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Spin correlation, entanglement and causality connection in top-antitop events

R. Demina (CMS collaboration)

University of Rochester

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Spin correlation and entanglement

Polarization, P and spin correlation matrix, C determine the angular distribution of the decay products in the helicity basis as in [\[1212.4888\]](#)

$$\frac{d\sigma}{d\Omega d\bar{\Omega}} = \sigma_{norm} (1 + \kappa \vec{P} \cdot \vec{\Omega} + \bar{\kappa} \vec{\bar{P}} \cdot \vec{\bar{\Omega}} - \kappa \bar{\kappa} \vec{\Omega} \cdot \vec{C} \cdot \vec{\bar{\Omega}})$$

κ - spin analyzing power of top/antitop decay products

$\vec{\Omega}$ – unit vector in the direction of the decay product

$2 \times 3(P) + 3 \times 3(C) = 15$ coefficients collectively called Q_m

Alternatively, we can define χ - opening angle between the two decay products, then

$$\frac{d\sigma}{d \cos \chi} = A(1 + D\kappa\bar{\kappa} \cos \chi) \quad C_{nn} + C_{rr} + C_{kk} = Tr(C) = -3D$$

and $\tilde{\chi}$, where the sign of n-component in one of the decay products is inverted

$$\frac{d\sigma}{d \cos \tilde{\chi}} = A(1 + \tilde{D}\kappa\bar{\kappa} \cos \tilde{\chi}) \quad C_{nn} - C_{rr} - C_{kk} = 3\tilde{D}$$

The system is considered **separable** if its density matrix can be factored into that of individual states

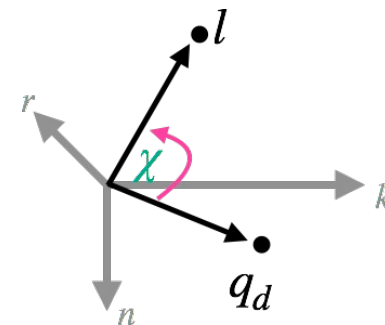
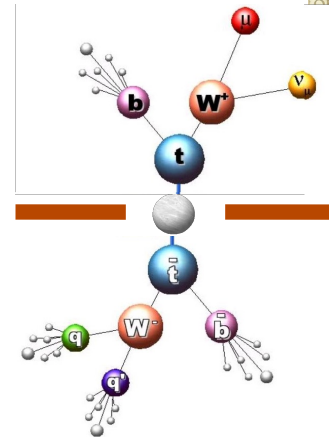
$$\rho = \sum_n p_n \rho_n^t \rho_n^{\bar{t}}$$

Otherwise, it is considered **entangled** \rightarrow **Peres-Horodecki criterion** [\[2003.02280\]](#)

Entanglement is a result of spin correlation.

Two approaches – both presented

- Use full angular information of two decay products (e.g. charged leptons, or a lepton and a d-type quark) to measure the full matrix C and then construct Δ_E
- Use the distribution in χ and $\tilde{\chi}$ to measure D and \tilde{D}



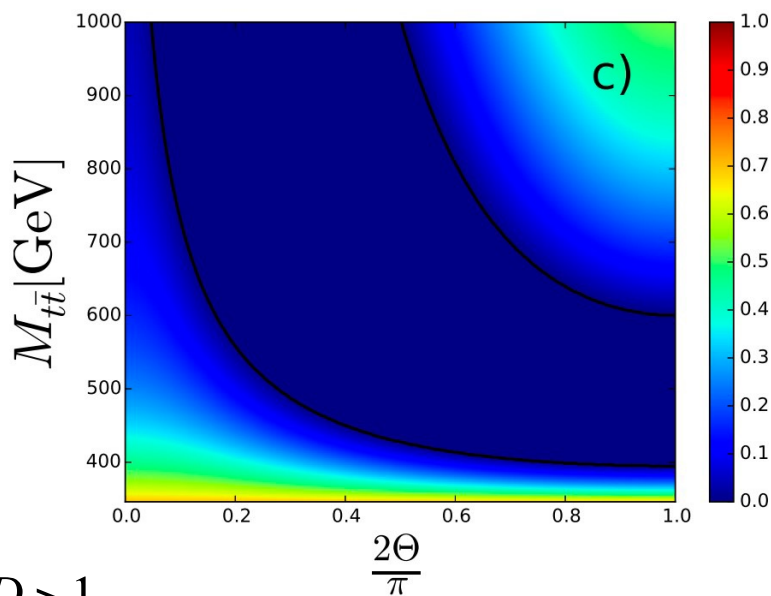
$$\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1$$

There are four maximally entangled states

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle),$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle).$$

at low $M_{t\bar{t}}$ **singlet**
pseudoscalar state Ψ^-
 $C_{nn}, C_{rr}, C_{kk} > 0$



at high $M_{t\bar{t}}$ **triplet** vector state
($\Phi^+ - \Phi^-$, Ψ^+ , $\Phi^+ + \Phi^-$)

$$C_{nn} > 0$$

$$C_{rr}, C_{kk} < 0$$

$$\Delta_E = C_{nn} - C_{rr} - C_{kk} = 3\tilde{D} > 1$$

$$\tilde{D} > \frac{1}{3}$$

$$\Delta_E = \text{Tr}(C) = -3D > 1$$

$$D < -\frac{1}{3}$$

Plot from Afik, De Nova
EPJP136(2021)9,907
hep-ph:2003.02280

Bell inequality

- **Bell inequality** is formulated based on conventional logic, which is violated in QM
- Entanglement is a necessary but not sufficient condition for Bell inequality violation
- It can be phrased in terms of Clauser, Horne, Shimony, and Holt (CHSH) inequality [PRL, 23(15), 1969] which states that measurements a, a' and b, b' on subsystems A and B, respectively (with absolute values ≤ 1) classically must satisfy:

$$|\langle ab \rangle - \langle ab' \rangle + \langle a'b \rangle + \langle a'b' \rangle| \leq 2$$

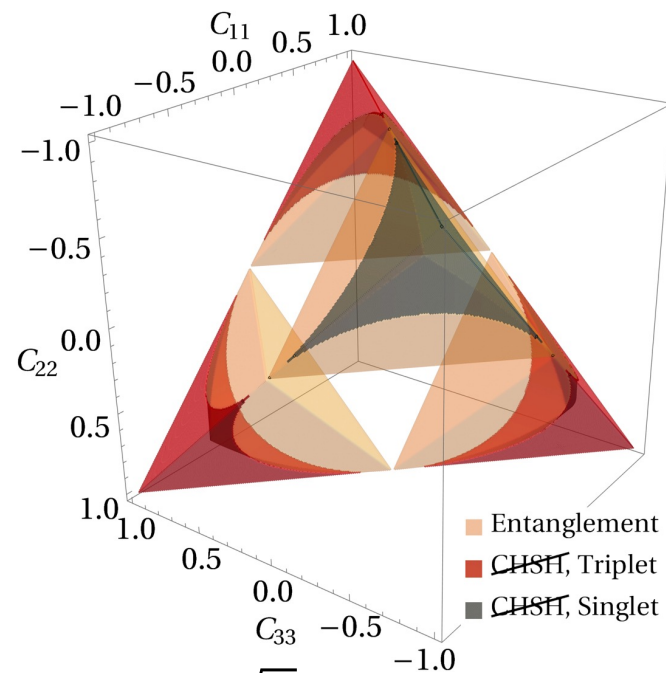
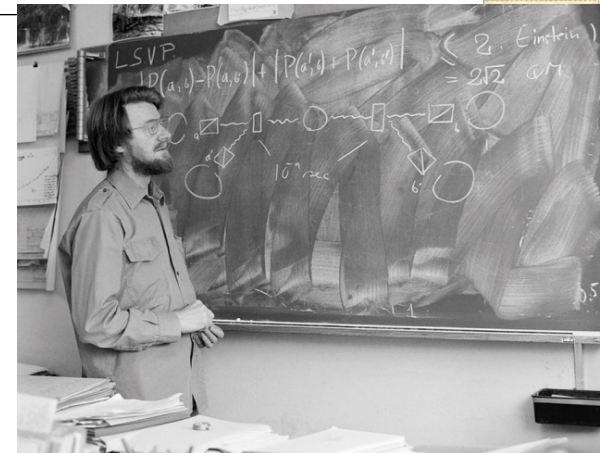
- For $t\bar{t}$ system the CHSH can be formatted as

