Testing the Born rule in high-energy collisions

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Main points

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-- high-energy collision experiments can be repurposed as tests of the Born rule

The Born rule

probability density: $\rho = |\psi|^2$

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high-energy physics, S-matrix elements

$$\langle f | \hat{S} | i \rangle \sim \delta^4(p_f - p_i)\mathcal{M}$$

Born rule implies cross sections

$$\frac{d\sigma}{d\Omega} \sim |\mathcal{M}|^2$$

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anomalous polarisation probabilities

De Broglie's Pilot-Wave Dynamics (1927)

$$i\frac{\partial\psi}{\partial t} = -\sum_{n=1}^{N}\frac{1}{2m_n}\nabla_n^2\psi + V\psi \qquad \frac{d\mathbf{x}_n}{dt} = \frac{\nabla_n S}{m_n}$$

$$\psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t) \qquad \psi = |\psi|\, e^{iS}$$

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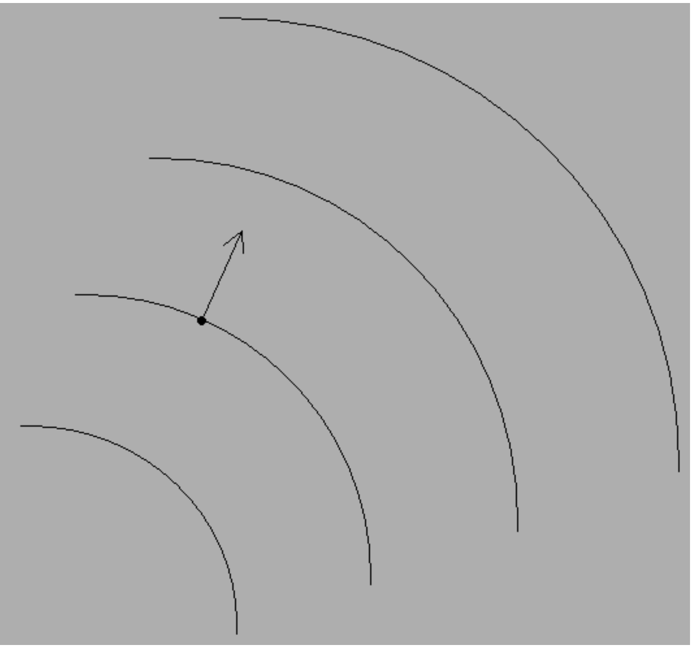


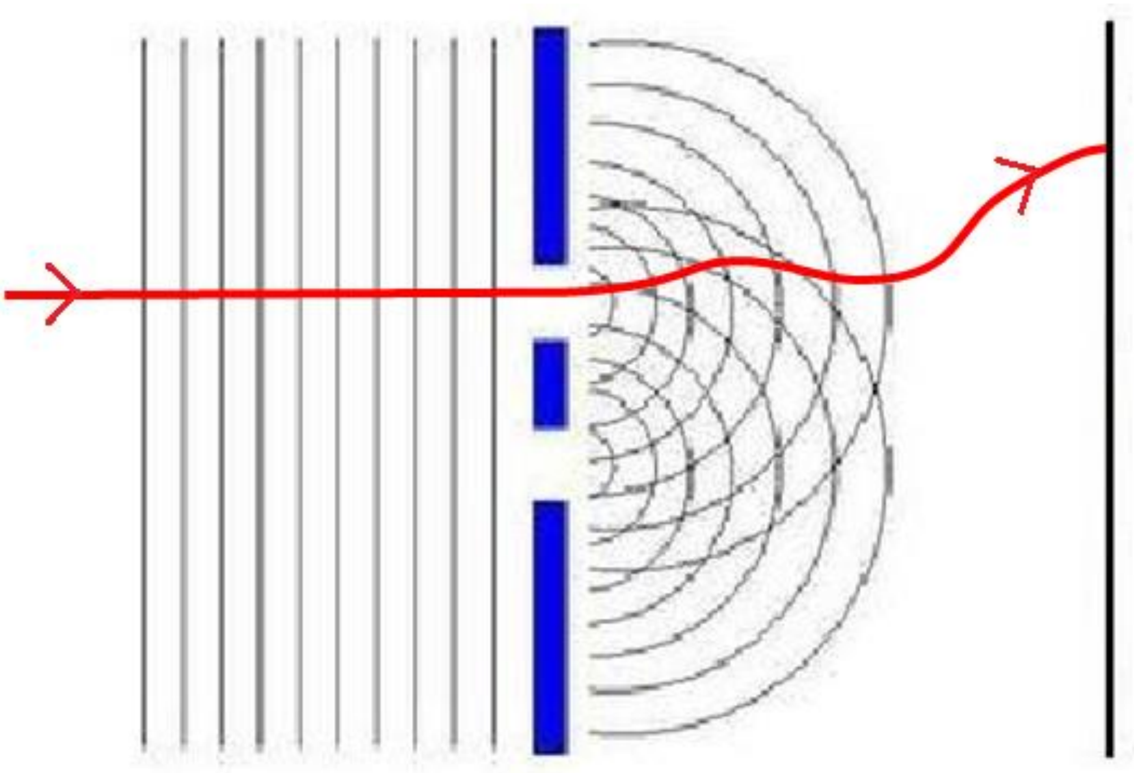
$$q = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N)$$

Motion of configuration $q(t)$
is determined by a 'pilot wave' ψ

(ψ is defined on configuration space)

An example: the two-slit experiment

Fire one particle at a two-slit screen



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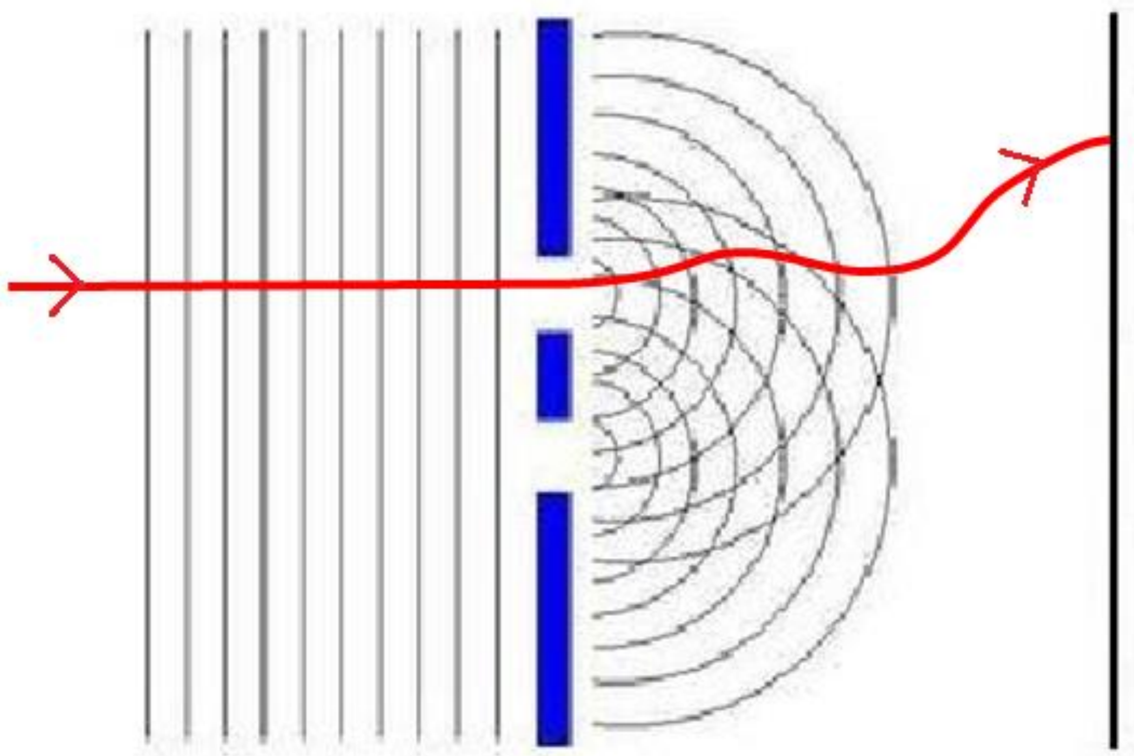

