Multi-Jet Merging in Deep Inelastic Scattering with PYTHIA

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UNIVERSITY OF JYVÄSKYLÄ

Research Council of Finland

Centre of Excellence in Quark Matter

Outline

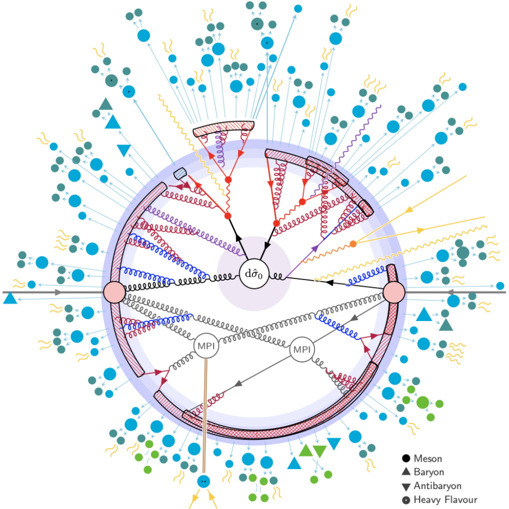


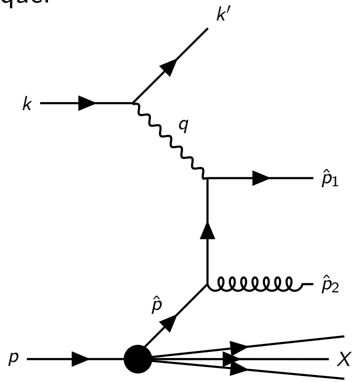
Image by P. Skands

Jet production in DIS

- DIS modelling lacking behind LHC-driven improvements in PYTHIA.
- Good description of DIS jets important with EIC in mind.
- Multiple hard scales, hardest scale not unique.

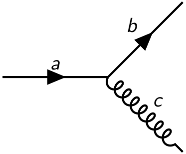
$$\mu_{\mathrm{F}}^2 = Q^2$$

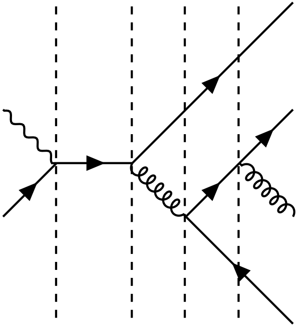
$$\mu_{\mathrm{F}}^2 = \left(Q^2 + \left(\frac{1}{N}\sum_{i=1}^{N} \hat{p}_{T,i}\right)^2\right)/2$$



seen success in [J. Currie et al., 1703.05977] and [S. Höche et al., 1809.04192]

Parton Showers





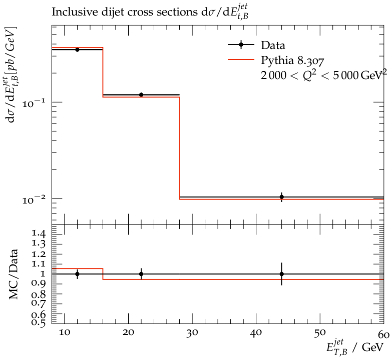
$$\mu_F^2 = \rho_0 > \rho_1 > \rho_2 > \rho_3$$

Differential probability for particle a splitting into b and c from DGLAP.
Evolution from large factorization scale μ_F to low hadronization regime Λ_{QCD}.

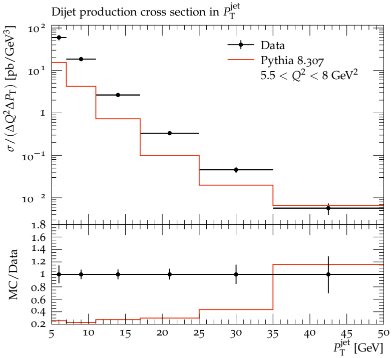
$$\mathrm{d}\mathcal{P}_a = \frac{\mathrm{d}\rho}{\rho}\frac{\alpha_s}{2\pi}\sum_{b,c=\{q,g\}}\hat{P}_{a\to bc}(z)\mathrm{d}z$$

Motivation - Parton shower approximation for dijets

- Good agreement in high-Q^2 using Power Shower.
- Poor description of low-Q^2 dijets.



[ZEUS Collaboration 1010.6167]

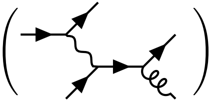


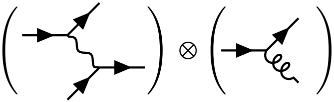
[H1 Collaboration, 1611.03421]

Multi-Jet Merging

- Objective: Combine parton showers with matrix elements
- Separate phase space using merging scale.

