Quantum Tomography of helicity states for general scattering processes.

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Quantum tests in collider physics

Motivation

From knowing the helicity density matrix of a quantum state we have access to all the spin information of the system, in particular:

- Spin polarizations
- Spin correlations
- Entanglement
- Violation of Bell inequalities
- Etc

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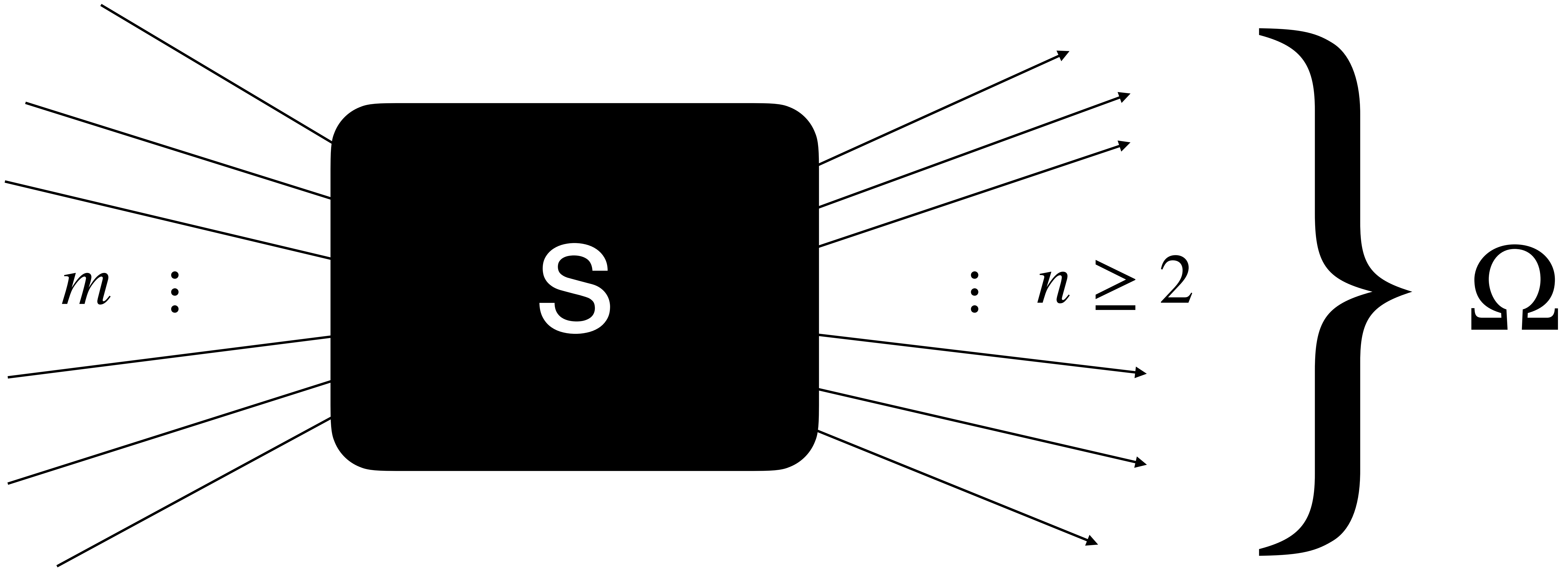
Simple and experimentally practical Quantum Tomography is an indispensable tool

Ashby-Pickering, Barr, Wierzchucka, 2023

Main goal

Determine the initial helicity state ρ of a general scattering process from the angular distribution data of the final particles



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Extra: Re-derivation from Quantum Information perspective (Weyl-Wigner-Moyal formalism)

Ashby-Pickering, Barr, Wierzchucka, 2023

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We consider an n-particle system with fixed λ_i and $\vec{p}_i$ such that $\vec{\chi} = \sum \vec{p}_i = \vec{0}$.