Given the wave function ψ and the initial position $\mathbf{x}(0)$, the particle trajectory $\mathbf{x}(t)$ is determined by de Broglie's *law of motion*

$$\frac{d\mathbf{x}}{dt} = \frac{\nabla S}{m} \qquad \psi = |\psi| \, e^{iS}$$

Postulated to be true, even if in practice we do not know $\mathbf{x}(0)$

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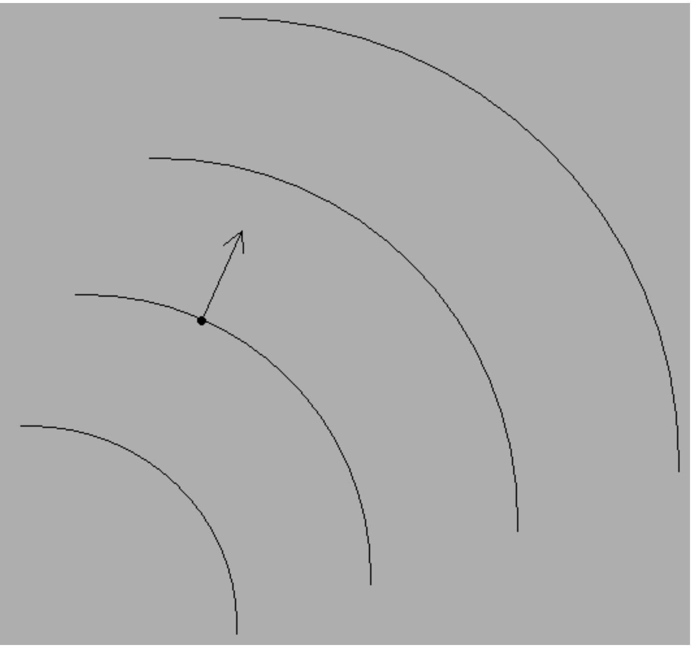

Consider an ensemble of systems with the same ψ and different *q*'s

Ensemble distribution $\rho(q,t)$

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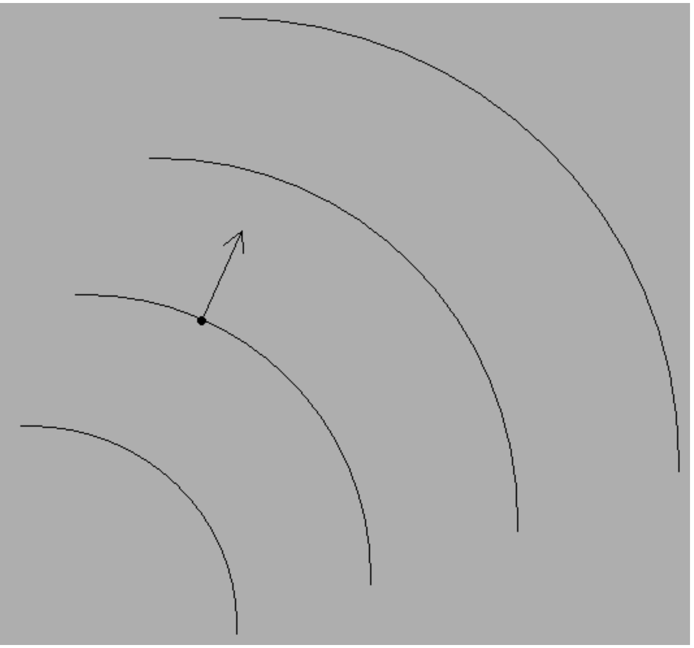

Consider an ensemble of systems with the same ψ and different *q*'s

Ensemble distribution $\rho(q,t)$

Recover quantum mechanics *if* *assume* initial distribution $\rho = |\psi|^2$ (preserved in time by dynamics)

(shown fully by Bohm in 1952)

Illustration for one particle

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + V\psi \quad \Rightarrow \quad \frac{\partial |\psi|^2}{\partial t} + \nabla \cdot \left(|\psi|^2 \frac{\nabla S}{m} \right) = 0$$

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Guidance equation $\frac{d\mathbf{x}}{dt} = \frac{\nabla S}{m}$ applied to an

ensemble (same ψ, different **x**'s)

Distribution $\rho(\mathbf{x},t)$ obeys $$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\frac{\nabla S}{m}\right) = 0$$

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THEOREM (quantum equilibrium):

If $\rho(\mathbf{x},0) = |\psi(\mathbf{x},0)|^2$, then $\rho(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2$ for all t

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(Generalisation: replace $\mathbf{x}$(t) by general configuration $q(t)$)

Similarly for a general system

System with configuration $q(t)$ and wave function(al) $\psi(q,t)$

$$i\frac{\partial\psi}{\partial t} = \hat{H}\psi \qquad\qquad \frac{dq}{dt} = v = \frac{j}{|\psi|^2}$$

where $j = j\,[\psi] = j(q,t)$ is the Schrödinger current

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By construction $\rho(q,t)$ will obey

$$\frac{\partial\rho}{\partial t} + \partial_q \cdot (\rho v) = 0$$ (*same* as $\frac{\partial\left|\psi\right|^2}{\partial t} + \partial_q \cdot (\left|\psi\right|^2 v) = 0$)

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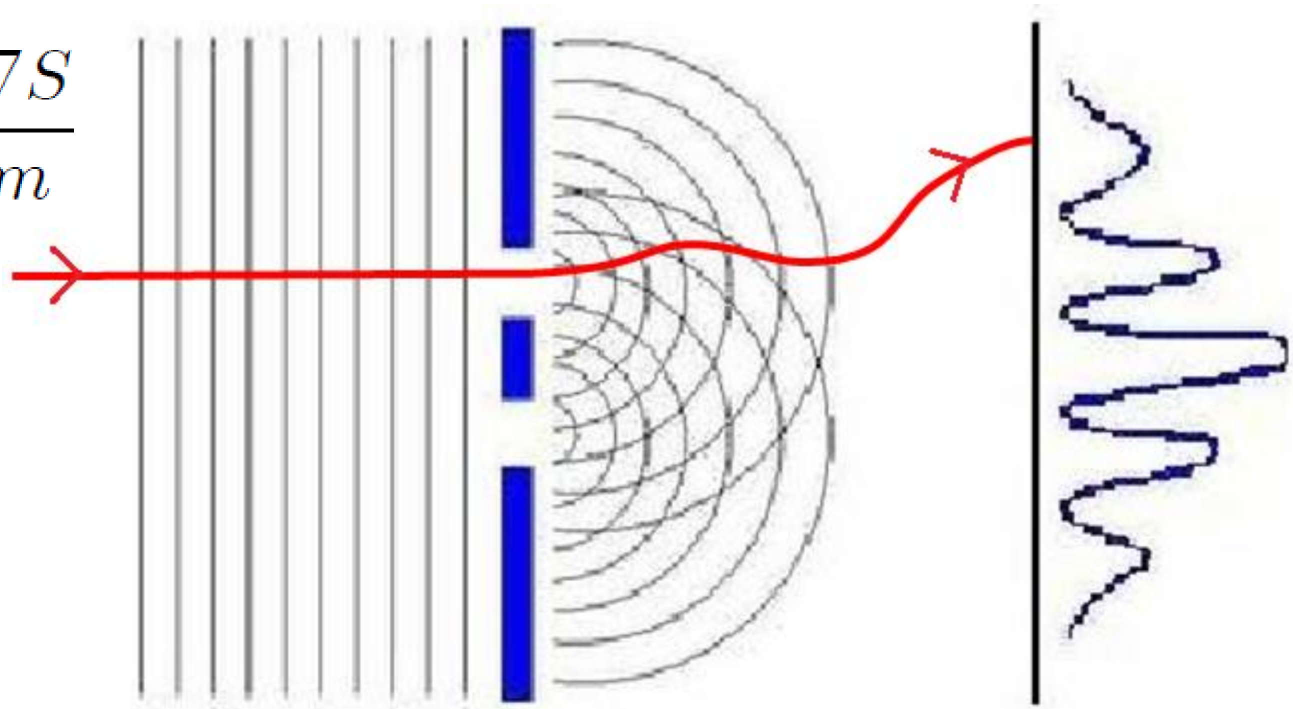
$$\frac{\partial\rho}{\partial t} + \partial_q\cdot(\rho v) = 0$$ (*same* as $\dfrac{\partial|\psi|^2}{\partial t} + \partial_q\cdot(|\psi|^2 v) = 0$)

and $\rho(q,t) = |\psi(q,t)|^2$ preserved in time (Born rule).

