Quantum entanglement and particle decay

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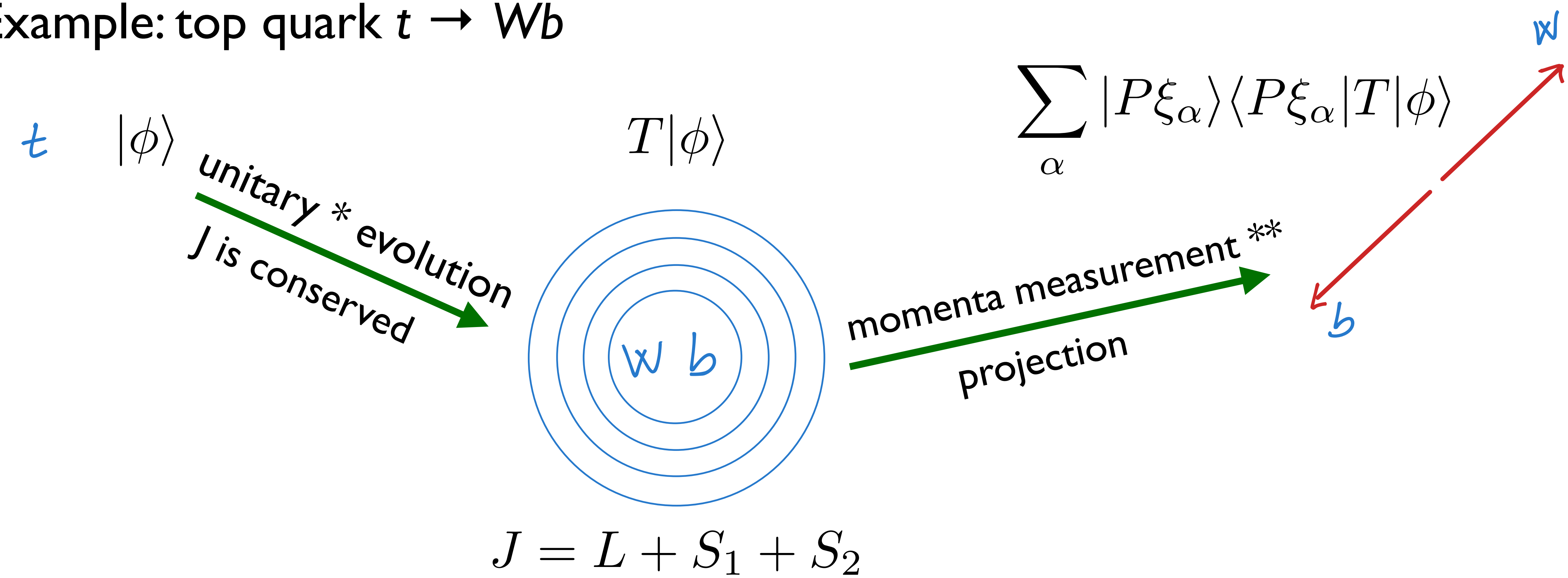
What is a decay?

Deconstructing particle decay

What does decay *mean* in a particle detector?

Example: top quark $t \to Wb$



* Strictly speaking, this is part of the unitary evolution, S = 1 + i T.

** This also involves the identification of the final state, *Wb* / …

Detectors measure momenta in the quantum-mechanical sense.

They do not measure spin.

The measurement of momenta influences the spin state but in general it does not collapse it as a Stern-Gerlach experiment would do.

This leads to novel entanglement effects that are yet untested:

- Post-decay entanglement — JAAS 2307.06991; JAAS, Casas 2401.06854; JAAS 2401.10988
- Entanglement and post-selection — JAAS 2308.07412

Post-decay entanglement

Post-decay entanglement

General states are described by a density operator.

One can fully characterise the effects of a particle decay

$$A \to A_1 \, A_2 \, \ldots$$

by specifying how the post-decay operator ρ' relates to the initial one ρ

➡ we will focus on spin degrees of freedom

Post-decay entanglement

Consider a system of two particles *A*, *B*, with spin state described by

$$\rho = \sum_{ijkl} \rho_{ij}^{kl} |\phi_i \chi_k\rangle\langle\phi_j \chi_l| \qquad |\phi_i\rangle \in \mathcal{H}_A\,, \quad |\chi_k\rangle \in \mathcal{H}_B$$

$\mathcal{H}$ are the spin spaces

Let *A* decay *A* → A_1 A_2 … with amplitudes

$$M_{\alpha j} = \langle P\,\xi_\alpha|T|\phi_j\rangle \qquad |\xi_\alpha\rangle \in \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \ldots$$

Then, the spin state of A_1 A_2 … and *B* is described by

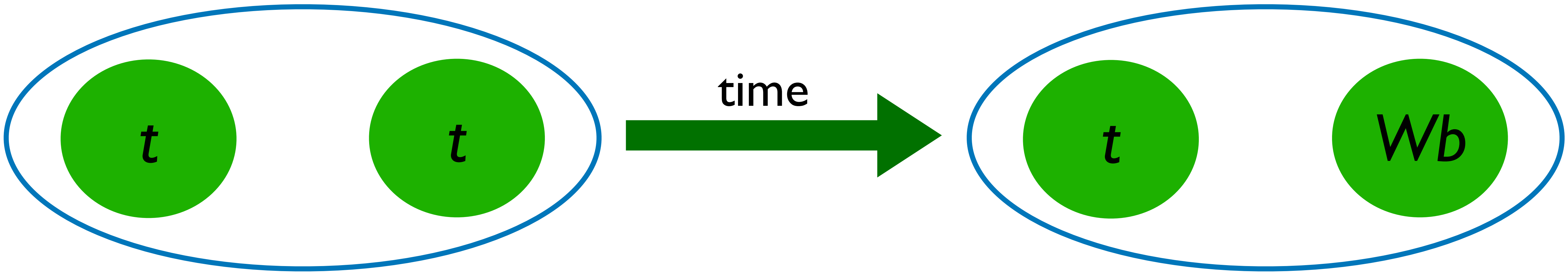
these come from the projector

$$\rho' = \frac{1}{\sum_{\alpha k}(M\rho^{kk}M^\dagger)_{\alpha\alpha}} \sum_{\alpha\beta kl} (M\rho^{kl}M^\dagger)_{\alpha\beta} |\xi_\alpha\,\chi_k\rangle\langle\xi_\beta\,\chi_l|$$

➡ in particular, the entanglement properties between *A* and *B* can be inherited by { the decay products of *A* } vs *B*

Post-decay entanglement

When t t-bar are entangled and t-bar decays into W^- b-bar, t is entangled with the W^- b-bar pair



Potential problem:

The b spin is, in principle, not measurable.