$$|-C_{rr} + C_{nn}| \leq \sqrt{2}$$

$$B_1 = |C_{rr} - C_{nn}| > \sqrt{2}$$

$$B_2 = |C_{rr} + C_{kk}| > \sqrt{2}$$

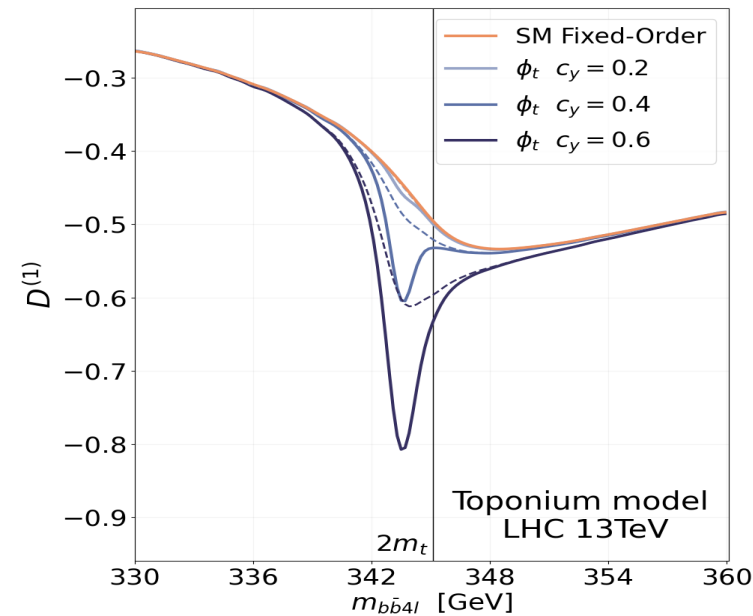
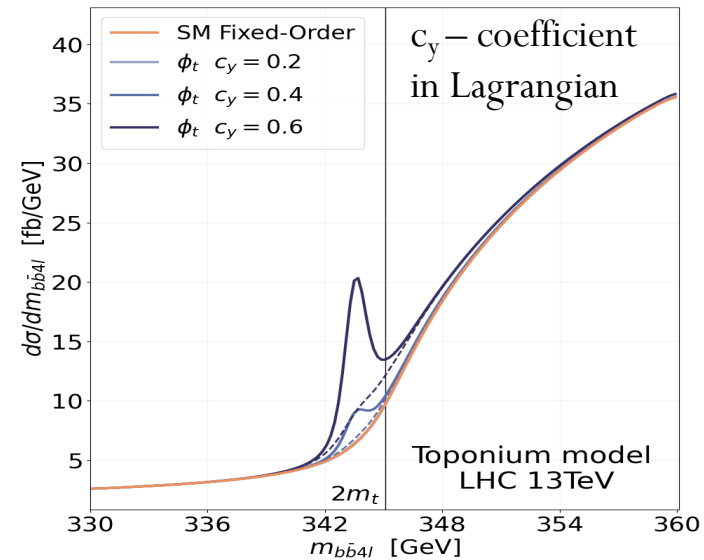
C Severi et al. arXiv:2110.10112v2 [hep-ph]



Dilepton vs l+jets channels

- Dilepton
- **Lower branching ratio**
- $|\kappa|=1$ for charged leptons, which are easy to ID \rightarrow Ideal channel for spin correlation
- Lower p_T cuts for leading/subleading lepton (25/20 GeV) \rightarrow higher efficiency at the threshold
- **Worse M_{tt} resolution, not ideal for differential measurement**
- **Best for threshold**
 - high entanglement
 - potential for “toponium” observation
 - mostly time-like separated events
- Lepton+jets
 - **Higher branching ratio**
 - $|\kappa|=1$ for down-type quarks, but they are harder to identify – employ AI ($\sim 66\%$)
 - Higher p_T cut for single lepton (30 GeV) and for 4 jets (30 GeV) \rightarrow lower efficiency at the threshold, but OK for high M_{tt}
 - **Better M_{tt} resolution, good for differential measurement**
 - **Advantage for high M_{tt}**
 - high entanglement
 - potential for observation of Bell Inequality Violation
 - mostly space-like separated events

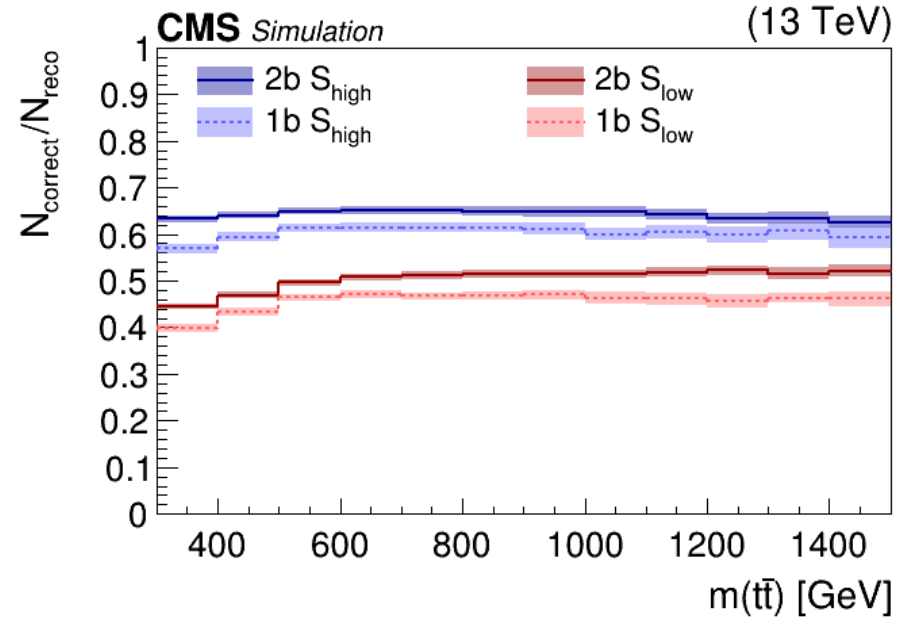
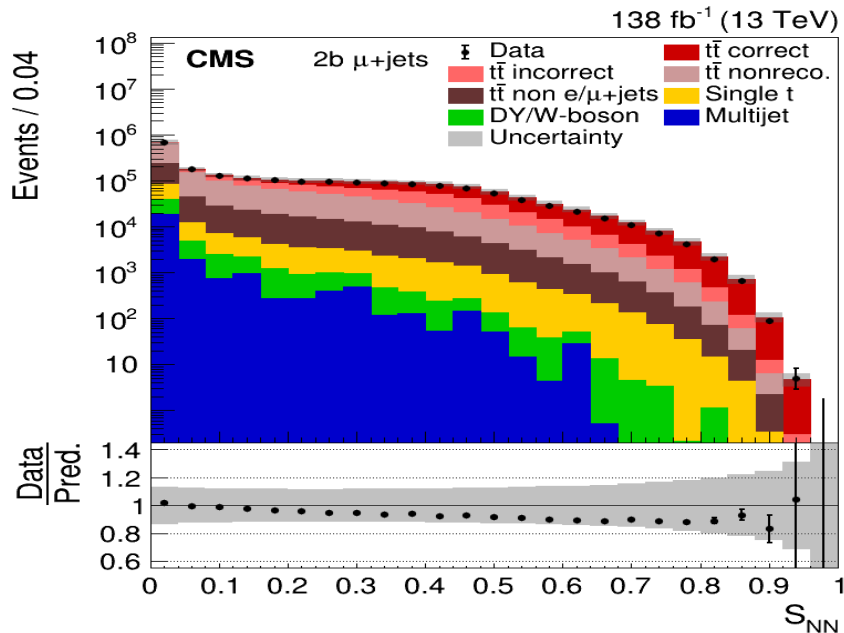
- NLO POWHEG+Pythia8
- Include EW corrections with HATHOR (*Comput. Phys. Commun.* 182 (2011) 10)
- NNLO (*Phys. Rev. Lett.* 127 (2021) 062001)
 - Dilepton: p_T reweighting to match the top quark p_T spectrum from a fixed order ME calculation at NNLO
 - Lepton+jets: NN-based reweighting to match NNLO distributions at reco level
- Include “toponium” (pseudo-scalar color singlet predicted by non-relativistic QCD) **as an uncertainty** $\pm 100\%$
 - $M(\text{toponium})\text{-}344\text{ GeV}$, $\sigma \sim 6.5\text{ pb}$
 - Sumino, Fujii, Hagiwara, Murayama & Ng (PRD'93)
 - Jezabek, Kuhn & Teubner (Z.Phys.C'92)
 - B. Fuks et al. (PRD 104 (2021) 034023)
 - affects the invariant mass distribution and the **spin correlations** at the threshold



