The experimental study of $K^0\overline{K}^0$ quantum entanglement at KLOE:

from past to future or from future to past?

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on behalf of the KLOE-2 collaboration
and Jose Bernabeu

Quantum Tests in Collider Physics

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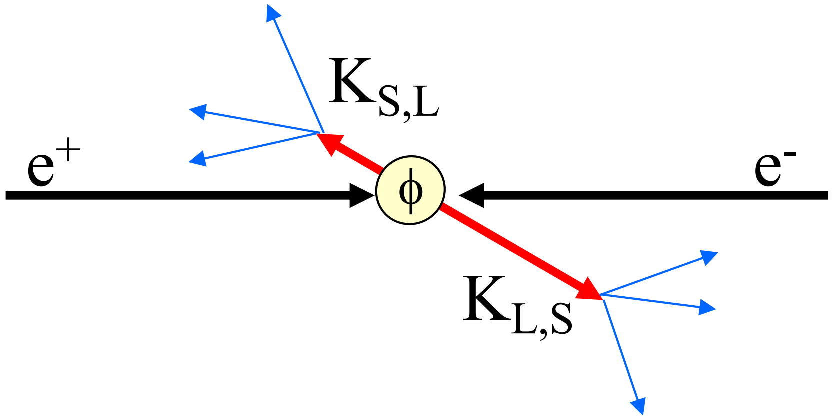
Entangled neutral kaons at a ϕ-factory

Production of the vector meson ϕ in e^+e^- annihilations:

- $e^+e^- \to \phi$ $\sigma_\phi \sim 3\ \mu b$
  $W = m_\phi = 1019.4$ MeV
- $BR(\phi \to K^0\bar{K}^0) \sim 34\%$
- $\sim 10^6$ neutral kaon pairs per pb^{-1} produced in an antisymmetric quantum state with $J^{PC} = 1^{--}$:



$$|i\rangle = \frac{1}{\sqrt{2}}\left[|K^0(\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle|K^0(-\vec{p})\rangle\right]$$

$$= \frac{N}{\sqrt{2}}\left[|K_S(\vec{p})\rangle|K_L(-\vec{p})\rangle - |K_L(\vec{p})\rangle|K_S(-\vec{p})\rangle\right]$$

$$N = \sqrt{\left(1+|\varepsilon_S|^2\right)\left(1+|\varepsilon_L|^2\right)} \Big/ \left(1-\varepsilon_S\varepsilon_L\right) \cong 1$$

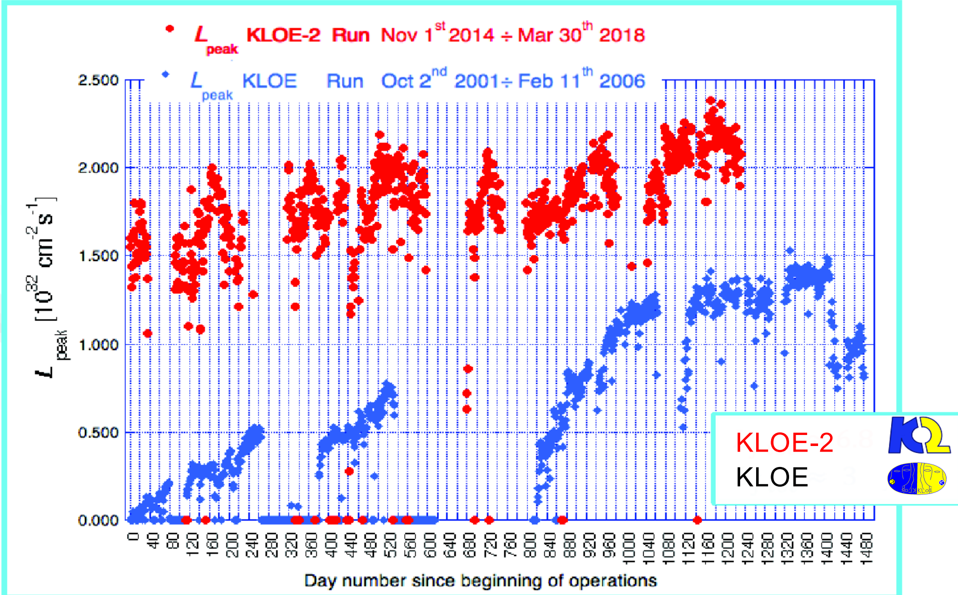
$p_K = 110$ MeV/c
$\lambda_S = 6$ mm $\quad \lambda_L = 3.5$ m

KLOE and KLOE-2 at the Frascati φ-factory DAΦNE

KLOE detector

DAΦNE e^+e^- collider

KLOE-2: L_{int} ~ 5.5 fb^{-1}
KLOE: L_{int} ~ 2.5 fb^{-1}



KLOE + KLOE-2 data sample:

- ~ 8 fb^{-1} ⇒ 2.4 × 10^{10} φ's produced
- ~ 8 x10^9 $K_S K_L$ pairs
- ~ 3 x10^8 η's
- ⇒ the largest sample ever collected at the φ(1020) peak in e^+e^- collisions

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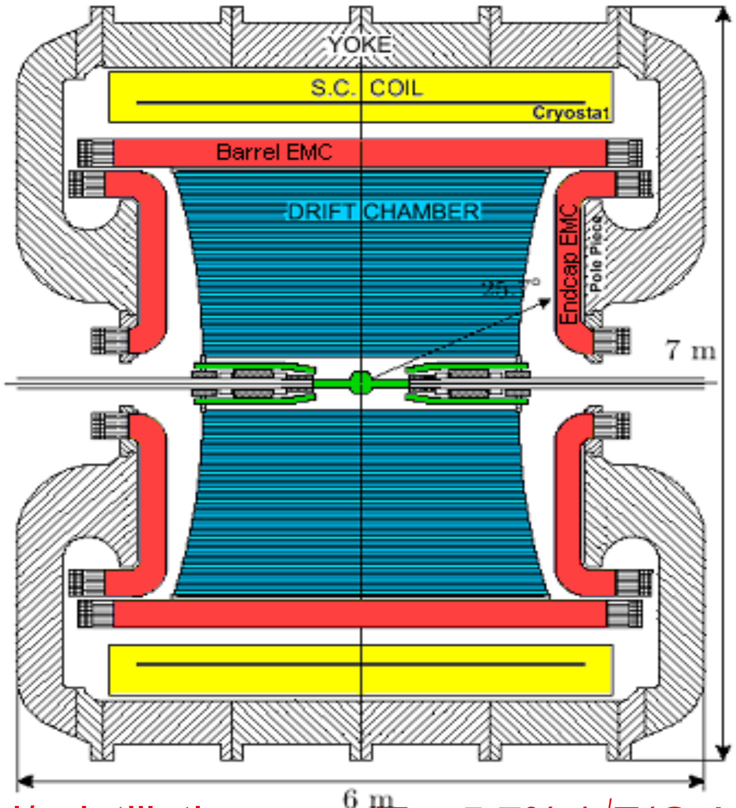
KLOE detector

Superconducting coil B = 0.52 T



Lead/scintillating fiber calorimeter $\sigma_E/E \cong 5.7\% / \sqrt{E(\mathrm{GeV})}$

$\sigma_t \cong 54\ \mathrm{ps} / \sqrt{E(\mathrm{GeV})} \oplus 50\ \mathrm{ps}$

drift chamber; 4 m diameter × 3.3 m length
90% He - 10% isobutane gas mixture

$\sigma(p_\perp)/p_\perp \simeq 0.4\ \%$ $\sigma_{xy} \simeq 150\ \mu\mathrm{m}$ $\sigma_z \simeq 2\ \mathrm{mm}$

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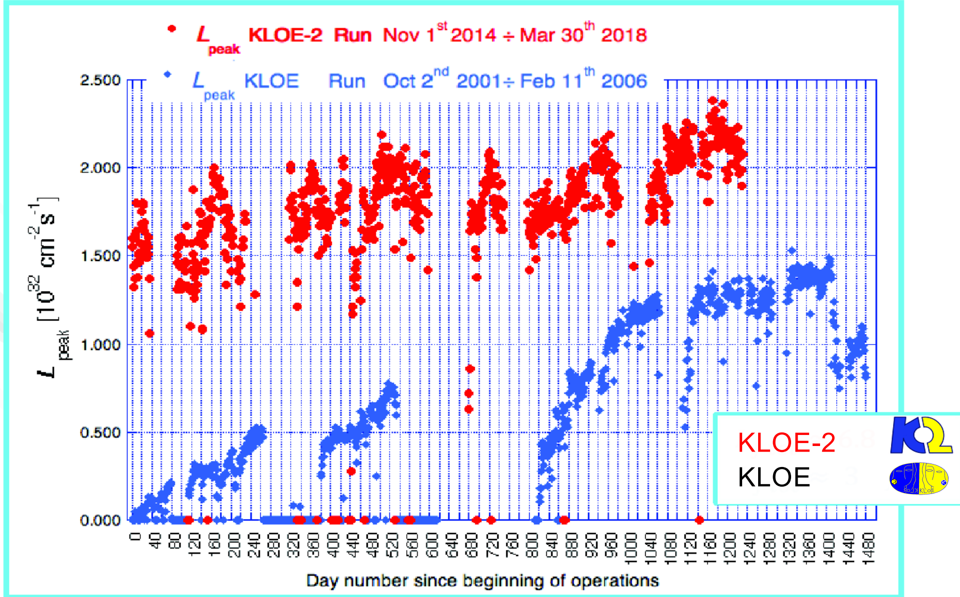

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EPR correlations in entangled neutral kaons

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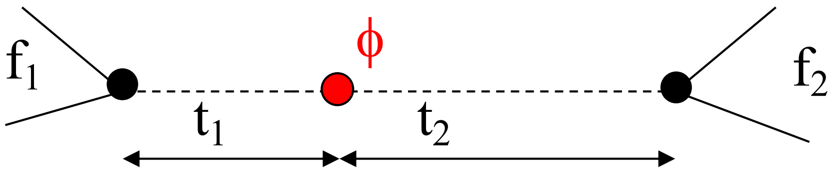

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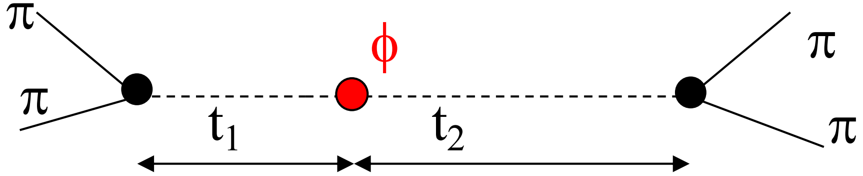

Same final state for both kaons: $f_1 = f_2 = \pi^+\pi^-$
(this specific channel is suppressed by CP viol.
$|\eta_{+-}|^2 = |A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)|^2 \sim |\varepsilon|^2 \sim 10^{-6}$)

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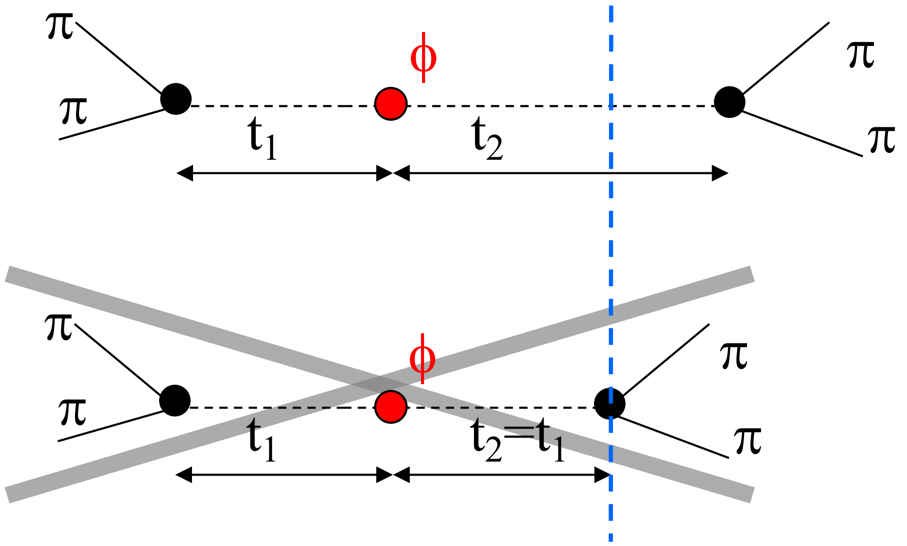

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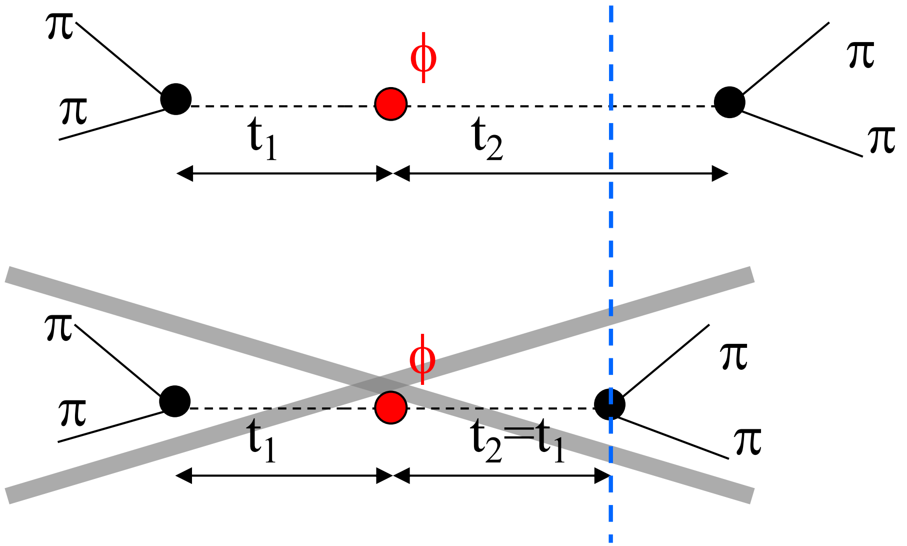

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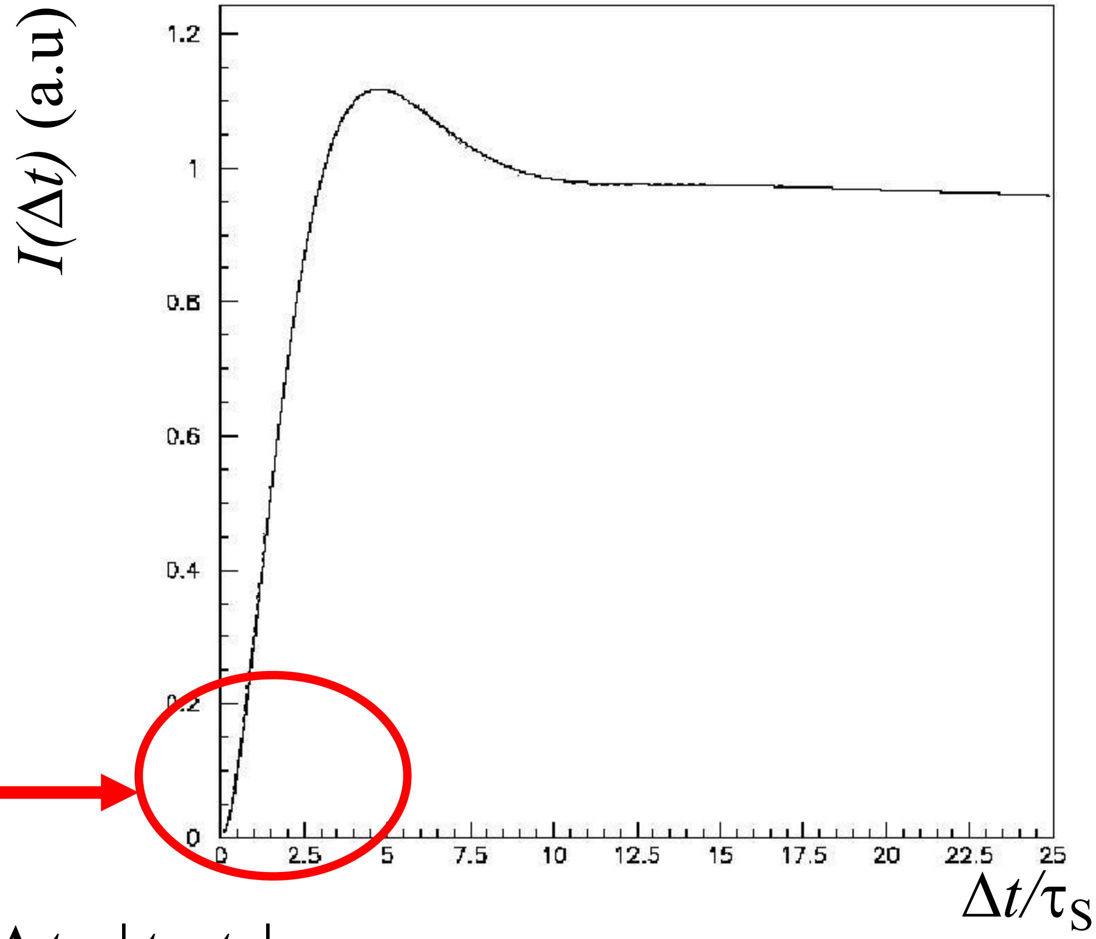

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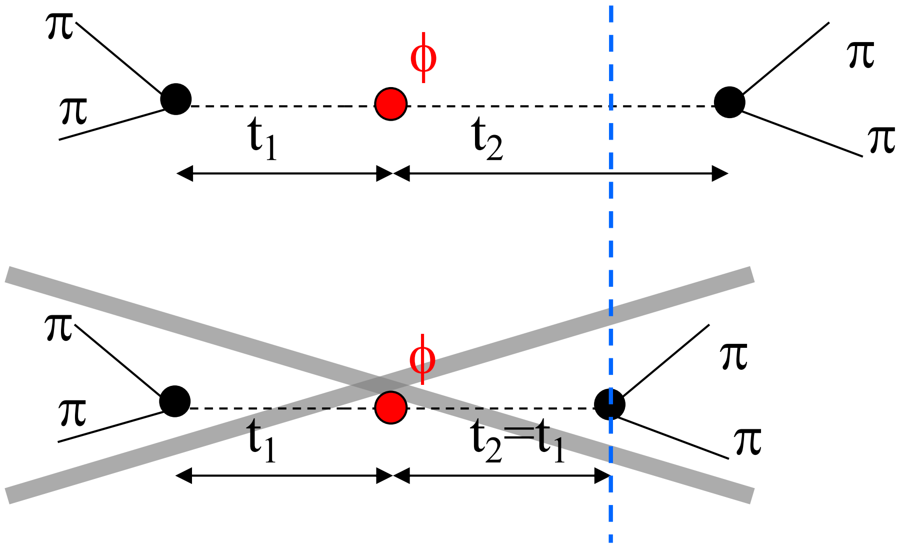

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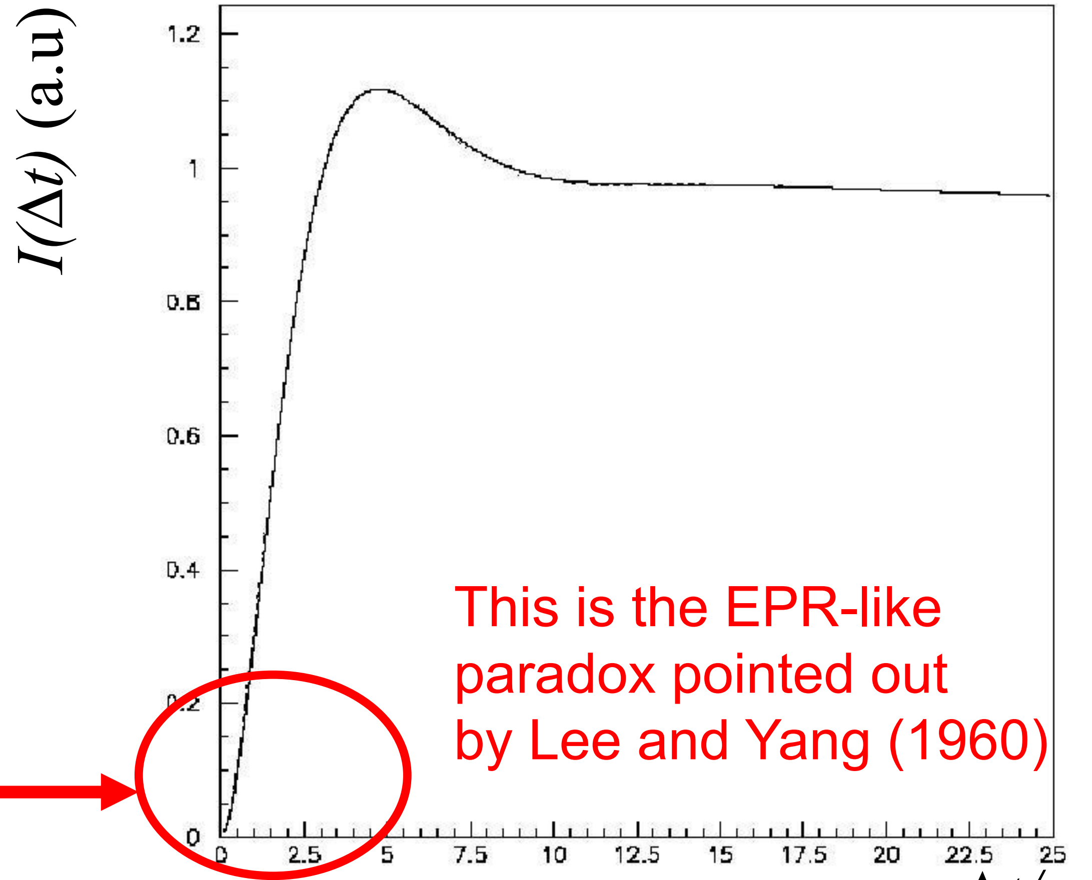

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$\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^- \ \pi^+\pi^-$: test of quantum coherence

$$|i\rangle = \frac{1}{\sqrt{2}}\left[\left|K^0\right\rangle\left|\overline{K}^0\right\rangle - \left|\overline{K}^0\right\rangle\left|K^0\right\rangle\right]$$

The EPR correlation suggested a simple test of quantum coherence

$$I\left(\pi^+\pi^-,\pi^+\pi^-;\Delta t\right) = \frac{N}{2}\left[\left|\left\langle\pi^+\pi^-,\pi^+\pi^-\left|K^0\overline{K}^0(\Delta t)\right.\right\rangle\right|^2 + \left|\left\langle\pi^+\pi^-,\pi^+\pi^-\left|\overline{K}^0K^0(\Delta t)\right.\right\rangle\right|^2\right.$$
$$\left. -2\Re\left(\left\langle\pi^+\pi^-,\pi^+\pi^-\left|K^0\overline{K}^0(\Delta t)\right.\right\rangle\left\langle\pi^+\pi^-,\pi^+\pi^-\left|\overline{K}^0K^0(\Delta t)\right.\right\rangle^*\right)\right]$$

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Decoherence parameter:

$\zeta_{0\bar{0}} = 0 \quad \rightarrow \quad$ QM

$\zeta_{0\bar{0}} = 1 \quad \rightarrow \quad$ total decoherence

(also known as Furry's hypothesis or spontaneous factorization)
W.Furry, PR 49 (1936) 393

Bertlmann, Grimus, Hiesmayr PR D60 (1999) 114032
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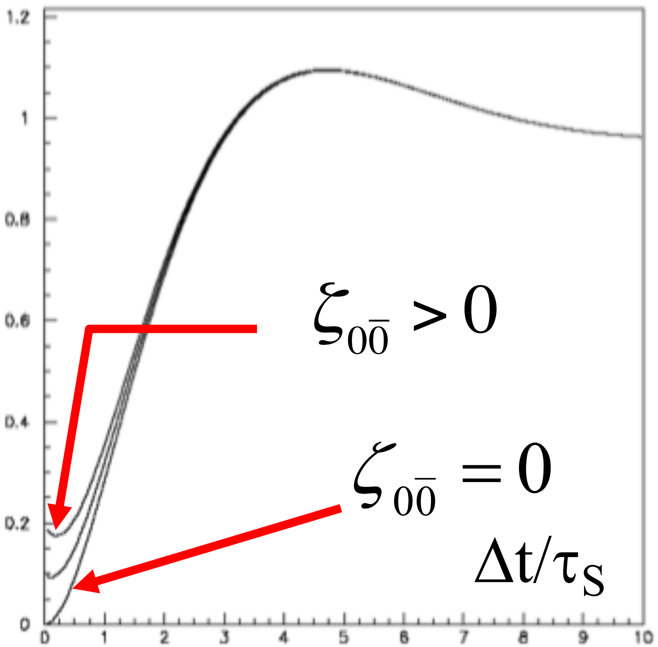
I(Δt) (a.u.)



