_2} \frac{d\mu}{\mu} \gamma_k(\mu)$$

$$Z(Q_1, Q_2) = \frac{\alpha_{em}^2(Q_1)|C_V(Q_1, \mu_{Q_1})|^2}{\alpha_{em}^2(Q_2)|C_V(Q_2, \mu_{Q_2})|^2}$$

where C_V is the hard coefficient function.

All terms except Σ_1/Σ_2 are perturbative and known up to up to N^{3}LO

- The extracted kernels in PB more than just the Δ_a^{NP}: it is a cumulative effect of many branchings, governed by α_s and z_M.
- different modelling of radiation can lead to a very different kernel behaviour, including different slopes.

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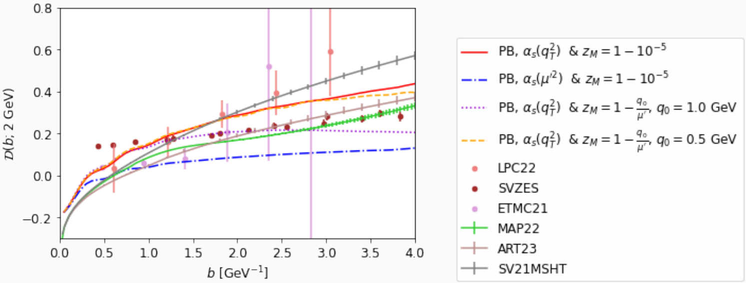
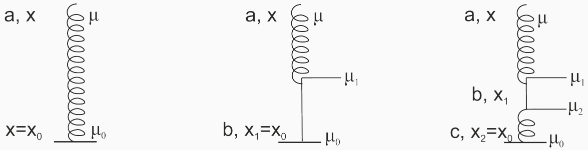


- The extracted kernels in PB more than just the Δ_a^{NP}: it is a cumulative effect of many branchings, governed by α_s and z_M.
- different modelling of radiation can lead to a very different kernel behaviour, including different slopes.
- The curves spread over a wide range, covering extractions from other groups

Exlusive observables

Backward evolution and TMD shower

S. Baranov et al., Eur.Phys.J.C 81 (2021) 5, 425



- For inclusive observables (e.g. DY) ME + TMDs

- For **exclusive observables: PS**

Cascade3: Initial State **TMD PS guided by the PB TMDs**

We start from a final parton a at a given x and μ and we it evolve back till μ_0

$$\Pi_a\left(\mu^2,\mu_0^2\right)=\exp\left(-\sum_b\int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\int_0^1\mathrm{d}z\ zP_{ab}^R\left(z,\mu^2\right)\frac{\widetilde{A}_b(x,k'_\perp,\mu')}{\widetilde{A}_a(x,k_\perp,\mu')}\right)$$

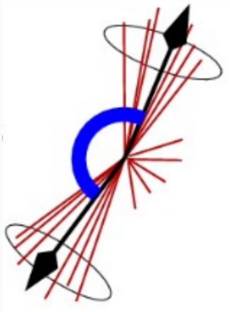
- currently the Final State PS, Hadronization via Pythia

Jet measurements

Measurements with jets allow to test our understanding of QCD by comparing predictions from different MCs

What do we look at?

- azimuthal correlations
- jet multiplicity
- jet $p_\perp$

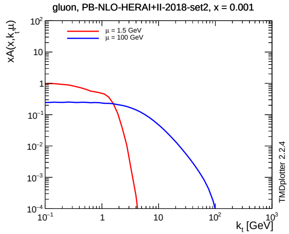


Are the TMDs important for high $p_\perp$ effects?

TMD effects at high $p_\perp$

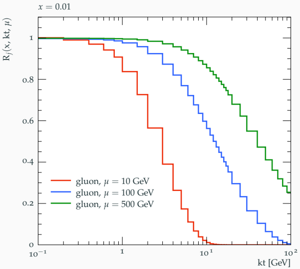
Bermudez Martinez, Hautmann, Mangano, Phys.Lett.B 822 (2021) 136700

It is commonly known that TMD effects play a role at scales $\mathcal{O}(\text{few GeV})$
Can TMDs also play a role at higher scales?



