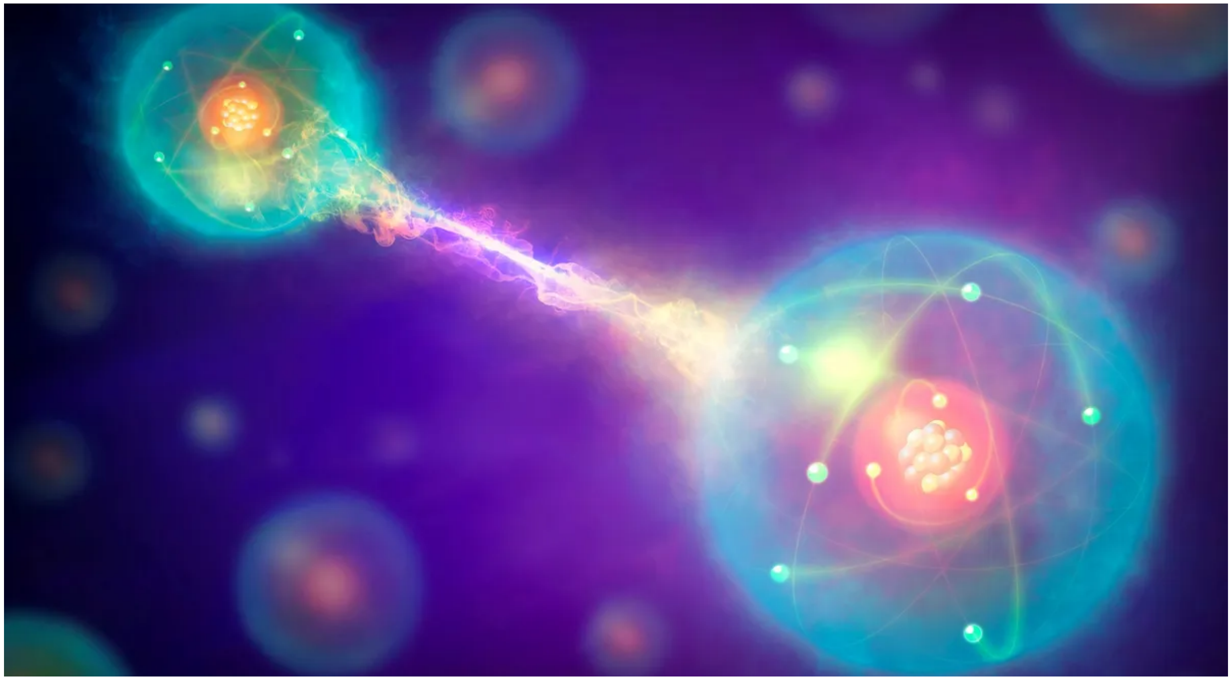
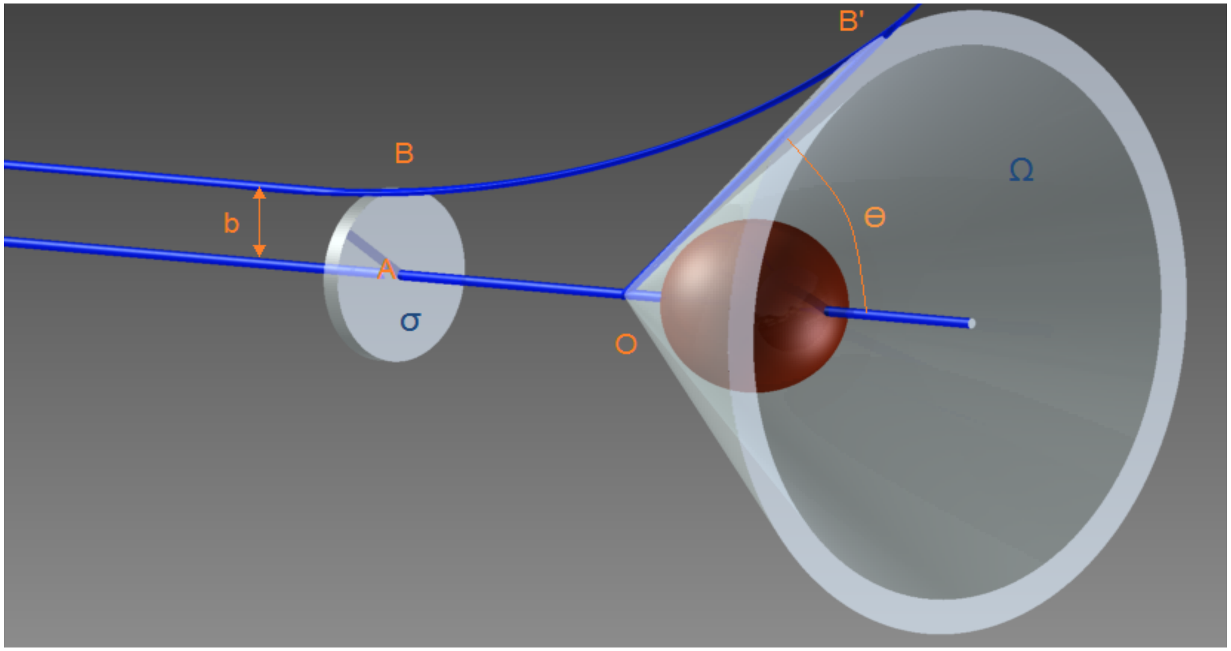
Oxford Workshop on Quantum Tests in Collider Physics



Entanglement Entropy = Cross Section

- Ian Low
- Argonne/Northwestern
- Oct. 1, 2024



Based on work collaborated with Zhewei Yin in 2405.085056 + to appear

Entropy is one of the oldest concepts in physics:

- Coarse-grained entropy –

  The underlying dynamics is deterministic, but our ignorance of microstates necessitates the use of probability.