When we have several entangled particles and trace over [unobserved] degrees of freedom, entanglement may be \`lost´.

→ but b-bar has RH helicity up to small mass effects, trace maintains entanglement between t and W^-

Post-decay entanglement

Example: threshold region $m_{tt} \leq 390$ GeV, $\beta \leq 0.9$, beamline basis z = (0,0,1)

θ 👉 angle between W^- momentum in *t*-bar rest frame and $\hat{z}$ axis *or any fixed axis*

Negativity: entanglement measure

phase space region	N(ρ)
$\theta = 0$	0.13
$\cos\theta > 0.9$	0.12
$\cos\theta > 0.5$	0.10
$\cos\theta > 0$	0.07
all θ	0

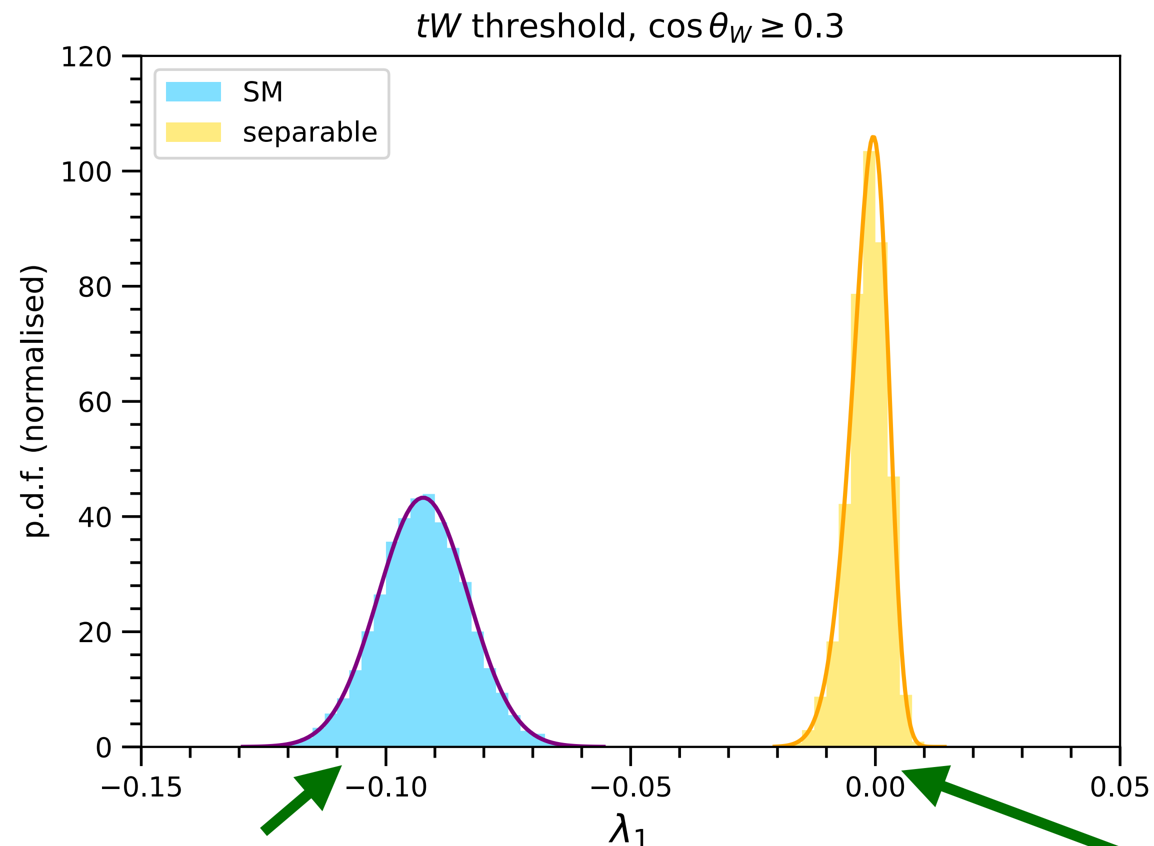
The amount of entanglement is the same in any direction but the quantum state is not, so integration washes out entanglement

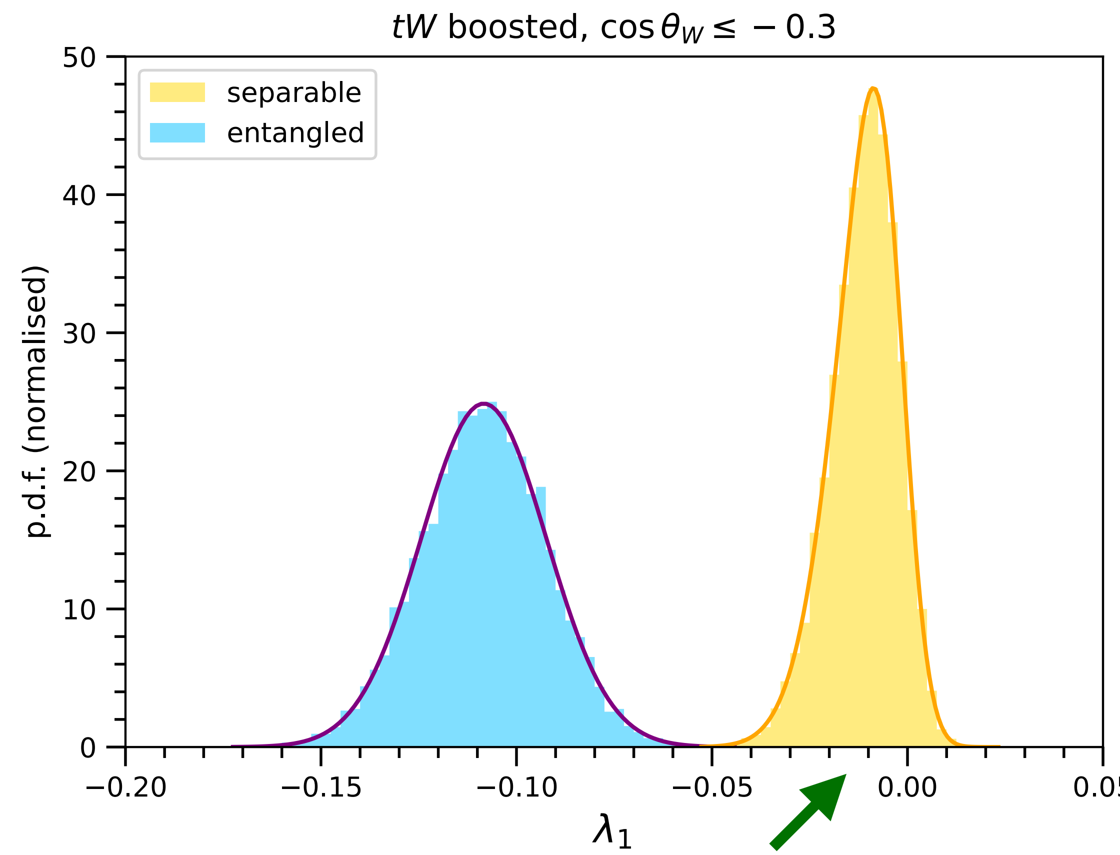
The projection is at work here: the spin quantum state depends on *t*-bar decay kinematics

