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# ENTANGLEMENT AND BELL INEQUALITY VIOLATION IN CHARMONIUM DECAYS









# Where have we already seen entanglement or Bell inequality violation at high energies?

*New York Times* headline May 4th, 1935

EINSTEIN ATTACKS  $\begin{array}{c}\n\text{Scientist and Two College}\n\end{array}$ \n
$$
\begin{array}{c}\n\text{Scientist and Two College}\n\end{array}
$$
\n
$$
\begin{array}{c}\n\text{Find it is Not 'Conpleacy}\n\end{array}
$$
Find It Is Not 'Complete'  $E_{Ven}$  Though 'Correct.'<br>E FILL Let  $E_{Ven}$ SEE FULLER ONE POSSIBLE Believe a Whole Description of<br>
the Physical Reality' Can Provided Figures "the Physical Reality" Can Be Provided Eventually.







![](_page_3_Picture_2.jpeg)

![](_page_3_Picture_3.jpeg)

**2**

MF, R. Floreanini, E. Gabrielli and L. Marzuolo, *Phys. Rev D 109 (2024) 3, L031104 B*  $\alpha$  **1**  $\alpha$  **1**  $\alpha$  **1** *V*  $\alpha$  *1 1 <i>V*  $\mathbf{P}_1 = \mathbf{P}_1 \cdot \mathbf{P}_2 \cdot \mathbf{P}_3 \cdot \mathbf{P}_4 \cdot \mathbf{P}_5 \cdot \mathbf{P}_6 \cdot \mathbf{P}_7 \cdot \mathbf{P}_8 \cdot \mathbf{P}_9 \cdot \mathbf{P}_1 \cdot \mathbf{P}_2 \cdot \mathbf{P}_1 \cdot \mathbf{P}_1 \cdot \mathbf{P$  $\text{r}$ zuolo, <u>arxiv:2400.17772 (2024</u>) E. Gabrielli and L. Marzuolo, arXiv:2406.17772 (2024) 2024) 3, L031104  $(2486, 17778, (9881)$ <u>IV:2400.17772 (2024)</u>

> matrices *U* and *V* fixed, we determine that *I*<sup>3</sup> for the \*\* K Chen et al, <u>Eur. Phys. J. C 84 (2024)</u> 580  $B_c^{\pm} \rightarrow J/\psi \rho^{\pm}$  $\frac{B0}{c}$   $B_c^{\pm}$   $\rightarrow$   $J/\psi \rho^{\pm}$ and A Szynkman, ar $X_{W}^{1}$ : 2409 13033  $B^0 \rightarrow K^* \mu^+ \mu^-$ <u>*Eur. Phys. J. C 84 (2024) 580 I*</u>  $\zeta$  iv: 2409 13033  $B^0 \to K^* \mu^+ \mu^$ entity and photographs are all the control of the state of the sta \*\* K Chen et al, <u>Eur. Phys. J. C 84 (2024) 580</u>  $B_c^{\pm} \rightarrow J/\psi \rho^{\pm}$ [6] C. P. Jessop *et al.* [CLEO], Phys. Rev. Lett. 79, 4533-  $25001 (2001) = 200$  $\overline{1}$ \*\* RA Morales and A Szynkman, <u>arXiv:2409.13033</u>  $B^0 \to K^* \mu^+ \mu^ B_c^{\pm} \rightarrow J/\psi \rho^{\pm}$

![](_page_4_Figure_1.jpeg)

the *f* mesons produced in *B*<sup>0</sup>

![](_page_4_Picture_1296.jpeg)

R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **131**, no.17, 171802 (2023)  $\text{arXiv:}2304.06198 \text{ [hep-ex]}.$ 

 $K^*(892)$ 

 $K^*(892)^0$ rest frame

 $\mathbf{\hat{n}}$ 

 $K^+$ 

This result demonstrates that the presence of quantum entanglement in *B*<sup>0</sup>

![](_page_4_Picture_6.jpeg)

## B-meson decays Pson decays not separated by a space-like interval, as it is the case of th

of the *J/ K*⇤ decays—one must consider decays in

K. Horodecki, Rev. Mod. Phys. 81, 865-942 (2009)

# Pairs of top quarks

ATLAS Collaboration, Nature 633 (2024) 542  $D = -0.547 \pm 0.002$  [stat]  $\pm 0.021$  [syst] CMS Collaboration, <u>arXiv[:2406.03976](https://arxiv.org/abs/2406.03976) (2024)</u>

![](_page_5_Figure_1.jpeg)

## Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus **136** (2021) 907

![](_page_5_Figure_4.jpeg)

 $D = -0.478_{-0.027}^{+0.025}$ 

**3**

CMS Collaboration, arXiv:2409.11067 (2024)

![](_page_5_Picture_8.jpeg)

Editor: J.-P. Blaizot

Hadronic decays

![](_page_6_Picture_4.jpeg)

# $\sim$  asymmetry parameters of Lambda and and and and and and and anti-Lambda and and and and anti-Lambda hyperons can be determined. © 2017 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license **4** Charmonium

MF, R. Floreanini, E. Gabrielli and L. Marzuolo, Phys. <u>Rev. D110 (2024) 053008</u>  $t_{\text{max}}$ ,  $t_{\text{max}}$  to find the fit therefore  $t_{\text{max}}$  or  $t_{\text{max}}$  and  $t_{\text{max}}$  (9094) 054019  $\frac{1}{2000}$ , and further edges in  $\frac{1}{2000}$ , and  $\frac{1}{2000}$ see, also: S. Wu et al., *Phys. Rev. D110 (2024) 054012* 

![](_page_6_Figure_1.jpeg)

sible, but seem to be curtailed. Incomplete distribution functions

*n*1, ~ *n*<sup>3</sup> for  $\tau$   $\pi$   $\alpha$  1 sell operator  $\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}_3]$  $\rho\mathscr{B}_3$  $\overline{1}$  $G$ iven a generic density matrix  $\sim$  of the bipartite state state is stated whether the state is stated when Eell operator  $\mathcal{L}_3 = \text{Tr} |\rho \mathcal{B}_3|$  $\sigma$   $\sigma$  0 0  $\sigma$ operator 1 Bell operator  $\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}_3]$ 

2.1.1 Qubits

density matrix of the form:

*<sup>i</sup>* and C*ij* need to be enforced to guarantee its positivity, as all eigenvalues

of a density matrix are necessarily non-negative.

concurrence of the state  $\alpha$  concurrence of the state  $\alpha$ 

$$
\rho = \frac{1}{4} \Big[ \mathbb{1}_2 \otimes \mathbb{1}_2 + \sum_{i=1}^3 \mathrm{B}_i^+(\sigma_i \otimes \mathbb{1}_2) + \sum_{i=1}^3 \mathrm{B}_j^-(\mathbb{1}_2 \otimes \sigma_j) + \sum_{i,j=1}^3 \mathrm{C}_{ij}(\sigma_i \otimes \sigma_j) \Big] \Bigg] \qquad \rho = \frac{1}{9} \Big[ \mathbb{1}_3 \otimes \mathbb{1}_3 \Big] + \sum_{a=1}^8 f_a \left[ T^a \otimes \mathbb{1}_3 \right] + \sum_{a=1}^8 f_a \left[ T^a \otimes \mathbb{1}_3 \right] \Big]
$$

concurrence can be analytically computed through the auxiliary matrix computed through the auxiliary matrix of<br>International computed through the auxiliary matrix of the auxiliary matrix of the auxiliary matrix of the aux

and 1 (maximally entangled states). In the case of two spin-1/2 system, a two spin-1/2 system, a two spin-1/2 s

The entanglement content of any bipartite system described with the density matrix  $\Gamma$ 

 $R = \rho \left( \sigma_y \otimes \sigma_y \right) \rho^* \left( \sigma_y \otimes \sigma_y \right),$ 

possesses non-negative eigenvalues *ri*, *i* = 1*,* 2*,* 3*,* 4, their square roots and denoting *r*<sup>1</sup> the largest, the

In quantum mechanics a statistical language is adopted for the description of the the behavior

where **ᆢ** $denotes a matrix with complex complex conjugated entries. Although non-Hermitian, the matrix  $R$$ 

<sup>1</sup>, *A*

ˆ

*n*2,

where  $\alpha$  matrix with complex conjugated entries. Although non-Hermitian, the matrix  $\alpha$ pondant  $\sigma$   $[\rho]$  = max  $(v, r_1 - r_2 - r_3 - r_4)$  $\mathscr{C}[\rho] = \max$  (  $0, r_1 - r_2 - r_3 - r_4$ Concurrence  $\mathscr{C}[\rho] = \max(0, r_1 - r_2 - r_3 - r_4)$ entanglement.

the correlation matrix *hab* appearing in the decomposition (2.13):

 $\mathbf{Entrom}$   $\mathcal{E}[o]$ In quantum mechanics a statistical language is adopted for the description of the the behavior of physical physical phenomena. Interestingly, the experimental verification  $\mathscr{E}[\rho] \equiv$ against alternative, fully deterministic, local description of natural phenomena through Bell locality deterministic,  $\mathbf{r}$  $F_n$  of  $\mathcal{E}[a] \equiv -\text{Tr}[a_A \ln a_A] = -\text{Tr}[a_B \ln a_B]$ **Entropy**  $\varphi[\mu] = -\ln[\mu_A \ln \mu_A] - -\ln[\mu_B \ln \mu_B]$ state (2.14) of a suitable Bell operator *B*3:  $\text{Eungley} \quad \text{e} \quad [\rho_1 - \text{H}[\rho_A \text{H} \rho_A] \quad \text{H}[\rho_B \text{H} \rho_B]$ **Entropy**  $\mathscr{E}[\rho] \equiv -\text{Tr}[\rho_A \ln \rho_A] = -\text{Tr}[\rho_B \ln \rho_B]$ giving the von Neumann entropy of the von Neumann entropy of the reduced density matrices and often called in t<br>The literature of the literature matrices and often called in the literature of the literature of the literatu matrix elements <sup>h</sup>*i*1*j*2*|*⇢*|i*2*j*1i; a similar expression holds for ⇢*T<sup>A</sup>* . Interestingly, if ⇢*T<sup>B</sup>* , or equivalently absolute sum of the negative eigenvalues of ⇢*T<sup>B</sup>* , called negativity,  $\mathcal{L}_{\text{interior}}$   $\mathcal{L}_{\text{inter$  $\text{Entropy}$   $\delta[\rho] \equiv -\text{Ir}[\rho_A \ln \rho_A] = -\text{Ir}[\rho_B \ln \rho_B]$ state (2.14) of a suitable Bell operator *B*3: Entropy

$$
\mathbb{1}_{2}\otimes \sigma_{j})+\sum_{i,j=1}^{3}\text{C}_{ij}(\sigma_{i}\otimes \sigma_{j})\bigg]\qquad\bigg[\rho=\frac{1}{9}\left[\mathbb{1}_{3}\otimes \mathbb{1}_{3}\right]+\sum_{a=1}^{8}f_{a}\left[T^{a}\otimes \mathbb{1}_{3}\right]+\sum_{a=1}^{8}g_{a}\left[\mathbb{1}_{3}\otimes T^{a}\right]+\sum_{a,b=1}^{8}h_{ab}\left[T^{a}\otimes T^{b}\right]\qquad \qquad \bigg]
$$

⇢ =

*<sup>f</sup><sup>a</sup>* [*T<sup>a</sup>* ⌦ **<sup>1</sup>**3] +<sup>X</sup>

where  $T$ a are the standard Gell-Mann matrix. While  $T$  is the unit 3  $\mu$  3  $\mu$ 

![](_page_7_Picture_0.jpeg)

 $c_1 = \rho \cos \theta$ 

#### *<sup>j</sup>* represent the polarization of the two spin-1/2 fermions, while the real matrix correlations. The toolbox the density matrix in  $\alpha$  is not the toolbox  $\alpha$ a measure of the amount of the amount of  $\alpha$  measure of  $\alpha$  and two composites sub-systems, can be systems, can be seen as  $\alpha$  $\alpha$   $\alpha$ <sup>1</sup> $\alpha$ <sup>1</sup> of *SB*.  $\frac{1}{2}$  toolbox the toolbox

that can be written as a can be written as  $\alpha$ 

that can be written as a contract of the written as a contract of the can be written as a contract of the case

![](_page_7_Figure_3.jpeg)