  → It tends to increase under unitary (time) evolution.

- Fine-grained entropy –

  Quantum mechanics is inherently probabilistic and there's intrinsic randomness even if the wavefunction is completely known.

  → It remains constant under unitary evolution.

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \; ; \qquad p_i \geq 0 \; , \quad \sum_i p_i = 1$$

von Neuman entropy: $E(\rho) = -\mathrm{Tr}(\rho \ln \rho)$

There is no unique definition of entropy. Two commonly adopted ones are

- Renyi entropy: $$E_R(\rho) = \frac{1}{1-n} \log \ \mathrm{Tr} \ \rho^n$$

- Tsallis entropy: $$E_T(\rho) = \frac{1 - \mathrm{Tr} \ \rho^n}{n-1}$$

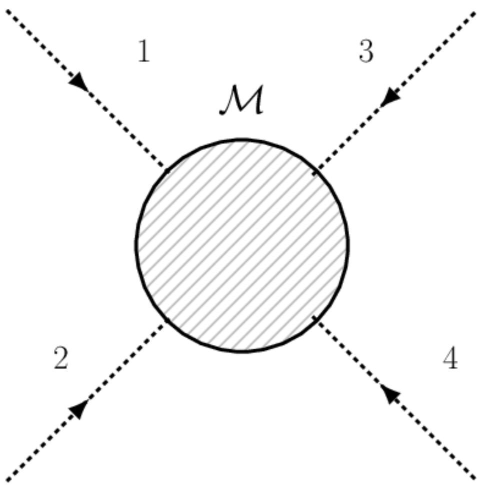
von Neuman entropy can be obtained in the limit n → 1 in both cases.

Linear entropy is the n=2 case of Tsallis entropy

$$E_2(\rho) = 1 - \mathrm{Tr} \ \rho^2$$

We are interested in 2-to-2 scattering of distinguishable particles in the S-matrix formalism:

A B → A B



$$|\text{out}\rangle \equiv S|\text{in}\rangle \qquad S = 1 + iT$$

$$\langle \{k_\text{f}\}, f_\text{f}|T|\{k_\text{i}\}, f_\text{i}\rangle$$
$$= (2\pi)^4 \delta^4 \left(\sum k_\text{f} - \sum k_\text{i} \right) M_{f_\text{i}, f_\text{f}}(\{k_\text{i}\}; \{k_\text{f}\})$$

We compute the quantum correlation between particle-A and particle-B and construct the bipartite system as

$$\mathcal{H}_\text{AB} = \mathcal{H}_\text{A} \otimes \mathcal{H}_\text{B}$$

$$\mathcal{H}_{A/B} = \mathcal{H}_\text{kinematic} \otimes \mathcal{H}_\text{flavor}$$

Kinematic = momentum and mass
Flavor = everything non-kinematic (could be spin!)

For now, we assume

- A pure initial state
- No entanglement between the incoming momenta
- No entanglement between momentum and flavor quantum numbers
- Allow possible entanglement among flavors

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In QFT textbooks it is customary to employ momentum eigenstates for the incoming particles.

(This is also how we prepare perform a high energy experiment!)

$$\langle p|q\rangle = (2\pi)^3\, 2E_p\, \delta^3(\vec{p}-\vec{q})$$

But then $\rho = |p\rangle\langle p| \qquad \mathrm{Tr}\,\rho = \langle p|p\rangle \;\propto\; \delta^3(0)$

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One possibility is to introduce finite-volume regularization:

$$\delta^3(0) = \int d^3x \;\longrightarrow\; V$$

We will instead introducing wave packets, which is really how we do the experiment!

$$|\mathrm{in}\rangle = \sum_{i,\bar{i}} \Omega_{i\bar{i}} |\psi_{\mathrm{A}}\rangle \otimes |i\rangle \otimes |\psi_{\mathrm{B}}\rangle \otimes |\bar{i}\rangle$$

$$|\psi_{\mathrm{A/B}}\rangle = \int_p \psi_{\mathrm{A/B}}(p)\,|p\rangle, \qquad \int_p \equiv \int \frac{d^3\vec{p}}{(2\pi)^3\sqrt{2E_p}}$$

$$\langle\psi|\psi\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3}\ |\psi(p)|^2 = 1$$

The initial density matrix is now properly normalized:

$$\rho^{\mathrm{i}} = |\mathrm{in}\rangle\langle\mathrm{in}|$$

$$\mathrm{tr}\rho^{\mathrm{i}} = \langle\mathrm{in}|\mathrm{in}\rangle = \langle\psi_{\mathrm{A}}|\psi_{\mathrm{A}}\rangle\langle\psi_{\mathrm{B}}|\psi_{\mathrm{B}}\rangle = 1$$

The out-state is in general a superposition of all outcome of the scattering:

$$|\mathrm{out}\rangle = S|\mathrm{in}\rangle = |\mathrm{outcome}_1\rangle + |\mathrm{outcome}_2\rangle + \cdots$$

We would like to focus on elastic scattering AB → AB and will insert a projection operator to select the AB final state:

$$|\mathrm{out}\rangle_{\mathrm{el}} = P_{\mathrm{AB}}|\mathrm{out}\rangle$$

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Applying the Luder's rule, the properly normalized final state density matrix is