Post-decay entanglement

Entanglement indicator:
lowest eigenvalue λ_l of the ρ^{T2} matrix for *tW*

$\lambda_l < 0 \Leftrightarrow$ Entanglement





stat uncertainty

Bias: even if $\lambda_l > 0$, in a small sample we may find it negative

Run 2	Significance [stat + 10% sys + bias]
Threshold	7.0 σ
Boosted	5.0 σ

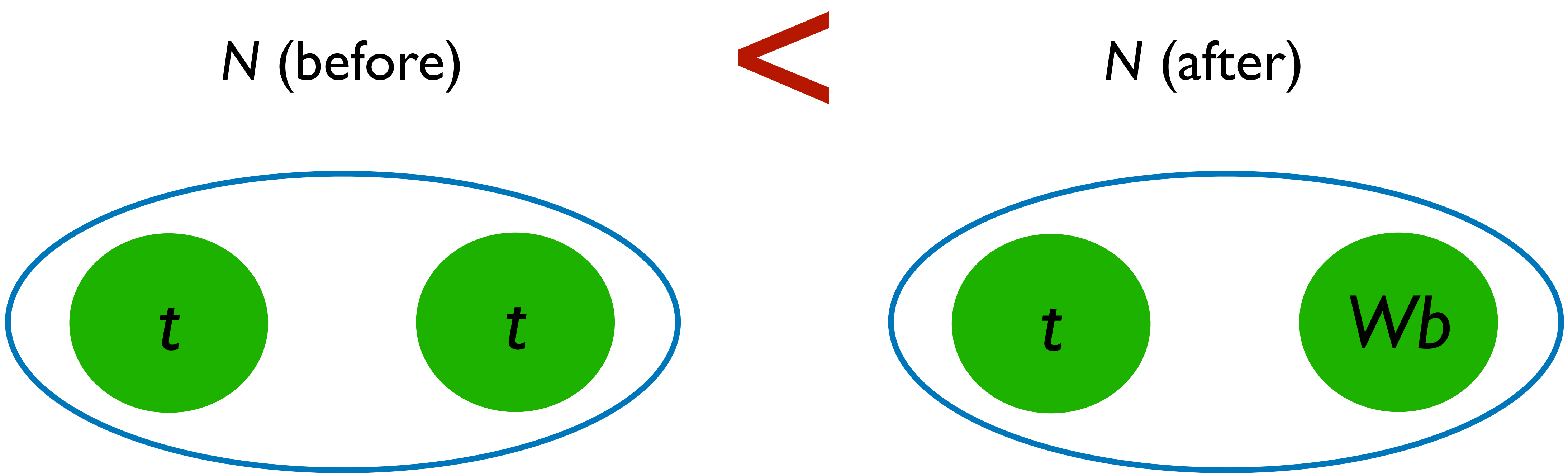
these numbers can possibly be improved by combining several regions...

Entanglement autodistillation

Entanglement decreases by measurements [collapse], interaction with environment [decoherence] …

Methods are known [distillation] to manipulate a sub-system and, if lucky, increase entanglement

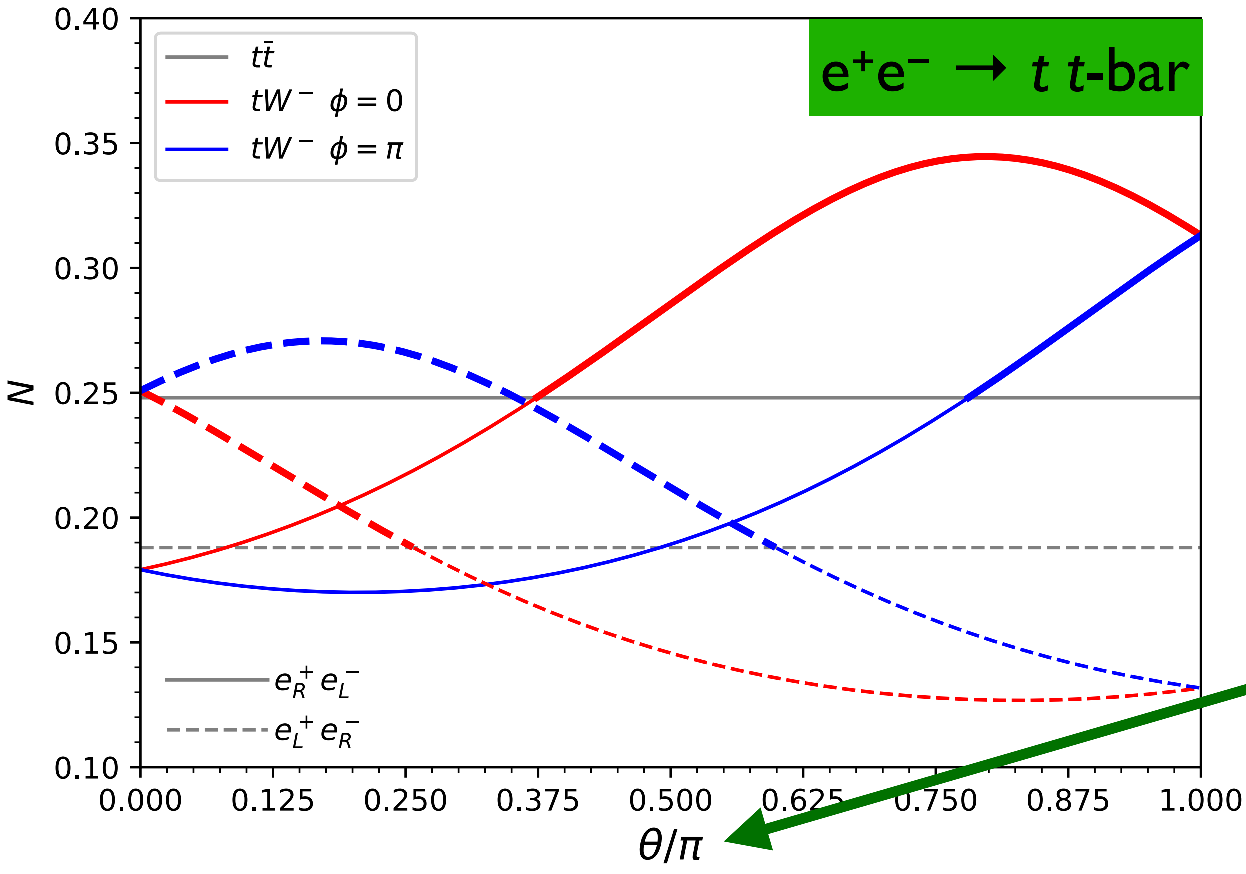
Most remarkably, the decay can increase entanglement spontaneously.