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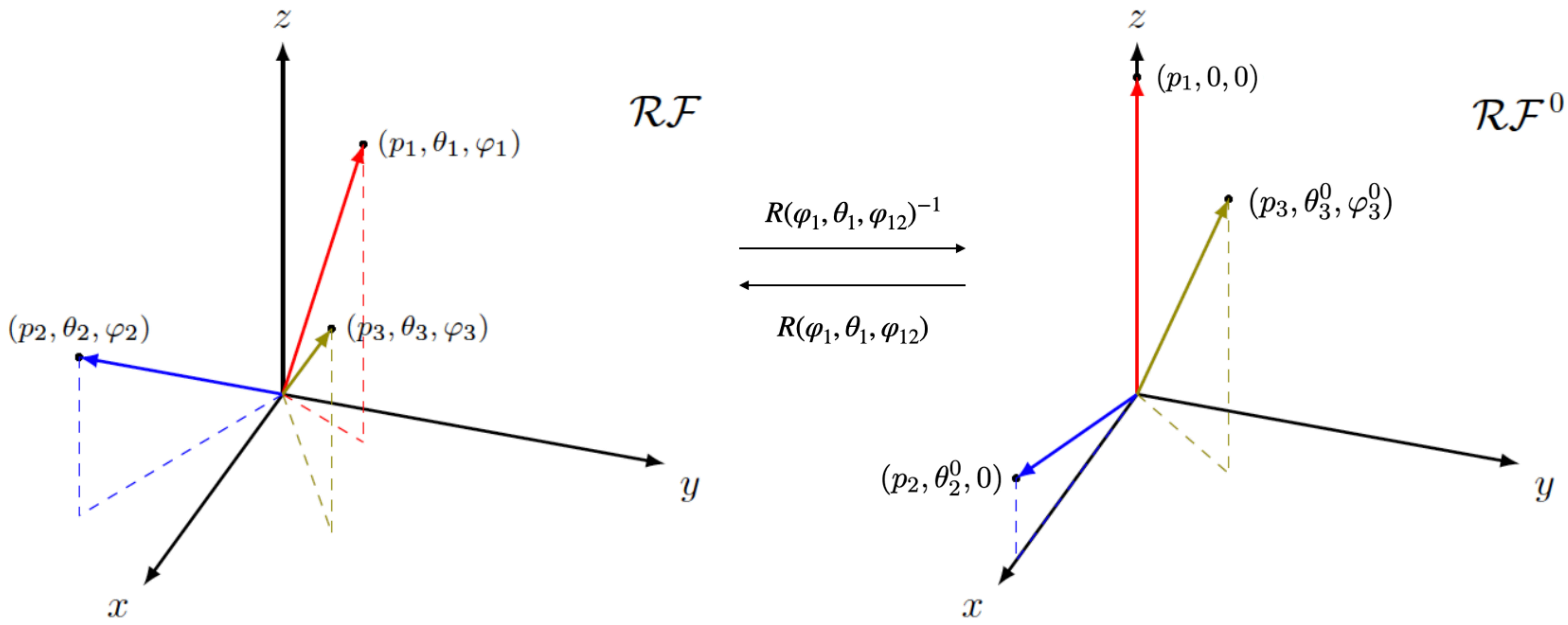
We distinguish 2 reference frames relevant for the work:



Following this setup, the quantum state of the n-particle system is given by

$$\prod_{i=1}^{n} |\vec{p}_i \lambda_i\rangle = \hat{D}(R) \prod_{i=1}^{n} |\vec{p}_i^{\,0} \lambda_i\rangle = |R p^0 \lambda\rangle$$

Here $\hat{D}(R)$ is the unitary representation of R, λ are the particle's helicities and p^0 are the $3n-3$ spherical coordinates in $\mathscr{R}\mathscr{F}^{\,0}$.

