Quantum Tomography @ Colliders

~~---To spin or not to spin---~~

decay or not decay

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Quantum Tests in Collider Physics

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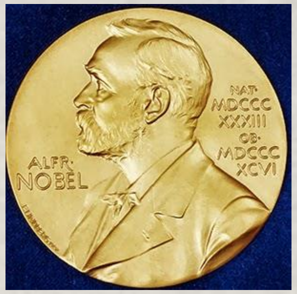
K. Cheng, TH, M. Low, arXiv: 2311.09166; 2407.01672; 2410.xxxxx

2022 Nobel Prize for physics:
"pioneering quantum information science”



EPR :

Go! QM Go!

QFT: most precise theory in science!

Our goals:

In the framework of QFT, in the HE regime at colliders,

- We lay out the QM predictions / information.
- We calculate the QM correlations / entanglement
- Hope to establish the quantum tomography.
- Understand quantum nature & seek for BSM effects.

Quantum State

For a state vector $|\phi_i\rangle$

Density matrix

a state

$$\rho = \sum_i n_i |\phi_i\rangle \langle\phi_i|$$

an observable

$$\langle\mathcal{O}\rangle = \mathrm{Tr}(\mathcal{O}\rho)$$

For a pure state: $n_i = 1$; for a mixed state: $\boldsymbol{\Sigma}_i n_i = 1$.

For a **single qubit** (*i.e.*, a doublet of spin, iso-spin etc.):

$$\rho = \frac{1}{2}\left(\mathbb{I}_2 + \sum_i B_i \sigma_i\right)$$

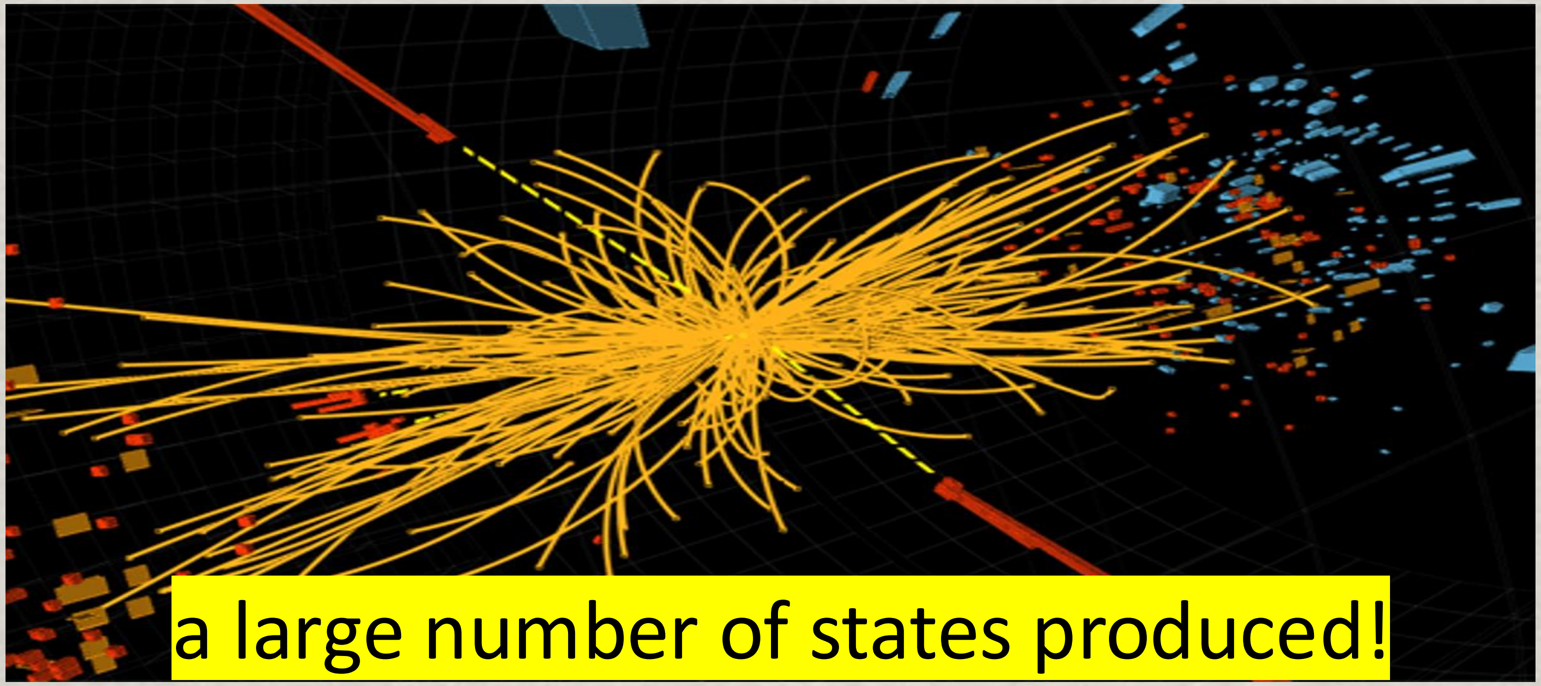
For a bipartite system (*i.e.*, ½⊗½)

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i \left(B_i^{\mathcal{A}}\,(\sigma_i \otimes \mathbb{I}_2) + B_i^{\mathcal{B}}\,(\mathbb{I}_2 \otimes \sigma_i)\right) + \sum_{i,j} C_{ij}\,(\sigma_i \otimes \sigma_j)\right)$$

$B_i^{A,B}$ the polarizations, C_{ij} the spin-correlation matrix

The 15 coefficients → **Quantum Tomography** for the bipartite.

Quantum Tomography @ Colliders



It is extremely challenging
for signal identification,
leave alone for the quantum information!