A. Barr, MF, R. Floreanini, E. Gabrielli and L. Marzuolo, Progress in P that in any, local, deterministic model cannot exceed a value of 2. In quantum mechanics, *I*<sup>2</sup> can be  $\frac{1}{\sqrt{2}}$ b. <u>Progress in Particle and Nuclear Physics 159 (20.</u>  $\frac{24}{\sqrt{2}}$ A B<sub>3</sub> depends on Flame in *B*3 depends on the form of the four measured operators *A B*<sub>3</sub> depends on the four measured operators *A B*<sub>3</sub> depends on the four measured operators *A B*<sub>3</sub> depends on the four measured opera case of the maximum correlations of the maximum of the problem of the problem of the problem of  $\frac{1}{202}$ One such a basis involves the operation. Given a basis of particle or particle and Nuclear Physics 139 (2024) 104134 Cohmellicity Marguele, Program in Perticle and Nuclear Physics 120 (2024) 104124 density matrix is independent of the scattering and scattering and the scattering and  $\begin{bmatrix} \mathbf{b} \\ \mathbf{b} \\ \hline \end{bmatrix}$ lo, <mark>Prog</mark> A. Barr, MF, R. Floreanini, E. Gabrielli and L. Marzuolo, <u>Progress in Particle and Nuclear Physics 139 (2024) 104134</u>  $\frac{1}{\sqrt{1}}$ 

Negativity 
$$
\mathcal{N}(\rho) = \sum_{k} \frac{|\lambda_k| - \lambda_k}{2}
$$

2.1.3 Additional observables

$$
\mathscr{C}_2 = 2 \max \left[ -\frac{2}{9} - 12 \sum_a f_a^2 + 6 \sum_a g_a^2 + 4 \sum_{ab} h_{ab}^2 ; -\frac{2}{9} - 12 \sum_a g_a^2 + 6 \sum_a f_a^2 + 4 \sum_{ab} h_{ab}^2, 0 \right]
$$

of *SB*.

![](_page_7_Picture_12.jpeg)

the correlation matrix *hab* appearing in the decomposition (2.13):

matrix.

⎠ <sup>d</sup>\*!d\*1d\*2*,* (5.46)

with *k* and *p* the initial- and final-state momenta; \*! the hyperon

scattering angle in the global c.m. system; \*<sup>1</sup> and \*<sup>2</sup> decay angles

measured in the rest system of  $\mathbb{R}^n$ 

Integration over the angles \*<sup>1</sup> makes the contributions from

![](_page_8_Picture_2.jpeg)

![](_page_8_Picture_3.jpeg)

the functions *G*<sup>05</sup> and *G*<sup>55</sup> disappear [2], and correspondingly for

the angles \*2. Integration over both angular variables results in

the cross-section distribution for the reaction *e*+*e*<sup>−</sup> → *J/*ψ → !!¯ .

![](_page_8_Figure_0.jpeg)

[3] H. Czy ˙z, A. Grzelinska, ´ J.H. Kühn, Phys. Rev. D 75 (2007) 074026.

fact, such decay distributions, only permit the decay distributions, only permit the determination of the determinati

The prefactor *K* contains on the mass shell delta functions for the

 $\mathbb{R}^n$ 

duction and decay parts. Repeating the manipulations of Ref. [2]

⎠ <sup>d</sup>\*!d\*1d\*2*,* (5.46)

with *k* and *p* the initial- and final-state momenta; \*! the hyperon

scattering angle in the global c.m. system; \*<sup>1</sup> and \*<sup>2</sup> decay angles

measured in the rest systems of ! and !¯ ; )! and )!¯ channel

Integration over the angles \*<sup>1</sup> makes the contributions from

the functions *G*<sup>05</sup> and *G*<sup>55</sup> disappear [2], and correspondingly for

the angles \*2. Integration over both angular variables results in

the cross-section distribution for the reaction *e*+*e*<sup>−</sup> → *J/*ψ → !!¯ .

#### $\eta_c \rightarrow \Lambda + \bar{\Lambda} \quad \text{and} \quad \chi_c^0 \rightarrow \Lambda + \bar{\Lambda}$ The scalar and pseudoscalar and pseudoscalar states of the charmonium can decay into a pair of strange ⊾ baryon and ⌘*<sup>c</sup>* ! ⇤ <sup>+</sup> ⇤¯ and <sup>0</sup> *<sup>c</sup>* ! ⇤ <sup>+</sup> ⇤¯ *,* (3.1)  $\eta_c \to \Lambda + \Lambda$  and  $\chi_c \to \Lambda + \Lambda$  $\overline{\Lambda}$  $\Lambda$ *time varying analyzers*, *Phys. Rev. Lett.* 49  $\mid \eta \mid -$ [3] G. W. Wein, T. J. Wein, C. Simon, C. **PERIODICAL ACTION**  $\sim$   $\sim$   $\rightarrow$ <u>[14] BESI</u>I Collaboration, M. Ablikim et al., M. Ablikim et al., M. Ablikim et al., M. Ablikim et al., M. Ablikim e

3 Charmonium spin 0 states de la charmonium spin 0 states de la charmonium spin 0 states de la charmonium spin<br>1980 : Charmonium spin 0 states de la charmonium spin 0 states de la charmonium spin 0 states de la charmonium

The states in Eq. (3.2) enter in Eq. (3.2) enter into the helicity density matrix  $\mathcal{L}(\mathcal{L})$ 

helicity density matrix only depends on one or, at most, two helicity amplitudes. Moreover, the

<sup>2</sup> <sup>i</sup> <sup>+</sup> *<sup>w</sup>* <sup>1</sup>

2

2 *,*

 $A_{\rm eff}$  optimization, the expectation, the expectation value of the Bell operator is  $\alpha$ 

The decays of the spin 0 states of the charmonium are the simplest to analyze because the

3.1 ⌘*<sup>c</sup>* ! ⇤ <sup>+</sup> ⇤¯ and <sup>0</sup>

*<sup>c</sup>* ! ⇤ <sup>+</sup> ⇤¯

with branching fraction (1*.*10  $\pm$  0.10  $\pm$  0.10

anti-baryon

which is the state

density matrix is independent of the scattering angle. The scattering angle of the s

The concept  $\mathcal{L}_\text{max}$  is maximal:  $\mathcal{L}_\text{max}$  is maximal:  $\mathcal{L}_\text{max}$  is maximal:  $\mathcal{L}_\text{max}$ 

in which *wij* are the helicity amplitudes and *|J, m*i the spin states. Parity fixes the relative sign

The final states are constrained are constrained by the conservation of the helicity—to be described by the states  $\mathcal{L}(\mathcal{L})$ 

between the two amplitudes: it is 1 for the pseudo-scalar ⌘*<sup>c</sup>* and 1 for the scalar <sup>0</sup>

3 Charmonium spin 0 states in 1980 in

*<sup>c</sup>* . Accordingly,

in textbooks. This property was already observed for the decay of the Higgs boson *<sup>H</sup>* ! ⌧ ⌧ <sup>+</sup> in [39].

