Entanglement and Bell inequality violation in vector diboson systems produced in decays of spin-0 particles

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- We assume the most general CPT conserving, Lorentz-invariant coupling of the spin-0 particle with the daughter bosons.
- We show that the two-boson state is entangled and violates the CGLMP inequality for all values of the (anomalous) coupling constants and that in this case the state is entangled iff it can violate the CGLMP inequality.

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- We assume the most general CPT conserving, Lorentz-invariant coupling of the spin-0 particle with the daughter bosons.
- We show that the two-boson state is entangled and violates the CGLMP inequality for all values of the (anomalous) coupling constants and that in this case the state is entangled iff it can violate the CGLMP inequality.
- As an exemplary process of this kind we use the decay $H \to ZZ$ with anomalous coupling.

Based on the following papers:

[1] A. Bernal, P. Caban, J. Rembieliński, "Entanglement and Bell inequalities violation in $H \to ZZ$ with anomalous coupling," Eur. Phys. J. C **83**, p. 1050, (2023).

[2] A. Bernal, P. Caban, J. Rembieliński, "Entanglement and the CGLMP inequality violation in a system of two vector bosons produced in the decay of a spin-0 particle with anomalous coupling," arXiv:2405.16525 [hep-ph] , (2024).

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Employs ideas from previous papers

[3] P. Caban, J. Rembieliński, M.Włodarczyk., Einstein–Podolsky–Rosen correlations of vector bosons," Phys. Rev. A 77, 012103 (2008);

[4] P. Caban, "Helicity correlations of vector bosons," Phys. Rev. A 77, 062101 (2008);

[5] A. Barr, P. Caban, J. Rembieliński, "Bell-type inequalities for systems of relativistic vector bosons," Quantum **7**, p. 1070, (2023).

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The amplitude corresponding to the most general Lorentz-invariant, CPT conserving coupling of X with two vector bosons can be written as

$$\mathcal{A}_{\lambda\sigma}(k,p) \propto \Big[v_1 \eta_{\mu\nu} + v_2 (k+p)_\mu (k+p)_\nu + v_3 \varepsilon_{\alpha\beta\mu\nu} (k+p)^\alpha (k-p)^\beta\Big] e^\mu_\lambda(k) e^\nu_\sigma(p), \quad (1)$$

$\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ — Minkowski metric tensor,
λ, σ — spin projections of the final states,
$v_1,\ v_2,\ v_3$ — real coupling constants,
$\varepsilon_{\alpha\beta\mu\nu}$ — a completely antisymmetric Levi-Civita tensor.

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$$e(q) = [e^\mu_\sigma(q)] = \begin{pmatrix} \frac{\mathbf{q}^T}{m} \\ I + \frac{\mathbf{q}\otimes\mathbf{q}^T}{m(m+q^0)} \end{pmatrix} V^T,$$

with

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}.$$

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These amplitudes fulfill standard transversality condition

$$e^\mu_\sigma(q) q_\mu = 0.$$

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Experimental data regarding Higgs decay admit nonzero v_2 and v_3 but give strong bounds on their values.

The VV state

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The most general pure VV state arising in the X decay ($v_1 \neq 0$) can be parametrized with the help of two parameters, c, $\tilde{c}$, as

$$|\psi_{VV}(k,p)\rangle = \Big[\eta_{\mu\nu} + \frac{c}{(kp)}(k_\mu p_\nu + p_\mu k_\nu) + \frac{\tilde{c}}{(kp)}\varepsilon_{\alpha\beta\mu\nu}(k+p)^\alpha(k-p)^\beta\Big] e^\mu_\lambda(k) e^\nu_\sigma(p) |(k,\lambda);(p,\sigma)\rangle,$$

where

$$c = (kp)\frac{v_2}{v_1}, \quad \tilde{c} = (kp)\frac{v_3}{v_1},$$

$|(k,\lambda);(p,\sigma)\rangle$ — the two-boson state, one boson with the four-momentum k and spin projection along z axis λ, second one with the four-momentum p and spin projection σ.