If $\rho = |\psi|^2$ at $t = 0$ then $\rho = |\psi|^2$ at $t > 0$

Consider the example of the two-slit experiment

$$\frac{d\mathbf{x}}{dt} = \frac{\nabla S}{m}$$



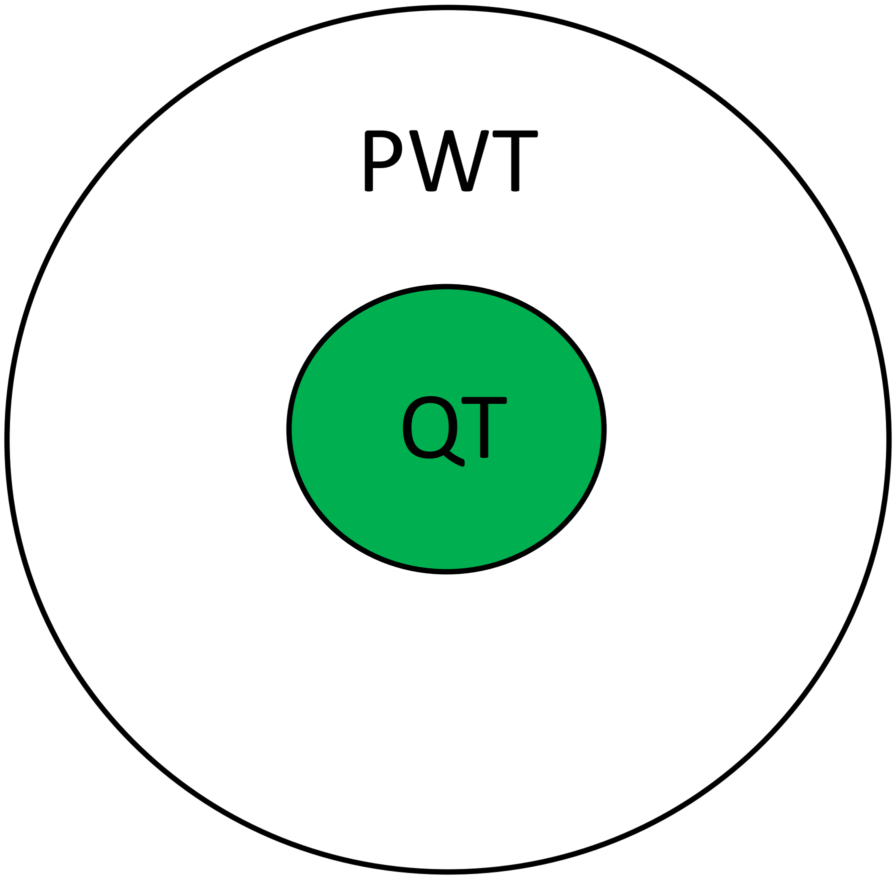
Quantum Equilibrium $\quad \rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2$

same statistical predictions as quantum mechanics

Quantum Nonequilibrium $\quad \rho(\mathbf{x}, t) \neq |\psi(\mathbf{x}, t)|^2$

statistical *deviations* from quantum mechanics

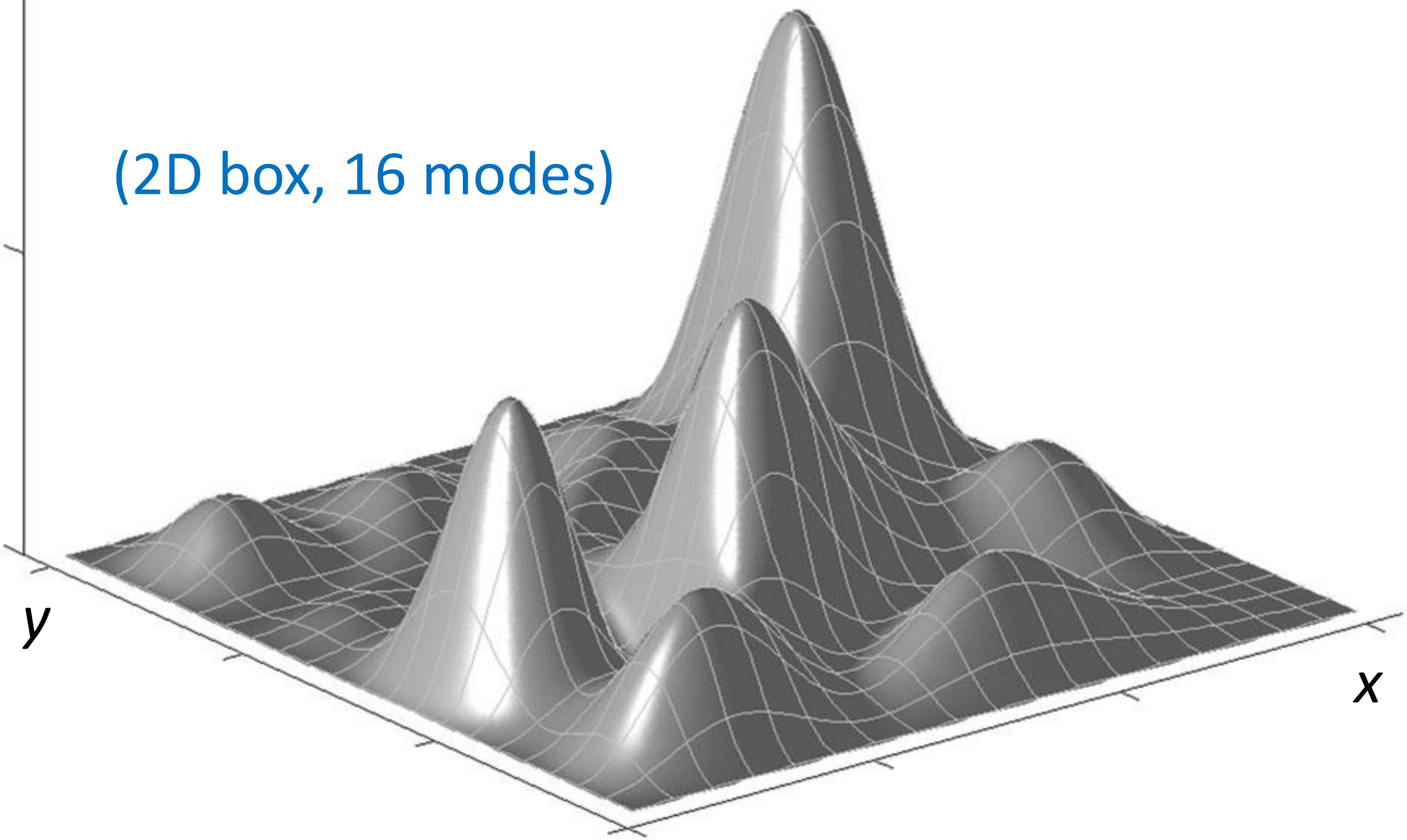
Quantum theory = special case of a wider physics



BUT: *experimentally* we always find the "quantum equilibrium" distribution:

$\rho = |\psi|^2$ (Born rule)

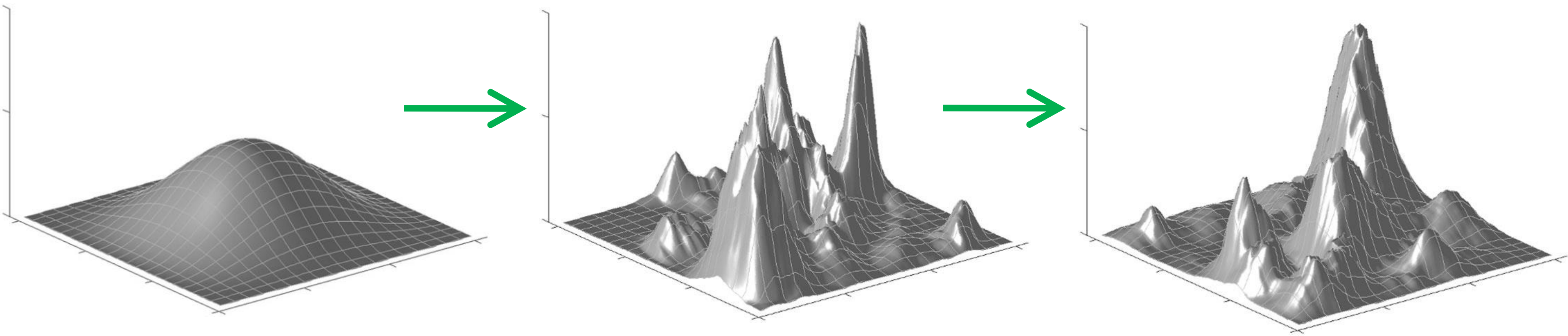
(2D box, 16 modes)



