The Beauty of Entanglement and Bell Non-locality

Quantum Tests in Collider Physics, Merton College, Oxford, UK

Based on YA, Kats, de Nova, Soffer, Uzan, 2406.04402

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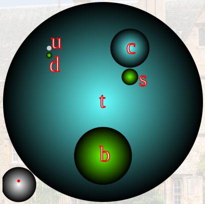
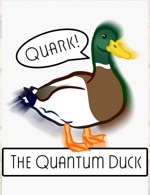
02.10.2024

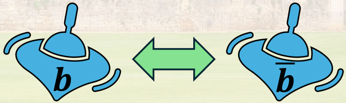
Overview

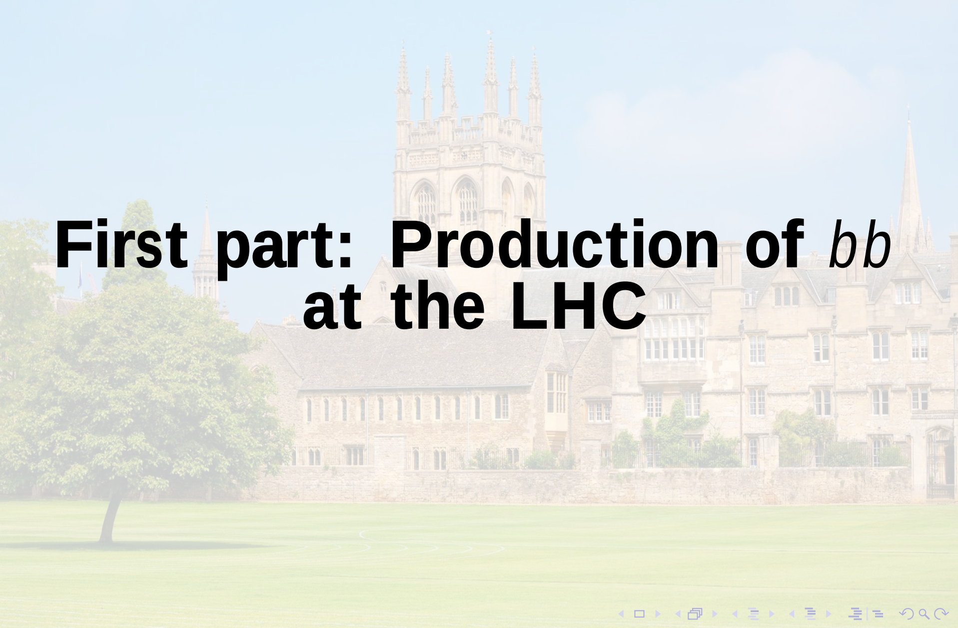
- Recently, it was shown that spin correlations can be measured in b-quark (beauty-quark) pairs at the LHC: Kats, Uzan, JHEP (2024).
- We have leveraged this work to study also Entanglement and Bell non-locality using $b\bar{b}$ pairs: YA, Kats, de Nova, Soffer, Uzan, 2406.04402.

- A unique system in many aspects:
  - Hadronizing system.
  - Low mass of the b-quark.
  - Highly boosted at the LHC.
  - b-jets can be tagged efficiently.
- Three main parts are in the talk:
  1. Production of $b\bar{b}$ at the LHC.
  2. Spin Correlations with $b\bar{b}$.
  3. Experimental Feasibility Study.



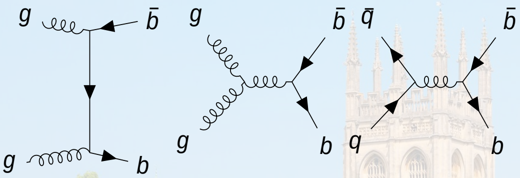


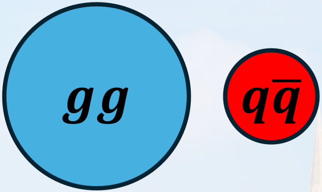




First part: Production of bb at the LHC

Production of $b\bar{b}$ at the LHC





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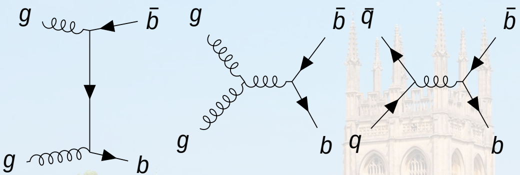

