Improving the measurement of the HZZ density matrix and QE

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Workshop on the Quantum Tests in Collider Physics

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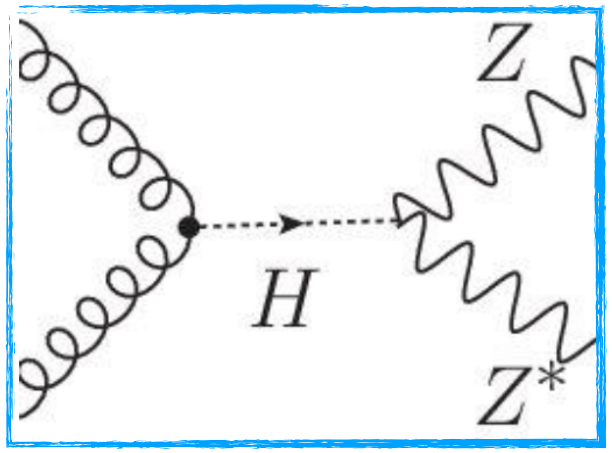
Introduction

- Study quantum mechanical concepts in $H \to ZZ$ **decays**, where the Z bosons decay into light leptons **at the LHC**



  - *Clean signature*
  - *Statistically limited*
  - *Two-quitrit system*

- An **experimentalist's view** of the theoretical papers on this decay channel

  - *Making it a reality*
  - *Work in progress*

Taken from https://www.questionpro.com/blog/experimental-research/

"Spherical basis" method

Ref: "*Testing entanglement and Bell inequalities in H → ZZ*" by **JAAS** et al. [arxiv:2209.13441]

- General form of the **spin density matrix for spin-1 particles** with 80 independent coefficients:

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}\; T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\; \mathbb{1}_3 \otimes T^L_M + C_{L_1M_1L_2M_2}\; T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

- Differential cross-section can be written:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + A^1_{LM} B_L Y^M_L(\theta_1,\varphi_1) + A^2_{LM} B_L Y^M_L(\theta_2,\varphi_2)$$
$$+ C_{L_1M_1L_2M_2} B_{L_1} B_{L_2} Y^{M_1}_{L_1}(\theta_1,\varphi_1) Y^{M_2}_{L_2}(\theta_2,\varphi_2)\Big] \,,$$

- Coefficients can be derived from the integration over the full phase space of angular distributions:

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^M_L(\Omega_j) d\Omega_j = \frac{B_L}{4\pi} A^j_{LM}, \qquad j = 1,2\,.$$
$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2) d\Omega_1 d\Omega_2 = \frac{B_{L_1}B_{L_2}}{(4\pi)^2} C_{L_1M_1L_2M_2}\,,$$

  - Relying on **integration over the full phase space** of the angular distributions

Spin density matrix

- The general form of the density operator of **the di-boson production in Higgs boson decays** (vanishing third-component) and assuming CP and P conservation

$$\rho = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

From **JAAS** et al. [arxiv:2209.13441]

where $$\frac{1}{\sqrt{2}}A^1_{2,0} + 1 = C_{2,2,2,-2}$$

- Our goal is to **measure the coefficients of this matrix**

Conditions for QE and violation of Bell inequalities

- **Entanglement conditions**
  - **Peres-Horodecki criterion** provides a necessary and sufficient (in the $H \to ZZ$ system) condition for entanglement which, given the form of spin density matrix, translates into

$$C_{2,1,2,-1} \neq 0 \qquad \text{or} \qquad C_{2,2,2,-2} \neq 0$$

- **Conditions for violation of Bell inequalities**
  - Once the spin density matrix is known, it is possible to test the **CGLMP inequality** which is a Bell inequality optimized for qutrits (e.g. massive spin-1 particles)
  - When using the *optimal choice for the Bell operator*, the system violates the CGLMP inequality when:

$$I_3 = \frac{1}{36}\left(18 + 16\sqrt{3} - \sqrt{2}\left(9 - 8\sqrt{3}\right) A^1_{2,0} - 8\left(3 + 2\sqrt{3}\right) C_{2,1,2,-1} + 6\, C_{2,2,2,-2}\right) > 2$$

- In the **context of quantum mechanics**

From **JAAS** et al. [arxiv:2209.13441]

Fully differential angular coefficients

- Dependence of the **fully differential cross-section of $H \to ZZ \to 4\ell$ on charged lepton decay angles** can be written analytically. The result is well-known at **LO**: [arxiv:2209.14033, arxiv:2105.07972].

$$
\begin{aligned}
\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2 dm_{Z_1} dm_{Z_2} dm_H} = \frac{1}{(4\pi)^2}[1 &+ A_{20}^{1\ \text{per-event}} B_2 Y_2^0(\theta_1,\phi_1) \\
&+ A_{20}^{1\ \text{per-event}} B_2 Y_2^0(\theta_2,\phi_2) \\
&- (1+\sqrt{1/2}\cdot A_{20}^{1\ \text{per-event}}) B_1^2 Y_1^0(\theta_1,\phi_1) Y_1^0(\theta_2,\phi_2) \\
&+ (1-\sqrt{1/2}\cdot A_{20}^{1\ \text{per-event}}) B_2^2 Y_2^0(\theta_1,\phi_1) Y_2^0(\theta_2,\phi_2) \\
&- C_{212-1}^{\text{per-event}} B_1^2 Y_1^1(\theta_1,\phi_1) Y_1^{-1}(\theta_2,\phi_2) \\
&+ C_{212-1}^{\text{per-event}} B_2^2 Y_2^1(\theta_1,\phi_1) Y_2^{-1}(\theta_2,\phi_2) \\
&+ C_{222-2}^{\text{per-event}} B_2^2 Y_2^2(\theta_1,\phi_1) Y_2^{-2}(\theta_2,\phi_2) \\
&- C_{212-1}^{\text{per-event}} B_1^2 Y_1^{-1}(\theta_1,\phi_1) Y_1^1(\theta_2,\phi_2) \\
&+ C_{212-1}^{\text{per-event}} B_2^2 Y_2^{-1}(\theta_1,\phi_1) Y_2^1(\theta_2,\phi_2) \\
&+ C_{222-2}^{\text{per-event}} B_2^2 Y_2^{-2}(\theta_1,\phi_1) Y_2^2(\theta_2,\phi_2)]
\end{aligned}
$$

$$
\begin{aligned}
A_{20}^{1\ \text{per-event}} &= \sqrt{2}\frac{1-K^2}{2+K^2} \\
C_{1010}^{\text{per-event}} &= \frac{3}{2+K^2} \\
C_{2020}^{\text{per-event}} &= \frac{1+2K^2}{2+K^2} \\
C_{111-1}^{\text{per-event}} &= \frac{3K}{2+K^2} \\
C_{212-1}^{\text{per-event}} &= -\frac{3K}{2+K^2} \\
C_{222-2}^{\text{per-event}} &= \frac{3}{2+K^2}
\end{aligned}
$$

$$K = \frac{m_H^2 - m_{Z_1}^2 - m_{Z_2}^2}{2 m_{Z_1} m_{Z_2}}$$

- Per-event values can be used to build **templates**

Spin density matrix

Reweighting and optimal observables

- **We want to measure C, defined as $C^{\text{per-event}}$ integrated over the full phase-space.**