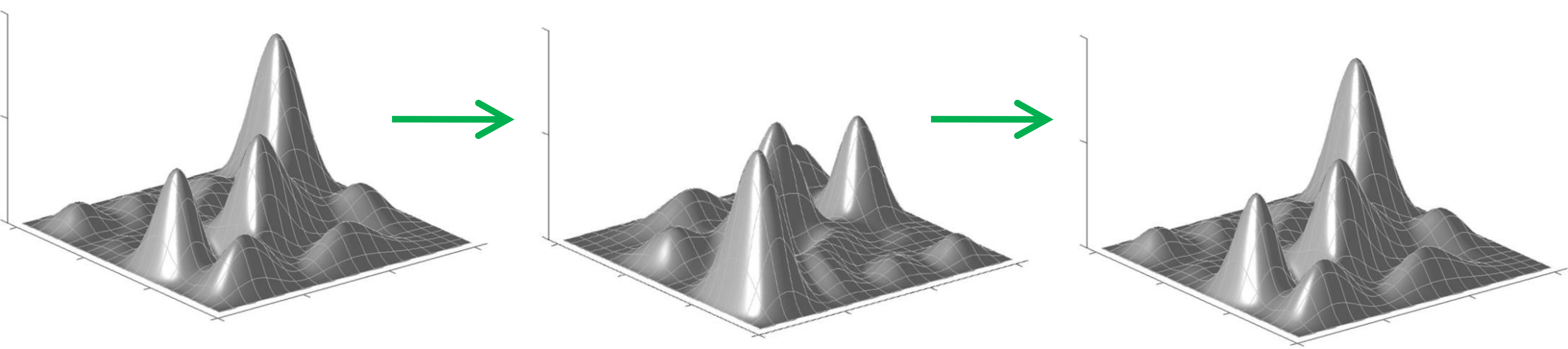
Why?

Quantum relaxation (cf. thermal relaxation)

Non-equilibrium ($\rho \neq |\psi|^2$) relaxes to equilibrium



Compare with time evolution of equilibrium $\rho = |\psi|^2$



(Valentini and Westman, Proc. Roy. Soc. A 2005)

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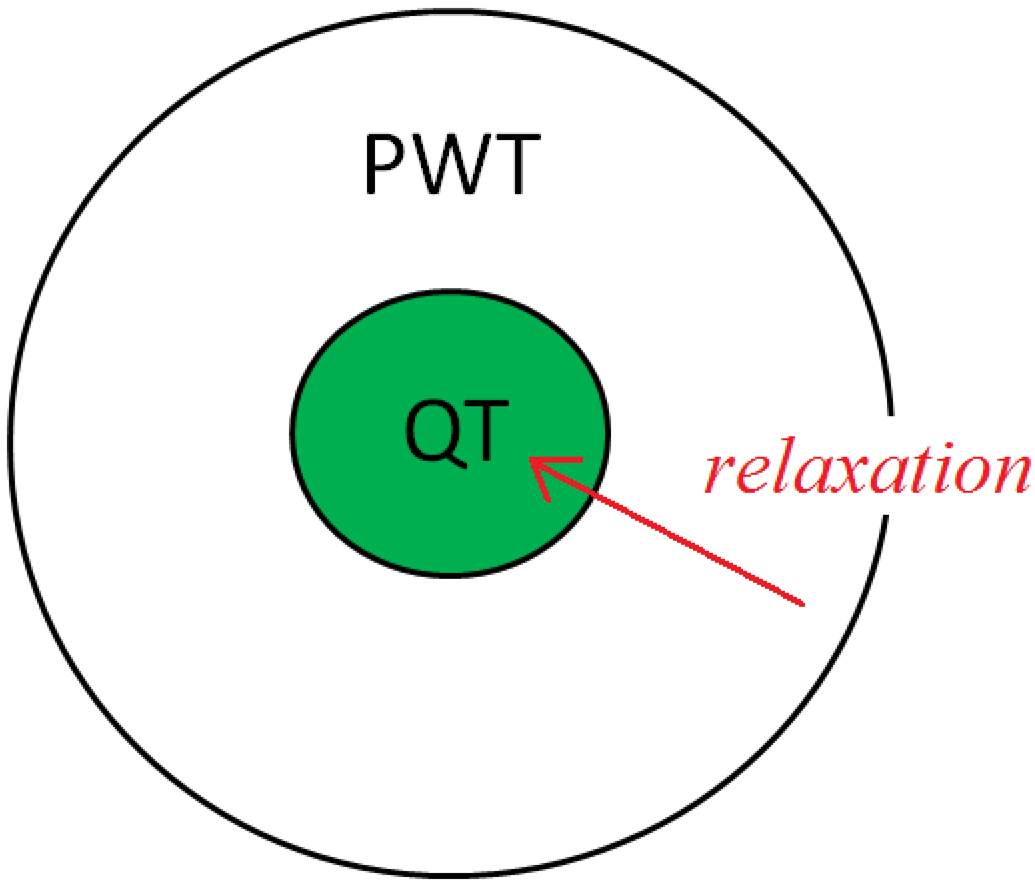


Quantum Theory is the effective description of a special state of statistical equilibrium

We are in that state now because of past relaxation

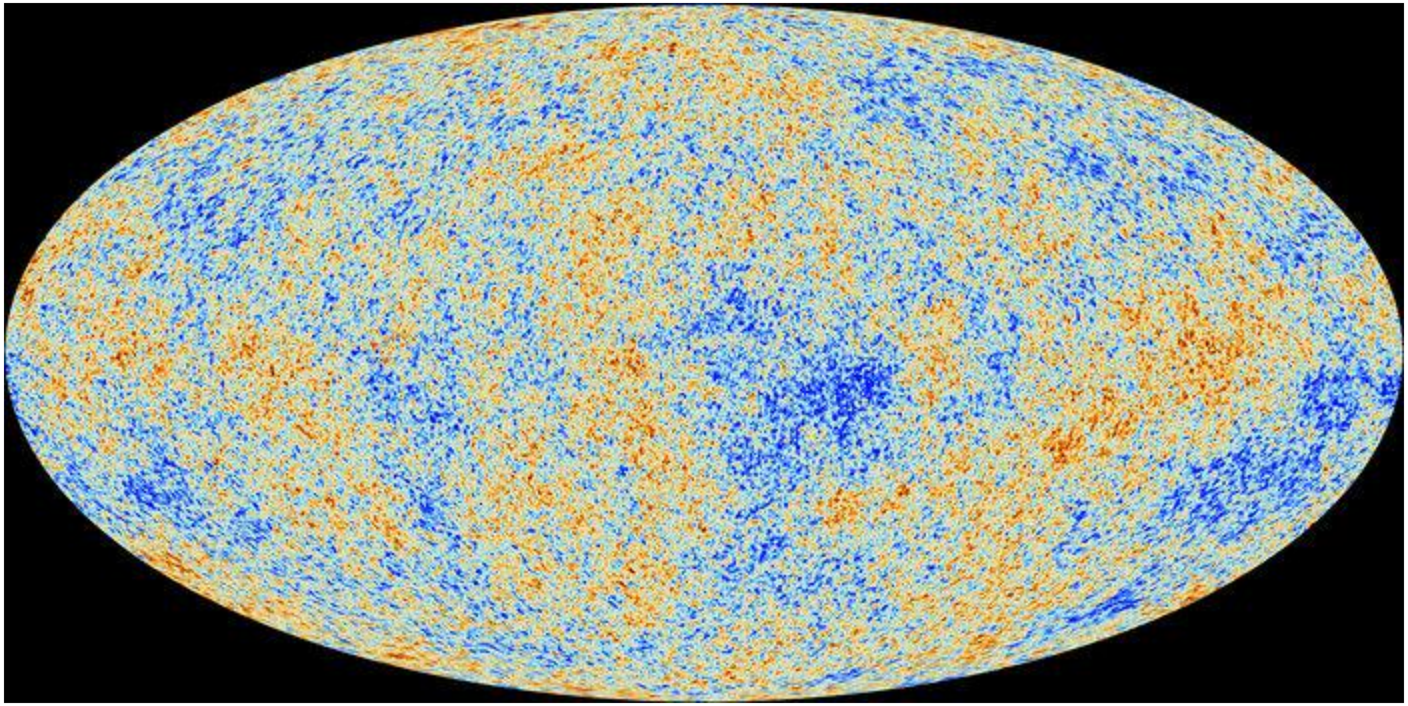
When did quantum relaxation happen?