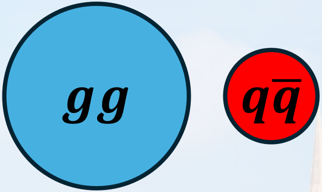

- Similar production mechanism as $t\bar{t}$, gg fusion is dominant at the LHC.
- Lower mass ! more boosted (m_b 5 GeV Vs. m_t 173 GeV), i.e. typically $M_{b\bar{b}}$ m_b.
- Large cross-section.
- Jets typically contain b-hadrons, which allow efficient tagging of b-jets.

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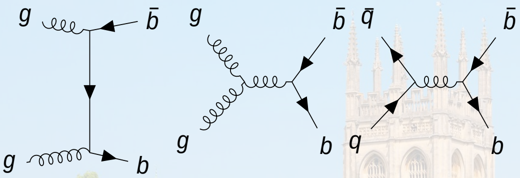

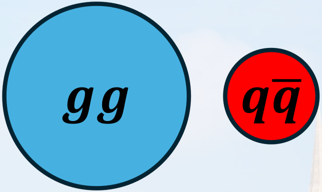

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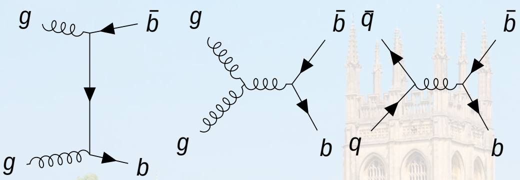

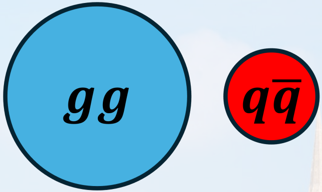

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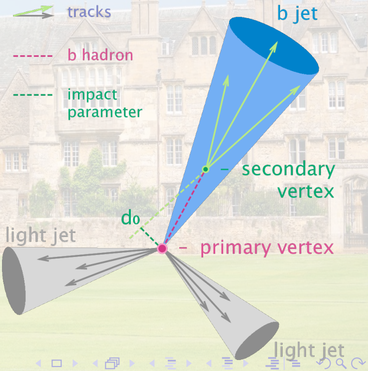
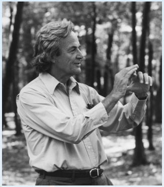
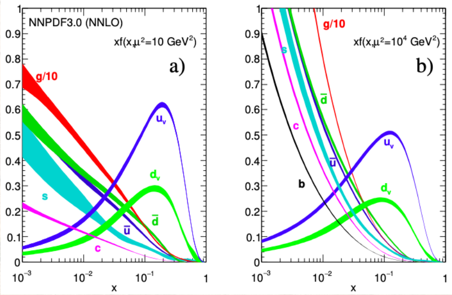


