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Entanglement and Bell inequality violation in charmonium decays

Oxford
2 October 2024

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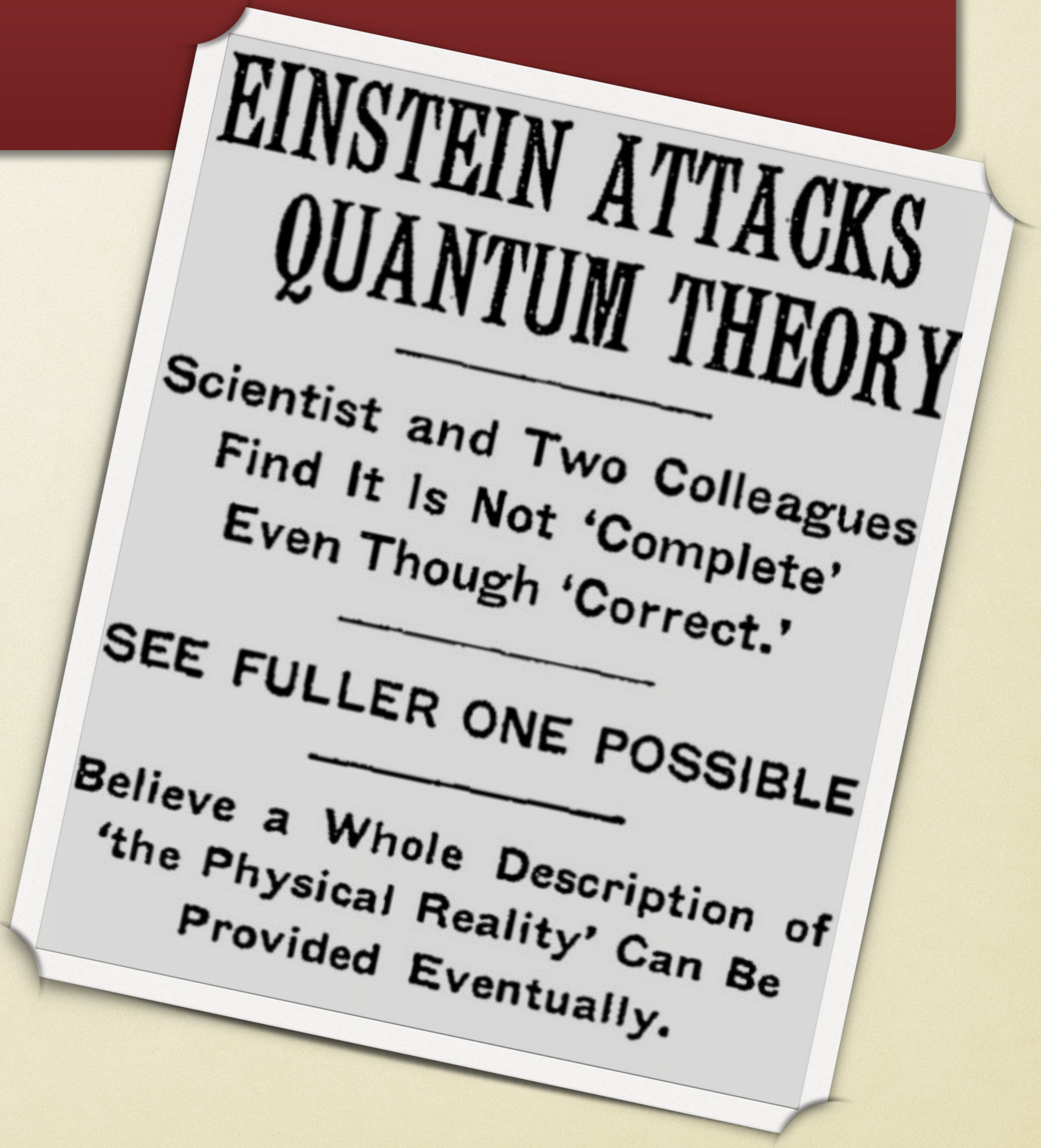
Where have we already seen entanglement or Bell inequality violation at high energies?

New York Times headline
May 4th, 1935


EINSTEIN ATTACKS QUANTUM THEORY

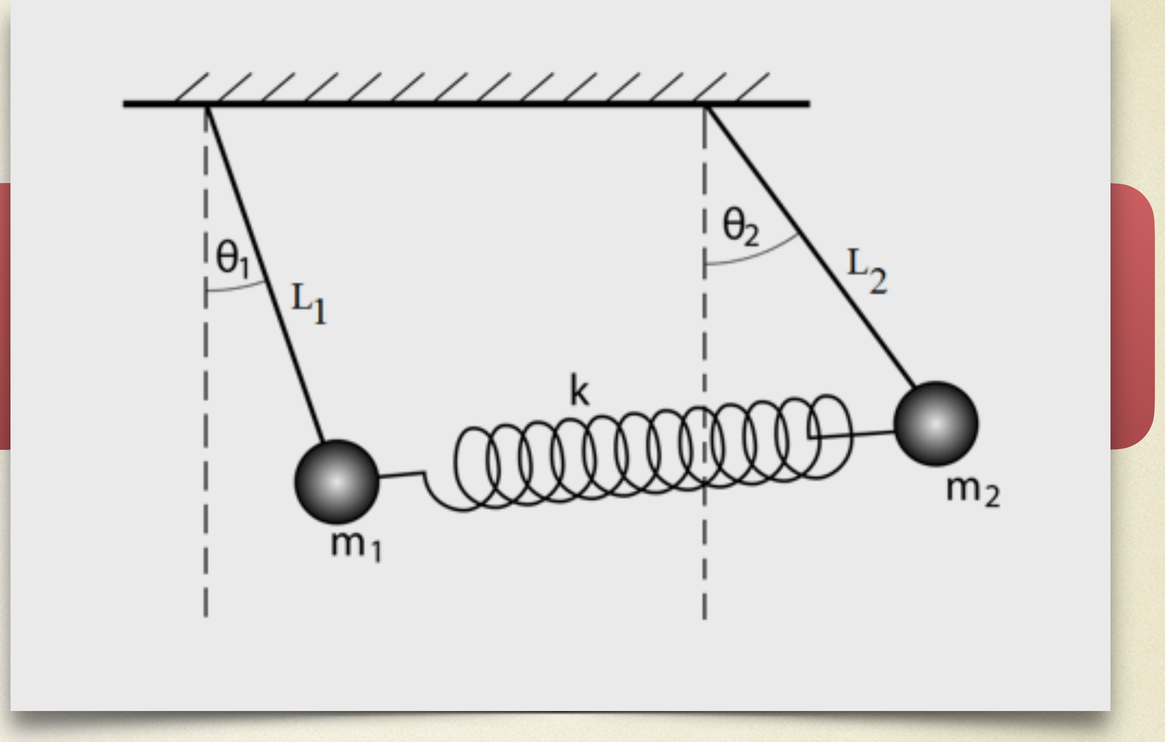
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

1

Flavor space



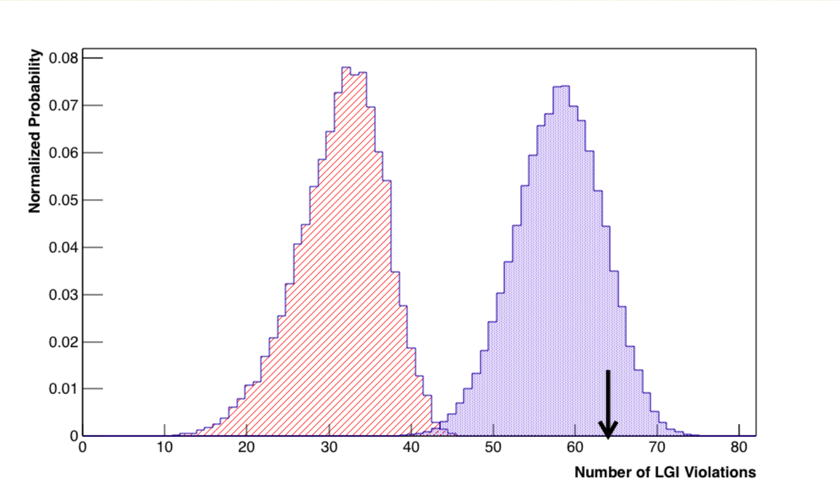
$K^0\bar{K}^0$ and $B^0\bar{B}^0$ oscillations

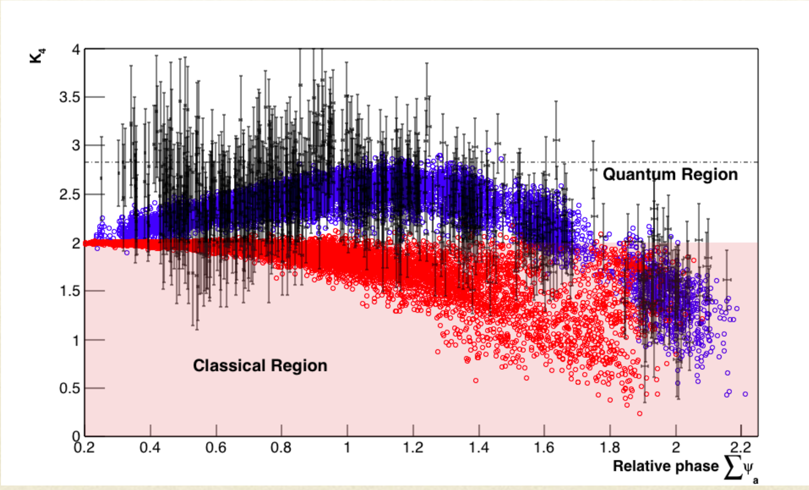
F Benatti and R Floreanini, Phys. Rev. D57 (1998) R1332, Eur. Phys. J. C13 (2000) 267