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KLOE-2 JHEP 04 (2022) 059

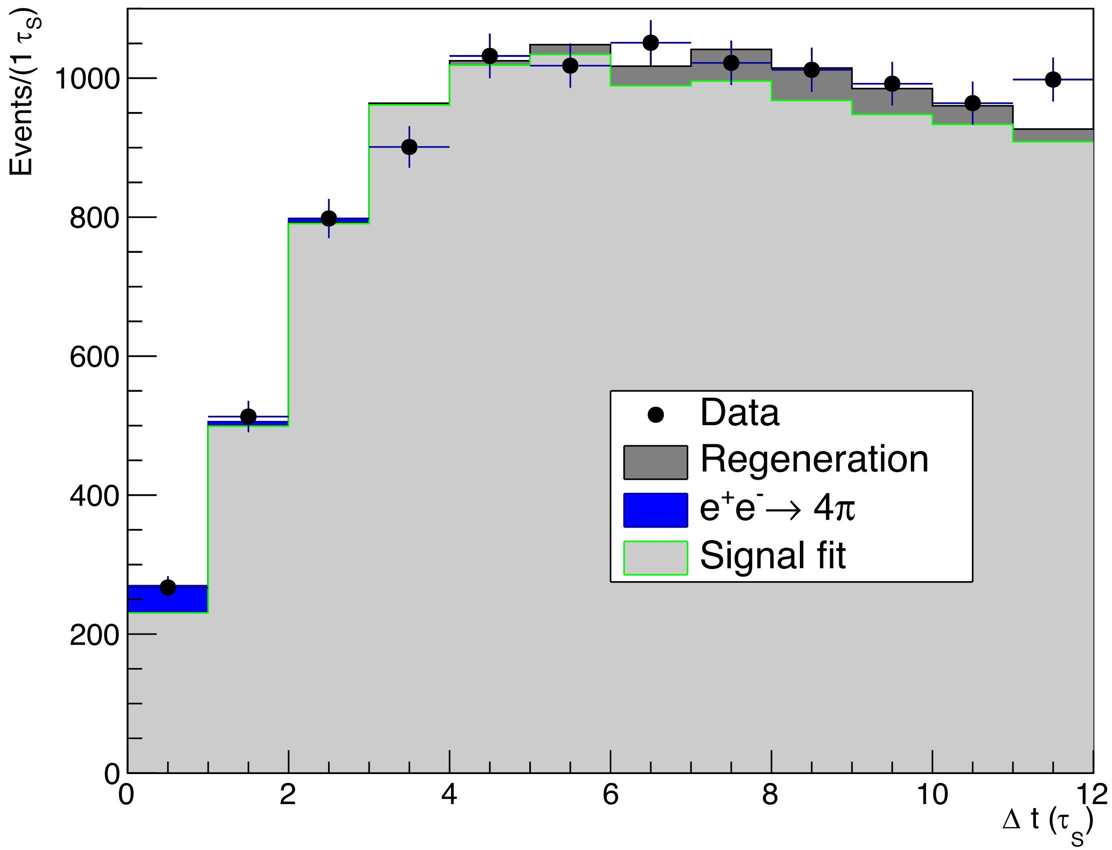
$$\zeta_{0\bar{0}} = \left(-0.5 \pm 8.0_{stat} \pm 3.7_{syst}\right)\times 10^{-7}$$

CP violating process:

terms $\zeta_{00}/|\eta_{+-}|^2$ with $|\eta_{+-}|^2 \sim |\varepsilon|^2 \sim 10^{-6}$

=> high sensitivity to ζ_{00} ;

CP violation in kaon mixing acts as amplification mechanism



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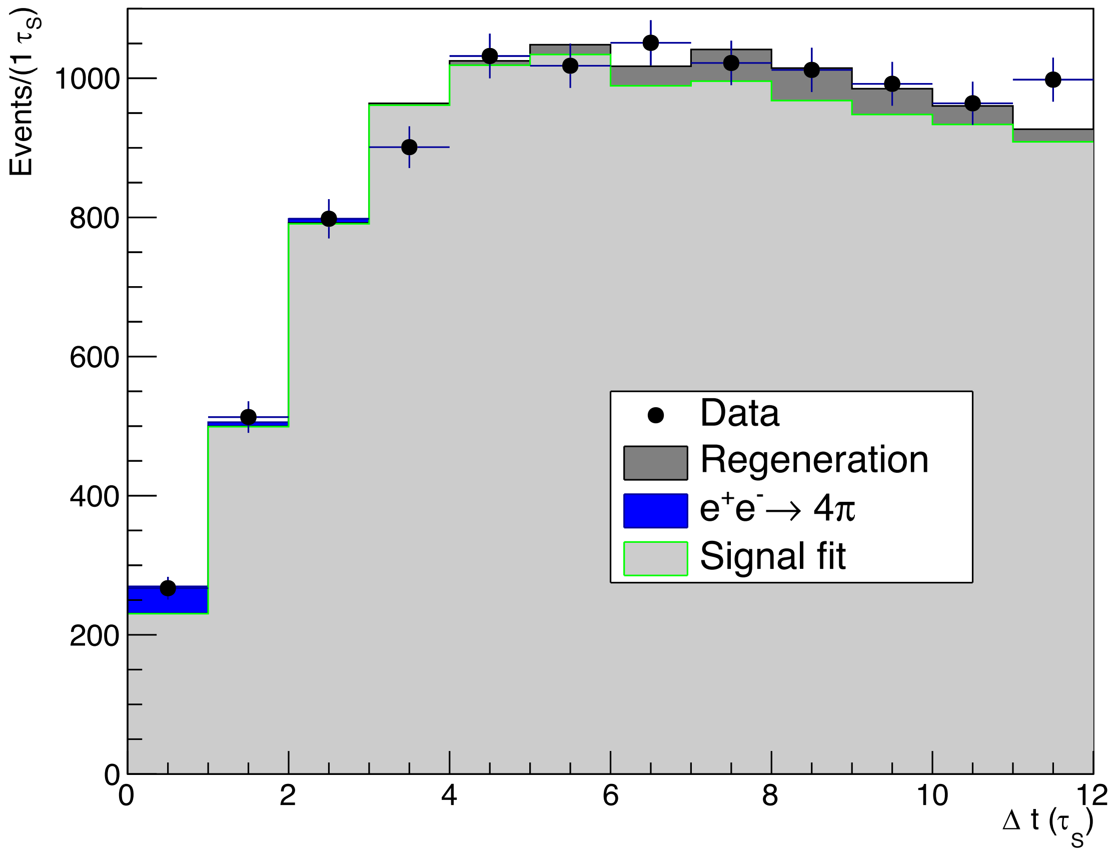
$$\zeta^{B}_{0\bar{0}} = 0.029 \pm 0.057$$



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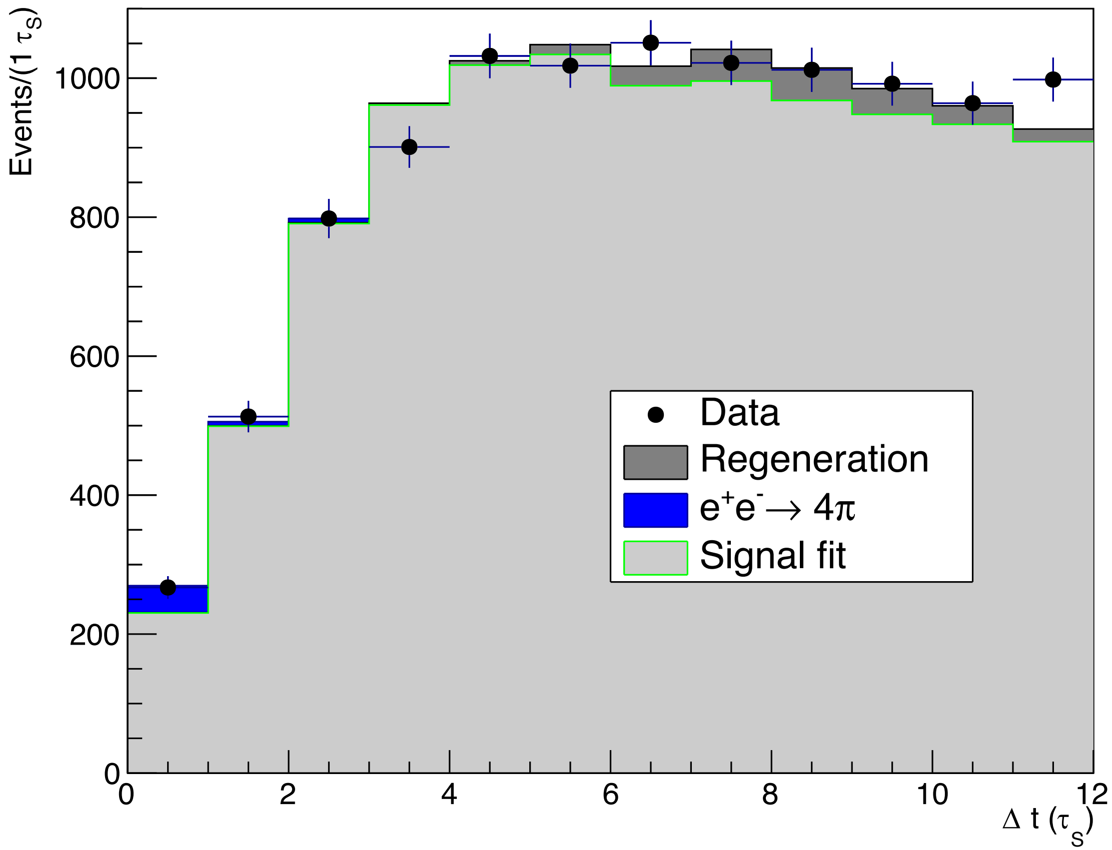


Possible decoherence due quantum gravity effects (apparent loss of unitarity) implying also **CPT** violation => modified Liouville – von Neumann equation for the density matrix of the kaon system depends on a CPTV parameter γ [J. Ellis et al. PRD53 (1996) 3846]

In this scenario γ can be at most:

$$O(m_K^2/M_{PLANCK}) = 2\times10^{-20}\ GeV$$

KLOE-2 result

$$\gamma = (1.3 \pm 9.4_{stat} \pm 4.2_{syst}) \times 10^{-22}\ \text{GeV}$$

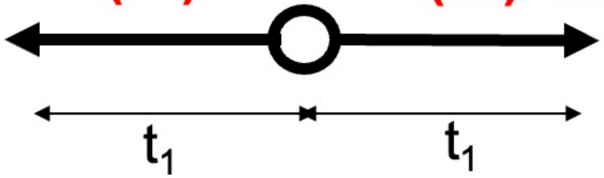
From past to future

Entanglement as a tool for discrete symmetries tests

Entangled neutral kaons

In QM the entangled state can be expressed in any base:

Flavor basis $\bar{K}^0$ (K^0) ϕ $(\bar{K}^0)$ K^0 $|i\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle]$



CP states basis (with CP=±1) K_- (K_+) (K_-) K_+ $|i\rangle = \frac{1}{\sqrt{2}}[|K_+\rangle|K_-\rangle - |K_-\rangle|K_+\rangle]$

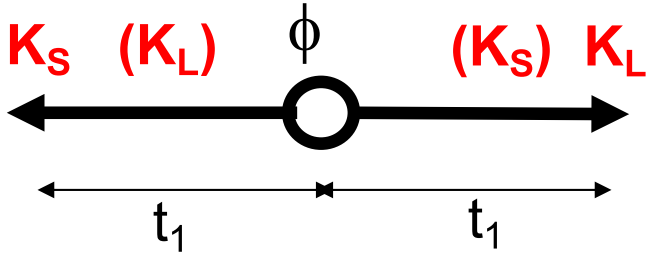
Physical states basis (non-orthogonal basis) K_L (K_S) (K_L) K_S $|i\rangle = \frac{\mathcal{N}}{\sqrt{2}}[|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle]$

The orthogonal basis $< K^{\perp}_{\nrightarrow f}|K_{\nrightarrow f} >= 0$ $\mathbf{K_{\nrightarrow f}(K^{\perp}_{\nrightarrow f})}$ $\mathbf{K^{\perp}_{\nrightarrow f}(K_{\nrightarrow f})}$ $|i\rangle = \frac{1}{\sqrt{2}}\{|K^{\perp}_{\nrightarrow f}\rangle|K_{\nrightarrow f}\rangle - |K_{\nrightarrow f}\rangle|K^{\perp}_{\nrightarrow f}\rangle\}$

with $|\mathrm{K}_{\nrightarrow f}\rangle = \mathcal{N}_{\nrightarrow f}\,[|\mathrm{K_L}\rangle - \eta_f|\mathrm{K_S}\rangle]$ implying $\langle f|T|\mathrm{K}_{\nrightarrow f}\rangle = 0$ $\eta_i \equiv |\eta_i|e^{i\phi_i} = \frac{\langle f_i|T|K_L\rangle}{\langle f_i|T|K_S\rangle}$

- In maximally entangled systems the complete knowledge of the system as a whole is encoded in the entangled state, the single subsystems are undefined.
- When the decay measurement to *f* is performed, the partner is instantaneously informed and tagged as $K_{\nrightarrow f}$ and the decay filters (projects) its orthogonal for the decayed meson.

Entangled neutral kaons



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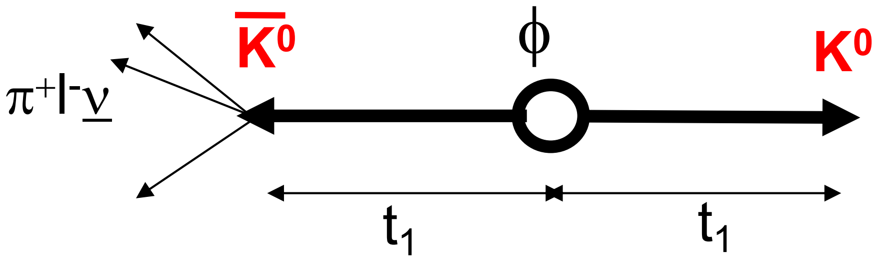

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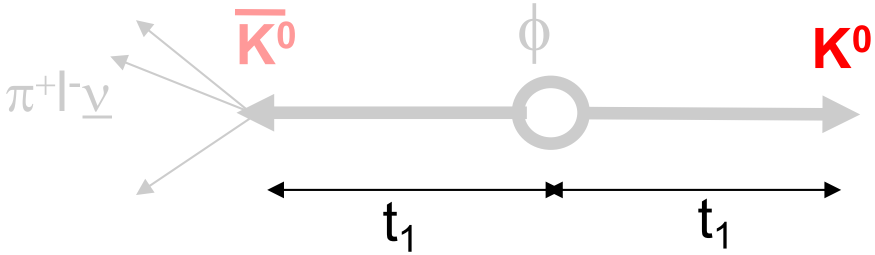

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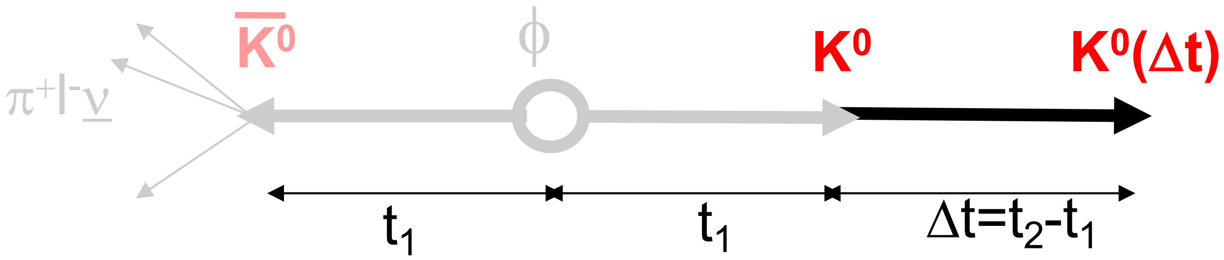

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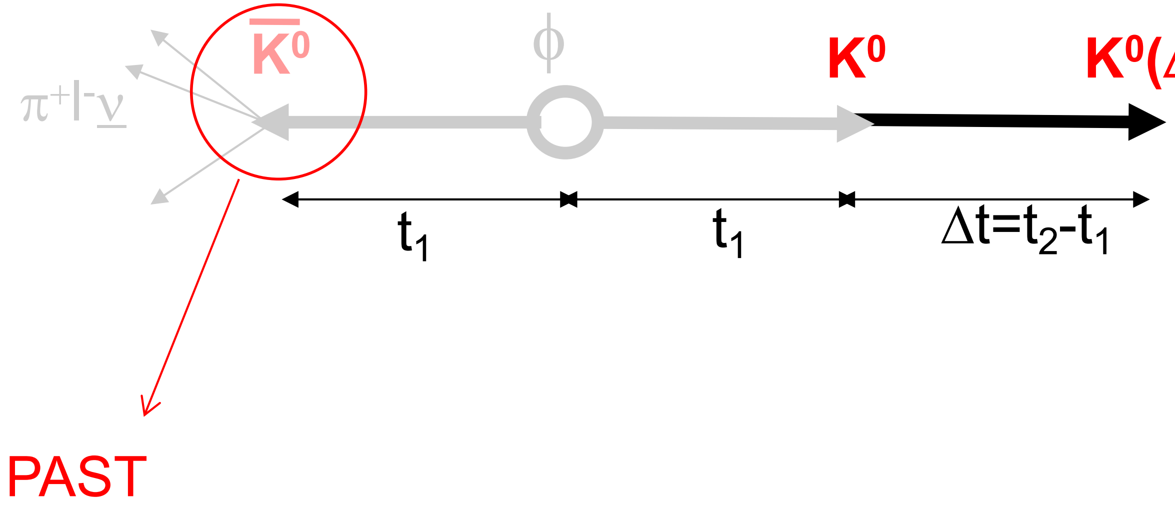

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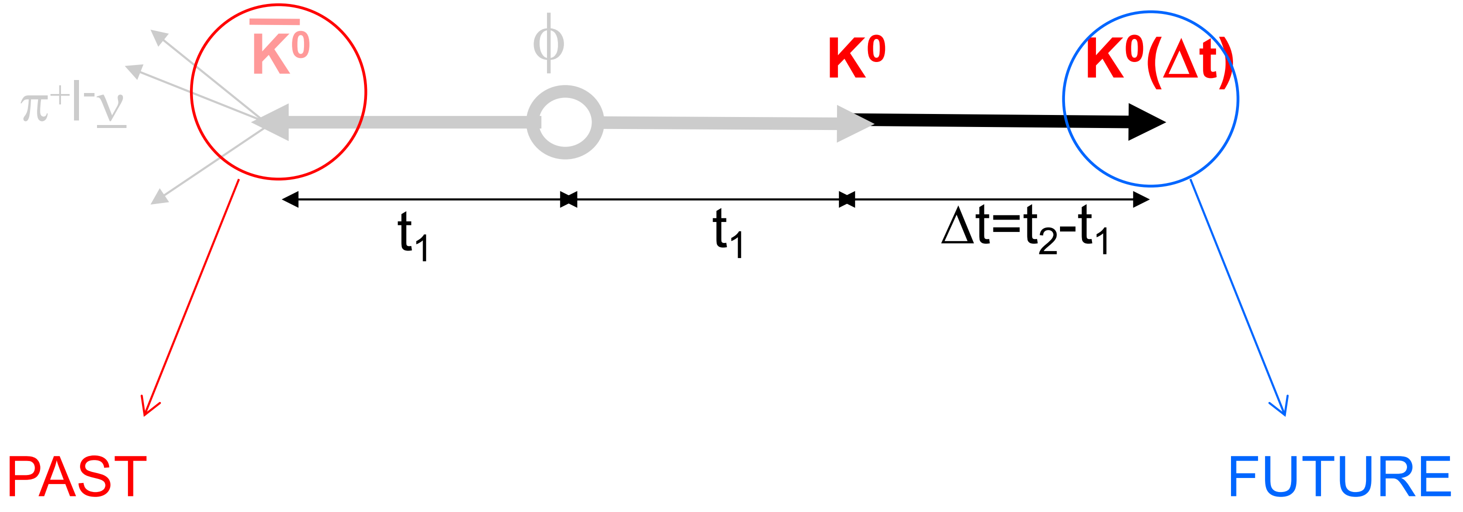

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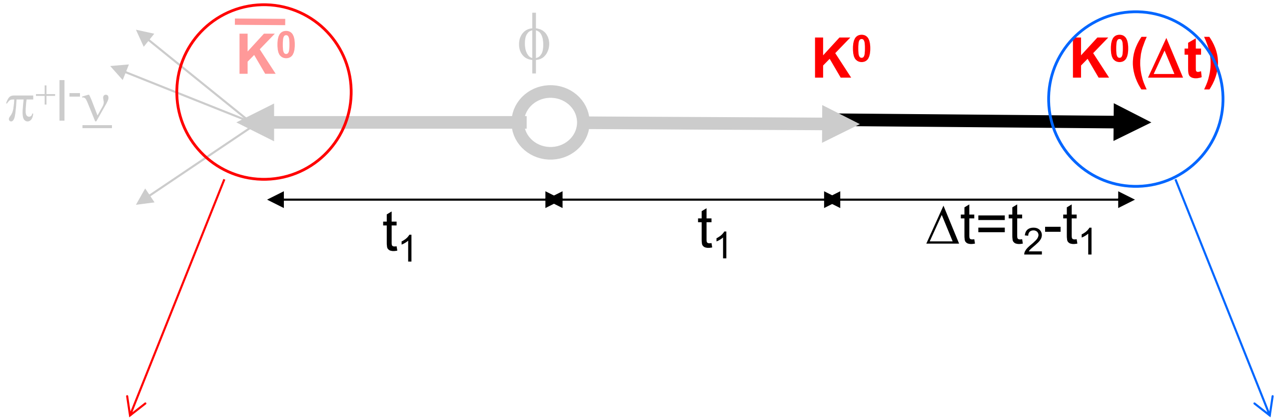