L+jets system reconstruction

- The measurements are done inclusively and differentially in $M_{t\bar{t}}$, $\cos\theta$ and $p_T(t)$
- **Jet-parton assignment, including d-type quark is performed using NN**
 - **Input – jet 4-vectors, b-tagging score (it carries some info about charm jets)**
- Remove events with NN score $S_{NN} < 0.1$;
- Divide events into categories based on lepton flavor, number of b-tags, and NN score

Fraction of $t\bar{t}$ events with correctly assigned jets to partons including d -type quark



Full matrix measurement

- Full measurement of the vectors P and matrix C is performed using templates defined based on the functions of angles of top and antitop decay products

$$\Sigma_m = \sigma_{norm} \left\{ \kappa \sin \theta_p \cos \phi_p, \dots - \kappa \bar{\kappa} \cos \theta_p \cos \theta_{\bar{p}} \right\}$$

- The total cross section is a linear combination of these templates with coefficients Q_m that are the components of P and C

$$\Sigma_{tot} = \Sigma_0 + \sum_{m=1}^{15} Q_m \Sigma_m$$

- **The templates T_m are defined at the reco level.**

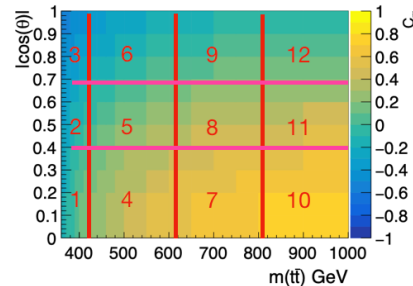
To avoid generating events with every possible combination of Q_m the events are reweighted with

weights defined at the gen level

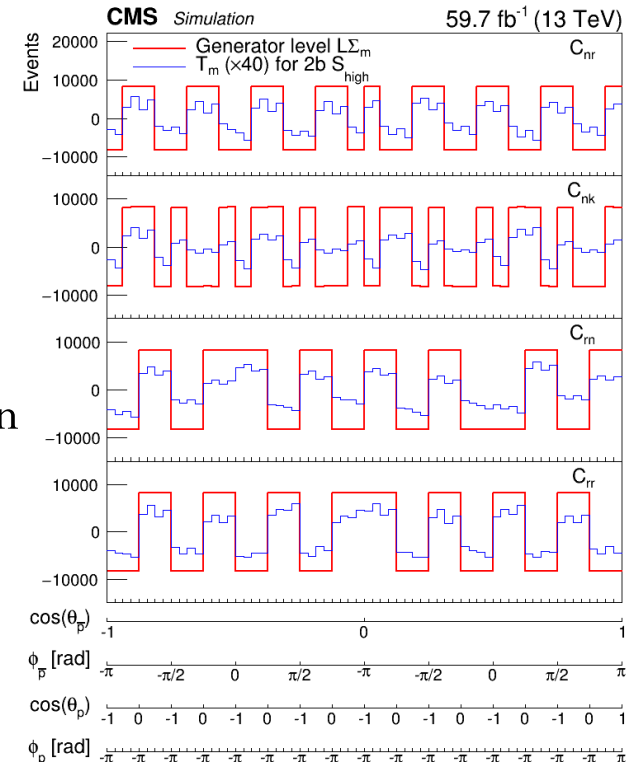
$$w_i = \frac{\Sigma_i}{\Sigma_{tot}}$$

To minimize the bias due to variation of Q_m or T_m within the bin we perform the measurement in finer bins in M_{tt} and $\cos \theta$,

then combine



Gen level Σ_m and reco level T_m



Example of the fit

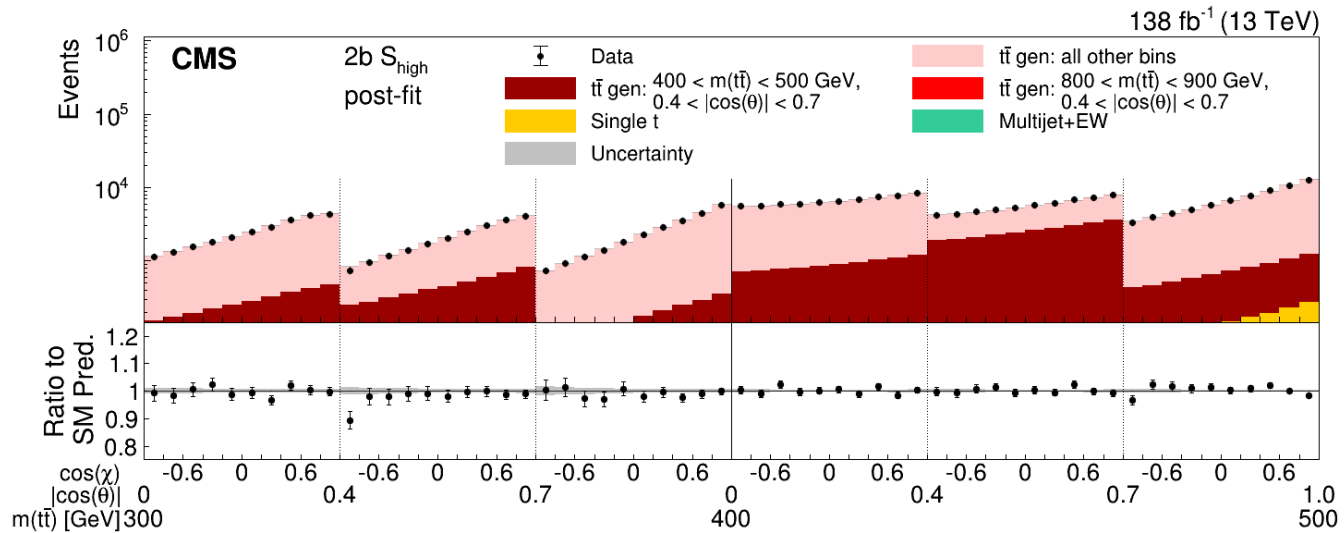
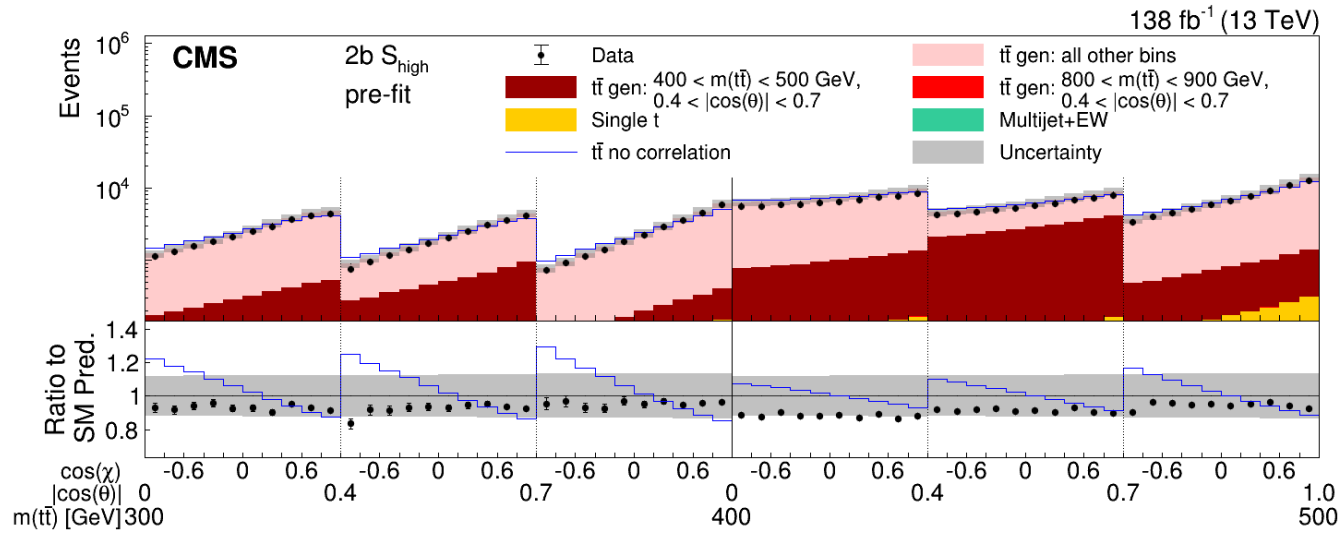
Fit is performed in different categories (e.g. 2b Shig, 1b Slow...)

simultaneously.