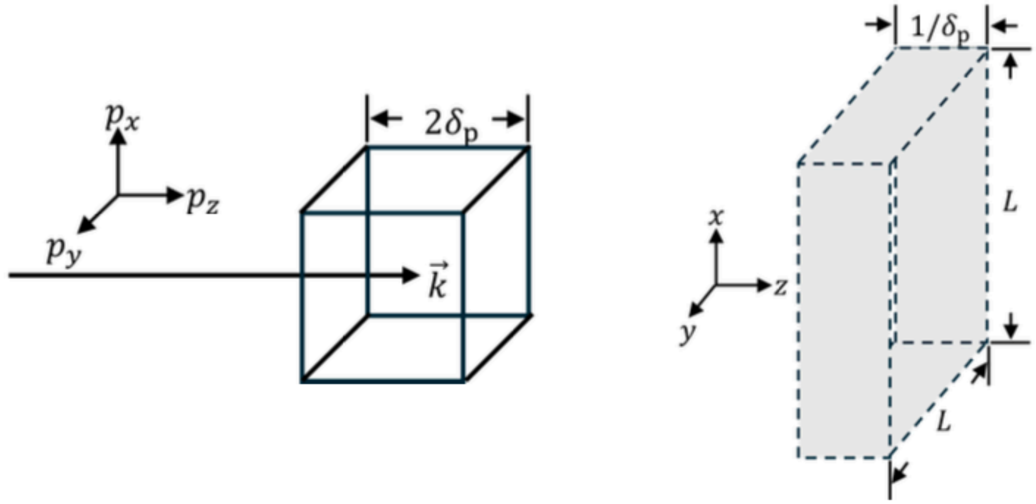
$$\rho^{\mathrm{f}} = \frac{P_{\mathrm{AB}}|\mathrm{out}\rangle\langle\mathrm{out}|P_{\mathrm{AB}}}{\mathrm{Tr}\left(P_{\mathrm{AB}}|\mathrm{out}\rangle\langle\mathrm{out}|\right)} = \frac{1}{1-\mathcal{P}_{\mathrm{inel}}}|\mathrm{out}\rangle_{\mathrm{el}\ \mathrm{el}}\langle\mathrm{out}|$$

$$\mathcal{P}_{\mathrm{inel}} = \langle\mathrm{out}|1-P_{\mathrm{AB}}|\mathrm{out}\rangle = \langle\mathrm{in}|T^\dagger(1-P_{\mathrm{AB}})T|\mathrm{in}\rangle$$

↑

The probability for AB to scatter inelastically into anything but AB.

We choose the following wave packet that is approximately uniform in the transverse plane in the position space:



(a) In momentum space (b) In position space

L^2 characterizes the transverse size of the wave packet in position space!

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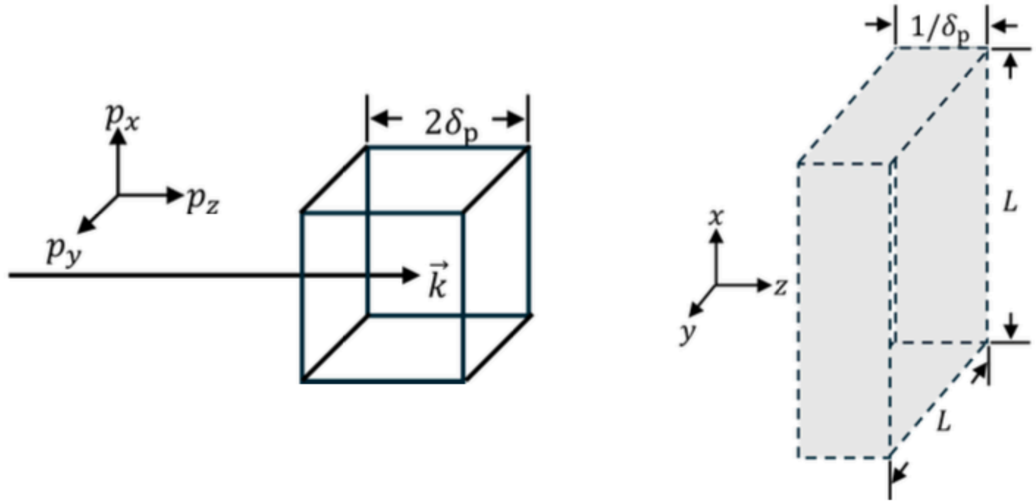

(a) In momentum space (b) In position space

L^2 characterizes the transverse size of the wave packet in position space!

In position space the plane wave limit is

$$\delta_p/|\vec{k}| \to 0 \ , \qquad \delta_p L \gg 1$$

Notice that in the position space the wave packet is a ”square pancake” like object, as we expect the longitudinal direction to be “Lorentz contracted.”