Entanglement autodistillation

Since the *b* spins are, in principle, not measurable, we can use the *t*-*W* entanglement as a proxy to probe the entanglement increase.

And this could be observed in e+ e− colliders [needs that tops are polarised]



Unique quantum effect that requires large luminosity to be observed

polar angle between *W* momentum in top rest frame and top direction in c.m. frame

To take away

☑ Particle decay and subsequent momenta projection is a very special kind of "measurement" in QM sense

☑ Post-decay entanglement never tested, 5σ sensitivity is possible at LHC with Run 2 data

☑ Spontaneous increase of entanglement [autodistillation] possible and testable at colliders

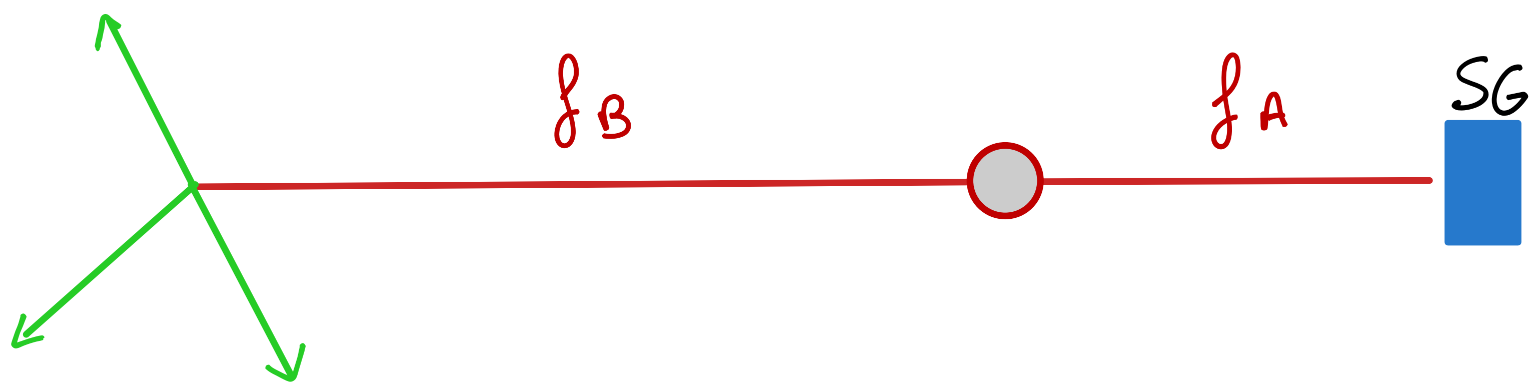
Entanglement and post-selection

Warming up: pre-selection experiment

Assume fermion pairs f_A f_B produced in an entangled state, say

$$\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$$

We perform a Stern-Gerlach experiment on f_A, and after that, f_B decays



We select those f_B for which the result of the SG experiment on f_A gives $|\uparrow\rangle$

The decay distribution of those pre-selected f_B corresponds to having spin $|\downarrow\rangle$

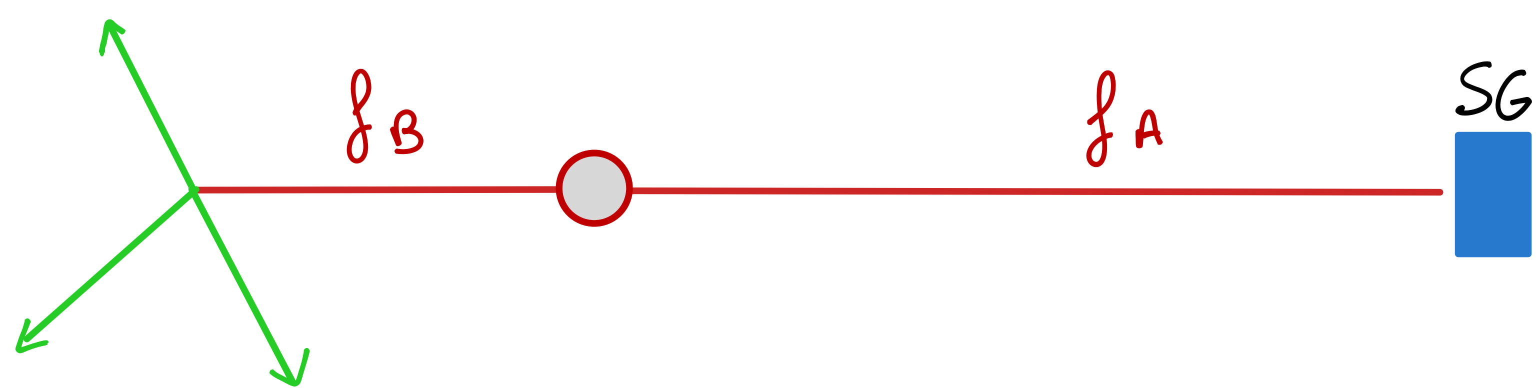
But… are we really using the fact that $t_B > t_A$?

What happens when $t_B < t_A$?

A post-selection experiment

Remarkably, the same happens time-backwards:

f_B decays and after that, we perform a Stern-Gerlach experiment on f_A



We select the subset of f_B for which the result of the posterior SG experiment on f_A gives $|\uparrow\rangle$