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A more convenient representation is

$$|RE\vec{\chi}\kappa\lambda\rangle = |R\kappa\lambda\rangle$$

↑

E and $\vec{\chi}$ fixed

Now, κ is a set of $3n-7$ parameters to be chosen depending on the case

Generalized decay matrix Γ

We define the decay matrix as:

$$\Gamma_{\bar{\lambda}\bar{\lambda}'} \propto \sum_{\lambda} \mathscr{M}_{\lambda\bar{\lambda}} \mathscr{M}^*_{\lambda\bar{\lambda}'}, \quad \mathscr{M}_{\lambda\bar{\lambda}} = \langle R\,\kappa\,\lambda\,|\,T\,|\,\bar{R}\,\bar{\kappa}\,\bar{\lambda}\rangle$$

with $\mathscr{M}_{\lambda\bar{\lambda}}$ the so-called helicity amplitudes.

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We are particularly interested in the transposed matrix Γ^T:

$$\Gamma^T = \Gamma^T(\bar{R}^{-1}R) \underset{\bar{R}=1}{\uparrow}{=} \Gamma^T(R) = \hat{D}(R)\,\Gamma^T(1)\,\hat{D}(R)^{-1}, \quad R = R(\varphi_1, \theta_1, \varphi_{12}) = R(\Omega)$$

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$$\uparrow$$
$$\bar{R} = 1$$

One only needs to compute the elements of $\Gamma^T(1)$ and then rotate the matrix accordingly. In general, in the canonical basis $e_{\sigma\sigma'}$

$$a_{\sigma\sigma'} = \sum_{\lambda} \langle \bar{1}\,\bar{\kappa}\,\sigma\,|\,T^\dagger\,|\,1\,\kappa\,\lambda\rangle\langle 1\,\kappa\,\lambda\,|\,T\,|\,\bar{1}\,\bar{\kappa}\,\sigma'\rangle, \quad a_{\sigma\sigma} = \sum_{\lambda} |\langle 1\,\kappa\,\lambda\,|\,T\,|\,\bar{1}\,\bar{\kappa}\,\sigma\rangle|^2$$

Red. helicity amplitudes

Reconstruction of density matrix ρ

We make use of the relation between ρ, Γ and the normalized differential cross section (narrow width approximation):

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega\, d\bar{\kappa}\, d\kappa} = \frac{d}{8\pi^2 \bar{K} K}\, Tr\{\rho\, \Gamma^T(R)\}, \qquad \bar{K} = \int d\bar{\kappa}, \quad K = \int d\kappa, \quad d = \prod_i (2s_i + 1)$$

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We expand both Γ and ρ over a convenient basis. We take the irreducible tensor operators $\{T^L_M\}$, defined by their transformation under rotations

$$\hat{D}(R) T^L_{M'} \hat{D}(R)^{-1} = \sum_M D^L_{M\,M'}(R) T^L_{M}, \qquad D^J_{M\,M'}(R)\, \delta_{J,J'} = \langle J\, M \,|\, \hat{D}(R) \,|\, J'\, M' \rangle$$

Properties

Dimensionality of the basis $\{T^L_M\}$: $L \in \{0,1,\ldots,(d-1)\}, \quad M \in \{-L,\ldots,L\}$

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Elements of each operator: $$\left[T^L_M\right]_{\sigma_T\,\sigma'_T} = (2L+1)^{1/2}\, C^{s_T\,\sigma_T}_{s_T\,\sigma'_T\,L\,M}$$

with s_T an "effective" spin for the whole system, $d = 2s_T + 1$.

Using the orthogonality condition, for any operator we get

$$\mathcal{O} = \frac{1}{d} \sum_{LM} \mathcal{O}_{LM} T^L_M, \text{ with } \mathcal{O}_{LM} = Tr\{\mathcal{O}\,(T^L_M)^\dagger\}$$

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Applying this result to $e_{\sigma\,\sigma'}$

$$Tr\{e_{\sigma\,\sigma'}(T^L_M)^\dagger\} = Tr\{e_{\sigma\,\sigma'}(T^L_M)^T\} = \left[T^L_M\right]_{\sigma_T\,\sigma'_T} = (2L+1)^{1/2} C^{s_T\,\sigma_T}_{s_T\,\sigma'_T\,L\,M}$$