A Go, Belle Collaboration, Phys. Phys. Lett. 99 (2007) 131802

neutrino oscillations

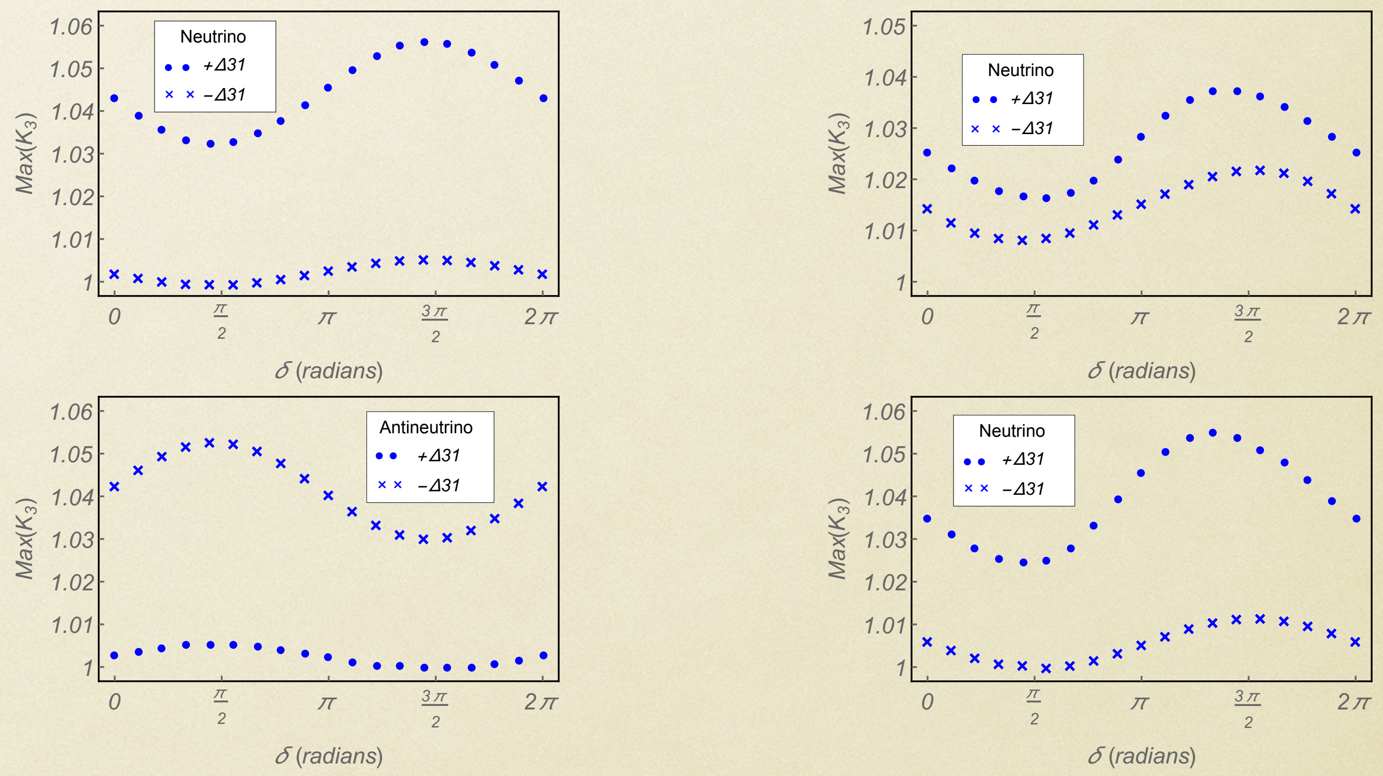
Leggett-Garg inequality violation





Minos (6σ)

JA Formaggio, DI Kaiser, MM Murskyj and TE Weiss, Phys. Rev. Lett. 117 (2016) 050402



Dune T2K/Noνa

J Naikoo et al, Phys. Rev. D 99 (2019) 095001

2

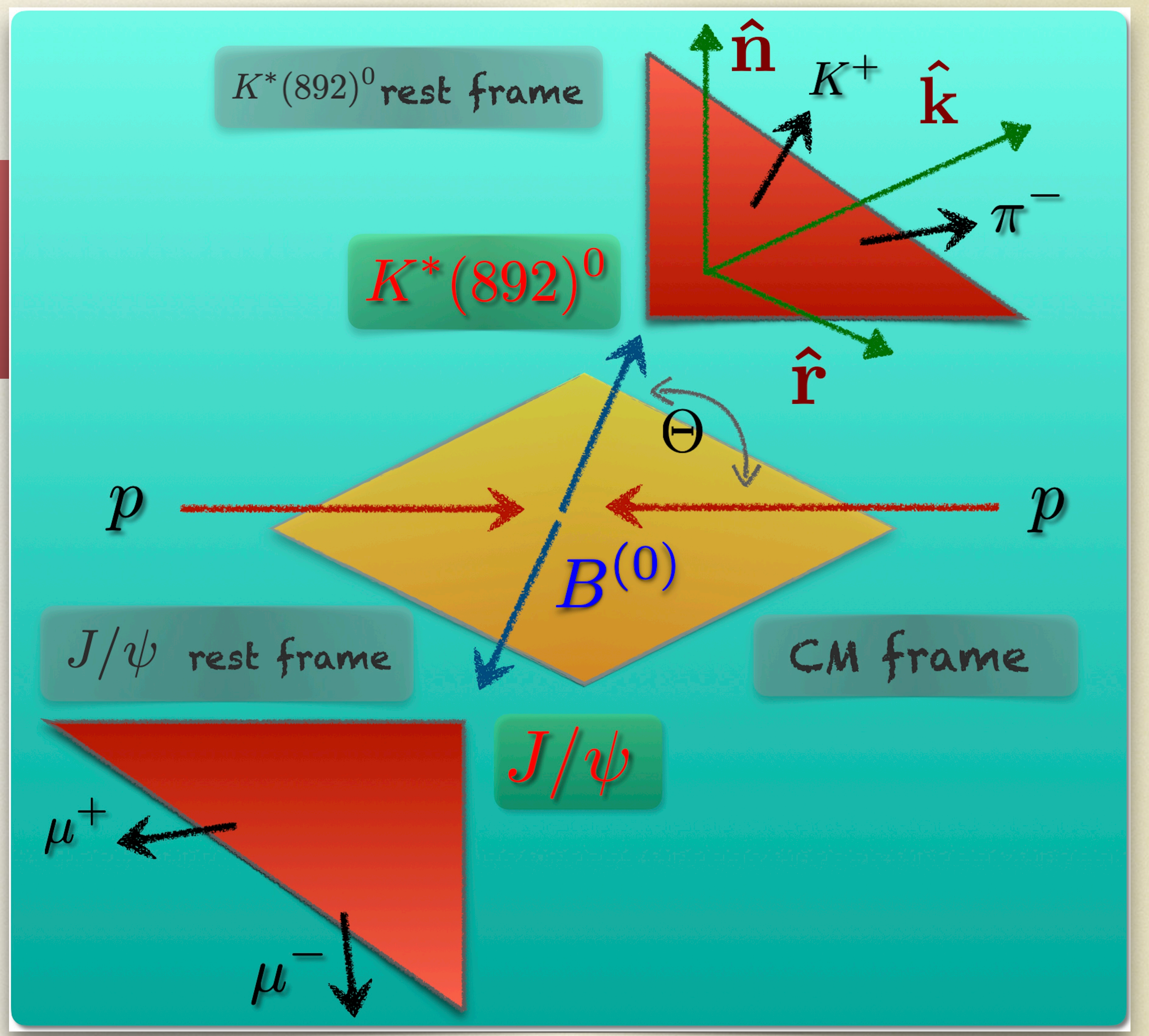
B-meson decays

	$\mathscr{E}$	$\mathcal{I}_3$
• $B^0 \to J/\psi\, K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015
• $B^0 \to \phi\, K^*(892)^0$ [22]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$
• $B^0 \to \rho\, K^*(892)^0$ [23]	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$
• $B_s \to \phi\,\phi$ [24]	0.734 ± 0.037	2.525 ± 0.064
• $B_s \to J/\psi\,\phi$ [25]	0.731 ± 0.032	2.462 ± 0.080

8.2σ

entanglement

Bell inequality



Parameter	Result
$\lvert A_0\rvert^2$	$0.384 \pm 0.007 \pm 0.003$
$\lvert A_\perp\rvert^2$	$0.310 \pm 0.006 \pm 0.003$
$\delta_\parallel$ [rad]	$2.463 \pm 0.029 \pm 0.009$
$\delta_\perp$ [rad]	$2.769 \pm 0.105 \pm 0.011$

	$\lvert A_0\rvert^2$	$\lvert A_\perp\rvert^2$	$\delta_\parallel$	$\delta_\perp$
$\lvert A_0\rvert^2$	1	-0.342	-0.007	0.064
$\lvert A_\perp\rvert^2$		1	0.140	0.088
$\delta_\parallel$			1	0.179
$\delta_\perp$				1

R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **131**, no.17, 171802 (2023) [arXiv:2304.06198 [hep-ex]].