*C* = 1 *.* (3.4)

0

0 0 1

1

⇢⇤ ⇤ = *|* <sup>0</sup>ih <sup>0</sup>*|* =

helicity density and the licity density matrix on only depends on one or, at most, two helicity amplitudes. Mo<br>In the licity amplitudes, two helicity amplitudes, at most, two helicity amplitudes, the licity amplitudes, th

0

BB@

00 00

0 *±*110

The scalar and pseudoscalar and pseudoscalar states of the charmonium can decay into a pair of strange  $\mathbb{R}$ 

1

*,* (3.3)

#### Concurrence *entanglement in top quark pair production in*

Neither the European Communication are polarized.<br>Neither the anti-baryon nor the anti-baryon are polarized.<br>Neither the anti-baryon are polarized.

and *w*1 1

depends on one complex number.

#### $\langle -1\ 1\ \vert\frac{1}{2}, -\frac{1}{2}\rangle\otimes\vert\frac{1}{2}, \frac{1}{2}\rangle$  $|\psi_0\rangle \propto w_{\frac{1}{2} - \frac{1}{2}}$  $\frac{1}{2}$ ,  $\frac{1}{2}$  $\frac{1}{2}$   $\rangle \otimes |$  $\frac{1}{2}, -\frac{1}{2} \rangle + w_{-\frac{1}{2}}$ 1  $\frac{1}{2}$  |  $\frac{1}{2},-\frac{1}{2}\rangle\otimes\vert$  $\frac{1}{2}$ ,  $\frac{1}{2}$  $\frac{1}{2}$  $\langle \nu_{\frac{1}{2}-\frac{1}{2}}\ket{\frac{1}{2},\frac{1}{2}}\otimes\ket{\frac{1}{2},-\frac{1}{2}}+\overline{w}_{-\frac{1}{2}}\frac{1}{2}\ket{\frac{1}{2},-\frac{1}{2}}\otimes\ket{\frac{1}{2}}$  $\frac{1}{2}$  $|\psi_0\rangle \propto w_{\frac{1}{2}-\frac{1}{2}}\,|\frac{1}{2},\frac{1}{2}\rangle \otimes |\frac{1}{2},-\frac{1}{2}\rangle + w_{-\frac{1}{2}}\frac{1}{2}\,|\frac{1}{2},-\frac{1}{2}\rangle \otimes |\frac{1}{2},\frac{1}{2}\rangle$ *Phys.* 81 (2009) 865–942, [quant-ph/0702225].  $\left[\psi\right]\left(\psi\right)\propto\omega_{\frac{1}{2}-\frac{1}{2}}\left|\bar{2},\bar{2}\right|\otimes$ L. Marzola, *Bell inequality is violated in*  $R_1$   $\ldots$   $1$   $1$  $\frac{1}{2}$ / T  $w_{-\frac{1}{2}}$ . [16] BESIII Collaboration, M. Ablikim et al., ⌃<sup>+</sup>

helicity density matrix ⇢ = *|* ih *|* is written as

 $Einstein-Podolsky-Rosen Experiment, Phys. Lett. A 117 (1986) 1–4.$ N. A. Tornqvist, *The Decay*  $J/\psi \to \Lambda \bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$  *as an*  $Einstein-Podolsky-Rosen Experiment, Phys. Lett.$ *A* 117 (1986) 1–4. *Experimental test of Bell's inequalities using*

 $(2008) 075002.$ *P*, round.<br>*Phys.*  $G$  3<sup>1</sup> by, *Bell's* inequali  $Phys. G$  35 (2008) 075002. **Observation of**  $S$ . P. Baranov, *Bell's inequality in charmonium*  $\vec{a}$  *i.e.*  $\vec{b}$   $\vec{c}$   $\vec{b}$   $\vec{c}$   $\vec{b}$   $\vec{c}$   $\vec{d}$   $\vec{c}$   $\vec{d}$   $\vec{c}$   $\vec{d}$   $\vec{d}$   $\vec{c}$   $\vec{d}$   $\vec{d}$ *Phys. G* 35 (2008) 075002.

$$
\mathscr{C}=1
$$

#### Charmonium spin-0 states The decay states of the spin-0 states of the spin-0 states of the simplest to analyze because the simplest the simples  $Charmonium cmin_0 star$ travel inside the beam pipe wall and the beam pipe wall and the first layers of the detector before decaying. T  $Charmonium cmin_0$  of the data in  $\alpha$ 0 0 *w*0 0*w*⇤ 1*,*<sup>1</sup> 0 *|w*0 0 *|* <sup>2</sup> 0 *w*0 0*w*⇤ 1 1 0 0 Charmonium spin-0 states 0 *w*<br>2 ∞ *w* 2 ∞ w 2 *entangled double-strange baryons*, *Nature* 606 ) nium spin-u stat 00 0 0 0 0 0 00 Unarmonium spin-U states 0 0 *|w*<sup>1</sup> <sup>1</sup> *|* <sup>2</sup> 0 *w*<sup>1</sup> <sup>1</sup>*w*⇤ 0 0 0 *w*<sup>1</sup> <sup>1</sup>*w*⇤

*w*1*,*<sup>1</sup>

*w*0 0

significance is 17*.*9.

$$
\ket{\psi_0}\propto w_{\frac{1}{2}-\frac{1}{2}}\ket{\frac{1}{2},\frac{1}{2}}\otimes\ket{\frac{1}{2},-\frac{1}{2}}+w_{-\frac{1}{2}\frac{1}{2}}\ket{\frac{1}{2},-\frac{1}{2}}\otimes\ket{\frac{1}{2},\frac{1}{2}}
$$

$$
\left| \frac{w_{1,1}}{w_{0,0}} \right| = 0.299 \pm 0.003_{\text{stat}} \pm 0.019_{\text{syst}}.
$$

