The Beauty of Entanglement and Bell Non-locality Quantum Tests in Collider Physics, Merton College, Oxford, UK Based on YA, Kats, de Nova, Soffer, Uzan, 2406.04402



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QI with bottom-quark pairs

Overview

- Recently, it was shown that spin correlations can be measured in b-quark (beauty-quark) pairs at the LHC: Kats, Uzan, JHEP (2024).
- We have leveraged this work to study also Entanglement and Bell non-locality using $b\bar{b}$ pairs: YA, Kats, de Nova, Soffer, Uzan, 2406.04402.

A unique system in many aspects:

- Hadronizing system.
- Low mass of the *b*-quark.
- Highly boosted at the LHC.
- *b*-jets can be tagged efficiently.

• Three main parts are in the talk:

- Production of $b\bar{b}$ at the LHC.
- Spin Correlations with *bb*.
- Experimental Feasibility Study.



First part: Production of $b\bar{b}$ at the LHC

First part: Production of $b\overline{b}$ at the LHC



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- Similar production mechanism as $t\bar{t}$, gg fusion is dominant at the LHC.
- Lower mass \rightarrow more boosted ($m_b \sim 5 \text{ GeV}$ Vs. $m_t \sim 173 \text{ GeV}$), i.e. typically $M_{b\bar{b}} \gg m_b$.
- Large cross-section.
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- Unlike *tt*, *bb* hadronize!

light jet

b hadron

impact parameter

do

b jet

secondary vertex

primary vertex

Collisions at the LHC



- At the LHC, protons are being collided at high energies.
- The proton is a complex creature!
- Proton: quarks and gluons (partons).
- Parton distribution function (PDF): the density of each parton in the proton.



Figure: Parton density at the proton. Figure is from JHEP 2015, 40 (2015).

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Leading-order Analytical Calculation



Analytical calculation at leading-order. The system is defined by:
 k: the direction of the *b*-quark with respect to the beam axis.

• The invariant mass $M_{b\bar{b}}$, $\beta = \sqrt{1 - \frac{4 \cdot m_{\bar{b}}^2}{M_{b\bar{b}}^2}}$.

- Each one of $I = q\bar{q}, gg$ gives rise to $\rho'(M_{b\bar{b}}, \hat{k})$ with probability $w_I(M_{b\bar{b}}, \hat{k})$, which is PDF dependent.
- The spin density matrix:

$$\rho(M_{b\bar{b}},\hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{b\bar{b}},\hat{k})\rho^I(M_{b\bar{b}},\hat{k})$$

• The total quantum state: $\rho(M_{b\bar{b}}) \equiv \int_{2m_b}^{M_{b\bar{b}}} \mathrm{d}M \int \mathrm{d}\Omega \ p(M,\hat{k})\rho(M,\hat{k}) = \int_{2m_b}^{M_{b\bar{b}}} \mathrm{d}M \ p(M)\rho_{\Omega}(M)$

Second part: Spin Correlations with $b\bar{b}$

Second part: Spin Correlations with *bb*

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in, W in all parts

Spin Correlations with $b\bar{b}$ - Calculations

- Spin correlations of *bb* are not included in MC generators.
 - Calculated analytically.
 - Cross section and efficiency are calculated from simulation.
- How can we calculate the spin correlations analytically?

Spin Correlations with $b\bar{b}$ - Calculations

- Spin correlations of *bb* are not included in MC generators.
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 - Cross section and efficiency are calculated from simulation.
- How can we calculate the spin correlations analytically?
 - Same calculation as for $t\bar{t}$, with $m_t \rightarrow m_b$.
- Using the helicity basis $\{\hat{k}, \hat{n}, \hat{r}\}$:
 - \hat{p} : the proton-beam axis.
 - *k*: the direction of the *b* in the *bb* COM frame.
 - $\hat{r} = (\hat{p} \cos \Theta \hat{k}) / \sin \Theta$.
 - $\hat{n} = \hat{r} \times \hat{k}$.
 - $\cos \Theta = \hat{k} \cdot \hat{p}.$





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Reaching the $b\bar{b}$ Polarizations and Spin Correlations

- Spin-correlation measurements can be performed with Λ_b and $\overline{\Lambda}_b$.
 - The lightest, most commonly produced b-baryon.
 - ud-quarks: spin-singlet, isospin-singlet.
 - *b*-quark: carries the baryon spin.
 - Since m_b ≫ Λ_{QCD}, Λ_b baryons are expected to carry a large fraction of the original b-quark polarization.