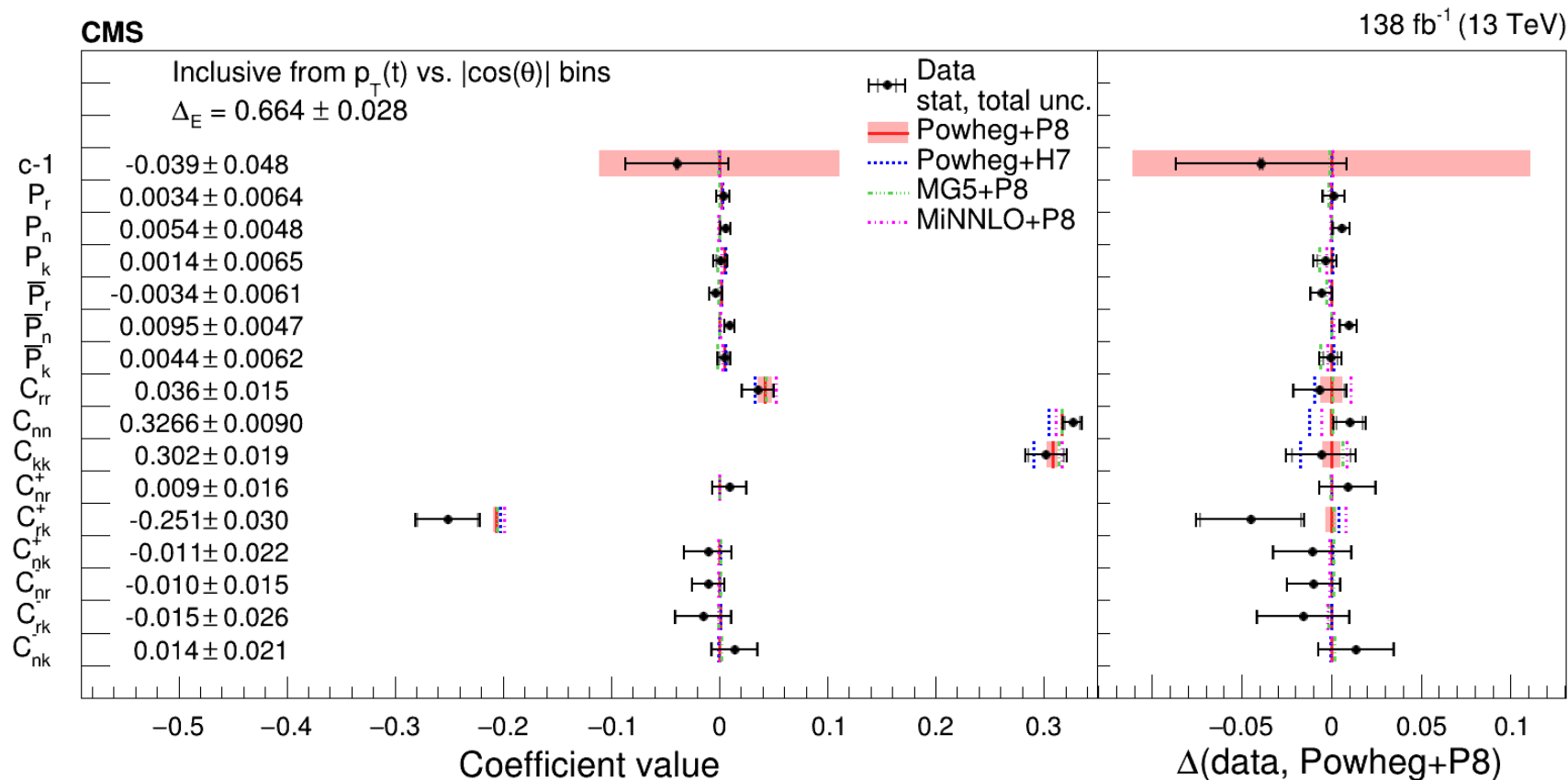
Bins are defined at the reconstruction level,

But contributions from all bins at gen level are included and fit for (different shades of red).

Thus, we present the result in bins of gen level variables.



- Full measurement of the P and C is performed inclusively and differentially in bins of M_{tt} , $\cos\theta$ and top p_T
- Include uncertainty on Powheg+p8 (in pink), the other models have similar uncertainties
- Full covariance matrix is provided
- A good agreement with the SM prediction is observed



Differential measurement

- You can see the transition from singlet to triplet state

at low p_T **singlet**

pseudoscalar state Ψ^-

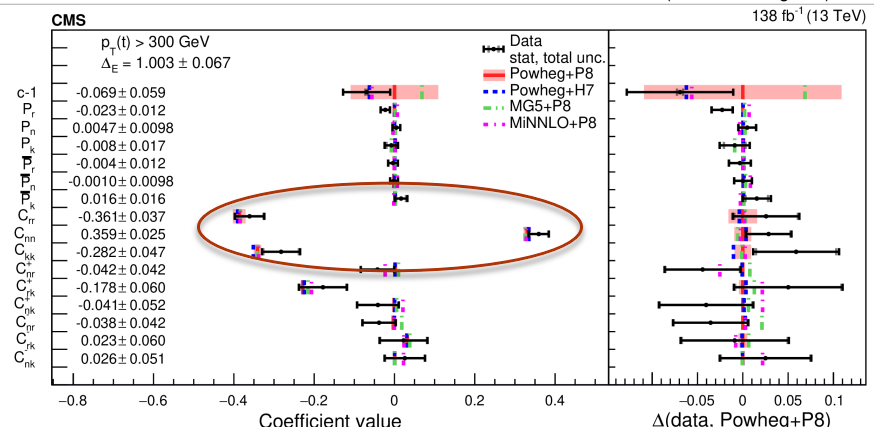
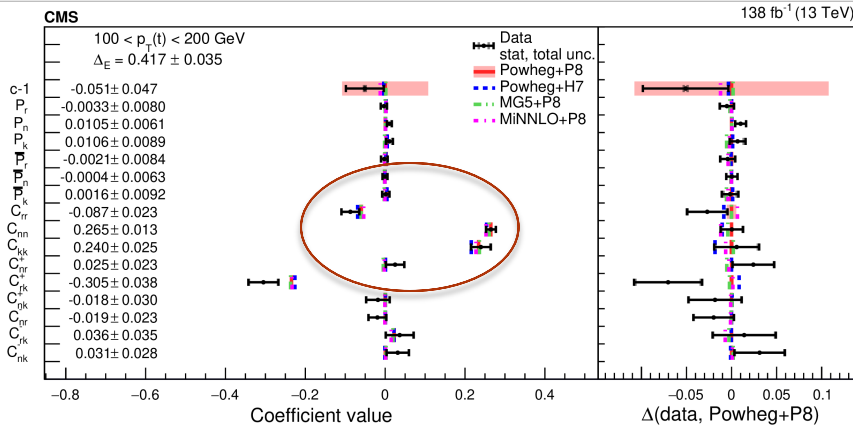
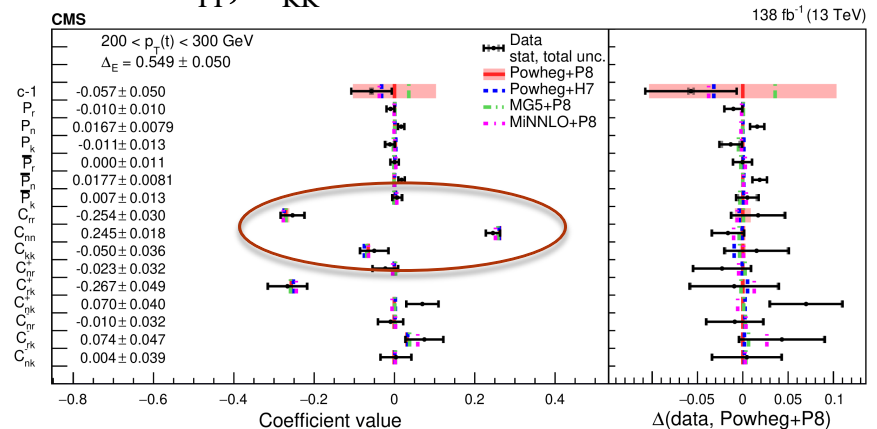
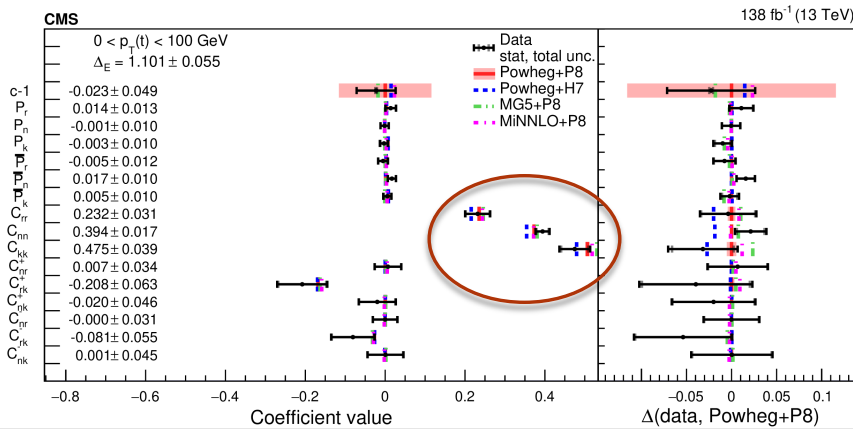
$$C_{nn}, C_{rr}, C_{kk} > 0$$