1) We build signal templates by reweighting the $H \to ZZ$ sample with:

$$w_{\text{gen}}(\mu) = \frac{\left[\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2 dm_{Z_1} dm_{Z_2} dm_H}\right](C^{\text{per-event}} = \mu C^{\text{per-event}}_{\text{SM}})}{\left[\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2 dm_{Z_1} dm_{Z_2} dm_H}\right](C^{\text{per-event}} = C^{\text{per-event}}_{\text{SM}})}$$

- The "gen" weight is calculated with born-level variables

2) We define optimal observables at the reconstruction level as $\log_{10}[w_{\text{reco}}(\mu)]$

- The "reco" weight is calculated with the same formula but with reconstructed variables
- Each template (each μ) has a **unique** optimal observable
- Does not include the background, since the selection has high purity

MadGraph
MadGraph+Delphes

Expected values of the full phase-space coefficients

Full phase-space integration as the **weighted average**:

- "**Analytical calculation**" method: [arxiv:2302.00683] (Emidio at al.)

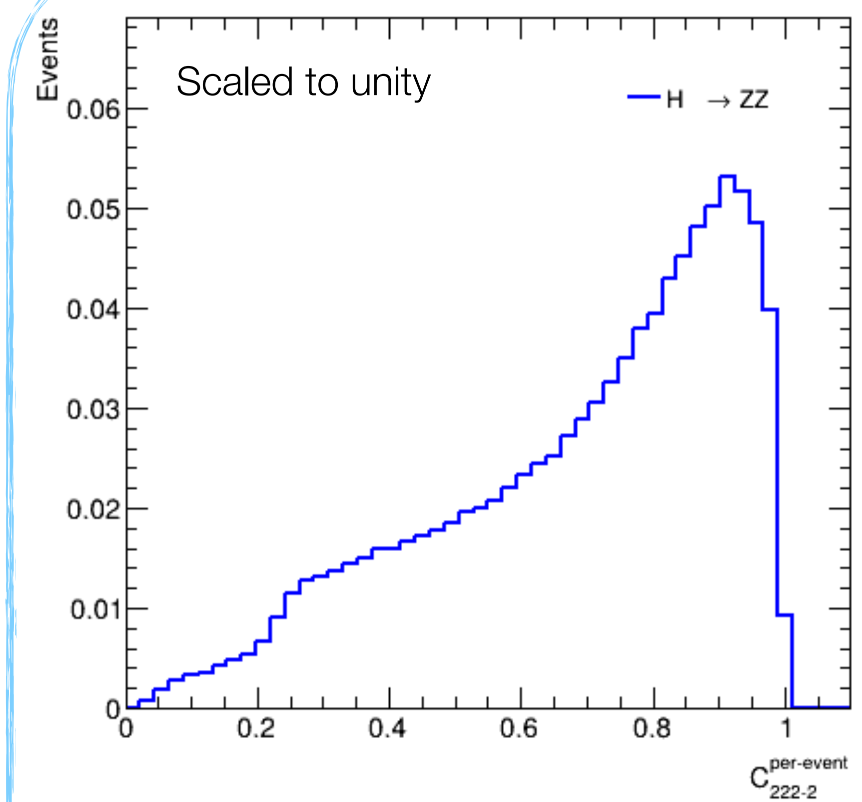
$$C^{\mathrm{SM}}_{222-2} = \frac{\sum w_{\mathrm{MC}} C^{\text{per-event}}_{222-2}}{\sum w_{\mathrm{MC}}}$$

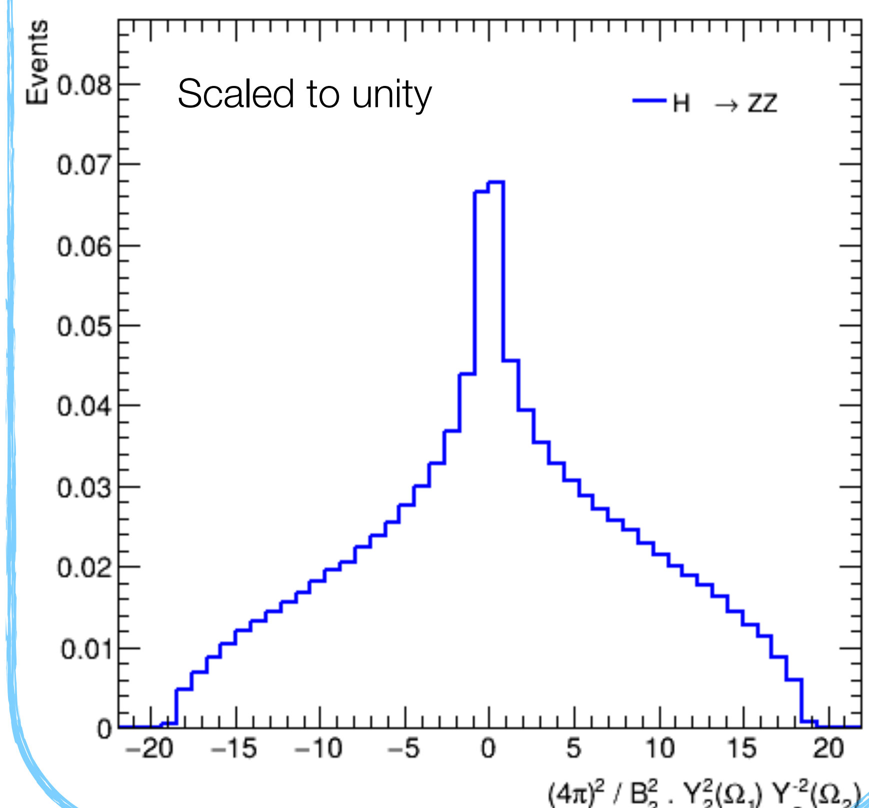
- "**Spherical basis**" method: [arxiv:2209.13441] (JAAS et a.)

$$C^{\mathrm{SM}}_{222-2} = \frac{(4\pi)^2}{B_2^2} \frac{\sum w_{\mathrm{MC}} Y_2^2(\Omega_1) Y_2^{-2}(\Omega_2)}{\sum w_{\mathrm{MC}}}$$

	Analytical calculation	Spherical basis
C^{SM}_{222-2}	0.54 +/- 0.09	0.54 +/- 0.09
C^{SM}_{212-1}	-0.89 +/- 0.09	-0.89 +/- 0.09
$A^{1\ \mathrm{SM}}_{20}$	-0.65 +/- 0.09	-0.65 +/- 0.09

Uncertainty from the number of MC events.