• The retention factors r_L and r_T :

$$r_{\hat{\mathcal{P}}} = \frac{\mathcal{P}(\Lambda_b)}{\mathcal{P}(b)}, \hat{\mathcal{P}} = L, T.$$

Determine how much of the polarization is transferred $b \rightarrow \Lambda_b$.



The Retention Factors

- In order to perform the measurement, we have to extract r_L, r_T .
 - Their values are expected to be roughly in the ranges $0.4 \lesssim r_L \lesssim 0.8$, $0.5 \lesssim r_T \lesssim 0.8$.
 - One possibility is to use dedicated control regions where significant entanglement is not expected while some of the elements C_{ij} are sizable.
- The polarizations have been measured in Z-boson decays at LEP, by ALEPH, OPAL, DELPHI.
 - An approximate combination gives $r_L = 0.47 \pm 0.14$.



DELPHI

Spin Measurement with $b\bar{b}$

• Most general density matrix for 2 qubits:

$$\rho = \frac{I_4 + \sum_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i \right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

 15 parameters B[±]_i, C_{ij} → Quantum tomography=Measurement of individual spin polarizations B[±] and spin correlation matrix C:

$$B_{i}^{+} = \langle \sigma^{i} \rangle, \ B_{i}^{-} = \langle \bar{\sigma}^{i} \rangle, \ C_{ij} = \langle \sigma^{i} \otimes \bar{\sigma}^{j} \rangle$$



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Spin Measurement with $b\bar{b}$

- We use $\Lambda_b \to X_c \ell^- \bar{\nu}_\ell$, where X_c denotes a charmed state containing a baryon, usually the Λ_c^+ .
- Neutrinos as spin analyzers $(\alpha \simeq 1)$:

 Λ_{h}

$$\frac{1}{\sigma} \frac{d\sigma}{dx_{ij}} = \frac{1}{2} \left(1 - c_{ij} x_{ij} \right) \ln \left(\frac{1}{|x_{ij}|} \right),$$

where $x_{ij} = \cos \theta_i^+ \cos \theta_j^-$, and

$$c_{ij} = \alpha^2 r_i r_j C_{ij}$$

• The retention factors: r_T goes for i, j = n, r and r_L for i, j = k indices.

Third part: Experimental Feasibility Study

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Experimental Observables

Quantum Entanglement:

- **Concurrence** $C[\rho]$: quantitative measurement of entanglement.
- $0 \leq C[\rho] \leq 1$, $C[\rho] \neq 0$ iff the state is entangled.
- Here, $\mathcal{C}[\rho] = \max(\Delta, 0); \Delta = \frac{-C_{nn} + |C_{kk} + C_{rr}| 1}{2}$.



Non-Separable

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Bell Non-locality:

• A violation of the CHSH inequality: $\sqrt{\mu_1 + \mu_2} \ge 1$, where $0 \le \mu_i \le 1$ are the eigenvalues of $\mathbf{C}^{\mathrm{T}}\mathbf{C}$. A sufficient criterion:

$$\mathcal{V} \equiv C_{kk}^2 + C_{rr}^2 - 1 \le \mu_1 + \mu_2 - 1$$

V > 0 is expected to accurately capture the Bell non-locality in the ultrarelativistic regime, in which **C** is diagonal, and C_{kk}^2 , $C_{rr}^2 > C_{nn}^2$.

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Non-Separable



Entanglement and Bell Non-locality Before Integration

- Full LHC $\rho(M_{b\bar{b}}, \hat{k})$ Concurrence.
- Solid white line: entanglement limit; Dashed black line: Bell non-locality limit.
- Regions with strong quantum correlations:
 - $M_{b\bar{b}} \simeq 2m_b$: maximally entangled spin singlet.
 - Ultra-relativistic regime: maximally entangled spin-triplet state for transverse production ($\cos \Theta \simeq 0$).
- In practice, most events are boosted.



$$\begin{split} |\psi\rangle_{singlet} &= \frac{1}{\sqrt{2}} (|\uparrow_{\hat{n}}\downarrow_{\hat{n}}\rangle - |\downarrow_{\hat{n}}\uparrow_{\hat{n}}\rangle) \\ |\psi\rangle_{triplet} &= \frac{1}{\sqrt{2}} (|\uparrow_{\hat{n}}\downarrow\rangle_{\hat{n}} + |\downarrow_{\hat{n}}\uparrow_{\hat{n}}\rangle) \end{split}$$

Experimental Setups

- ATLAS:
 - Large data size.
 - High trigger thresholds.
- CMS B-parking data:
 - Storing a large amount of raw detector data, with low trigger thresholds.
 - Processed when sufficient computational power is available to handle such data.
 - High statistics thanks to the low *p*_T thresholds.
- LHCb:
 - Smaller data size.
 - Low trigger thresholds and better reconstruction.