at high p_T **triplet** vector state

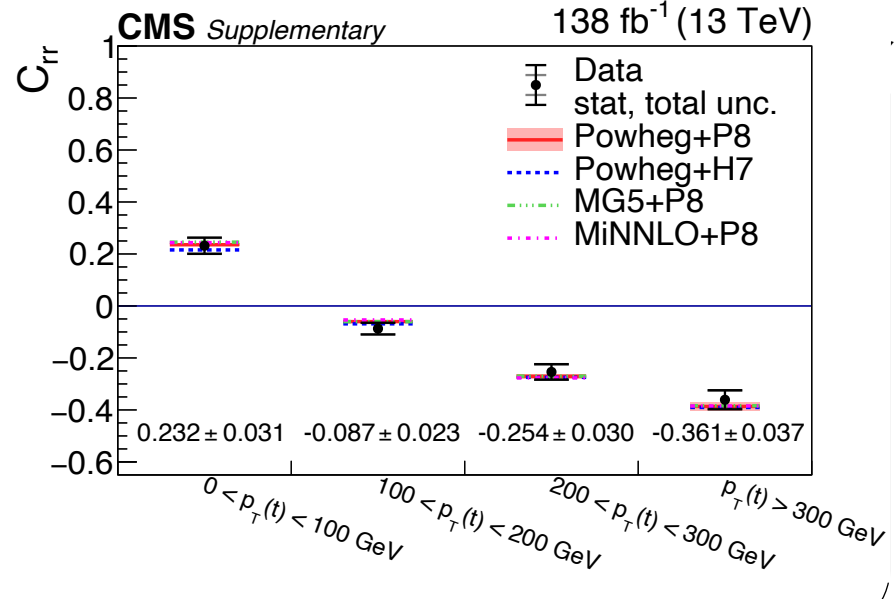
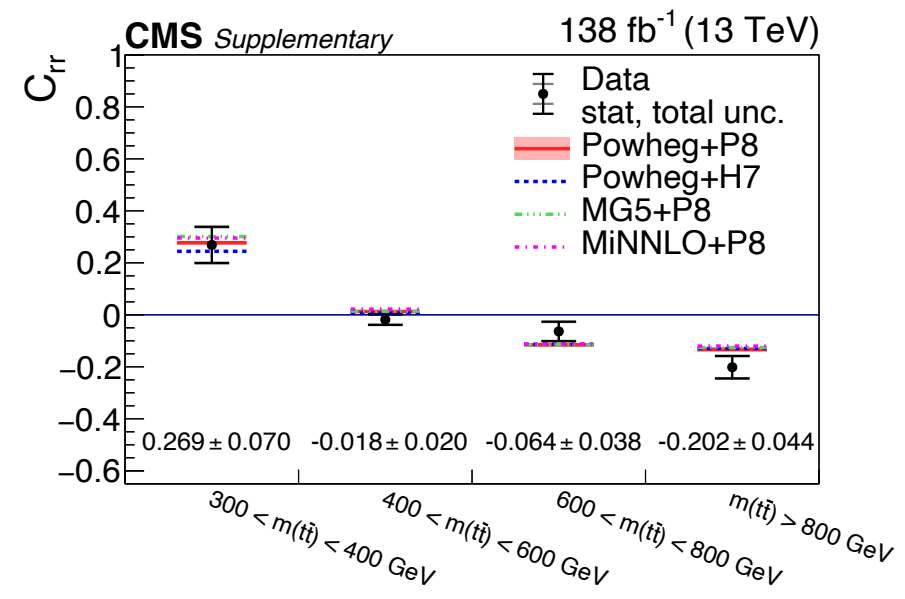
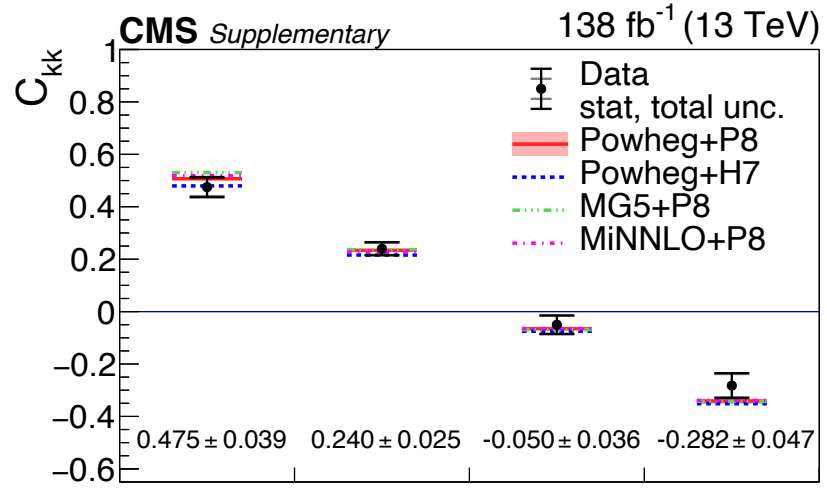
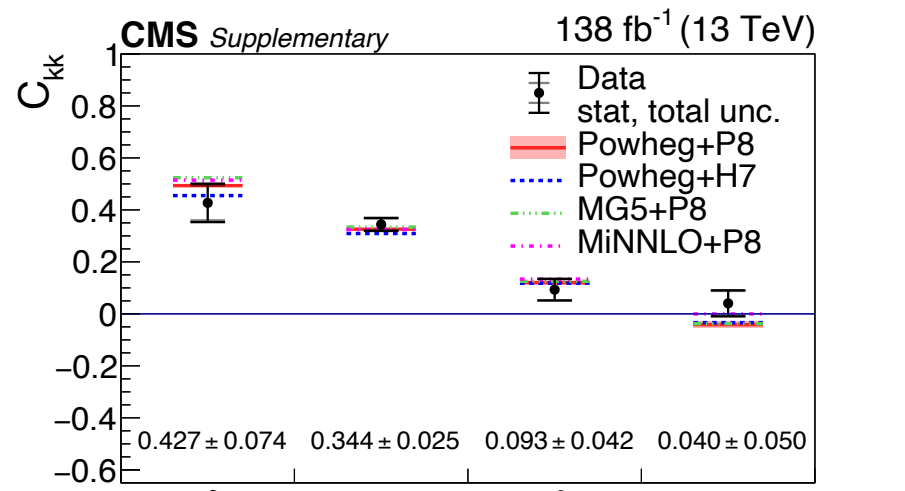
$(\Phi^+ - \Phi^-, \Psi^+, \Phi^+ + \Phi^-)$