Hope to do well :

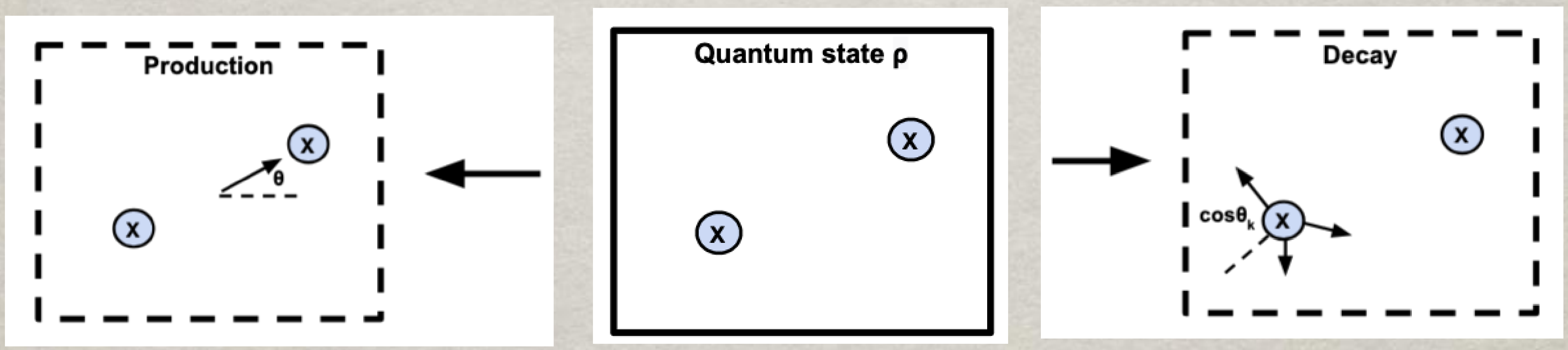
- Energy & momentum
- Angles & directions
- Charges & particle IDs
- Position / timing information

Quantum Tomography @ Colliders

- All "classical observables"
- No direct spin measurement

→ inferred by angular distributions, and statistically: "fictitious states"!

Two paths to proceed to quantum tomography



$$\rho = \frac{1}{4}\Big(\mathbb{I}_4 + \sum_i \left(B_i^{\mathcal{A}}\,(\sigma_i \otimes \mathbb{I}_2) + B_i^{\mathcal{B}}\,(\mathbb{I}_2 \otimes \sigma_i)\right) + \sum_{i,j} C_{ij}\,(\sigma_i \otimes \sigma_j)\Big)$$

Both the state before decay & the final state decay products inherit the SAME quantum information!

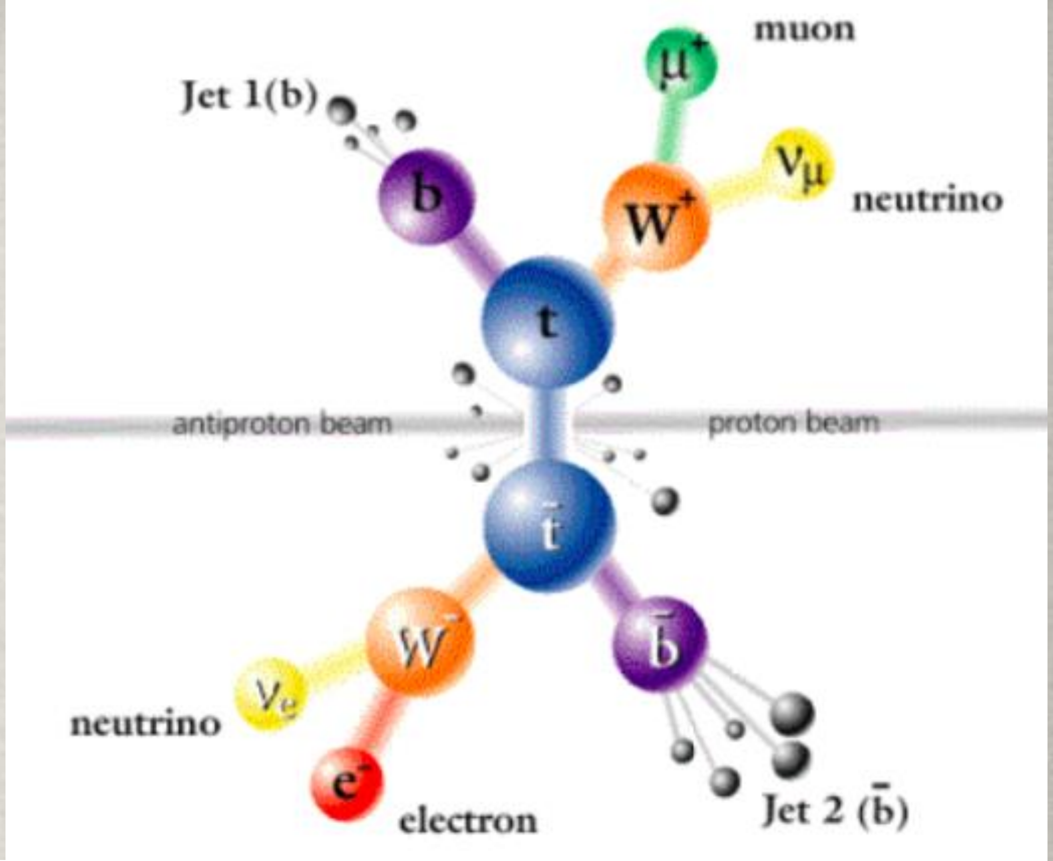
Top decay & spin correlation

decaying to $A_{1,2,3}$, $B_{1,2,3}$

$$\sigma(XY \to t\bar{t} \to (A_1A_2A_3)(B_1B_2B_3)) = \int d\Omega^A d\Omega^B \left(\frac{d\Gamma_{ab}}{d\Omega^A}\right) R_{ab,\bar{a}\bar{b}} \left(\frac{d\bar{\Gamma}_{\bar{a}\bar{b}}}{d\Omega^B}\right)$$

$$\frac{d\Gamma_{ab}}{d\Omega} \propto \delta_{ab} + \kappa\, \sigma^i_{ab}\, \Omega^i$$