Figure: A schematic view of the typical Run 2 data flow (up) and comparison of the typical HLT rates (down) in the CMS experiment (CMS, 2403.16134).

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Analysis Selection

• We apply similar selections to the ones applied in the experiments.

	ATLAS	CMS B-parking	LHC <i>b</i>						
Trigger	$2\mu^{\pm}$	displaced $1\mu^{\pm}$	$1\mu^{\pm}$						
$p_{T}(\mu_{1})$	$> 15 { m GeV}$	> 7 - 12 GeV	$> 1.8 { m GeV}$						
η (μ_1)	$ \eta < 2.4$	$ \eta < 1.5$	$2 < \eta < 5$						
p _T (μ ₂)	> 15 GeV	> 5 GeV	> 0.5 GeV						
η (μ_2)	$ \eta < 2.4$	$ \eta < 2.4$	$2 < \eta < 5$						
N _{b-tagged}	≥ 1	- /	≥1						
M _{bb}		-	> 20 GeV						
$p_{T}^{\mu}/p_{T}^{\mathrm{jet}}$	$>$ 0.2 for at least 1μ								
Tracks	Non-		2-4, displaced						
Additional	-111		$p_{T}(X^{\pm}) > 1.6$ GeV, displaced						
Λ_c^+ reco	Full reco on one of the sides								

For HL-LHC the selections are the same, besides the ATLAS 2μ[±] muon threshold: p_T (μ_{1,2}) > 10 GeV, |η(μ_{1,2})| < 2.5.

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Feasibility Study - Determine the Statistics

• For ATLAS and LHCb, we use:

$$N = 2 \sigma \epsilon_{\mu\mu} \mathcal{L} f^2(b \to \Lambda_b) BR^2(\Lambda_b \to X_c \mu^- \bar{\nu}_{\mu})$$
$$\times BR(\Lambda_c^+ \to \text{reco.}) \epsilon_{\text{reco.}} \epsilon_{b,2}$$

- $\sigma \epsilon_{\mu\mu}$: the $b\bar{b}$ production cross section with muon cuts efficiency.
- *L*: integrated luminosity.
- $f(b \rightarrow \Lambda_b) \approx 7\%$: fragmentation fraction for Λ_b .
- BR $(\Lambda_b \to X_c \mu^- \bar{\nu}_{\mu}) \approx 11\%$ and BR $(\Lambda_c^+ \to \text{reco.}) \approx 18\%$.
- $\epsilon_{\rm reco.} \approx 50\%$: estimate for the average Λ_c^+ decay reconstruction efficiency.
- *ϵ_{b,2}*: the efficiency for at least one of the two jets to pass the *b*-tagging condition.



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Feasibility Study - Determine the Statistics

For the CMS B-parking data, we use:

$$N = 2f^{2}(b \to \Lambda_{b}) \operatorname{BR}(\Lambda_{b} \to X_{c}\mu^{-}\bar{\nu}_{\mu}) \epsilon_{\mu_{2}}$$
$$\times \operatorname{BR}(\Lambda_{c}^{+} \to \operatorname{reco.}) \epsilon_{\operatorname{reco}} N_{0}$$

N₀ ≈ 10¹⁰: the number of bb events in the CMS B parking dataset.

 ϵ_{μ2} ≈ 38%: the efficiency of selecting the muon on the non-triggering side of the event.



Feasibility Study - A Glimpse to the Present

	$\sigma \epsilon_{\mu\mu}$ [pb]	$\mathcal{L} \; [fb^{-1}]$	Ν	C _{kk}	C _{rr}	C _{nn}	Δ	ν	rL	$\sigma^{\rm stat}_{\Delta}$	$\sigma_{\mathcal{V}}^{\rm stat}$	$\frac{\Delta}{\sigma^{\rm stat}_{\Delta}}$	$\frac{\mathcal{V}}{\sigma_{\mathcal{V}}^{stat}}$	$\frac{\Delta}{\sigma_{\Delta}^{\rm tot}}$	$\frac{\mathcal{V}}{\sigma_{\mathcal{V}}^{\text{tot}}}$
	Run 2, $\sqrt{s} = 13$ TeV														
ATLAS	9.6×10^3	140	1.4×10^{4}	0.96	0.62	-0.61	0.60	0.31	0.75	0.19	0.48	3.1	0.6	2.6	0.6
									0.45	0.32	1.11	1.8	0.3	1.7	0.3
LHCb, $\Delta > 0.4$	$2.6 imes10^6$	5.7	4.2×10^{4}	0.62	0.76	-0.66	0.52	-0.04	0.75	0.11	0.25	4.6	-0.1	3.4	-0.1
									0.45	0.19	0.46	2.7	-0.1	2.4	-0.1
CMS B parking	$1.1 imes 10^5$	41.6	3.7×10^{5}	0.88	0.61	-0.58	0.53	0.14	0.75	0.038	0.089	> 10	1.6	4.7	1.5
									0.45	0.064	0.20	8.4	0.7	4.3	0.7

Table: Sensitivity studies: $r_T = 0.7$, systematic uncertainty of 20%.