More specifically, the properly normalized wave packet is:

$$\psi_{\mathrm{A/B}}(\vec{p}) = \frac{8\sqrt{\pi^5 \delta_{\mathrm{p}}}}{L} \tilde{\delta}^3(\vec{p} - \vec{k}_{\mathrm{A/B}})$$

$$\begin{aligned}
\tilde{\delta}^3(\vec{p}) &= \tilde{\delta}_0(p_x)\tilde{\delta}_0(p_y)\tilde{\delta}_z(p_z)\,, \\
\tilde{\delta}_0(k) &= \frac{\pi}{2}\frac{\Theta(k+\delta_{\mathrm{p}}) - \Theta(k-\delta_{\mathrm{p}})}{\mathrm{Si}(\delta_{\mathrm{p}}L/2)}\frac{L}{2\pi}\mathrm{sinc}\,\frac{kL}{2}\,, \\
\tilde{\delta}_z(k) &= \frac{\Theta(k+\delta_{\mathrm{p}}) - \Theta(k-\delta_{\mathrm{p}})}{2\delta_{\mathrm{p}}}\,,
\end{aligned}$$

One can show that, in the limit

$$\delta_{\mathrm{p}}/|\vec{k}| \to 0\ , \qquad\qquad \delta_{\mathrm{p}}L \gg 1$$

$$\tilde{\delta}^3(k) \;\to\; \delta^3(k)$$

We find it convenient to set $1/(\delta_{\mathrm{p}}L) \lesssim \delta_{\mathrm{p}}/|\vec{k}|$ so that there is a single small parameter to expand.

Now we have carefully set up a wave packet formalism to compute the cross section and entanglement entropy in the plane wave limit (i.e. momentum eigenstates).

We are going to compute everything to the leading order in $\delta_{\mathrm{p}}/|\vec{k}|$

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Recall the final state density matrix

$$\rho^{\rm f} = \frac{1}{1-\mathcal{P}_{\rm inel}}|{\rm out}\rangle_{\rm el}\ {}_{\rm el}\langle{\rm out}| \qquad \mathcal{P}_{\rm inel} = \langle{\rm in}|T^\dagger(1-P_{\rm AB})T|{\rm in}\rangle$$

Let's insert a complete basis $1 = \sum_f \int d\Pi_f |f\rangle\langle f|$

We get

$$\mathcal{P}_{\rm inel} = I_0(|\vec{k}|)\left[\sigma_{\rm inel} + \mathcal{O}(\delta_{\rm p}/|\vec{k}|)\right]$$

$$\begin{aligned} I_0(|\vec{k}|) \ &= \ 4|\vec{k}|\sqrt{s}\int_{p_1,p_2,q_1,q_2} \psi_{\rm A}(p_1)\psi_{\rm B}(p_2)\psi^*_{\rm A}(q_1) \\ &\quad \times\psi^*_{\rm B}(q_2)(2\pi)^4\delta^4(q_1+q_2-p_1-p_2) \end{aligned}$$

IL and Zhewei Yin: 2405.085056

After expanding around the plane wave limit,

$$I_0(|\vec{k}|) = \frac{1}{L^2}\left(1 + \mathcal{O}(\delta_\mathrm{p}/|\vec{k}|)\right)$$

$$\mathcal{P}_\mathrm{inel} = \frac{\sigma_\mathrm{inel}}{L^2} + \mathcal{O}({\delta_\mathrm{p}}^5/|\vec{k}|^5)$$

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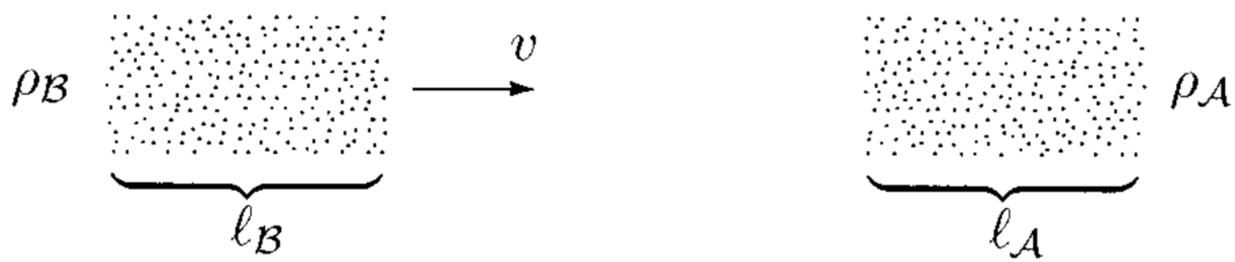
There's an intuitive understanding of this result. Let's go back to Chapter 4 in Peskin and Schroeder:



$$\sigma \equiv \frac{\text{Number of scattering events}}{\rho_{\mathcal{A}}\, \ell_{\mathcal{A}}\, \rho_{\mathcal{B}}\, \ell_{\mathcal{B}}\, A}$$

We are scattering only two particles head on, so

$$\rho_{\mathcal{A}}\ell_{\mathcal{A}}A = \rho_{\mathcal{B}}\ell_{\mathcal{B}}A = 1\ , \quad A = L^2\ , \quad N_{\mathrm{inel}} = \mathcal{P}_{\mathrm{inel}}$$

Similarly one can show

$$\mathcal{P}_{\rm tot} \;=\; \langle {\rm in}|T^\dagger T|{\rm in}\rangle = \frac{\sigma_{\rm tot}}{L^2} + \mathcal{O}({\delta_{\rm p}}^5/|\vec{k}|^5),$$

$$\mathcal{P}_{\rm el} \;=\; \langle {\rm in}|T^\dagger P_{\rm AB} T|{\rm in}\rangle = \frac{\sigma_{\rm el}}{L^2} + \mathcal{O}({\delta_{\rm p}}^5/|\vec{k}|^5).$$

$$\mathcal{P}_{\rm tot} = \mathcal{P}_{\rm el} + \mathcal{P}_{\rm inel}$$

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Now let's consider an initial that is unentangled in both momentum and flavors,

$$|{\rm in}\rangle = \sum_{i,\bar{i}} \Omega_{i\bar{i}} |\psi_{\rm A}\rangle \otimes |i\rangle \otimes |\psi_{\rm B}\rangle \otimes |\bar{i}\rangle$$

$$\Omega_{i\bar{i}} = \omega_i \omega'_{\bar{i}} \qquad\qquad \omega = (1, 0, 0, \cdots), \quad \omega' = (1, 0, 0, \cdots)$$