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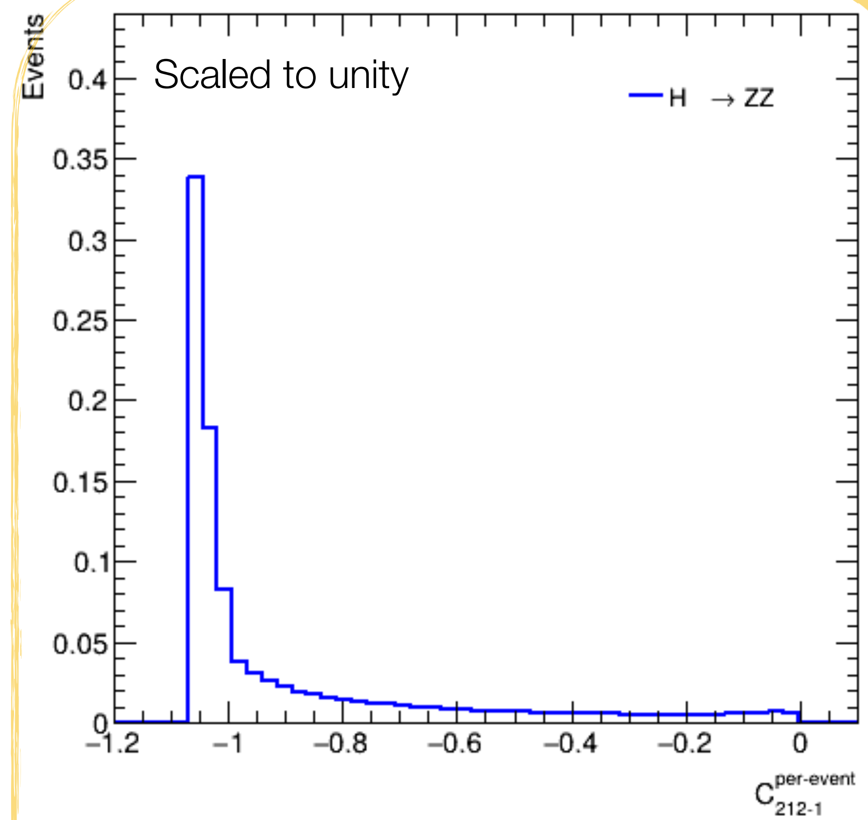