Spin analyzing power (κ)



$$\Rightarrow \frac{1}{\sigma}\frac{d\sigma}{d\Omega^A d\Omega^B} = \frac{1}{(4\pi)^2}\left(1 + \kappa^A P^A_i \Omega^A_i + \kappa^B P^B_i \Omega^B_i + \kappa^A\kappa^B \Omega^A_i C_{ij} \Omega^B_j\right)$$

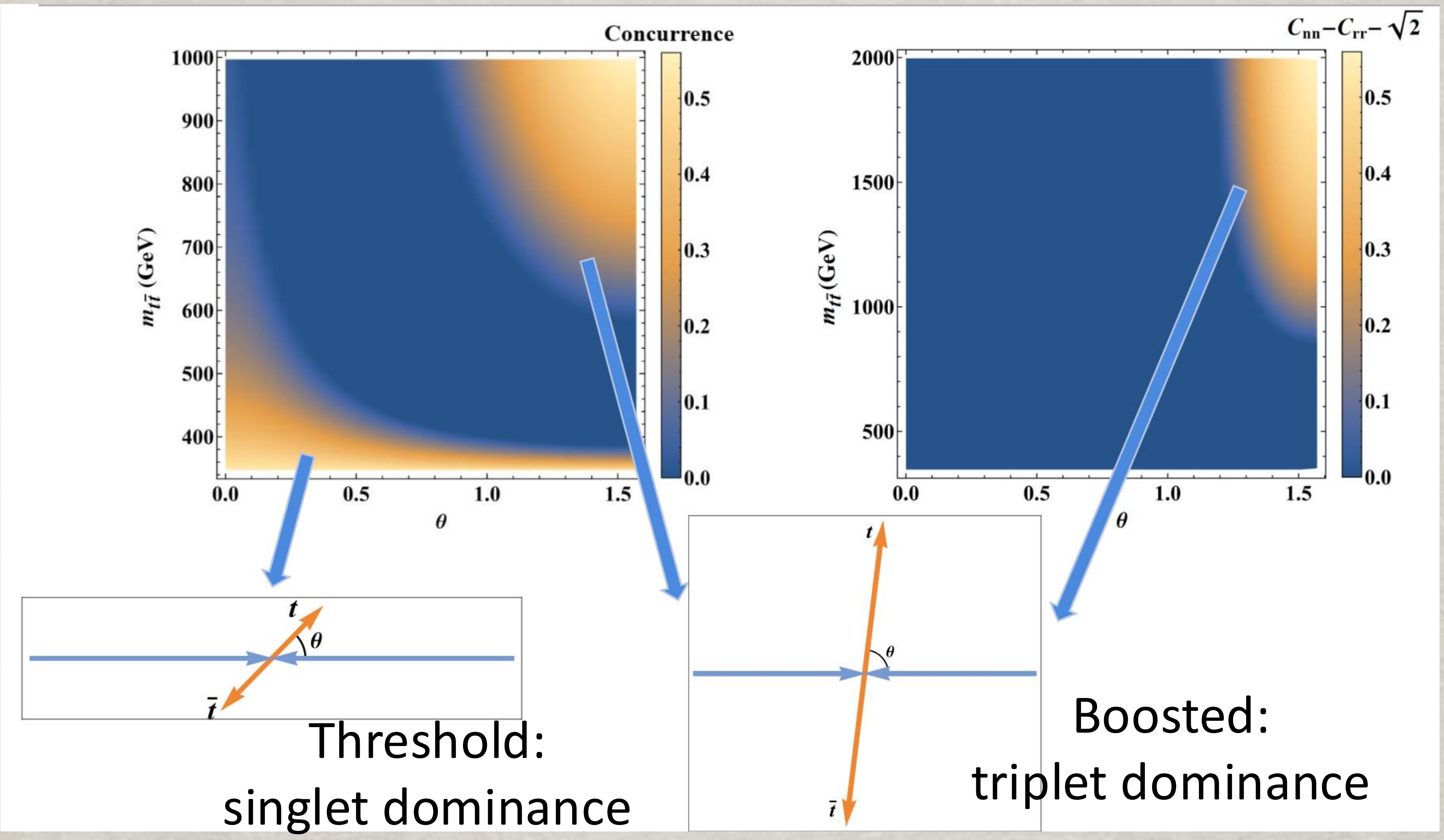
Direction of A, B (Ω^A, Ω^B)

$$\Rightarrow \frac{1}{\sigma}\frac{d\sigma}{d(\cos\theta^A_i \cos\theta^B_j)} = -\frac{1 + \kappa^A\kappa^B C_{ij}\cos\theta^A_i\cos\theta^B_j}{2}\log|\cos\theta^A_i\cos\theta^B_j|$$

Polar angle of A with respect to the i-th axis (θ^A_i)

$$\Rightarrow C_{ij} = \frac{4}{\kappa^A\kappa^B}\frac{N(\cos\theta^A_i\cos\theta^B_j > 0) - N(\cos\theta^A_i\cos\theta^B_j < 0)}{N(\cos\theta^A_i\cos\theta^B_j > 0) + N(\cos\theta^A_i\cos\theta^B_j < 0)}$$

Theory & Observation:



- Threshold:

high rate, low sensitivity

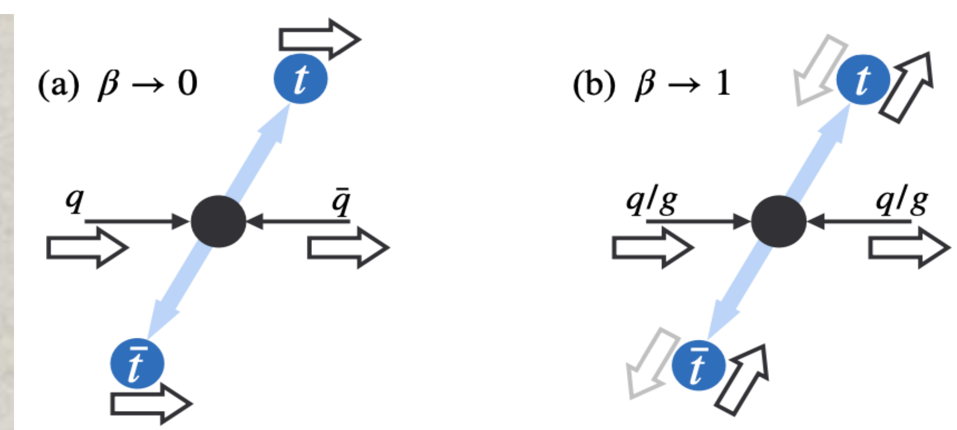
- Highly boosted:

Low rate, high sensitivity

(Many theory papers; ATLAS; CMS …)

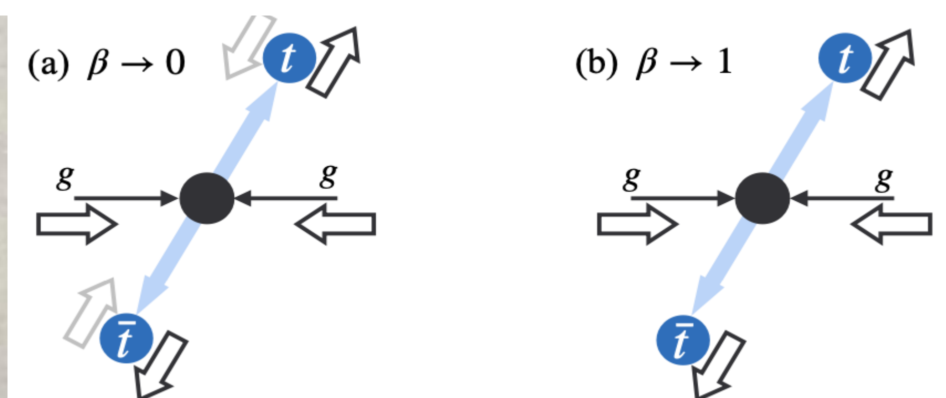
Kinematic Approach for 2 → 2 Production

QCD production	$\overline{\sum}\lvert\mathcal{M}\rvert^2$	spin correlation matrix C_{ij}	ξ
$q\bar{q} \to t\bar{t}$	$\kappa_q\left(2-\beta^2 s_\theta^2\right)$	$\begin{pmatrix} \frac{(2-\beta^2)s_\theta^2}{2-\beta^2 s_\theta^2} & 0 & -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} \\ 0 & \frac{-\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} & 0 \\ -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} & 0 & \frac{2c_\theta^2+\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} \end{pmatrix}$	$\tan\xi = \frac{1}{\gamma}\tan\theta$
$g_L g_R \to t\bar{t}$	$\kappa_g \beta^2 s_\theta^2(2-\beta^2 s_\theta^2)$		$\tan\xi = \frac{1}{\gamma}\tan\theta$
$g_L g_L/g_R g_R \to t\bar{t}$	$\kappa_g(1-\beta^4)$	$\begin{pmatrix} \frac{\beta^2-1}{\beta^2+1} & 0 & 0 \\ 0 & \frac{\beta^2-1}{\beta^2+1} & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\xi = 0$



All C_{ij} encoded in the production kinematics:

θ & **β**$=(1-4m_t^2/m_{tt}^2)^{1/2}$

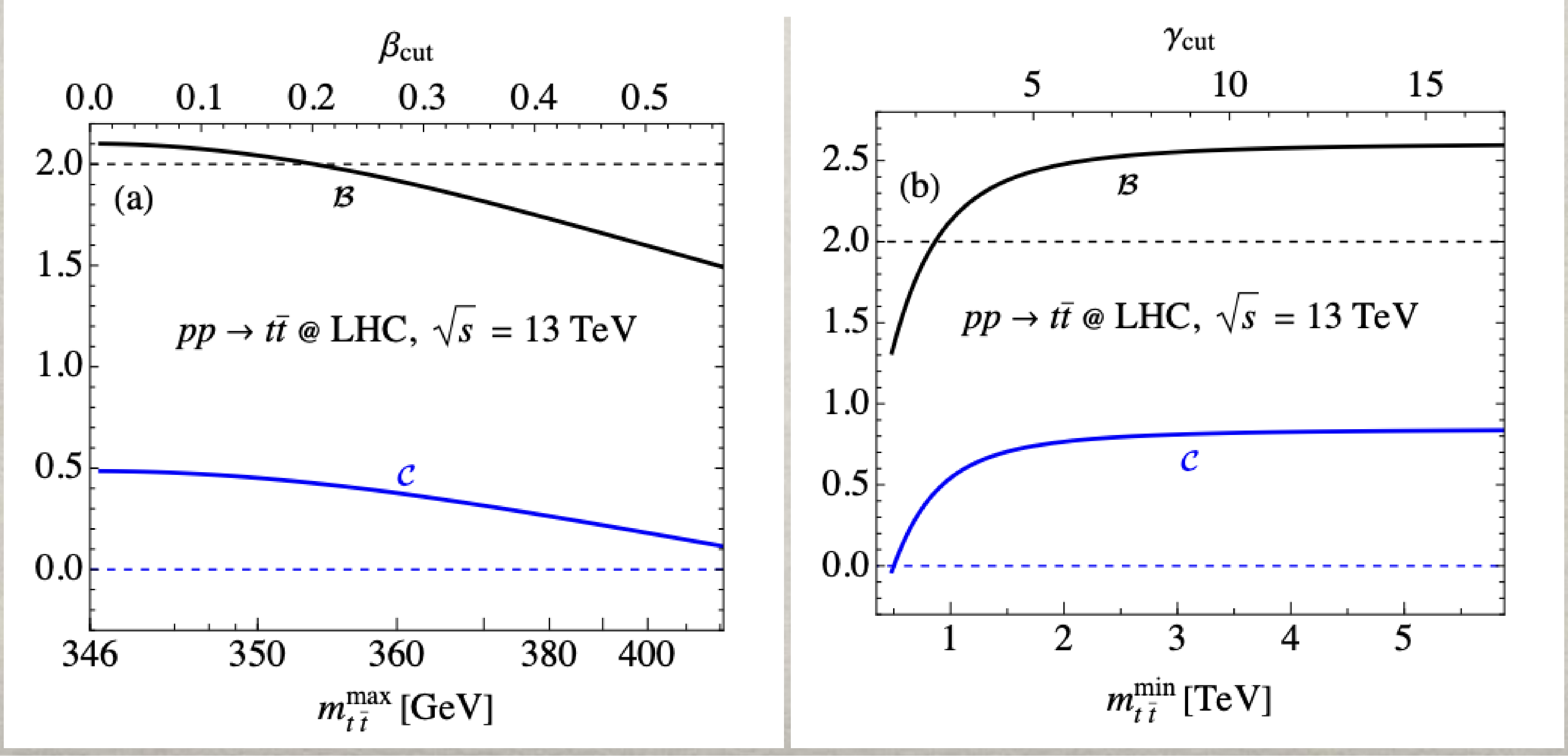


Quantum Entanglement from production: without decay measurement

$$\rho(\Theta,\beta)=\frac{L_{q\bar q}|\mathcal{M}_{q\bar q\to t\bar t}|^2\rho_{q\bar q\to t\bar t}+L_{gg}|\mathcal{M}_{gg\to t\bar t}|^2\rho_{gg\to t\bar t}}{L_{q\bar q}|\mathcal{M}_{q\bar q\to t\bar t}|^2+L_{gg}|\mathcal{M}_{gg\to t\bar t}|^2},$$

$$F_{\mathcal{C}}(\Theta,\beta)=\frac{C_{11}(\Theta,\beta)+C_{22}(\Theta,\beta)-C_{33}(\Theta,\beta)-1}{2}=\frac{\beta^2 s_\Theta^2}{2-\beta^2 c_\Theta}$$

$$F_{\mathcal{B}}(\Theta,\beta)=\sqrt{2}|C_{22}(\Theta,\beta)-C_{33}(\Theta,\beta)|=\frac{2\sqrt{2}}{2-\beta^2 s_\Theta^2}$$



Observability: Error Estimation

Kinematic Approach:

$$\frac{\Delta\mathcal{C}}{\mathcal{C}} \leq \frac{1}{4\langle F_{\mathcal{C}}(\Theta,\beta)\rangle}\frac{1}{\sqrt{N}}$$

$$\frac{\Delta\mathcal{B}}{\mathcal{B}} \leq \frac{1}{\langle F_{\mathcal{B}}(\Theta,\beta)\rangle}\frac{1}{\sqrt{N}}.$$

Decay Approach:

$$\frac{\Delta\mathcal{C}}{\mathcal{C}} = \frac{3\sqrt{3}}{(|C_{11}+C_{22}|-C_{33}-1)}\frac{1}{\sqrt{N}}$$

$$\frac{\Delta\mathcal{B}}{\mathcal{B}} = \frac{3\sqrt{2}}{|C_{22}-C_{33}|}\frac{1}{\sqrt{N}}.$$

	cuts	$\mathcal{C}$	$\Delta\mathcal{C}^{\rm stat}$	$\mathcal{B}$	$\Delta\mathcal{B}^{\rm stat}$
Production	$m_{t\bar{t}} < 350\,\mathrm{GeV}$	0.45	1.0×10^{-4}	2.04	1.4×10^{-4}
Decay			1.4×10^{-2}		6.3×10^{-2}
Production	$m_{t\bar{t}} > 1.5\,\mathrm{TeV}$	0.70	2.6×10^{-3}	2.37	3.8×10^{-3}
Decay	$\lvert\cos\Theta\rvert < 0.5$		5.6×10^{-2}		0.26

TABLE I. Statistical uncertainties on the $\mathcal{C}$ and $\mathcal{B}$ measurements for $pp\to t\bar{t}$ with represetative selection cuts. The production rate is given by $N_{t\bar{t}} = \mathcal{L}\sigma_{pp\to t\bar{t}}$ with 300 fb^{-1} luminosity. The di-leptonic decay branching fractions are included in the decay approach without other kinematic cuts.

Kinematic approach is optimal!