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And compute the subsystem *linear* entropy using the reduced density matrix,

$$\begin{aligned}
\mathcal{E}_2^{\rm f} \;&=\; I_0(|\vec{k}|)\frac{{\rm Im}(M^{\rm F}_{1\bar{1},1\bar{1}}) - 2|\vec{k}|\sqrt{s}\,\sigma_{\rm inel} + \mathcal{O}(\delta_{\rm p}/|\vec{k}|)}{|\vec{k}|\sqrt{s}} \\
&=\; 2I_0(|\vec{k}|)\left[\sigma_{\rm tot} - \sigma_{\rm inel} + \mathcal{O}(\delta_{\rm p}/|\vec{k}|)\right] \\
&=\; 2\frac{\sigma_{\rm el}}{L^2} + \mathcal{O}(\delta_{\rm p}{}^5/|\vec{k}|^5)\,,
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$$= 2I_0(|\vec{k}|)\left[\sigma_{\rm tot} - \sigma_{\rm inel} + \mathcal{O}(\delta_{\rm p}/|\vec{k}|)\right]$$

$$= 2\frac{\sigma_{\rm el}}{L^2} + \mathcal{O}(\delta_{\rm p}{}^5/|\vec{k}|^5)\,,$$

Optical theorem: $2\,{\rm Im}\,T = T^\dagger T$

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Next we consider the case of a mixed initial state, but unentangled:

$$\rho^{\mathrm{i}} = \rho^{\mathrm{i}}_{f_{\mathrm{A}}} \otimes \rho^{\mathrm{i}}_{p_{\mathrm{A}}} \otimes \rho^{\mathrm{i}}_{f_{\mathrm{B}}} \otimes \rho^{\mathrm{i}}_{p_{\mathrm{B}}}$$

If the subsystem density matrix satisfies

$$(\rho^{\mathrm{i}}_{f_{\mathrm{A}}})^2 \ \propto \ \rho^{\mathrm{i}}_{f_{\mathrm{A}}}$$

which is the case of unpolarized scattering, the subsystem entanglement entropy is

$$\mathcal{E}^{\mathrm{f}}_{2,\mathrm{A}} = \frac{2}{n_{\mathrm{A}}} \frac{\overline{\sigma_{\mathrm{el}}}}{L^2} + \mathcal{O}({\delta_{\mathrm{p}}}^5/|\vec{k}|^5)$$

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The number of flavor d.o.f. occupied in the beam

Unpolarized elastic cross section

In the end there's a surprisingly simple statement:

$$\mathrm{AB} \to \mathrm{AB}$$

$$\mathcal{H}_{\mathrm{AB}} = \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$$

The entanglement entropy in the wave packet formalism is

$$\mathcal{E}_n(\rho_{\mathrm{A}}^{\mathrm{f}}) = \frac{n}{n-1} \frac{\sigma_{\mathrm{el}}}{L^2}$$

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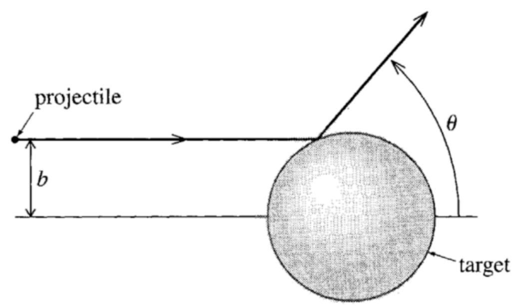
In plain English,

The entanglement entropy is the cross section in unit of the transverse size of the wave packet.

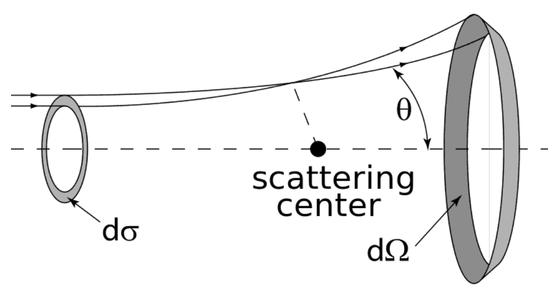
Duality in the cross section:

- It is an effective area characterizing the strength of interaction when two particles collide:



$$\sigma = \pi r^2$$

- Quantum-mechanically, it is a probability measure of a specific process taking place.