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and plugging this expression in $\Gamma^T(1)$ leads to

$$\Gamma^T(1) = \frac{1}{d} \sum_{LM} \tilde{B}_{LM} T^L_M, \quad \tilde{B}_{LM} \equiv \tilde{B}_{LM}\left(a_{\sigma\sigma'}, C^{s_T\,\sigma_T}_{s_T\,\sigma'_T\,LM}\right)$$

From the transformation of $\{T^L_M\}$ under rotations we get the expansion of $\Gamma^T(R)$

$$\hat{D}(R)T^L_{M'}\hat{D}(R)^{-1} = \sum_M D^L_{MM'}(R)T^L_M \implies \Gamma^T(R) = \frac{1}{d}\sum_{LM}\left[\sum_{M'}\tilde{B}_{LM'}D^L_{MM'}(R)\right]T^L_M$$

We have factorized the kinematic dependence as

$$\tilde{B}_{LM'} = \tilde{B}_{LM'}(\bar{\kappa},\kappa), \quad D^L_{MM'}(R) = D^L_{MM'}(\Omega)$$

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In the same fashion,

$$\rho = \frac{1}{d} \sum_{LM} A_{LM}\, T^L_M \implies \frac{1}{\sigma} \frac{d\sigma}{d\Omega\, d\bar{\kappa}\, d\kappa} = \frac{1}{8\pi^2 K \bar{K}} \sum_{LM} A_{LM} \sum_{M'} \tilde{B}^*_{LM'} D^L_{MM'}(R)^*$$

Finally, from the orthogonality conditions for the Wigner D-matrices we get

$$\boxed{\int d\Omega \left[\frac{1}{\sigma}\frac{d\sigma}{d\Omega\, d\bar{\kappa}\, d\kappa}\right]\left(\frac{2L+1}{4\pi}\right)^{1/2} D^{L}_{M M'}(\Omega) = \frac{B_{L M'}(\bar{\kappa},\kappa)^*}{4\pi} A_{L M}(\bar{\kappa})}$$

with

$$B_{L M'}(\bar{\kappa},\kappa) \equiv \left(\frac{4\pi}{2L+1}\right)^{1/2} \frac{\tilde{B}_{L M'}(\kappa,\bar{\kappa})}{\bar{K}K}$$

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For $M' = 0$

$$\int d\Omega \left[\frac{1}{\sigma}\frac{d\sigma}{d\Omega\, d\bar{\kappa}\, d\kappa}\right] Y_L^{M\,*}(\Omega) = \frac{B_{L M'}(\bar{\kappa},\kappa)^*}{4\pi} A_{L M}(\bar{\kappa})$$

Factorizable case

Let us consider a scattering process of the form

$$(\bar{A}_1\,\bar{B}_1\,\bar{C}_1\ldots)\,(\bar{A}_2\,\bar{B}_2\,\bar{C}_2\ldots)\ldots\,(\bar{A}_N\,\bar{B}_N\,\bar{C}_N\ldots) \to (A_1\,B_1\,C_1\ldots)\,(A_2\,B_2\,C_2\ldots)\ldots\,(A_N\,B_N\,C_N\ldots)$$

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The decay matrix Γ and the diff. cross section are in this case

$$\Gamma = \bigotimes_{j=1}^{N} \Gamma_j(R_j) \implies \frac{1}{\sigma}\frac{d\sigma}{d\Omega\, d\bar{\kappa}\, d\kappa} = \mathcal{N} Tr\left\{\rho\left(\bigotimes_{j=1}^{N}\Gamma_j^T(R_j)\right)\right\}$$

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In this context instead of using $\{T_M^L\}$, it is convenient to use the factorized one:

$$\left\{\bigotimes_{j=1}^{N} T_{M_j}^{L_j}\right\}_{L_j, M_j} \implies \rho = \frac{1}{d}\sum_{L_1 L_2 \ldots L_N}\;\sum_{M_1 M_2 \ldots M_N} A_{L_1 M_1, L_2 M_2, \ldots, L_N M_N}\;\bigotimes_{j=1}^{N} T_{M_j}^{L_j}$$