Collisions at the LHC

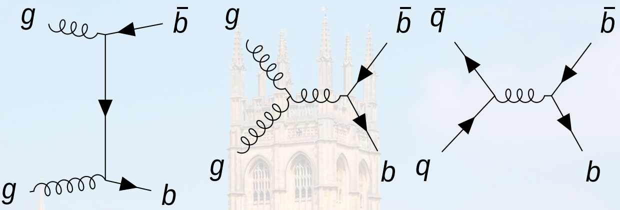




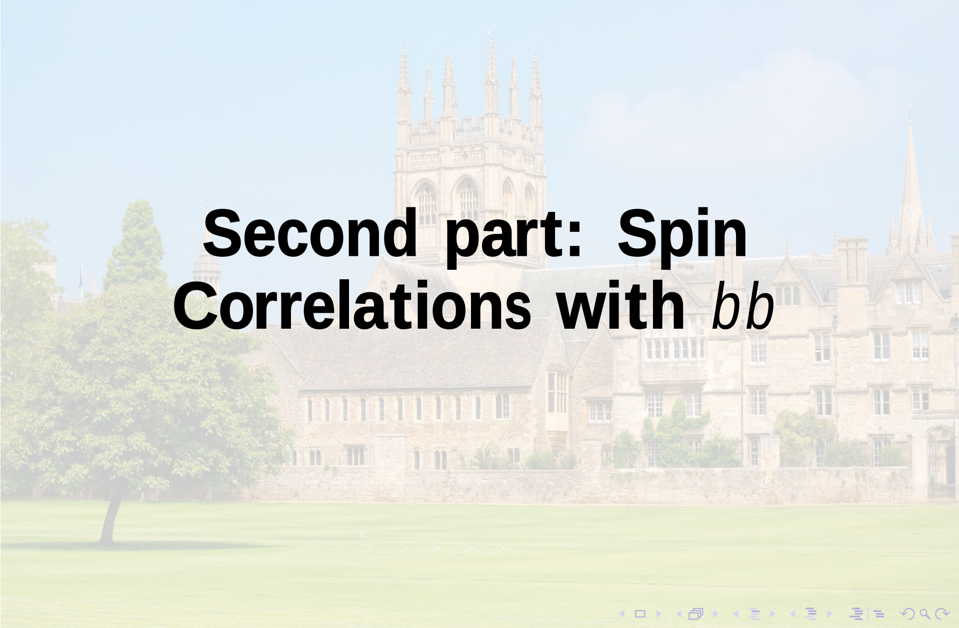
- At the LHC, protons are being collided at high energies.
- The proton is a complex creature!
- Proton: quarks and gluons (partons).
- Parton distribution function (PDF): the density of each parton in the proton.

Figure: Parton density at the proton. Figure is from JHEP 2015, 40 (2015).

Leading-order Analytical Calculation



- Analytical calculation at leading-order. The system is defined by:
  - $\hat{k}$: the direction of the b-quark with respect to the beam axis.
  - The invariant mass $M_{b\bar{b}}$, $\quad = \sqrt{1 - \frac{4\,m_b^2}{M_{b\bar{b}}^2}}$.
- Each one of $I = q\bar{q}; gg$ gives rise to $\;^{I}(M_{b\bar{b}}; \hat{k})$ with probability $w_I(M_{b\bar{b}}; \hat{k})$, which is PDF dependent.
- The spin density matrix:
  $$(M_{b\bar{b}}; \hat{k}) = \sum_{I=q\bar{q};gg} w_I(M_{b\bar{b}}; \hat{k}) \;^{I}(M_{b\bar{b}}; \hat{k}).$$
- The total quantum state:
  $$(M_{b\bar{b}}) \quad \int_{2m_b}^{M_{b\bar{b}}} \mathrm{d}M \int \mathrm{d}\Omega\, p(M; \hat{k}) \;(M; \hat{k}) = \int_{2m_b}^{M_{b\bar{b}}} \mathrm{d}M\, p(M) \;_{\Omega}(M)$$



Second part: Spin Correlations with bb

Spin Correlations with $b\bar{b}$ - Calculations

- Spin correlations of $b\bar{b}$ are not included in MC generators.
  - Calculated analytically.
  - Cross section and e ciency are calculated from simulation.
- How can we calculate the spin correlations analytically?

Spin Correlations with $b\bar{b}$ - Calculations

- Spin correlations of $b\bar{b}$ are not included in MC generators.
  - Calculated analytically.
  - Cross section and efficiency are calculated from simulation.
- How can we calculate the spin correlations analytically?
  - Same calculation as for $t\bar{t}$, with $m_t \to m_b$.
- Using the helicity basis $\{\hat{k}, \hat{n}, \hat{r}\}$:
  - $\hat{p}$: the proton-beam axis.
  - $\hat{k}$: the direction of the b in the $b\bar{b}$ COM frame.
  - $\hat{r} = (\hat{p} - \cos\theta\,\hat{k})/\sin\theta$.
  - $\hat{n} = \hat{r} \times \hat{k}$.
  - $\cos\theta = \hat{k} \cdot \hat{p}$.

Reaching the $b\bar{b}$ Polarizations and Spin Correlations

Spin-correlation measurements can be performed with $_b$ and $\bar{\ }_b$.

- The lightest, most commonly produced b-baryon.
- ud-quarks: spin-singlet, isospin-singlet.
- b-quark: carries the baryon spin.
- Since m_b $\quad_{QCD}$, $_b$ baryons are expected to carry a large fraction of the original b-quark polarization.