PB TMD: at $\mu \sim \mathcal{O}(1\ \text{GeV})$ TMD is a gaussian with $\Lambda_{QCD} < \sigma < \mathcal{O}(1\ \text{GeV})$. Effect of the evolution: $k_\perp$ accumulated in each step $\rightarrow$ **TMD broadening**

in PB: iTMDs (=PDFs) from TMD: $\widetilde{f}_a(x, \mu^2) = \int \mathrm{d}k_\perp^2\, \widetilde{A}_a(x, k_\perp, \mu^2)$



What is the contribution to the emission of an extra jet of $p_\perp < \mu$ from the $k_\perp$-broadening of the TMD?

$$R_j(x, k_\perp, \mu^2) = \frac{\int_{k_\perp^2}^{\infty} \mathrm{d}k'^2_\perp \widetilde{A}_j(x, k'_\perp, \mu^2)}{\int \mathrm{d}k'^2_\perp \widetilde{A}_j(x, k'_\perp, \mu^2)}$$

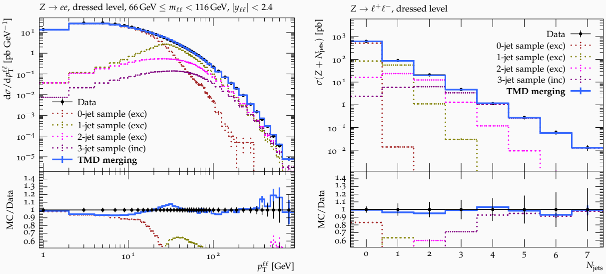
at LHC the contribution from high $k_\perp$ tail to jet emission comparable to perturbative emissions via hard ME!

TMDs and MLM Multi-Jet Merging

Bermudez Martinez, Hautmann, Mangano, Phys.Lett.B 822 (2021) 136700

Recall: DY at high $p_\perp$: large corrections from higher orders

TMD merging procedure developed (at LO)
extension of MLM method NPB 632 (2002) 343–362 to the TMD case



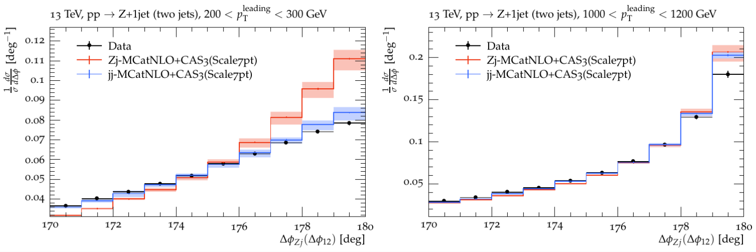
- The **merged prediction** provides **good description** of the data in the **whole DY $p_\perp$** spectrum
- jet multiplicity in $Z+$ jets production well described, also for multiplicities larger than the maximum nb of jets in MEs

PB TMD in azimuthal correlations

H. Yang et al., Eur.Phys.J.C 82 (2022) 8, 755

PB was used to compare azimuthal correlations in dijets and Z+jets

- sensitive to soft radiation
- probe of colour/spin correlations:
  different initial state, different FSR → potential interference between initial and final state different
  → Comparing these two processes one can look for the hints of factorization breaking



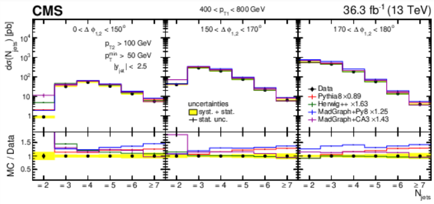
dijet data well described by PB TMD + MCatNLO
small deviation in $\Delta\Phi = \pi$ - to be studied further
Still missing: data for Z+jets at high $p_\perp$

PB at CMS

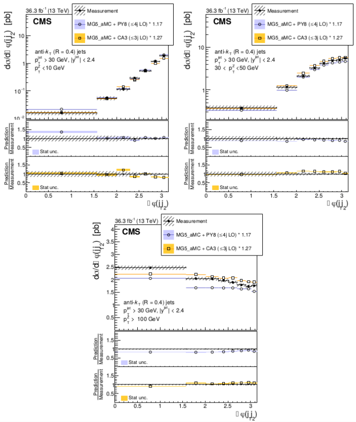
PB at CMS

Predictions from the PB method used in several CMS publications, e.g.:

- Measurements of jet multiplicity and jet transverse momentum in multijet events in proton-proton collisions at $\sqrt{13}$ TeV Eur.Phys.J.C 83 (2023) 8, 742
- Azimuthal correlations in Z+jets events in proton-proton collisions at $\sqrt{13}$ TeV Eur.Phys.J.C 83 (2023) 8, 722



Madrgrap +Cascade3 merged prediction (for $N > 2$) agree with data, similarly to HERWIG ++.



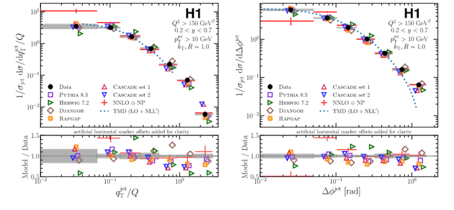
The predictions from Madgraph+Cascade agree with the measurements (in the regions where MPI effects are negligible)

PB at HERA

Lepton-Jet Correlation & 1-jettiness in DIS

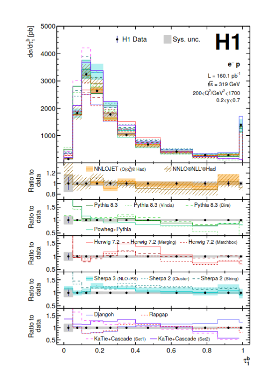
Predictions from the PB method used in two H1 publications

- Measurement of Lepton-Jet Correlation in Deep-Inelastic Scattering with the H1 Detector Using Machine Learning for Unfolding Phys.Rev.Lett. 128 (2022) 13, 132002
- Measurement of the 1-jettiness event shape observable in deep-inelastic electron-proton scattering at HERA 2403.10109



Lepton-jet production $e + p \to e + jet + X$ sensitive to TMDs when lepton-jet imbalance $q_\perp^{\text{jet}} = |\overrightarrow{p_\perp}^e + \overrightarrow{p_\perp}^{\text{jet}}|$ small $\longleftrightarrow$ small deviation from π in azimuthal angle $\Delta\Phi^{\text{jet}} = |\pi - (\Phi^e - \Phi^{\text{jet}})|$

Cascade + KaTie describe the data reasonably well at lower $q_\perp^{\text{jet}}/Q$ and $\Delta\Phi$



$\tau \to 0$: 2 jets, one along the beam direction from ISR and the other by the hard collision with the electron
$\tau \to 1$: > 2 jets
Cascade+KaTie gives good description at lower τ

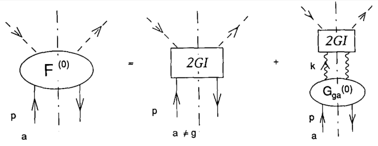
High-energy factorization

TMD Splitting functions

- Concept from **high-energy factorization** (Catani & Hautmann 94′)

  $k_\perp$ - factorization for DIS:

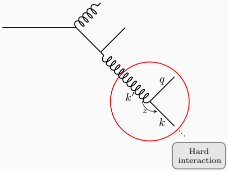
  $F^0(x, Q^2) = \int [\mathrm{d}\mathbf{k}] \int \frac{\mathrm{d}z}{z} \hat{\sigma}(z, \mathbf{k}, Q^2, \mu) G^0\left(\frac{x}{z}, \mathbf{k}, \mu\right)$
  G^0 - solution of BFKL equation



- originally TMD Pqg calculated

$$P_{qg}\left(\alpha_s, z, k'_\perp, \tilde{q}_\perp\right) = \frac{\alpha_s T_F}{2\pi} \frac{\tilde{q}_\perp^2\, z(1-z)}{(\tilde{q}_\perp^2 + z(1-z) k'^2_\perp)^2} \left[\frac{\tilde{q}_\perp^2}{z(1-z)} + 4(1-2z)\tilde{q}_\perp \cdot k'_\perp - 4\frac{(\tilde{q}_\perp \cdot k'_\perp)^2}{k'^2_\perp} + 4z(1-z)k'^2_\perp\right]$$

where $\tilde{q}_\perp = k_\perp - zk'_\perp$



Properties:

- **well defined collinear and high energy limits**:

  - for $k'^2_\perp \ll k^2_\perp$, after angular average:
  TMD $P_{qg} \rightarrow$ LO DGLAP P_{qg}

  - for finite $k'^2_\perp$, $k'^2_\perp \sim \mathcal{O}(k^2_\perp)$:
  expansion in $(k'^2_\perp / \tilde{q}^2_\perp)^n$, with z-dependent coefficients

  resummation of $\ln \frac{1}{z}$ at all orders in α_s via convolution with TMD gluon Green's functions

- positive definite

Other channels by Gituliar, Hentschinski, Kusina, Kutak & Serino (2015 – 2017)

High energy kt-factorization & PB

Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek, Phys.Lett.B 833 (2022) 137276

Idea: **replace DGLAP P by TMD P**

goal: incorporate **both small-x and Sudakov** contributions

$$\widetilde{A}_a\left(x, k_\perp^2, \mu^2\right) = \Delta_a\left(\mu^2, \mu_0^2\right)\widetilde{A}_a\left(x, k_\perp^2, \mu_0^2\right) +$$

$$\sum_b \int \frac{\mathrm{d}^2\mu_{\perp 1}}{\pi\mu_{\perp 1}^2}\Theta\left(\mu_{\perp 1}^2 - \mu_0^2\right)\Theta\left(\mu^2 - \mu_{\perp 1}^2\right)\Delta_a\left(\mu^2, \mu_{\perp 1}^2\right)\int_x^{z_M}\mathrm{d}z P_{ab}^R\left(z, k_\perp + (1-z)\mu_{\perp 1}, \mu_{\perp 1}\right)\widetilde{A}_b\left(\frac{x}{z}, |k_\perp + (1-z)\mu_{\perp 1}|^2, \mu_{\perp 1}^2\right)$$

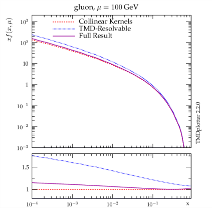
What to do with the Sudakov form factor?

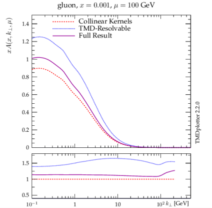
- collinear $\Delta_a\left(\mu^2, \mu_0^2\right)$
- **newly constructed TMD Sudakov**

$$\Delta_a\left(\mu^2, \mu_0^2\right) \;\rightarrow\; \Delta_a\left(\mu^2, \mu_{\perp 1}^2, k_\perp^2, \right) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\int_0^{z_M}\mathrm{d}z\, z\overline{P}_{ba}^R\left(z, k_\perp^2, \mu'^2\right)\right),$$ $\overline{P}$ - angular averaged P

momentum sum rule & unitarity crucial

Only with TMD Sudakov momentum sum rule satisfied





First parton branching algorithm to TMDs and PDFs which **includes TMD P and fulfils momentum sum rule**

first step towards a full TMD MC covering the small-x

Conclusions

Summary & Conclusions

- **TMD Parton Branching**: a MC method to obtain QCD collider predictions based on TMDs
- PB: **TMD evolution** equation to obtain TMDs; **TMDs can be used in TMD MC generators** to obtain predictions:
  - fits with xFitter
  - matching NLO ME with PB TMDs
  - merging
  - TMD PS

Discussed today:

- the **PB TMD evolution equation** and its relation to other approaches (DGLAP, CMW, CSS, high energy factorization)
- the soft gluon **resolution scale** and its interplay with the **intrinsic kt**
- fits of the PB (i)TMDs to HERA and DY data
- examples of the PB method applications: **DY** at different $\sqrt{s}$, m_{DY}, **DY+jets**, azimuthal correlations in Z+jest and **multijets**, **jets at DIS**

The TMD PB method: **flexible & widely applicable MC approach** to obtain QCD high energy predictions

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Thank you!