PAST

FUTURE

The past (kaon decay at t_1) tags the future partner kaon state at t_2 before its decay;

THE RELEVANT TIME DEPENDENCE HERE IS IN $\Delta t = t_2 - t_1$

i.e. from the preparation of the tagged state until its decay

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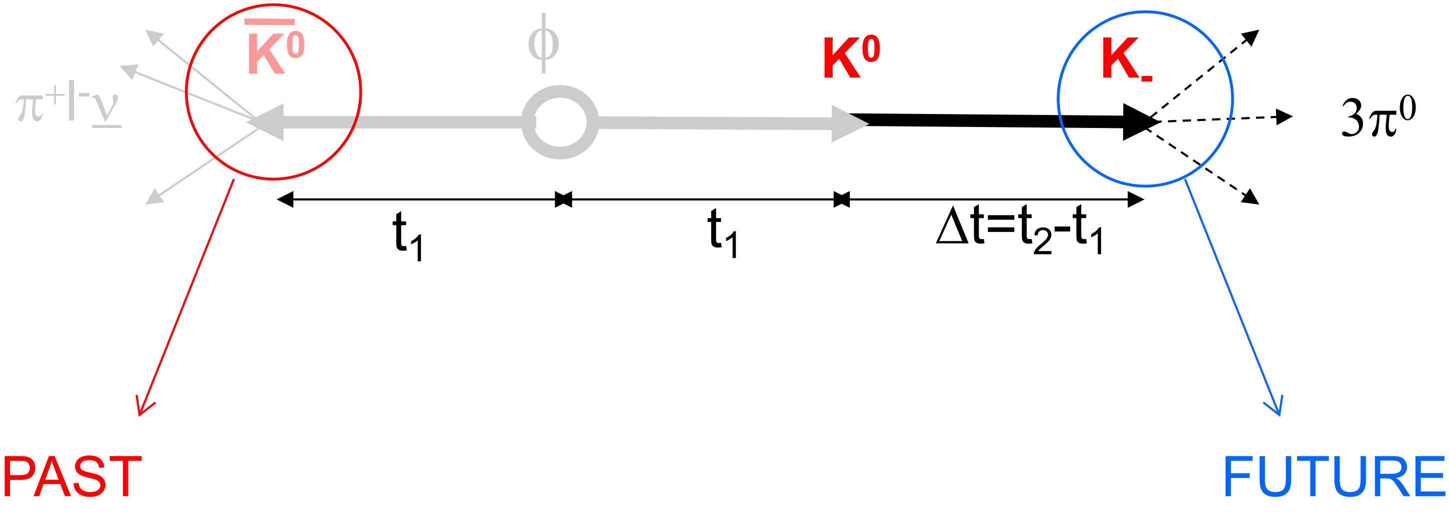

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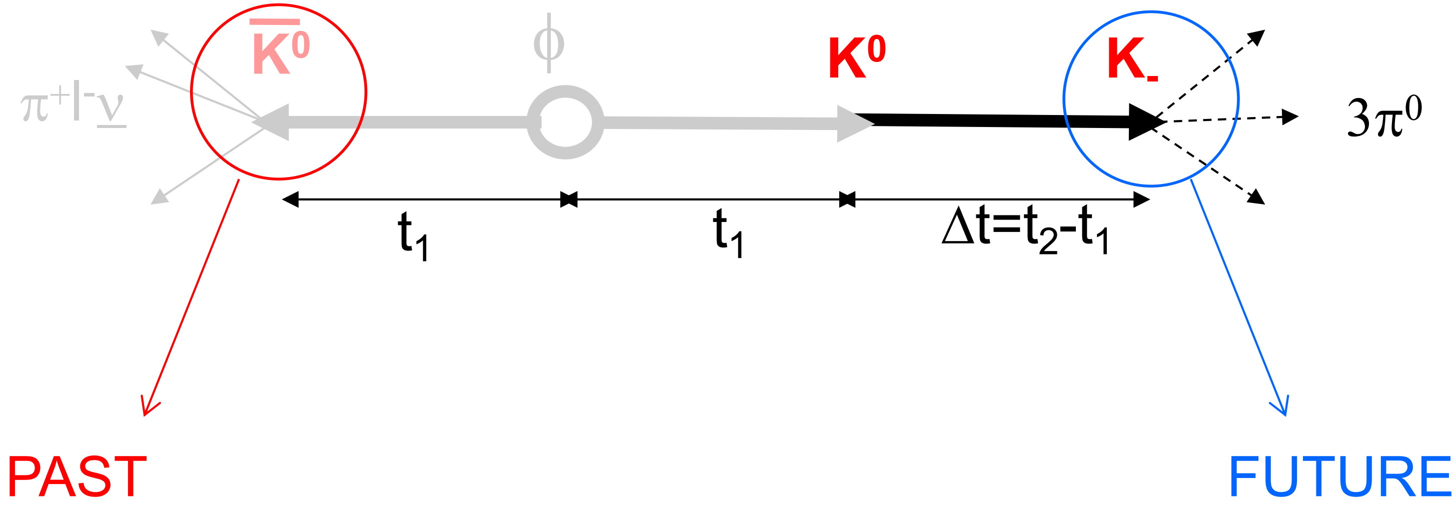
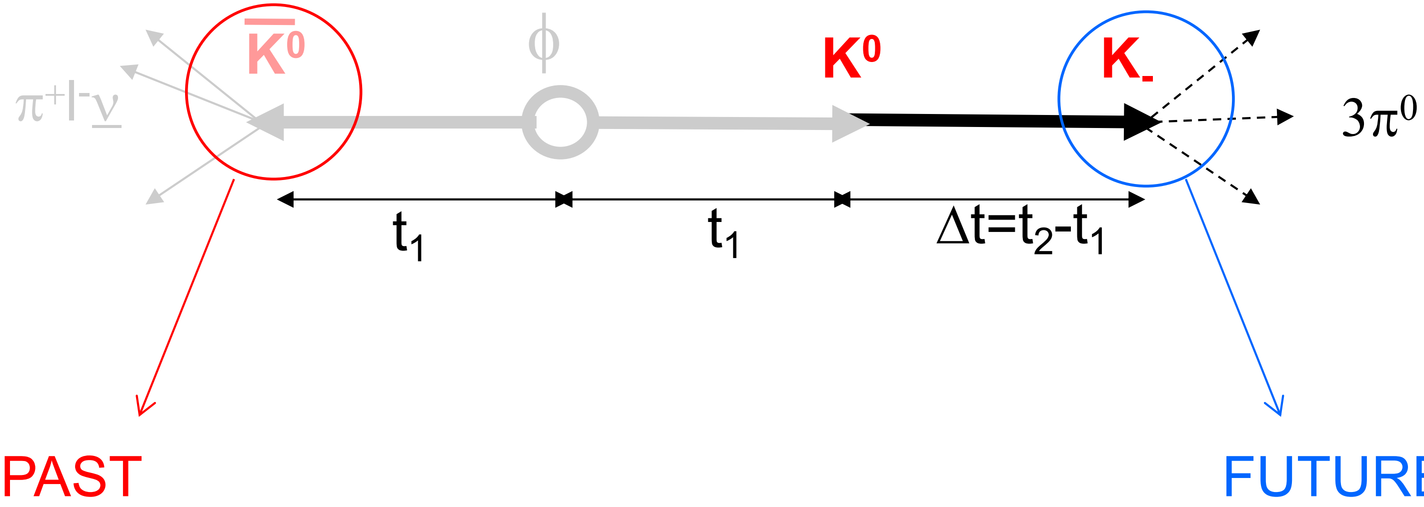


Double decay intensity calculation



QM calculation of double decay intensity: two alternatives

(I) Time History approach (TH), from past to future

(1) The time evolution of the state $|i\rangle$ from time $t = 0$ to time $t = t_1$, with definite total width Γ;

(2) The projection of the state $|i(t = t_1)\rangle$ onto the orthogonal pair $|K^{\perp}_{\nrightarrow f_1}\rangle |K_{\nrightarrow f_1}\rangle$, filtered by the decay f_1, times the decay amplitude of the state $|K^{\perp}_{\nrightarrow f_1}\rangle$ into the f_1 channel;

(3) The time evolution of the surviving (single) kaon state $|K_{\nrightarrow f_1}\rangle$ from time $t = t_1$ to time $t = t_2$;

(4) The projection at time $t = t_2$ of the evolved state $|K_{\nrightarrow f_1}(\Delta t)\rangle$ onto the state $|K^{\perp}_{\nrightarrow f_2}\rangle$ filtered by the decay f_2, times the decay amplitude of the state $|K^{\perp}_{\nrightarrow f_2}\rangle$ into the f_2 channel.

$$I(f_1, t_1; f_2, t_2)_{\mathrm{TH}} = \left| \underbrace{\langle f_2|T|\mathrm{K}^{\perp}_{\nrightarrow f_2}\rangle}_{(4)} \underbrace{\langle \mathrm{K}^{\perp}_{\nrightarrow f_2}|\mathrm{K}_{\nrightarrow f_1}(\Delta t)\rangle}_{(3)} \underbrace{\langle f_1|T|\mathrm{K}^{\perp}_{\nrightarrow f_1}\rangle}_{(2)} \underbrace{\langle \mathrm{K}^{\perp}_{\nrightarrow f_1}\mathrm{K}_{\nrightarrow f_1}|i(t = t_1)\rangle}_{(1)} \right|^2$$

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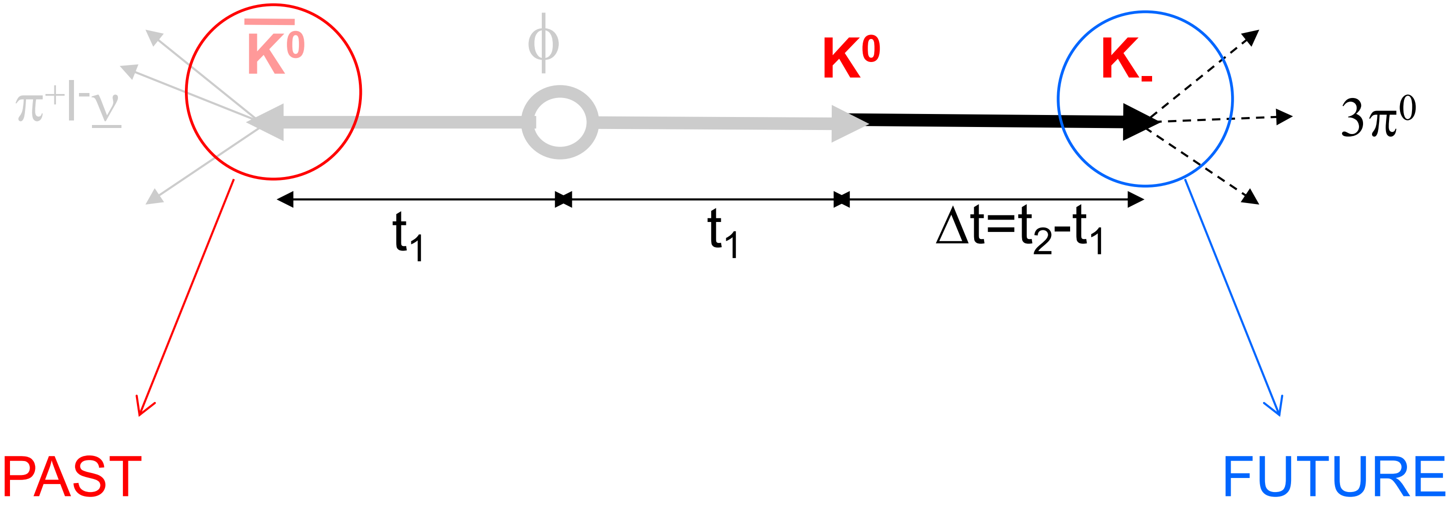

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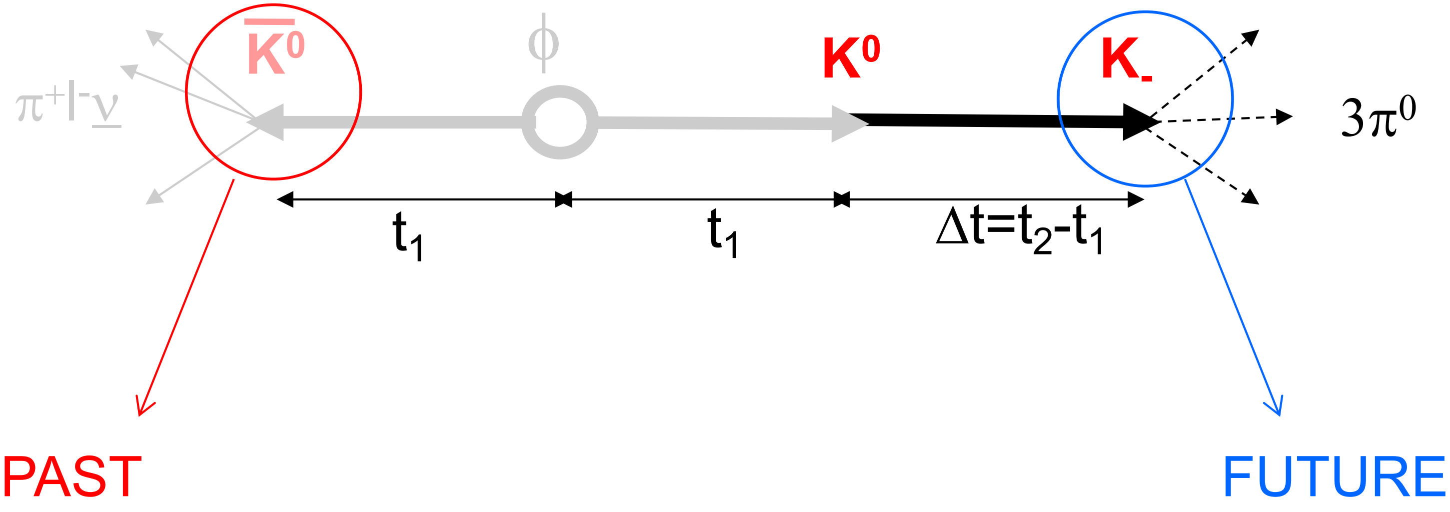
$$I(\pi^+\ell^-\bar{\nu}, t_1; 3\pi^0, t_2)_{TH} = |\langle 3\pi^0|T|K_-\rangle\langle K_-|K^0(\Delta t)\rangle\langle \pi^+\ell^-\bar{\nu}|T|\overline{K}^0\rangle\langle \overline{K}^0 K^0|i(t_1)\rangle|^2$$

(4) (3) (2) (1)

Double decay intensity calculation



QM calculation of double decay intensity: two alternatives

(I) Time History approach (TH), from past to future

(1) The time evolution of the state $|i\rangle$ from time $t = 0$ to time $t = t_1$, with definite total width Γ;

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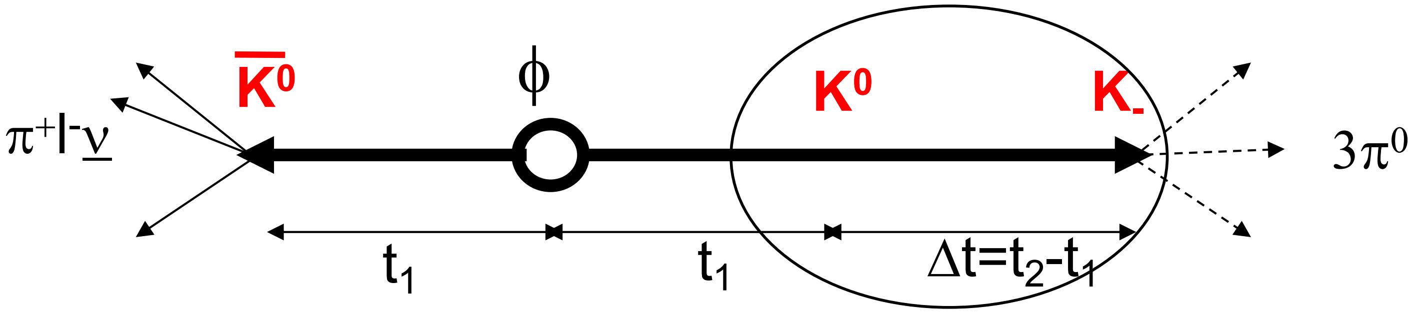
$$K^0 \to K_-$$

$$I(\pi^+\ell^-\bar{\nu}, t_1; 3\pi^0, t_2)_{TH} = |\langle 3\pi^0|T|K_-\rangle\langle K_-|K^0(\Delta t)\rangle\langle \pi^+\ell^-\bar{\nu}|T|\overline{K}^0\rangle\langle \overline{K}^0 K^0|i(t_1)\rangle|^2$$

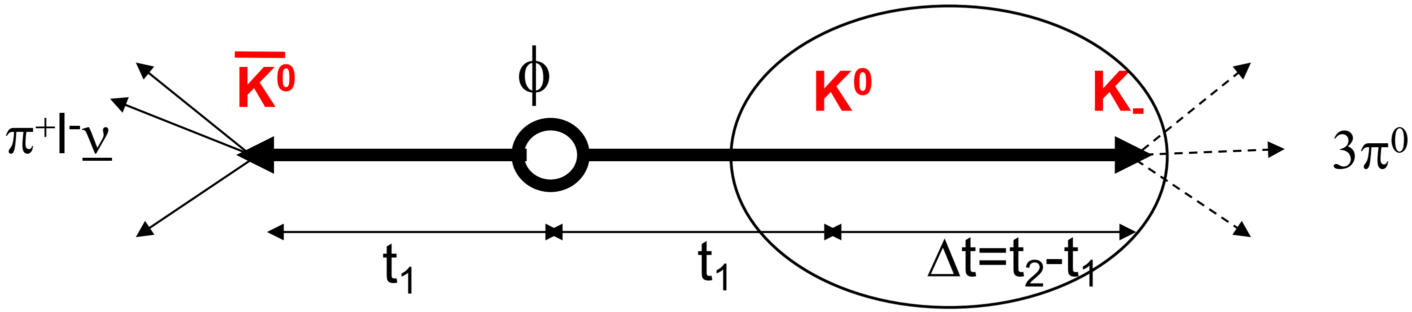
(4) (3) (2) (1)

T,CP, CPT tests in transitions



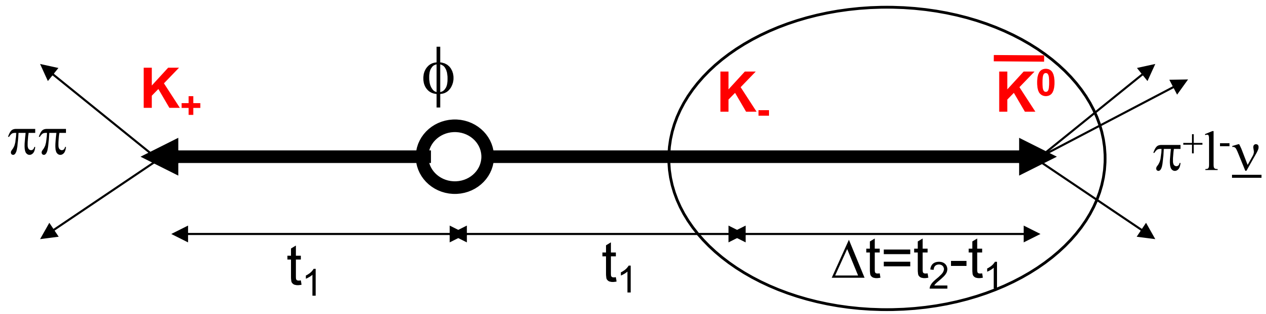
$K^0 \to K_-$ reference process

T,CP, CPT tests in transitions



$K^0 \rightarrow K_-$ reference process

$K_- \rightarrow \overline{K}^0$ CPT-conjugated process



Direct test of CPT symmetry in neutral kaon transitions

CPT symmetry test

Reference		$\mathcal{CPT}$-conjugate	
Transition	Decay products	Transition	Decay products
$K^0 \to K_+$	$(\ell^-, \pi\pi)$	$K_+ \to \bar{K}^0$	$(3\pi^0, \ell^-)$
$K^0 \to K_-$	$(\ell^-, 3\pi^0)$	$K_- \to \bar{K}^0$	$(\pi\pi, \ell^-)$
$\bar{K}^0 \to K_+$	$(\ell^+, \pi\pi)$	$K_+ \to K^0$	$(3\pi^0, \ell^+)$
$\bar{K}^0 \to K_-$	$(\ell^+, 3\pi^0)$	$K_- \to K^0$	$(\pi\pi, \ell^+)$

One can define the following ratios of probabilities:

$$R_{1,\mathcal{CPT}}(\Delta t) = P\left[K_+(0) \to \bar{K}^0(\Delta t)\right] / P\left[K^0(0) \to K_+(\Delta t)\right]$$
$$R_{2,\mathcal{CPT}}(\Delta t) = P\left[K^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to \bar{K}^0(\Delta t)\right]$$
$$R_{3,\mathcal{CPT}}(\Delta t) = P\left[K_+(0) \to K^0(\Delta t)\right] / P\left[\bar{K}^0(0) \to K_+(\Delta t)\right]$$
$$R_{4,\mathcal{CPT}}(\Delta t) = P\left[\bar{K}^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to K^0(\Delta t)\right]$$

Any deviation from $R_{i,CPT}$=1 constitutes a violation of CPT-symmetry

J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139

Direct test of T symmetry in neutral kaon transitions

T symmetry test

Reference		T-conjugate	
Transition	Final state	Transition	Final state
$\bar{K}^0 \to K_-$	$(\ell^+, \pi^0\pi^0\pi^0)$	$K_- \to \bar{K}^0$	$(\pi^0\pi^0\pi^0, \ell^-)$
$K_+ \to K^0$	$(\pi^0\pi^0\pi^0, \ell^+)$	$K^0 \to K_+$	$(\ell^-, \pi\pi)$
$\bar{K}^0 \to K_+$	$(\ell^+, \pi\pi)$	$K_+ \to \bar{K}^0$	$(\pi^0\pi^0\pi^0, \ell^-)$
$K_- \to K^0$	$(\pi\pi, \ell^+)$	$K^0 \to K_-$	$(\ell^-, \pi\pi)$