 $\begin{aligned} \textit{Helicity amplitude analysis of }\chi^J_c\ \textbf{05}\ (2023)\ 069, \texttt{[arXiv:2301.1292]}\ \end{aligned}$ **BESIII** Collaboration, N.<br>  $\text{Helicity amplitude analysis}$  $C = 1$  **05** (2023) 06 **BESIII** Collaboration, M. Ablikim et al.,<br>*Helicity amplitude analysis of*  $\chi_c^J \to \phi \phi$ , *JHEP* **OF THE FINAL PARTICI** Collaboration, M. Ablikim et al.,  $\frac{1}{2020}$   $\frac{1}{2020}$   $\frac{1}{2020}$   $\frac{1}{2020}$ ,  $\frac{1}{20201}$ ,  $\frac{1}{20222}$ .  $\frac{1}{10000}$   $\frac{1}{2000}$   $\frac{1}{2000}$   $\frac{1}{2000}$   $\frac{1}{2000}$   $\frac{1}{2000}$   $\frac{1}{2000}$   $\frac{1}{2000}$ . [arXiv:2301.12922]. case of zoro phase. As pointed out in the International state strong phase comes from the final state strong from the final state strong phase comes from the final strong phase comes from the final state strong phase comes

**Example 3** Horodecki condition  $m_{12} = 2$  $\mathcal{E}[\rho]=0.5$  $\mathscr{E}[\rho] = 0.531 \pm 0.0021$  (255  $m_{12} = 2$  , and  $m_{12} = 3$  , and  $m_{13} = 3$  , and  $m_{14} = 3$  , and  $m_{15} = 3$  , and  $m_{16} = 3$  , and  $m_{17} = 3$  , and  $m_{18} = 3$  , and  $m_{19} = 3$  , and  $m_{18} = 3$  , and  $m_{19} = 3$  , and  $m_{10} = 3$  , and  $m_{11} = 3$  , and For the case of the experimental values of the experimental values of the experimental values of the experimental values of the becay  $\frac{N_A}{N_A}$  is  $\frac{N_B}{N_A}$  in  $\frac{N_B}{N_A}$  in  $\frac{N_B}{N_A}$  in  $\frac{N_B}{N_A}$  is  $\frac{N_B}{N_A}$  **Bell operator**  $\text{Tr } \rho_{\phi\phi} \mathscr{B} = 2.$ 0 to<br>Lite Entropy  $\mathscr{E}[\rho] = 0.531 \pm 0.0021$  (255 $\sigma$ ) 1 1 0 0 or  $Tr \alpha_{\mu}$   $\mathscr{B} - 2.2961 + 0.0165$  (18 or  $\text{Tr } \rho_{\phi\phi} \mathscr{B} = 2.2961 \pm 0.0165$  (18 $\sigma$ )  $(255\sigma)$  $\sigma$ )  $T = 2$  , and the significance of  $\sim$   $\sim$  $\overrightarrow{AB}$ ,  $\chi_c \rightarrow \Lambda \overline{\Lambda}$  and  $\overrightarrow{AB}$  and  $\overrightarrow{AB}$  a significance of  $\overrightarrow{AB}$  significance of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$   $\mathcal{D}^{\text{U1U02}}$  Dell operator is  $p_{\phi\phi} = 2.2901 \pm 0.0100$  (100) Bell operator  $(18\sigma)$  $P_{\text{intercept}}$   $P_{\text{a}} = 0.521 \pm 0.0021$  (255 $\sigma$ )  $\text{F1}$  is  $\text{F2}$  is pure. We find  $\text{F1}$  is propagating the errors, and the errors, after propagating the errors, and t  $D_{\phi\phi} \mathscr{B} = 2.2901 \pm 0.0103$  (100)

⇢ /

BBBBBBBBBBBB@

 

*w*0 0

 

$$
\frac{\text{Total Indi Information spin-0 states}}{ \eta_c \to \Lambda + \bar{\Lambda} \quad \text{and} \quad \chi_c^0 \to \Lambda + \bar{\Lambda}}
$$

*Probing CP symmetry and weak phases with*

[19] BESIII Collaboration, M. Ablikim et al., *Tests*

*of CP symmetry in entangled* ⌅<sup>0</sup> *and* ⌅¯<sup>0</sup> *pairs*,

*simultaneous measurement of* ⌅<sup>0</sup> *and* ⌅¯<sup>0</sup>

![](_page_9_Picture_15.jpeg)

#### is pure. We find  $\sum_{n=1}^{\infty}$   $\sum_{n=1}^{\infty}$ *of CP symmetry in entangled* ⌅<sup>0</sup> *and* ⌅¯<sup>0</sup> *pairs*,

necessary to assess the significance of the significance of the result. We understand that they are forthcoming.<br>The significance of the result in the significance of the results in the significance of the significance of

which is the same for both decay processes. Accordingly, the Horodecki condition is found to be  $\mathcal{A}$ 

References

*philosophy*. Cambridge University Press, 1987.

*Experimental test of Bell's inequalities using*

*conditions*, *Phys. Rev. Lett.* 81 (1998)

not additional metal in die note additions among the helicity among the helicity among the helicity among the<br>The helicity among the helicity among the helicity amplitudes. The helicity among the helicity among the helic

Neither the Library of the anti-baryon normal control to the anti-baryon are polarized.<br>Neither the anti-baryon are polarized to the anti-baryon are polarized. The anti-baryon are polarized to the a

[11] S. P. Baranov, *Bell's inequality in charmonium*

*Testing Bell's Inequality using Charmonium*

*Electron-Positron Annihilation*, *Nature Phys.* 15

component of the triplet. Charge parity conservation implies the same conservation in planet of the same condition as parity and does not be same condition as parity and does not be a parity and does not be a parity and do

#### E. Gabrielli, and L. Marzola, *Quantum*  $\alpha$  or deck is condition the geometry of the geometry of the BESI is supported by the Estonian Research Council grants PRG803, RVTT3 and by the CoE program grant TK202