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For the decay $H \to ZZ$ from [CMS Collaboration, Phys. Rev. D 99, 112003 (2019)] we obtain the following experimental bounds:

$$|c| < 0.23, \qquad |\tilde{c}| < 0.5.$$

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We do not restrict ranges of c and $\tilde{c}$ to the above intervals.

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For $k \neq p$ states $|(k,\lambda);(p,\sigma)\rangle$ are orthonormal:

$$\langle (k,\lambda);(p,\sigma)|(k,\lambda');(p,\sigma')\rangle = \delta_{\lambda\lambda'}\delta_{\sigma\sigma'}. \qquad (*)$$

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The state $|\psi_{VV}(k,p)\rangle$ is not normalized, with the help of (*) we find

$$\langle \psi_{VV}(k,p)|\psi_{VV}(k,p)\rangle = 2 + \left[(1+c)\frac{(kp)}{m_1 m_2} - c\,\frac{m_1 m_2}{(kp)}\right]^2 + 8\tilde{c}^2\left[1 - \left(\frac{m_1 m_2}{(kp)}\right)^2\right].$$

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$k^\mu = (\omega_1, \mathbf{k})$, $\omega_1^2 - \mathbf{k}^2 = m_1^2$ and $p^\mu = (\omega_2, -\mathbf{k})$, $\omega_2^2 - \mathbf{k}^2 = m_2^2$.

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In the CM frame we have

$$
\begin{aligned}
M &= \omega_1 + \omega_2, \\
\mathbf{k}^2 &= \frac{1}{4M^2} \lambda(M^2, m_1^2, m_2^2), \\
kp &= \frac{1}{2} \Big[M^2 - m_1^2 - m_2^2 \Big], \\
\omega_1 &= \frac{1}{2M} \Big[M^2 + (m_1^2 - m_2^2) \Big], \\
\omega_2 &= \frac{1}{2M} \Big[M^2 - (m_1^2 - m_2^2) \Big],
\end{aligned}
$$

where

$$
\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.
$$

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In the CM frame normalization of the state $|\psi_{VV}(k,p)\rangle$ depends only on masses M, m_1, m_2 and the parameters c, $\tilde{c}$:

$$\langle \psi_{VV}(k,p)|\psi_{VV}(k,p)\rangle|_{CM} = 2(1+\tilde{\kappa}^2)+\kappa^2,$$

where

$$\kappa = \beta + c(\beta - 1/\beta), \quad \tilde{\kappa} = 2\tilde{c}\sqrt{1-1/\beta^2}$$

and

$$\beta = \left.\frac{(kp)}{m_1 m_2}\right|_{CM} = \frac{M^2-(m_1^2+m_2^2)}{2m_1 m_2}.$$

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The ranges of possible values of κ, $\tilde{\kappa}$:

$$
\begin{aligned}
&\kappa \in (-\infty, 1] && \text{for} && c \in (-\infty, -1),\\
&\kappa \in [0, 1] && \text{for} && c = -1,\\
&\kappa \in [2\sqrt{-c(1+c)}, \infty) && \text{for} && c \in (-1, -\tfrac{1}{2}),\\
&\kappa \in [1, \infty] && \text{for} && c \in [-\tfrac{1}{2}, \infty),
\end{aligned}
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and

$$
\begin{aligned}
&\tilde{\kappa} \in (2\tilde{c}, 0] && \text{for} && \tilde{c} \in (-\infty, 0),\\
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We simplify the notation of basis two-boson states in this case

$$|\lambda, \sigma\rangle \equiv |(\omega_1, 0, 0, |\mathbf{k}|); (\omega_2, 0, 0, -|\mathbf{k}|)\rangle.$$

In this notation the normalized state of two bosons reads

$$|\psi_{VV}^{\mathrm{norm}}(m_1, m_2, c, \tilde{c})\rangle = \frac{1}{\sqrt{2(1+\tilde{\kappa}^2)+\kappa^2}}\big[(1-i\tilde{\kappa})|+,-\rangle - \kappa|0,0\rangle + (1+i\tilde{\kappa})|-,+\rangle\big].$$