MF, R. Floreanini, E. Gabrielli and L. Marzola, Phys. Rev D 109 (2024) 3, L031104

E. Gabrielli and L. Marzola, arXiv:2406.17772 (2024)

** K Chen et al, Eur. Phys. J. C 84 (2024) 580 $B_c^\pm \to J/\psi\, \rho^\pm$

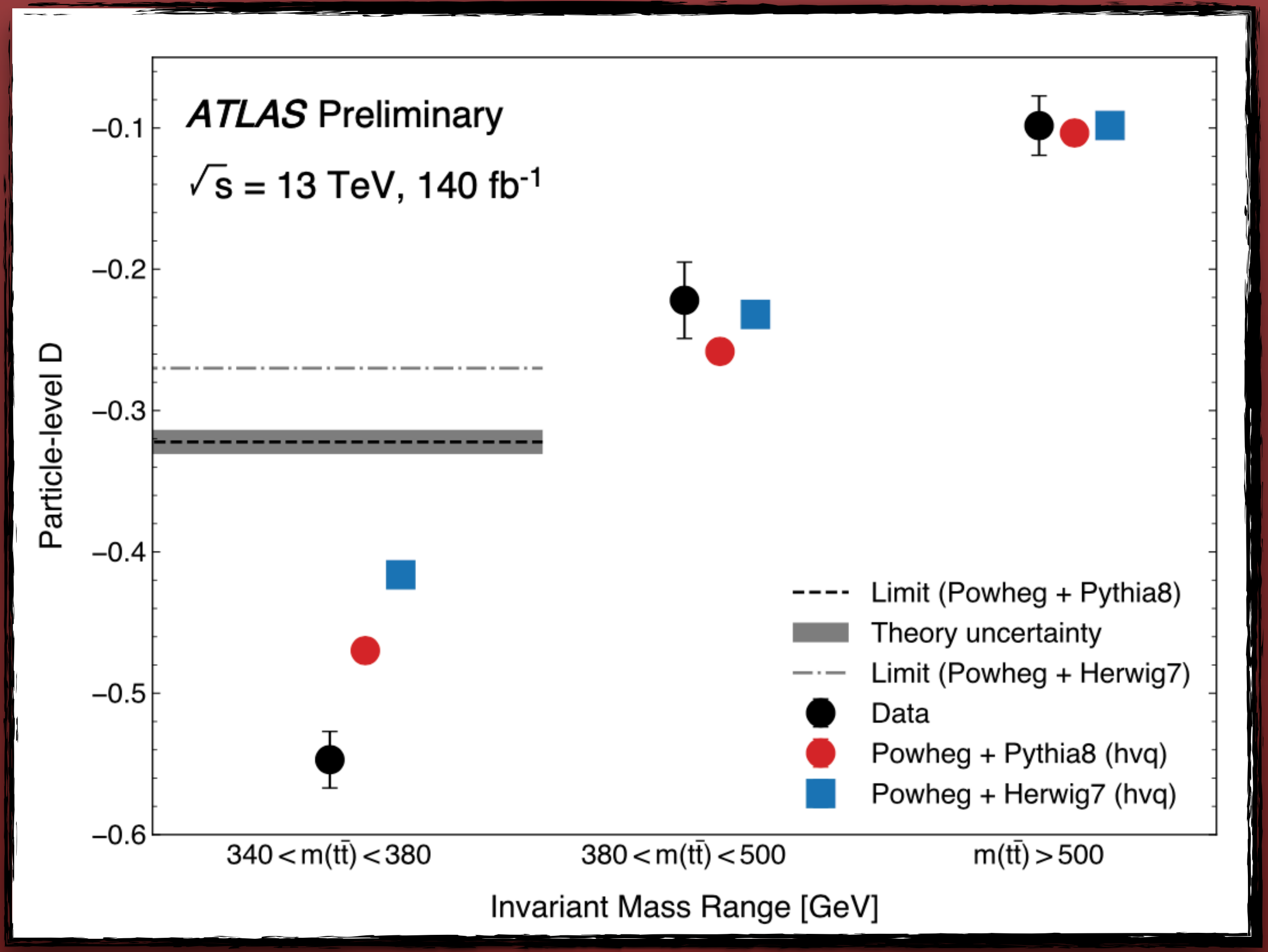
** RA Morales and A Szynkman, arXiv:2409.13033 $B^0 \to K^* \mu^+ \mu^-$

Pairs of top quarks

Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus **136** (2021) 907

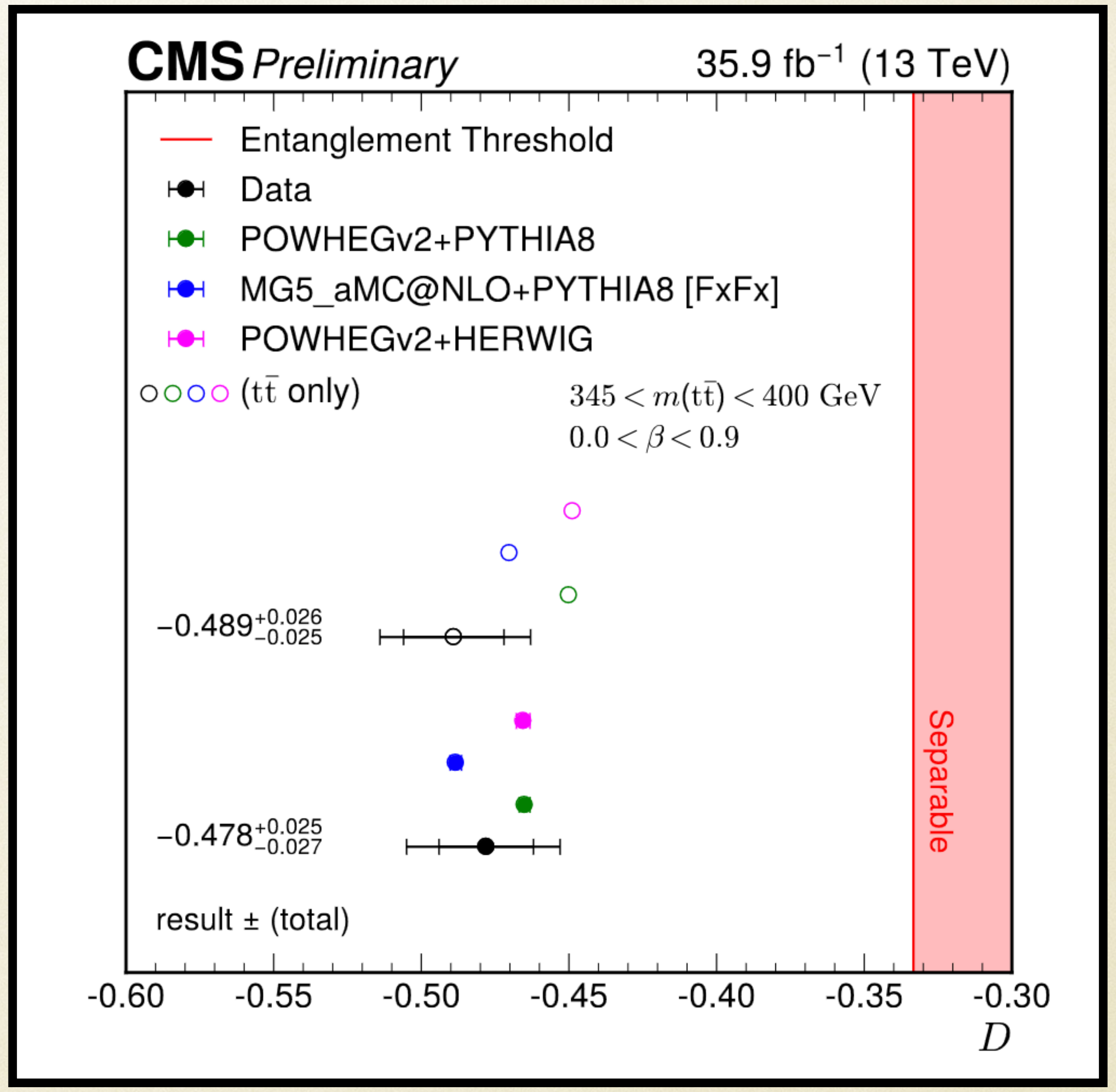
$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\Big(1 - D\cos\phi\Big)$$