One can define the following ratios of probabilities:

$$
\begin{aligned}
R_1(\Delta t) &= P\left[K^0(0) \to K_+(\Delta t)\right] / P\left[K_+(0) \to K^0(\Delta t)\right] \\
R_2(\Delta t) &= P\left[K^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to K^0(\Delta t)\right] \\
R_3(\Delta t) &= P\left[\bar{K}^0(0) \to K_+(\Delta t)\right] / P\left[K_+(0) \to \bar{K}^0(\Delta t)\right] \\
R_4(\Delta t) &= P\left[\bar{K}^0(0) \to K_-(\Delta t)\right] / P\left[K_-(0) \to \bar{K}^0(\Delta t)\right] \ .
\end{aligned}
$$

Any deviation from R_i=1 constitutes a violation of T-symmetry

J. Bernabeu, A.D.D., P. Villanueva: NPB 868 (2013) 102

T, CP, CPT tests in neutral kaon transitions at KLOE

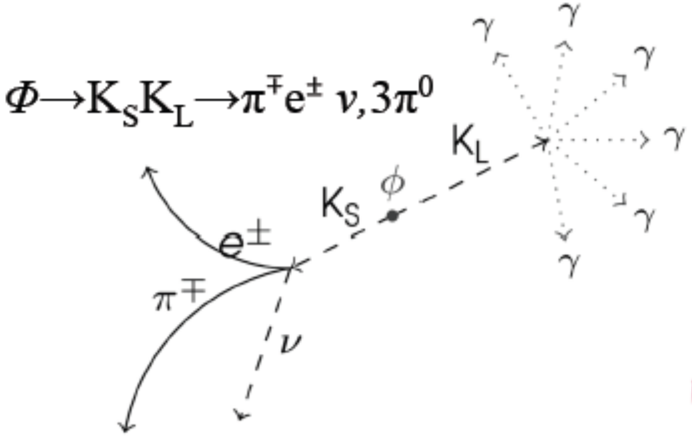
CPT **T** **CP** **observables**

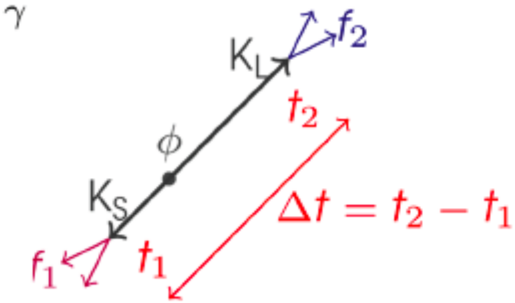
$$R^{\text{exp}}_{2,\mathcal{CPT}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} \qquad R^{\text{exp}}_{2,\mathcal{T}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} \qquad R^{\text{exp}}_{2,\mathcal{CP}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\ell^+, 3\pi^0; \Delta t)}$$

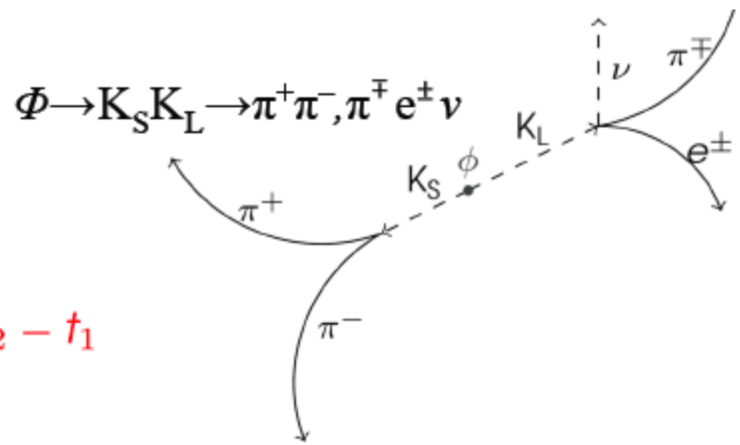
$$R^{\text{exp}}_{4,\mathcal{CPT}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} \qquad R^{\text{exp}}_{4,\mathcal{T}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} \qquad R^{\text{exp}}_{4,\mathcal{CP}}(\Delta t) \equiv \frac{I(\pi\pi, \ell^+; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)}$$

$$\mathcal{DR}_{\mathcal{CPT}}(\Delta t \gg \tau_S) \equiv \frac{R^{\text{exp}}_{2,\mathcal{CPT}}(\Delta t \gg \tau_S)}{R^{\text{exp}}_{4,\mathcal{CPT}}(\Delta t \gg \tau_S)} \qquad \mathcal{DR}_{\mathcal{T},\mathcal{CP}}(\Delta t \gg \tau_S) \equiv \frac{R^{\text{exp}}_{2,\mathcal{T}}(\Delta t \gg \tau_S)}{R^{\text{exp}}_{4,\mathcal{T}}(\Delta t \gg \tau_S)} \equiv \frac{R^{\text{exp}}_{2,\mathcal{CP}}(\Delta t \gg \tau_S)}{R^{\text{exp}}_{4,\mathcal{CP}}(\Delta t \gg \tau_S)}$$

Corresponding to study the following processes at KLOE:



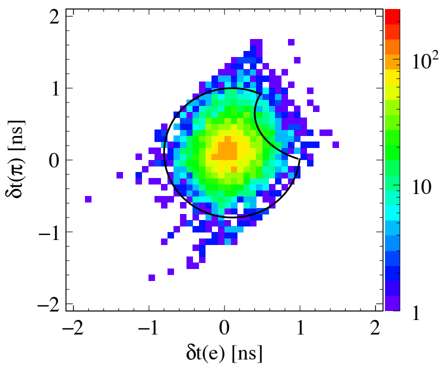




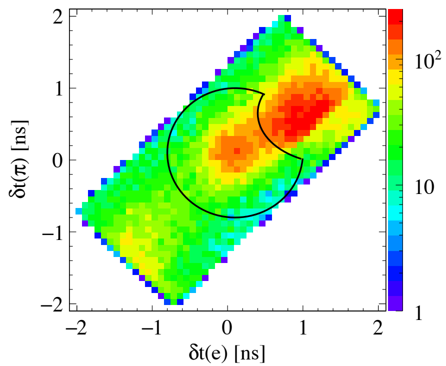
T, CP, CPT tests in neutral kaon transitions at KLOE

- Analysed data $L=1.7\ \mathrm{fb}^{-1}$
- Four processes studied: $\phi \to K_S K_L \to \pi e^{\pm}\nu\ 3\pi^0$ and $\pi^+\pi^-\ \pi e^{\pm}\nu$ in the asympotic regime: $\Delta t \gg \tau_S$
- Time of flight technique to identify semileptonic decays

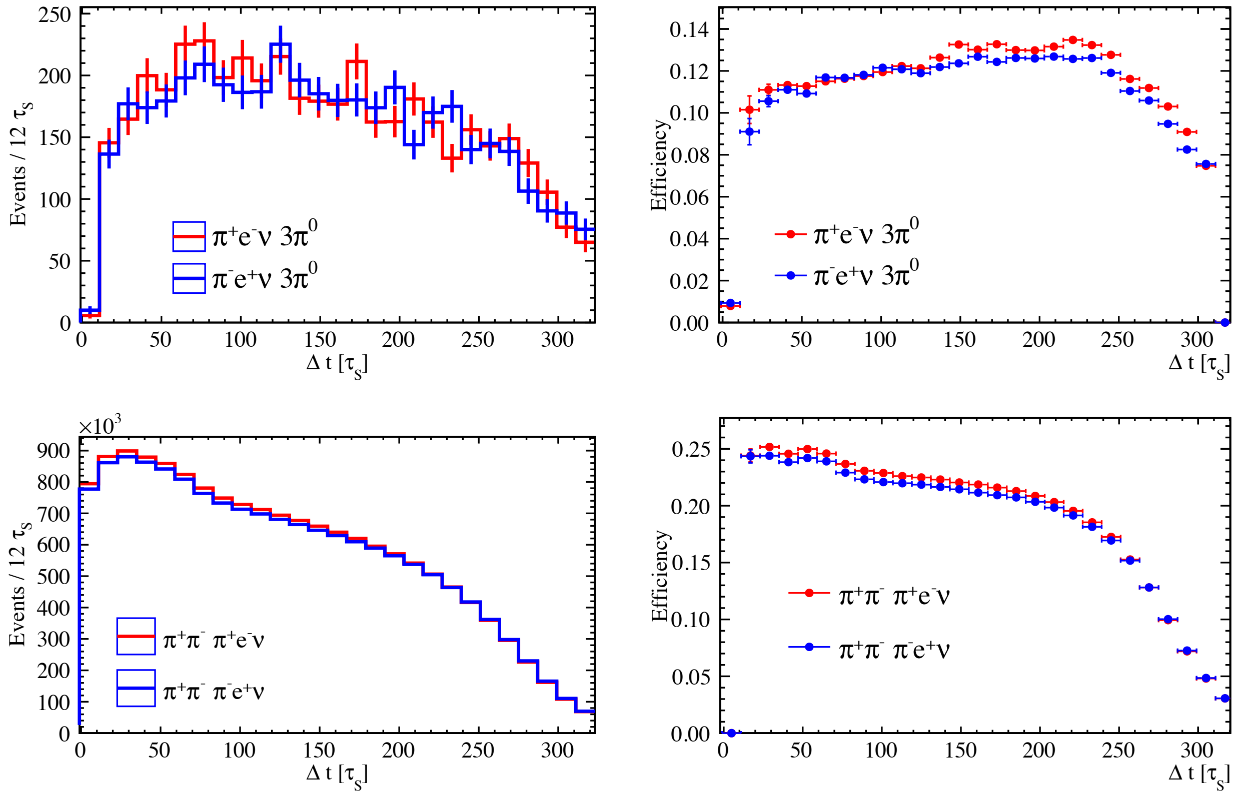
MC K_S signal



Data



Measured double kaon decay intensities



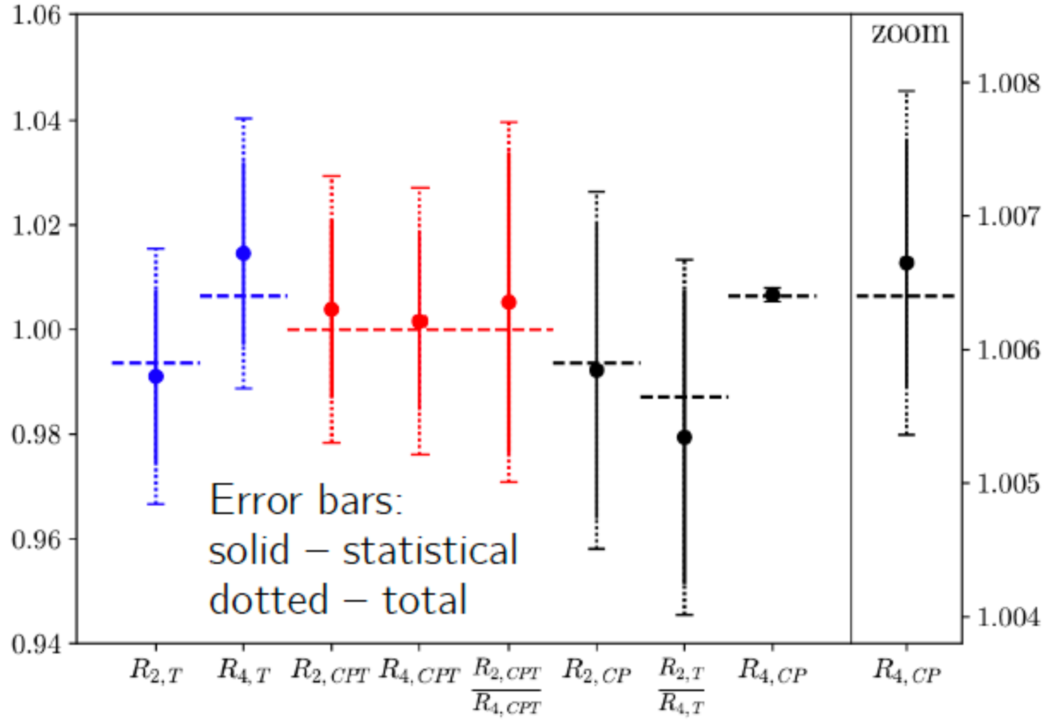
- residual background subtraction for $\pi e^{\pm}\nu\ 3\pi^0$ channel
- MC selection efficiencies corrected from data with 4 independent control samples

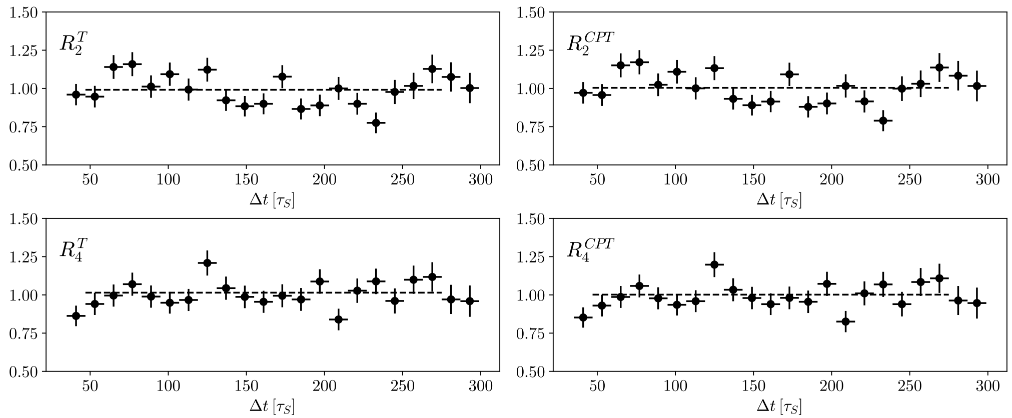
$K_S K_L \to \pi e^{\pm}\nu\ 3\pi^0$ → $K_S \to \pi e\nu$ Klcrash ; $K_S K_L \to \pi^+\pi^-\ 3\pi^0$

$K_S K_L \to \pi^+\pi^-\ \pi e^{\pm}\nu$ → $K_S \to \pi^+\pi^-$ Klcrash ; $K_S K_L \to \pi^0\pi^0\ \pi e\nu$

T, CP, CPT tests in neutral kaon transitions at KLOE

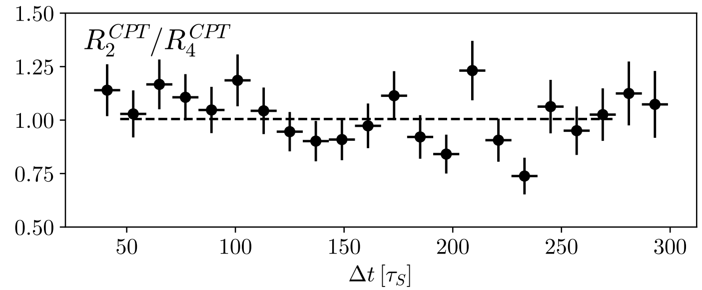
horizontal dashed lines denote expected values:
CPT invariance and TV extrapolated from observed CPV (PDG)





KLOE-2 result
PLB 845 (2023) 138164

$$R_{2,\mathrm{T}} = 0.991 \pm 0.017_{stat} \pm 0.014_{syst} \pm 0.012_D\ ,$$
$$R_{4,\mathrm{T}} = 1.015 \pm 0.018_{stat} \pm 0.015_{syst} \pm 0.012_D\ ,$$
$$R_{2,\mathrm{CPT}} = 1.004 \pm 0.017_{stat} \pm 0.014_{syst} \pm 0.012_D\ ,$$
$$R_{4,\mathrm{CPT}} = 1.002 \pm 0.017_{stat} \pm 0.015_{syst} \pm 0.012_D\ ,$$
$$R_{2,\mathrm{CP}} = 0.992 \pm 0.028_{stat} \pm 0.019_{syst}\ ,$$
$$R_{4,\mathrm{CP}} = 1.00665 \pm 0.00093_{stat} \pm 0.00089_{syst}\ ,$$
$$DR_{\mathrm{T,CP}} = R_{2,\mathrm{T}}/R_{4,\mathrm{T}} = 0.979 \pm 0.028_{stat} \pm 0.019_{syst}\ ,$$
$$DR_{\mathrm{CPT}} = R_{2,\mathrm{CPT}}/R_{4,\mathrm{CPT}} = 1.005 \pm 0.029_{stat} \pm 0.019_{syst}\ .$$

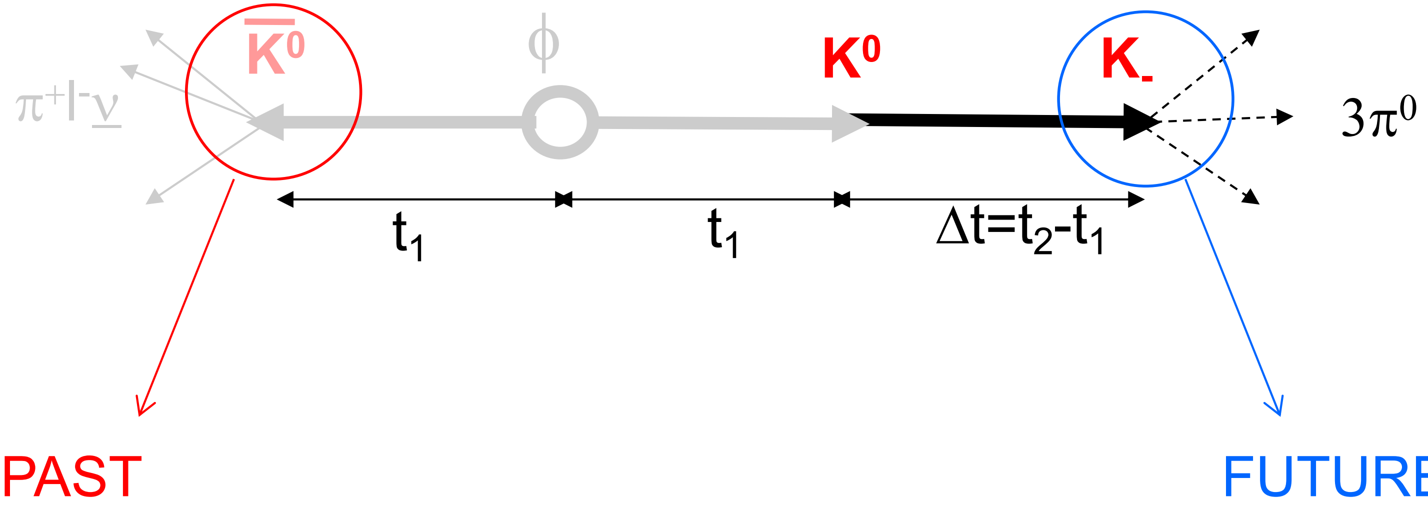


First T and CPT test in kaon transitions

From future to past

Further studies of the properties of entanglement for neutral K mesons

Double decay intensity calculation



QM calculation of double decay intensity: two alternatives

(I) Time History approach (TH), from past to future

(1) The time evolution of the state $|i\rangle$ from time $t = 0$ to time $t = t_1$, with definite total width Γ;

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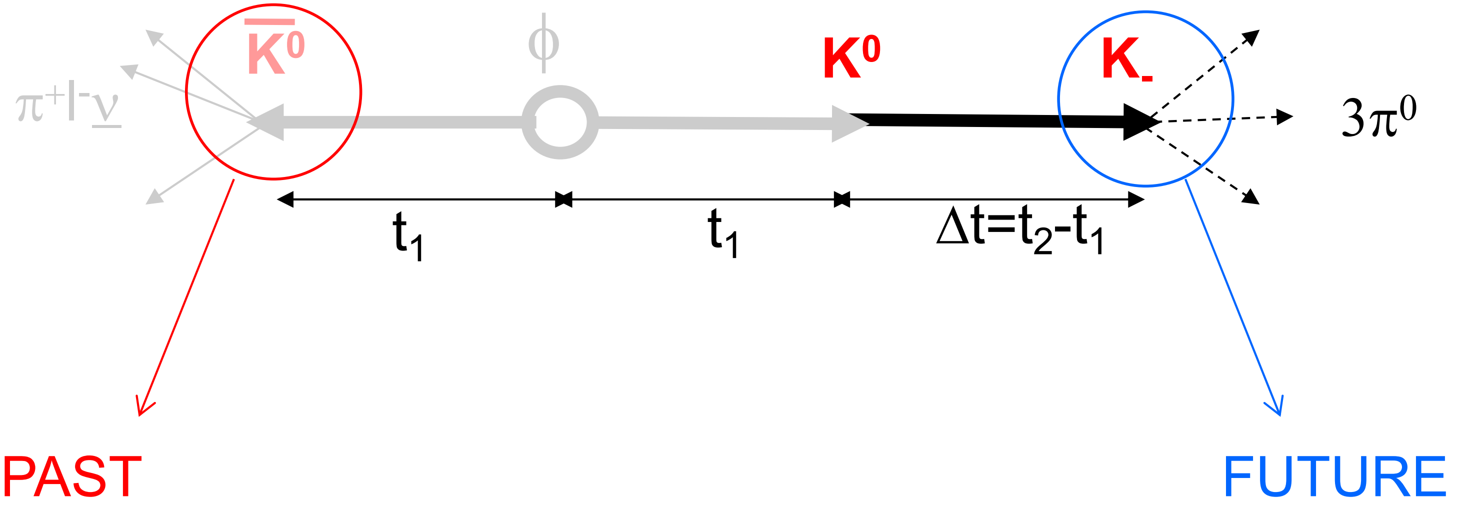
$$I(f_1, t_1; f_2, t_2)_{\rm TH} = \left| \underbrace{\langle f_2|T|K^{\perp}_{\nrightarrow f_2}\rangle}_{(4)} \underbrace{\langle K^{\perp}_{\nrightarrow f_2}|K_{\nrightarrow f_1}(\Delta t)\rangle}_{(3)} \underbrace{\langle f_1|T|K^{\perp}_{\nrightarrow f_1}\rangle}_{(2)} \underbrace{\langle K^{\perp}_{\nrightarrow f_1} K_{\nrightarrow f_1}|i(t = t_1)\rangle}_{(1)} \right|^2$$

Double decay intensity calculation



QM calculation of double decay intensity: two alternatives

(II) T.D. Lee and C.N. Yang (LY) two decay times state formalism (1961)