$$C_{nn} > 0$$

$$C_{rr}, C_{kk} < 0$$

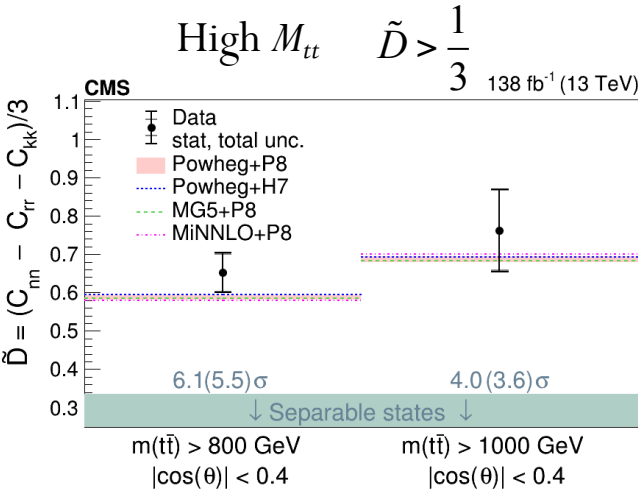
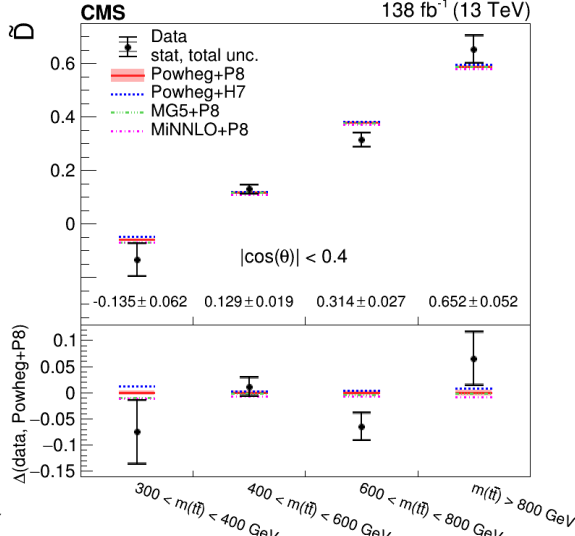
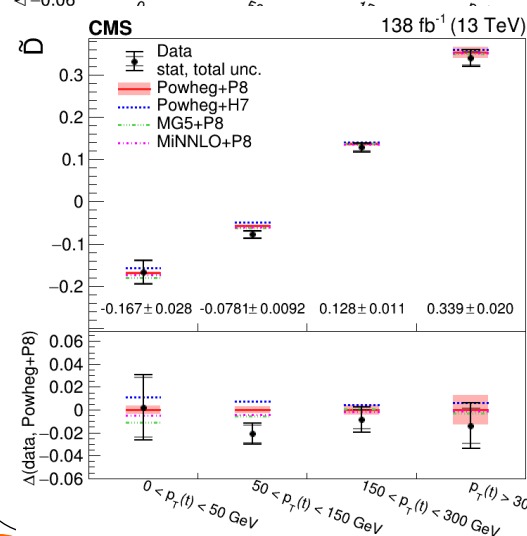
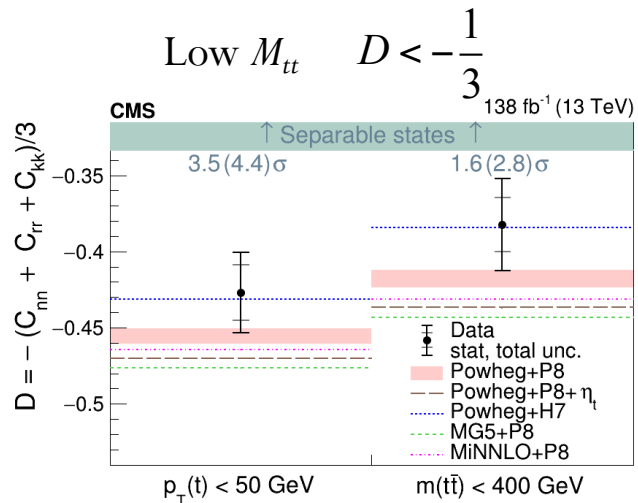
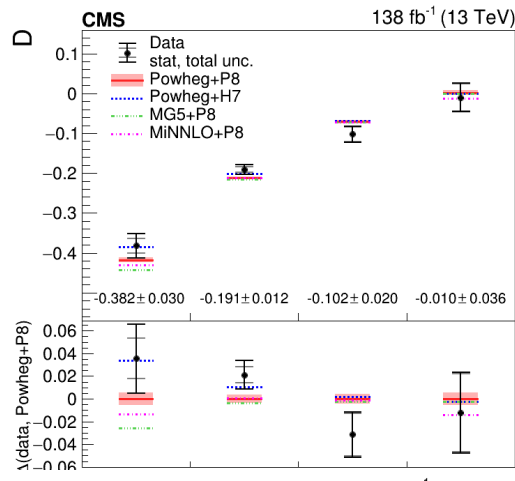
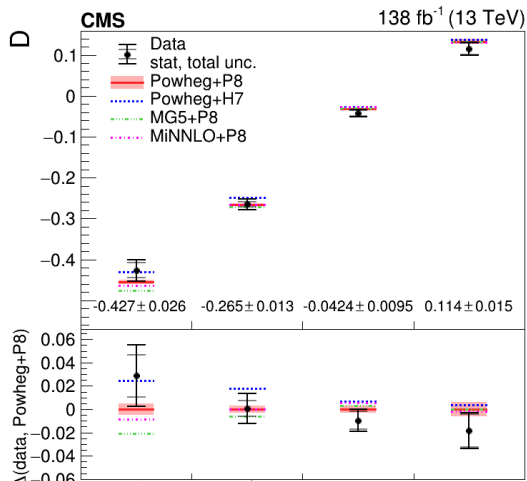


- You can see the transition from singlet to triplet state

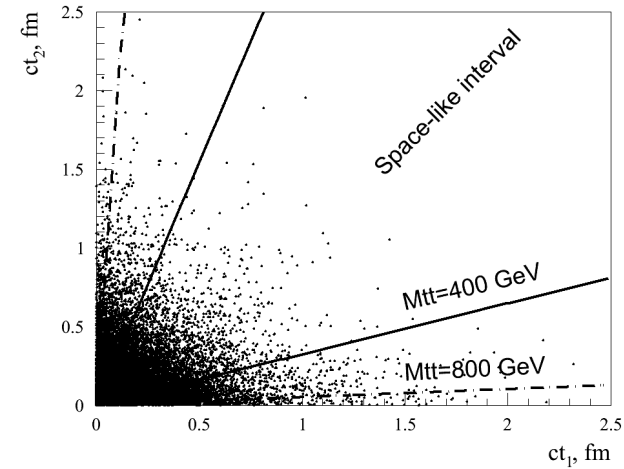


D and \tilde{D} measurements

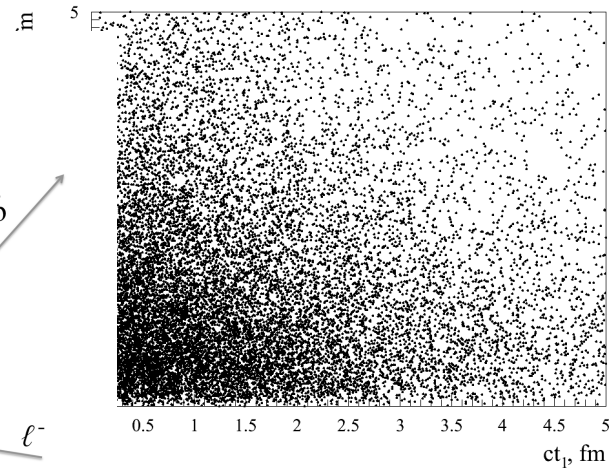
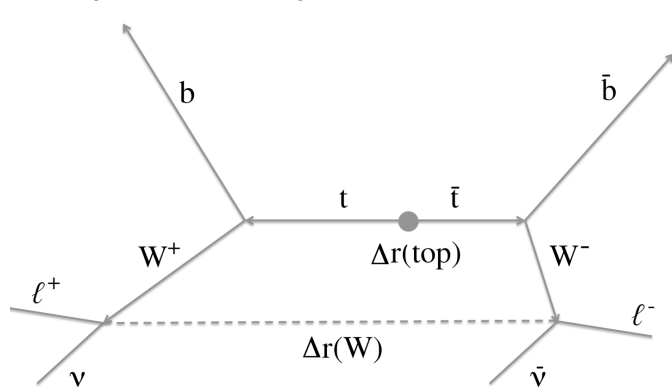
- Full measurement of the P and C is performed inclusively and differentially in bins of $M_{t\bar{t}}$, $\cos\theta$ and top p_T
- Full covariance matrix will be provided with the published result
- A good agreement with the SM prediction is observed



Causal connection in ttbar events



$$\frac{1-\beta}{1+\beta}t_1 < t_2 < \frac{1+\beta}{1-\beta}t_1$$

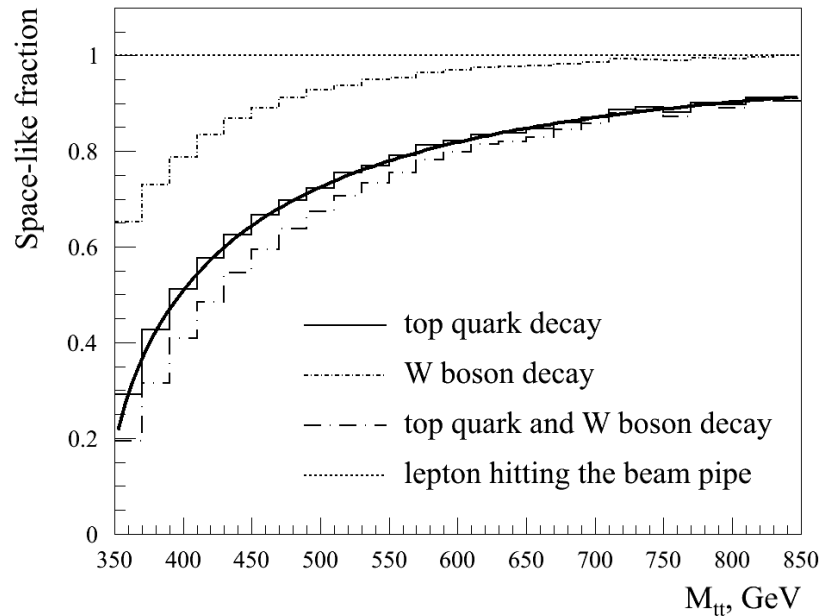


Could top and antitop or their decay products have communicated?