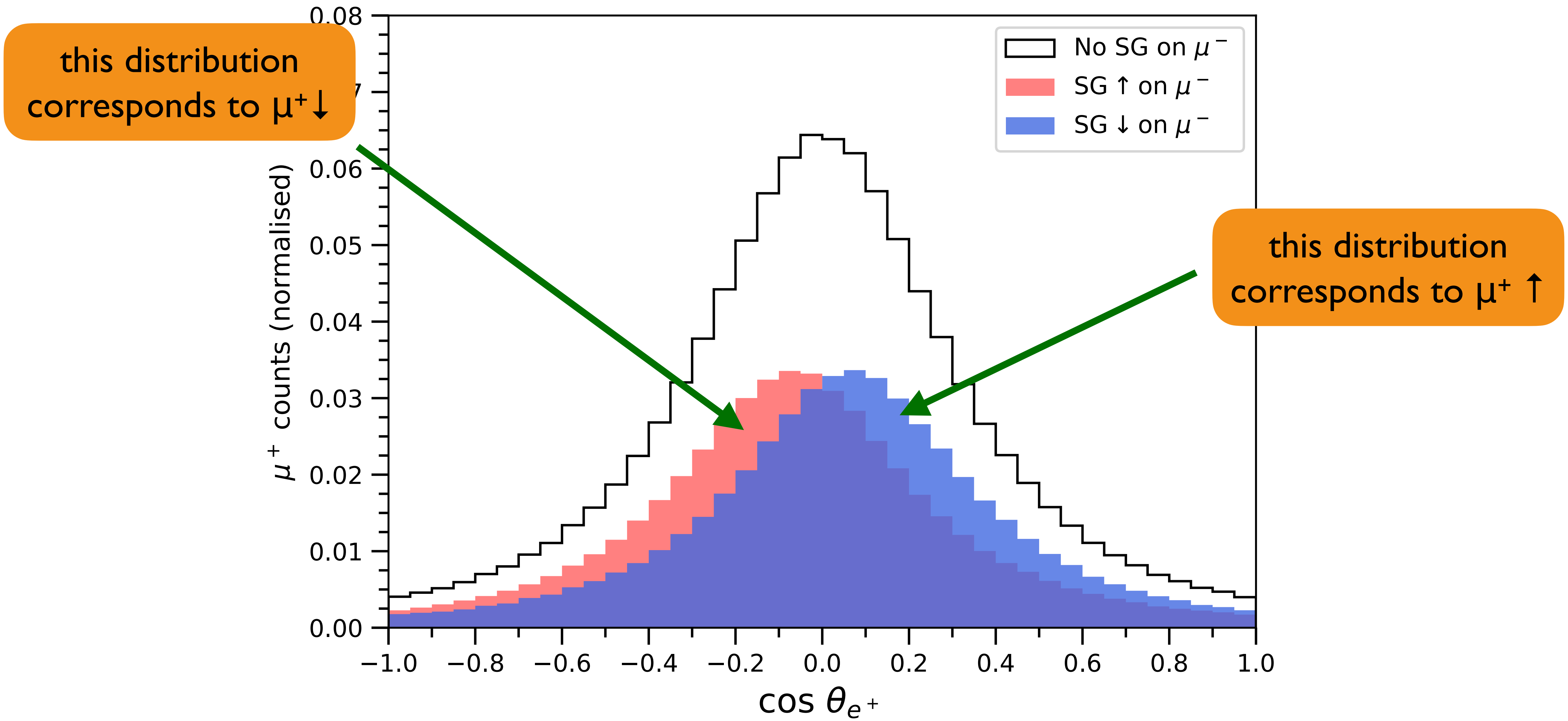
Then, the decay distribution of those f_B that had decayed before the outcome of the SG experiment corresponds to having spin $|\downarrow\rangle$

[no experimental evidence yet, verified with Monte Carlo]

A post-selection experiment

This experiment could be performed with low-energy $\mu^+\mu^-$ pairs produced in Drell-Yan or from the decay of a η meson

The muon polarisation can be measured from the daughter electron



Comment #1

From what I have shown, one cannot state that μ^+ in the sub-sample

for which the posterior SG on μ^- gives $|\downarrow\rangle$

have spin $|\uparrow\rangle$

What one can say is that this sub-sample

is physically equivalent to a sample of μ^+ with spin $|\uparrow\rangle$

This is related to the fact that

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

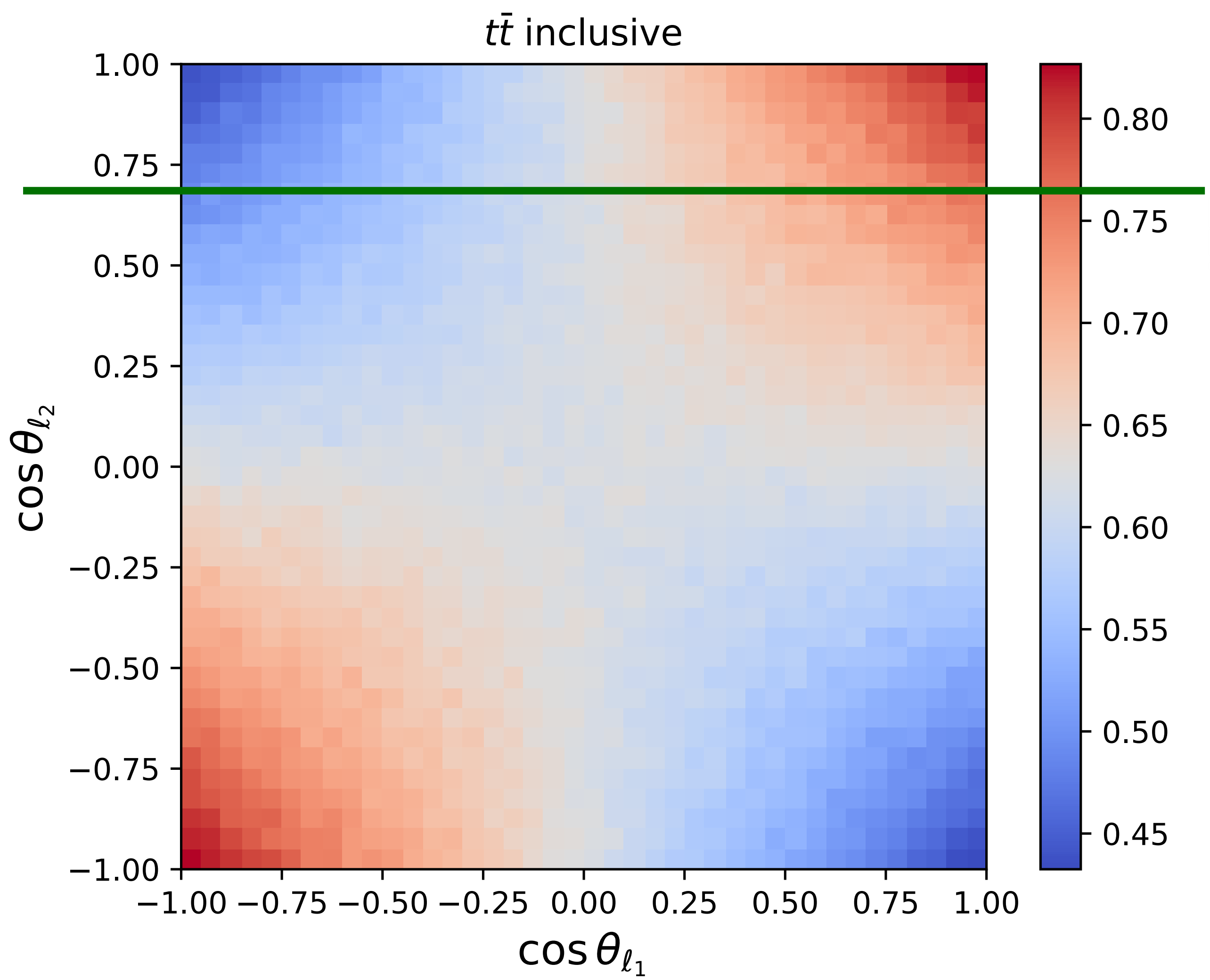
$$= \ldots$$

Comment #2

I have not *yet* shown that this effect is genuinely quantum.