Applying a similar reasoning than for the general case

$$\int d\Omega \left[\frac{1}{\sigma}\frac{d\sigma}{d\Omega\, d\bar{\kappa}\, d\kappa}\right] \left[\prod_{j=1}^{N}\left(\frac{2L_j+1}{4\pi}\right)^{1/2} D^{L_j}_{M_j\, M'_j}(\Omega_j)\right] = \frac{\prod_{j=1}^{N} B_{L_j\, M'_j}(\bar{\kappa},\kappa)^*}{4\pi} A_{L_1\, M_1,\, L_2\, M_2, \ldots,\, L_N\, M_N}(\bar{\kappa})$$

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When all the processes are decays ($N = m$)

$$\bar{A}_1\, \bar{A}_2 \ldots \bar{A}_m \to (A_1\, B_1\, C_1 \ldots)\, (A_2\, B_2\, C_2 \ldots) \ldots\ (A_m\, B_m\, C_m \ldots),$$

$L_j = L_{j_0}\delta_{j\, j_0} \longrightarrow A_{0\,0, \ldots, L_{j_0}\, M_{j_0}, \ldots, 0\,0}$ spin polarization vector of particle j_0

$L_j = 0$ except for $L_{j_1}, L_{j_2} \longrightarrow A_{0\,0, \ldots, L_{j_1}\, M_{j_1}, \ldots, L_{j_2}\, M_{j_2}, \ldots, 0\,0}$ spin correlation matrix of particles j_1 and j_2

$\vdots$

$L_j \neq 0 \quad \forall j \longrightarrow A_{L_1\, M_1,\, L_2\, M_2, \ldots,\, L_m\, M_m}$ spin correlation tensor of the whole system

Physical examples

$$\cdot\ t\bar{t} \to (bW^+)(\bar{b}W^-) \to (bl^+\nu_l)(\bar{b}l^-\bar{\nu}_l), \qquad \rho = \frac{1}{4}\sum_{L_1 L_2=0}^{1}\sum_{M_1 M_2} A_{L_1 M_1, L_2 M_2}\, T^{L_1}_{M_1}(1/2) \otimes T^{L_2}_{M_2}(1/2)\,.$$

$$A_{L_1 M_1, L_2 M_2} = \frac{4\pi}{B_{L_1}B_{L_2}}\int d\Omega_1 d\Omega_2 \left[\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1\, d\Omega_2}\right] Y^{M_1\,*}_{L_1}(\Omega_1) Y^{M_2\,*}_{L_2}(\Omega_2)$$

$$B_0 = \sqrt{4\pi}, \quad B_{L_i=1} = \sqrt{\frac{4\pi}{3}}\alpha_i$$

$$\alpha_b \simeq -0.41, \quad \alpha_l \simeq 1$$

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$\cdot\ V_1 V_2 \to (f_1\bar{f}_1)(f_2\bar{f}_2),$ $\quad \rho = \frac{1}{9}\sum_{L_1 L_2=0}^{2}\sum_{M_1 M_2} A_{L_1 M_1, L_2 M_2}\, T^{L_1}_{M_1}(1)\otimes T^{L_2}_{M_2}(1)\,.$

$$B_0 = \sqrt{4\pi}, \quad B_{L_i=1} = \sqrt{2\pi}\alpha_i, \quad B_{L_i=2} = \sqrt{\frac{2\pi}{5}}(1-3\delta_i) \qquad \alpha_Z \simeq -0.13, \quad \alpha_W \simeq -1, \quad \delta_i \simeq 0$$