The concurrence can be concurrence can be computed and it is maximal: it is maximal: it is maximal: it is maximal:

 $\begin{array}{l} \text{N. A. Tornqvist, } \emph{Suggestion for} \ \emph{Einstein-podolsky-rosen Experiments} \emph{Using} \ \emph{Reactions Like } e^+e^- \rightarrow \Lambda \bar{\Lambda} \rightarrow \pi^-p\pi^+\bar{p}, \emph{Found.} \end{array}$ [9] N. A. Tornqvist, *Suggestion for N. A. Tornqvist, Suggestion for*<br>*Einstein-podolsky-rosen Experiments Using Phys.* **11** (1981) 171–177.

density matrix ⇢ = *|* ih *|* is written as

⇢ /

0

CCA

0 0 *w*0 0*w*⇤

The final state of the final state of the two states of the two states of the two states of the two states of

1*,*<sup>1</sup> 0 *|w*0 0 *|*

<sup>2</sup> 0 *w*0 0*w*⇤

*<sup>|</sup>* #i / *<sup>w</sup>* <sup>1</sup>  $\Psi = 0.1321 \pm 0.0042$  $\alpha = 0.4748 \pm 0.0022$  stated to  $\alpha = 0.4748 \pm 0.0031$  syst and Figure 4.1: Concurrence (left) and Horodecki condition m<sup>12</sup> (right) for *J/* ! ⇤⇤. Both quantities are the largest for  $\Delta \Phi = 0.7521 + 0.0042$ <sub>stat</sub>  $+ 0.0066$ <sub>s</sub> (2019) 631–634, [arXiv:1808.08917].  $\alpha = 0.4748 \pm 0.0022|_{\text{stat}} \pm 0.0031|_{\text{syst}}$  and  $\Delta \Phi = 0.7521 \pm 0.0042|_{\text{stat}} \pm 0.0066|_{\text{syst}}$ .

the *J/* is produced polarized. The correlation matrix of the two baryons depends on the scattering

angle ⇥ because the polarization of the *J/* does.

$$
|\psi_{\uparrow}\rangle \propto w_{\frac{1}{2} \frac{1}{2}} |\frac{1}{2} \frac{1}{2} \rangle \otimes |\frac{1}{2} \frac{1}{2} \rangle
$$
  
\n $|\psi_{\downarrow}\rangle \propto w_{-\frac{1}{2} - \frac{1}{2}} |\frac{1}{2} - \frac{1}{2} \rangle \otimes |\frac{1}{2} - \frac{1}{2} \rangle$   
\n $|\psi_{0}\rangle \propto w_{\frac{1}{2} - \frac{1}{2}} |\frac{1}{2} \frac{1}{2} \rangle \otimes |\frac{1}{2} - \frac{1}{2} \rangle + w_{-\frac{1}{2} \frac{1}{2}} |\frac{1}{2} - \frac{1}{2} \rangle \otimes |\frac{1}{2} \frac{1}{2} \rangle$ ,  
\n0.0

## Charmonium spin-1 states The decay *J/* ! ⇤⇤¯ has branching fraction 1*.*<sup>8</sup> ⇥ <sup>10</sup><sup>3</sup> [27]. From [15] we can obtain the values

fall in the triplet representation of the product <sup>1</sup>

the angular momentum to be described by the three states

BBB<br>BBBBBBBB<br>BBBBBBBBBB

to the 0 helicity (*J<sup>z</sup>* = 0), that is, the *J/* being longitudinally polarized. The states in Eq. (??) are

In the process

angle ⇥ because the polarization of the *J/* does.

The elements of the spin density matrix can be written as

instead contains a mixture of the states discussed in Eq. (4.4). The states discus

**|**<br>|
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| Precise Measurements<br>
CP Asymmetry with  $\overrightarrow{Precise\ Measurements\ of\ Decay\ Parameters\ and\ CP\ Asymmetry\ with\ Entangled\ \Lambda-\bar{\Lambda} \ Pairs, \ Phys.}$ ווי<br>ד *Precise Measurements of Decay Parameters and* 2 i ⌦ *| <sup>e</sup>*+*e* ! ! *cc*¯ ! *J/* ! ⇤⇤¯ *,* (4.5) the *J/*  $$ (0*,* ⇥*,* 0) (4.6) **BESIII** Collaboration, M. Ablikim et al.,<br>*Precise Measurements of Decay Parameters and*  $\quad \quad \texttt{[arXiv:2204.11058]}.$ are found at  $\frac{1}{2}$ , for which at  $\frac{1}{2}$ , for wh *Rev. Lett.* 129 (2022), no. 13 131801,

Horodecki condition concurrence  $\ell = 0.4$ <sup>1</sup>  $\mathscr{C} = 0.475 \pm 0.0039$   $(199\pi)$   $J/\psi \rightarrow \Sigma^{-\bar{\Sigma}^+}$  1.  $\psi(3686) \rightarrow$ **Horodeck** *i,j* is the Wigner matrix for the spin 1 representation of *SO*(3) and the sum is only over the the electron and positron taken to be massless and, therefore, with only the helicities *±*1. Of the four helicity amplitudes, only two are independent. The density matrix is given by *w*⇤ l Q decki c :on dition  $\mathbf{m}$ *s*2 *c*⇥ *s*⇥  $m_{10} = 1.995 \pm 0.004$ CCCCCA  $\% = 0.475 \pm 0.1$ ↵ = 0*.*<sup>690</sup> *<sup>±</sup>* <sup>0</sup>*.*07*|*stat *<sup>±</sup>* <sup>0</sup>*.*02*|*syst and = 0*.*401+0*.*<sup>154</sup> 0*.*140*|*stat *<sup>±</sup>* <sup>0</sup>*.*028*|*syst *,* (4.13)  $A = 1.220 \pm 0.004$   $(000)$  $\text{Horodecki condition}$   $\mathfrak{m}_{12} = 1.225 \pm 0.004$   $(56\sigma)$   $\frac{J/\psi \rightarrow \Sigma^0 \Sigma^0}{\psi^0 \rightarrow \Sigma^0 \Sigma^0}$   $\frac{1.171 \pm 0.007}{1.62 \pm 0.065}$  $\mathcal{A}$ are found at ⇥ = ⇡*/*2, for which  $A$  **Forodecki condition**  $m_{12} = 1.225 \pm 0.004$  (56 $\sigma$ )  $0.475 \pm 0.0039$ [17] BESIII Collaboration, M. Ablikim et al., *Strong* [18] BESIII Collaboration, M. Ablikim et al.,

↵ = 0*.*4748 *±* 0*.*0022*|*stat *±* 0*.*0031*|*syst and = 0*.*7521 *±* 0*.*0042*|*stat *±* 0*.*0066*|*syst *.* (4.10)

No correlation in the uncertainties are given.