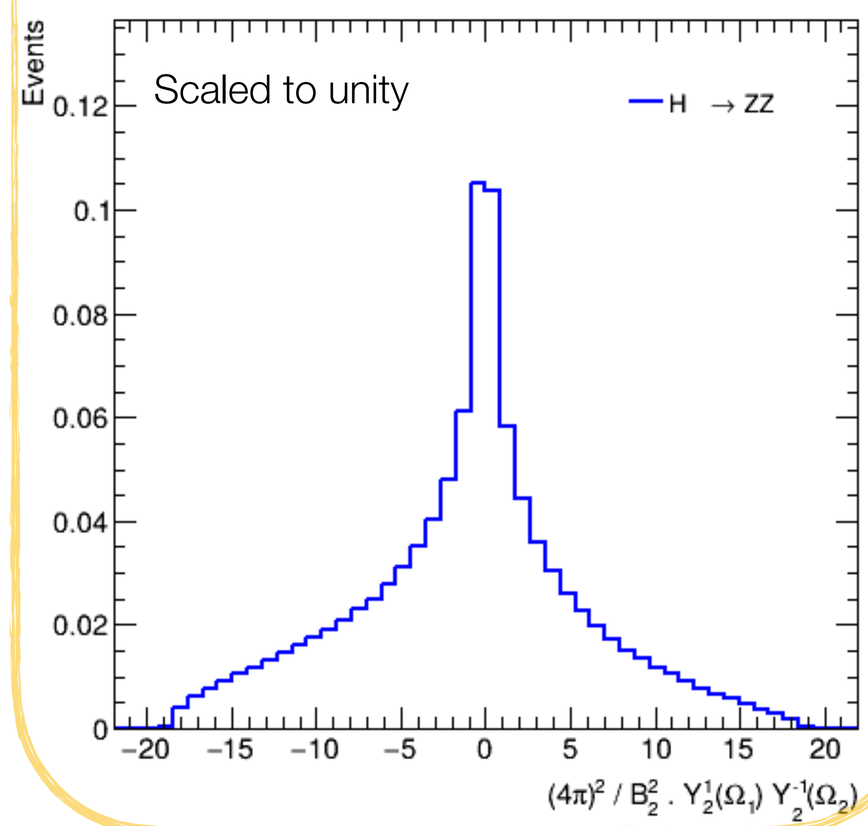

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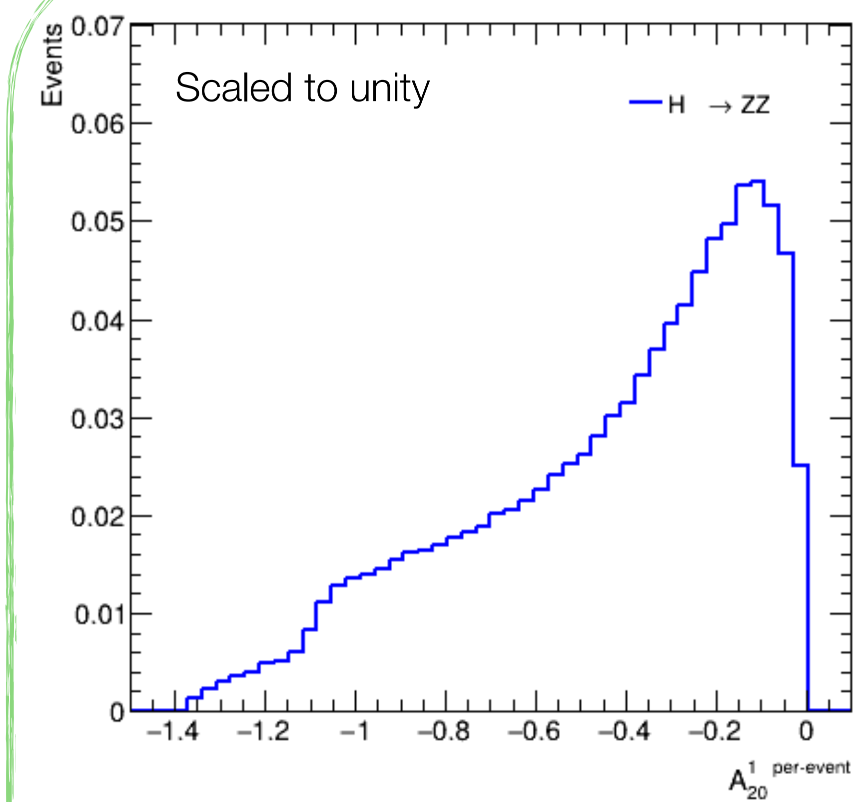
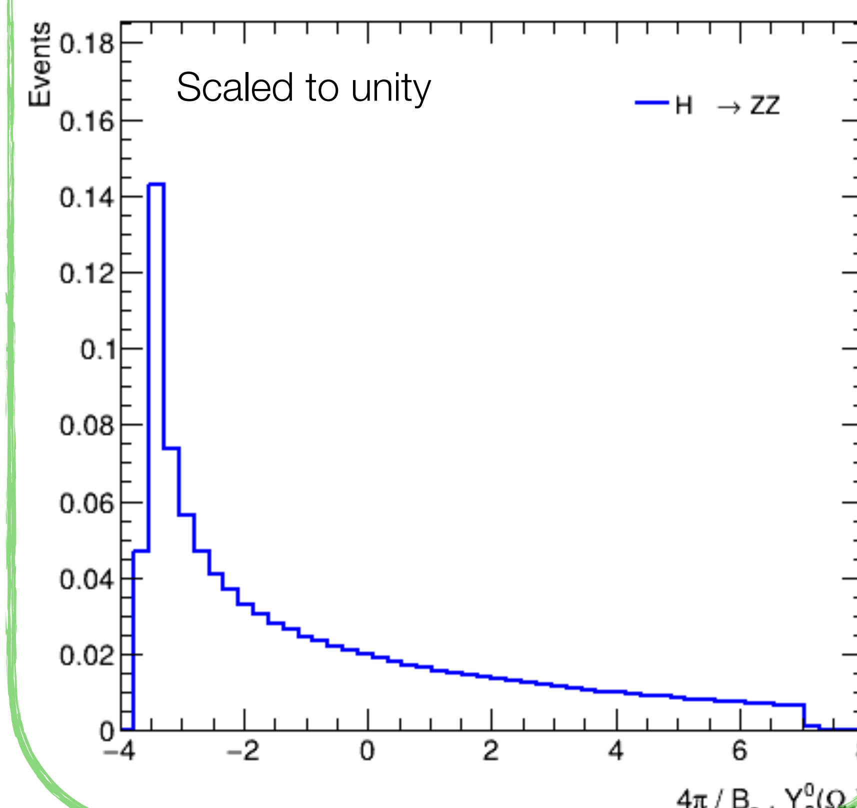
Uncertainty from the number of MC events.

Identity (slide 4) is perfectly valid.





Event selections

- Events generated at LO for gluon-induced production $gg \rightarrow H \rightarrow e^+e^-\mu^+\mu^-$ (distinguishable) and for $q\bar{q} \rightarrow ZZ$ with `MadGraph` and reconstructed using `Delphes`
  - Cross-section reweighted to match the expected number of events in the Higgs boson production cross-section measurements paper [hepdata]

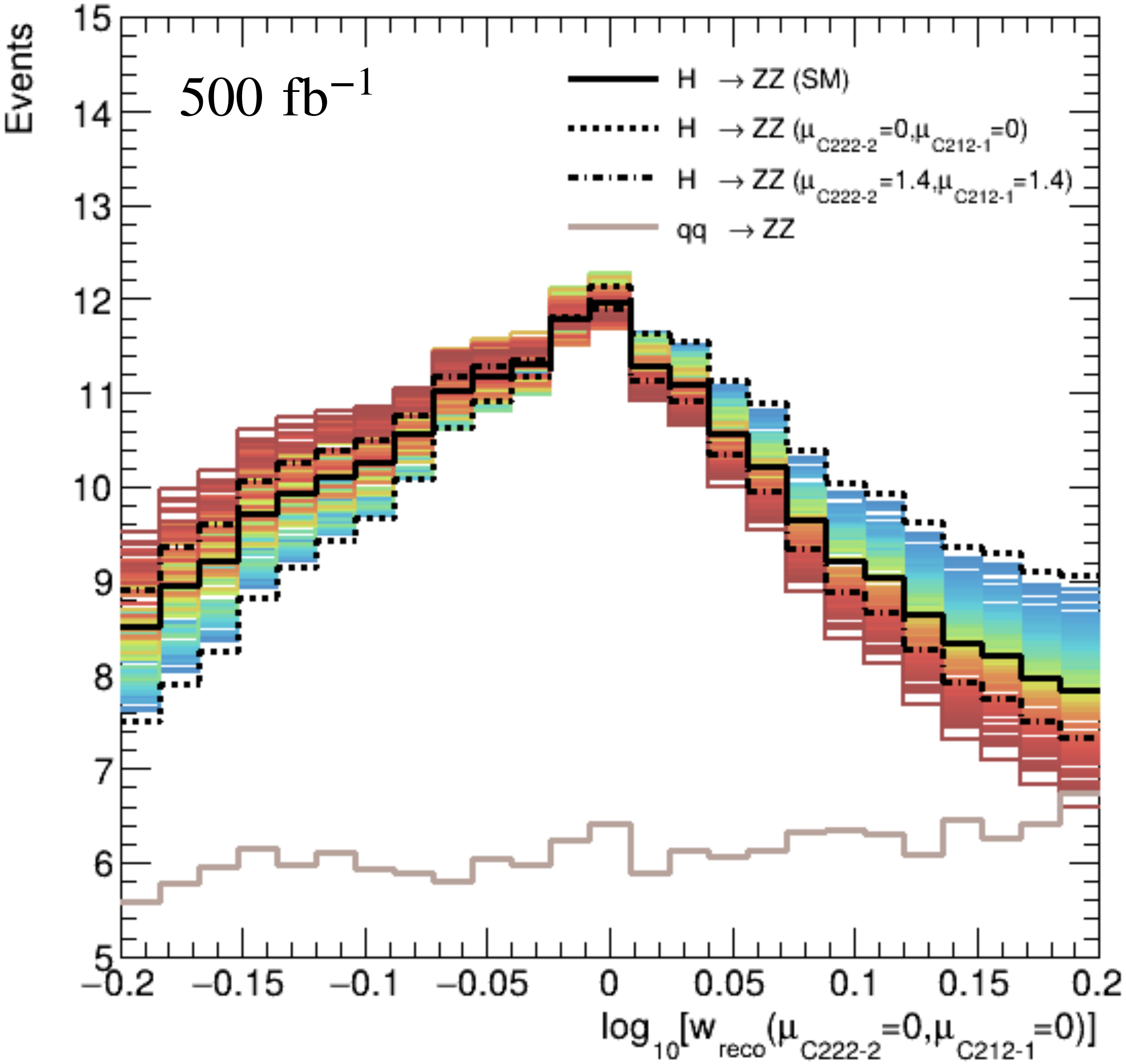
- Basic selection for leptons and quadruplets
  - Lepton momentum: $p_{\mathrm{T}}^{\ell\ 1\ (2)\ [3]\ \{4\}} > 20\ (15)\ [10]\ \{5\}$ GeV
  - Higgs mass: $115 < m_H < 130$ GeV
  - Z-boson mass: $50 < m_{Z_1} < 106$ GeV and $12 < m_{Z_2} < 115$ GeV