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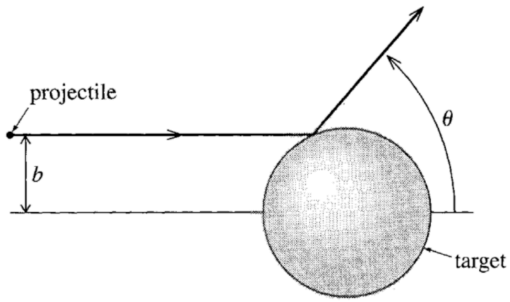

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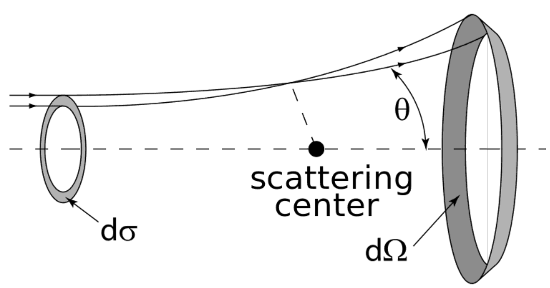
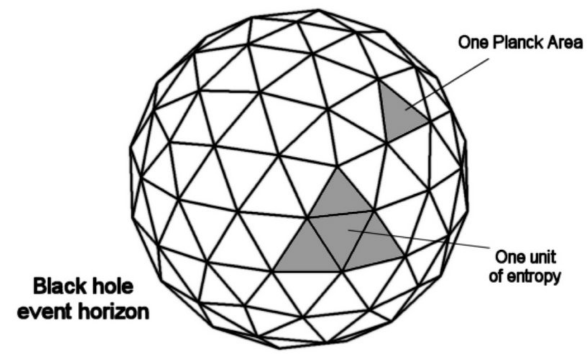


This is an area law: Entropy ~ Area

Area laws for entropy have been a subject of fascination:

- Bekenstein-Hawking entropy

$$S_{BH} = \frac{A}{4G}$$



- Massless free field theory:

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Entropy and Area

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(Received 15 March 1993)

The ground-state density matrix for a massless free field is traced over the degrees of freedom residing inside an imaginary sphere; the resulting entropy is shown to be proportional to the area (and not the volume) of the sphere. Possible connections with the physics of black holes are discussed.

- Quantum many-body systems:

arXiv: 0808.3773

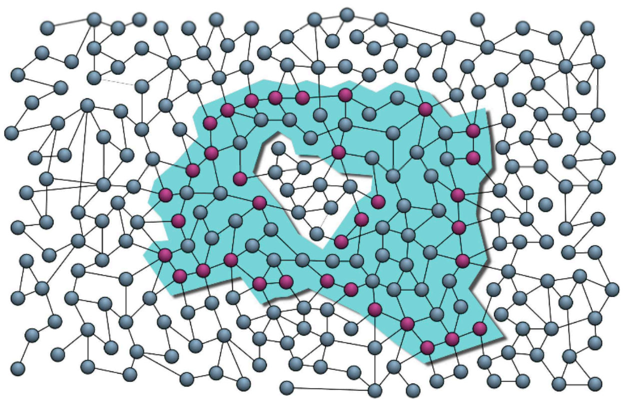
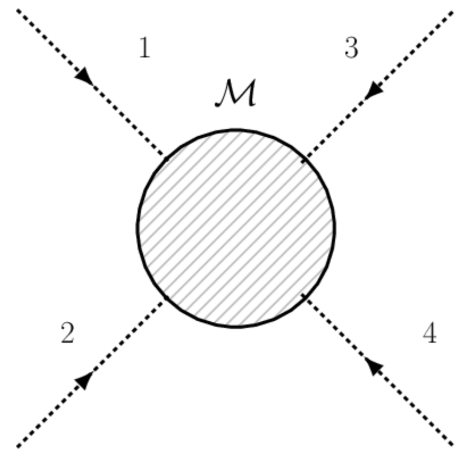


FIG. 1 A lattice L with a distinguished set $I \subset L$ (shaded area). Vertices depict the boundary ∂I of I with surface area $s(I) = |\partial I|$.

$$s(I) = |\partial I|$$

All examples involve macroscopic (black hole) or many-body systems and there is a **clearly defined boundary** to construct an area.

In our case, it is a simple 2-to-2 scattering without a priori a **clearly defined boundary:**



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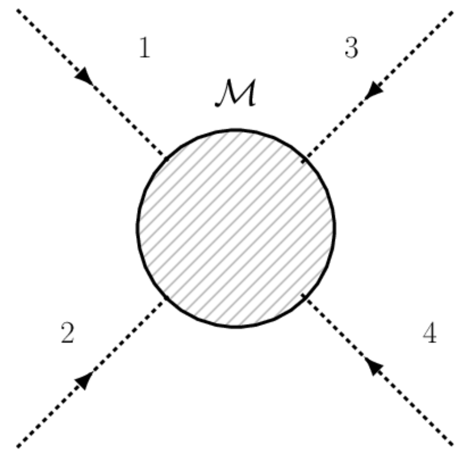
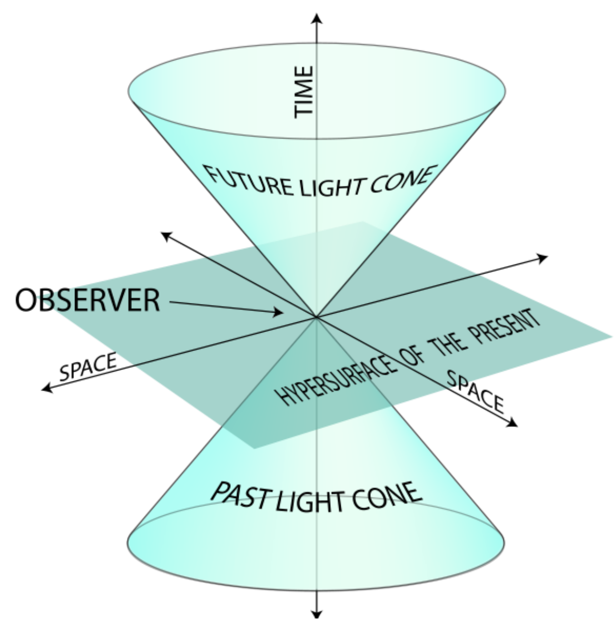


With the benefit of hindsight, perhaps we can view the cross section as the space-like boundary between future and past light-cones.