Consider separation between:

- Top and antitop
- W+ and W-
- Both tops and Ws are separated
- Leptons contact with macroscopic device

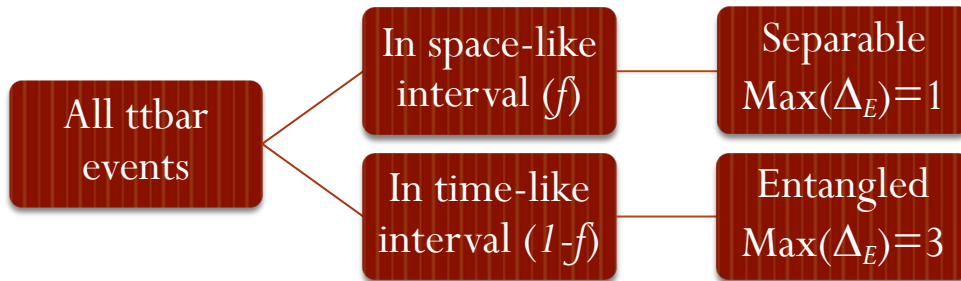
RD, G. Landi arXiv2407.15223v1



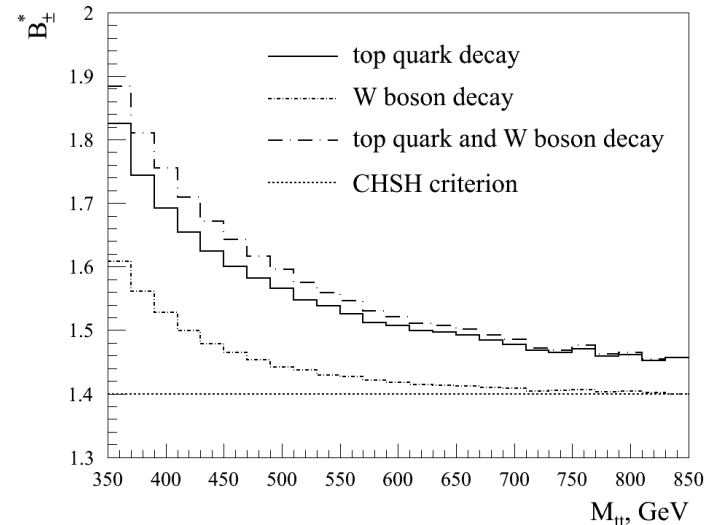
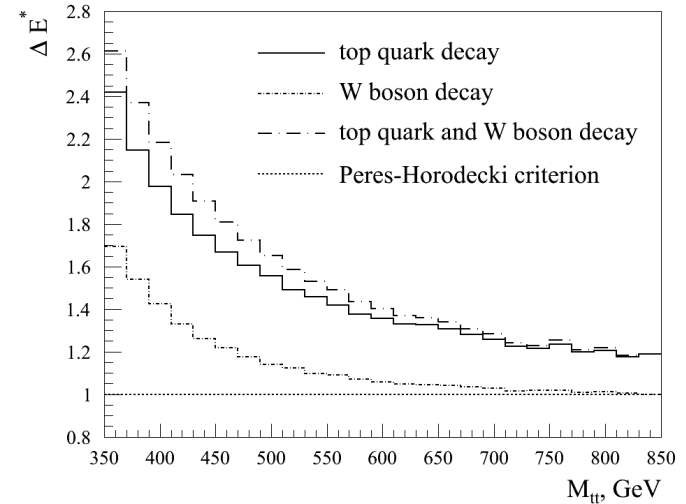
Causal connection in $t\bar{t}$ events

f - fraction of space-like separated top and antitop

$$\Delta_{Ecritical} = f(\Delta_E = 1) + (1-f)(\Delta_E = 3)$$

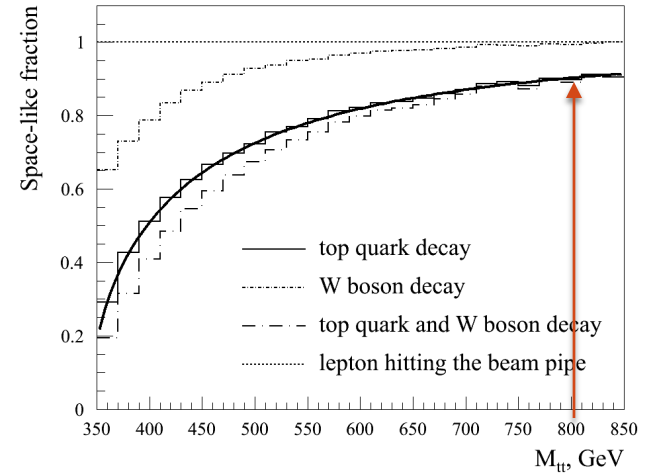


$$B_{Ecritical} = f(B_E = \sqrt{2}) + (1-f)(B_E = 2)$$

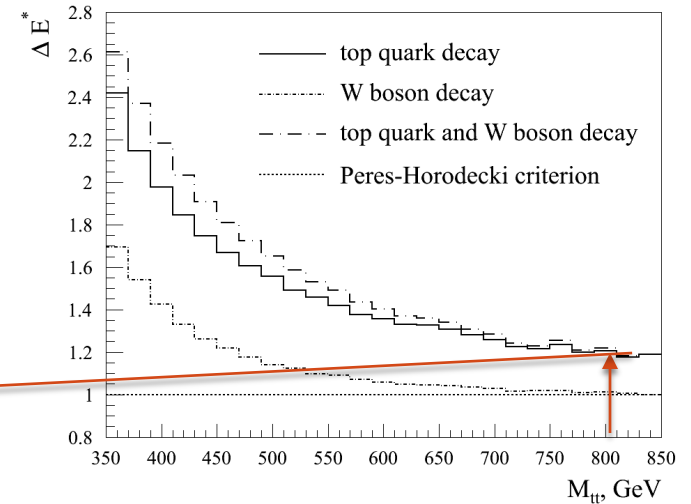
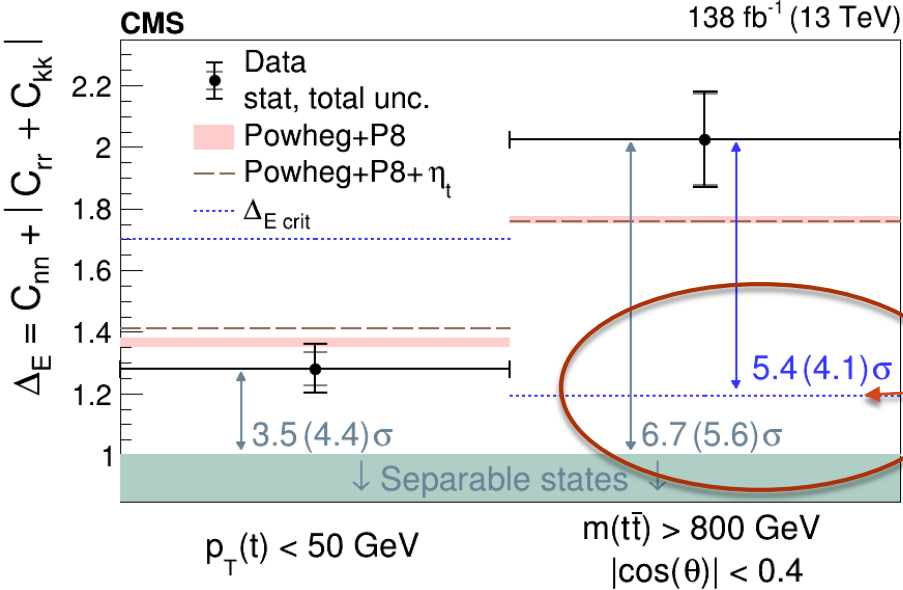