$$D = -0.547 \pm 0.002\ [\text{stat}] \pm 0.021\ [\text{syst}]$$

ATLAS Collaboration, Nature 633 (2024) 542

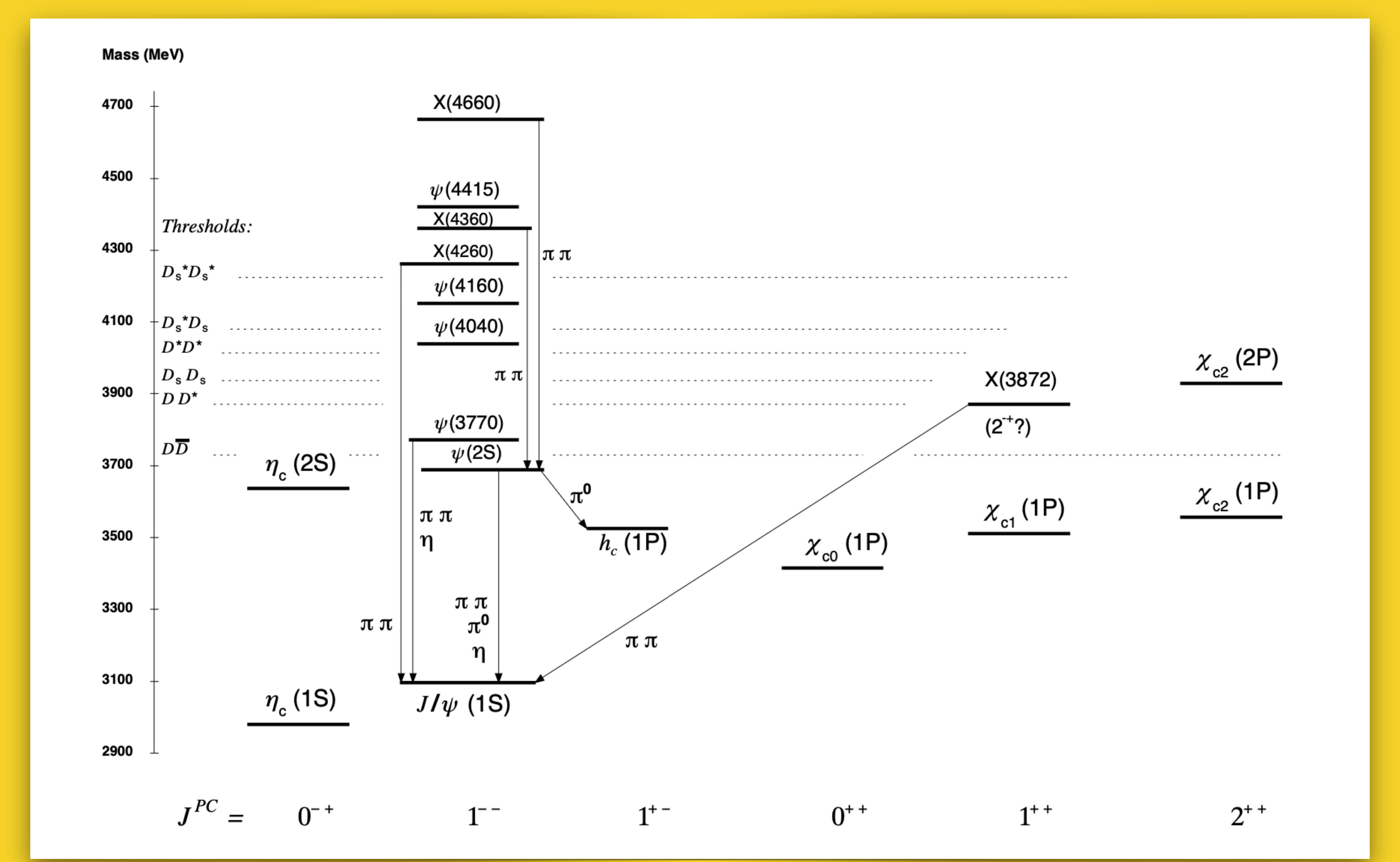


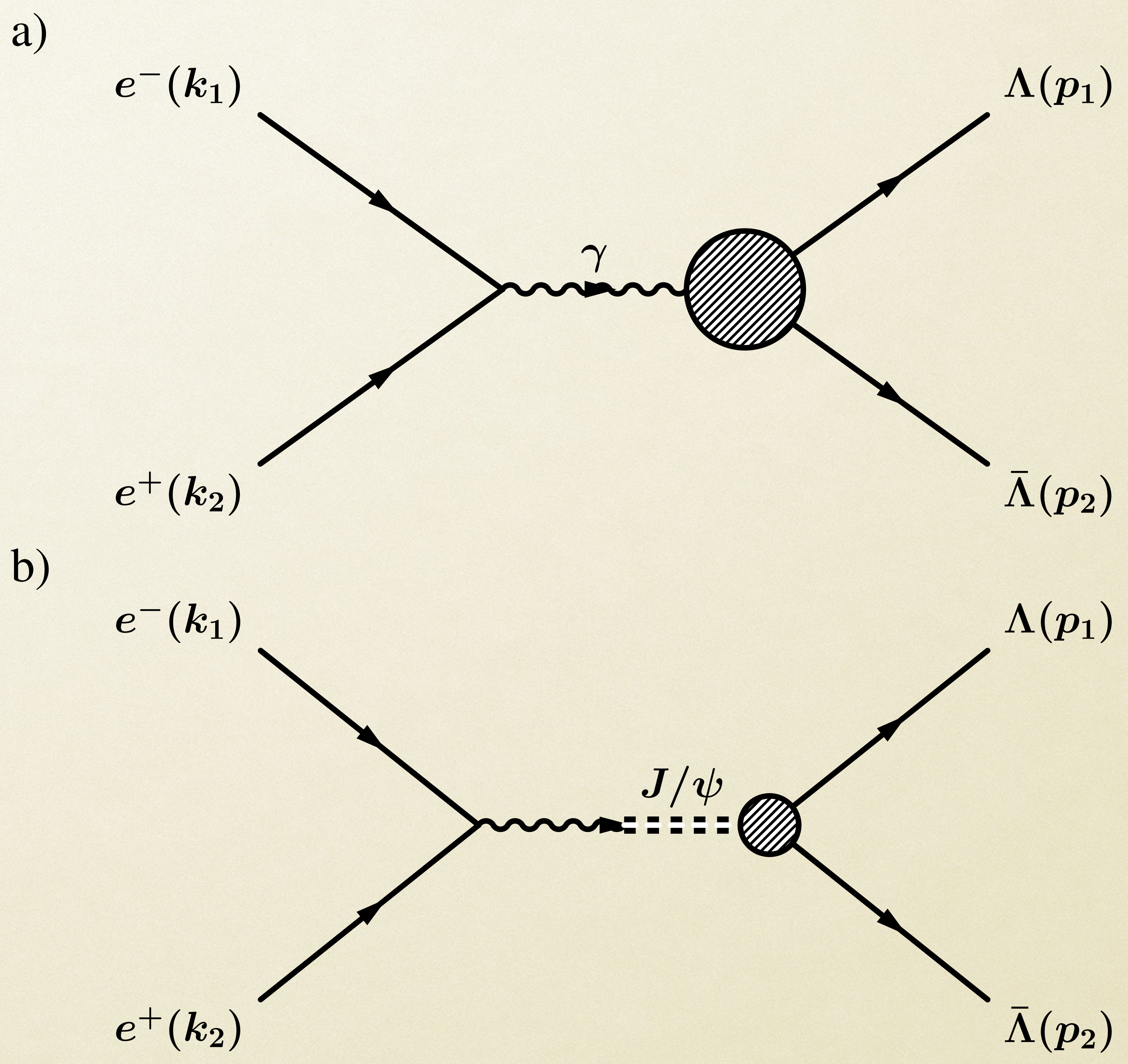
$$D = -0.478^{+0.025}_{-0.027}$$

CMS Collaboration, arXiv:2406.03976 (2024)

CMS Collaboration, arXiv:2409.11067 (2024)

Charmonium





MF, R. Floreanini, E. Gabrielli and L. Marzola, Phys. Rev. D110 (2024) 053008
see, also: S. Wu et al., Phys. Rev. D110 (2024) 054012

the toolbox

Qubits

$$\rho = \frac{1}{4}\Big[\mathbb{1}_2 \otimes \mathbb{1}_2 + \sum_{i=1}^{3} \mathrm{B}_i^{+}(\sigma_i \otimes \mathbb{1}_2) + \sum_{i=1}^{3} \mathrm{B}_j^{-}(\mathbb{1}_2 \otimes \sigma_j) + \sum_{i,j=1}^{3} \mathrm{C}_{ij}(\sigma_i \otimes \sigma_j)\Big]$$

$$R = \rho\,(\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y)$$

Concurrence $\mathscr{C}[\rho] = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$

CC^T $[m_1,\, m_2,\, m_3]$

Horodecki condition $\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$

Qutrits

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3\right] + \sum_{a=1}^{8} f_a \left[T^a \otimes \mathbb{1}_3\right] + \sum_{a=1}^{8} g_a \left[\mathbb{1}_3 \otimes T^a\right] + \sum_{a,b=1}^{8} h_{ab} \left[T^a \otimes T^b\right]$$

$$\begin{aligned}\mathscr{C}_2 \;=\; & 2\max\Big[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,; \\ & -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2,\; 0\Big]\end{aligned}$$

Entropy $\mathscr{E}[\rho] \equiv -\mathrm{Tr}[\rho_A \ln \rho_A] = -\mathrm{Tr}[\rho_B \ln \rho_B]$