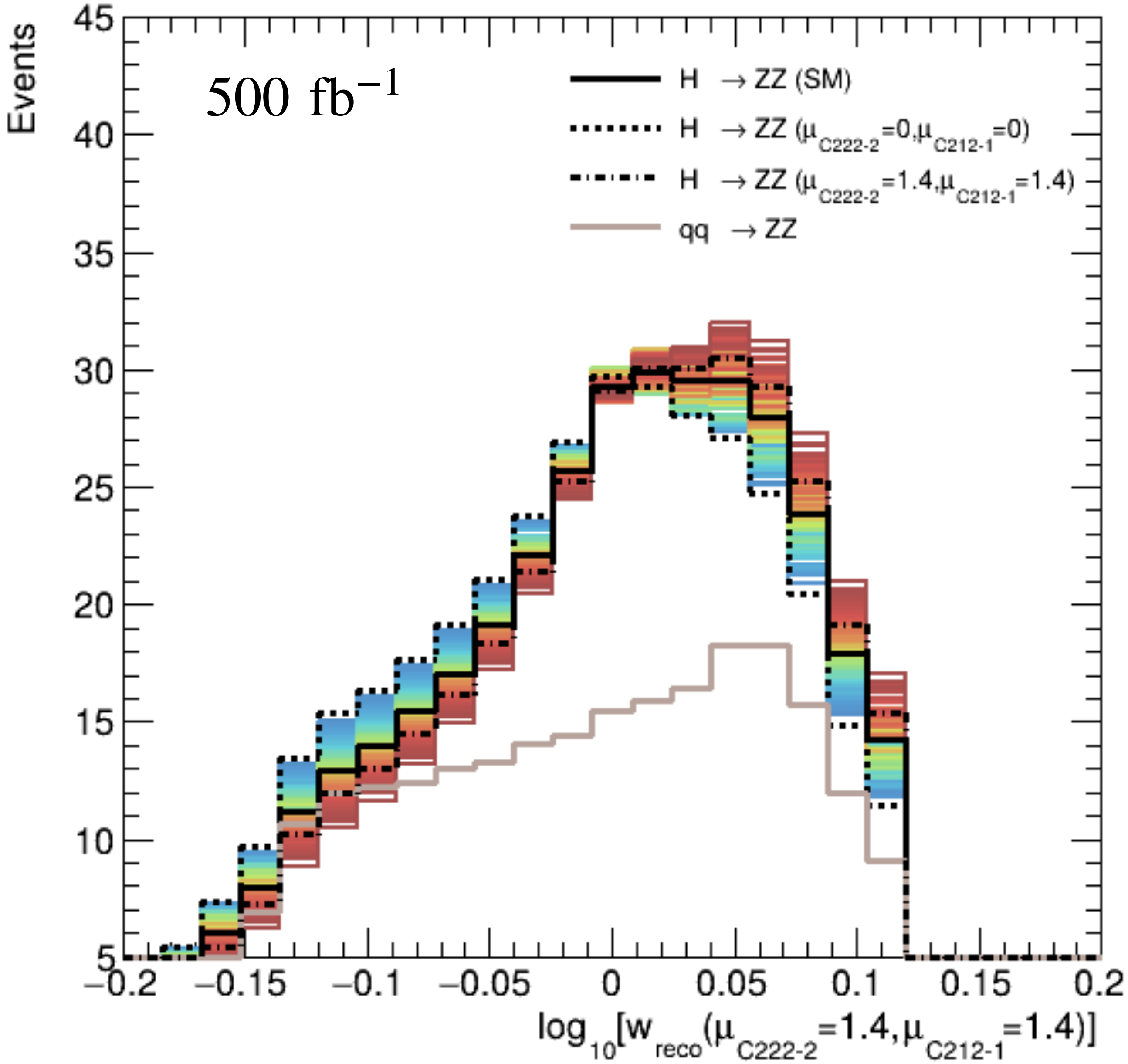
- Luminosity scaled up to **500 ifb** (Run 2 + Run 3)