Presumably, a long time ago, in the very early universe, soon after the big bang

If so, we can expect that quantum theory will break down close to the big bang.

This can be tested using inflationary cosmology

CMB anisotropies are ultimately generated by early quantum noise (inflationary vacuum)



We are testing the Born rule in the early universe

This can be tested using inflationary cosmology

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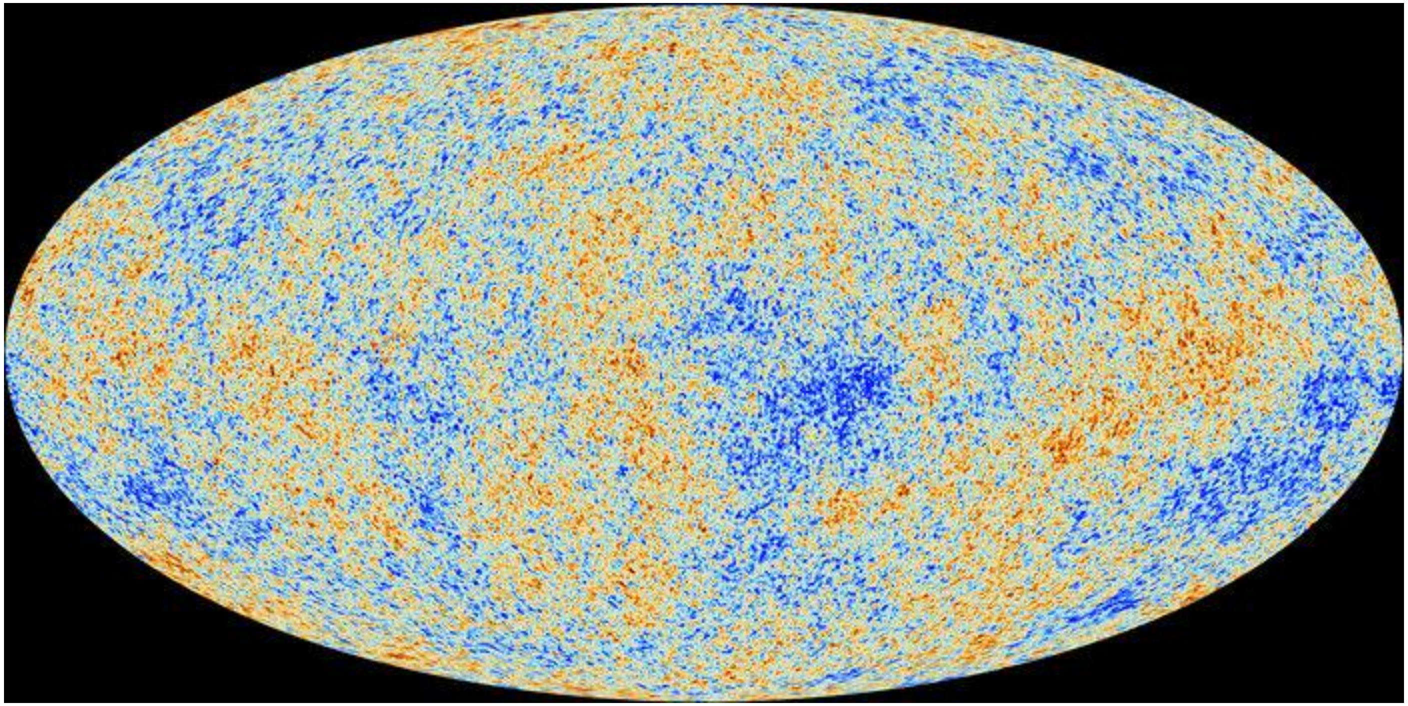
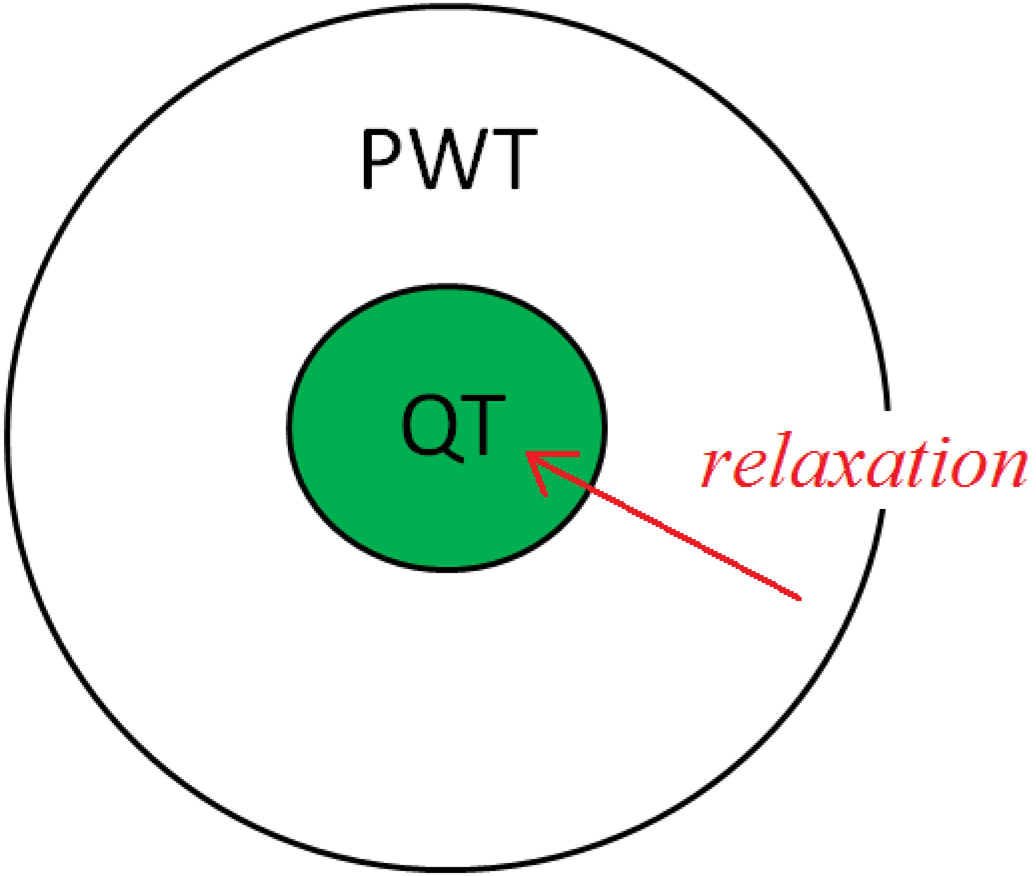


Signatures of quantum relaxation in the CMB?

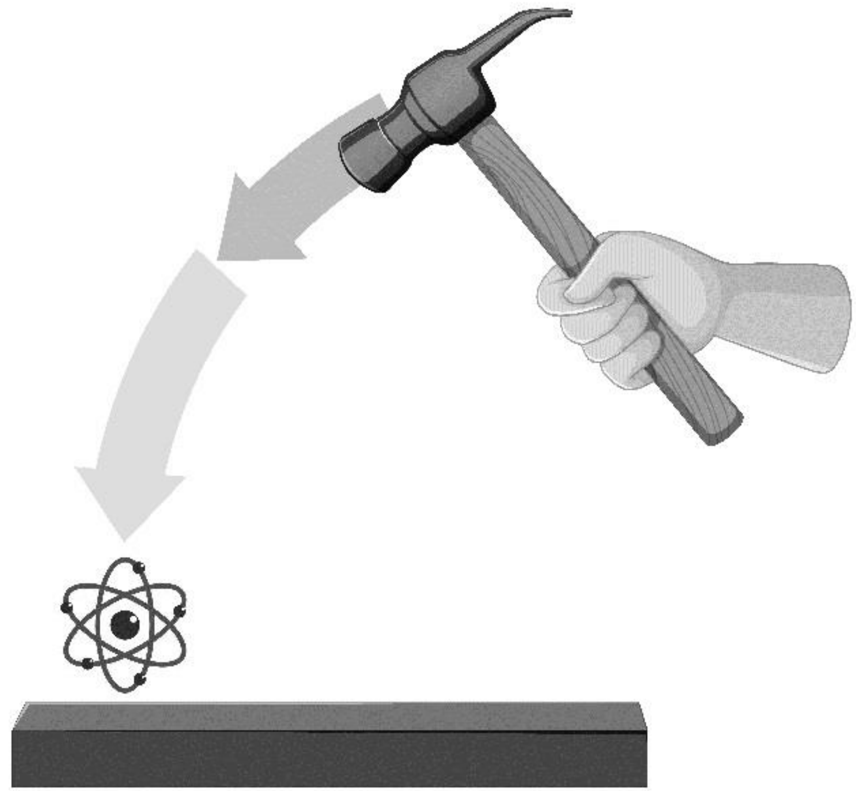
(AV, PRD 2010; Colin and AV, PRD 2013; Colin and AV, PRD 2015; Vitenti, Peter and AV, PRD 2019)

Trapped in quantum equilibrium



Is there a way to escape?