Hard jets : $p_T > t_{\mathrm{MS}}$ Soft jets : $p_T \leq t_{\mathrm{MS}}$

(ME) 

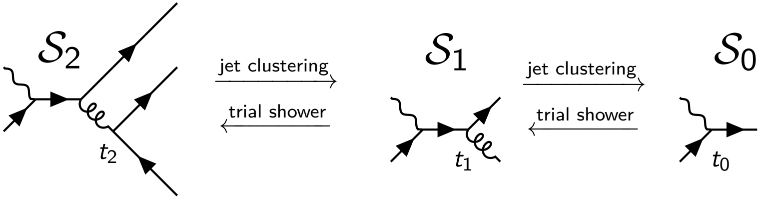
(PS) 

SHERPA generated parton level events (ME) up to +4 jets. MADGRAPH5 reliably up to +2 jets.
[T. Gleisberg et al., 0811.4622] [J. Alwall et al., 1405.0301]

VINCIA as the parton shower (PS)
[C.T.Preuss et al., 2003.00702]

```
("PartonShowers:model = 2");
```

Merging - Algorithm visually



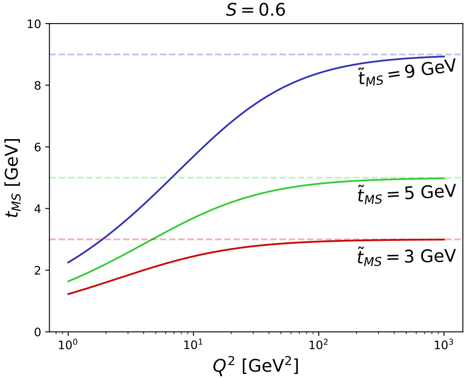
1. Start from ME event which passed the t_{MS} cut.
2. Cluster from shower state $\mathcal{S}_n$ to obtain most probable shower history $\{\mathcal{S}_1, \mathcal{S}_2 \ldots, \mathcal{S}_{n-1}\}$ and corresponding scales $\{t_1, t_2, \ldots, t_n\}$.
3. Starting from $\mathcal{S}_0$, generate trial emissions.
4. Reweight with merging weight, consisting of $\Pi_{\mathcal{S}_i}(t_i, t_{i+1})$, $\frac{f(x_i,t_i)}{f(x_i,t_{i+1})}$ and $\frac{\alpha_{s,PS}(t_i)}{\alpha_{s,ME}}$.

Merging - t_{MS}-dependence

Phase space separation with dynamic merging scale[1].

- Attempt to solve problem of low-Q^2 jet production.
- Let MEs contribute more in low-Q^2 region.

$$t_{\mathrm{MS}} = \tilde{t}_{\mathrm{MS}} \frac{1}{\sqrt{1 + \frac{\tilde{t}_{\mathrm{MS}}^2}{S^2 Q^2}}}$$



[1] Sherpa DIS jet merging [T. Carli, T. Gehrmann, S. Höche, 0912.3715]

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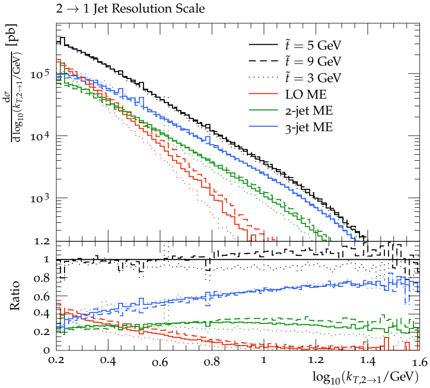
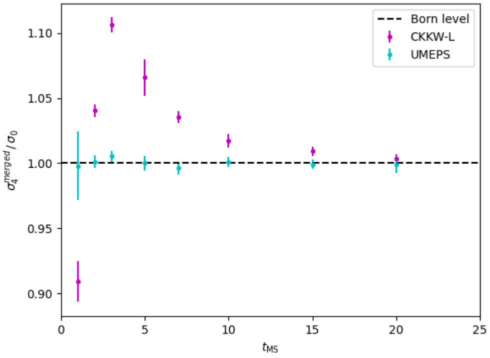


[1] SHERPA DIS jet merging [T. Carli, T. Gehrmann, S. Höche, 0912.3715]

Merging - Algorithms

Merging algorithm effect on inclusive cross section. DIS +4 jets.

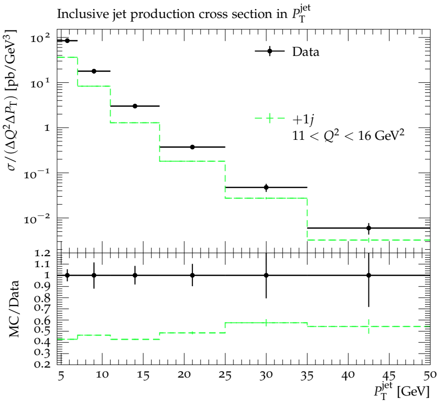
- CKKW-L[1], dynamic merging
  - phase space separation t_{MS} and reweighting using PS histories to account for double-counting.
- UMEPS[2], fixed merging
  - similar, but preserves unitarity.