- Event selection and number of bins (currently 25) can be further optimised

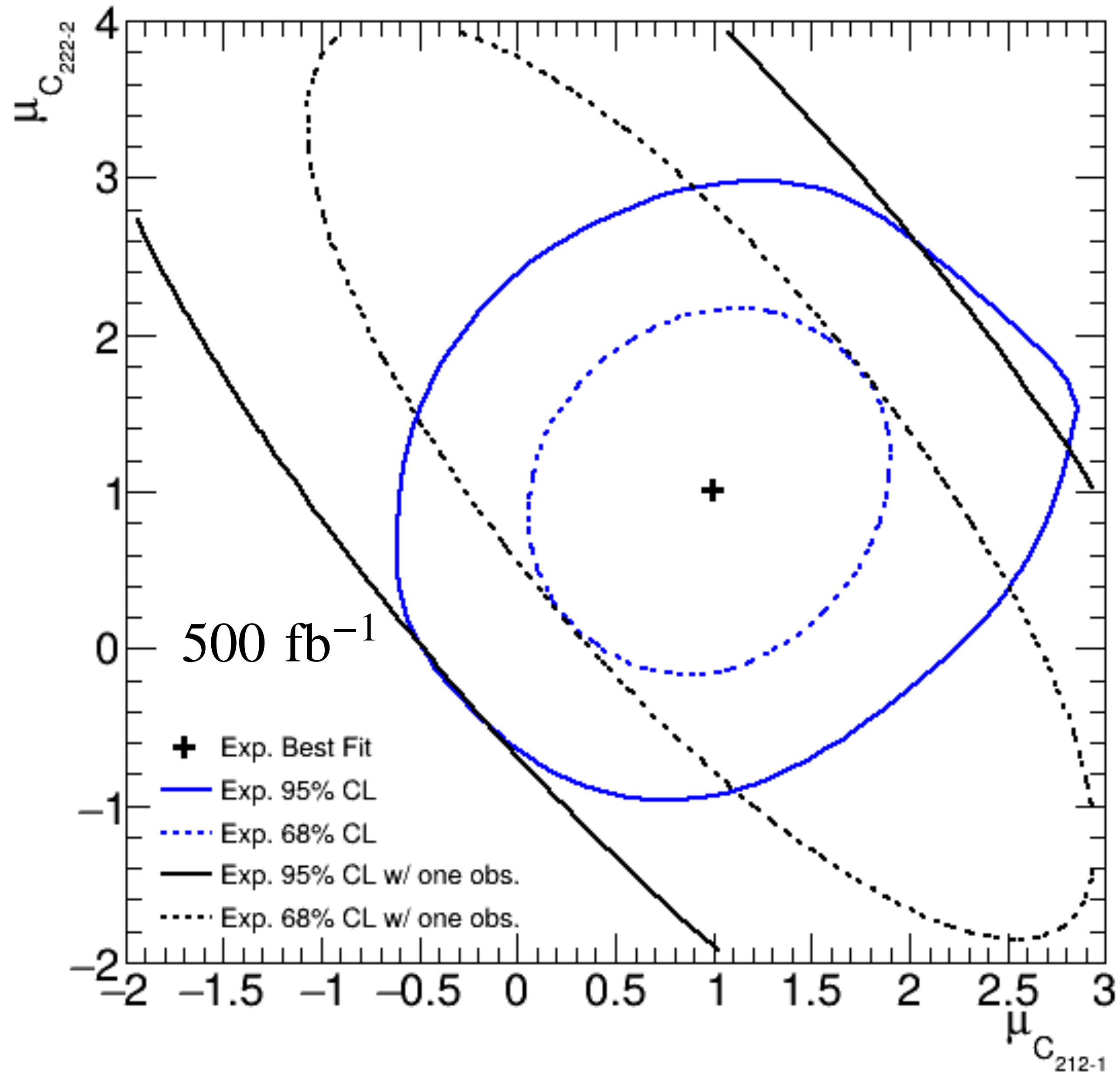
Optimal observables

- Optimal observables for two different signal templates build for $(\mu_{C_{212-1}} = 0,\ \mu_{C_{222-2}} = 0)$ and $(\mu_{C_{212-1}} = 1.4,\ \mu_{C_{222-2}} = 1.4)$





2D NLL scan: C_{212-1}vs C_{222-2}



Only statistical uncertainty

- **No-entanglement hypothesis** ($\mu_{C_{212-1}} = 0, \mu_{C_{222-2}} = 0$) excluded at the 2σ level
- Using one observable per POI eliminated correlations and reduced the uncertainty of the C_{222-2} coefficient
  - $C_{212-1} = -0.89 \pm 0.80$ and $C_{222-2} = 0.54 \pm 0.60$

Expected stat-only results for spin density matrix

- Predicted spin density matrix

$$\rho^{\text{Sph. basis}}_{\text{Th.}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.18 \pm 0.03\ (\text{MC}) & 0 & -0.30 \pm 0.03\ (\text{MC}) & 0 & 0.18 \pm 0.03\ (\text{MC}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & -0.30 \pm 0.03\ (\text{MC}) & 0 & 0.64 \pm 0.06\ (\text{MC}) & 0 & -0.30 \pm 0.03\ (\text{MC}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.18 \pm 0.03\ (\text{MC}) & 0 & -0.30 \pm 0.03\ (\text{MC}) & 0 & 0.18 \pm 0.03\ (\text{MC}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Measured (stat-only) spin density matrix

$$\rho^{\text{Asimov}}_{\text{Meas.}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.18 \pm 0.20\ (\text{stat.}) & 0 & -0.30 \pm 0.27\ (\text{stat.}) & 0 & 0.18 \pm 0.20\ (\text{stat.}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & -0.30 \pm 0.27\ (\text{stat.}) & 0 & 0.64 \pm 0.40\ (\text{stat.}) & 0 & -0.30 \pm 0.27\ (\text{stat.}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.18 \pm 0.20\ (\text{stat.}) & 0 & -0.30 \pm 0.27\ (\text{stat.}) & 0 & 0.18 \pm 0.20\ (\text{stat.}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Only two coefficients were measured (scanned) simultaneously: C_{212-1} and C_{222-2}
  - Relation between A^1_{20} and C_{222-2} was assumed: $C_{222-2} = 1 + 1/\sqrt{2}\ A^1_{20}$
- Results are very **preliminary** and can be **improved**

Bell inequality

Bell inequality interpretation

- Again from **JAAS** et al. [arxiv:2209.13441]: $I_3 = \mathrm{Tr}\left\{\rho\ \mathcal{O}_{\mathrm{Bell}}\right\} > 2$

$$I_3 = \frac{1}{36}\left(18+16\sqrt{3}-\sqrt{2}\left(9-8\sqrt{3}\right)A^1_{2,0}-8\left(3+2\sqrt{3}\right)C_{2,1,2,-1}+6\,C_{2,2,2,-2}\right)$$

$$I_3 = \frac{\sqrt{A^2+B^2+C^2}}{36}D_3+\frac{18+16\sqrt{3}}{36}$$

where

$$D_3 = \frac{A\cdot A^1_{20}+B\cdot C_{212-1}+C\cdot C_{222-2}}{\sqrt{A^2+B^2+C^2}}$$

$$A=-\sqrt{2}(9-8\sqrt{3}),\ B=-8(3+2\sqrt{3}),\ C=6$$

- *Rotation*:

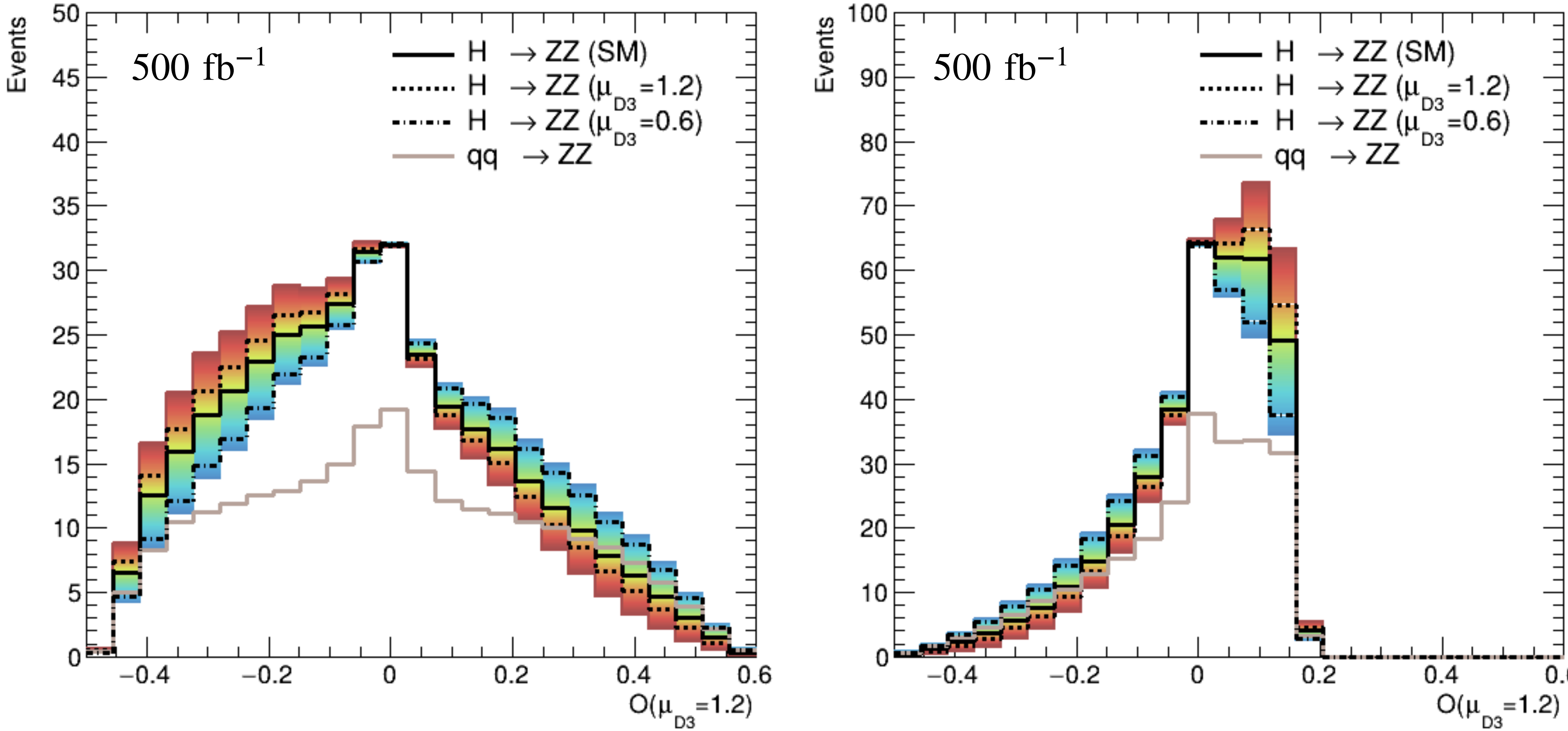
$$\begin{pmatrix} D_1^{\mathrm{per-event}} \\ D_2^{\mathrm{per-event}} \\ D_3^{\mathrm{per-event}} \end{pmatrix} = \mathbf{R}\begin{pmatrix} A_{20}^{1\ \mathrm{per-event}} \\ C_{212-1}^{\mathrm{per-event}} \\ C_{222-2}^{\mathrm{per-event}} \end{pmatrix} = \frac{1}{\sqrt{A^2+B^2+C^2}}\begin{pmatrix} \frac{AC\sqrt{A^2+B^2+C^2}}{\sqrt{1+C^2}\sqrt{A^2+B^2}} & \frac{BC\sqrt{A^2+B^2+C^2}}{\sqrt{1+C^2}\sqrt{A^2+B^2}} & -\frac{\sqrt{A^2+B^2+C^2}}{\sqrt{1+C^2}} \\ -\frac{B\sqrt{A^2+B^2+C^2}}{\sqrt{A^2+B^2}} & \frac{A\sqrt{A^2+B^2+C^2}}{\sqrt{A^2+B^2}} & 0 \\ A & B & C \end{pmatrix}\begin{pmatrix} A_{20}^{1\ \mathrm{per-event}} \\ C_{212-1}^{\mathrm{per-event}} \\ C_{222-2}^{\mathrm{per-event}} \end{pmatrix}$$

$$\begin{pmatrix} A_{20}^{1\ \mathrm{per-event}} \\ C_{212-1}^{\mathrm{per-event}} \\ C_{222-2}^{\mathrm{per-event}} \end{pmatrix} = \mathbf{R}^{-1}\begin{pmatrix} D_1^{\mathrm{per-event}} \\ D_2^{\mathrm{per-event}} \\ D_3^{\mathrm{per-event}} \end{pmatrix} = \begin{pmatrix} \sqrt{2}\frac{1-K^2}{2+K^2} \\ \frac{-3K}{2+K^2} \\ \frac{3}{2+K^2} \end{pmatrix} \Rightarrow \begin{aligned} A_{20}^{1\ \mathrm{per-event}} &= A_{20\ D1+D2}^{1\ \mathrm{per-event}} + \mu\cdot A_{20\ D3}^{1\ \mathrm{per-event}} \\ C_{212-1}^{\mathrm{per-event}} &= C_{212-1\ D1+D2}^{\mathrm{per-event}} + \mu\cdot C_{212-1\ D3}^{1\ \mathrm{per-event}} \\ C_{222-2}^{\mathrm{per-event}} &= C_{222-2\ D1+D2}^{1\ \mathrm{per-event}} + \mu\cdot C_{222-2\ D3}^{1\ \mathrm{per-event}} \end{aligned}$$

- Coefficients (A^1_{20}, C_{212-1} and C_{222-2}) expressed as linear combinations of three terms from which one represents D_3

Optimal observables

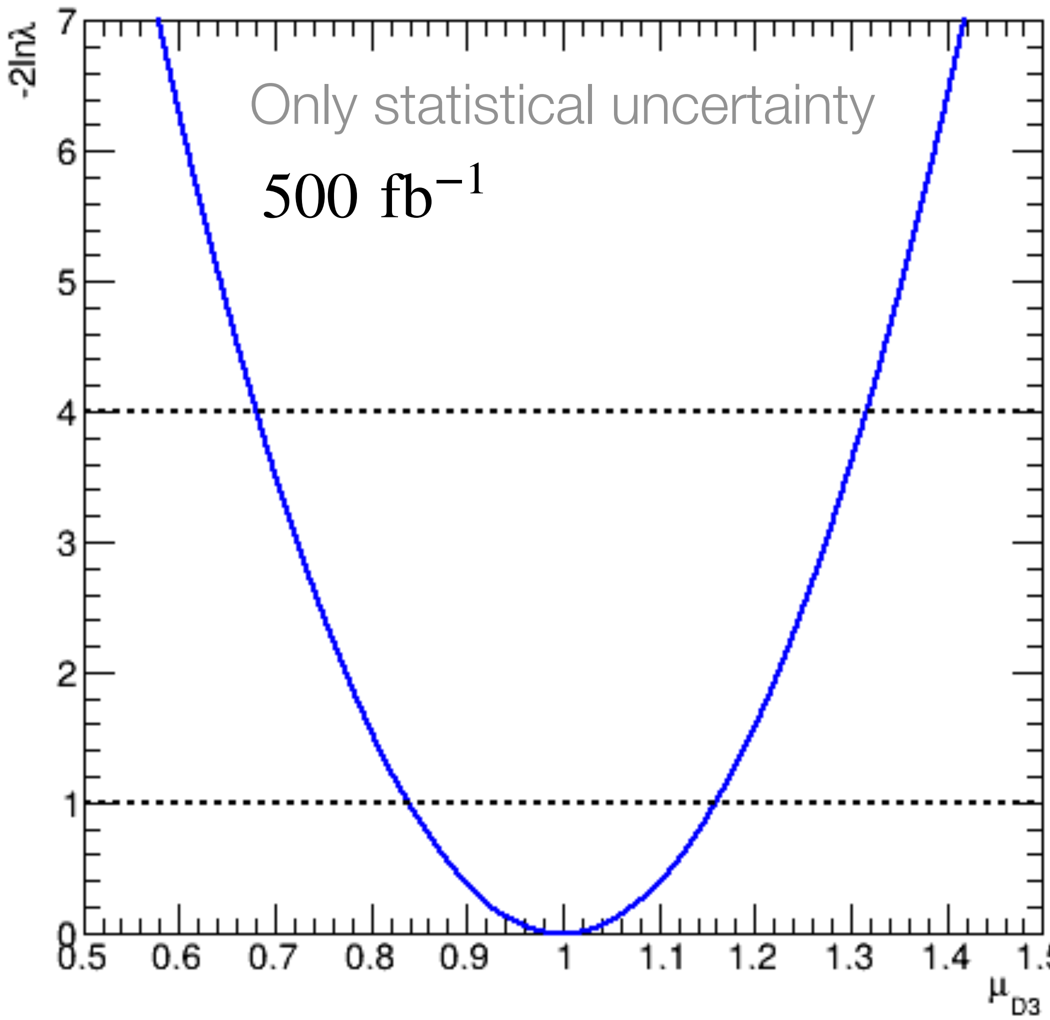
- Defined in the same way as for the measurement of the spin density matrix
- Optimal observables for two different signal templates build for $\mu_{D_3} = 0.6$ and $\mu_{D_3} = 1.2$