Can we *create* nonequilibrium (from equilibrium)?



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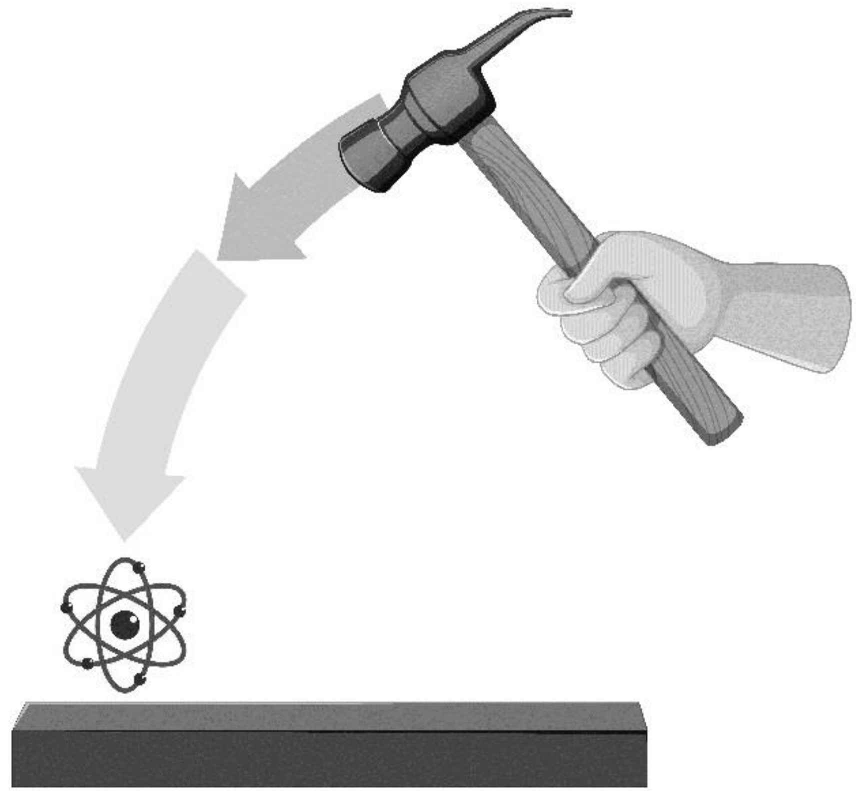

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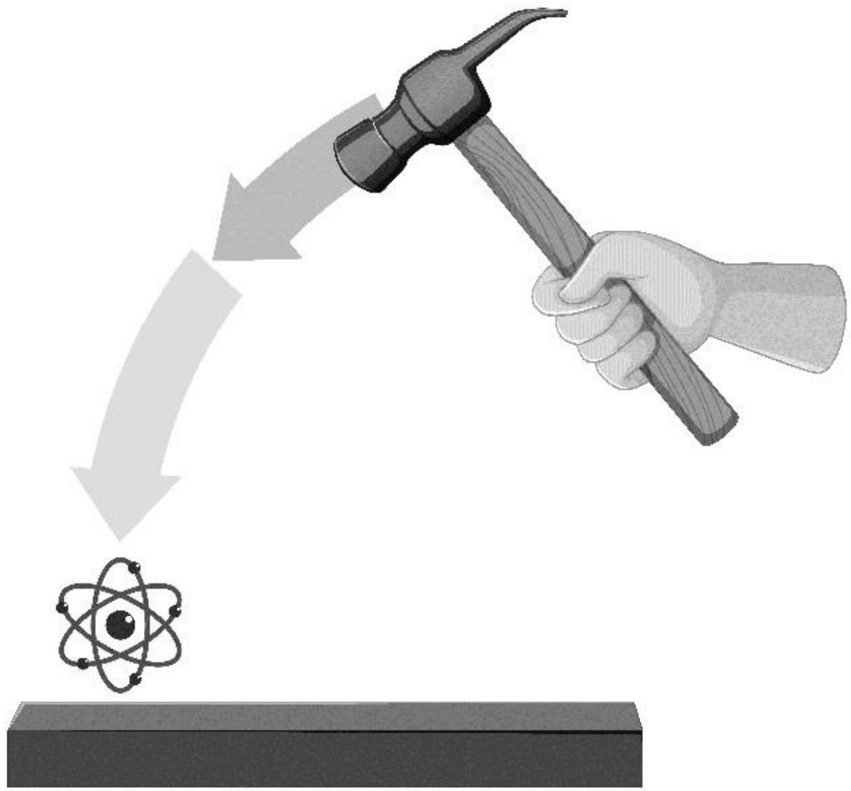


Apparently not: $P = |\Psi|^2$ is preserved by the dynamics.

At least in non-gravitational physics…

Quantum gravity may change the game

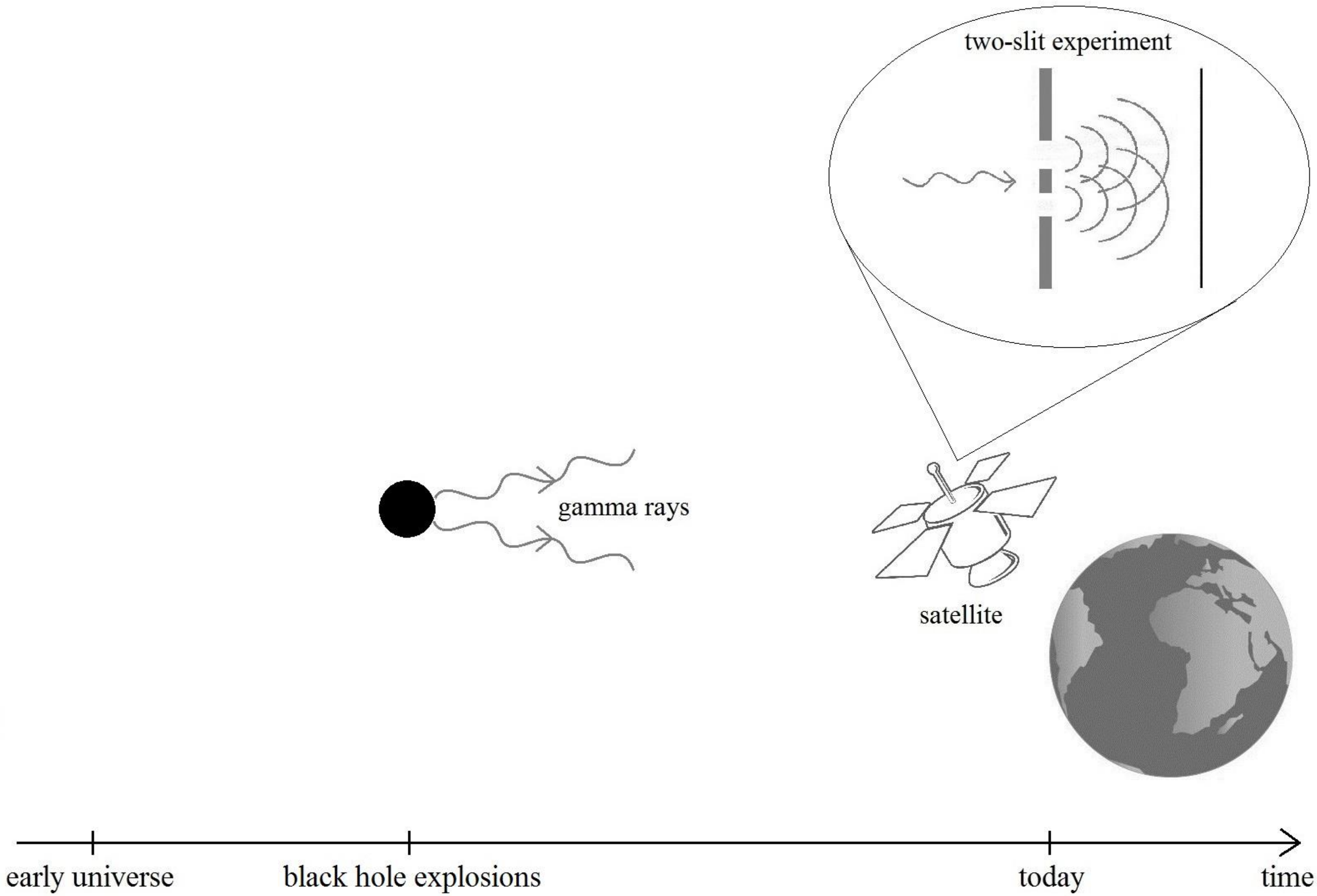
Tiny quantum-gravitational corrections to the Schrödinger equation can make the Born rule unstable ($\rho = |\psi|^2$ evolves to $\rho \neq |\psi|^2$).