Consider for example inclusive t t-bar production, label as t_1 the quark that decays first, t_2 the one that decays last, and ℓ_1, ℓ_2 their daughter leptons.

$\theta_{\ell_1}, \theta_{\ell_2}$: angles between leptons and top helicity direction



A selection on the latter decay results in a selection on the first decay.

But this happens despite the t t-bar spin state state being separable.

Nothing to do with quantum entanglement!

Comment #2

Definitive test: CHSH inequalities

$$A = \sigma_3\,, \quad A' = \sigma_2\,, \quad B = \frac{1}{\sqrt{2}}(\sigma_2 + \sigma_3)\,, \quad B' = \frac{1}{\sqrt{2}}(\sigma_2 - \sigma_3)$$

How are they measured?

- Bob registers μ^+ decay
- Alice chooses whether to measure spin in $\hat{z}$ or $\hat{y}$ axis for μ^-
- Expected values for Bob are calculated for each choice of Alice, e.g.

$$\langle AB \rangle = \frac{1}{2}\left[\langle B \rangle_\uparrow - \langle B \rangle_\downarrow\right]$$

$\langle B \rangle$ when Alice gets $\uparrow$

$\langle B \rangle$ when Alice gets $\downarrow$

- It turns out that $\langle AB \rangle = -\langle AB' \rangle = \langle AB' \rangle = \langle A'B' \rangle = -\frac{1}{\sqrt{2}}$

⇒ $|\cdots| = 2\sqrt{2} > 2$ CHSH inequality violated

From past to future

Experiment agrees with future-to-past interpretation

But… as said, the phenomenon is time-unaware…

Is it possible a past-to-future interpretation?

yes, it is!

The initial state is $\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$ and if we do a SG on f_A before f_B decays, we get up or down with equal probability.

The decay of f_B projects f_A into a state that depends on the decay configuration.

$$\sum_\alpha \left[M_{\alpha\,\frac{1}{2}} |\xi_\alpha\, \text{-}\tfrac{1}{2}\rangle - M_{\alpha\,\text{-}\frac{1}{2}} |\xi\ \tfrac{1}{2}\rangle \right]$$

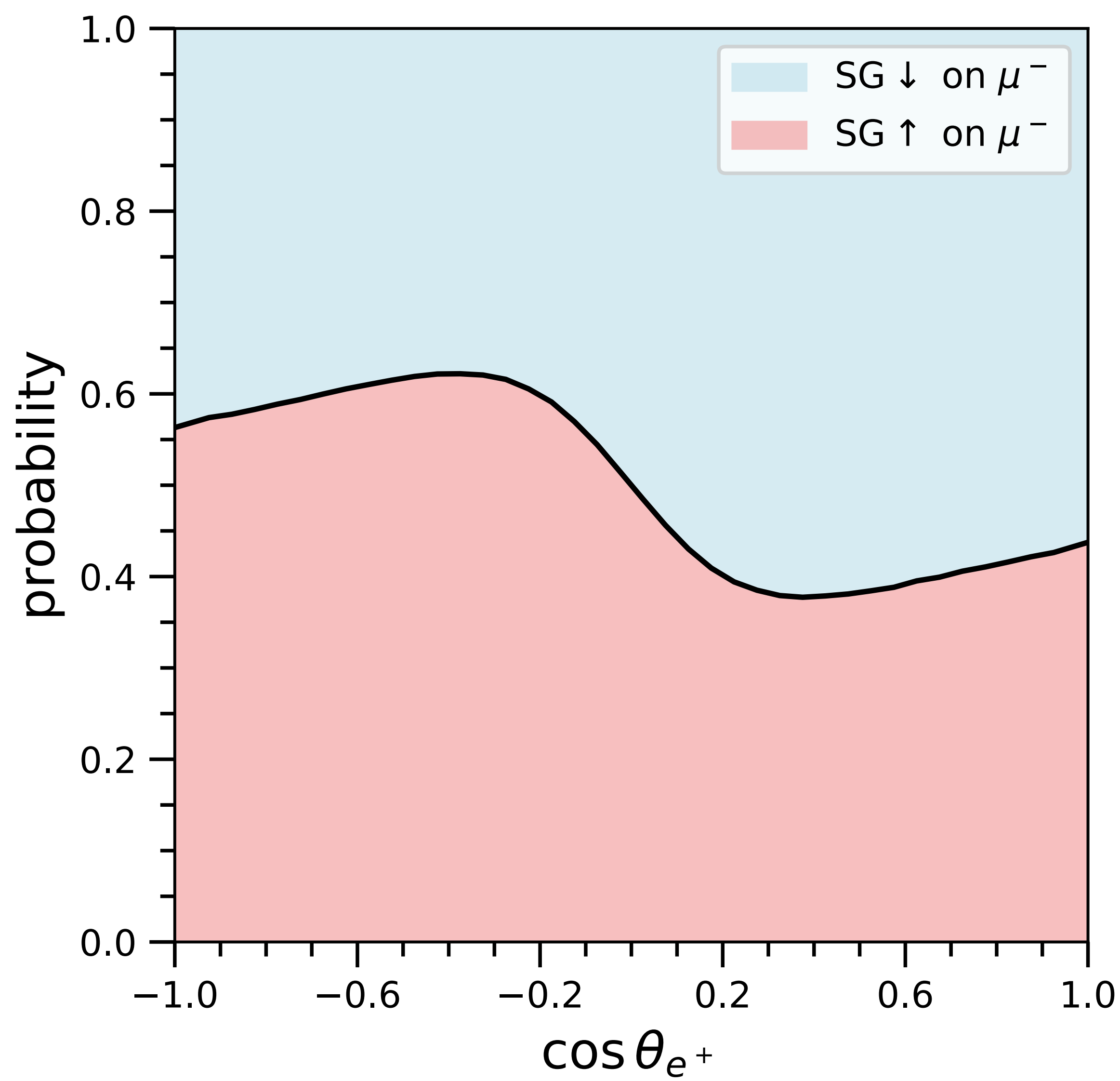
The probability to have SG up or down is not the same.