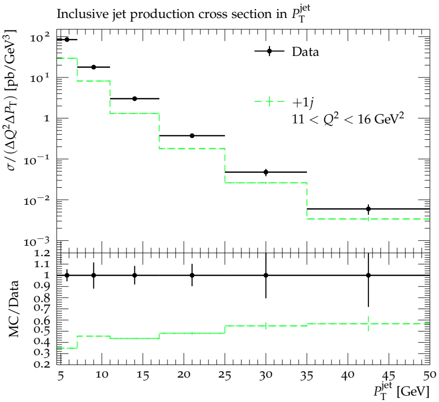


[1] [L. Lönnblad, 0112284]
[2] [L. Lönnblad, S. Prestel, 1211.4827]

Results - Inclusive jets

We compare the results to HERA data. [H1 Collaboration, 1611.03421]





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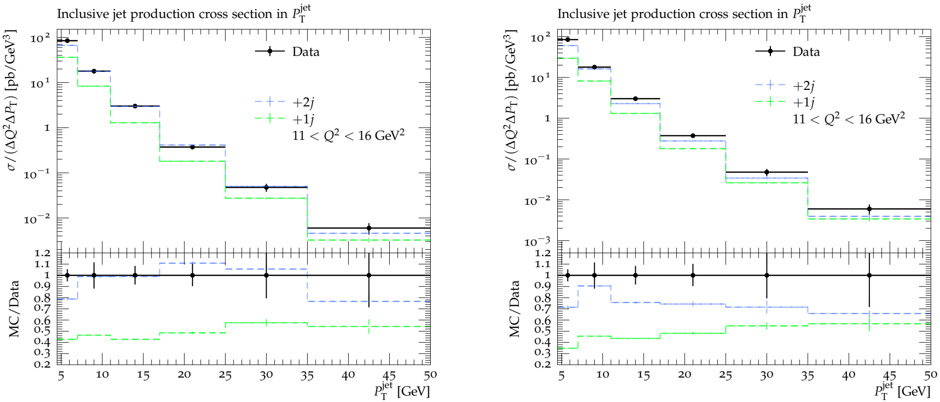

Satisfactory result with just 2 additional jets.

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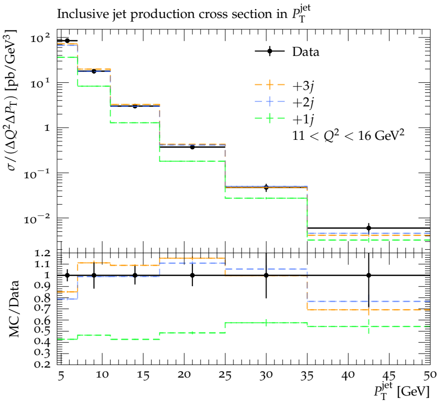

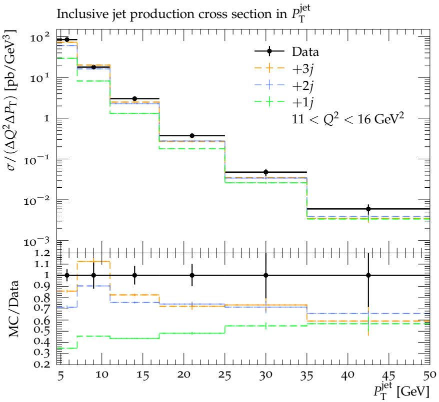

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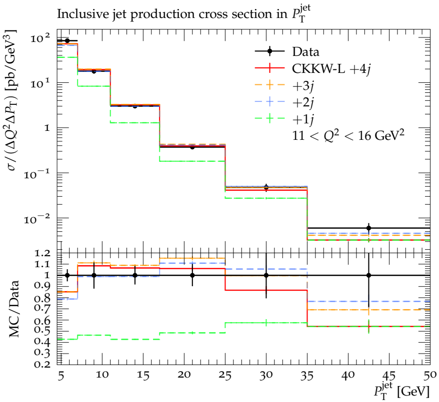
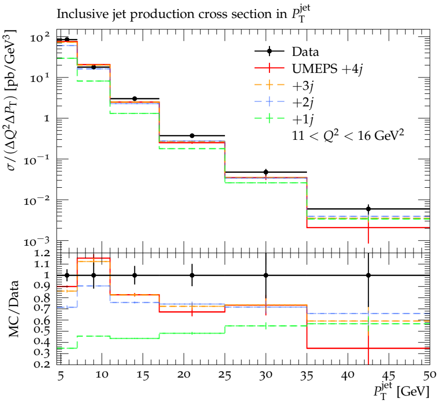
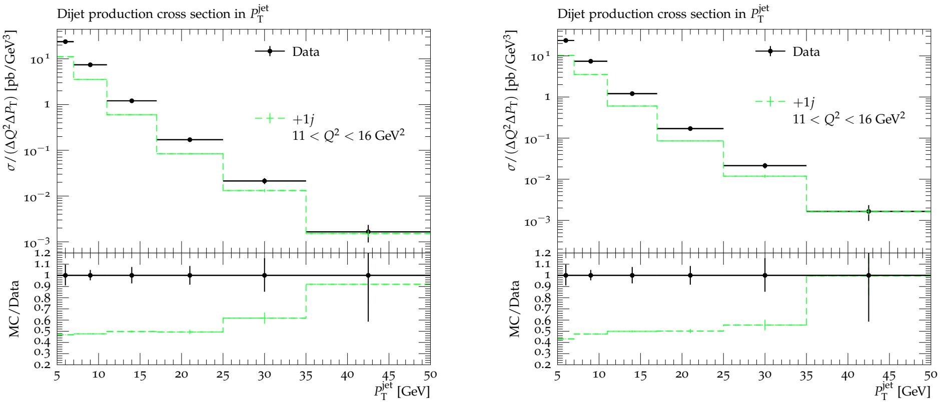