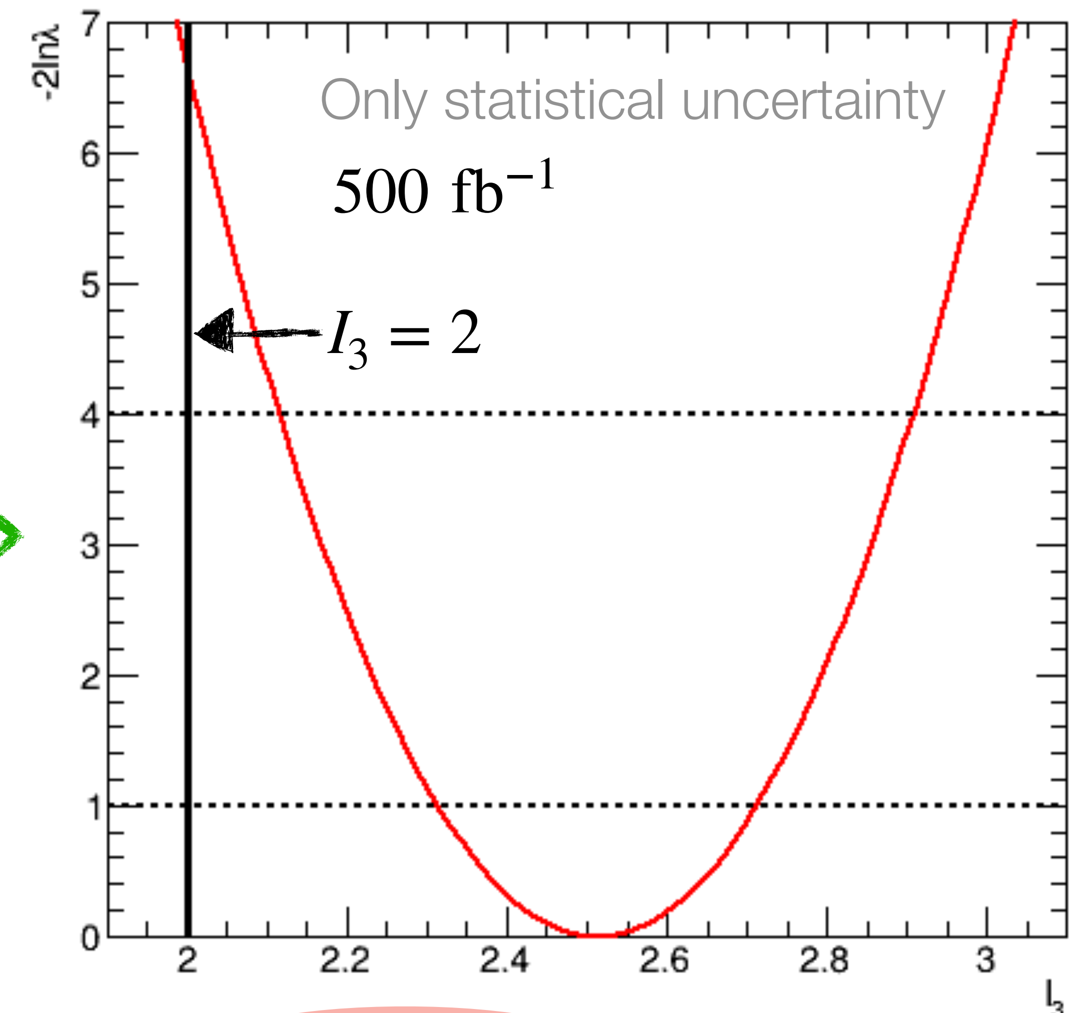
1D NLL scans

$$\hat{I}_3 = \frac{1}{36}\left(-\sqrt{2}(9-8\sqrt{3})A^1_{20} - 8(3+2\sqrt{3})C_{212-1} + 6C_{222-2}\right)\cdot \hat{\mu}_{D_3} + \frac{18+16\sqrt{3}}{36}$$

$$\sigma_{\hat{I}_3} = \frac{1}{36}\left(-\sqrt{2}(9-8\sqrt{3})A^1_{20} - 8(3+2\sqrt{3})C_{212-1} + 6C_{222-2}\right)\cdot \sigma_{\hat{\mu}_{D_3}}$$



$$\hat{\mu}_{D_3} \pm \sigma_{\hat{\mu}_{D_3}} = 1 \pm 0.16$$



$$\hat{I}_3 \pm \sigma_{\hat{I}_3} = 2.51 \pm 0.20$$

Only statistical uncertainty

- Sensitivity to a **violation of the Bell inequalities** is at the $\sim 2.6\sigma$ level

Conclusions

- A spherical method approach for measuring the coefficients of the $H \to ZZ \to 2e2\mu$ **spin density matrix** was presented based on the paper by JAAS et al. [arxiv:2209.13441]
  - Run2+Run3 projections (~500 ifb) and only-statistical uncertainties
  - This method can be used to test the *entanglement* condition (at the 2σ level) and the violation of the *Bell inequalities* (at the 2.6σ level)
  - Validate templates
- Quantum *entanglement* (only) can be also probed as a binary test: SM versus **longitudinal polarisation** (JAAS [arxiv:2209.14033])
  - A clear experimental approach
  - Relies on the reliability of MC generators: validate samples

Thank you for your attention and stay tuned!