Ultimately, systematic dominance!

perhaps ~ 1% level

Quantum Entanglement from production: With stable particles

Entanglement in Drell-Yan Production

$pp \to Z \to \mu^+\mu^-$

$$C_{ij} = \begin{pmatrix} \frac{s_\Theta^2((2-\beta^2)g_V^2-\beta^2 g_A^2)}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta+g_V^2(2-\beta^2 s_\Theta^2)} & 0 & -\frac{2g_V s_\Theta\sqrt{1-\beta^2}(g_V c_\Theta-g_A\beta)}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta+g_V^2(2-\beta^2 s_\Theta^2)} \\ 0 & \frac{(g_V^2-g_A^2)\beta^2 s_\Theta^2}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta+g_V^2(2-\beta^2 s_\Theta^2)} & 0 \\ -\frac{2g_V s_\Theta\sqrt{1-\beta^2}(g_V c_\Theta-g_A\beta)}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta+g_V^2(2-\beta^2 s_\Theta^2)} & 0 & \frac{g_V^2(2c_\Theta^2+\beta^2 s_\Theta^2)+g_A^2\beta^2(1+c_\Theta^2)-4g_V g_A\beta}{\beta^2 g_A^2(1+c_\Theta^2)-4\beta g_A g_V c_\Theta+g_V^2(2-\beta^2 s_\Theta^2)} \end{pmatrix}$$

Taking **β** =1 →

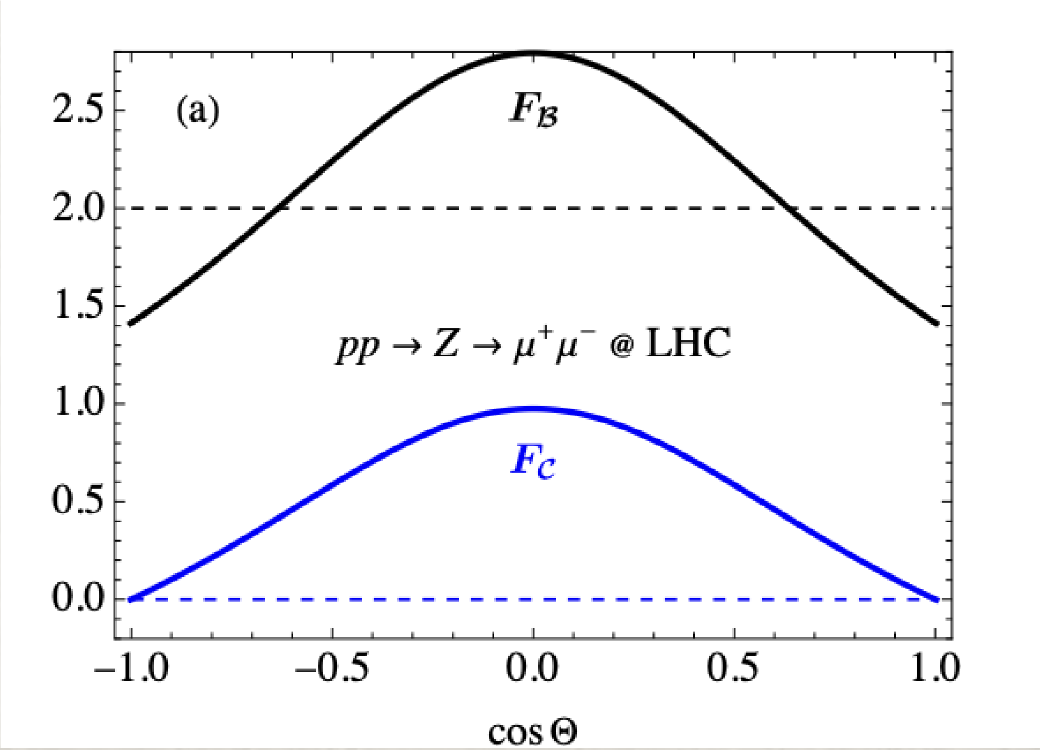
$$C_{ij} = \begin{pmatrix} \frac{s_\Theta^2(g_V^2-g_A^2)}{(1+c_\Theta^2)(g_A^2+g_V^2)} & 0 & 0 \\ 0 & -\frac{s_\Theta^2(g_V^2-g_A^2)}{(1+c_\Theta^2)(g_A^2+g_V^2)} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

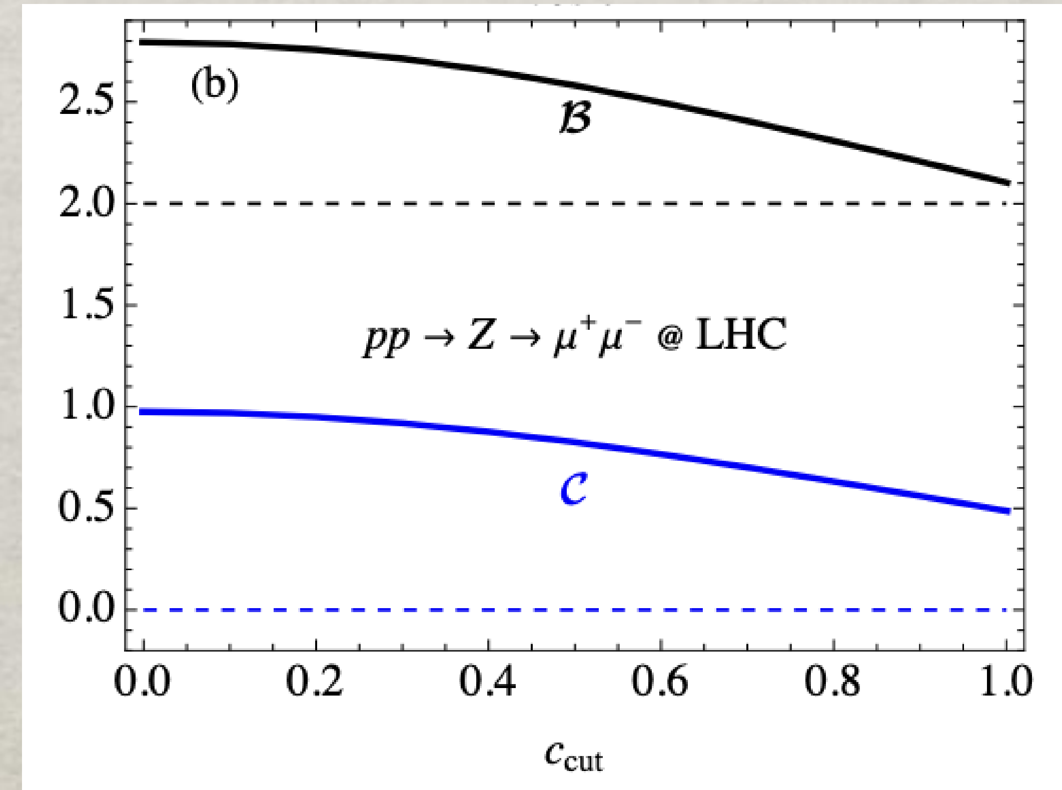
$$F_{\mathcal{C}}(\Theta,\beta=1) = \frac{1}{2}(|C_{rr}-C_{nn}|+C_{kk}-1) = \frac{s_\Theta^2(g_A^2-g_V^2)}{(1+c_\Theta^2)(g_A^2+g_V^2)}$$