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Bosons arising in a single decay $X \to V_1 V_2$ have definite masses m_1 and m_2; thus two-boson state is pure and has the following form

$$\rho(m_1, m_2, c, \tilde{c}) = |\psi_{VV}^{\text{norm}}(m_1, m_2, c, \tilde{c})\rangle\langle\psi_{VV}^{\text{norm}}(m_1, m_2, c, \tilde{c})|.$$

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When one determines two-boson state from experimental data then averaging over various kinematical configurations is necessary and the state becomes mixed

$$\rho_{VV}(c, \tilde{c}) = \int dm_1\, dm_2\, \mathcal{P}_{c,\tilde{c}}(m_1, m_2)\rho(m_1, m_2, c, \tilde{c}),$$

where $\mathcal{P}_{c,\tilde{c}}(m_1, m_2)$ is a normalized probability distribution.

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The explicit form of this probability distribution can be determined in the case when the daughter VV bosons subsequently decay into massless fermions

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Following [T. Zagoskin, A. Korchin, J. Exp. Theor. Phys. 122, 663 (2016); A. Bernal, P. Caban, J. Rembieliński, Eur. Phys. J. C 83, 1050 (2023)] we find

$$\mathcal{P}_{c,\tilde{c}}(m_1, m_2) = N \frac{\lambda^{\frac{1}{2}}(M^2, m_1^2, m_2^2) m_1^3 m_2^3}{D(m_1)D(m_2)} \left[2(1+\tilde{\kappa}^2) + \kappa^2\right],$$

with

$$D(m) = \left(m^2 - m_V^2\right)^2 + (m_V \Gamma_V)^2,$$

where m_V, Γ_V denotes the mass and decay width of the on-shell V boson and the normalization factor N can be determined numerically for given values c and $\tilde{c}$.

The VV state in the center of mass frame

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The state averaged over kinematical configurations has the following structure

$$\rho_{VV}(c,\tilde{c}) = \frac{1}{\mathrm{b}+2\mathrm{e}}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{\mathrm{e}} & 0 & \boxed{\mathrm{f}} & 0 & \boxed{\mathrm{h}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{\mathrm{f}^*} & 0 & \boxed{\mathrm{b}} & 0 & \boxed{\mathrm{f}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{\mathrm{h}^*} & 0 & \boxed{\mathrm{f}^*} & 0 & \boxed{\mathrm{e}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

where star denotes complex conjugation.

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Non-zero matrix elements are equal to

$$\begin{aligned}
\mathrm{b} &= -2c(1+c)\mathrm{B}(0) + (1+c)^2\mathrm{B}(2) + c^2\mathrm{B}(-2),\\
\mathrm{e} &= (1+4\tilde{c}^2)\mathrm{B}(0) - 4\tilde{c}^2\mathrm{B}(-2),\\
\mathrm{f} &= -(c+1)\mathrm{B}(1) + c\mathrm{B}(-1)\\
&\qquad\qquad + 2i\tilde{c}\left[(1+c)\tilde{\mathrm{B}}(0) - c\tilde{\mathrm{B}}(-2)\right],\\
\mathrm{h} &= (1-4\tilde{c}^2)\mathrm{B}(0) + 4\tilde{c}^2\mathrm{B}(-2) - 4i\tilde{c}\tilde{\mathrm{B}}(-1),
\end{aligned}$$

where

$$\mathrm{B}(n) = \int\limits_S dm_1 dm_2 \frac{\lambda^{1/2}(M^2, m_1^2, m_2^2) m_1^3 m_2^3}{D(m_1)D(m_2)} \beta^n,$$

$$\tilde{\mathrm{B}}(n) = \int\limits_S dm_1 dm_2 \frac{\lambda^{1/2}(M^2, m_1^2, m_2^2) m_1^3 m_2^3}{D(m_1)D(m_2)} \beta^n (\beta^2 - 1)^{1/2},$$

for $n = -2, -1, 0, 1, 2$, and
$S = \{(m_1, m_2) : m_1 \geq 0, m_2 \geq 0, m_1 + m_2 \leq M\}$.