$\cdot\ t\bar{t}W \to (bW^+)(\bar{b}W^-)(l\nu_l) \to (bl^+\nu_l)(\bar{b}l^-\bar{\nu}_l)(l\nu_l)$

$$\rho = \frac{1}{12} \sum_{L_1 L_2=0}^{1} \sum_{L_3=0}^{2} \sum_{M_1 M_2 M_3} A_{L_1 M_1, L_2 M_2, L_3 M_3}\, T^{L_1}_{M_1}(1/2) \otimes T^{L_2}_{M_2}(1/2)\ T^{L_3}_{M_3}(1)$$

$$A_{L_1 M_1, L_2 M_2, L_3 M_3} = \frac{4\pi}{B_{L_1} B_{L_2} B_{L_3}} \int d\Omega_1 d\Omega_2 d\Omega_3 \left[\frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_2 d\Omega_3}\right] Y^{M_1\,*}_{L_1}(\Omega_1) Y^{M_2\,*}_{L_2}(\Omega_2) Y^{M_3\,*}_{L_3}(\Omega_3)$$

$$B_0 = \sqrt{4\pi}, \quad B_{L_1=1} = B_{L_2=1} = \sqrt{\frac{4\pi}{3}}, \quad B_{L_3=1} = -\sqrt{2\pi}, \quad B_{L_3=2} = \sqrt{\frac{2\pi}{5}}\,.$$

Weyl-Wigner-Moyal formalism

Re-derivation using concepts from quantum information:

· Positive Operator-Valued Measure (POVM):
positive semi-definite hermitian operators $\{F_l = \mathscr{K}_l^\dagger \mathscr{K}_l\}_l$, with $\sum_l \mathscr{K}_l^\dagger \mathscr{K}_l = \sum_l F_l = 1.$

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· Generalized Wigner Q and P symbols: $\Phi_A^Q \equiv Tr\{A\, F_l\} = Tr\{A\, \Gamma^T\}$, $\qquad Tr\{A\, B\} = \dfrac{d}{8\pi^2 \bar{K} K} \displaystyle\int d\Omega\, \Phi_B^Q\, \Phi_A^P$.

$$\Phi^Q_{T^L_M} = \sum_{M'} \tilde{B}_{L\,M'}(\bar{\kappa}, \kappa)^*\, D^L_{M\,M'}(\Omega)^*, \qquad \Phi^P_{T^L_M,\, M'}(\Omega, \bar{\kappa}, \kappa) = \frac{4\pi}{B_{L\,M'}(\bar{\kappa}, \kappa)} \left(\frac{2L+1}{4\pi}\right)^{1/2} D^L_{M\,M'}(\Omega)^*, \qquad \Phi^Q_\rho = \frac{8\pi^2 \bar{K} K}{d} \frac{1}{\sigma} \frac{d\sigma}{d\Omega\, d\bar{\kappa}\, d\kappa}$$

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$$\Phi^Q_{T^L_M} = \sum_{M'} \tilde{B}_{L\,M'}(\bar{\kappa},\kappa)^*\,D^L_{M\,M'}(\Omega)^*,\qquad \Phi^P_{T^L_M,\,M'}(\Omega,\bar{\kappa},\kappa) = \frac{4\pi}{B_{L\,M'}(\bar{\kappa},\kappa)}\left(\frac{2L+1}{4\pi}\right)^{1/2} D^L_{M\,M'}(\Omega)^*,\qquad \Phi^Q_\rho = \frac{8\pi^2\bar{K}K}{d}\frac{1}{\sigma}\frac{d\sigma}{d\Omega\,d\bar{\kappa}\,d\kappa}$$

· Applying the definition of the Q and P symbols as well as the decomposition of ρ in terms of T^L_M:

$$\int d\Omega\,\left[\frac{1}{\sigma}\frac{d\sigma}{d\Omega\,d\bar{\kappa}\,d\kappa}\right]\left(\frac{2L+1}{4\pi}\right)^{1/2} D^L_{M\,M'}(\Omega) = \frac{B_{L\,M'}(\bar{\kappa},\kappa)^*}{4\pi}A_{L\,M}(\bar{\kappa})$$

Conclusions

- We have developed a practical way of performing the Quantum Tomography of the initial helicity state ρ in general scattering processes.
- The method is based on computing the coefficients of the expansion over $\{T^L_M\}$ by averaging the angular distribution of the final particles under Wigner D-matrices kernels.
- We have further given explicit formulas for the angular dependence of both a generalization of the production/decay matrix Γ and of the diff. cross section, elaborating on the factorizable case with some examples.
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Thank you for your attention!