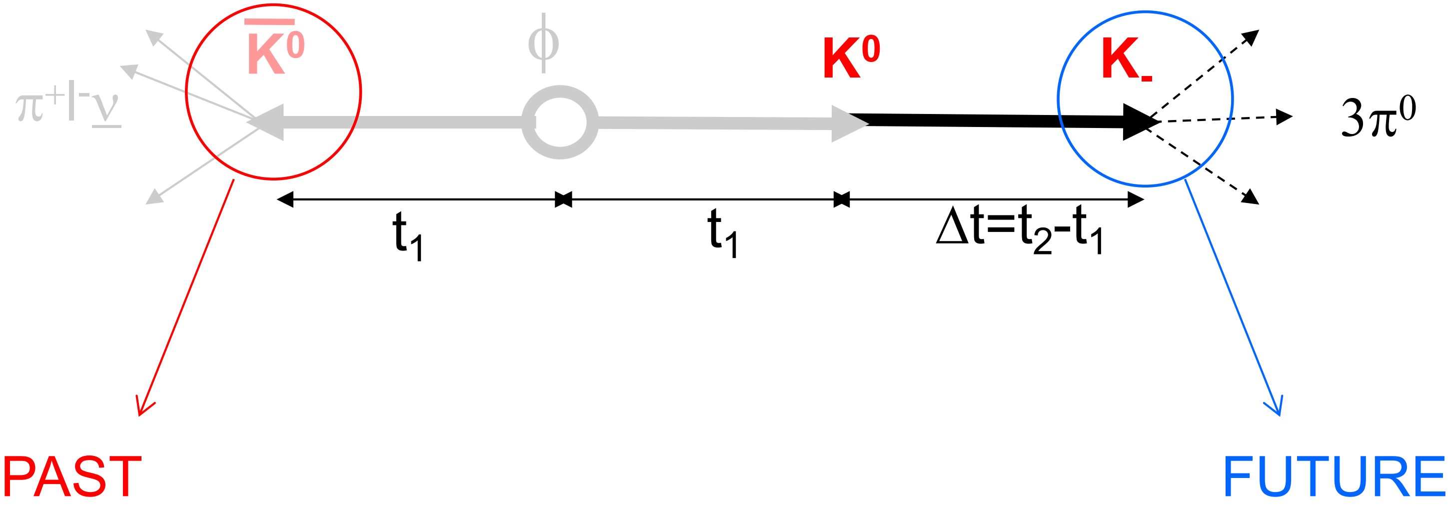
[see e.g. T.Day PR121, 1204 (1961), D. Inglis RMP 33, 1 (1961)]

$$|i(t)\rangle = \frac{\mathcal{N}}{\sqrt{2}}\{|\mathrm{K_S}\rangle e^{-i\lambda_S t}|\mathrm{K_L}\rangle e^{-i\lambda_L t} - |\mathrm{K_L}\rangle e^{-i\lambda_L t}|\mathrm{K_S}\rangle e^{-i\lambda_S t}\}$$

$$|i_{t_1,t_2}\rangle = \frac{\mathcal{N}}{\sqrt{2}}\{|\mathrm{K_S}\rangle e^{-i\lambda_S t_1}|\mathrm{K_L}\rangle e^{-i\lambda_L t_2} - |\mathrm{K_L}\rangle e^{-i\lambda_L t_1}|\mathrm{K_S}\rangle e^{-i\lambda_S t_2}\}$$

$$I(f_1,t_1;f_2,t_2)_{\mathrm{LY}} = |\langle f_1 f_2|T|i_{t_1,t_2}\rangle|^2$$

Double decay intensity calculation



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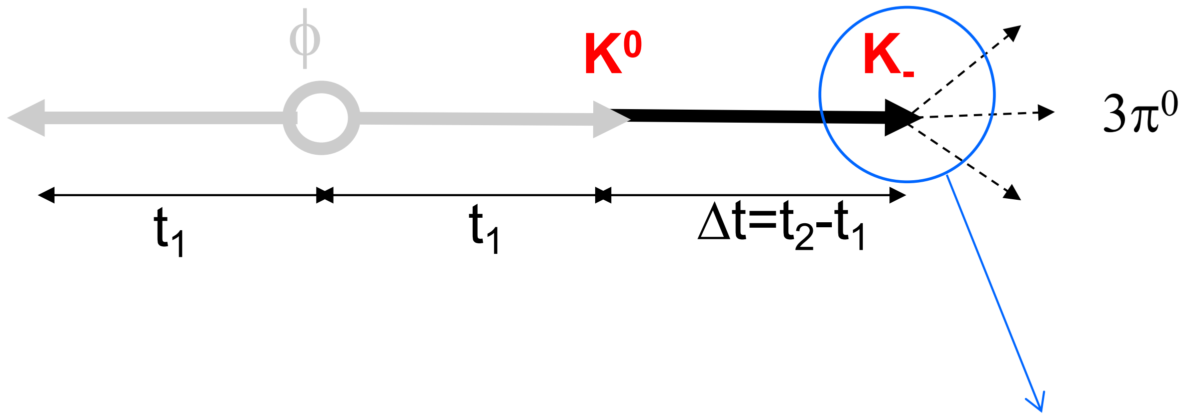
$$I(f_1,t_1;f_2,t_2)_{\mathrm{LY}} = |\langle f_1 f_2|T|i_{t_1,t_2}\rangle|^2$$

TH and LY approaches are fully equivalent

$$I(f_1,t_1;f_2,t_2)_{\mathrm{TH}} = I(f_1,t_1;f_2,t_2)_{\mathrm{LY}} \equiv I(f_1,t_1;f_2,t_2)$$

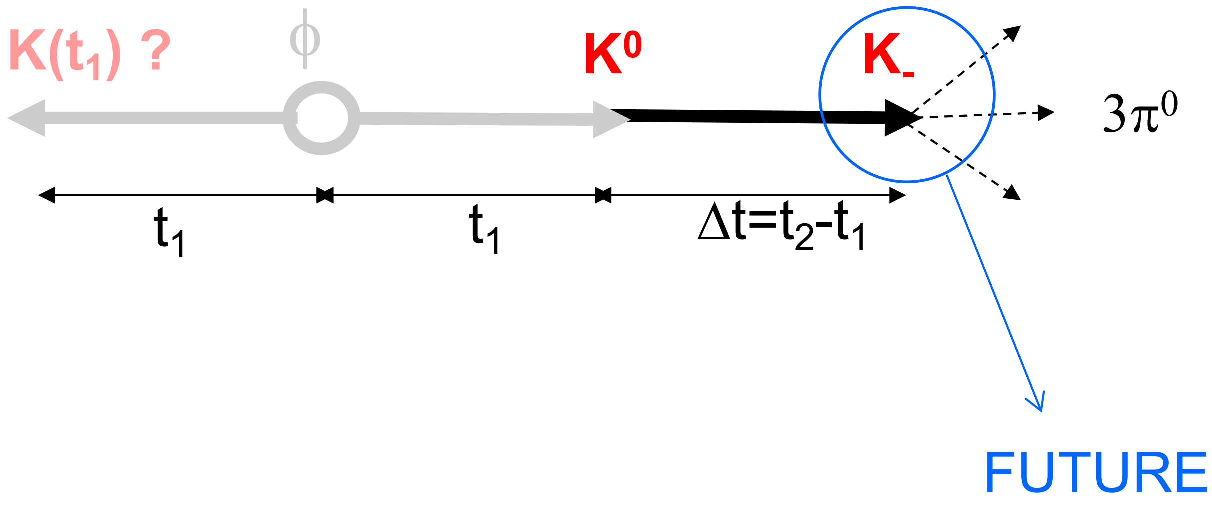
Can Future post-tag the Past?



FUTURE

If past tags the future, the t_1 , t_2 symmetry of the correlated state in the LY approach demands the exploration of the question: can future post-tag the past?

Can Future post-tag the Past?



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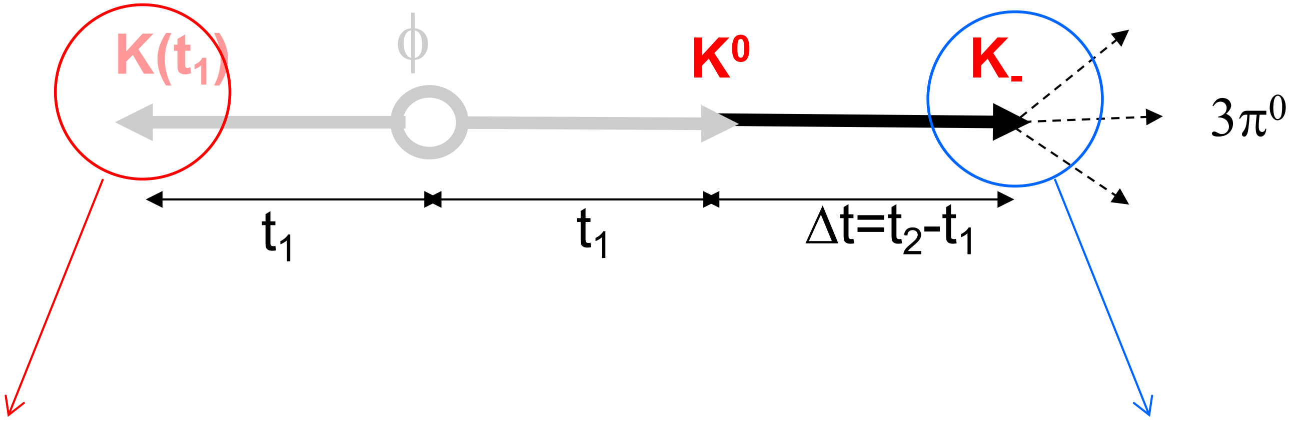

PAST

The future (kaon decay at t_2) post-tags the past partner kaon state at t_1, before the decay, when it was entangled !

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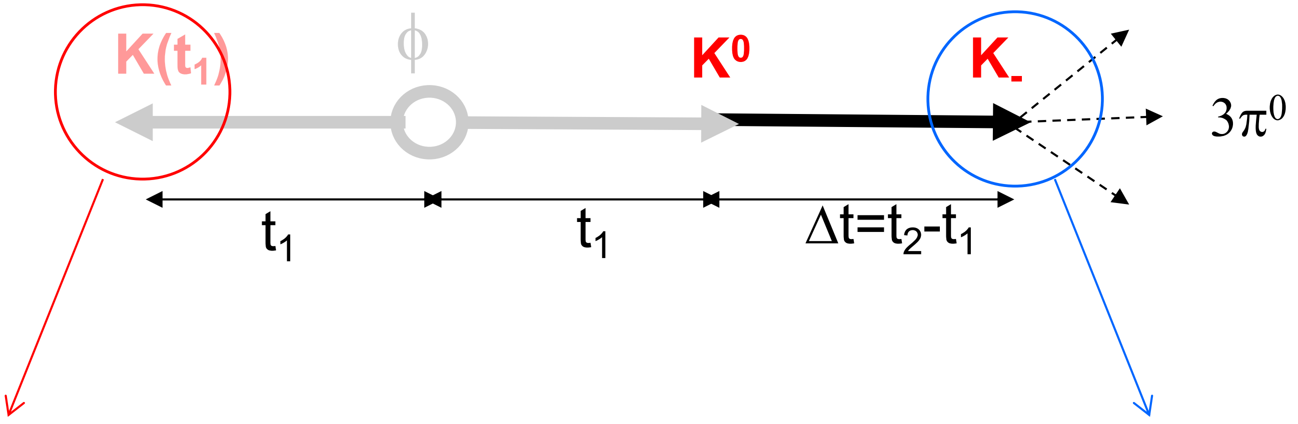

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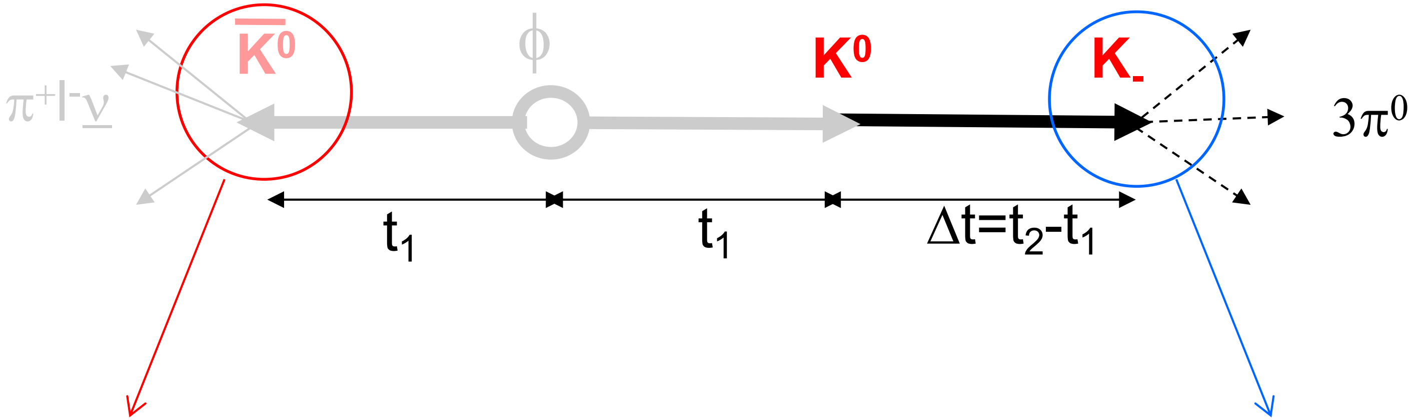
$$
\begin{aligned}
|K^{(1)}(t=t_1)\rangle =& \langle f_2|T|i_{t_1,t_2}\rangle \\
=& \frac{\mathcal{N}}{\sqrt{2}}\{\langle f_2|T|\mathrm{K_L}\rangle e^{-i\lambda_L t_2} e^{-i\lambda_S t_1}|\mathrm{K_S}\rangle - \langle f_2|T|\mathrm{K_S}\rangle e^{-i\lambda_S t_2} e^{-i\lambda_L t_1}|\mathrm{K_L}\rangle\} \\
=& \frac{\mathcal{N}}{\sqrt{2}}\langle f_2|T|\mathrm{K_S}\rangle\{e^{-i\lambda_S t_1}\left[\eta_2\ e^{-i\lambda_L t_2}|\mathrm{K_S}\rangle\right] - e^{-i\lambda_L t_1}\left[e^{-i\lambda_S t_2}|\mathrm{K_L}\rangle\right]\}\ .
\end{aligned}
$$

PAST

FUTURE

Can Future post-tag the Past?



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&= \frac{\mathcal{N}}{\sqrt{2}}\{\langle f_2|T|\mathrm{K_L}\rangle e^{-i\lambda_L t_2} e^{-i\lambda_S t_1}|\mathrm{K_S}\rangle - \langle f_2|T|\mathrm{K_S}\rangle e^{-i\lambda_S t_2} e^{-i\lambda_L t_1}|\mathrm{K_L}\rangle\} \\
&= \frac{\mathcal{N}}{\sqrt{2}}\langle f_2|T|\mathrm{K_S}\rangle\{e^{-i\lambda_S t_1}\left[\eta_2\, e^{-i\lambda_L t_2}|\mathrm{K_S}\rangle\right] - e^{-i\lambda_L t_1}\left[e^{-i\lambda_S t_2}|\mathrm{K_L}\rangle\right]\}\ .
\end{aligned}
$$

PAST

FUTURE

Post-tagging: summary

From past to future:

The state of the last decaying particle (particle-2) is tagged (prepared) at $t = t_1$ as:

$|K^{(2)}(t = t_1)\rangle \;=\; \mathcal{N}_2\left[|\mathrm{K_L}\rangle - \eta_1|\mathrm{K_S}\rangle\right]$ a state which depends on η_1 of particle-1.

From future to past:

The state of the first decaying particle (particle-1) is ***post-tagged*** at $t = 0$ as:

$|K^{(1)}(t = 0)\rangle \;=\; \mathcal{N}_1\{\eta_2 e^{-i\lambda_L t_2}|\mathrm{K_S}\rangle - e^{-i\lambda_S t_2}|\mathrm{K_L}\rangle\}$ a state which depends on η_2 **and** $\boldsymbol{t_2}$ of particle-2.

The explicit dependence on the future time t_2, and the other unique features of neutral kaons with respect to other physical systems, like $\Delta\Gamma \neq 0$ and $\langle K_L|K_S\rangle \neq 0$, naturally lead to this peculiar quantum effect:
a definite time correlation (not symmetric comparing "from past to future" to "from future to past") between the outcome at a given future time of the observed decay and the state of the unobserved partner in the past, at entanglement time.

J. Bernabeu and A.D.D., Phys. Rev. D 105, 116004 (2022)

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a definite time correlation (not symmetric comparing "from past to future" to "from future to past") between the outcome at a given future time of the observed decay and the state of the unobserved partner in the past, at entanglement time.
K_S tag: due to CP violation $\langle K_L|K_S\rangle \neq 0$, the time correlation "from future to past" with condition $e^{-\Delta\Gamma\Delta t/2}/\eta_2 \ll 1$ is the only known method **to *post-tag*** a K_S beam with arbitrary high purity.

see Bernabeu's talk

J. Bernabeu and A.D.D., Phys. Rev. D 105, 116004 (2022)

Post-tagging: summary

From past to future:

The state of the last decaying particle (particle-2) is tagged (prepared) at $t = t_1$ as:

$$|K^{(2)}(t=t_1)\rangle \;=\; \mathcal{N}_2\,[|\mathrm{K_L}\rangle - \eta_1|\mathrm{K_S}\rangle]$$ a state which depends on η_1 of particle-1.

From future to past:

Back *from* the future

The state of the first decaying partic … as:

$|K^{(1)}(t=0)\rangle \;=\; \mathcal{N}_1\{\eta_2 e^{-i\lambda_L t_2}|\mathrm{K_S}\rangle - e$ … $_2$ **and t_2** of particle-2.

The explicit dependence on the fu … ures of neutral kaons with respect to other physical … aturally lead to this peculiar quantum effect:
a definite time correlation (not symm … "from future to past") between the outcome at a giv … nd the state of the unobserved partner in the past, at entanglement time.
K_S tag: due to CP violation $\langle K_L|K_S\rangle \neq 0$, the time correlation "from future to past" with condition $e^{-\Delta\Gamma\Delta t/2}/\eta_2 \ll 1$ is the only known method **to *post-tag*** a K_S beam with arbitrary high purity.

see Bernabeu's talk

J. Bernabeu and A.D.D., Phys. Rev. D 105, 116004 (2022)

Parametrization of the "Back from the future" effect

post-tagged state: $|K^{(1)}(t_1 = 0)\rangle = \mathcal{N}\{|K_S\rangle - \rho(t_2)|K_L\rangle\}$

see Bernabeu's talk

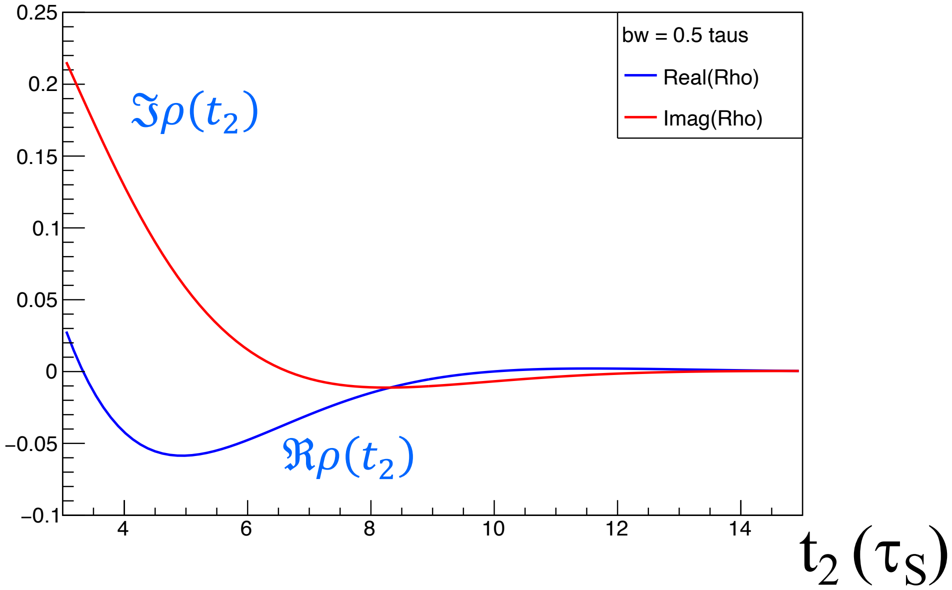
in the case $f = f_1 = f_2$ at fixed t_2: $\rho(t_2) = e^{-i(\lambda_S - \lambda_L)t_2}$

$$\left|\langle f|K^{(1)}(t_1)\rangle\right|^2 = |\mathcal{N}|^2 \, |\langle f|K_S(t_1)\rangle - \rho(t_2)\langle f|K_L(t_1)\rangle|^2$$

$$= |\mathcal{N}|^2 \left\{e^{-\Gamma_S t_1} + |\rho(t_2)|^2 \, e^{-\Gamma_L t_1} - \right.$$

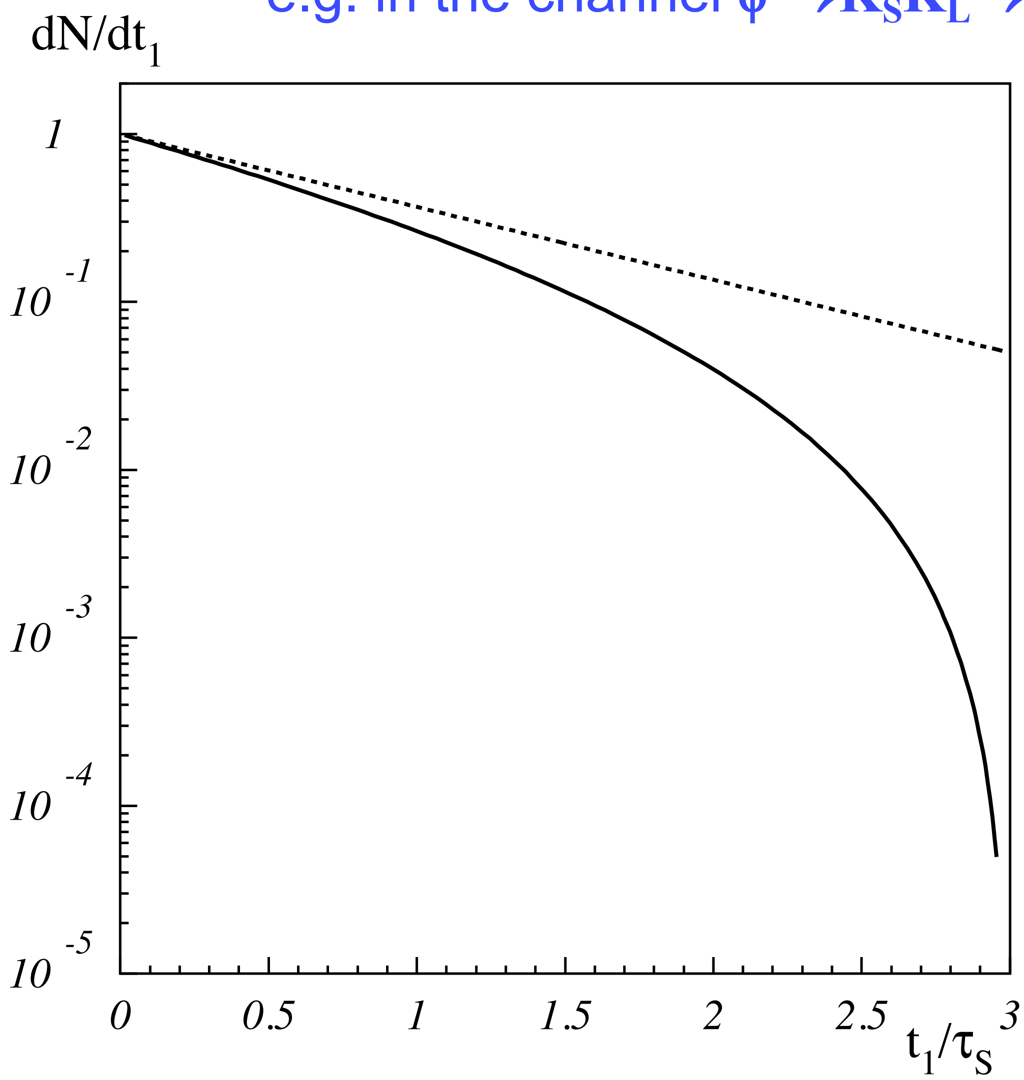
$$\left. -2e^{-\frac{\Gamma_S+\Gamma_L}{2}t_1}[\Re\rho(t_2)\cos\Delta m\, t_1 + \Im\rho(t_2)\sin\Delta m\, t_1]\right\}$$