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$$\begin{aligned}
\mathrm{b}_Z &= 9431.55 + 12883.6c + 4983.07c^2, \\
\mathrm{e}_Z &= 2989.76 + 5834.84\tilde{c}^2, \\
\mathrm{f}_Z &= -4819.07 - 2752.19c + 7052.85i\tilde{c} + 4477.64ic\tilde{c}, \\
\mathrm{h}_Z &= 2989.76 - 8031.86i\tilde{c} - 5834.84\tilde{c}^2.
\end{aligned}$$

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And this implies that the state $\rho_{VV}(c, \tilde{c})$ is entangled iff at least one off-diagonal matrix entry is non-zero.

To quantify entanglement of the state $\rho_{VV}(c, \tilde{c})$ we use the logarithmic negativity which is a computable entanglement measure and is defined as

$$E_N(\rho_{AB}) = \log_3(||\rho^{T_B}||_1),$$

where T_B denotes partial transposition with respect to the subsystem B and $||A||_1 = \mathrm{Tr}(\sqrt{A^\dagger A})$ is the trace norm of a matrix A. $||A||_1$ is equal to the sum of all the singular values of A; when A is Hermitian then it is equal to the sum of absolute values of all eigenvalues of A. $E_N(\rho) > 0$ implies that the state ρ is entangled.

Entanglement

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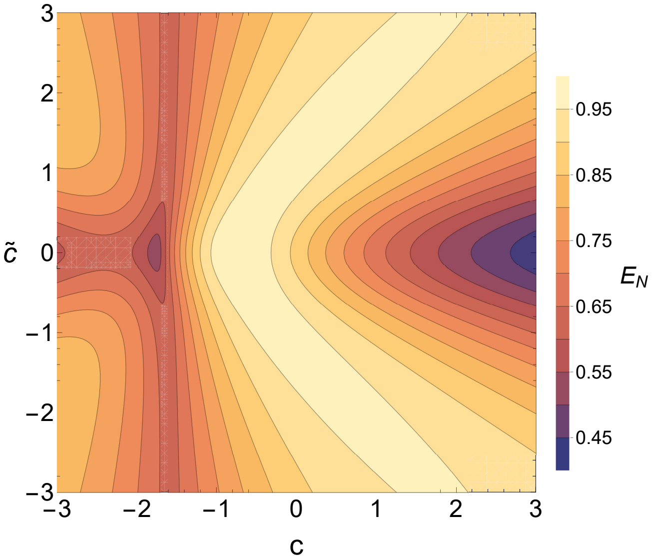

Figure: The logarithmic negativity of the state $\rho_{ZZ}(c, \tilde{c})$ as a function of c, $\tilde{c}$.

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Moreover, $E_N > 0$ for all values of c, $\tilde{c}$ and in the limit $c \to \infty$ the logarithmic negativity tends to zero.

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Here we are interested in the CGLMP inequality for a two-qutrit system (for spin-1 particle there are three possible outcomes of a spin projection measurements).

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For two qutrits the CGLMP inequality has the following form

$$\mathcal{I}_3 \leq 2,$$

where

$$\mathcal{I}_3 = \big[P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)\big] \\ - \big[P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)\big]$$

and A_1, A_2 (B_1, B_2) are possible measurements that can be performed by Alice (Bob).

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$P(A_i = B_j + k)$ — the probability that the outcomes A_i and B_j differ by k modulo 3, i.e., $P(A_i = B_j + k) = \sum_{l=0}^{l=2} P(A_i = l, B_j = l + k \mod 3)$.

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As usual, we assume that Alice can perform measurements on one of the bosons, Bob on the second one, i.e., we take Alice (Bob) observables as $A \otimes I$ ($I \otimes B$).

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The CGLMP inequality can be written as

$$\mathrm{Tr}\left(\rho \mathcal{O}_{\mathrm{Bell}}\right) \leq 2,$$

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Each Hermitian 3×3 matrix A can be represented with the help of the 3×3 unitary matrix U_A, columns of U_A are normalized eigenvectors of A in a given basis.