Negativity $\mathcal{N}(\rho) = \sum_k \frac{|\lambda_k| - \lambda_k}{2}$

Bell operator $\mathcal{I}_3 = \mathrm{Tr}\left[\rho\, \mathscr{B}_3\right]$

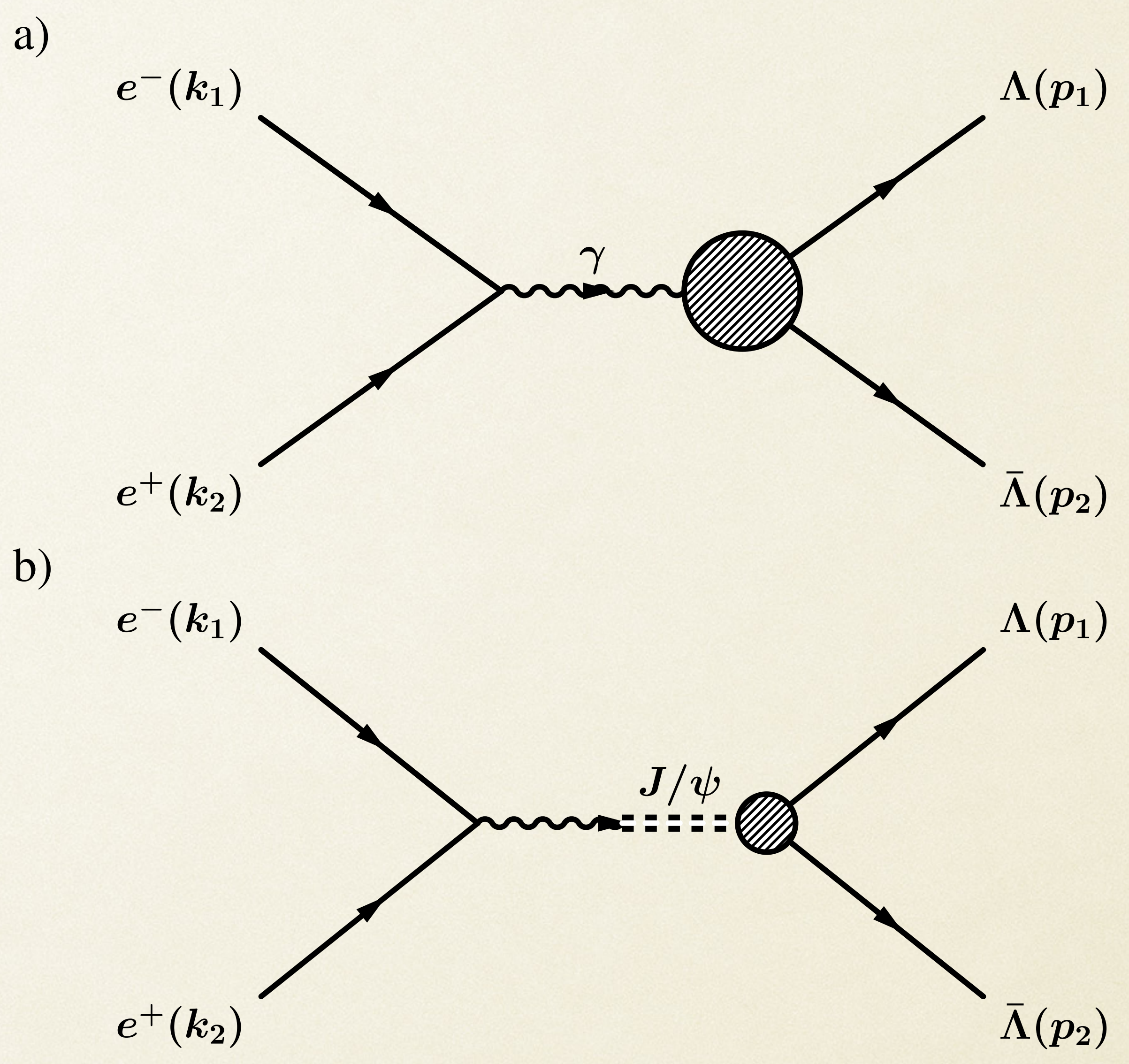
A. Barr, MF, R. Floreanini, E. Gabrielli and L. Marzola, Progress in Particle and Nuclear Physics 139 (2024) 104134

$$\boldsymbol{\xi} = (\theta, \Omega_1, \Omega_2),$$

$$\begin{aligned}\mathcal{W}(\boldsymbol{\xi}) &= \mathcal{F}_0(\boldsymbol{\xi}) + \alpha \mathcal{F}_5(\boldsymbol{\xi}) \\ &\quad + \alpha_1\alpha_2 \left(\mathcal{F}_1(\boldsymbol{\xi}) + \sqrt{1-\alpha^2}\cos(\Delta\Phi)\mathcal{F}_2(\boldsymbol{\xi}) + \alpha\mathcal{F}_6(\boldsymbol{\xi})\right) \\ &\quad + \sqrt{1-\alpha^2}\sin(\Delta\Phi)\left(\alpha_1\mathcal{F}_3(\boldsymbol{\xi}) + \alpha_2\mathcal{F}_4(\boldsymbol{\xi})\right), \qquad (6.55)\end{aligned}$$

$$\begin{aligned}\mathcal{F}_0(\boldsymbol{\xi}) &= 1 \\ \mathcal{F}_1(\boldsymbol{\xi}) &= \sin^2\theta \sin\theta_1 \sin\theta_2 \cos\phi_1 \cos\phi_2 + \cos^2\theta \cos\theta_1 \cos\theta_2 \\ \mathcal{F}_2(\boldsymbol{\xi}) &= \sin\theta\cos\theta\,(\sin\theta_1\cos\theta_2\cos\phi_1 + \cos\theta_1\sin\theta_2\cos\phi_2) \\ \mathcal{F}_3(\boldsymbol{\xi}) &= \sin\theta\cos\theta\sin\theta_1\sin\phi_1 \\ \mathcal{F}_4(\boldsymbol{\xi}) &= \sin\theta\cos\theta\sin\theta_2\sin\phi_2 \\ \mathcal{F}_5(\boldsymbol{\xi}) &= \cos^2\theta \\ \mathcal{F}_6(\boldsymbol{\xi}) &= \cos\theta_1\cos\theta_2 - \sin^2\theta\sin\theta_1\sin\theta_2\sin\phi_1\sin\phi_2. \qquad (6.56)\end{aligned}$$

G. Faldt and A. Kupsc, Phys. Lett B 772 (2017) 16



maximum likelihood fit

more events
SM needed

$$\rho_{\lambda_1\lambda_2,\lambda_1'\lambda_2'} \propto w_{\lambda_1\lambda_2} w^*_{\lambda_1'\lambda_2'} \sum_k D^{(J)*}_{k,\lambda_1-\lambda_2}(0,\Theta,0)\, D^{(J)}_{k,\lambda_1'-\lambda_2'}(0,\Theta,0)$$

Charmonium spin-0 states

$$\boxed{\eta_c \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \chi_c^0 \to \Lambda + \bar{\Lambda}}$$

$$|\psi_0\rangle \propto w_{\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},\tfrac{1}{2}\rangle$$

Concurrence $\mathscr{C} = 1$

Horodecki condition $\mathfrak{m}_{12} = 2$

N. A. Tornqvist, *Suggestion for Einstein-podolsky-rosen Experiments Using Reactions Like $e^+e^- \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$, Found. Phys.* **11** (1981) 171–177.

N. A. Tornqvist, *The Decay $J/\psi \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$ as an Einstein-Podolsky-Rosen Experiment, Phys. Lett. A* **117** (1986) 1–4.

S. P. Baranov, *Bell's inequali decays $\eta_c \to \Lambda\bar{\Lambda}$, $\chi_c \to \Lambda\bar{\Lambda}$ an Phys. G* **35** (2008) 075002.

$$\boxed{\chi_c^0 \to \phi + \phi}$$

$$|\Psi\rangle = w_{-1\,-1}\,|-1,\,-1\rangle + w_{0\,0}\,|0\,0\rangle + w_{1\,1}\,|1,\,1\rangle$$

$$\left|\frac{w_{1,1}}{w_{0\,0}}\right| = 0.299 \pm 0.003|_{\text{stat}} \pm 0.019|_{\text{syst}}\,.$$