Experimentally t_2 is averaged on a bin width => e.g. bin width ½ τ_S



“Back from the future”: observable effects

This quantum effect is directly observable at KLOE/KLOE-2
e.g. in the channel $\phi \to K_S K_L \to \pi^+\pi^- \pi^+\pi^-$ to maximize the effect



Distributions normalized to unity at t_1=0

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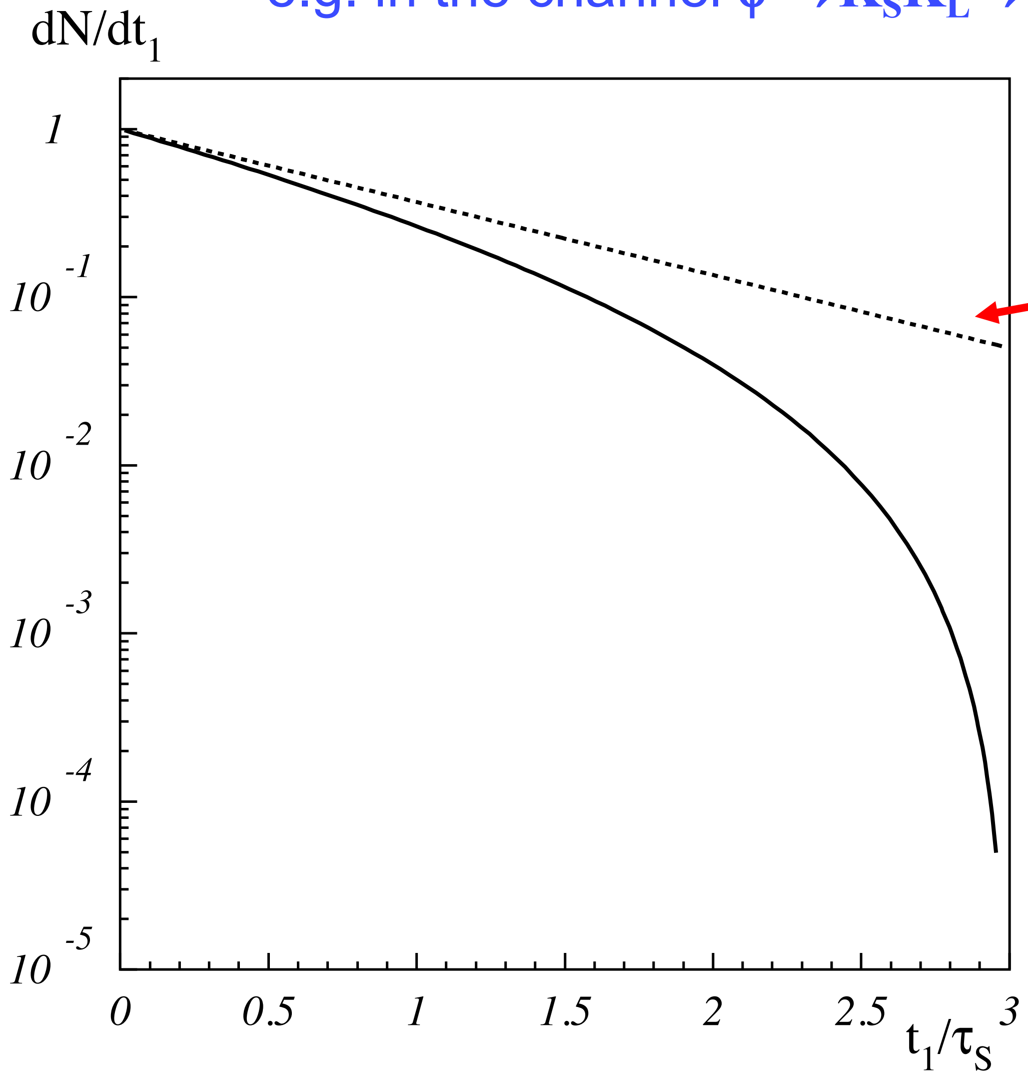

DECOHERENCE REGIME

$I(t_1)$ with $t_2 >> t_1$ and $|\eta_{+-}|, \Delta\Gamma$
such that the K_S post-tag condition is fulfilled =>
definite width: Γ_S i.e. a K_S state

Distributions normalized to unity at t_1=0

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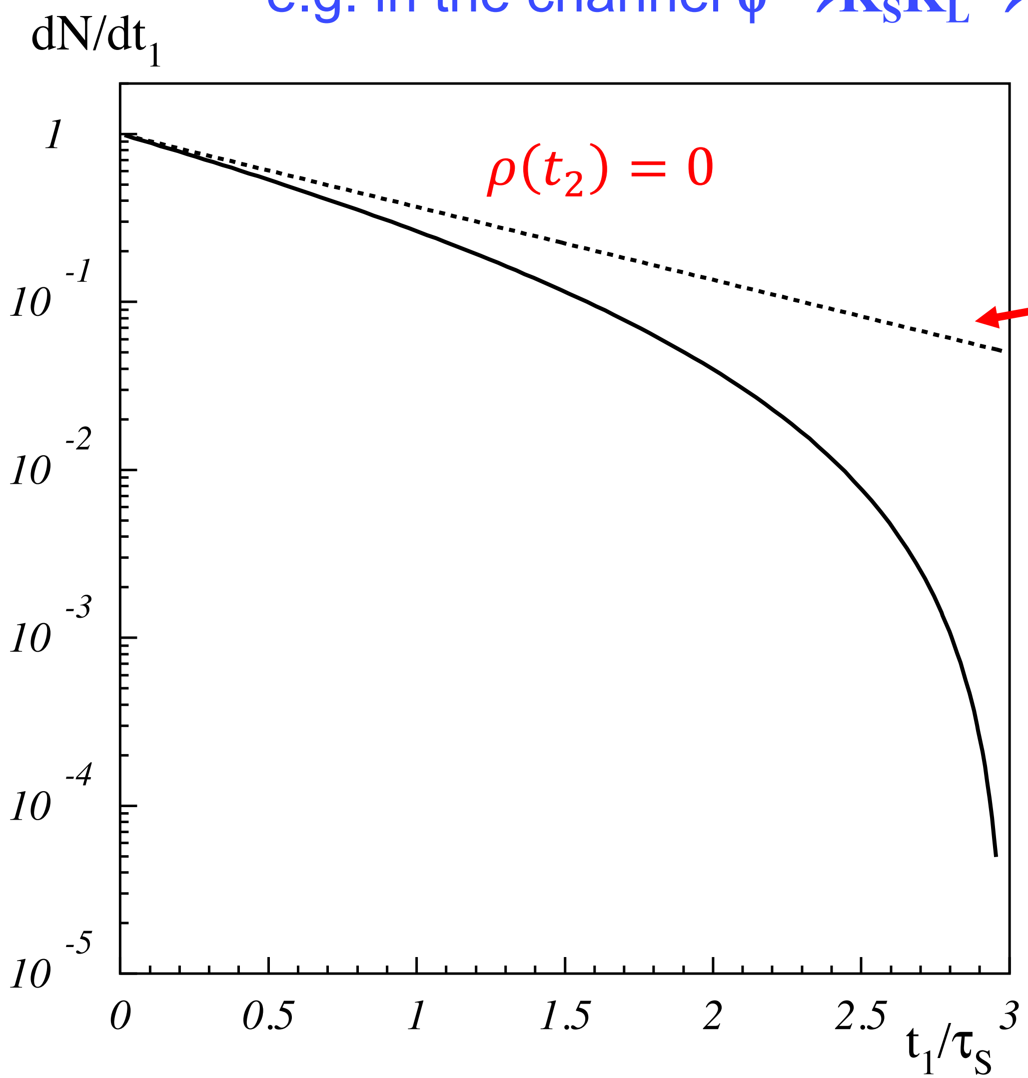

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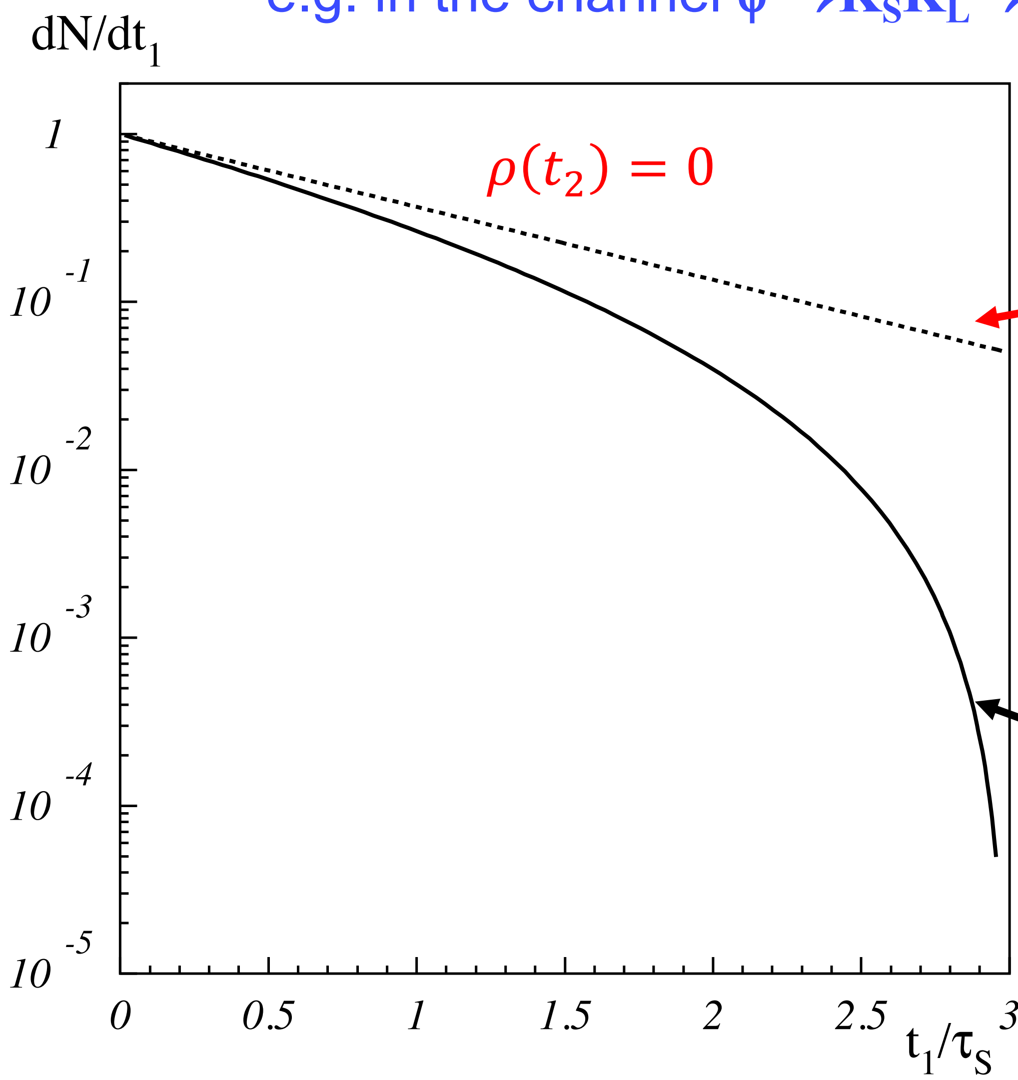

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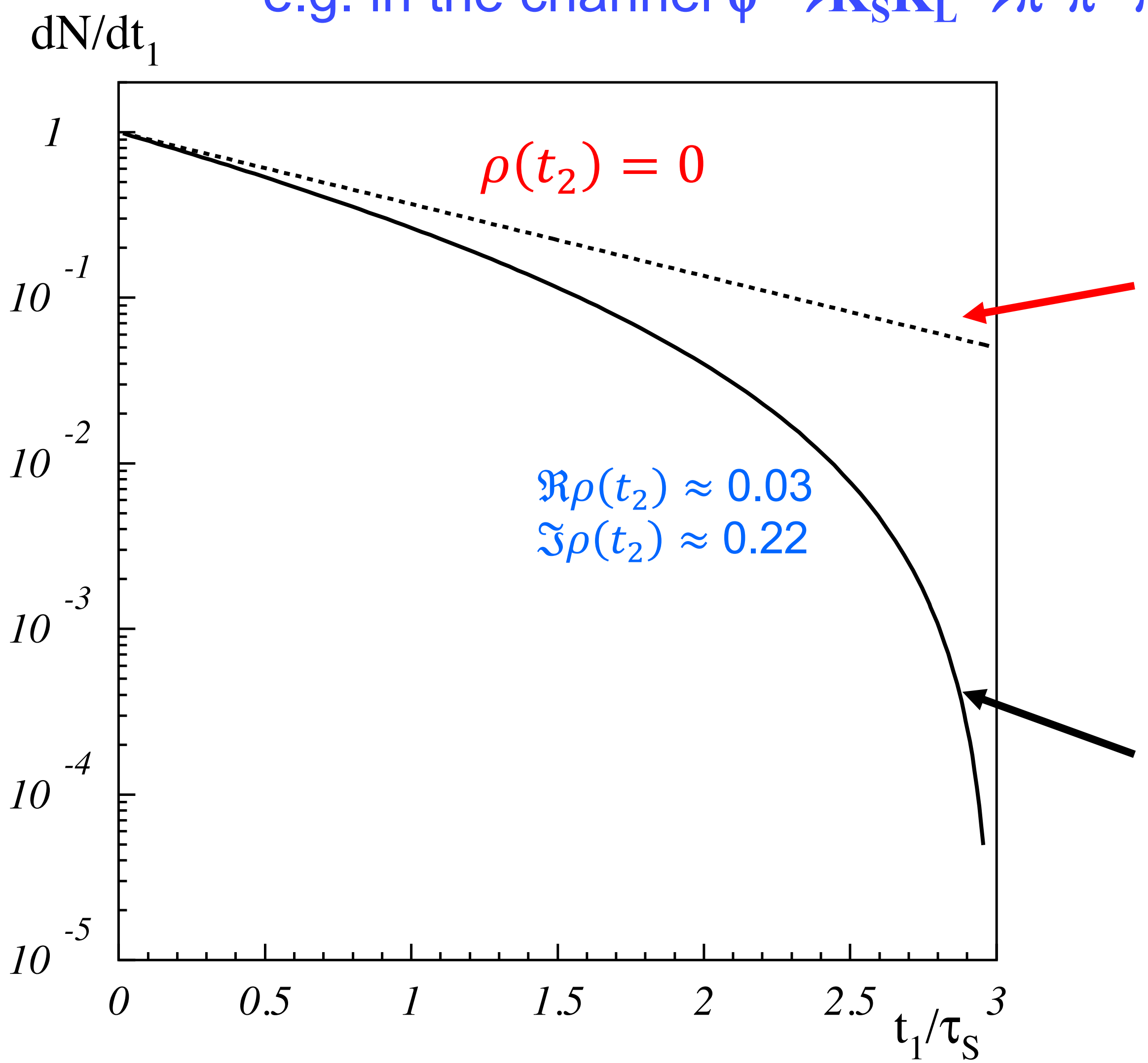
INTERFERENCE REGIME

$I(t_1)$ with $t_2 = 3\tau_S$ (> t_1)
K_S post-tag condition
is NOT fulfilled => no definite width

Distributions normalized to unity at $t_1=0$

"Back from the future": observable effects

This quantum effect is directly observable at KLOE/KLOE-2
e.g. in the channel $\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^- \pi^+\pi^-$ to maximize the effect



DECOHERENCE REGIME

$I(t_1)$ with $t_2 >> t_1$ and $|\eta_{+-}|, \Delta\Gamma$
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definite width: Γ_S i.e. a K_S state

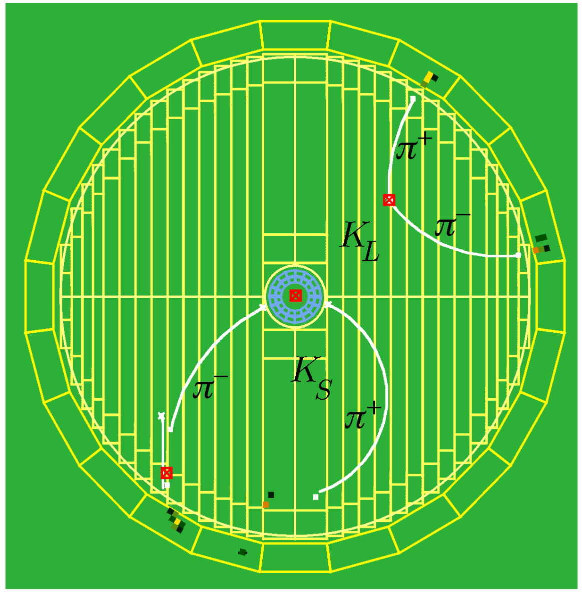
INTERFERENCE REGIME

$I(t_1)$ with $t_2=3\tau_S$ (> t_1)
K_S post-tag condition
is NOT fulfilled => no definite width

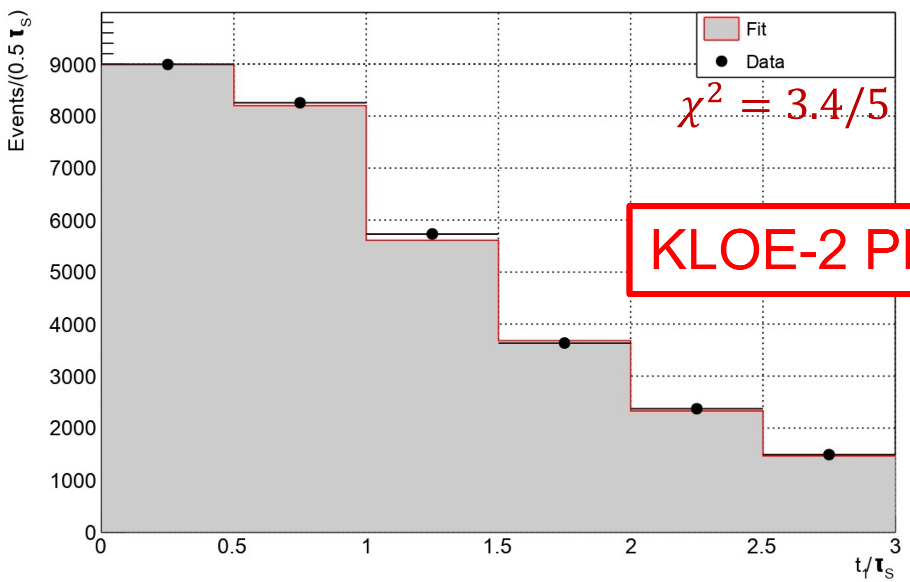
Distributions normalized to unity at $t_1=0$

"Back from the future" effect at KLOE-2

- Analysed data: 1.7 fb^{-1} - selection of $K_S K_L \to \pi^+\pi^-\pi^+\pi^-$ events as for search for decoherence/CPTV effects [KLOE-2 - JHEP 04 (2022) 059];
- Fit of t_1 distribution with QM theory taking into account resolution and efficiency through a 4-dimensional smearing matrix $(t_{1,true}\ , t_{1,reco}, t_{2\ true}\ , t_{2,reco})$;
- Negligible background from $e^+e^- \to 4\pi$ process and regeneration on beam pipe;
- histogram normalization as single fit parameter.