## *Measurement of the parity-violating asymmetry parameter* ↵*<sup>b</sup> and the helicity amplitudes for the* **b Bell** inequality violation

The significance of the violation of the Bell inequality is nominally 56*.*2. This is a lower bound

![](_page_10_Picture_2291.jpeg)

![](_page_10_Picture_12.jpeg)

The significance of the violation of the Bell inequality is nominally 56*.*2. This is a lower bound

$$
J/\psi \to \Lambda + \bar{\Lambda} \quad \text{and} \quad \psi(3686) \to \Lambda + \bar{\Lambda}
$$

![](_page_10_Figure_2.jpeg)

1 ↵<sup>2</sup> sin 2 ⇥ cos 0 (1 + 2↵ + cos 2⇥)

Again, it agrees with [40].

The entanglement is not maximal even for ⇥ = ⇡*/*2 because the final state is not a pure state and

![](_page_10_Picture_8.jpeg)

 $(56\sigma)$ 

[19] BESIII Collaboration, M. Ablikim et al., *Tests*

that could be improved once that could be improved once the uncertainties are included. In the uncertainties are included. In the uncertainties are included in the uncertainties are included. In the uncertainties are inclu

[30] V. B. Berestetskii, E. M. Lifshitz, and L. P.

*<sup>f</sup>*⇥ *<sup>w</sup>* <sup>1</sup>

*w*⇤

⇥

<sup>2</sup> *w* <sup>1</sup>

*w*⇤

*s*⇥

*w*<sup>1</sup>

<sup>2</sup> <sup>1</sup>

<sup>1</sup>

<sup>2</sup> <sup>1</sup>

<sup>2</sup> *w*<sup>1</sup>

<sup>2</sup> <sup>1</sup>

<sup>1</sup>

1

<sup>2</sup> *w*<sup>1</sup>

<sup>2</sup> <sup>1</sup>

1 <sup>2</sup> <sup>1</sup> *f*⇥ *w*<sup>1</sup>

<sup>2</sup> <sup>1</sup>

1 1 2

CCCCCA

*,* (4.5)

in which *f*⇥ ⌘ (3 cos 2⇥)*/*4, *s*⇥ ⌘ sin ⇥ and *c*⇥ ⌘ cos ⇥.

<sup>p</sup><sup>1</sup> ↵

1 + ↵ exp[*i*] *.* (4.6)

The correlation matrix can be computed from the density matrix  $\mathcal{L}$  is given by  $\mathcal{L}$ 

# Horodecki condition

## Entanglement

*<sup>c</sup>* are produced in

The 2002

3/2 fermions is not yet available. The state of the state o<br>3/2 fermions is not yet available. The state of the state<br>

5 Charlotte and the charmonium spin 2 state and the charmonium spin 2 state and the charmonium spin 2 state and<br>The charmonium spin 2 state and the charmonium spin 2 state and the charmonium spin 2 state and the charmonium

*<sup>e</sup>*+*e* ! (3686) ! <sup>2</sup>

chiral coupling.

 $\text{Horodecki condition} \quad \text{Tr } \mathscr{B} \rho_{\phi\phi} = 1.202 \pm 0.032$  BESIII Collaboration, M. Ablikim et al.,  $\Omega_{\perp} = 1.202 + 0.032$ 

*w*<sup>1</sup> <sup>1</sup>

= *w*1 1 *, w*1 1

= *w*<sup>1</sup> <sup>1</sup> *,* and *w*<sup>0</sup> <sup>1</sup>

= *w*1 0

= *w*1 0

$\chi_c^2 \rightarrow \phi \phi$	Entanglement
Horodecki condition	

⇤*<sup>b</sup>* ! *J/* + ⇤ (B.1)

There is no particle is no particle is no particle independent non-vanishing helicity amplitudes:  $\mathcal{A}$ 

#### $p_y = 0.00 \pm 0.00$ ,  $JV(\rho) = 0.03 \pm 0.00$ 05 (2023) 069, [arXiv:2301.12922].

**PTEP** 2022 (2022) 083CON [29] BESIII Collaboration, M. Ablikim et al., *Design Measurement of the parity-violating asymmetry*  $\alpha$  *decay*  $\Lambda_b^0 \to J/\psi + \Lambda^0$  *with the ATLAS detector*, *Phys. Rev. D* 89 (2014), no. 9 092009,  $[{\tt arXiv:1404.1071}].$ *Phys. Rev. D* **89** (2014), no. 9 092009,<br>[arXiv:1404.1071]. **ATLAS** Collaboration, G. Aad et al., *parameter*  $\alpha_b$  *and the helicity amplitudes for the* 

![](_page_11_Picture_14.jpeg)

![](_page_11_Figure_3.jpeg)

The area of the state of the sta<br>The state of the st

2 representation of the product 1 representation of the product 1 representation of the product 1  $\alpha$ 

## Charmonium spin-2 states There is no particle is no particle is no particle independent non-vanishing helicity amplitudes:  $\sim$ . Charmoniu

fall in the <sup>1</sup>

<sup>2</sup> =

3

<sup>2</sup> <sup>1</sup>

#### 11 and n  $\mathscr{C}[\rho] = 0.12 \pm 0.11$  and  $\mathfrak{m}_{12} = 1.01 \pm 0.04$ .  $2+0.11$  and  $m_1$ *Esinstein-Podolsky-Rosen correlations and*  $\mathscr{C}[\rho] = 0.12 \pm 0.11$  and  $\mathfrak{m}_{12} = 1.01 \pm 0.04$ .  $\mathscr{C}[\rho] = 0.12 \pm 0.11 \text{ and } \mathfrak{m}_{12} = 1.01 \pm 0.04$ .