- The expected significance of entanglement with CMS *B*-parking Run 2 data.
- Scanning the unknown r_L, r_T .
- White dotted polygon: plausible values for r_L and r_T.
- Vertical yellow lines: central value of r_L (thick line) and its $\pm 1\sigma$ uncertainties from LEP measurements.







Feasibility Study - A Glimpse to the Future

	$\sigma \epsilon_{\mu\mu}$ [pb]	$\mathcal{L} \; [fb^{-1}]$	Ν	C _{kk}	Crr	Cnn	Δ	v	rL	$\sigma^{\rm stat}_{\Delta}$	$\sigma_{\mathcal{V}}^{stat}$	$\frac{\Delta}{\sigma^{\rm stat}_{\Delta}}$	$\frac{\mathcal{V}}{\sigma_{\mathcal{V}}^{\text{stat}}}$	$\frac{\Delta}{\sigma_{\Delta}^{\rm tot}}$	$\frac{\mathcal{V}}{\sigma_{\mathcal{V}}^{\rm tot}}$
	HL-LHC , $\sqrt{s} = 14$ TeV														
ATLAS, $V > 0.3$	$3.7 imes10^4$	3000	6.2×10^{5}	0.94	0.86	-0.85	0.82	0.63	0.75	0.03	0.08	> 10	7.5	4.9	4.2
									0.45	0.05	0.17	> 10	3.7	4.8	3.0
LHCb, $\mathcal{V} > 0.3$	$3.0 imes10^6$	200	3.3×10^{5}	0.83	0.88	-0.83	0.77	0.48	0.75	0.040	0.11	> 10	4.3	4.8	3.3
		300							0.45	0.067	0.21	> 10	2.2	4.6	2.0
CMS <i>B</i> parking, $V > 0.2$	1 2 105 000 2 2	2 2 ~ 106	0.94	0.05	0.00	0.75	0.42	0.75	0.013	0.035	> 10	> 10	5.0	4.6	
	1.2 × 10	1.2 × 10° 000 3.2	3.2 × 10	0.04	0.05	-0.00	0.75	0.45	0.45	0.022	0.068	> 10	6.3	4.9	3.9

Table: Sensitivity studies: $r_T = 0.7$, systematic uncertainty of 20%.

- The expected significance of Bell non-locality with HL-LHC ATLAS expected data.
- Scanning the unknown r_{I}, r_{T} .
- White dotted polygon: plausible values for r_1 and r_T .
- Vertical yellow lines: central value of r_1 (thick line) and its $\pm 1\sigma$ uncertainties from LEP measurements.



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Summary

- So far, proposals to study quantum information theory in high-energy physics included mostly non-hadronizing systems, which decay quickly.
- We show that Entanglement and Bell non-locality can be measured with $b\bar{b}$ pairs, an hadronizing system.
- This possibility was almost explicitly rejected in many introductions of previous papers (including my own), so it is a rather surprising result.
- The most promising experimental setup for this purpose, using current data, is the CMS *B*-parking data.
- Experimentally challenging:
 - Reconstruction of a specific decay inside the jets to identify $\Lambda_b \overline{\Lambda}_b$.
 - Non-isolated leptons.
 - Unmeasured neutrino.
 - Loss of statistics due to fragmentation fraction, BR and efficiency.
- Theoretically interesting:
 - The *bb* system is boosted in low invariant mass.
 - Quantum correlations are key tools used for studying hadronizing systems, such as the quark-gluon plasma.

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Thank You



Backup Slides

Backup

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The Retention Factors

• In the heavy-quark limit:

$$r_L \approx \frac{1 + A(0.23 + 0.38w_1)}{1 + A}, \quad r_T \approx \frac{1 + A(0.62 - 0.19w_1)}{1 + A}$$

The above expressions describe the dominant polarization loss effect, due to the contribution to the Λ_b sample from $\Sigma_b^{(*)} \to \Lambda_b \pi$ decays.

$$1 \leq A \leq 5, \qquad 0 \leq w_1 \leq 1.$$

where the chosen range for A reflects a large systematic uncertainty.

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P(b)