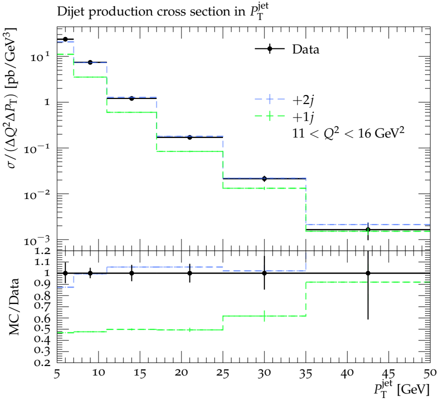


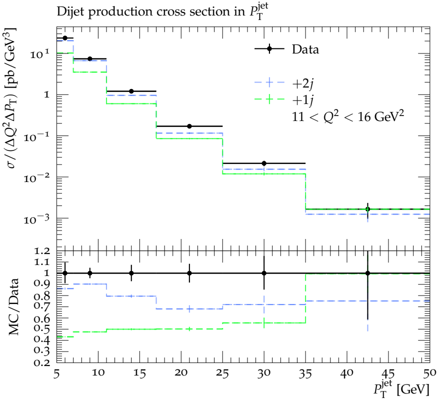
Satisfactory result with just 2 additional jets.
Result converges with addition of higher multiplicity samples.

Results - Dijets



Results - Dijets





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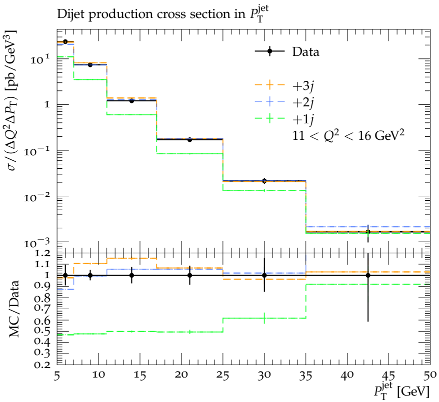

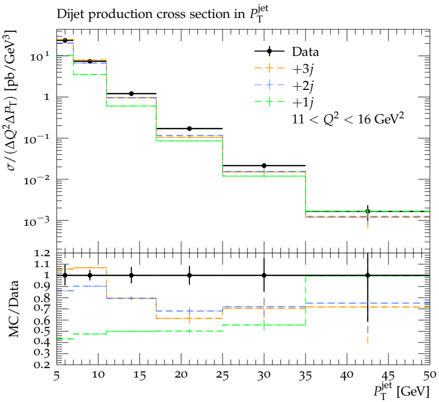

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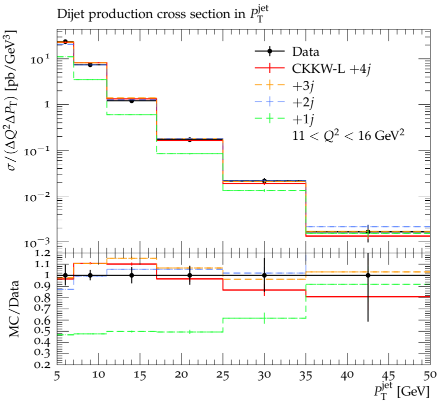
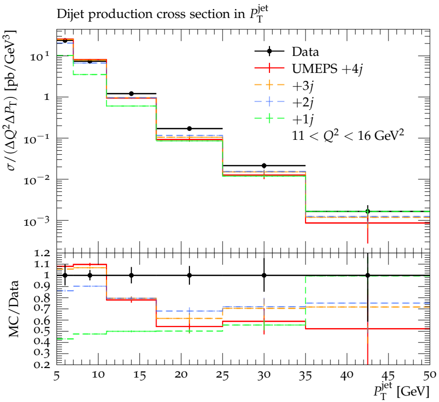
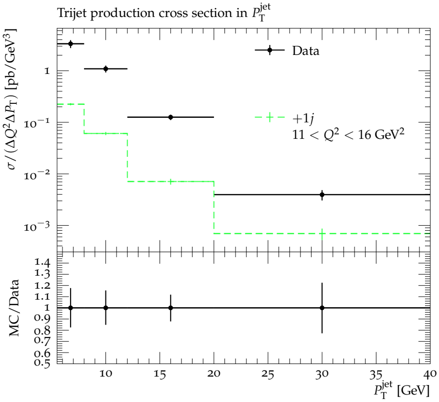
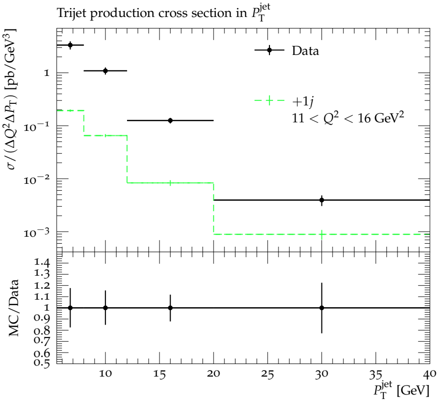




[1611.03421] RIVET analysis to be released by us (along with [hep-ex/0206029])