Small non-Hermitian terms in the effective Hamiltonian (Kiefer and Singh 1991; Brizuela et al. 2016).

Consistent with pilot-wave theory: non-Hermitian terms generate a small instability of the Born rule (AV 2021, 2023).

Final burst of Hawking radiation breaks the Born rule

Exploding primordial black holes



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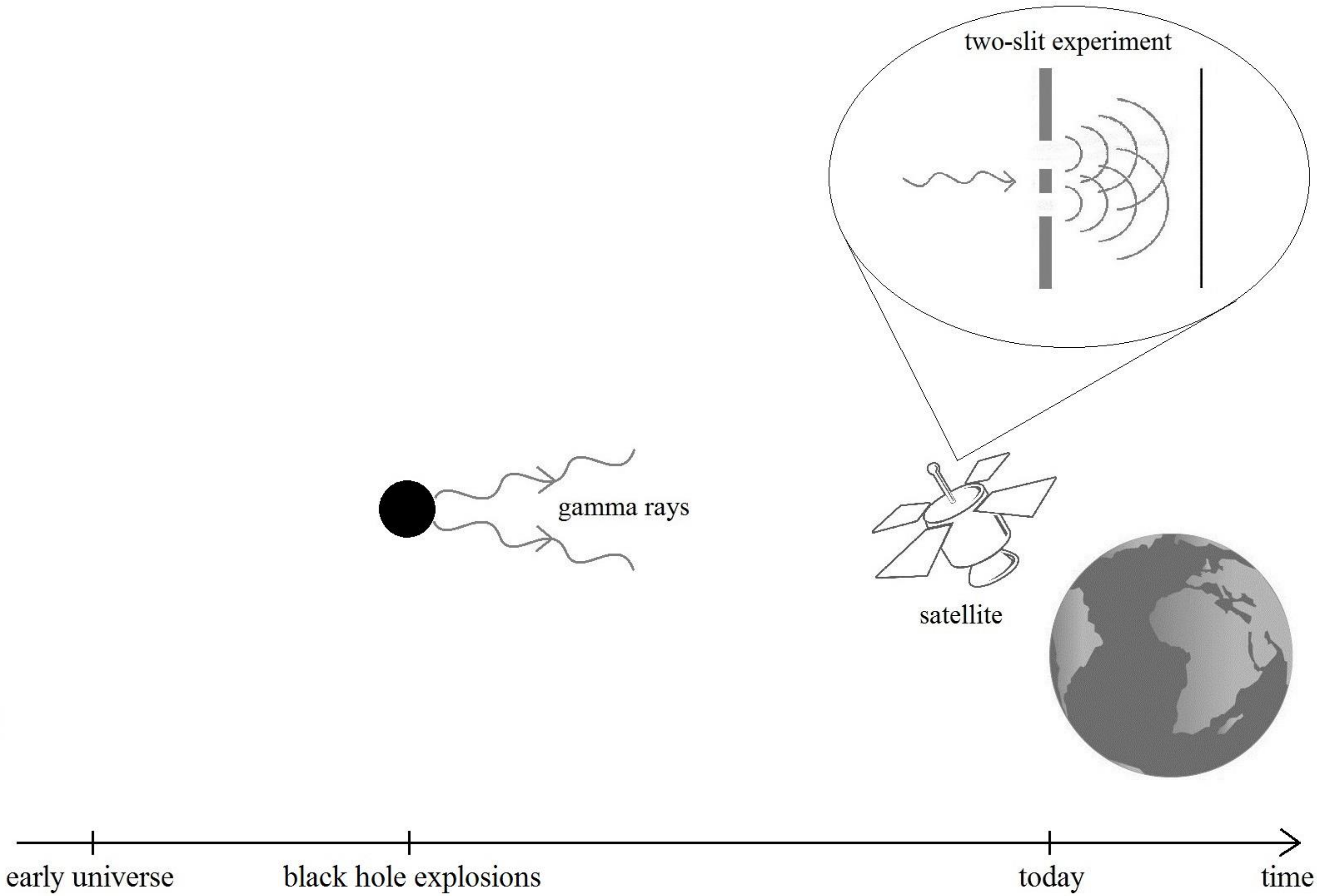

QUICK³ satellite mission (launch in 2024) has an interferometer to test the Born rule in space

The Born rule is being tested:

-- in the early universe (inflationary cosmology)

-- in space (upcoming satellites)

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What about in the lab? In high-energy collisions?

De Broglie-Bohm quantum mechanics has another natural suggestion for where new physics might lie…

Pilot-wave dynamics predicts its own demise at $\psi = 0$

For Hamiltonians quadratic in momenta, de Broglie velocity is proportional to a phase gradient:

$$\partial_q S = \mathrm{Im}\,\frac{\partial_q \psi}{\psi}$$

This generally diverges at $\psi = 0$

Usually ignored, nodal regions $\mathrm{Re}\,\psi = 0,\ \mathrm{Im}\,\psi = 0$ are of measure zero (dimension $n - 2$ in *n*-D space)

New physics must set in close to $\psi = 0$

(cf. classical gravitational singularities)

Example of hydrogen atom:

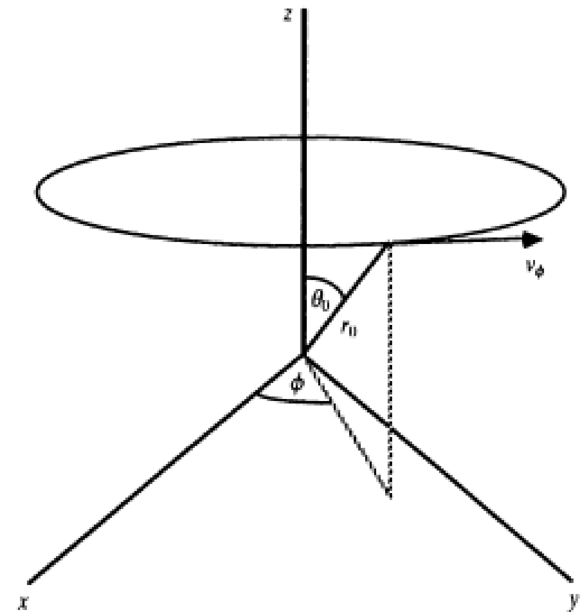
stationary state $\psi(r, \theta, \phi, t) = \psi_{nlm}(r, \theta, \phi) e^{-iE_n t}$

nodal line $\psi_{nlm} = 0$ along *z*-axis (for $m \neq 0$)

trajectories circle round *z*-axis at distance $d = r \sin\theta$

$$\mathbf{v} = \frac{1}{m_0} \boldsymbol{\nabla} S$$

$$|\mathbf{v}| = \frac{m}{m_0} \frac{1}{d}$$



velocity diverges as $d \to 0$

Is this an artifact of the low-energy theory?