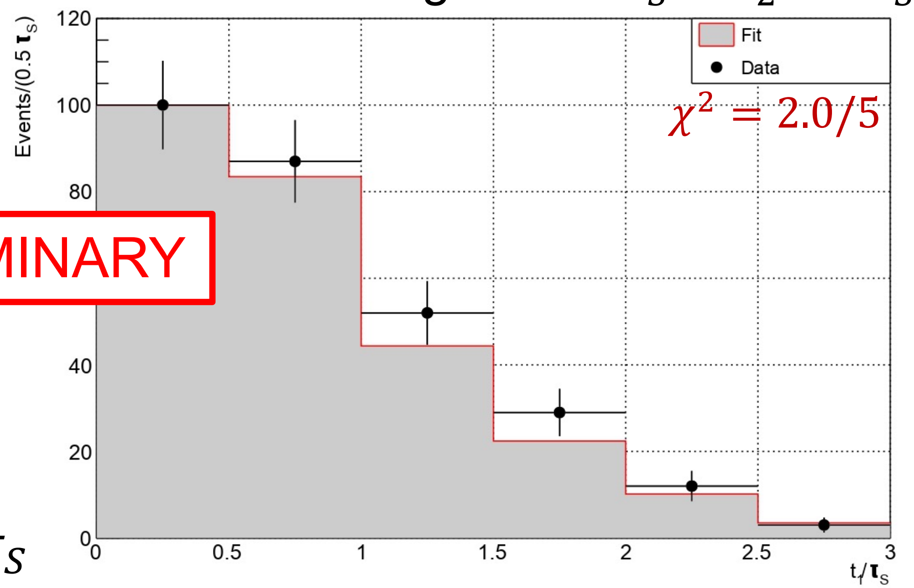
Decoherence regime: $t_2 > 30\tau_S$



t_1/τ_S

KLOE-2 PRELIMINARY

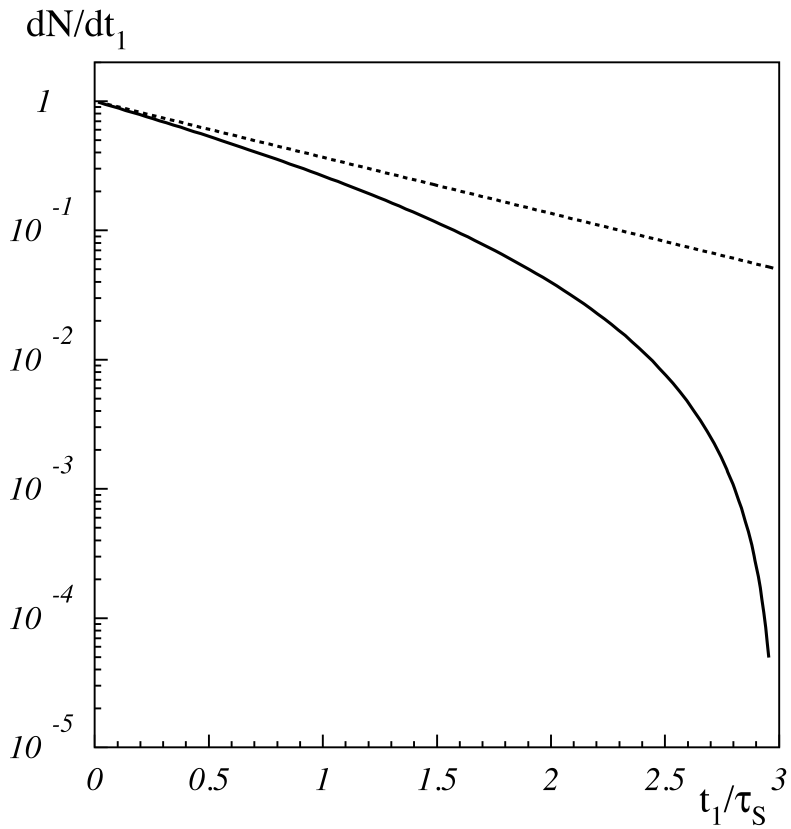
Interference regime: $2.5\tau_S < t_2 < 3\tau_S$

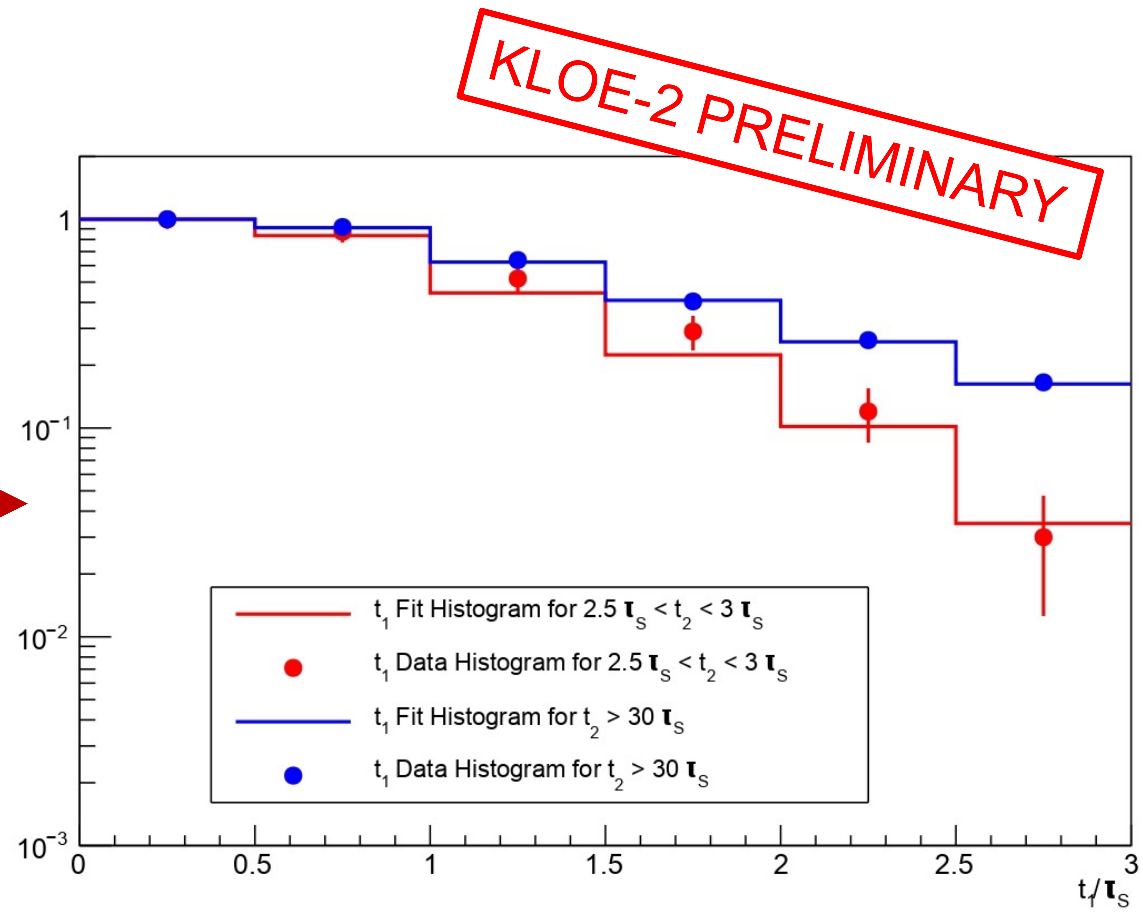


t_1/τ_S

"Back from the future" effect at KLOE-2

- normalizing the distributions to unity at $t_1=0$, we get a first evidence of the effect





- The analysis to extract the ρ parameter as a function of t_2 is being finalized

Conclusions

- The entanglement of neutral kaon pairs at a ϕ-factory has unique features.
- Search for decoherence and CPT violation effects in $\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^- \pi^+\pi^-$ at KLOE/KLOE-2 => stringent limits on model parameters, in some cases with a precision reaching the interesting Planck's scale region.

FROM PAST TO FUTURE:

- Exploiting the maximal entanglement of the initial state for the necessary exchange of *in* and *out* states, it is possible to directly test T and CPT in transition processes.
- The KLOE-2 collaboration performed the first direct test of T and CPT in neutral kaon transitions with a precision of few percent on the corresponding observables.
- No CPT violation observed, T violation at limit, CP violation is observed with a significance of 5.2σ.

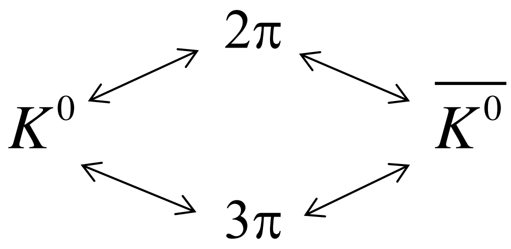
FROM FUTURE TO PAST:

- Novel time quantum correlation effect in the entangled kaon system [PRD 105, 116004 (2022)].
- This surprising **"Back from the future"** effect is fully observable at KLOE/KLOE-2 and naturally leads to the tagging of the K_S state, and to the definition of new observables.
- A preliminary analysis of the ϕ->$K_S K_L$ -> $\pi^+\pi^-\pi^+\pi^-$ events with KLOE data shows a first evidence of this effect. Finalization of the analysis to extract the ρ parameter as a function of t_2.
- The Back from the future effect cannot be a causal influence, independently of time-like or space-like intervals. This result seems to confirm the counterintuitive feature of time in quantum mechanics, and goes beyond other phenomena, like delayed choice experiments with entangled photon systems, that are stationary at all times, and have the result independent on whether the choice is made in the past or in the future.

SPARE SLIDES

The neutral kaon two-level oscillating system in a nutshell

K^0 and $\overline{K^0}$ can decay to common final states due to weak interactions: strangeness oscillations



$$|\Psi\rangle = a|K^0\rangle + b|\overline{K}^0\rangle$$

$$i\frac{\partial}{\partial t}\Psi(t) = \mathbf{H}\Psi(t)$$

H is the effective hamiltonian (non-hermitian), decomposed into a Hermitian part (mass matrix **M**) and an anti-Hermitian part (i/2 decay matrix **Γ**) :

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\Gamma = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

Diagonalizing the effective Hamiltonian:

eigenvalues

$$\lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L}$$

$$|K_{S,L}(t)\rangle = e^{-i\lambda_{S,L}t}|K_{S,L}(0)\rangle$$

τ_S ~ 90 ps $\quad$ τ_L ~ 51 ns

$K_L \to \pi\pi$ violates CP

eigenstates: physical states

$$|K_{S,L}\rangle = \frac{1}{\sqrt{2\left(1+|\varepsilon_{S,L}|\right)}}\left[\left(1+\varepsilon_{S,L}\right)|K^0\rangle \pm \left(1-\varepsilon_{S,L}\right)|\overline{K}^0\rangle\right]$$

$$= \frac{1}{\sqrt{\left(1+|\varepsilon_{S,L}|\right)}}\left[|K_{1,2}\rangle + \varepsilon_{S,L}|K_{2,1}\rangle\right]$$

$|K_{1,2}\rangle$ are CP=±1 states

$$\langle K_S|K_L\rangle \cong \varepsilon_S^* + \varepsilon_L \neq 0$$

small CP impurity ~2 x 10^{-3}

The neutral kaon two-level oscillating system in a nutshell

$$|K_{S,L}\rangle \propto \left[(1+\varepsilon_{S,L})|K^0\rangle \pm (1-\varepsilon_{S,L})|\bar{K}^0\rangle\right]$$

CP violation:

$$\varepsilon_{S,L} = \varepsilon \pm \delta$$

T violation:

$$\varepsilon = \frac{H_{12} - H_{21}}{2(\lambda_S - \lambda_L)} = \frac{-i\Im M_{12} - \Im\Gamma_{12}/2}{\Delta m + i\Delta\Gamma/2}$$

CPT violation:

$$\delta = \frac{H_{11} - H_{22}}{2(\lambda_S - \lambda_L)} = \frac{1}{2}\frac{(m_{\bar{K}^0} - m_{K^0}) - (i/2)(\Gamma_{\bar{K}^0} - \Gamma_{K^0})}{\Delta m + i\Delta\Gamma/2}$$

- $\delta \neq 0$ implies CPT violation
- $\varepsilon \neq 0$ implies T violation
- $\varepsilon \neq 0$ or $\delta \neq 0$ implies CP violation

(with a phase convention $\Im\Gamma_{12} = 0$)

$$\Delta m = m_L - m_S \quad , \quad \Delta\Gamma = \Gamma_S - \Gamma_L$$

$$\Delta m = 3.5 \times 10^{-15} \text{ GeV}$$

$$\Delta\Gamma \approx \Gamma_S \approx 2\Delta m = 7 \times 10^{-15} \text{ GeV}$$

The neutral kaon two-level oscillating system in a nutshell

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CPT violation:

$$\delta = \frac{H_{11}-H_{22}}{2\left(\lambda_S-\lambda_L\right)} = \frac{1}{2}\frac{\left(m_{\bar{K}^0}-m_{K^0}\right)-(i/2)\left(\Gamma_{\bar{K}^0}-\Gamma_{K^0}\right)}{\Delta m + i\Delta\Gamma/2}$$

huge amplification factor!!

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neutral kaons vs other oscillating meson systems

	$\langle m \rangle$ (GeV)	Δm (GeV)	$\langle \Gamma \rangle$ (GeV)	$\Delta\Gamma/2$ (GeV)
K^0	0.5	3×10^{-15}	3×10^{-15}	3×10^{-15}
D^0	1.9	6×10^{-15}	2×10^{-12}	1×10^{-14}
B^0_d	5.3	3×10^{-13}	4×10^{-13}	$O(10^{-15})$ (SM prediction)
B^0_s	5.4	1×10^{-11}	4×10^{-13}	3×10^{-14}

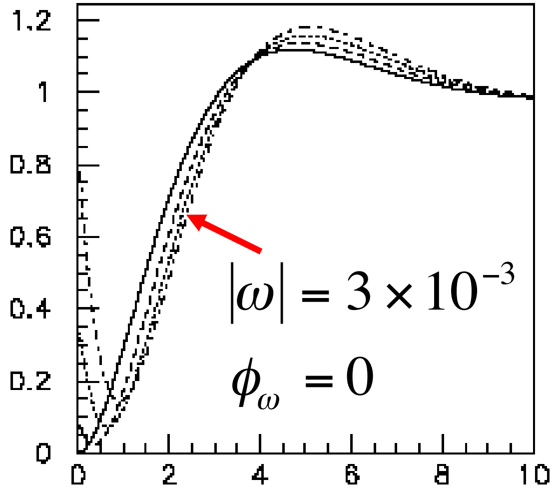
$\phi \to K_S K_L \to \pi^+\pi^- \, \pi^+\pi^-$: CPT violation in entangled K states

In presence of decoherence and CPT violation induced by quantum gravity (CPT operator "ill-defined") the definition of the particle-antiparticle states could be modified. This in turn could induce a breakdown of the correlations imposed by Bose statistics (EPR correlations) to the kaon state:

[Bernabeu, et al. PRL 92 (2004) 131601, NPB744 (2006) 180].

$$|i\rangle \propto \left(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\right) + \omega\left(|K^0\rangle|\bar{K}^0\rangle + |\bar{K}^0\rangle|K^0\rangle\right)$$

$I(\pi^+\pi^-, \pi^+\pi^-; \Delta t)$ (a.u.)



$\Delta t/\tau_S$

at most one expects:

$$|\omega|^2 = O\left(\frac{E^2/M_{PLANCK}}{\Delta\Gamma}\right) \approx 10^{-5} \Rightarrow |\omega| \sim 10^{-3}$$

In some microscopic models of space-time foam arising from non-critical string theory [Bernabeu, Mavromatos, Sarkar PRD 74 (2006) 045014] : $|\omega| \sim 10^{-4} \div 10^{-5}$

The maximum sensitivity to ω is expected for $f_1 = f_2 = \pi^+\pi^-$ (terms: $|\omega|/|\eta_{+-}|$)
All CPTV effects induced by QG $(\alpha, \beta, \gamma, \omega)$ could be simultaneously disentangled.

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$$\propto \left(|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle\right) + \omega\left(|K_S\rangle|K_S\rangle - |K_L\rangle|K_L\rangle\right)$$

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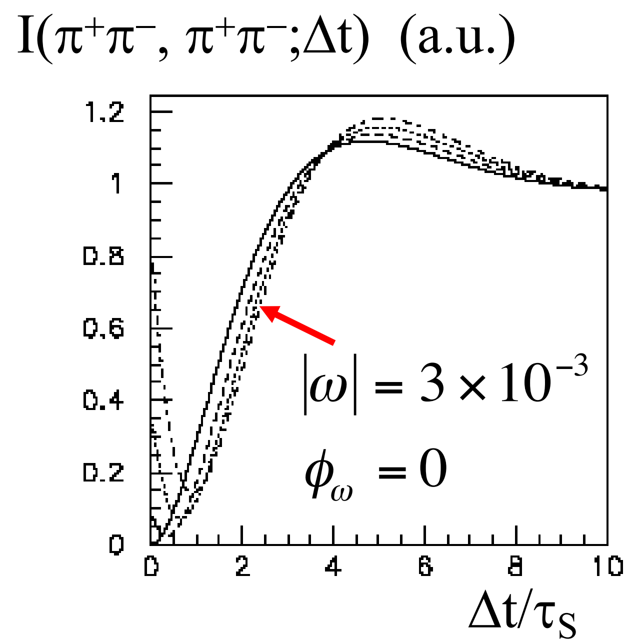

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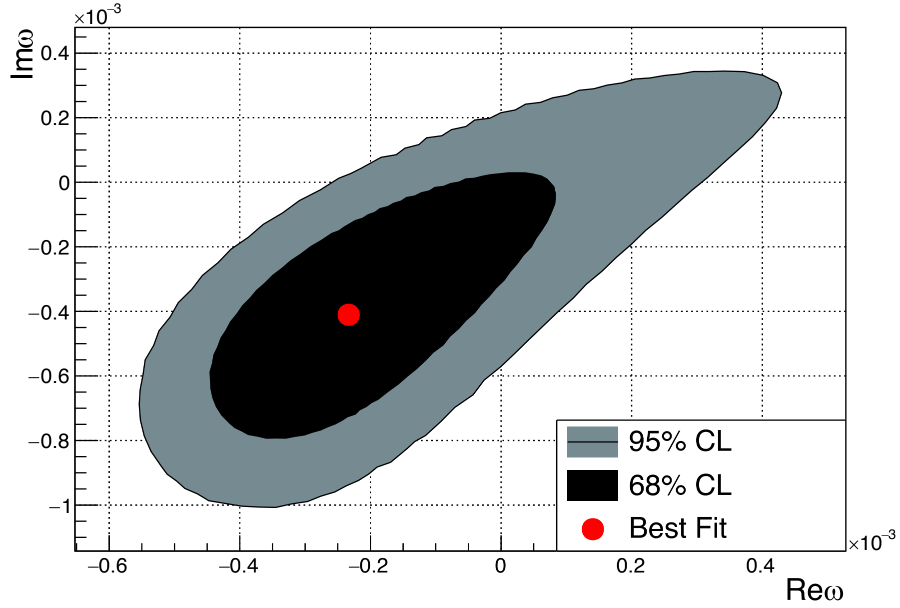
$\phi \to K_S K_L \to \pi^+\pi^- \; \pi^+\pi^-$: CPT violation in entangled K states

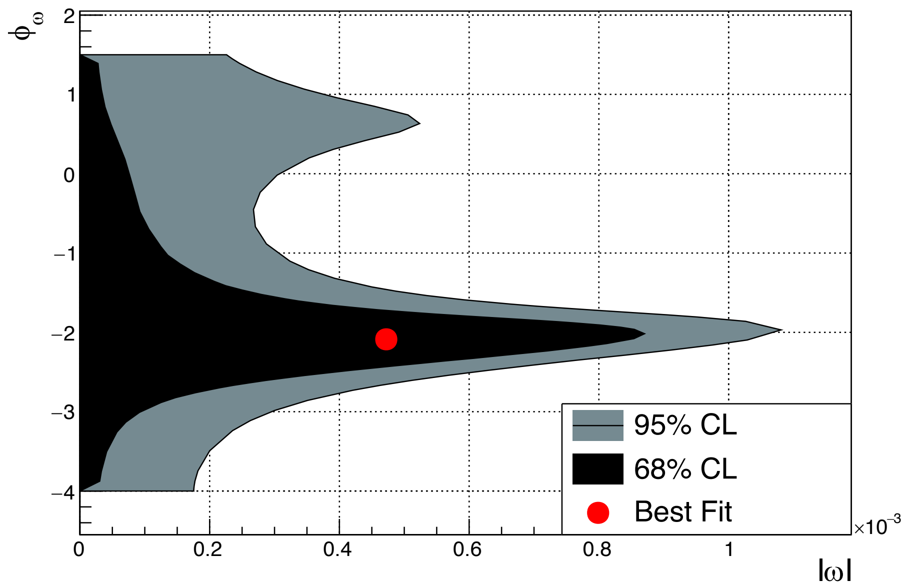
The fit with $I(\pi^+\pi^-, \pi^+\pi^-; \Delta t, \omega)$ yields (1.7 fb-1):

$$\Re\omega = \left(-2.3^{+1.9}_{-1.5}{}_{stat} \pm 0.6_{syst}\right)\times 10^{-4}$$
$$\Im\omega = \left(-4.1^{+2.8}_{-2.6}{}_{stat} \pm 0.9_{syst}\right)\times 10^{-4}$$
$$|\omega| = \left(4.7 \pm 2.9_{stat} \pm 1.0_{syst}\right)\times 10^{-4}$$
$$\phi_\omega = -2.1 \pm 0.2_{stat} \pm 0.1_{syst} \text{ rad}$$



from $|\omega|^2 = \dfrac{BR(\phi \to K_S K_S, K_L K_L)}{BR(\phi \to K_S K_L)}$

$$BR(\phi \to K_S K_S, K_L K_L) < 2.4\times 10^{-7}$$
at 90% C.L.



KLOE-2 JHEP 04 (2022) 059

In the B system:
$-0.0084 \le \Re\omega \le 0.0100$ at 95% C.L.
Alvarez, Bernabeu, Nebot JHEP 11 (2006) 087
(see also Bernabeu et al, EPJC (2017) 77:865)

$\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^- \pi^+\pi^-$: decoherence & CPTV limits

$$\zeta_{0\bar{0}} = (-0.5 \pm 8.0_{stat} \pm 3.7_{syst})\times 10^{-7}$$
$$\zeta_{SL} = (0.1 \pm 1.6_{stat} \pm 0.7_{syst})\times 10^{-2}$$
$$\gamma = (1.3 \pm 9.4_{stat} \pm 4.2_{syst})\times 10^{-22}\ \mathrm{GeV}$$
$$\Re\omega = \left(-2.3^{+1.9}_{-1.5}{}_{stat} \pm 0.6_{syst}\right)\times 10^{-4}$$
$$\Im\omega = \left(-4.1^{+2.8}_{-2.6}{}_{stat} \pm 0.9_{syst}\right)\times 10^{-4}$$
$$|\omega| = (4.7 \pm 2.9_{stat} \pm 1.0_{syst})\times 10^{-4}$$
$$\phi_\omega = -2.1 \pm 0.2_{stat} \pm 0.1_{syst}\ \mathrm{rad}$$

$$\lambda \cong \frac{\zeta_{SL}}{\Gamma_S} = (0.1 \pm 1.2_{stat} \pm 0.5_{syst})\times 10^{-16}\ \mathrm{GeV}$$

$$\mathrm{BR}(\phi \rightarrow K_S K_S, K_L K_L) < 2.4\times 10^{-7}$$
at 90% C.L.

KLOE-2 JHEP 04 (2022) 059

[improvement x2 wrt KLOE PLB 642(2006) 315]

Systematic uncertainties

	$\delta\zeta_{SL}$ $\cdot 10^2$	$\delta\zeta_{0\bar{0}}$ $\cdot 10^7$	$\delta\gamma$ $\cdot 10^{21}$ GeV	$\delta\Re\omega$ $\cdot 10^4$	$\delta\Im\omega$ $\cdot 10^4$	$\delta\|\omega\|$ $\cdot 10^4$	$\delta\phi_\omega$ (rad)
Cut stability	0.56	2.9	0.33	0.53	0.65	0.78	0.07
4π background	0.37	1.9	0.22	0.32	0.19	0.32	0.04
Regeneration	0.17	0.9	0.10	0.06	0.63	0.58	0.05
Δt resolution	0.18	0.9	0.10	0.15	0.09	0.15	0.02
Input phys. const.	0.04	0.2	0.02	0.03	0.09	0.07	0.01
Total	0.71	3.7	0.42	0.64	0.93	1.04	0.10

Direct CPT test in transitions

- CPT theorem holds for any QFT formulated on flat space-time which assumes: (1) Lorentz invariance (2) Locality (3) Unitarity
- Extension of CPT theorem to a theory of quantum gravity far from obvious (e.g. CPT violation appears in several QG models)
- Consequences of CPT symmetry: equality of masses, lifetimes, |q| and |μ| of a particle and its anti-particle.

- Is it possible to test the CPT symmetry directly in transition processes between kaon states, rather than comparing masses, lifetimes, or other intrinsic properties of particle and anti-particle states?

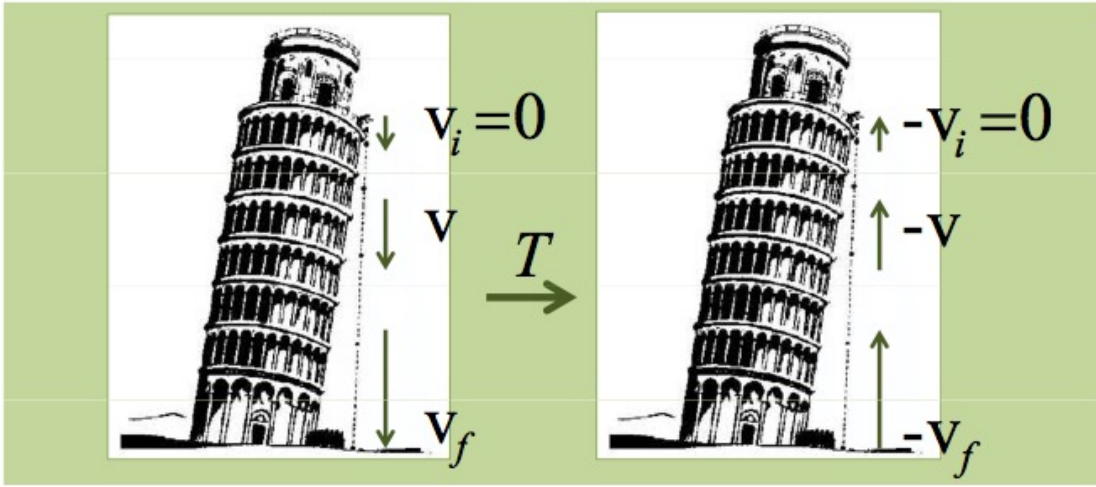
- CPT violating effects may not appear at first order in diagonal mass terms (survival probabilities) while they can manifest at first order in transitions (non-diagonal terms).