Results -Trijets





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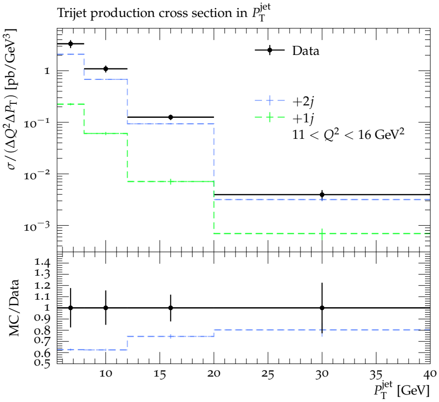

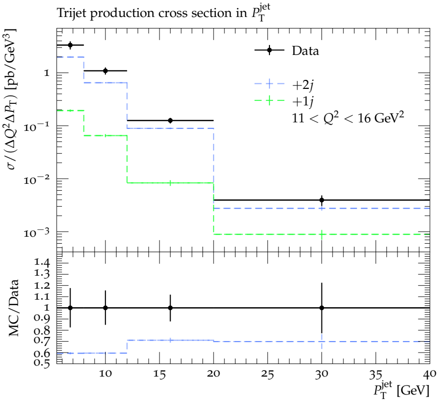

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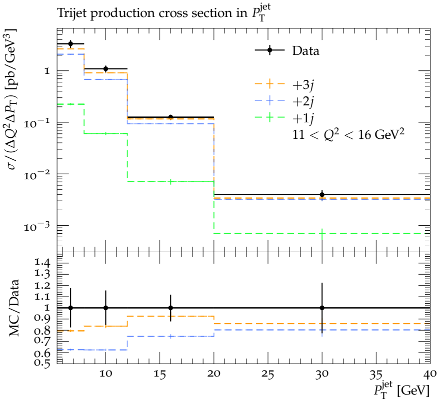

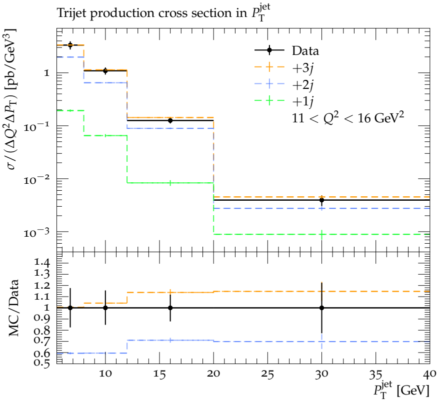

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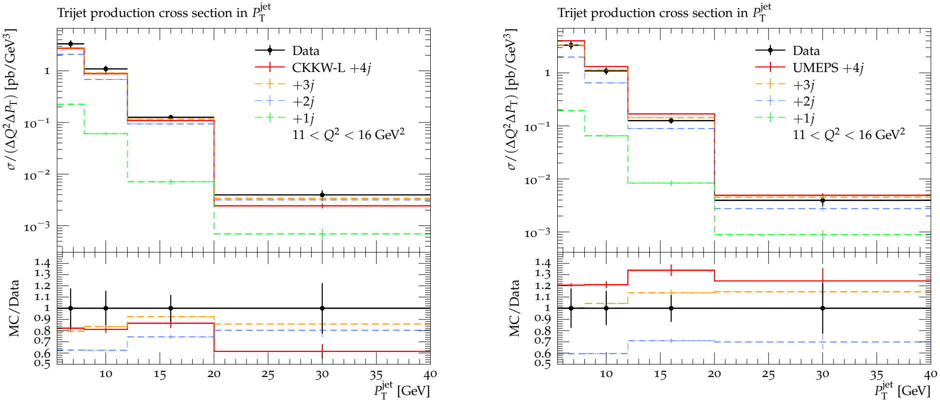
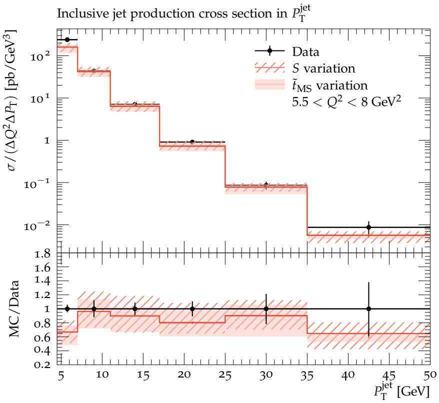
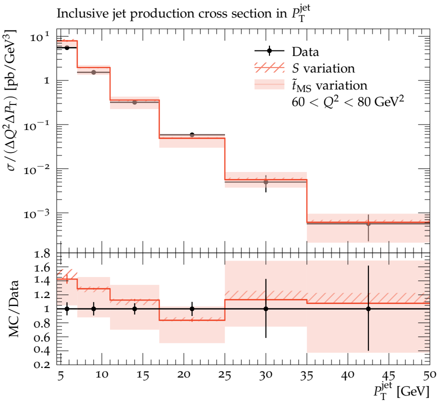


Competent trijet modeling requires at least +3-jet merging.