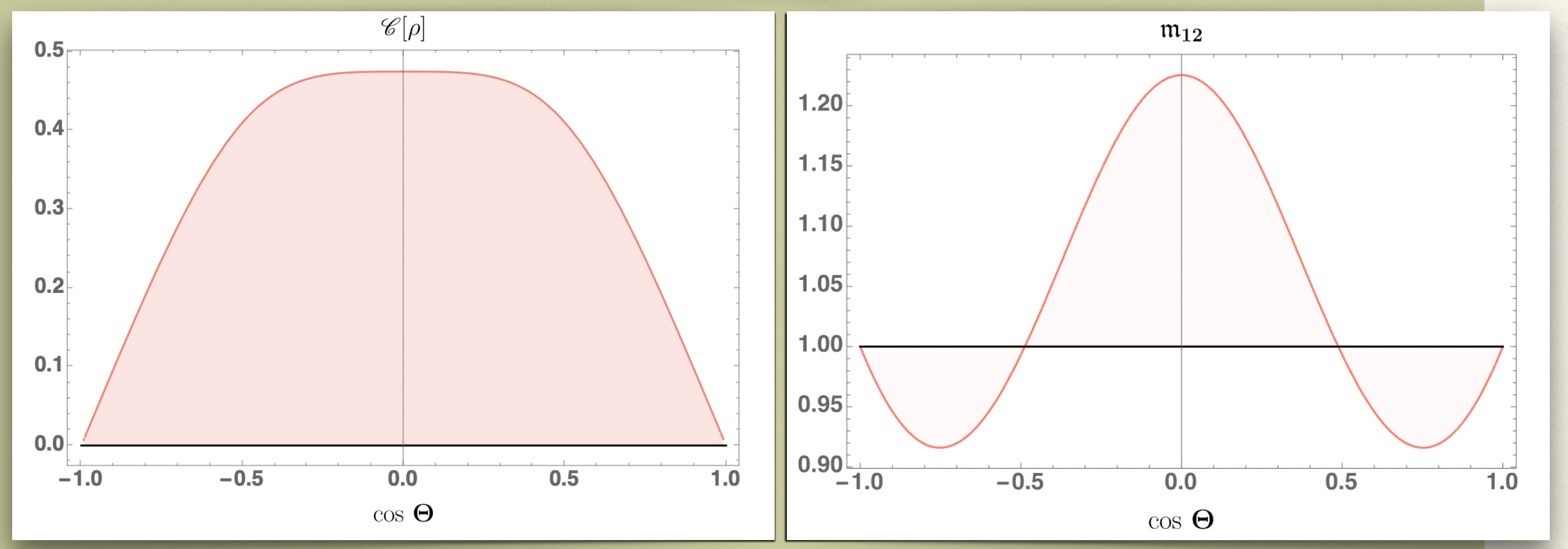
BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of $\chi_c^J \to \phi\phi$, JHEP* **05** (2023) 069, [arXiv:2301.12922].

Entropy	$\mathscr{E}[\rho] = 0.531 \pm 0.0021$	(255σ)
Bell operator	$\text{Tr}\,\rho_{\phi\phi}\,\mathscr{B} = 2.2961 \pm 0.0165$	(18σ)

Charmonium spin-1 states

$$\boxed{J/\psi \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \psi(3686) \to \Lambda + \bar{\Lambda}}$$

$$\begin{aligned}
|\psi_\uparrow\rangle &\propto w_{\frac{1}{2}\frac{1}{2}} \, |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \\
|\psi_\downarrow\rangle &\propto w_{-\frac{1}{2}-\frac{1}{2}} \, |\tfrac{1}{2}\, -\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\, -\tfrac{1}{2}\rangle \\
|\psi_0\rangle &\propto w_{\frac{1}{2}-\frac{1}{2}} \, |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\, -\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\frac{1}{2}} \, |\tfrac{1}{2}\, -\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \,,
\end{aligned}$$



$$\alpha = 0.4748 \pm 0.0022|_{\text{stat}} \pm 0.0031|_{\text{syst}} \quad \text{and} \quad \Delta\Phi = 0.7521 \pm 0.0042|_{\text{stat}} \pm 0.0066|_{\text{syst}} \,.$$

BESIII Collaboration, M. Ablikim et al., *Precise Measurements of Decay Parameters and CP Asymmetry with Entangled Λ-$\bar{\Lambda}$ Pairs, Phys. Rev. Lett.* **129** (2022), no. 13 131801, [arXiv:2204.11058].

Concurrence $\quad \mathscr{C} = 0.475 \pm 0.0039 \quad (122\sigma)$

Horodecki condition $\quad \mathfrak{m}_{12} = 1.225 \pm 0.004 \quad (56\sigma)$

Bell inequality violation

decay	$\mathfrak{m}_{12}$	significance
$J/\psi \to \Lambda\bar{\Lambda}$	1.225 ± 0.004	56.3
$\psi(3686) \to \Lambda\bar{\Lambda}$	1.476 ± 0.100	4.8
$J/\psi \to \Xi^-\bar{\Xi}^+$	1.343 ± 0.018	19.1
$J/\psi \to \Xi^0\bar{\Xi}^0$	1.264 ± 0.017	15.6
$\psi(3686) \to \Xi^-\bar{\Xi}^+$	1.480 ± 0.095	5.1
$\psi(3686) \to \Xi^0\bar{\Xi}^0$	1.442 ± 0.161	2.7
$J/\psi \to \Sigma^-\bar{\Sigma}^+$	1.258 ± 0.007	36.9
$\psi(3686) \to \Sigma^-\bar{\Sigma}^+$	1.465 ± 0.043	10.8
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	1.171 ± 0.007	24.4
$\psi(3686) \to \Sigma^0\bar{\Sigma}^0$	1.663 ± 0.065	10.2

Charmonium spin-2 states

$$\boxed{\chi_c^2 \to \phi\phi}$$

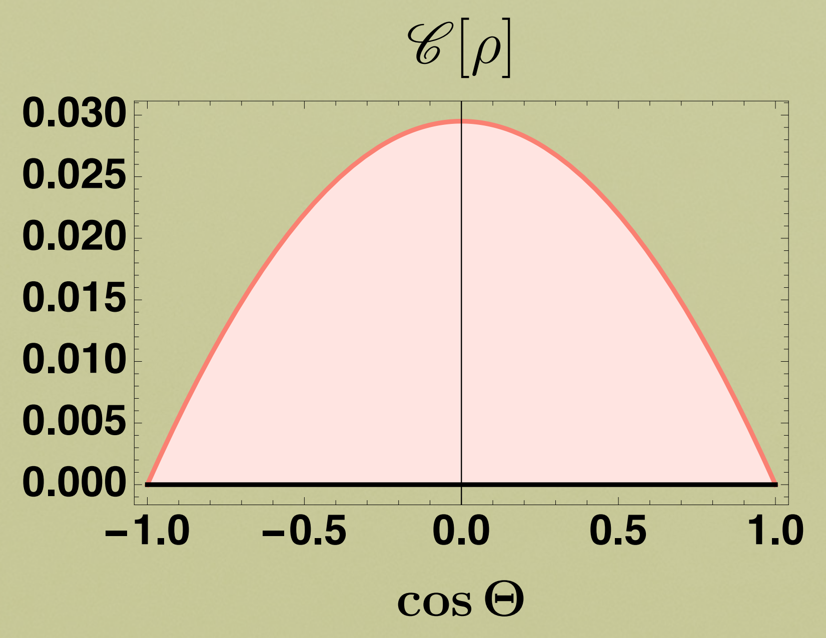
Entanglement $\mathcal{N}(\rho) = 0\,, \quad \text{and} \quad \mathscr{C}_2 = 0\,.$

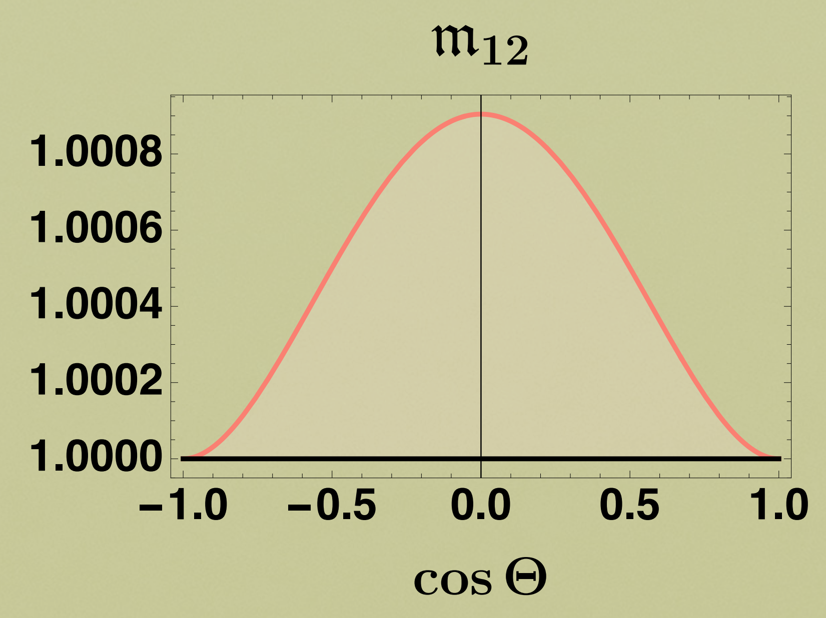
Horodecki condition $\mathrm{Tr}\,\mathscr{B}\rho_{\phi\phi} = 1.202 \pm 0.032$

BESIII Collaboration, M. Ablikim et al., *Helicity amplitude analysis of $\chi_c^J \to \phi\phi$, JHEP* **05** (2023) 069, [arXiv:2301.12922].

Other processes

$$\boxed{e^+ + e^- \to \Lambda + \bar{\Lambda}}$$





$$\mathscr{C}[\rho] = 0.12 \pm 0.11 \quad \text{and} \quad \mathfrak{m}_{12} = 1.01 \pm 0.04\,.$$

BESIII Collaboration, M. Ablikim et al., *Complete Measurement of the Λ Electromagnetic Form Factors, Phys. Rev. Lett.* **123** (2019), no. 12 122003, [arXiv:1903.09421].