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$$\mathcal{O}_{\text{Bell}}(U_{A_1}, U_{A_2}, U_{B_1}, U_{B_2}) = -[U_{A_1} \otimes U_{B_1}]P_1[I \otimes S^3]P_1^\dagger[U_{A_1} \otimes U_{B_1}]^\dagger$$
$$+[U_{A_1} \otimes U_{B_2}]P_0[I \otimes S^3]P_0^\dagger[U_{A_1} \otimes U_{B_2}]^\dagger + [U_{A_2} \otimes U_{B_1}]P_1[I \otimes S^3]P_1^\dagger[U_{A_2} \otimes U_{B_1}]^\dagger$$
$$- [U_{A_2} \otimes U_{B_2}]P_1[I \otimes S^3]P_1^\dagger[U_{A_2} \otimes U_{B_2}]^\dagger,$$

S^3 — the standard spin z component matrix, $S^3 = \mathrm{diag}(1, 0, -1)$,
P_0, P_1 — $3^2 \times 3^2$ block-diagonal permutation matrices:

$$P_n = \begin{pmatrix} C^n & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & C^{n+1} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & C^{n+2} \end{pmatrix}, \quad n = 0, 1,$$

$\mathcal{O}$ — the 3×3 null matrix, C — the 3×3 cyclic permutation matrix

$$C = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

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Thus, usually, one applies a certain optimization procedure in order to find optimal observables.

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We define unitary matrices

$$U_V(t, \theta) = \begin{pmatrix} \cos\frac{t}{2} & 0 & e^{i\theta}\sin\frac{t}{2} \\ 0 & 1 & 0 \\ -e^{-i\theta}\sin\frac{t}{2} & 0 & \cos\frac{t}{2} \end{pmatrix}$$

and

$$O_A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

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Now, we calculate the mean value of the operator

$$(O_A \otimes I)\mathcal{O}_{\mathrm{Bell}}(U_V(0,0), U_V(\tfrac{\pi}{2},0), U_V(t,\theta), U_V(-t,\theta))(O_A \otimes I)$$

in the state $\rho_{VV}(c,\tilde{c})$.

The violation of the CGLMP inequality

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The result can be written as

$$\mathcal{I}_3 = 2 + \tfrac{3}{2}\left[a(\cos t - 1) - 2|r|\cos(\alpha + \theta)\sin t\right], \tag{2}$$

where

$$a = \frac{2\mathrm{e}}{\mathrm{b} + 2\mathrm{e}}, \qquad r = \frac{\mathrm{h}}{\mathrm{b} + 2\mathrm{e}} = |r|e^{i\alpha}.$$

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The maximal value of (2) is attained for

$$\begin{aligned}
&\sin(\alpha+\theta) = 0, \qquad \cos(\alpha+\theta) = \pm 1,\\
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and is equal to

$$(\mathcal{I}_3)_{\max} = 2 + \tfrac{3}{2}\left[\sqrt{a^2+4|r|^2} - a\right].$$

Thus, we see that $(\mathcal{I}_3)_{\max} > 2$ for $\mathrm{h} \neq 0$.

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Using very similar method one can show that when the state has the following structure

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{a_{11}} & 0 & \boxed{a_{12}} & 0 & \boxed{a_{13}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{a_{12}^*} & 0 & \boxed{a_{22}} & 0 & \boxed{a_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{a_{13}^*} & 0 & \boxed{a_{23}^*} & 0 & \boxed{a_{33}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

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Therefore, a state of such a form violates the CGLMP inequality iff it is entangled.

It is a non-trivial observation since for an arbitrary 3×3 quantum state ρ such a statement is true only if ρ is pure.