Results - CKKW-L parameter variations

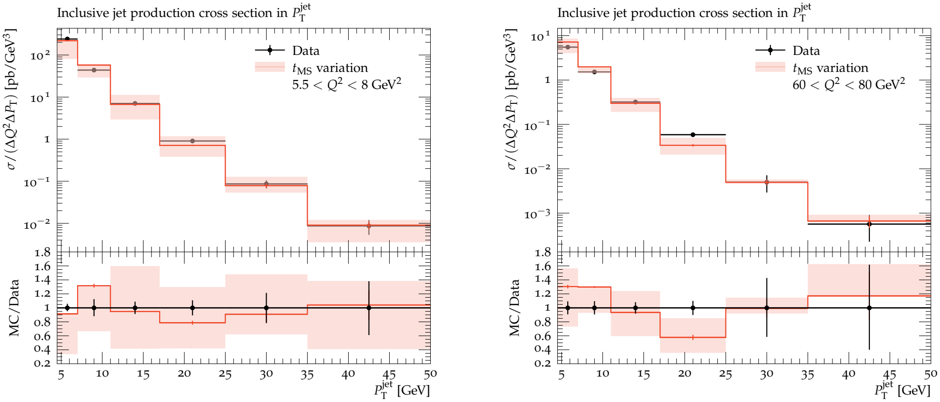
Variation of $S = 0.8 \pm 0.1$ and $\tilde{t}_{\mathrm{MS}} = 5$ between 3 and 9 GeV.





S-parameter mostly influences low-Q^2 region.

Results - UMEPS parameter variations

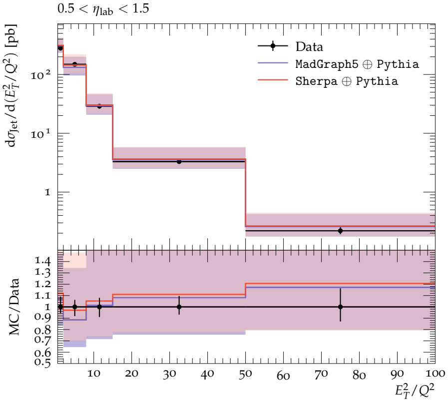


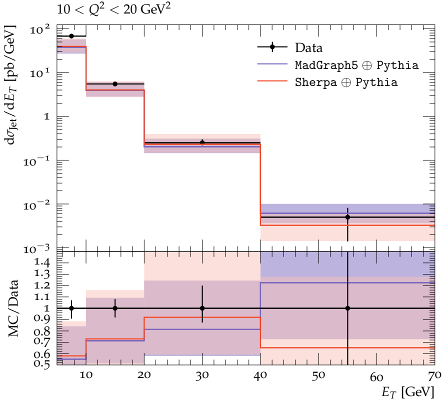
Moderate sensitivity to fixed merging scale variations.

Results - Scale variations

Also possible to use MG5 input. CKKW-L +2 jets.

Uncertainty bands by varying the scales μ_R and μ_F by a factor of 2.

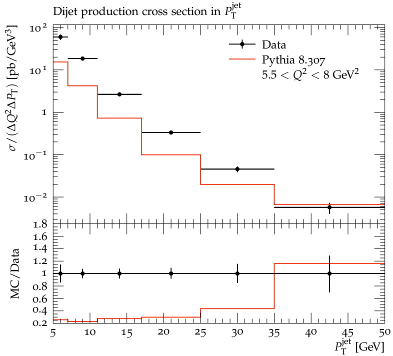




[H1 Collaboration, hep-ex/0206029]

Summary

- Starting point: default PYTHIA shower not enough to describe jets in DIS $\rightarrow$ merging implementation.
- Problem: Hardest scale not unique $\rightarrow$ use dynamic merging scale or different factorization scale choice.
- Multi-jet merging provides good description of HERA data also in low-Q^2 region.



Outlook: Working DIS jet merging setup for an upcoming PYTHIA release, possibly QED-clusterings to VINCIA merging.
Upcoming projects: Matrix element corrections.

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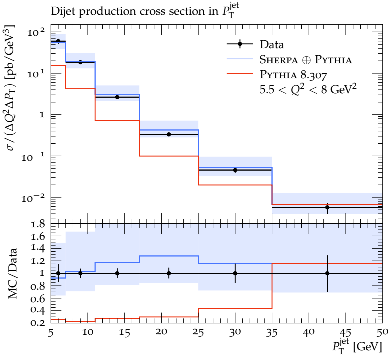

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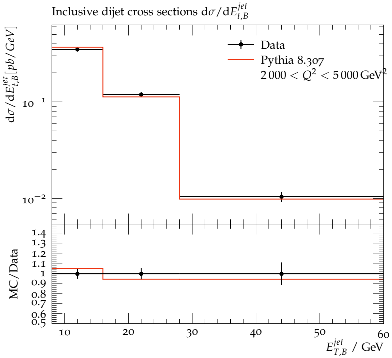

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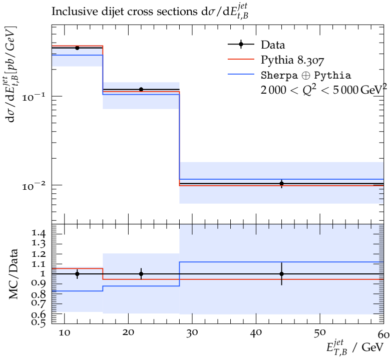

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Fixed order calculations / Matrix elements

Why not just use exact fixed order calculations all the way to a given final state multiplicity? Why stop at +4 jets?