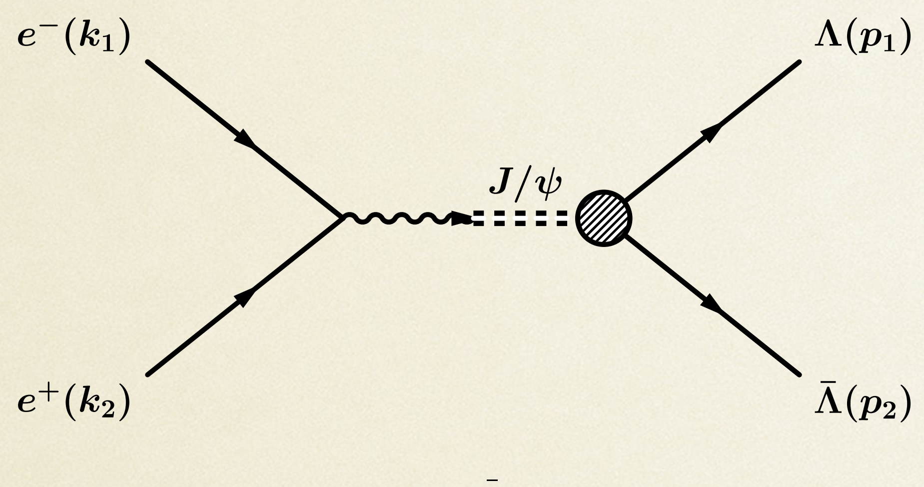
$$\boxed{\Lambda_b \to J/\psi + \Lambda}$$

$$\rho_{\Lambda\, J/\psi} \propto p_\uparrow \,|\psi_\uparrow\rangle\langle\psi_\uparrow| + p_\downarrow\,|\psi_\downarrow\rangle\langle\psi_\downarrow|\,,$$

$$\mathcal{N}(\rho) = 0.05 \pm 0.06\,,$$

ATLAS Collaboration, G. Aad et al., *Measurement of the parity-violating asymmetry parameter α_b and the helicity amplitudes for the decay $\Lambda_b^0 \to J/\psi + \Lambda^0$ with the ATLAS detector, Phys. Rev. D* **89** (2014), no. 9 092009, [arXiv:1404.1071].

Entanglement as a function of the form factors



$$\bar{u}_\Lambda \left[F_1 \gamma^\mu + \frac{1}{2m_\Lambda} \sigma^{\mu\nu} q^\nu F_2 \right] u_\Lambda A_\mu^{J/\psi}$$

$$G_M = F_1 + F_2 \text{ and } G_E = F_1 + s/2m_\Lambda^2 F$$

$$\frac{G_M}{G_E} = \left| \frac{G_M}{G_E} \right| e^{i\Delta\Phi} \quad \text{and} \quad \alpha = \frac{s|G_M|^2 - 4m_\Lambda^2 |G_E|^2}{s|G_M|^2 + 4m_\Lambda^2 |G_E|^2} \, .$$

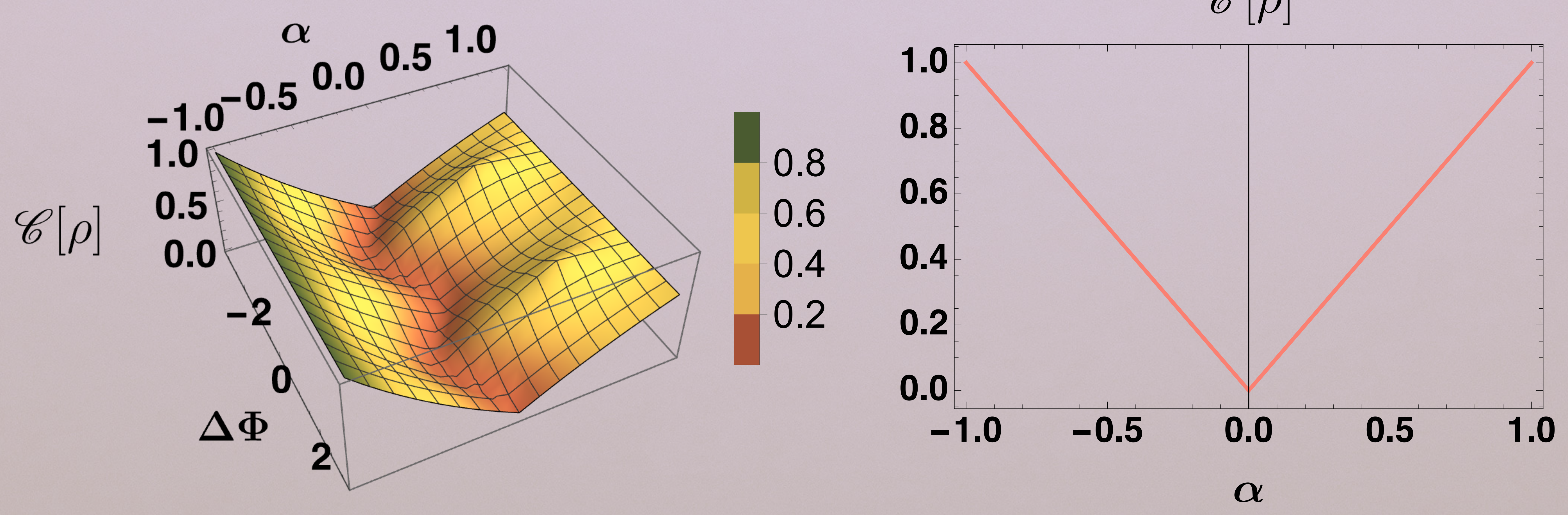


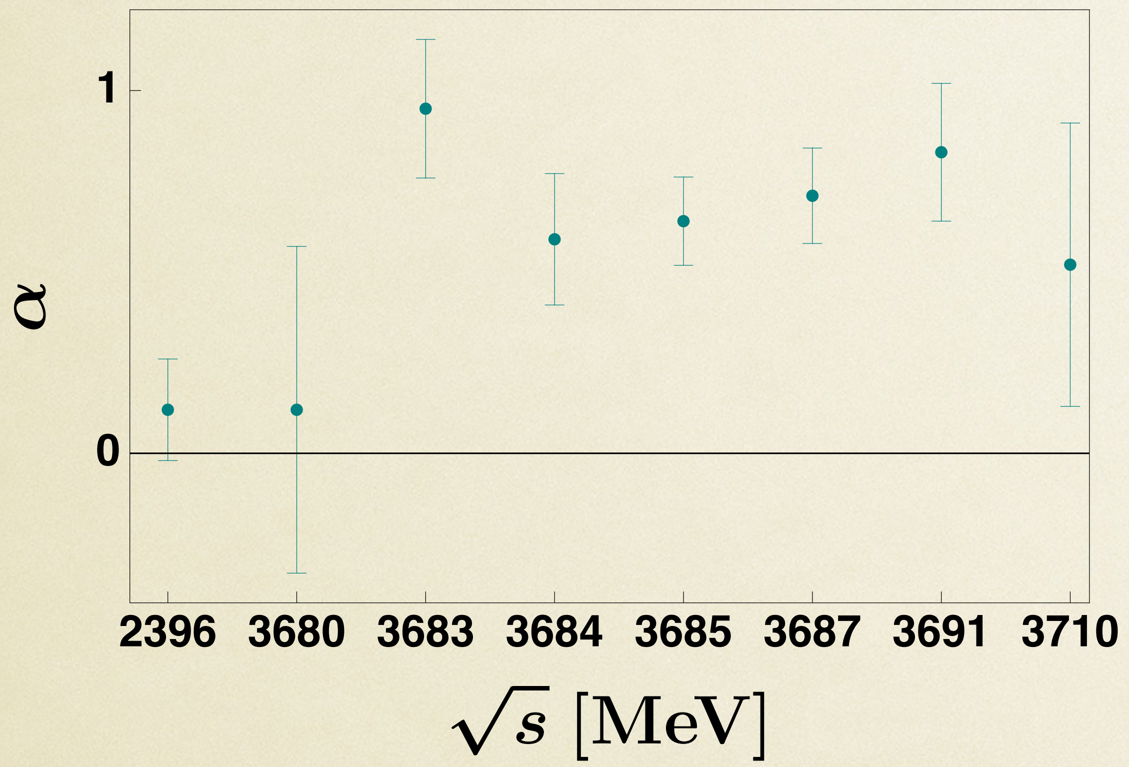
Figure 4.6: Left side: Entanglement as a function of the form factor parameters α and $\Delta\Phi$. The concurrence is computed at $\Theta = \pi/3$. Right side: The concurrence as a function of the parameter α at $\Theta = \pi/2$ is $\mathscr{C}[\rho] = |\alpha|$. There is no dependence on the other parameter $\Delta\Phi$.