$$F_{\mathcal{B}}(\Theta,\beta=1) = \sqrt{2}(C_{kk}+C_{nn}) = \frac{2\sqrt{2}\,(g_A^2+g_V^2 c_\Theta^2)}{(1+c_\Theta^2)\,(g_A^2+g_V^2)}.$$

Entanglement in Drell-Yan Production

$$pp \to Z \to \mu^+\mu^-$$

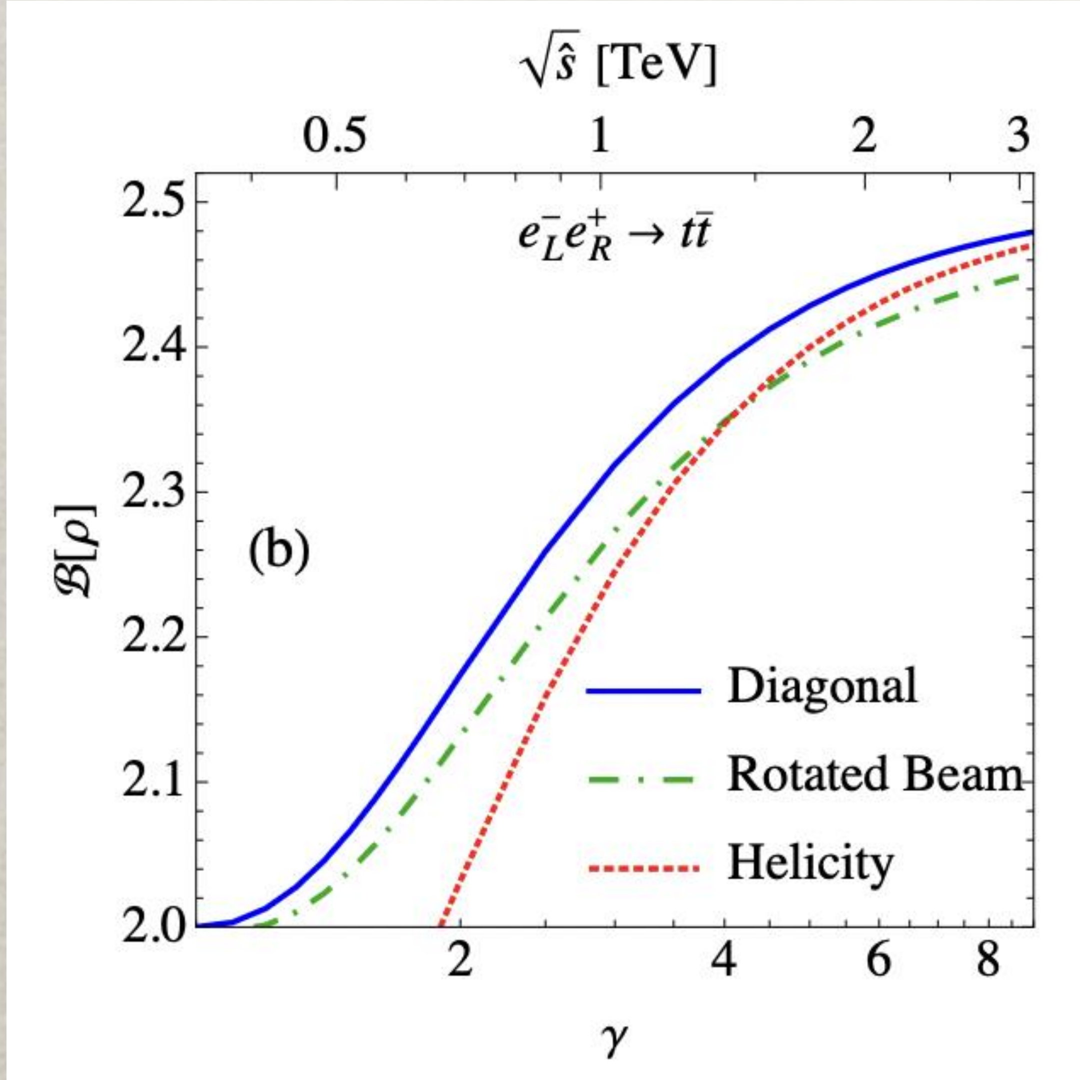


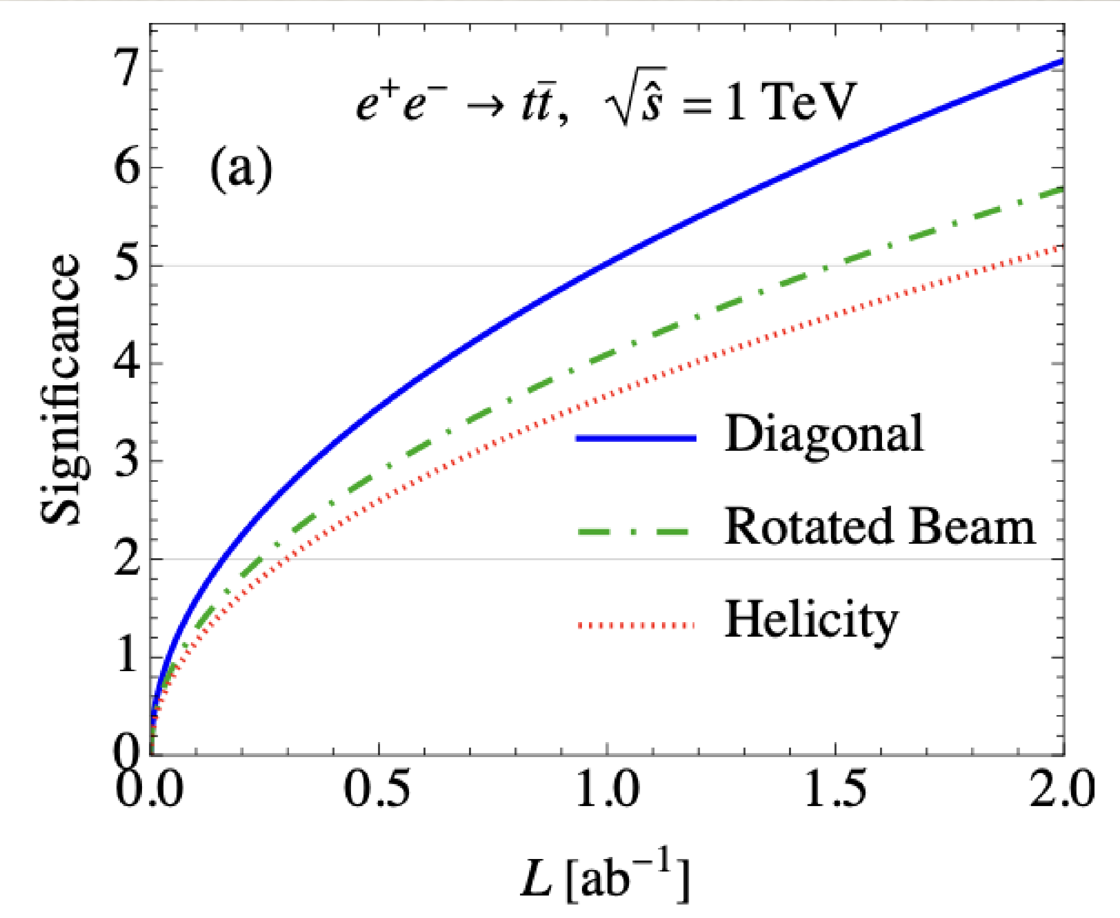


Such a simple process, simple kinematics!
(who would have thought about this?!)

Kinematic approach equally applicable to $e^+e^- \rightarrow f\bar{f}$

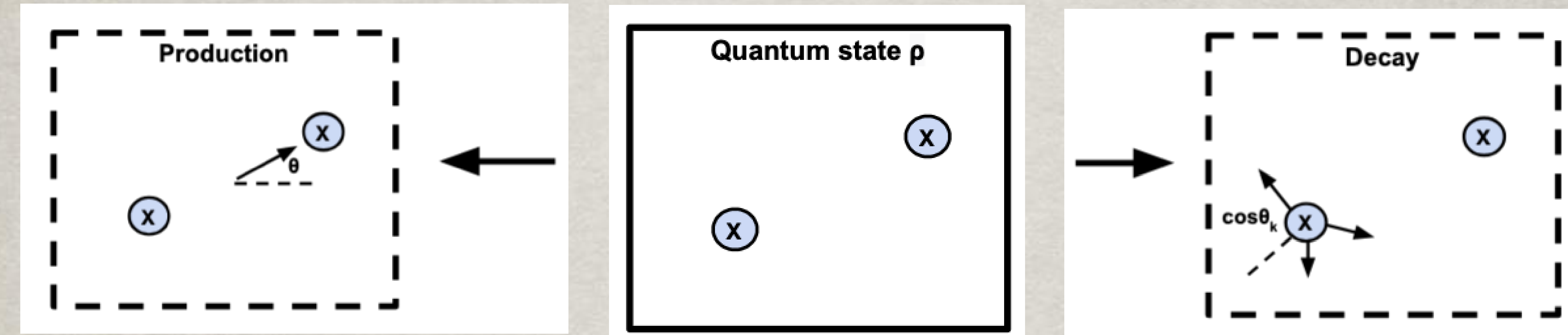
$$C_{ij} \sim \begin{pmatrix} s_\theta^2(f_V^2(2-\beta^2) - f_A^2\beta^2) & 0 & -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} \\ 0 & (f_A^2 - f_V^2)\beta^2 s_\theta^2 & 0 \\ -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} & 0 & f_V^2(2c_\theta^2 + \beta^2 s_\theta^2) + f_A^2\beta^2(1+c_\theta^2) \pm 4 f_V f_A \beta \end{pmatrix}$$





Conclusions

Two paths to proceed to quantum tomography



Production kinematic approach -- **exciting new avenue!**

- enables using particles not to decay
- easier to implement: only the speed and moving direction
- applicable to other 2 → 2 systems: W^+W^-,… **two-qudits.**
- more theoretical input on C_{ij} relations

Decay approach:

- more measurements, fewer theory relations
- lower statistics and more systematics

Significant complementarity!