The retention factors r_L and r_T:

$$r_{\hat{P}} = \frac{P(\ _b)}{P(b)}\ ; \hat{P} = L; T :$$

Determine how much of the polarization is transferred $b \rightarrow\ _b$.

The Retention Factors

- In order to perform the measurement, we have to extract r_L, r_T.
  - Their values are expected to be roughly in the ranges $0.4 \lesssim r_L \lesssim 0.8$, $0.5 \lesssim r_T \lesssim 0.8$.
  - One possibility is to use dedicated control regions where significant entanglement is not expected while some of the elements C_{ij} are sizable.
- The polarizations have been measured in Z-boson decays at LEP, by ALEPH, OPAL, DELPHI.
  - An approximate combination gives $r_L = 0.47 \pm 0.14$.

Spin Measurement witbb

Most general density matrix for 2 qubits:

$$= \frac{I_4 + \sum_i B_i^+ \;^i \;\; I_2 + B_i \;\; I_2 \;\; ^i + \sum_{i;j} C_{ij} \;^i \;\; ^j}{4}$$

15 parameters B_i ; C_{ij} ! Quantum tomography=Measurement of individual spin polarizations B and spin correlation matrix C:

$$B_i^+ = h \;^i i \; ; \; B_i = h \;^i i \; ; \; C_{ij} = h \;^i \;\; ^j i$$

Spin Measurement witbb

We use $_b$! X_c ` `, where X_c denotes a charmed state containing a baryon, usually the ^+_c.

Neutrinos as spin analyzers (' 1):

$$\frac{1}{}\frac{d}{dx_{ij}} = \frac{1}{2}(1 \quad c_{ij}\, x_{ij})\ln \frac{1}{j x_{ij}\, j} \quad ;$$

where $x_{ij} = \cos\ ^+_i \cos\ _j$, and

$$c_{ij} = \ ^2 r_i\, r_j\, C_{ij} :$$

The retention factors: r_T goes for $i;j = n;r$ and r_L for $i;j = k$ indices.

Third part: Experimental Feasibility Study

Experimental Observables

Quantum Entanglement

Concurrence $C[\;]$: quantitative measurement of entanglement.

$0 \quad C[\;] \quad 1$, $C[\;] \neq 0$ i the state is entangled.

Here, $C[\;] = \max(\;\;; 0)$; $\;\; = \dfrac{C_{nn} + jC_{kk} + C_{rr}j \quad 1}{2}$.

Experimental Observables

Quantum Entanglement:

- Concurrence C[ ]: quantitative measurement of entanglement.
- 0 C [ ] 1, C[ ] 6= 0 i the state is entangled.
- Here, C[ ] = max(; 0); = $\frac{C_{nn} + |C_{kk} + C_{rr}| \quad 1}{2}$.

Bell Non-locality:

- A violation of the CHSH inequality: p $\overline{\quad_1 + \quad_2}$ 1, where 0 $_i$ 1 are the eigenvalues of $C^T C$. A su cient criterion:

$$V \quad C_{kk}^2 + C_{rr}^2 \quad 1 \qquad _1 + \quad _2 \quad 1:$$

- $V > 0$ is expected to accurately capture the Bell non-locality in the ultrarelativistic regime, in which C is diagonal, and $C_{kk}^2 ; C_{rr}^2 > C_{nn}^2$.

Entanglement and Bell Non-locality Before Integration

Full LHC $(M_{bb}, \hat{k})$ Concurrence.

Solid white line: entanglement limit; Dashed black line: Bell non-locality limit.

Regions with strong quantum correlations:

- $M_{bb} \simeq 2m_b$: maximally entangled spin singlet.
- Ultra-relativistic regime: maximally entangled spin-triplet state for transverse production ($\cos\theta \simeq 0$).