From past to future

μ^+ decay configuration *more compatible* with spin $|\downarrow\rangle$

time

higher probability that SG on μ^- gives $|\uparrow\rangle$

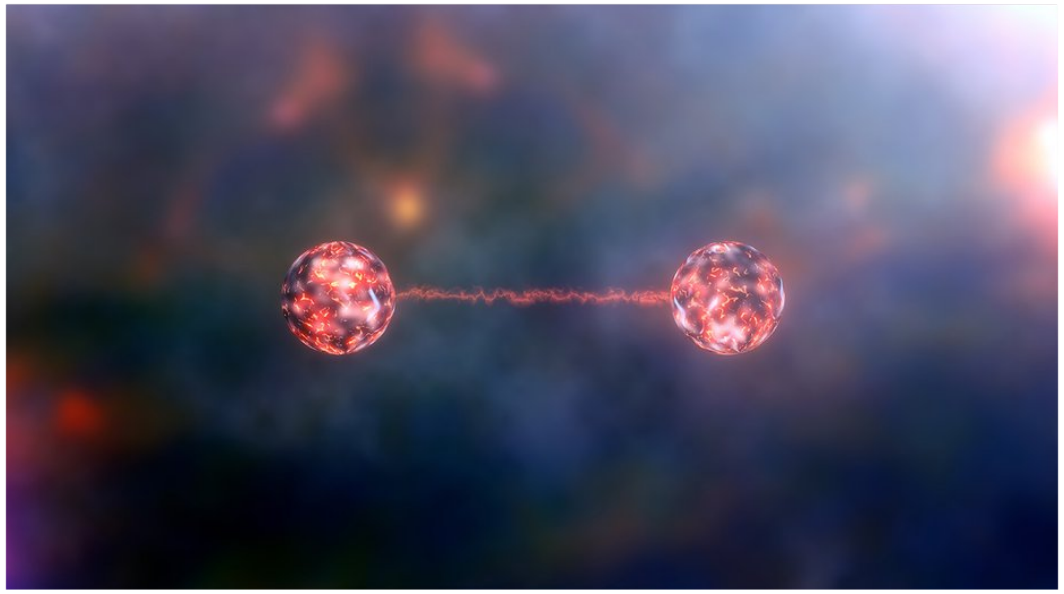


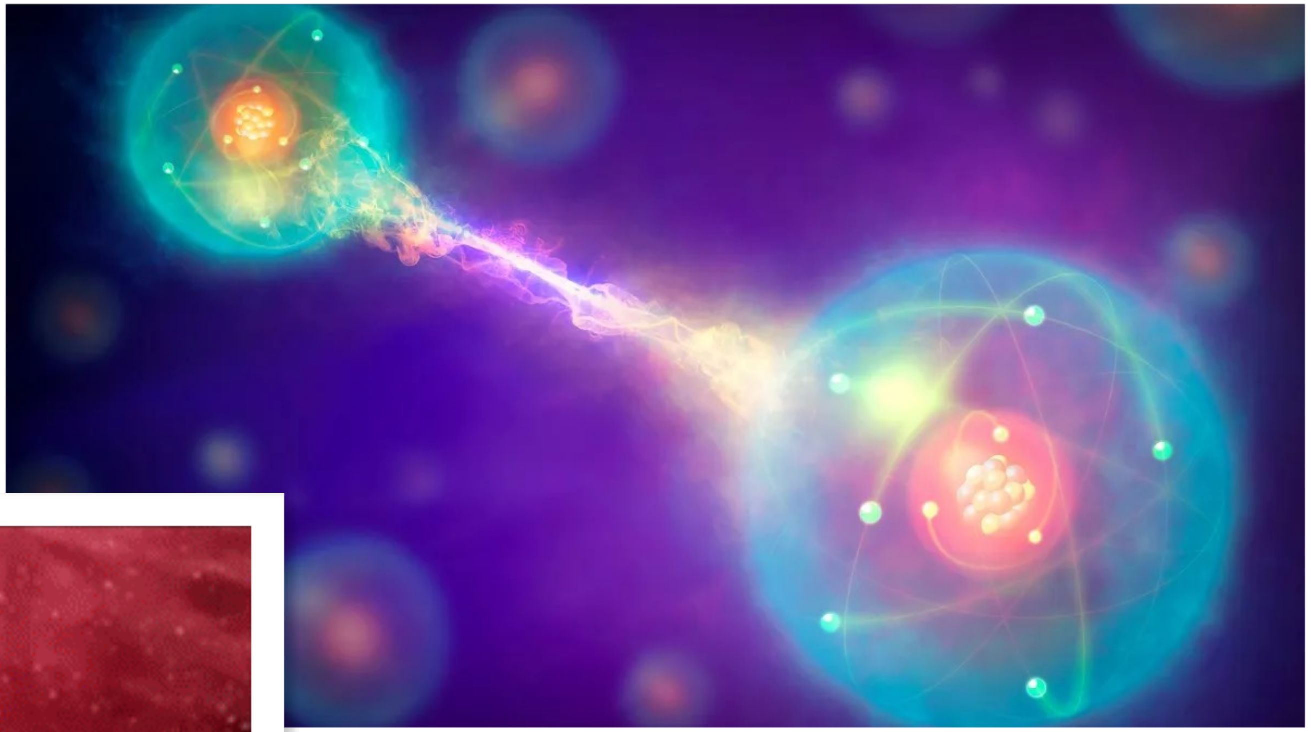
Then, when we post-select events where SG gives $|\downarrow\rangle$, we recover μ^+ decay distributions just as if μ^+ had spin $|\uparrow\rangle$ when it decayed. Beautiful and amazing.