How general is this viewpoint?



So far we considered the subsystem entropy between particle-A and particle-B:

$$\mathcal{H}_{\rm AB} = \mathcal{H}_{\rm A} \otimes \mathcal{H}_{\rm B} \qquad\qquad \mathcal{H}_{A/B} = \mathcal{H}_{\rm kinematic} \otimes \mathcal{H}_{\rm flavor}$$

There are other possibilities of constructing a bipartite system:

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There are other possibilities of constructing a bipartite system:

- $\mathcal{H}_{AB} = \mathcal{H}_{\rm kinematic} \otimes \mathcal{H}_{\rm flavor}$

$$\mathcal{H}_{\rm kinematic} = \mathcal{H}_{p_{\rm A}} \otimes \mathcal{H}_{p_{\rm B}}$$

$$\mathcal{H}_{\rm flavor} = \mathcal{H}_{\rm flavor_A} \otimes \mathcal{H}_{\rm flavor_B}$$

$$\mathcal{E}_n^{\rm f} = \frac{n}{n-1} \frac{\sigma_{\rm el,fc}}{L^2}$$

Elastic cross section where at least one of the particles changes "flavor"

IL and Zhewei Yin: to appear

So far we considered the subsystem entropy between particle-A and particle-B:

$$\mathcal{H}_{\rm AB} = \mathcal{H}_{\rm A} \otimes \mathcal{H}_{\rm B} \qquad \mathcal{H}_{A/B} = \mathcal{H}_{\rm kinematic} \otimes \mathcal{H}_{\rm flavor}$$

There are other possibilities of constructing a bipartite system:

- $$\mathcal{H}_{AB} = \mathcal{H}_{\rm kinematic} \otimes \mathcal{H}_{\rm flavor} \qquad \mathcal{H}_{\rm kinematic} = \mathcal{H}_{p_{\rm A}} \otimes \mathcal{H}_{p_{\rm B}}$$

$$\mathcal{H}_{\rm flavor} = \mathcal{H}_{\rm flavor_A} \otimes \mathcal{H}_{\rm flavor_B}$$

$$\mathcal{E}_n^{\rm f} = \frac{n}{n-1}\frac{\sigma_{\rm el,fc}}{L^2}$$

Elastic cross section where at least one of the particles changes "flavor"

- $$\mathcal{H}_{AB} = \overline{\mathcal{H}_{\rm flavor_A}} \otimes \mathcal{H}_{\rm flavor_A} \qquad \overline{\mathcal{H}_{\rm flavor_A}} = \mathcal{H}_{p_{\rm A}} \otimes \mathcal{H}_{p_{\rm B}} \otimes \mathcal{H}_{\rm flavor_B}$$

$$\mathcal{E}_n^{\rm f} = \frac{n}{n-1}\frac{\sigma_{\rm el,fc(A)}}{L^2}$$

Elastic cross section where particle-A changes "flavor"

IL and Zhewei Yin: to appear

There are intriguing consequences of relating ”entropy” to “cross section”:

- Total and elastic cross sections are known to increase with respect to energy:

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LIMIT OF CROSS SECTIONS AT INFINITE ENERGY*

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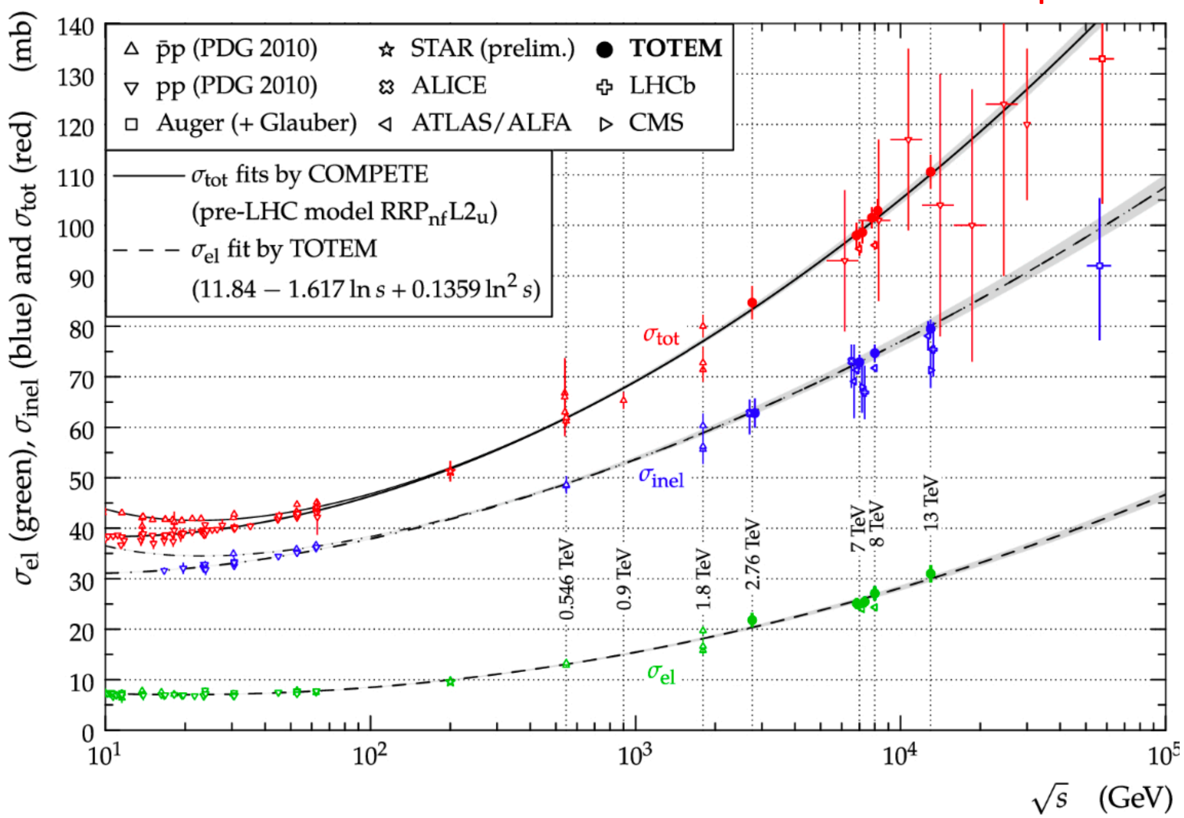
(Received 15 May 1970)