In practice, most events are boosted.

$$|\psi\rangle_{\text{singlet}} = \frac{1}{\sqrt{2}}\left(|\uparrow_{\hat{n}}\downarrow_{\hat{n}}\rangle - |\downarrow_{\hat{n}}\uparrow_{\hat{n}}\rangle\right)$$

$$|\psi\rangle_{\text{triplet}} = \frac{1}{\sqrt{2}}\left(|\uparrow_{\hat{n}}\downarrow_{\hat{n}}\rangle + |\downarrow_{\hat{n}}\uparrow_{\hat{n}}\rangle\right)$$

Experimental Setups

- ATLAS:
  - Large data size.
  - High trigger thresholds.
- CMS B-parking data:
  - Storing a large amount of raw detector data, with low trigger thresholds.
  - Processed when su cient computational power is available to handle such data.
  - High statistics thanks to the low p_T thresholds.
- LHCb:
  - Smaller data size.
  - Low trigger thresholds and better reconstruction.

Figure: A schematic view of the typical Run 2 data ow (up) and comparison of the typical HLT rates (down) in the CMS experiment (CMS, 2403.16134).

Analysis Selection

We apply similar selections to the ones applied in the experiments. Selections with Run 2 data:

	ATLAS	CMS B-parking	LHCb
Trigger	2	displaced 1	1
p_T ($_1$)	> 15 GeV	> 7 12 GeV	> 1:8 GeV
($_1$)	j j < 2:4	j j < 1:5	2 < < 5
p_T ($_2$)	> 15 GeV	> 5 GeV	> 0:5 GeV
($_2$)	j j < 2:4	j j < 2:4	2 < < 5
N_b $_{tagged}$	1	-	1
M_{bb}	-	-	> 20 GeV
$p_T=p_T^{jet}$	> 0:2 for at least 1	-	-
Tracks	-	-	2-4, displaced
Additional	-	-	p_T(X) > 1:6 GeV, displaced
^+_c reco	Full reco on one of the sides		

For HL-LHC the selections are the same, besides the ATLAS 2 muon threshold:p_T ($_{1;2}$) > 10 GeV,j ($_{1;2}$)j < 2:5.

Feasibility Study - Determine the Statistics

For ATLAS and LHCb, we use:

$$N = 2 \qquad L\, f^{2}(b\,! \quad _{b})\, BR^{2}(\; _{b}\,! \quad X_{c} \qquad)$$
$$BR(\; ^{+}_{c}\; ! \quad \text{reco.}) \;_{\text{reco.}} \;\; _{b;2}$$

: the bb production cross section with muon cuts e ciency.

L: integrated luminosity.

$f(b\,! \quad _{b})$ 7%: fragmentation fraction for $_{b}$.

$BR(\; _{b}\,! \quad X_{c} \qquad)$ 11% and $BR(\; ^{+}_{c}\,! \quad \text{reco.})$ 18%.

$_{\text{reco.}}$ 50%: estimate for the average $^{+}_{c}$ decay reconstruction e ciency.

$_{b;2}$: the e ciency for at least one of the two jets to pass the b-tagging condition.

Feasibility Study - Determine the Statistics

For the CMSB-parking data, we use:

$$N = 2 f^{2}(b \to \quad _{b})\, BR(\, _{b} \to \; X_{c} \qquad) \quad _{2}$$
$$BR(\, _{c}^{+} \to \; reco) \; _{reco} N_{0}$$

- N_0 $\; 10^{10}$: the number of bb events in the CMS B parking dataset.
- $_2$ $\;$ 38%: the e ciency of selecting the muon on the non-triggering side of the event.

Feasibility Study - A Glimpse to the Present

	[pb]	L [fb ^{1}]	N	C_{kk}	C_{rr}	C_{nn}		V	r_L	stat	stat / V	/ stat	V / stat V	/ tot	V / tot V
	Run 2, $\sqrt{s} = 13$ TeV														
ATLAS	9:6 10^3	140	1:4 10^4	0:96	0:62	0:61	0:60	0:31	0.75	0.19	0.48	3.1	0.6	2.6	0.6
									0.45	0.32	1.11	1.8	0.3	1.7	0.3
LHCb, > 0:4	2:6 10^6	5:7	4:2 10^4	0:62	0:76	0:66	0:52	0:04	0.75	0.11	0.25	4.6	0:1	3.4	0:1
									0.45	0.19	0.46	2.7	0:1	2.4	0:1
CMS B parking	1:1 10^5	41:6	3:7 10^5	0:88	0:61	0:58	0:53	0:14	0.75	0.038	0.089	> 10	1.6	4.7	1.5
									0.45	0.064	0.20	8.4	0.7	4.3	0.7

Table: Sensitivity studies $r_T = 0{:}7$, systematic uncertainty of 20%.