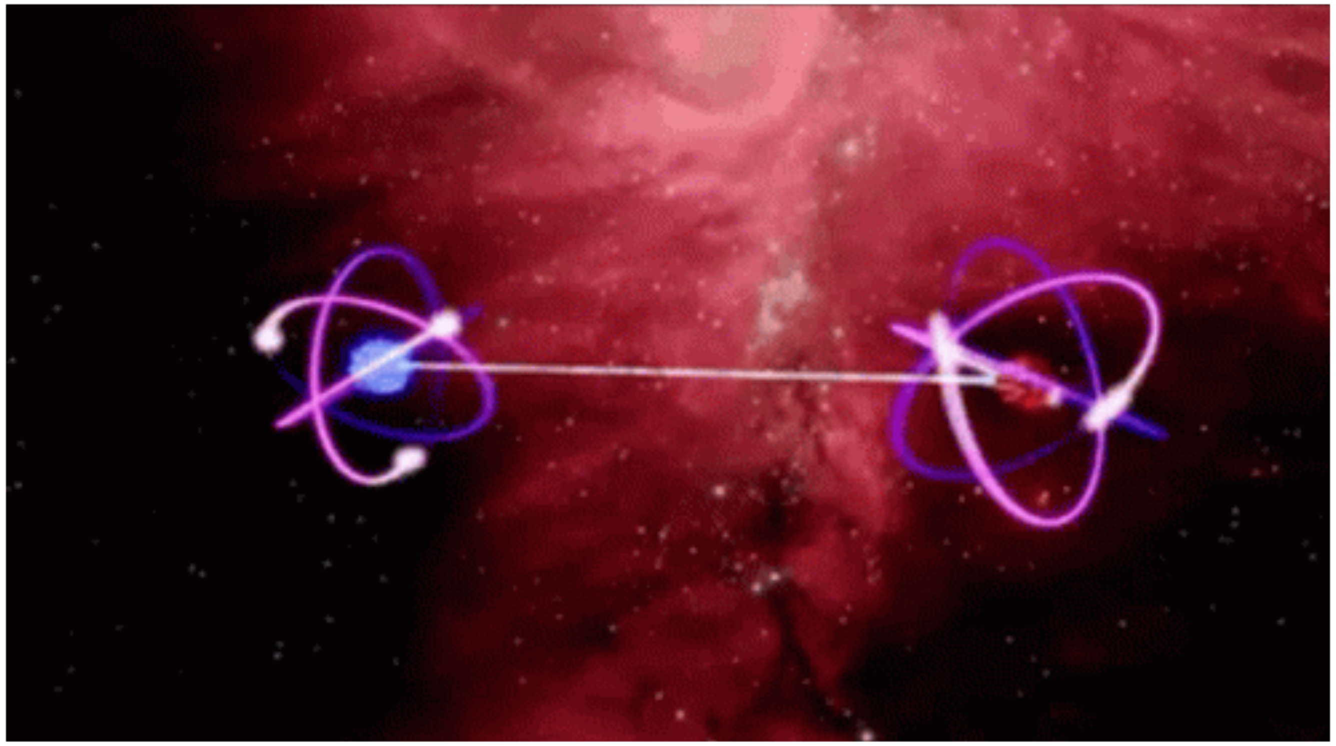
To take away

- ☑ Post-selection is a genuine entanglement effect, yet untested
- ☑ Past-to-future and future-to-past interpretations are equivalent and lead to the very same predictions

Quantum entanglement in the media







🤔 does not exactly look like a spooky action at a distance ...

End

What?

If we want to study quantum information stuff with the spin of elementary particles, we have to measure it. All of it! → density operator

As we all know, top quarks, *W*/*Z* bosons, … even τ leptons decay before one can pass them through a Stern-Gerlach experiment to measure spin.

But: the spin leaves its imprint in angular distributions.

$$\rho = \frac{1}{2}\begin{pmatrix} 1+P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1-P_3 \end{pmatrix}$$

density operator for spin 1/2

take daughter particle as spin analyser

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi}(1+\alpha\vec{P}\cdot\hat{n})$$

constant

momentum direction of daughter particle

$$\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

What?

Top pair: two spin-1/2 particles, simplest example of quantum correlation

$$\rho = \frac{1}{4}\left(1 \otimes 1 + \sum_i B_i^+ \sigma_i \otimes 1 + \sum_i B_i^- 1 \otimes \sigma_i + \sum_{ij} C_{ik}\sigma_i \otimes \sigma_j\right)$$

$$\hat{n}_a = (\sin\theta_a \cos\varphi_a, \sin\theta_a \sin\varphi_a, \cos\theta_a)$$
$$\hat{n}_b = (\sin\theta_b \cos\varphi_b, \sin\theta_b \sin\varphi_b, \cos\theta_b)$$

normalisation

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}\left[1 + \alpha_a \vec{B}^+ \cdot \hat{n}_a + \alpha_b \vec{B}^- \cdot \hat{n}_b + \alpha_a \alpha_b \hat{n}_a^T C \hat{n}_b\right]$$

3 coefficients corresponding to top polarisation

3 coefficients corresponding to anti-top polarisation

9 spin correlations

Measured by ATLAS and CMS since some time

A post-selection experiment

Definitive test: CHSH inequalities

A useful formulation of Bell-like inequalities for spin-1/2 systems is provided by the so-called CHSH inequalities for two systems A (Alice) and B (Bob).

Clauser, Horne, Shimony, Holt, '69

Alice measures two spin observables A, A'. Bob measures two spin observables B, B'. [Both normalised to unity]. Then, clasically:

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \leq 2$$

these are spin correlation observables

What?

The CHSH inequalities involve spin correlations. Therefore, for a particle of spin 1/2, they involve the C_{ij} spin-correlation coefficients [already measured for top pair production]

It can be shown that the maximum of the l.h.s.

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle|$$

is given by

$$2\sqrt{\lambda_1 + \lambda_2}$$

where λ_1 and λ_2 are the two largest eigenvalues of the positive definite matrix $C^T C$

Horodecki, Horodecki, Horodecki, ‘ 95

What?

Simpler but equally effective: Take judicious choice of [non-commuting] spin observables

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(2S_i + 2S_j)$$
$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(-2S_i + 2S_j)$$

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \quad \to \quad |C_{ii} + C_{jj}|$$

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \quad \to \quad |C_{ii} - C_{jj}|$$

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(-2S_i - 2S_j)$$
$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(2S_i - 2S_j)$$

CHSH violation is probed by testing if $|C_{ii} \pm C_{jj}| > \sqrt{2}$

These estimators are optimal when off-diagonal C_{ij} vanish

Post-decay entanglement

A density operator describing a composite system is separable if it can be written as

$$\rho_{\rm sep} = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the transpose in subspace of B [for example] the resulting density operator ρ^{T2} is valid.

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \quad \longrightarrow \quad \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

Post-decay entanglement

It is quite complicated to prove that a composite system is in a separable state [extensive work on PPT entangled states]

However, we are interested in showing that the system is entangled.

For this, one can use the counter-reciprocal of Peres-Horodecki necessary condition

ρ^{T2} non-positive ⇒ ρ^{T2} not valid ⇒ system entangled

Associated to it, there is a measure of entanglement that can be used for general systems

$$N(\rho) = \frac{\|\rho^{T_2}\| - 1}{2}$$

← equals the sum of negative eigenvalues of ρ^{T2}

now this is the
end