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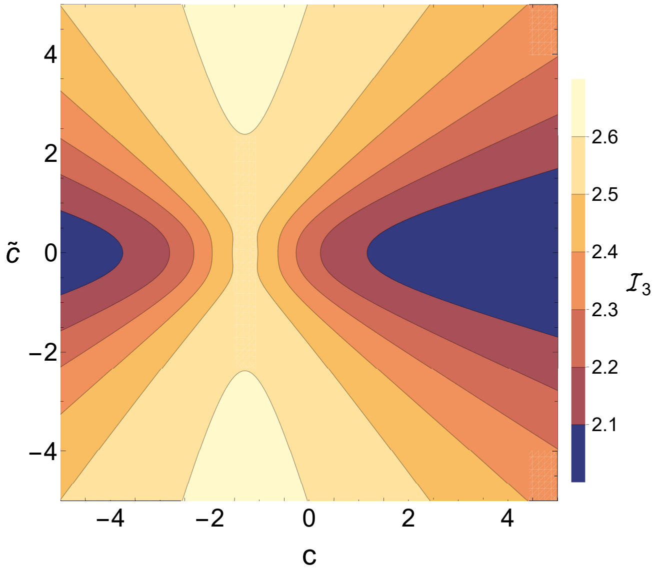

Figure: The maximal value of $\mathcal{I}_3$ in the state $\rho_{ZZ}(c, \tilde{c})$ as a function of c, $\tilde{c}$.

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The state with the highest entanglement does not correspond to the state with the highest violation of the CGLMP inequality.

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This observation is consistent with the general property of CGLMP inequality [A. Acín, T. Durt, N. Gisin, J.I. Latorre, Phys. Rev. A 65, 052325 (2002)].

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The noise resistance we define as a minimal value of λ, λ_{min}, for which the state

$$\lambda \rho_{ZZ}(c, \tilde{c}) + (1-\lambda)\tfrac{1}{9} I_9, \qquad \lambda \in (0, 1]$$

violates the CGLMP inequality. Inserting the state (31) into the CGLMP inequality (23) we obtain

$$\lambda_{\text{min}} = \frac{2}{\max\{\text{Tr}(\rho_{ZZ}(c, \tilde{c}) \mathcal{O}_{\text{Bell}})\}}. \tag{3}$$

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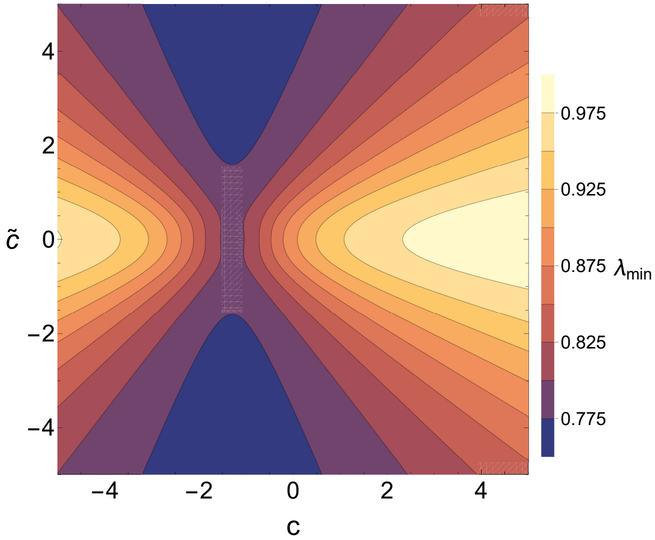

Figure: λ_{min} as a function of c and $\tilde{c}$.

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From the plot we can see that for values c, $\tilde{c}$ close to 0 we can tolerate up to almost a 20% of noise and still attain a violation of the CGLMP inequality and hence an entangled state.

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Values of c, $\tilde{c}$ close to 0 are expected for the decay $H \to ZZ$ due to experimental bounds on anomalous couplings for the HZZ vertex.

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    Thus, we assumed that $v_1 \neq 0$.
- In such a case, the state of produced bosons, beyond four-momenta and spins, can be characterized by two parameters c, $\tilde{c}$ which, up to normalization are equal to v_2/v_1 and v_3/v_1, respectively.

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- In the center-of-mass frame, we have determined the most general pure state of a $V\,V$ boson pair for a particular event $X \to V\,V$.
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- Moreover, even tests with current technology can be useful—they at least can serve as tests of internal consistency of quantum mechanics under completely new conditions.

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- This relation between entanglement and the violation of the CGLMP inequality is solely based on the texture of the matrix (which is a consequence of the symmetries involved in the decay) and not on the experimental way of getting the density matrix itself.