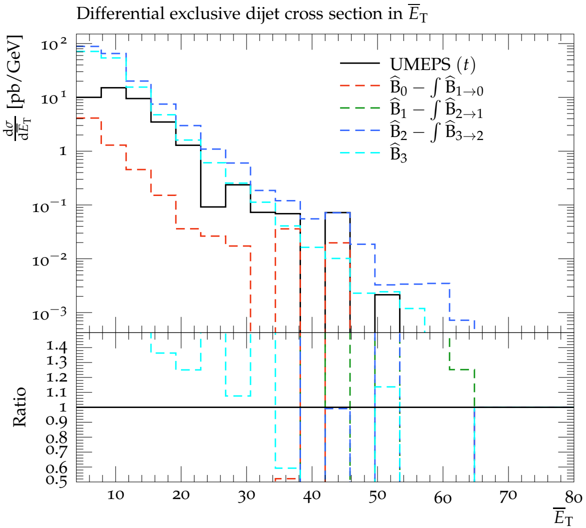
Number of diagrams grows almost **factorially** as a function of N.

Final states are observed to contain **hundreds** of particles.

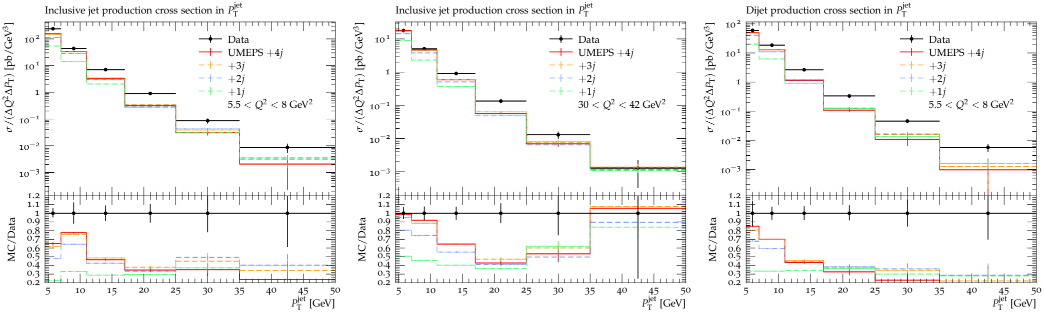
Process	# of diagrams
$e^- p \to e^- j$	20
$e^- p \to e^- 2j$	60
$e^- p \to e^- 3j$	720
$e^- p \to e^- 4j$	5 340
$e^- p \to e^- 5j$	71 080

Access description of up to 5 hard jets with matrix elements.

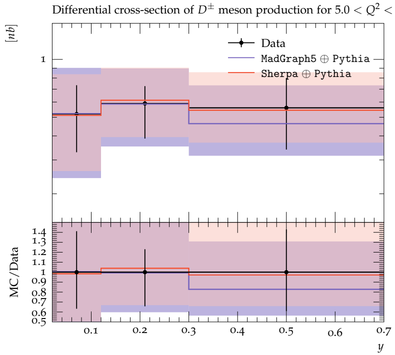
UMEPS dynamical merging scale

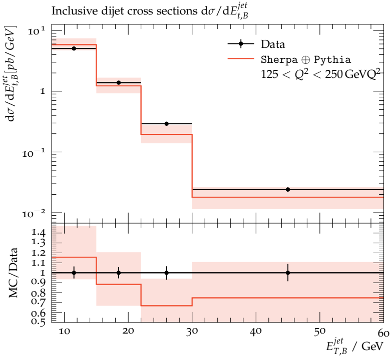


UMEPS $\mu_F^2 = \left(Q^2 + \overline{\hat{p}_T^2}\right) / 2$ scale



Other analyses: ZEUS_2008_I810112 and ZEUS_2010_I875006





Hard events are generated at leading order in QCD using a HPC-enabled version of the event generator[2] and stored using the efficient and scalable HDF5 event-file format. We use the G-scheme with electroweak input parameters

$$m_W = 80.419\ e\,, \quad m_Z = 91.188\ e\,, \quad G_{\mathrm{F}} = 1.16639 \times 10^{-5}\ e^{-2}\,. \qquad (1)$$

Quark masses are taken in the four-flavour scheme and the b-quark mass is set to $m_b = 4.7\ e$. The PDF choice is `NNPDF40_lo_pch_as_01180`.

[2]Available at `https://gitlab.com/hpcgen/me`.

I. Produce *Les Houches* event files (LHEF) [25] with a matrix element generator for $n = 0, 1 \dots N$ extra jets with a regularisation cut-off, $\rho_{\rm MS}$, typically using a fixed factorisation scale, μ_F, and a fixed $\alpha_{\rm s}(\mu_R)$.

II. Pick a jet multiplicity, n, and a state S_{+n} according to the cross sections given by the matrix element generator.