- Clean formulation required. Possible spurious effects induced by CP violation in the decay and/or a violation of the $\Delta S = \Delta Q$ rule have to be well under control. Genuine effect must be independent of $\Delta\Gamma$, i.e. not requiring the decay as an essential ingredient.

Probing CPT: J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139
Time-reversal violation: J. Bernabeu, A.D.D., P. Villanueva, NPB 868 (2013) 102

Time Reversal

•The transformation of a system corresponding to the inversion of events in time, or reversed dynamics, with the formal substitution $\Delta t \rightarrow -\Delta t$, is usually called '**time reversal**', but a more appropriate name would actually be **motion reversal**.



•Exchange of in ↔ out states and reversal of all momenta and spins tests time reversal, i.e. the symmetry of the responsible dynamics for the observed process under time reversal (transformation implemented in QM by an antiunitary operator)

•Similarly for CPT tests: the exchange of in ↔ out states etc.. is required.

Direct test of symmetries with neutral kaons

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	$K^0 \to K^0$	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$	$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$
$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$	$K^0 \to K^0$	$K^0 \to K^0$
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	$K^0 \to K_+$	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	$K^0 \to K_-$	$K_- \to K^0$
$K_+ \to K^0$	$K^0 \to K_+$	$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$
$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$	$K_+ \to K^0$	$K^0 \to K_+$
$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$
$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$
$K_- \to K^0$	$K^0 \to K_-$	$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$
$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$	$K_- \to K^0$	$K^0 \to K_-$
$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$
$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$

Direct test of symmetries with neutral kaons

Conjugate= reference

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	~~$K^0 \to K^0$~~	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	~~$K^0 \to \bar{K}^0$~~
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$	$K^0 \to \bar{K}^0$	~~$\bar{K}^0 \to K^0$~~
$\bar{K}^0 \to \bar{K}^0$	~~$\bar{K}^0 \to \bar{K}^0$~~	$K^0 \to K^0$	$K^0 \to K^0$
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	$K^0 \to K_+$	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	$K^0 \to K_-$	$K_- \to K^0$
$K_+ \to K^0$	$K^0 \to K_+$	$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$
$K_+ \to \bar{K}^0$	$\bar{K}^0 \to K_+$	$K_+ \to K^0$	$K^0 \to K_+$
$K_+ \to K_+$	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~
$K_+ \to K_-$	$K_- \to K_+$	~~$K_+ \to K_-$~~	$K_- \to K_+$
$K_- \to K^0$	$K^0 \to K_-$	$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$
$K_- \to \bar{K}^0$	$\bar{K}^0 \to K_-$	$K_- \to K^0$	$K^0 \to K_-$
$K_- \to K_+$	$K_+ \to K_-$	~~$K_- \to K_+$~~	$K_+ \to K_-$
$K_- \to K_-$	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~

Direct test of symmetries with neutral kaons

Conjugate= reference (red)

already in the table with conjugate as reference (black)

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	~~$K^0 \to K^0$~~	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	~~$K^0 \to \bar{K}^0$~~
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	~~$K^0 \to \bar{K}^0$~~	~~$K^0 \to \bar{K}^0$~~	~~$\bar{K}^0 \to K^0$~~
$\bar{K}^0 \to \bar{K}^0$	~~$\bar{K}^0 \to \bar{K}^0$~~	~~$K^0 \to K^0$~~	~~$K^0 \to K^0$~~
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	~~$K^0 \to K_+$~~	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	~~$K^0 \to K_-$~~	$K_- \to K^0$
$K_+ \to K^0$	~~$K^0 \to K_+$~~	$K_+ \to \bar{K}^0$	~~$\bar{K}^0 \to K_+$~~
$K_+ \to \bar{K}^0$	~~$\bar{K}^0 \to K_+$~~	~~$K_+ \to K^0$~~	~~$K^0 \to K_+$~~
$K_+ \to K_+$	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~	~~$K_+ \to K_+$~~
$K_+ \to K_-$	$K_- \to K_+$	~~$K_+ \to K_-$~~	$K_- \to K_+$
$K_- \to K^0$	~~$K^0 \to K_-$~~	$K_- \to \bar{K}^0$	~~$\bar{K}^0 \to K_-$~~
$K_- \to \bar{K}^0$	~~$\bar{K}^0 \to K_-$~~	~~$K_- \to K^0$~~	~~$K^0 \to K_-$~~
$K_- \to K_+$	~~$K_+ \to K_-$~~	~~$K_- \to K_+$~~	~~$K_+ \to K_-$~~
$K_- \to K_-$	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~	~~$K_- \to K_-$~~

Direct test of symmetries with neutral kaons

Conjugate= reference

already in the table with conjugate as reference

Two identical conjugates for one reference

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	$K^0 \to K^0$	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	[redacted]	[redacted]	$\bar{K}^0 \to K^0$
$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$	[redacted]	[redacted]
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	[redacted]	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	[redacted]	$K_- \to K^0$
$K_+ \to K^0$	[redacted]	$K_+ \to \bar{K}^0$	[redacted]
$K_+ \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$
$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$
$K_- \to K^0$	[redacted]	$K_- \to \bar{K}^0$	[redacted]
$K_- \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_- \to K_+$	[redacted]	$K_- \to K_+$	[redacted]
$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$

Direct test of symmetries with neutral kaons

Conjugate= reference (red)

already in the table with conjugate as reference (black)

Two identical conjugates for one reference (blue)

Reference	T-conjugate	CP-conjugate	CPT-conjugate
$K^0 \to K^0$	$K^0 \to K^0$	$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$
$K^0 \to \bar{K}^0$	$\bar{K}^0 \to K^0$	$\bar{K}^0 \to K^0$	$K^0 \to \bar{K}^0$
$K^0 \to K_+$	$K_+ \to K^0$	$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$
$K^0 \to K_-$	$K_- \to K^0$	$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$
$\bar{K}^0 \to K^0$	[redacted]	[redacted]	$\bar{K}^0 \to K^0$
$\bar{K}^0 \to \bar{K}^0$	$\bar{K}^0 \to \bar{K}^0$	[redacted]	[redacted]
$\bar{K}^0 \to K_+$	$K_+ \to \bar{K}^0$	[redacted]	$K_+ \to K^0$
$\bar{K}^0 \to K_-$	$K_- \to \bar{K}^0$	[redacted]	$K_- \to K^0$
$K_+ \to K^0$	[redacted]	$K_+ \to \bar{K}^0$	[redacted]
$K_+ \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$	$K_+ \to K_+$
$K_+ \to K_-$	$K_- \to K_+$	$K_+ \to K_-$	$K_- \to K_+$
$K_- \to K^0$	[redacted]	$K_- \to \bar{K}^0$	[redacted]
$K_- \to \bar{K}^0$	[redacted]	[redacted]	[redacted]
$K_- \to K_+$	[redacted]	$K_- \to K_+$	[redacted]
$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$	$K_- \to K_-$

4 distinct tests of T symmetry

4 distinct tests of CP symmetry

4 distinct tests of CPT symmetry

Direct test of CPT symmetry in neutral kaon transitions

Two observable ratios of double decay intensities at ϕ-factory

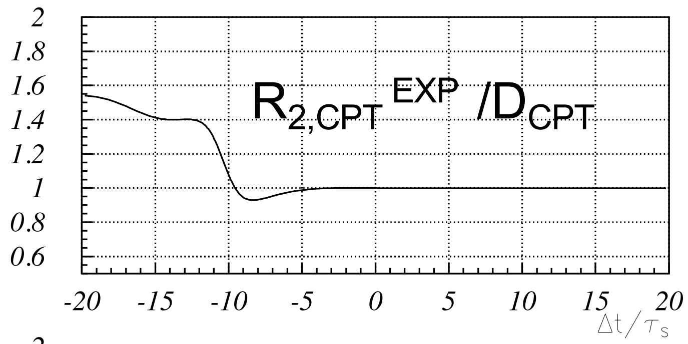
for $\Delta t<0$ | for $\Delta t>0$

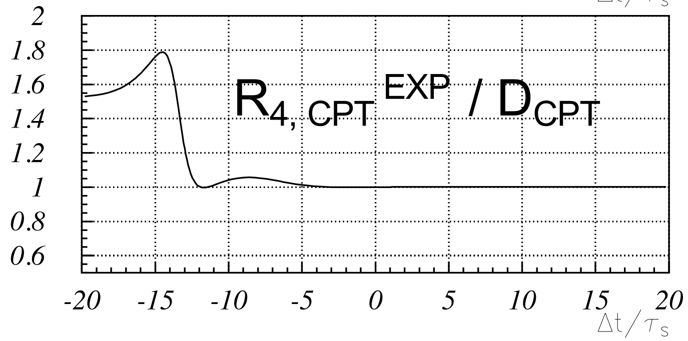
$$R^{\text{exp}}_{2,\text{CPT}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_{1,\text{CPT}}(|\Delta t|) \times D_{\text{CPT}} \quad = R_{2,\text{CPT}}(\Delta t) \times D_{\text{CPT}}$$

$$R^{\text{exp}}_{4,\text{CPT}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_{3,\text{CPT}}(|\Delta t|) \times D_{\text{CPT}} \quad = R_{4,\text{CPT}}(\Delta t) \times D_{\text{CPT}}$$

with D_{CPT} constant

$$D_{\text{CPT}} = \frac{\text{BR}\left(\text{K}_\text{L} \to 3\pi^0\right)}{\text{BR}\left(\text{K}_\text{S} \to \pi\pi\right)} \frac{\Gamma_L}{\Gamma_S}$$





The region $\Delta t>0$ is statistically most populated at KLOE

for visualization purposes, plots with $\text{Re}(\delta)=3.3\ 10^{-4}$ $\text{Im}(\delta)=1.6\ 10^{-5}$

Direct test of CPT symmetry in neutral kaon transitions

Two observable ratios of double decay intensities at ϕ-factory

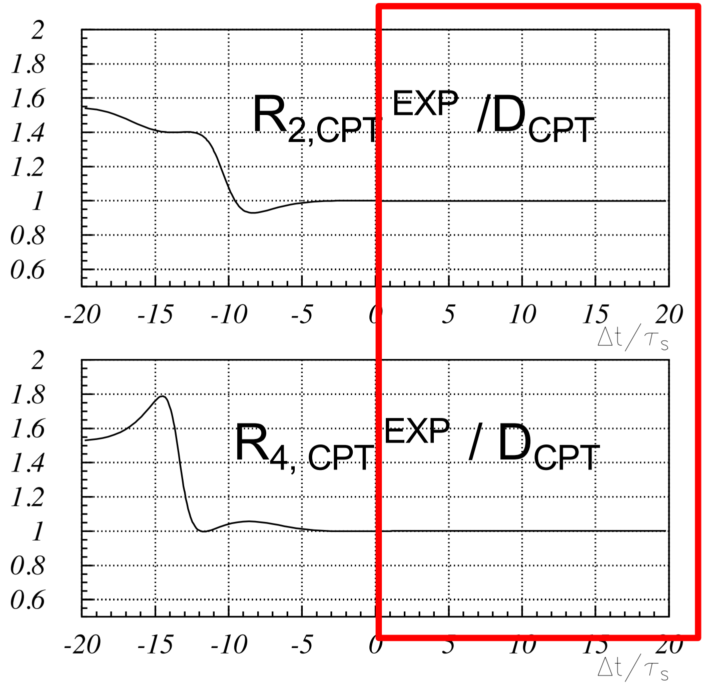
for $\Delta t<0$ | for $\Delta t>0$

$$R_{2,\mathrm{CPT}}^{\mathrm{exp}}(\Delta t) \equiv \frac{I(\ell^-,3\pi^0;\Delta t)}{I(\pi\pi,\ell^-;\Delta t)} = R_{1,\mathrm{CPT}}(|\Delta t|) \times D_{\mathrm{CPT}} \quad = R_{2,\mathrm{CPT}}(\Delta t) \times D_{\mathrm{CPT}}$$

$$R_{4,\mathrm{CPT}}^{\mathrm{exp}}(\Delta t) \equiv \frac{I(\ell^+,3\pi^0;\Delta t)}{I(\pi\pi,\ell^+;\Delta t)} = R_{3,\mathrm{CPT}}(|\Delta t|) \times D_{\mathrm{CPT}} \quad = R_{4,\mathrm{CPT}}(\Delta t) \times D_{\mathrm{CPT}}$$

with D_{CPT} constant

$$D_{\mathrm{CPT}} = \frac{\mathrm{BR}\left(\mathrm{K_L} \to 3\pi^0\right)}{\mathrm{BR}\left(\mathrm{K_S} \to \pi\pi\right)}\frac{\Gamma_L}{\Gamma_S}$$



The region $\Delta t>0$ is statistically most populated at KLOE

for visualization purposes, plots with
$Re(\delta)=3.3\ 10^{-4}$ $Im(\delta)=1.6\ 10^{-5}$

Direct test of T symmetry in neutral kaon transitions

Two observable ratios of double decay intensities at ϕ-factory

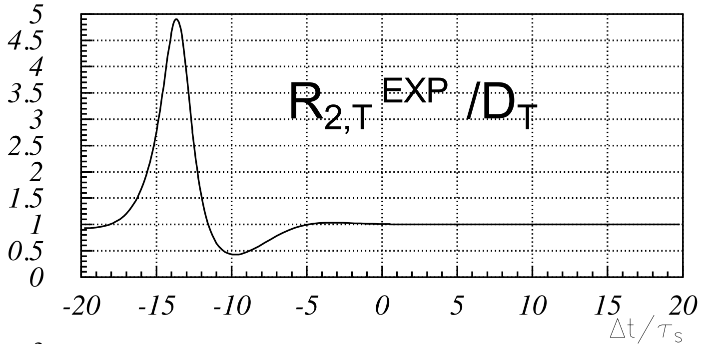
for $\Delta t<0$ for $\Delta t>0$

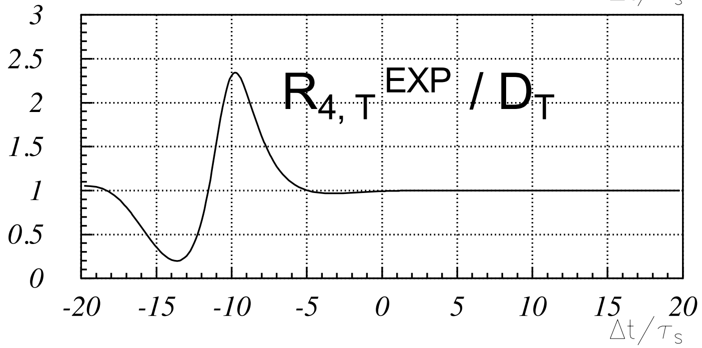
$$R_{2,\mathcal{T}}^{\exp}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_{1,\mathrm{T}}(|\Delta t|) \times D_\mathrm{T} = R_{2,\mathrm{T}}(\Delta t) \times D_\mathrm{T}$$

$$R_{4,\mathcal{T}}^{\exp}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_{3,\mathrm{T}}(|\Delta t|) \times D_\mathrm{T} = R_{4,\mathrm{T}}(\Delta t) \times D_\mathrm{T}$$

with $D_T = D_{CPT}$ constant

$$D_{\mathrm{CPT}} = \frac{\mathrm{BR}\left(\mathrm{K_L} \to 3\pi^0\right)}{\mathrm{BR}\left(\mathrm{K_S} \to \pi\pi\right)} \frac{\Gamma_L}{\Gamma_S}$$





The region $\Delta t>0$ is statistically most populated at KLOE

for visualization purposes, plots with CP violation in the mixing from PDG and CPT invariance

Direct test of T symmetry in neutral kaon transitions

Two observable ratios of double decay intensities at ϕ-factory

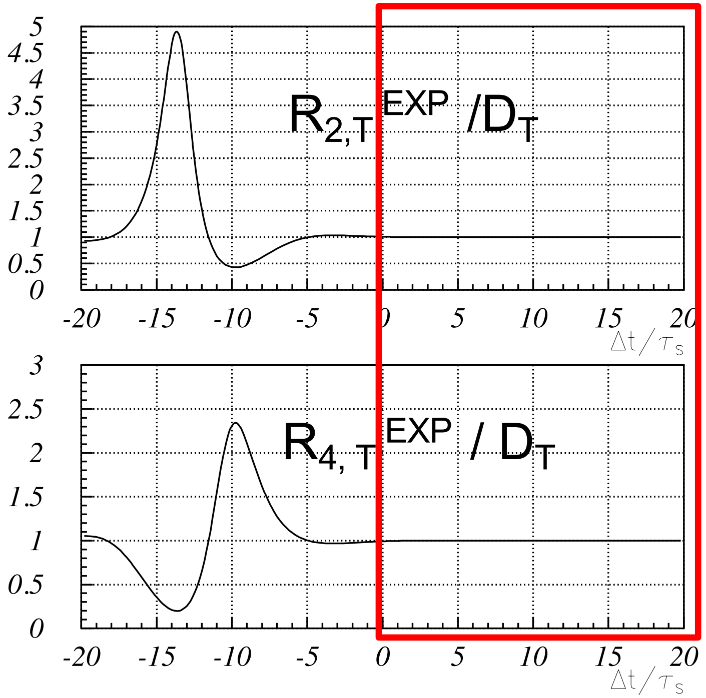
for $\Delta t<0$ | for $\Delta t>0$

$$R^{\mathrm{exp}}_{2,\mathcal{T}}(\Delta t) \equiv \frac{I(\ell^-,3\pi^0;\Delta t)}{I(\pi\pi,\ell^+;\Delta t)} = R_{1,\mathrm{T}}(|\Delta t|)\times D_{\mathrm{T}} = R_{2,\mathrm{T}}(\Delta t)\times D_{\mathrm{T}}$$

$$R^{\mathrm{exp}}_{4,\mathcal{T}}(\Delta t) \equiv \frac{I(\ell^+,3\pi^0;\Delta t)}{I(\pi\pi,\ell^-;\Delta t)} = R_{3,\mathrm{T}}(|\Delta t|)\times D_{\mathrm{T}} = R_{4,\mathrm{T}}(\Delta t)\times D_{\mathrm{T}}$$

with $D_T = D_{CPT}$ constant

$$D_{\mathrm{CPT}} = \frac{\mathrm{BR}\left(\mathrm{K_L}\to 3\pi^0\right)}{\mathrm{BR}\left(\mathrm{K_S}\to\pi\pi\right)}\frac{\Gamma_L}{\Gamma_S}$$



The region $\Delta t>0$ is statistically most populated at KLOE

for visualization purposes, plots with CP violation in the mixing from PDG and CPT invariance

Direct test of T symmetry in neutral kaon transitions

Two observable ratios of double decay intensities at ϕ-factory

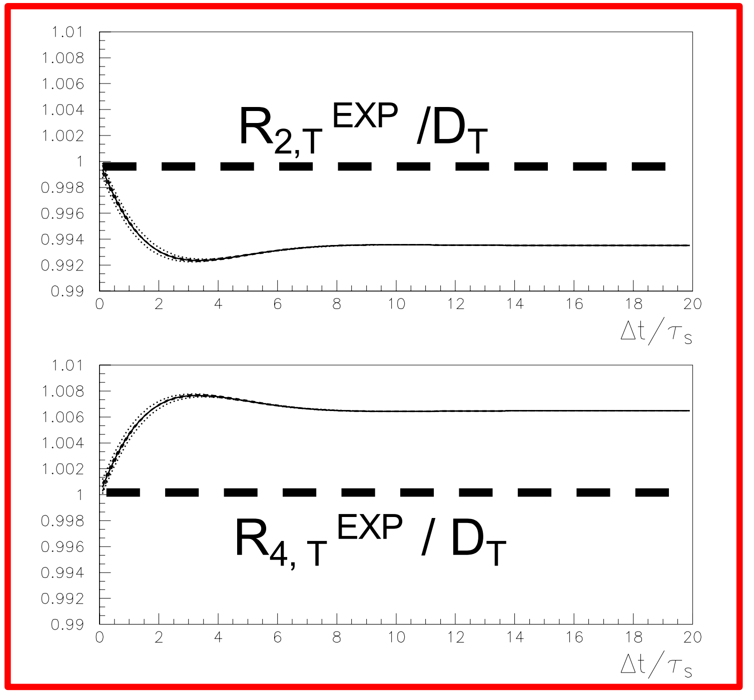
for Δt<0 | for Δt>0

$$R_{2,\mathcal{T}}^{\rm exp}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_{1,\rm T}(|\Delta t|) \times D_{\rm T} = R_{2,\rm T}(\Delta t) \times D_{\rm T}$$

$$R_{4,\mathcal{T}}^{\rm exp}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_{3,\rm T}(|\Delta t|) \times D_{\rm T} = R_{4,\rm T}(\Delta t) \times D_{\rm T}$$

with $D_T = D_{CPT}$ constant

$$D_{\rm CPT} = \frac{{\rm BR}\left({\rm K_L} \to 3\pi^0\right)}{{\rm BR}\left({\rm K_S} \to \pi\pi\right)} \frac{\Gamma_L}{\Gamma_S}$$



The region Δt>0 is statistically most populated at KLOE

for visualization purposes, plots with CP violation in the mixing from PDG and CPT invariance

Impact of the approximations on the tests

In general K_+, K_- and $K^0, \overline{K}^0$ can be non-orthogonal bases

T test

Assumes $\Delta S = \Delta Q$ rule and negligible direct CP/CPT violation.
In the limit $\Delta t \gg \tau_S$ negligible contaminations from direct CP violation.

CPT test

Assumes $\Delta S = \Delta Q$ rule and negligible direct CP/CPT violation.
In the limit $\Delta t \gg \tau_S$ negligible contaminations from direct CP violation.
The double ratio constitutes one of the most robust observables for the proposed CPT test. In the limit $\Delta t \gg \tau_S$ it exhibits a pure and genuine CPT violating effect, without assuming negligible contaminations from direct CP violation and/or $\Delta S = \Delta Q$ rule violation.

$$DR_{CPT} = \frac{R^{\exp}_{2,\mathrm{CPT}}(\Delta t \gg \tau_S)}{R^{\exp}_{4,\mathrm{CPT}}(\Delta t \gg \tau_S)} = 1 - 8\Re\delta - 8\Re x_-$$

CLEANEST MODEL INDEPENDENT CPT OBSERVABLE