At infinite energy, we predict: (1) σ_{tot} approaches infinity; (2) the ratio of the real part to the imaginary part of the forward elastic amplitude approaches zero; (3) σ_{el}/σ_{tot} approaches $\frac{1}{2}$; (4) the width of diffraction peak approaches zero; its product with σ_{tot} is a constant. We give theoretical evidence based on massive quantum electrodynamics as well as experimental evidence in support of these predictions, and a physical picture for high-energy scattering.

This is a counter-intuitive result, and controversial at the time, as the partonic rate usually decreases with 1/s.

The growth has since been verified experimentally,

Particle Data Group 2024



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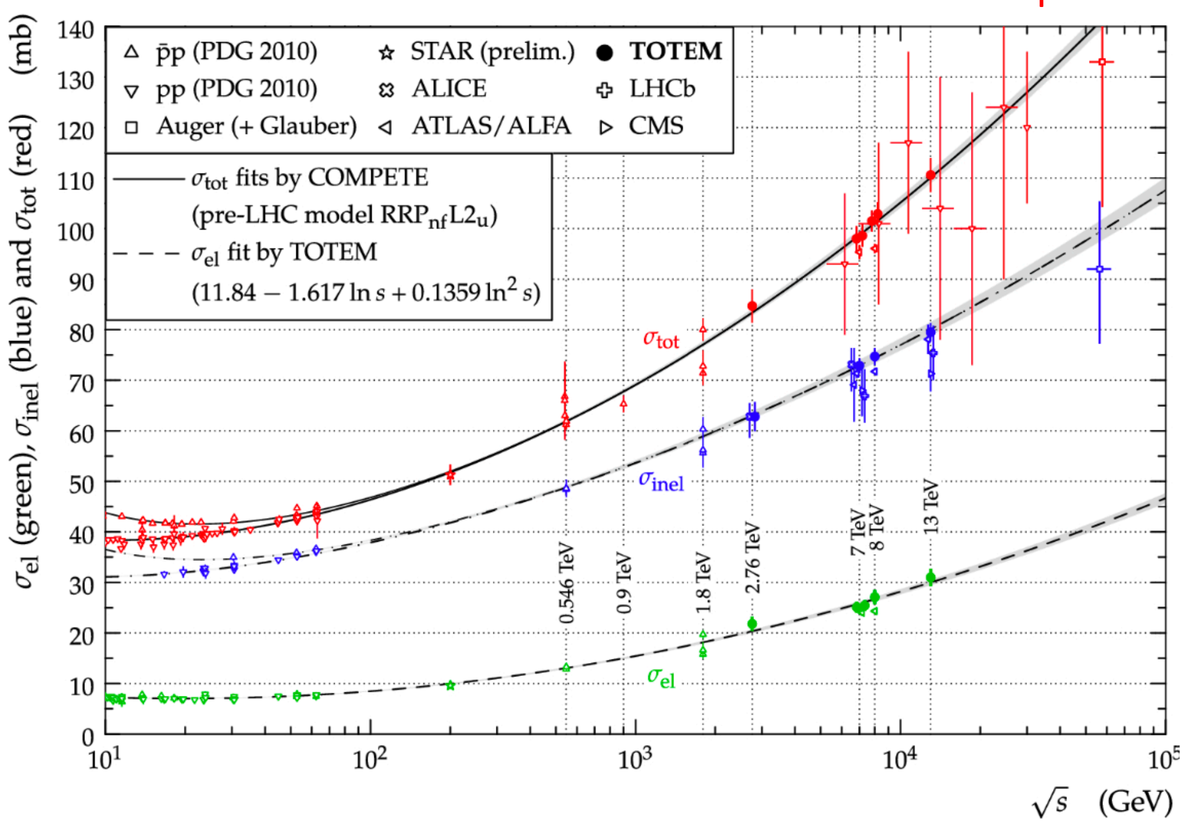

- Froissiart and Martin showed there's a universal bound on the total cross section:

$$\sigma_{\text{tot}} \leq \log^2 s$$

Concluding Remarks/Questions:

- In 2-to-2 scattering,

Entanglement Entropy is Cross Section!

- Entanglement entropy grows with energy in high energy scattering.

- The growth is bounded logarithmically by the Foissart bound.

Does this suggest some sort of thermodynamics law in particle scattering?

- Can we interpret other examples of area laws as ``probability'' or ''cross sections"?
- Can we measure the size of the wave packet experimentally?