No such divergence for Dirac fermions:

Dirac equation $$i\frac{\partial\psi}{\partial t} = -i\boldsymbol{\alpha}\cdot\nabla\psi + m\beta\psi$$

Continuity equation $$\frac{\partial(\psi^\dagger\psi)}{\partial t} + \boldsymbol{\nabla}\cdot(\psi^\dagger\boldsymbol{\alpha}\psi) = 0$$

Finite de Broglie velocity $$\frac{d\mathbf{x}}{dt} = \frac{\psi^\dagger\boldsymbol{\alpha}\psi}{\psi^\dagger\psi} \qquad (\,|\mathbf{v}| \leq 1\,)$$

Problem solved? No…

Not an artifact of the low-energy theory

Diverging phase gradient for bosonic fields:

Scalar field $$i\frac{\partial\Psi}{\partial t} = \int d^3x \; \frac{1}{2}\left(-\frac{\delta^2}{\delta\phi^2} + (\nabla\phi)^2 + m^2\phi^2\right)\Psi$$

Continuity equation $$\frac{\partial|\Psi|^2}{\partial t} + \int d^3x \; \frac{\delta}{\delta\phi}\left(|\Psi|^2\frac{\delta S}{\delta\phi}\right) = 0$$

De Broglie velocity $$\frac{\partial\phi}{\partial t} = \frac{\delta S}{\delta\phi}$$

Again diverges at nodes (on field configuration space)

Regularisation of the de Broglie velocity

Simple suggestion (Bell 1987): smearing of j and $|\psi|^2$

$$j(q,t)_{\mathrm{reg}} = \int dq' \ \mu(q'-q) j(q',t)$$

$$\left(|\psi(q,t)|^2\right)_{\mathrm{reg}} = \int dq' \ \mu(q'-q) |\psi(q',t)|^2$$

(narrow and positive, with $\int dq' \ \mu(q'-q) = 1$)

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Regularised de Broglie velocity

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Modified equilibrium:
(smeared Born rule) $\rho(q,t) = \left(|\psi(q,t)|^2\right)_{\mathrm{reg}}$

Lowest order correction to the Born rule

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1-D: $$I_a(x) = \int dx' \; \delta_a(x'-x) f(x') = \int ds \; \delta_a(s) f(x+s)$$

$$f(x+s) = f(x) + s f'(x) + \frac{1}{2} s^2 f''(x) + \ldots$$

$$I_a(x) = f(x) + \frac{1}{2} a^2 f''(x) + \ldots$$

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Corrected Born rule: $$\left(|\psi|^2\right)_{\mathrm{reg}} = |\psi|^2 + \frac{1}{2} a^2 \frac{\partial^2 |\psi|^2}{\partial x^2} + \ldots$$

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n-D: $$I_a(q) = \int d^n q'\ \delta_a^n(q'-q) f(q')$$

$$I_a(q) = f(q) + \frac{1}{2} a^2 \nabla^2 f(q) + \ldots$$

(correction is proportional to local Laplacian)

Example

1-D oscillator (first excited state, node at $x = 0$):

$$\psi_1(x) = \left(\frac{4}{\pi}\right)^{1/4} (m\omega)^{3/4} \, x \exp\left(-\frac{m\omega x^2}{2}\right)$$

node is replaced by a minimum

$$\min\left(|\psi|^2\right)_{\mathrm{reg}} = a^2 \frac{2}{\pi^{1/2}} (m\omega)^{3/2} + \ldots$$

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Applies realistically to high-energy field theory:

Fourier mode $\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}} (q_{\mathbf{k}1} + i q_{\mathbf{k}2})$ ($m = 1$, $\omega = k$)

Wave function $\psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)$, 2D oscillator, has nodes

de Broglie velocities

$$\dot{q}_{\mathbf{k}1} = \frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}1}}, \quad \dot{q}_{\mathbf{k}2} = \frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}2}} \quad \text{(can diverge)}$$

Expect smearing of cross sections at small scales

-- outcomes of measurements are determined
by initial conditions

-- distributions of outcomes are determined
by distributions of initial conditions

-- expect to see smearing of differential cross sections

$$\frac{d\sigma}{d\Omega} \sim |\mathcal{M}|^2$$

(as functions of scattering angles, small angular scales)

(*work in progress*)

Extended regularised theory

-- time-dependent $\mu(q'-q,t)$ (during collisions?)

-- $(|\psi|^2)_{\mathrm{reg}}$ now satisfies

$$\frac{\partial(|\psi|^2)_{\mathrm{reg}}}{\partial t} + \partial_q \cdot \left((|\psi|^2)_{\mathrm{reg}} v_{\mathrm{reg}}\right) = s$$

$$s(q,t) = \int dq' \, \frac{\partial \mu(q'-q,t)}{\partial t} |\psi(q',t)|^2$$

-- no longer matches continuity equation for ρ

$$\frac{\partial \rho}{\partial t} + \partial_q \cdot (\rho v_{\mathrm{reg}}) = 0$$

-- initial $\rho = (|\psi|^2)_{\mathrm{reg}}$ can evolve into $\rho \neq (|\psi|^2)_{\mathrm{reg}}$

Creation of nonequilibrium (at high energies?)

Possible effects to look for

Smeared zeros (nodes) of differential cross sections

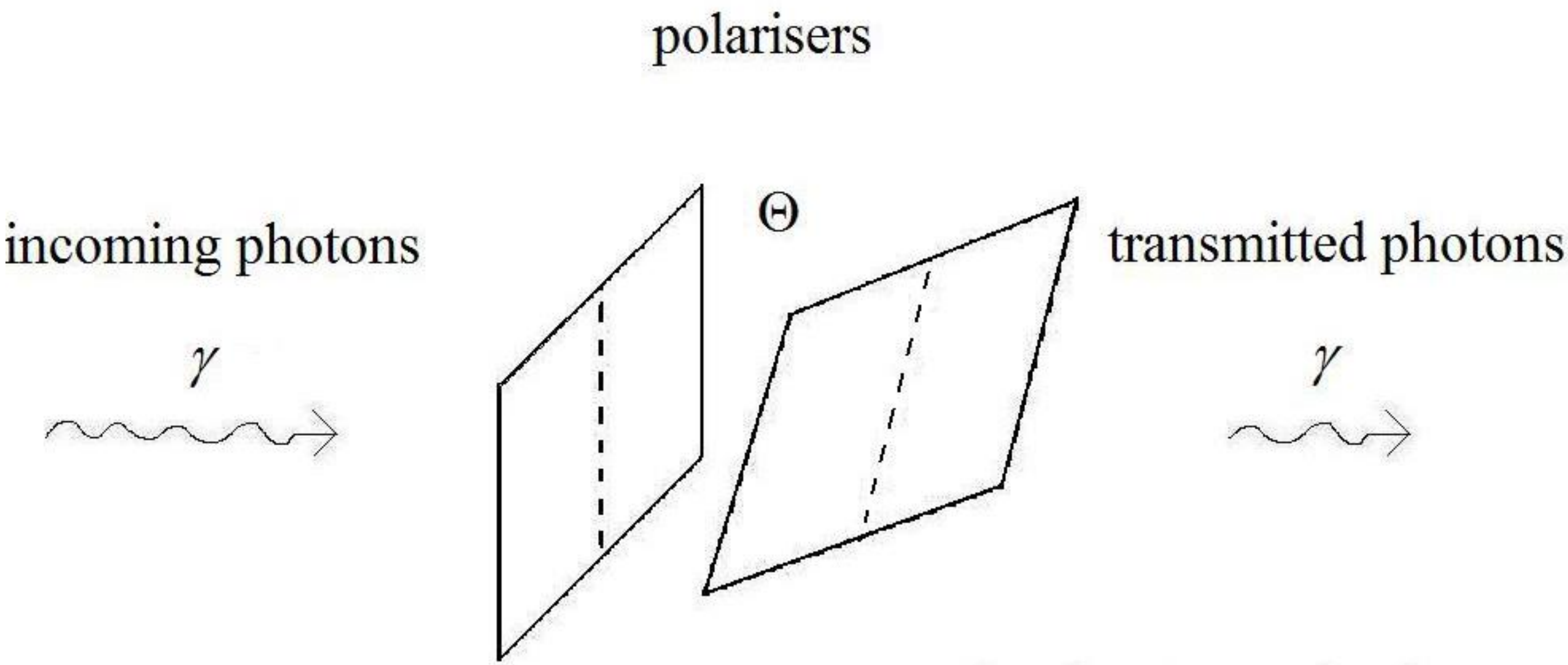
$$\frac{d\sigma}{d\Omega} \sim |\mathcal{M}|^2$$

(as functions of scattering angles, small angular scales)

Does the cross section really go to zero
at certain angles?

Possible effects to look for

Anomalous polarisation probabilities:



fraction transmitted

$$p^{+}(\Theta) \neq p^{+}_{\mathrm{QT}}(\Theta) = \cos^{2}\Theta \quad ?$$

General signature of quantum nonequilibrium
(AV 2004, cf. Timpson 2004)

Conclusions

-- The Born rule can and should be tested
in high-energy collisions

-- de Broglie-Bohm quantum mechanics suggests
how the Born rule may be broken in some conditions

-- repurpose collision data to test the Born rule:

regions of low probability (nodes)

polarisation probabilities