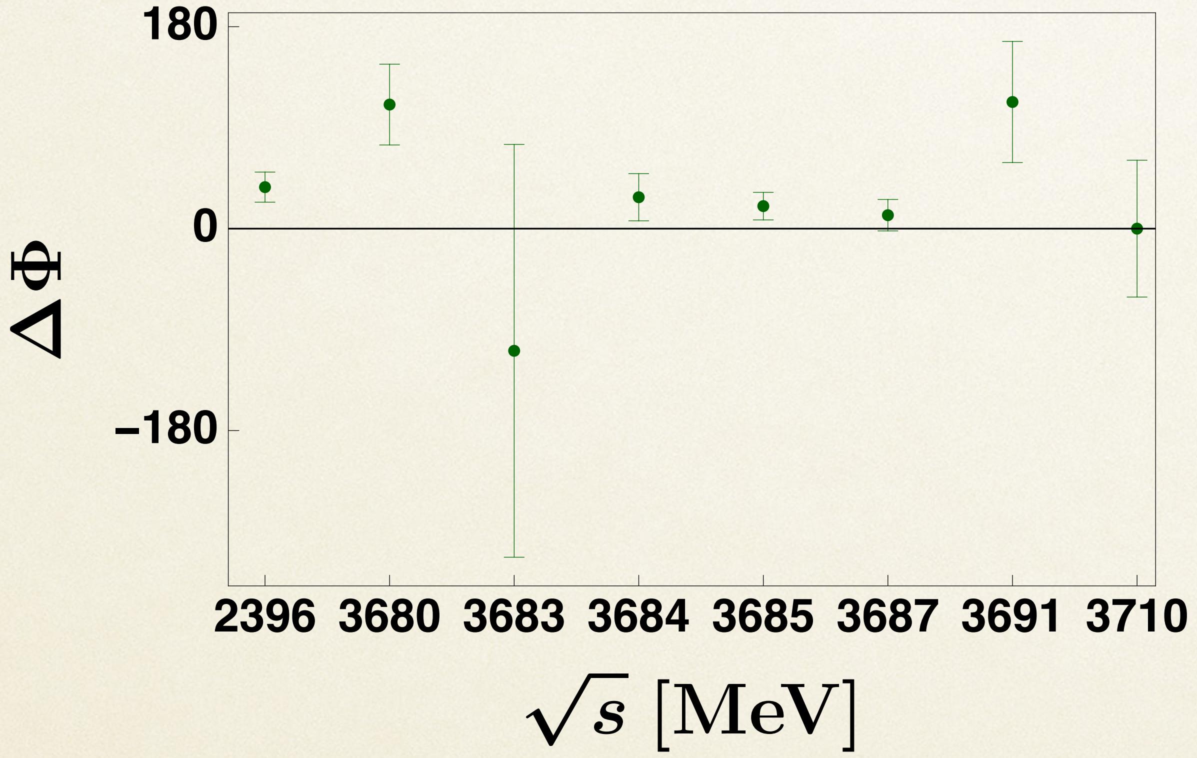
what is still missing

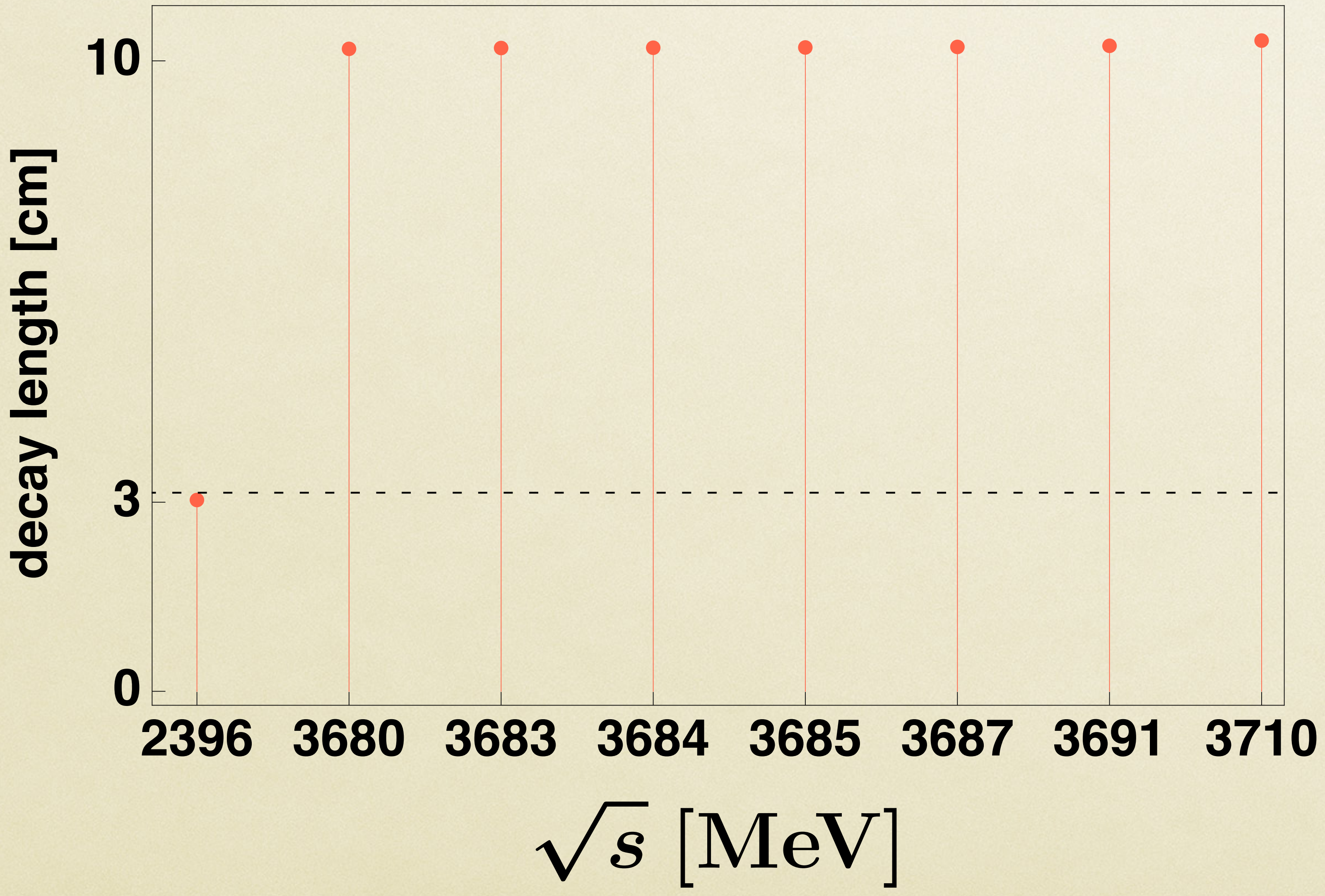
- some amplitudes (e.g. scalar states into hyperons)
- better data (e.g. for spin 2 or direct hyperon production)
- phases of most amplitudes
- uncertainty correlations

Quantum correlations and decoherence

	mass (MeV)	lifetime (s)
Λ	1116	2.6×10^{-10}
$\Sigma^{\pm}$	1189	0.8×10^{-10}
Σ^{0}	1193	7.4×10^{-20}
$\Xi^{\pm}$	1322	2.9×10^{-10}
$\Omega^{\pm}$	1673	0.8×10^{-10}







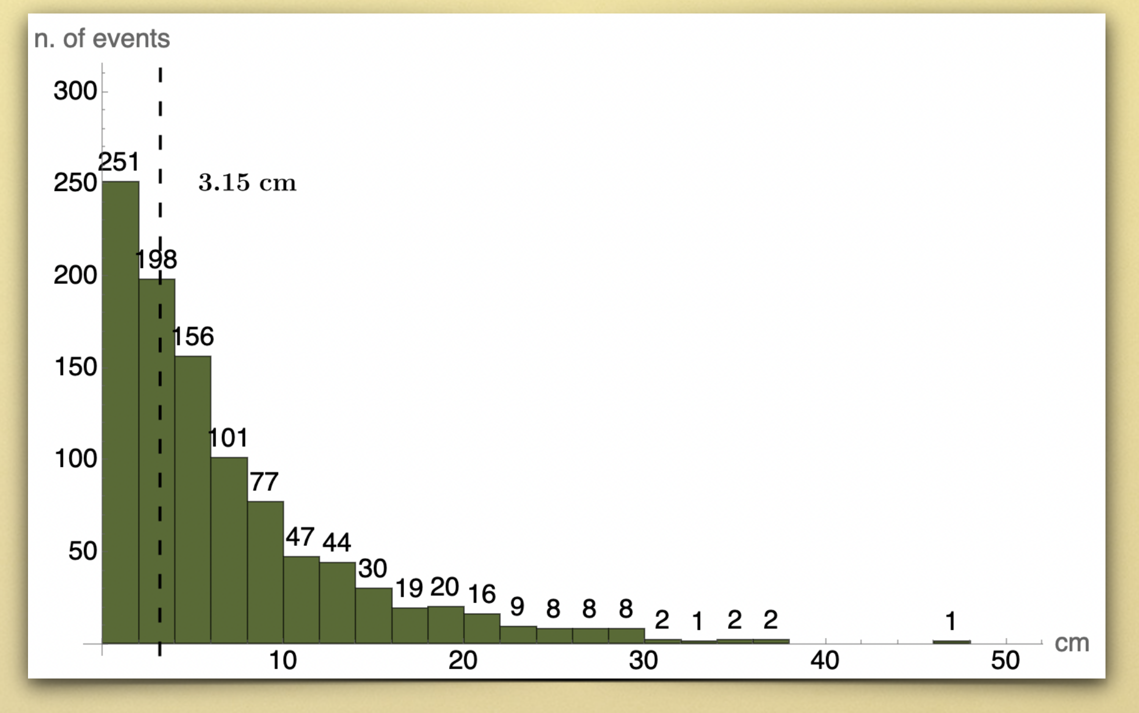


Figure 6.1: Decay $\eta_c \to \Lambda\bar{\Lambda}$: Fraction (out of 1000) of Λ baryons decaying at different lengths from the primary vertex. The Vertical dashed line stands for the inner surface of the beam pipe (3.15 cm from the primary vertex).