2 i 1922 (provinci pod stanovni stanovni stanovni stanovni stanovni stanovni stanovni stanovni stanovni stanov<br>2 i provinci pod stanovni sta

*Measurement* of the U.S. *Measurement <sup>b</sup>* ! *J/* ⇤ *decay amplitudes and the* ⇤<sup>0</sup> *<sup>b</sup> polarisation in pp* The analysis of the data taken at p*s* = 3*.*710 GeV gives [42] *complete measurement of the N Electromagnetic*<br>*Form Factors, Phys. Rev. Lett.* **123** (2019), **|**  $\qquad \qquad \textbf{BESIII}$  Collaboration, M. Ablikim et al., *Complete Measurement of the*  $\Lambda$  *Electromagnetic* 

in which *p*" =

BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of*  $\chi_c^J \rightarrow \phi \phi$ , *JHEP*<br>
05 (2023) 060 [*w*Niv:2301, 12922]  $Helicity amplitude analysis of  $\chi_c^* \rightarrow \varphi \varphi$ , JHEP \n**05** (2023) 069, [arXiv:2301.12922].$ *ampun*<br>023) 069,

<sup>2</sup> + *P<sup>b</sup>* and *p*# =

<sup>2</sup> *Pb*.

 $\mathcal{N}(\rho) = 0$ , and  $\mathcal{C}_2 = 0$ . ⇢⇤ *J/* /  $\bigcup_{i=1}^n\mathbb{R}^d$ 

with no angular dependence. The helicity amplitude *w*

1 1

#### $\blacksquare$  Other processes *Pr* Drocesses Other processes 2.4 cm from the production production production production production production and before any pipe <sup>2</sup> . It is constrained by the conservation of the  $\blacksquare$  Denote of the experimental collaborations  $\blacksquare$

$$
\langle\psi_{\uparrow}| + p_{\downarrow}|\psi_{\downarrow}\rangle\langle\psi_{\downarrow}| \, ,
$$

*J/* ! ⇤⇤¯ 1.225 *<sup>±</sup>* 0.004 56.3

![](_page_12_Figure_2.jpeg)

Figure 4.6: Left side: Entanglement as a function of the form factor parameters  $\alpha$  and  $\Delta\Phi$ . The concurrence is computed at  $\Theta = \pi/3$ . Right side: The concurrence as a function of the parameter  $\alpha$  at  $\Theta = \pi/2$  is  $\mathscr{C}[\rho] = |\alpha|$ . There is no dependence on the other parameter  $\Delta\Phi$ .

#### $\Gamma$  , summary of Bell inequality violation in spin  $\Gamma$  in spin  $\Gamma$  into baryons. In spin  $\Gamma$  $\mathcal{A} = \mathcal{A} \cup \mathcal{A}$  . For the form factors of the form fa We have seen that the amount of entanglement in the final state spin correlations depends on the kind of baryons and on the charmonium state these come from. *J/*  $\frac{1}{\sqrt{2}}$  2.171 **−** 0.007 24.44.44 24.4 Entanglement as a function of the form factors

in which *G<sup>M</sup>* = *F*<sup>1</sup> + *F*<sup>2</sup> and *G<sup>E</sup>* = *F*<sup>1</sup> + *s/*2*m*<sup>2</sup>

$$
G_M=F_1+F_2\,\,\text{and}\,\,G_E=F_1+s/2m_{\Lambda}^2F
$$

![](_page_12_Picture_6.jpeg)

$$
{}^{\nu}F_2 \bigg] \, u_{\Lambda} A_{\mu}^{J/\psi} \qquad \qquad \frac{G_M}{G_E} = \bigg| \frac{G_M}{G_E} \bigg| \, e^{i \Delta \Phi} \quad \text{and} \quad \alpha = \frac{s |G_M|^2 - 4 m_{\Lambda}^2 |G_E|^2}{s |G_M|^2 + 4 m_{\Lambda}^2 |G_E|^2} \, .
$$

![](_page_12_Figure_1.jpeg)

Two hadronic form factors, commonly called *G <sup>M</sup> (s)* and *GE (s)*,

are needed for the description of the annihilation process *e*−*e*<sup>+</sup> → !!¯ , Fig. 1a, and by varying the c.m. energy <sup>√</sup>*s*, their numerical

values can in principle be determined for all *s* values above !!¯ threshold. For the general case of annihilation via an intermediate

photon, the joint !*(*→ *<sup>p</sup>*π−*)*!¯ *(*→ *<sup>p</sup>*¯π+*)* decay distributions were calculated and analyzed in Ref. [1], using methods developed in  $[2,3]$ . Recently, a first attempt to calculate the hyperon form factors at the hyperon factors  $\mathcal{L}$ *GM (s)* and *GE (s)* in the time-like region was reported in Ref. [4].

Previously, the interesting special case of annihilation through

an intermediate *J/*ψ or ψ*(*2*S)*, Fig. 1b, has been investigated in

cess has also been used for determination of the anti-Lambda decay-asymmetry parameter and for CP symmetry tests in the hyperon system. A precise knowledge of the Lambda decayasymmetry parameter is needed for studies of spin polarization

Presently, <sup>a</sup> collected data sample of <sup>1</sup>*.*<sup>31</sup> × 109 *<sup>J</sup>/*ψ events

[10] by the BESIII detector [11] permits high-precision studies of

In the experimental work referred to above, the joint-hyperon-

decay distributions considered are not the most general ones possible, but seem to be curtailed. Incomplete distribution functions do not permit a reliable determination of the form factors and we

# • better data (e.g. for spin 2 or direct hyperon production)

- some amplitudes (e.g. scalar states into hyperons)
- 
- phases of most amplitudes
- uncertainty correlations

![](_page_13_Picture_7.jpeg)

# what is still missing

![](_page_14_Picture_5.jpeg)

## $100 \text{ seconds}$ The energy are to the energy and the decay of the decay of the decay of the decay  $\sim$ Quantum correlations and decoherence

![](_page_14_Figure_2.jpeg)

Figure 6.1: Decay  $\eta_c \to \Lambda \bar{\Lambda}$ : Fraction (out of 1000) of  $\Lambda$  baryons decaying at different lengths from the primary vertex. The Vertical dashed line stands for the inner surface of the beam pipe (3.15 cm from the primary vertex).

![](_page_14_Figure_4.jpeg)

![](_page_14_Figure_0.jpeg)