- The expected signi cance of entanglement with CMS B-parking Run 2 data.
- Scanning the unknown $r_L; r_T$.
- White dotted polygon: plausible values for r_L and r_T.
- Vertical yellow lines: central value of r_L (thick line) and its 1 uncertainties from LEP measurements.

Feasibility Study - A Glimpse to the Future

	[pb]	L [fb ^{1}]	N	C_{kk}	C_{rr}	C_{nn}		V	r_L	stat	stat V	stat	$\frac{V}{\text{stat } V}$	tot	$\frac{V}{\text{tot } V}$
	HL-LHC, $\sqrt{s}$ = 14 TeV														
ATLAS, V > 0:3	3:7 10^4	3000	62 10^5	0:94	0:86	0:85	0:82	0:63	0.75	0.03	0.08	> 10	7.5	4.9	4.2
									0.45	0.05	0.17	> 10	3.7	4.8	3.0
LHCb, V > 0:3	3:0 10^6	300	33 10^5	0:83	0:88	0:83	0:77	0:48	0.75	0.040	0.11	> 10	4.3	4.8	3.3
									0.45	0.067	0.21	> 10	2.2	4.6	2.0
CMS B parking, V > 0:2	1:2 10^5	800	32 10^6	0:84	0:85	0:80	0:75	0:43	0.75	0.013	0.035	> 10	> 10	5.0	4.6
									0.45	0.022	0.068	> 10	6.3	4.9	3.9

Table: Sensitivity studies $r_T = 0{:}7$, systematic uncertainty of 20%.

- The expected signi cance of Bell non-locality with HL-LHC ATLAS expected data.
- Scanning the unknown $r_L; r_T$.
- White dotted polygon: plausible values for r_L and r_T.
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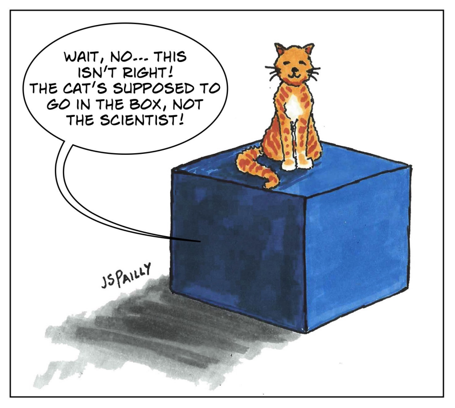
Summary

- So far, proposals to study quantum information theory in high-energ physics included mostly non-hadronizing systems, which decay quic
- We show that Entanglement and Bell non-locality can be measured with bb pairs, an hadronizing system.
- This possibility was almost explicitly rejected in many introductions c previous papers (including my own), so it is a rather surprising resu
- The most promising experimental setup for this purpose, using current data, is the CMSB-parking data.
- Experimentally challenging:
  - Reconstruction of a speci c decay inside the jets to identify $_b$.
  - Non-isolated leptons.
  - Unmeasured neutrino.
  - Loss of statistics due to fragmentation fraction, BR and e ciency.
- Theoretically interesting:
  - The bb system is boosted in low invariant mass.
  - Quantum correlations are key tools used for studying hadronizing systems, such as the quark-gluon plasma.

Thank You


WAIT, NO... THIS ISN'T RIGHT! THE CAT'S SUPPOSED TO GO IN THE BOX, NOT THE SCIENTIST!
JSPAILLY

Backup Slides

Backup

The Retention Factors

- In the heavy-quark limit:

$$r_L \quad \frac{1+A\,(0.23+0.38w_1)}{1+A}\,; \quad r_T \quad \frac{1+A\,(0.62 \quad 0.19w_1)}{1+A}\,.$$

The above expressions describe the dominant polarization loss effect, due to the contribution to the Λ_b sample from $\Sigma_b^{(\ )}$! Λ_b decays.

$$1 \quad A \quad 5\,; \qquad 0 \quad w_1 \quad 1\,.$$

where the chosen range for A reflects a large systematic uncertainty.