Entanglement results

- We quantify the entanglement using Peres Horodecki criterion
- Significant entanglement is observed in the $M_{tt} > 800$ GeV region
- Apply the Demina-Landi criterion to see if this entanglement could be explained by classical communication



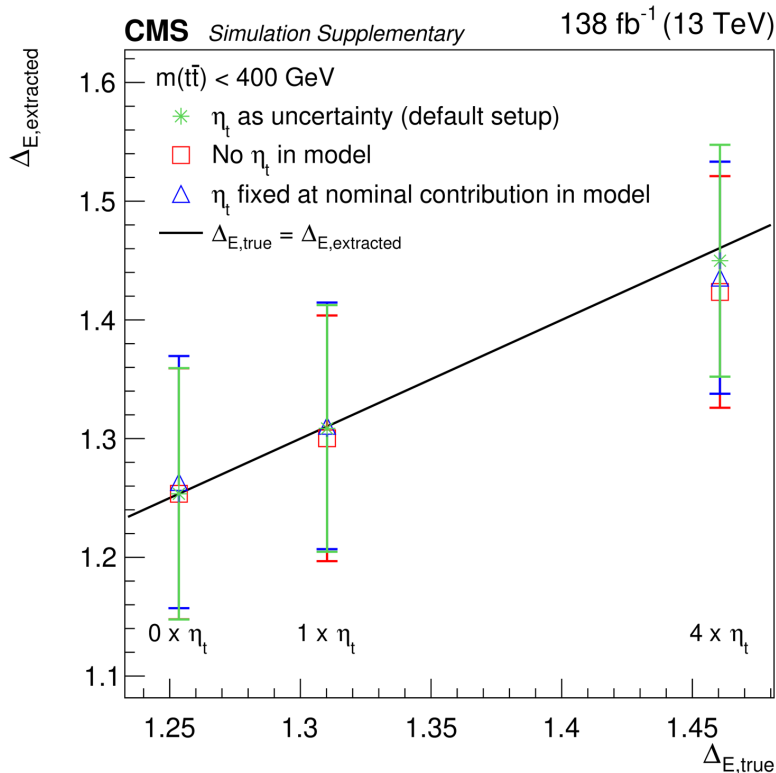
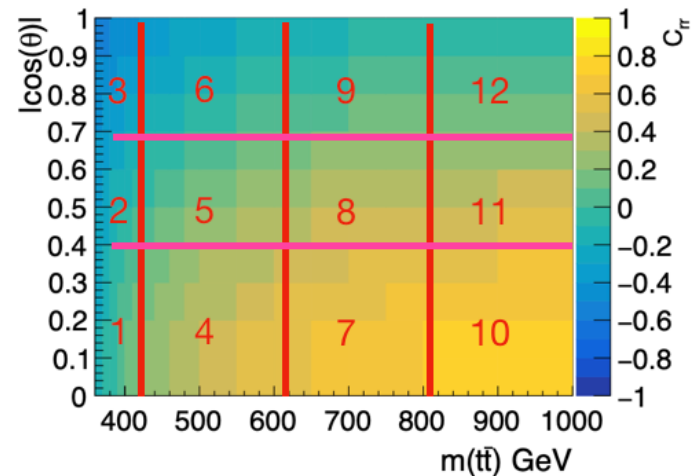
Based on full matrix $\Delta_E = C_{mm} + |C_{rr} + C_{kk}| > 1$



- Angular distributions of the top and antitop quarks were used to measure their **polarization** and **spin correlation matrix**, C_{ij} inclusively and in bins of M_{tt} , $\cos\theta$ and top quark p_T
- In some regions of phase space top and antitop get entangled, which can be demonstrated using **Peres-Horodecki criterion** based on their spin correlation matrix
- Maximally entangled states are a singlet produced at the threshold, and a triplet produced at high M_{tt}
- Using full matrix measurement the entanglement was observed at **6.7σ** level in $M_{tt} > 800 \text{ GeV}$, $|\cos\theta| < 0.4$ region
- The later result was found to exceed the maximum entanglement achievable by classical communication between top and antitop decays by **$> 5\sigma$**

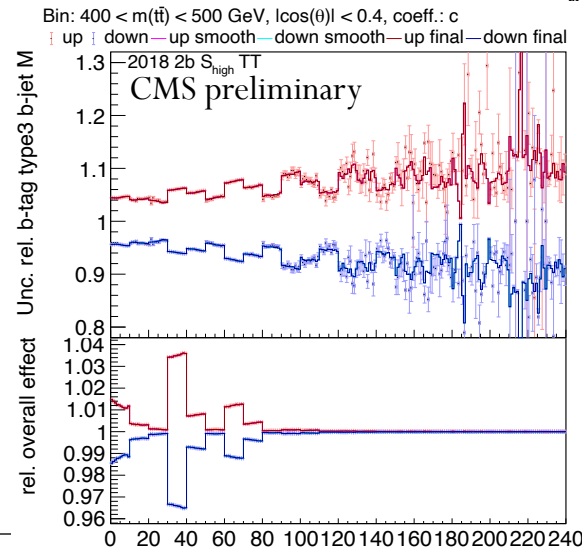
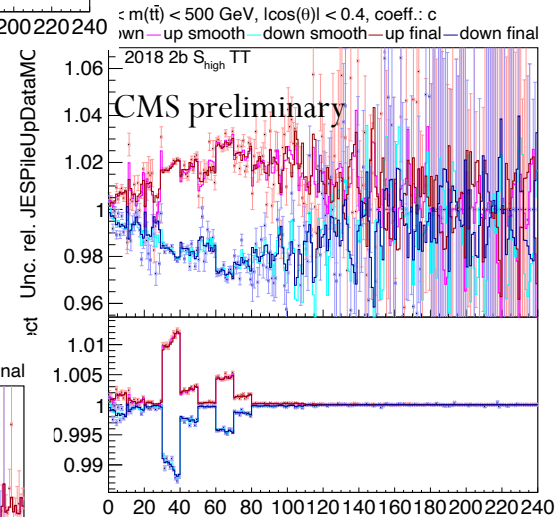
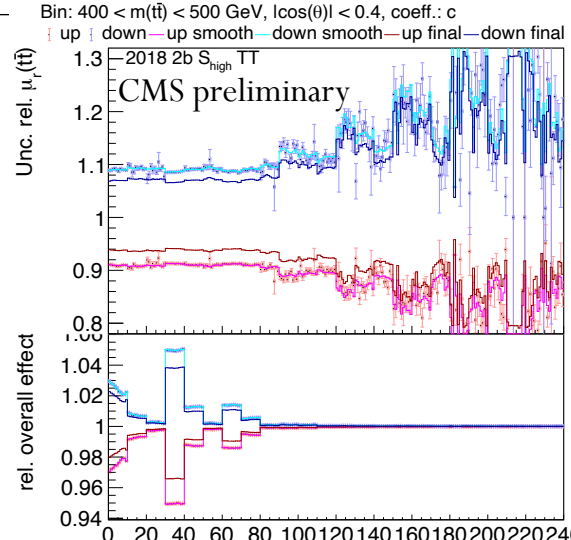
To minimize the bias due to variation of Q_m or T_m within the bin we perform the measurement in finer bins in $M_{t\bar{t}}$ and $\cos q$, then combine

We perform calibration of the procedure by injecting the variation of Q_m different from the SM, e.g. using h_t model:



Systematics

- The analysis is statistics limited
- Theoretical uncertainties
 - M_{top} , renormalization/factorization scale, NNLO, EW
 - NB. Toponium effect is small for lepton+jets $\sim 5E-04$
- Experimental uncertainties:
 - Jet energy scale, b-tagging efficiency



Impacts

Measurement: $m(t\bar{t})$ vs $l\cos(\theta)l$, D

Value (exp.)
 Impact -1σ (exp.)
 Impact $+1\sigma$ (exp.)
 Value (obs.)
 Impact -1σ (obs.)
 Impact $+1\sigma$ (obs.)

CMS preliminary

• Pull (obs)