1. Find all shower histories for the state S_{+n}, pick a sequence according to the product of splitting probabilities. Only pick un-ordered sequences if no ordered sequence was found. Only pick incomplete paths if no complete path was constructed.
2. Perform reweighting: For each $0 \leqslant i < n$,
   i. Start the shower off the state S_{+i} at ρ_i, generate a trial state R with scale ρ_R. If $\rho_R > \rho_{i+1}$, veto the event and start again from II.
   ii. Calculate the weight factor

$$w_i = \frac{\alpha_{\rm s}(\rho_{i+1})}{\alpha_{\rm s}(\mu_R)} \frac{x_i^+ f_i^+(x_i^+, \rho_i)}{x_i^+ f_i^+(x_i^+, \rho_{i+1})} \frac{x_i^- f_i^-(x_i^-, \rho_i)}{x_i^- f_i^-(x_i^-, \rho_{i+1})} \tag{4.16}$$

3. Start the shower from S_{+n}.
   i. If $n < N$, start the shower at ρ_n, veto any shower emission producing an additional resolved jet.
   ii. If $n = N$, start the shower at ρ_n.

III. If the event was not rejected, multiply the event weight by

$$w_n' = \frac{x_n^+ f_n^+(x_n^+, \rho_n)}{x_n^+ f_n^+(x_n^+, \mu_F)} \times \frac{x_n^- f_n^-(x_n^-, \rho_n)}{x_n^- f_n^-(x_n^-, \mu_F)} \times \prod_{i=0}^{n-1} w_i \tag{4.17}$$

V. Start again from II.

The second part, i.e. producing $\int_s \widehat{\rm B}_{n \to m}$−events to effect lower-multiplicity PS resummation, requires only two changes:

II.3 Replace the matrix-element state by S_{+n-1}, or the first state S_{+l} with all $l \leq n-1$ partons above the merging scale. If no integrated state can be constructed, i.e. if only incomplete paths were found, reject the event. For valid events, start the shower at ρ_n, veto any shower emission producing an additional resolved jet.

1. Calculate cross sections and generate events according to step 1 and 2 in section 2.1. $Q_{\rm MS}$ denotes the merging scale which is equal to the matrix element cutoff and may be defined using any choice of scale. The events are generated using a fixed strong coupling, $\alpha_{\rm sME}$, and a maximum parton multiplicity, N.
2. Construct a full cascade history by considering all possible ordered histories and selecting one randomly with a probability proportional to the product of the branching probabilities. If no ordered histories can be constructed, unordered ones are considered. This results in a set of intermediate states $(S_2, S_3, ...S_n)$ and scales $(\rho_2 = \rho_{\max}, \rho_3, ..., \rho_n)$. S_2 denotes here denotes the constructed $2 \to 2$ process and S_n is the state given by the matrix element. n is the parton multiplicity in the event, $\rho_{\max}$ is the maximum scale of the process and ρ_i is the constructed scale where the state S_{i-1} emits a parton to produce the state S_i.
3. Reweight the events with $\prod_{i=3}^n \alpha_s(\rho_i)/\alpha_{\rm sME}^{n-2}$.
4. For each state S_i (except S_n), generate an emission with ρ_i as starting scale and if this emission occurred at a scale larger than ρ_{i+1} reject the event. This is equivalent to reweighting with a factor $\prod_{i=2}^{n-1} \Delta_{S_i}(\rho_i, \rho_{i+1})$.
5. For the last step there are two cases.
   - If the event does not have the highest multiplicity $n < N$, generate an emission from the state S_n with ρ_n as starting scale. If the emission is above the merging scale $Q_{\rm MS}$, reject the event. Otherwise accept the event and continue the cascade.
   - If the event has the highest possible multiplicity $n = N$, accept the event and start the cascade from the state S_n with the scale ρ_n.

For CKKW-L 1-jet merging to preserve unitarity, the first PS emission has to exactly match the 1-jet ME. This can be done (matching), but then there would be no need for 1-jet CKKW-L. For 2-jet merging, ”the splitting kernels need to exactly reproduce the matrix element, phase space must be fully covered by the parton shower, and the no-emission probabilities need to be produced identically in both cases. Particularly the requirement that the phase space is completely covered is problematic, since parton showers commonly fill only phase space regions in which consecutive emissions are ordered in a decreasing evolution variable.”[3]

[3] [L. Lönnblad, S. Prestel, 1211.4827]

UMEPS preserves the Born level cross section by integrating over $n+1$-jet phase space to induce resummation in n-jet cross section. The idea is to subtract in n what you add in $n-1$.

$$\sigma^{\mathrm{inc}} = \int \mathrm{d}\phi_0 \left(\frac{\mathrm{d}\sigma_0^{\mathrm{exc}}}{\mathrm{d}\phi_0} + \frac{\mathrm{d}\sigma_1^{\mathrm{inc}}}{\mathrm{d}\phi_0} \right) \tag{2}$$

$$= \int \mathrm{d}\phi_0 \left(\frac{\mathrm{d}\sigma_0^{\mathrm{inc}}}{\mathrm{d}\phi_0} - \frac{\mathrm{d}\sigma_{1\to 0}^{\mathrm{inc}}}{\mathrm{d}\phi_0} + \frac{\mathrm{d}\sigma_1^{\mathrm{inc}}}{\mathrm{d}\phi_0} \right) \tag{3}$$